Spatial Localization - II

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Lecture #9 - Learning Objectives

- Describe the three steps required for spatial localization.
- Be able to explain the role of RF and gradients during slice selection.
- Learn to define B_{eff} for various combinations of B-fields.
- Identify the complexity of the Bloch equations for forced precession in the presence of a gradient field.
- Understand the small tip angle approximation.
- Appreciate that the small tip angle approximation works for intermediate flip angles!
- Understand what truncation artifacts are and one way to reduce them.

Lecture #10 - Learning Objectives

- **Understand the small tip angle approximation.**
- **Appreciate that the small tip angle approximation works for intermediate flip angles!**
- **Understand what truncation artifacts are and one way to reduce them. Learn to describe *k*-space in words and mathematically.**
- **Appreciate what different points in *k*-space represent.**
- **Understand the connection between Fourier encoding and image acquisition.**
- **Be able to describe the roll of phase and frequency encoding.**

Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.

Slice Selective Excitation

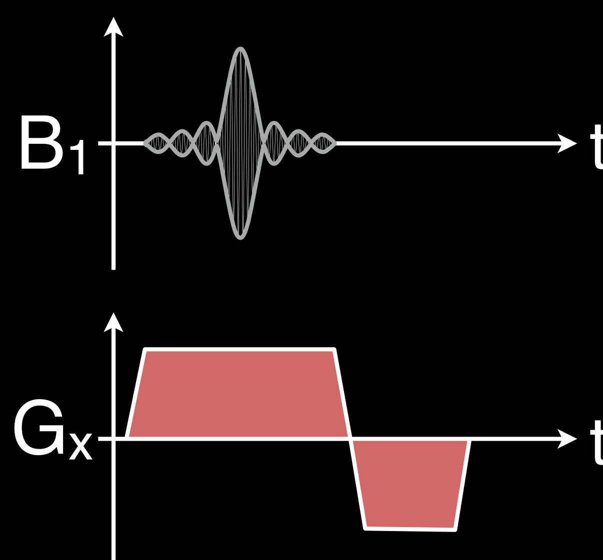
Selective Excitation

- **What factors control slice selection?**

$$B_1^e(t)$$ **Pulse envelope function**
(e.g. $B_{1,max}$ and $\Delta\omega$)

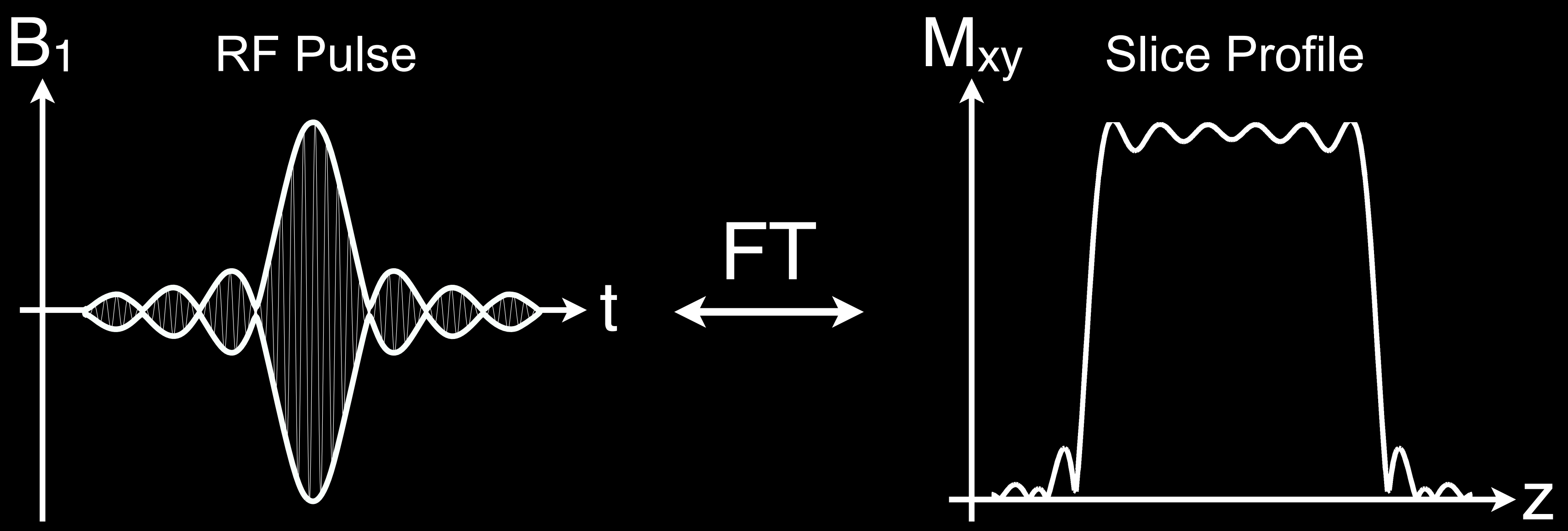
$$\omega_{RF}$$ **Excitation carrier frequency**

$$\vec{G}$$ **Gradient amplitude**



Slice Selective Excitation

- **What is the ideal slice profile?**
- **Changing the shape (envelope function) of the pulse affects the *excitation bandwidth* of excitation.**
- **How do we know which shape to use?**
  - **Small Tip Angle Approximation**
    - ➡ **Slice profile depends on the FT of the shape.**



Small Tip Angle Approximation

Forced Precession with a Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff}(z,t) = \begin{bmatrix} B_1(t) \\ 0 \\ \cancel{B_0 +} G_z \cdot z \cancel{- \frac{\omega_{RF}}{\gamma}} \end{bmatrix}$$

Effective B-Field in the Rotating Frame

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \gamma B_1(t) & 0 & \gamma G_z \cdot z \end{vmatrix} \Rightarrow \begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \omega_1(t) & 0 & \omega(z) \end{vmatrix}$$

Coupled system of differential equations!

Small Tip Approximation

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \omega_1(t) & 0 & \omega(z) \end{vmatrix}$$

$$\frac{dM_x}{dt} = \omega(z) M_y$$

$$\frac{dM_y}{dt} = -\omega(z) M_x + \omega_1(t) M_z$$

$$\frac{dM_z}{dt} = -\omega_1 M_y$$

Coupled system of differential equations!

$$\sin\alpha \approx \alpha$$

$$\cos\alpha \approx 1$$

$$M_z(t) \approx M_0$$ Constant!

Small Tip Angle Approximation

Small Tip Approximation

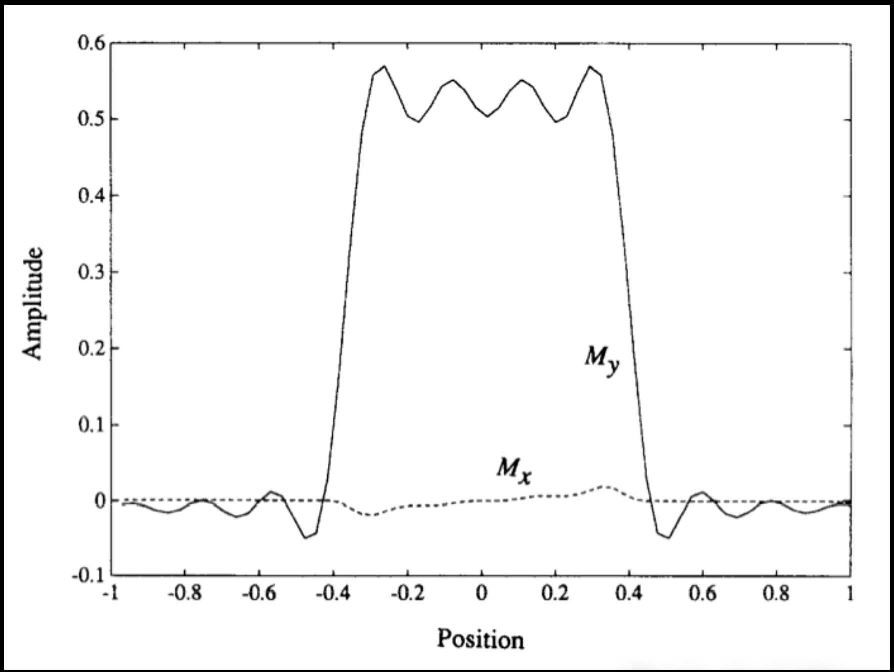
To the board ...

Small Tip Approximation

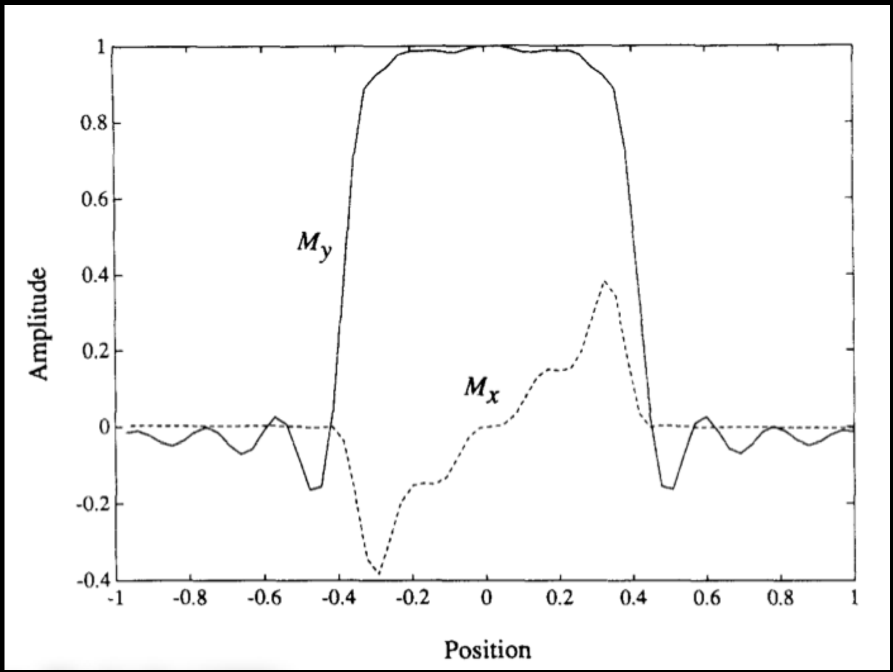
1. The excitation profile, within the small angle approximation, is just the Fourier transform of the pulse.
2. Remember that the Bloch equations are non-linear and thus cannot be expected to behave linearly.
3. The approximation works surprisingly well even for flip angles up to 90°!

Shaped Pulses

30°



90°



Pauly, J. *J. Magn. Reson.* **81** 43-56 (1989)

The small flip angle approximation still works reasonably well for flip angles that aren't necessarily "small".

Truncation Artifacts

In MRI we want pulses to be as short as possible:

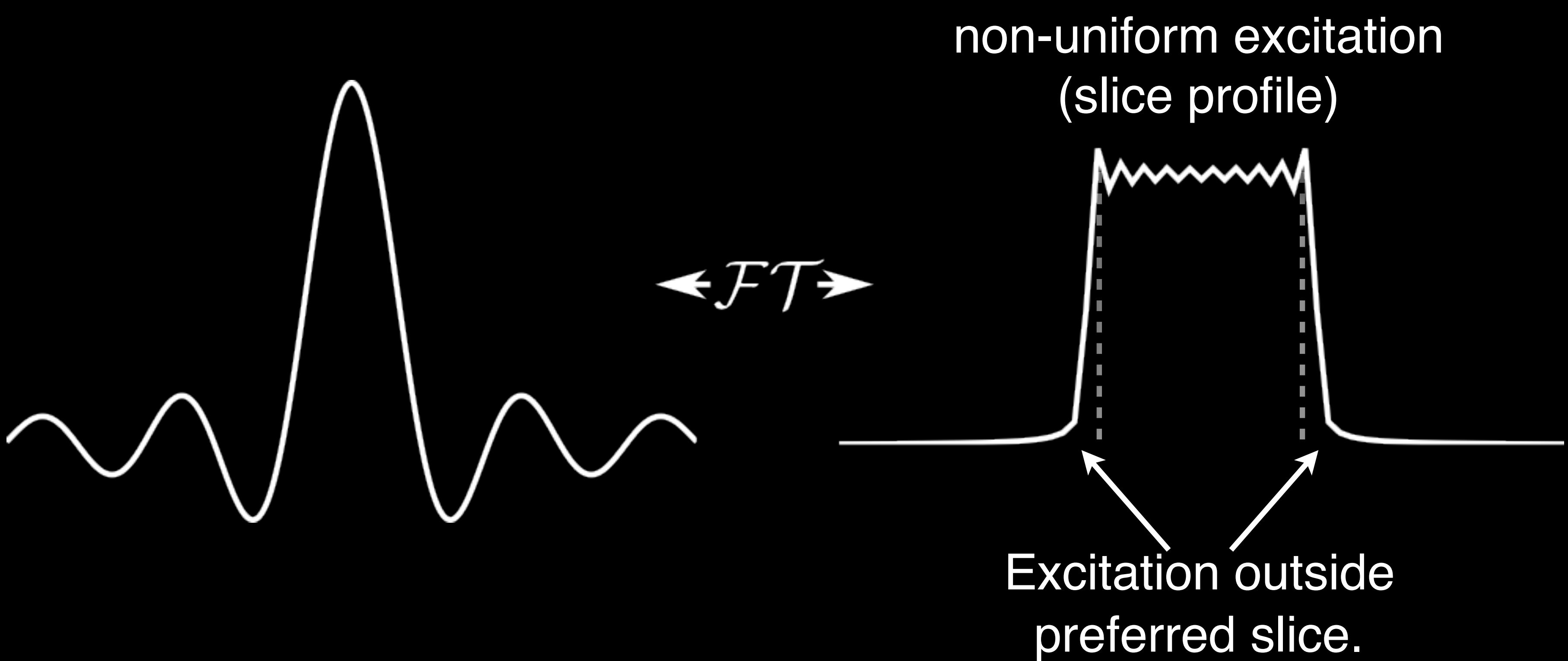
1) To avoid relaxation effects.
2) To improve scan efficiency.

The ***sinc*** function is defined over all time, which is impractical in any experiment.

The ***sinc*** pulse needs to be truncated to be appropriate for clinical scans.

Truncation Artifacts

What happens when we truncate our pulses?



Deviations from the ideal slice profile
are known as truncation artifacts.

Reducing Truncation Artifacts

Alternative Pulse Shapes

$$B_x(t) = A \exp\left[-a(t-\tau/2)^2\right]$$ Gaussian

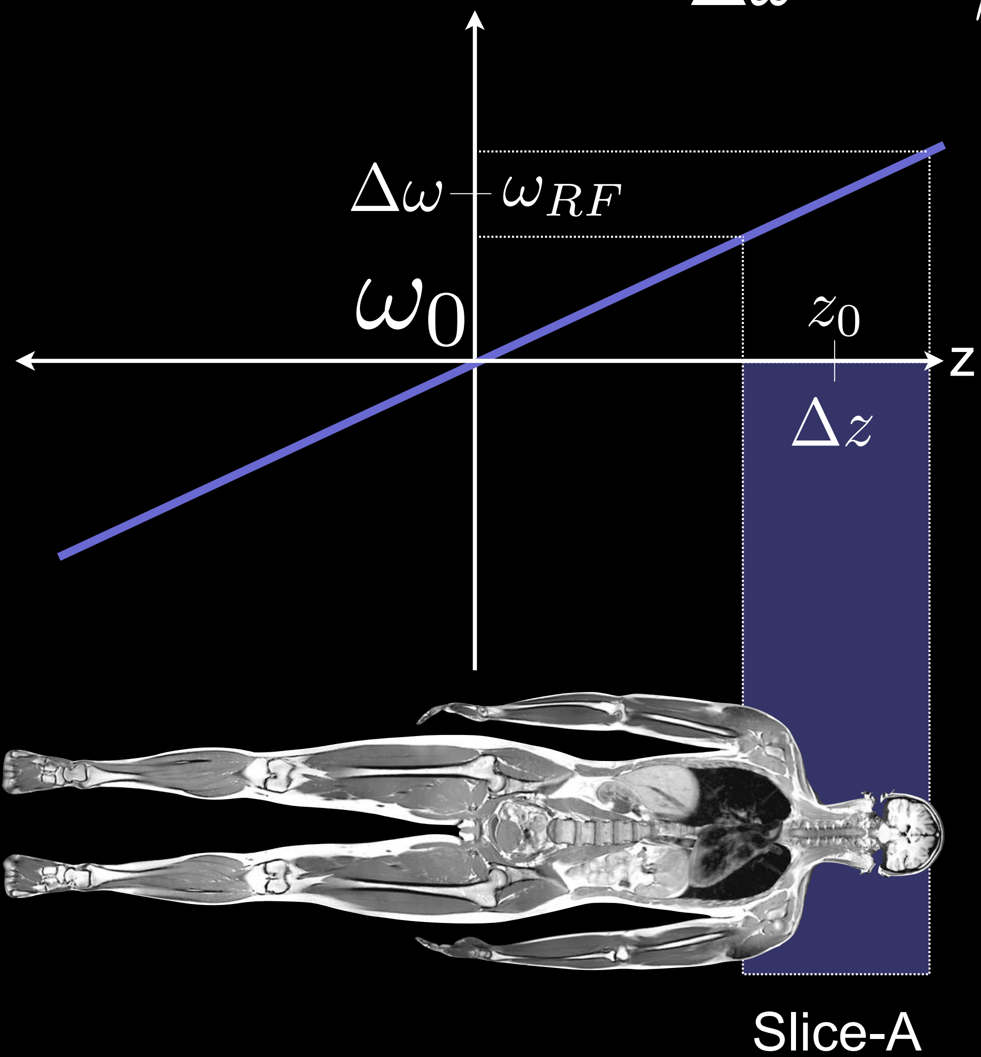
Reduced side-lobes, but not as flat of a slice profile.

Time Bandwidth Product (TBW)

- **Time bandwidth (TBW) product:**
  - **Pulse Duration [s] x Pulse Bandwidth [Hz]**
  - **Unitless**
  - **# of zero crossings**
  - **High TBW**
    - Large # of zero crossings ∴ fewer truncation artifacts
    - Longer duration pulse
- **Examples:**
  - **TBW = 4, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?
  - **TBW = 16, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?

Slice Selective Excitation - Example

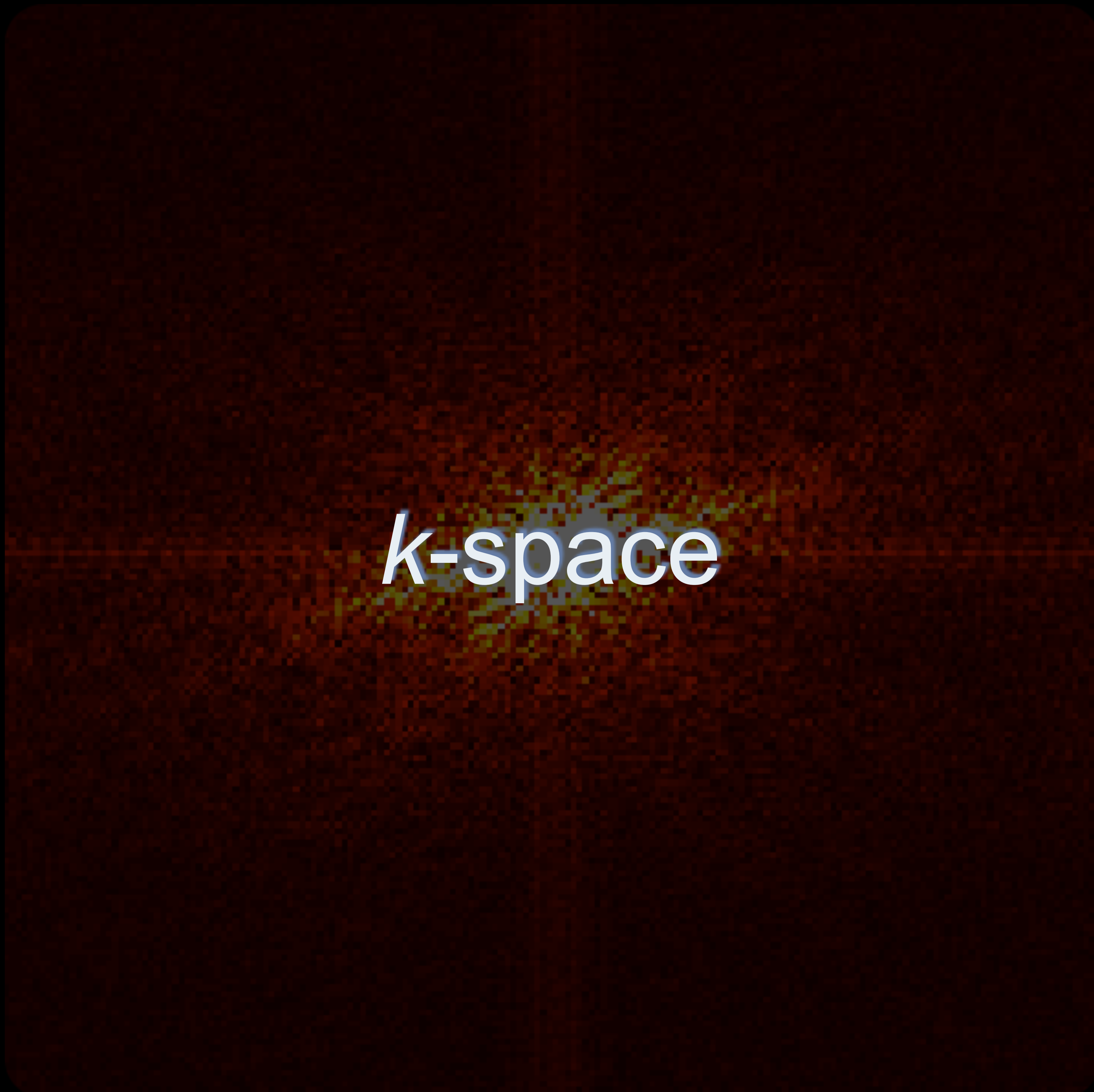
$$\Delta\omega = -\gamma\left(G_z \cdot \Delta z\right)$$ Excitation (BW_{RF}) Bandwidth



$$\mathrm{TBW} = \tau_{RF} \cdot \mathrm{BW}_{RF}$$

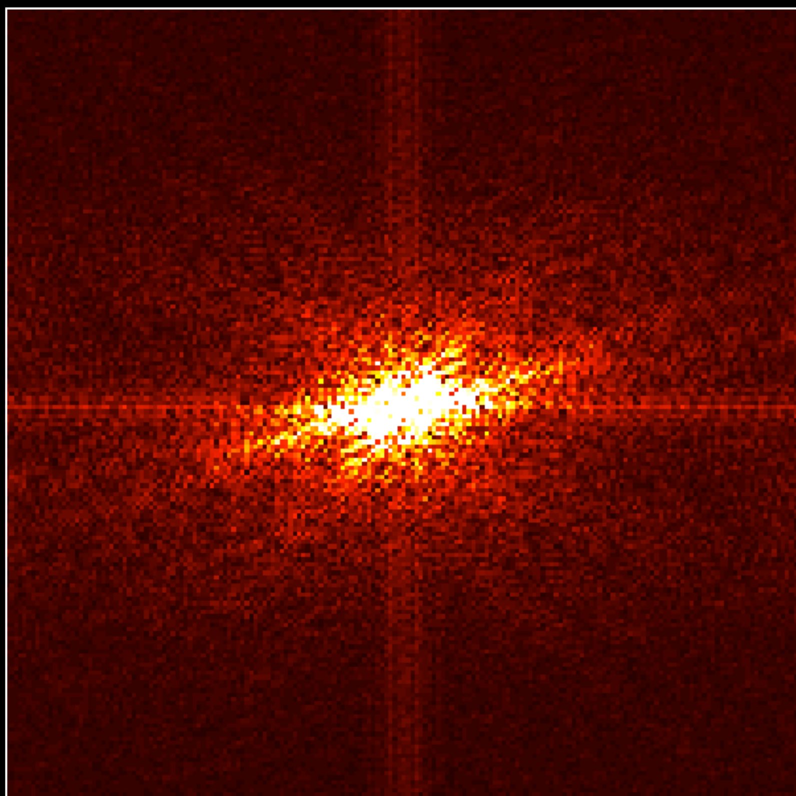
$$\mathrm{BW}_{RF} = \frac{\mathrm{TBW}}{\tau_{RF}} = \frac{4}{1\mathrm{ms}} = 4\mathrm{kHz}$$

$$G_z = \frac{\Delta f}{\gamma \Delta z} = \frac{4000\mathrm{Hz}}{42.57e6\frac{Hz}{T}\frac{1\mathrm{T}}{10000\mathrm{G}} \cdot 10\mathrm{mm}} = 0.94\frac{\mathrm{G}}{\mathrm{cm}}$$


k-space

What is *k*-space?

k-space



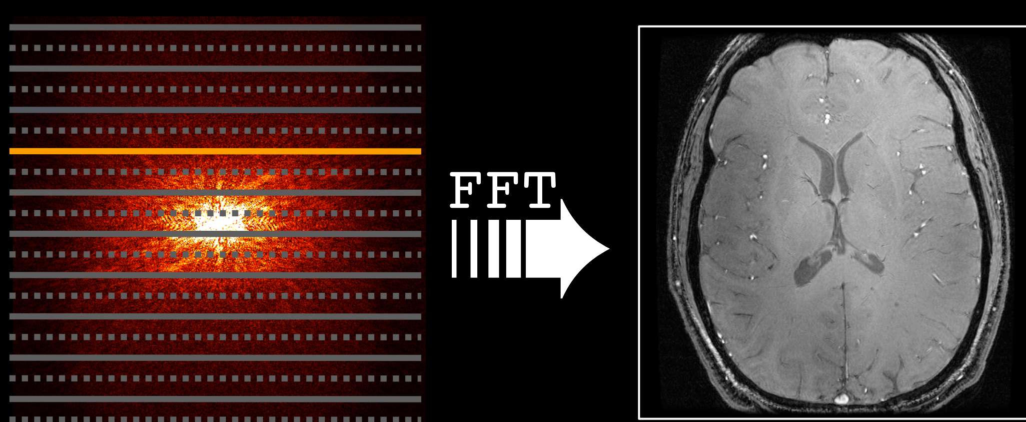
FFT

image space

***k*-space is the raw data collected by the scanner.**

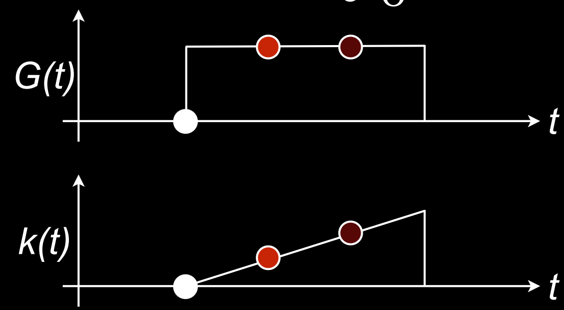
What is *k*-space?

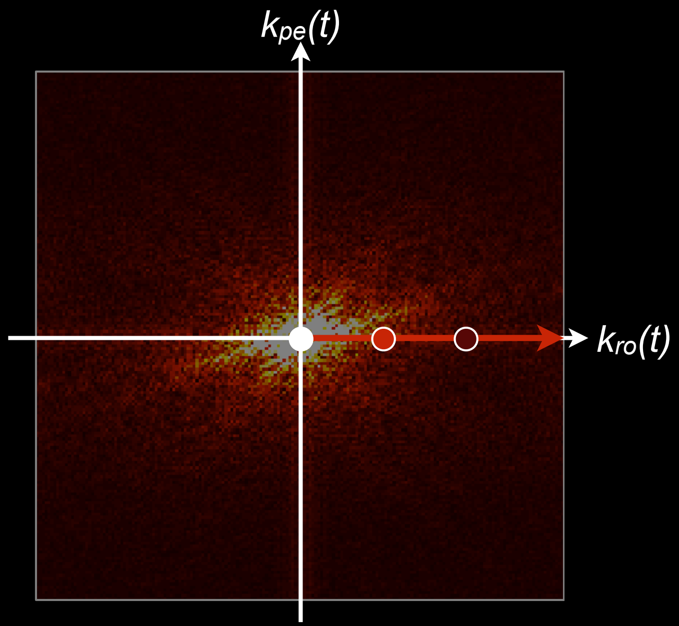
- ***k*-space is the raw data collected by the scanner.**
  - A point in k-space tells us about the presence/absence of a spatial frequency (pattern) in the acquired image.
  - Each echo measures *many* of the spatial frequencies that comprise the object.
  - *k*-space has units of cm^{-1} or mm^{-1}
    - Audio signals have units of Hertz (s^{-1})
- **Gradients**
  - Help extract spatial frequency information
  - Move us around in k-space
- **A line of *k*-space is filled by an echo**



What is *k*-space?

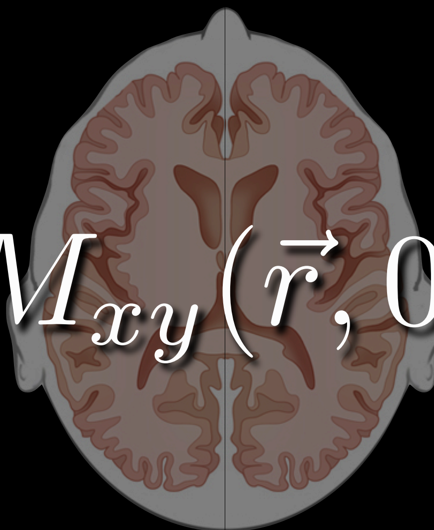
$$\vec{k}(t) = \frac{\gamma}{2\pi} \int_0^{\tau} \vec{G}(t) d\tau$$

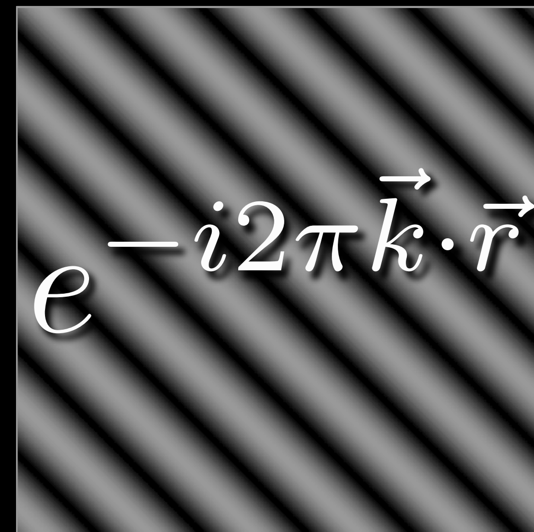




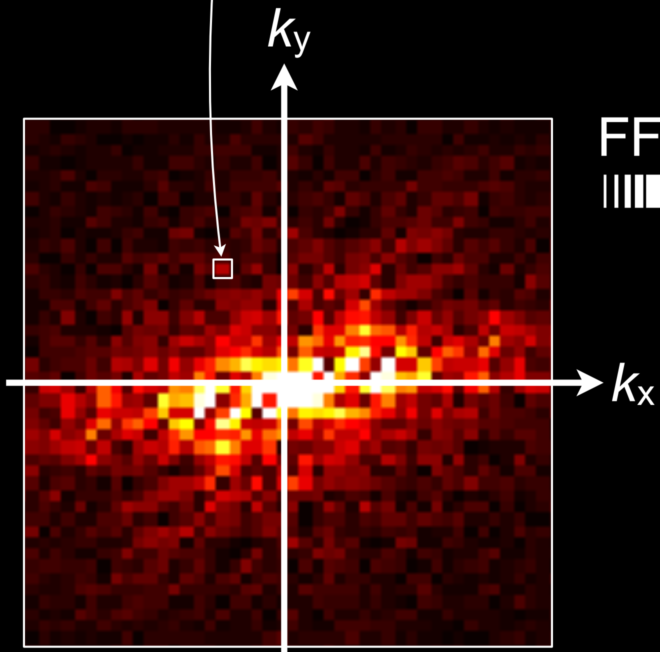
Gradients move us through k-space.

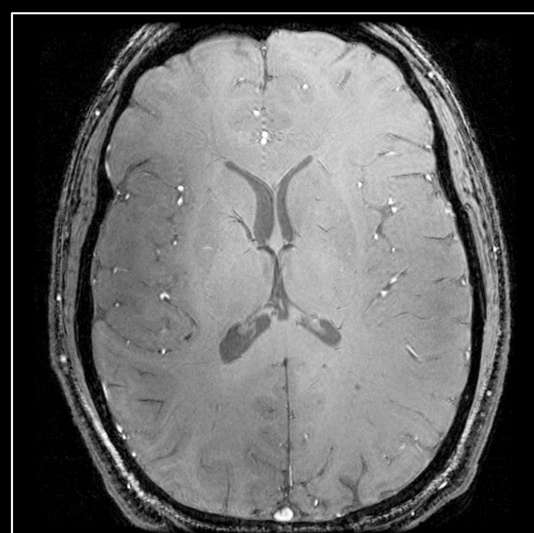
MRI Signal Equation





$$S(\vec{k}) = \int_{object} M_{xy}(\vec{r},0)\; e^{-i2\pi\vec{k}\cdot\vec{r}}\; d\vec{r}$$





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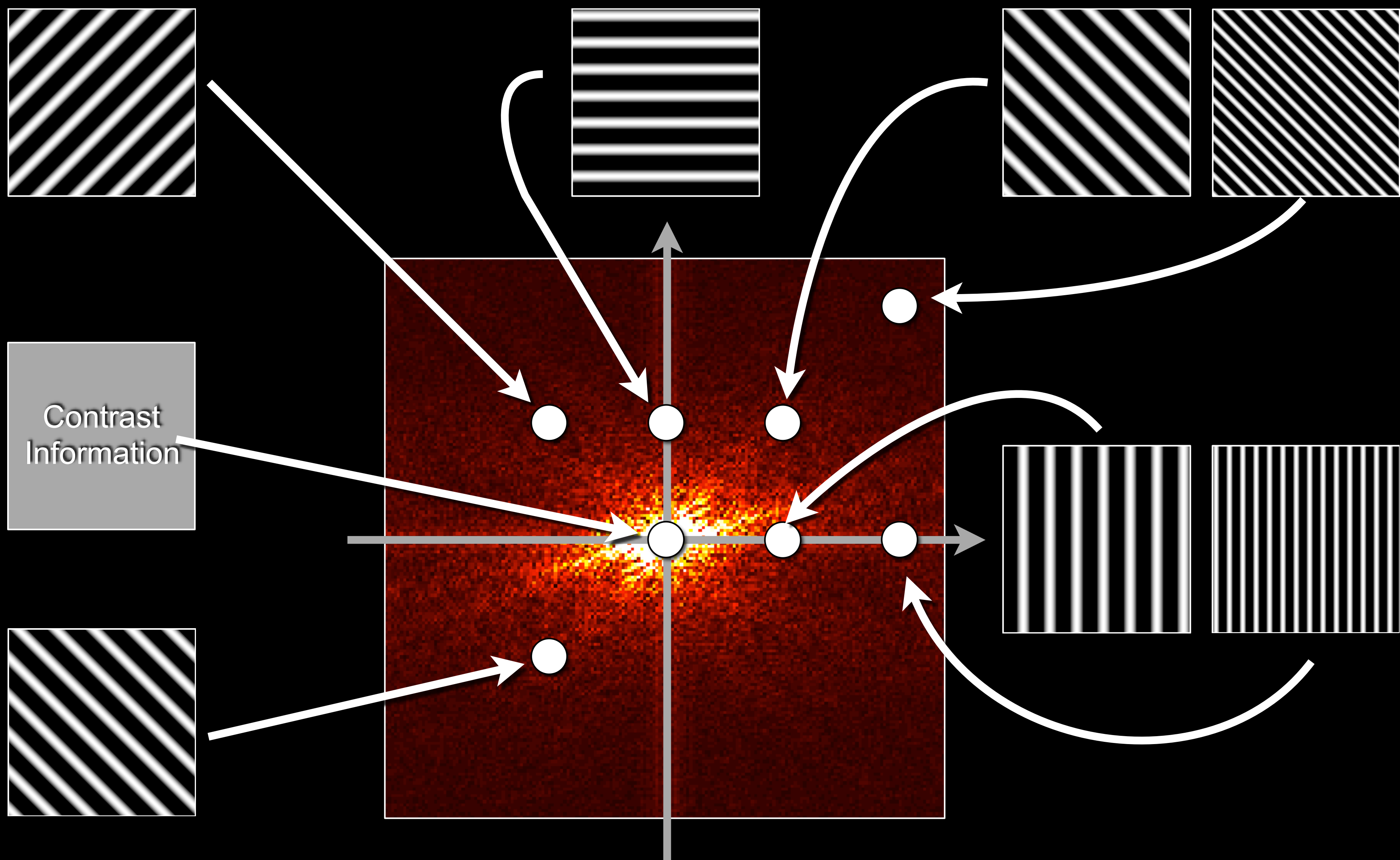
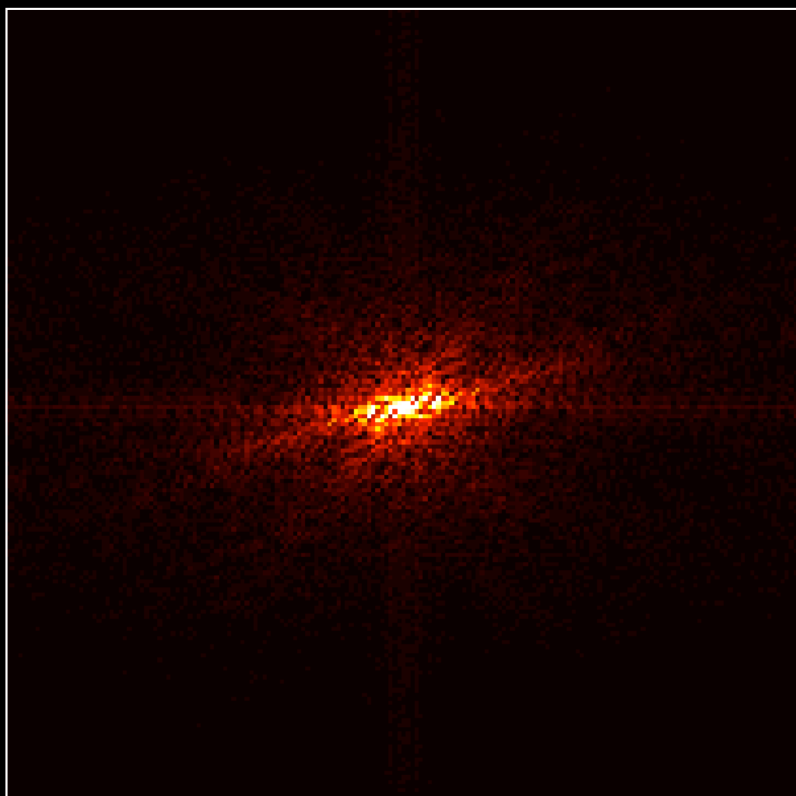
What is *k*-space?



A point in *k*-space tells us about the presence/absence of a spatial frequency (pattern) in the acquired image.

k-space spikes

k-space

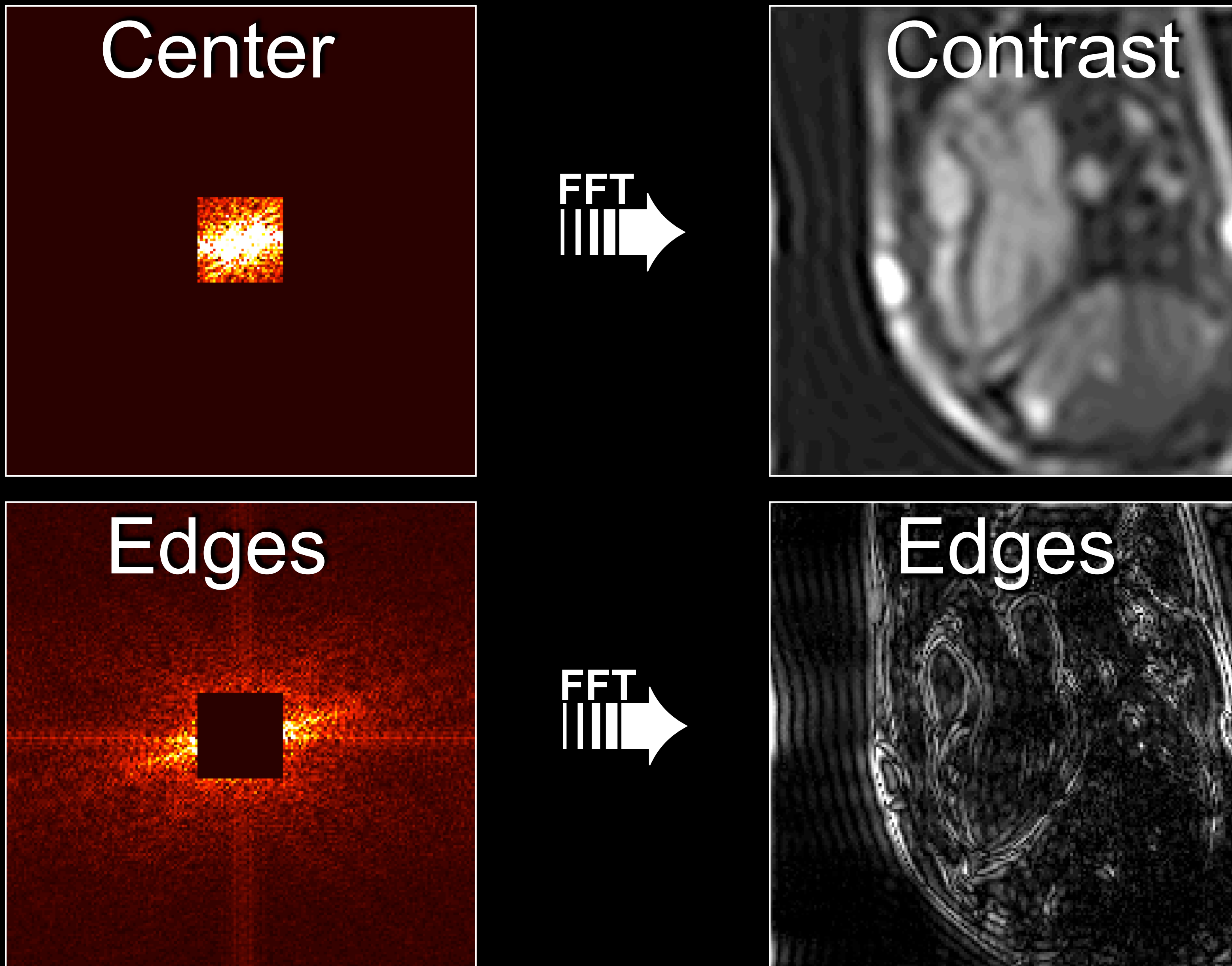


FFT

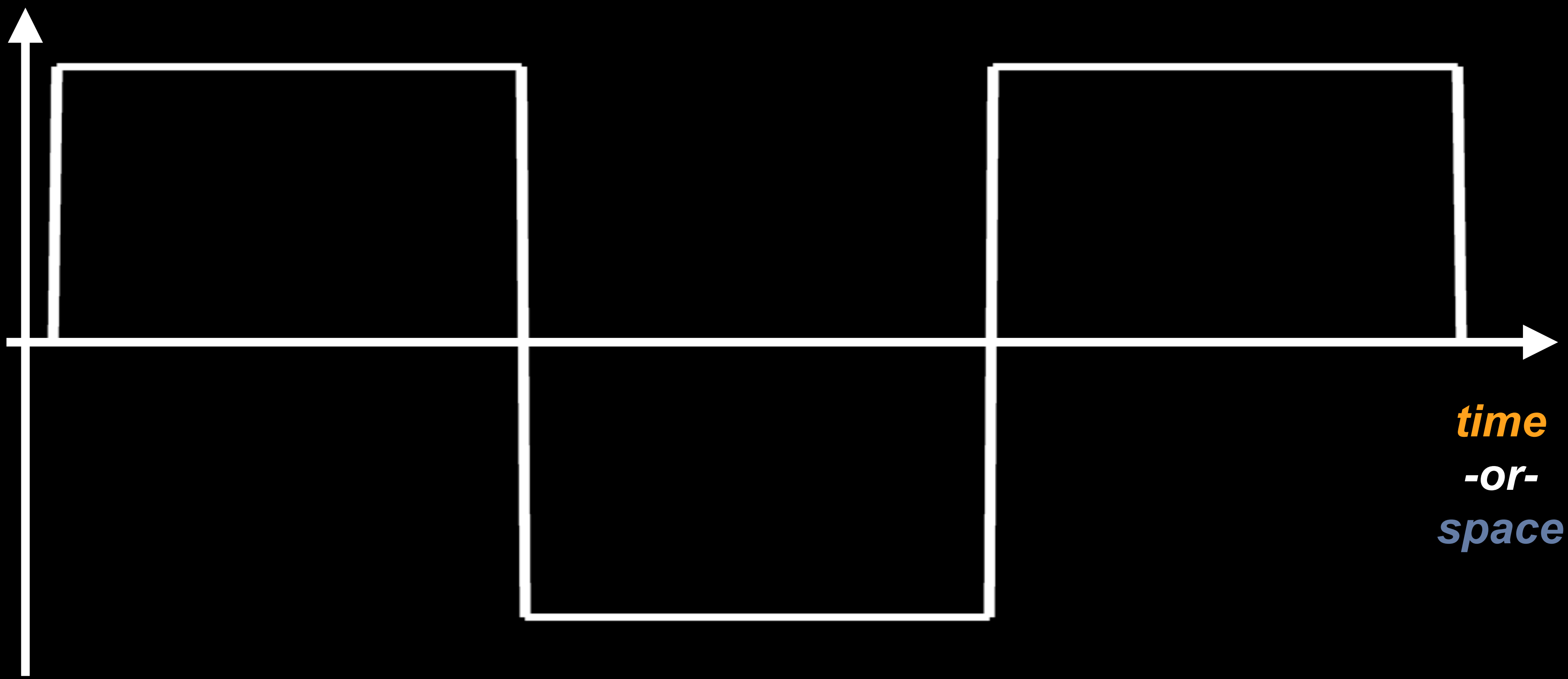
image space

A *k*-space spike creates a banding artifact.

What is *k*-space?

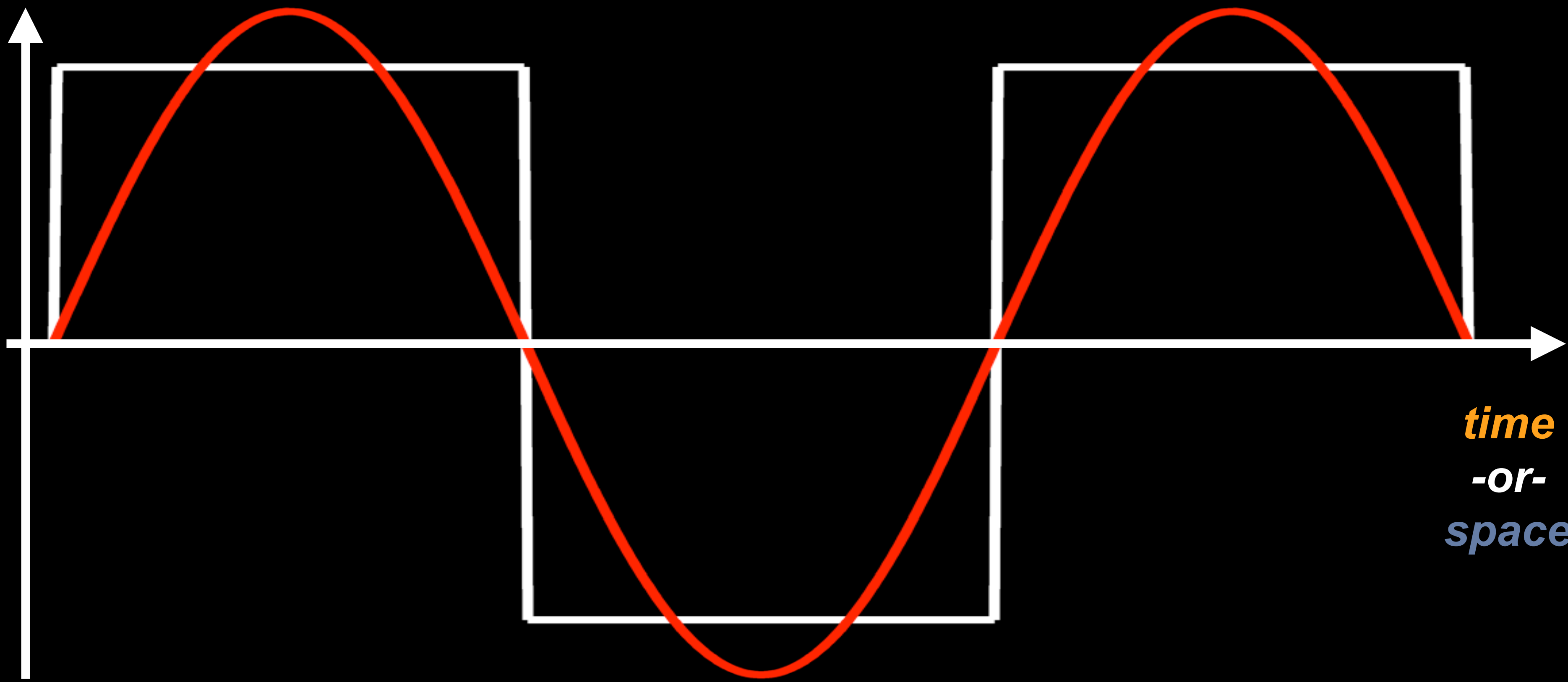


1D *k*-space



Any signal/image can be decomposed into a summation of sine waves of appropriate amplitude.

1D *k*-space



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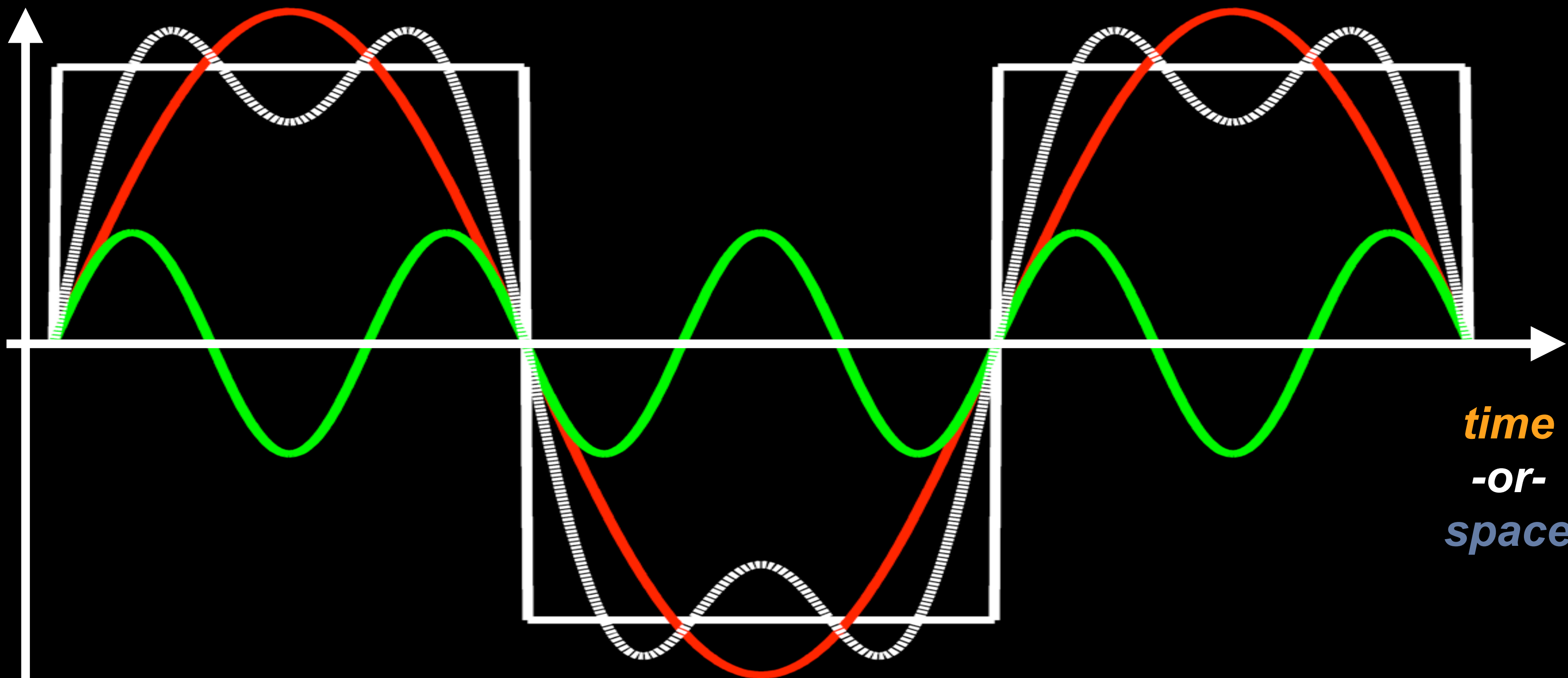
1D *k*-space



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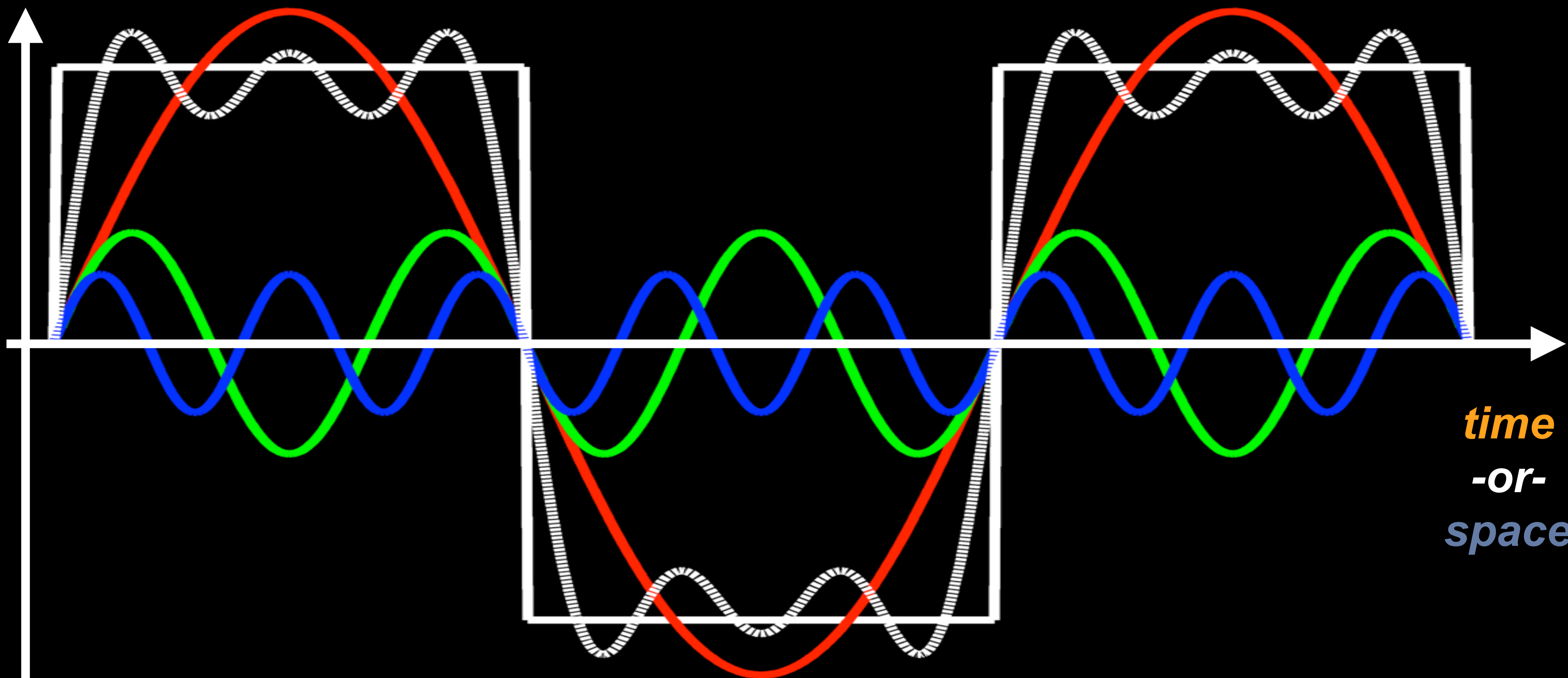
1D *k*-space



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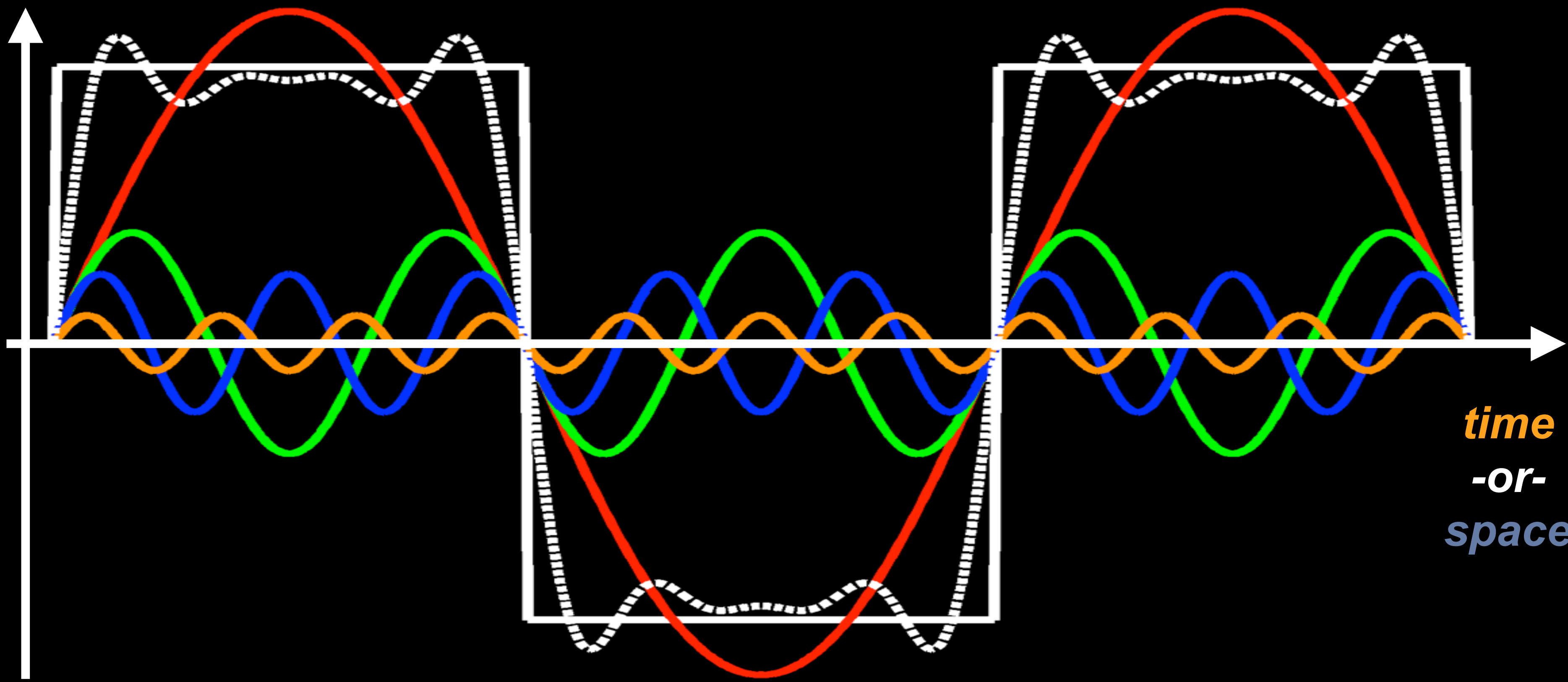
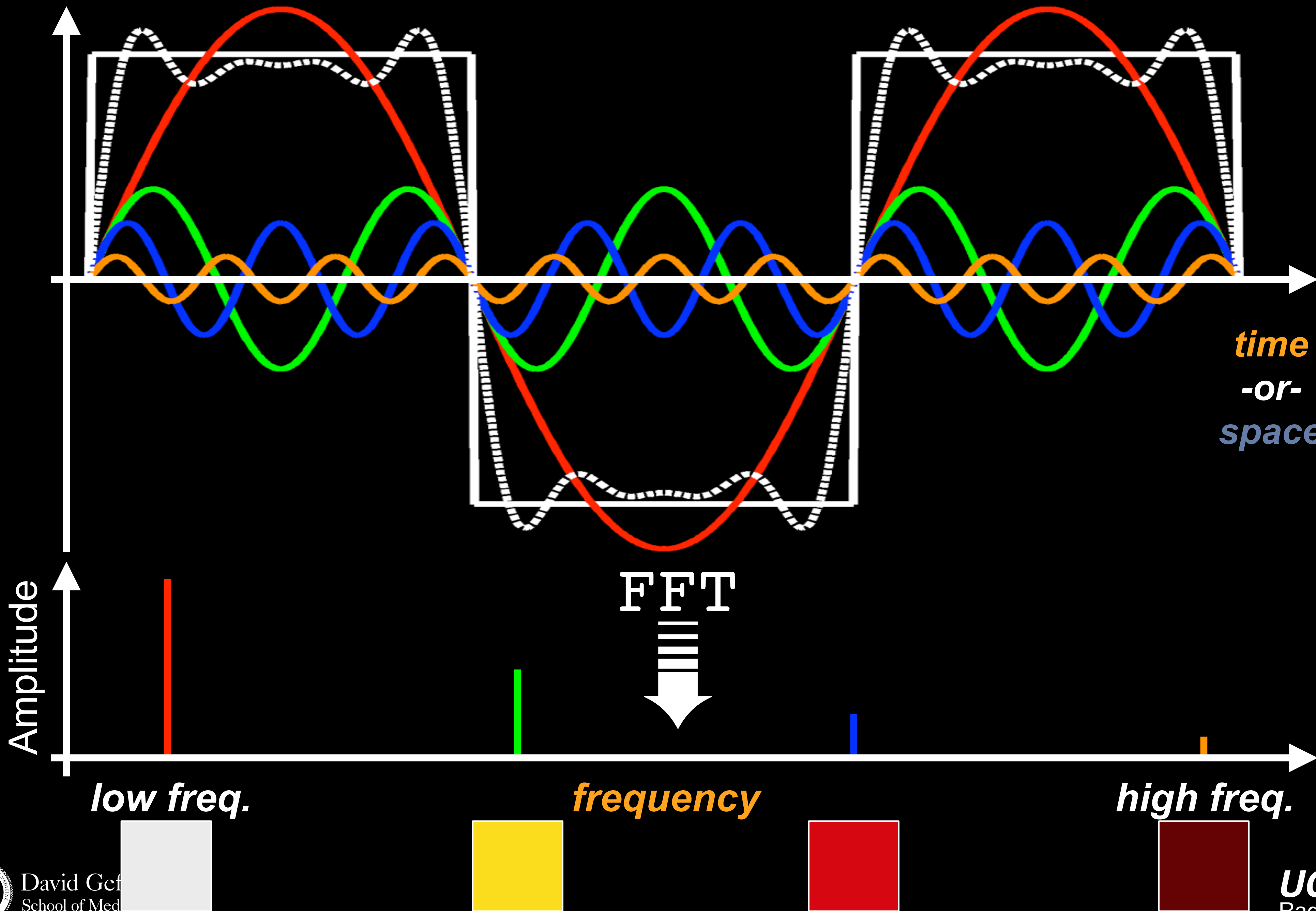
1D *k*-space

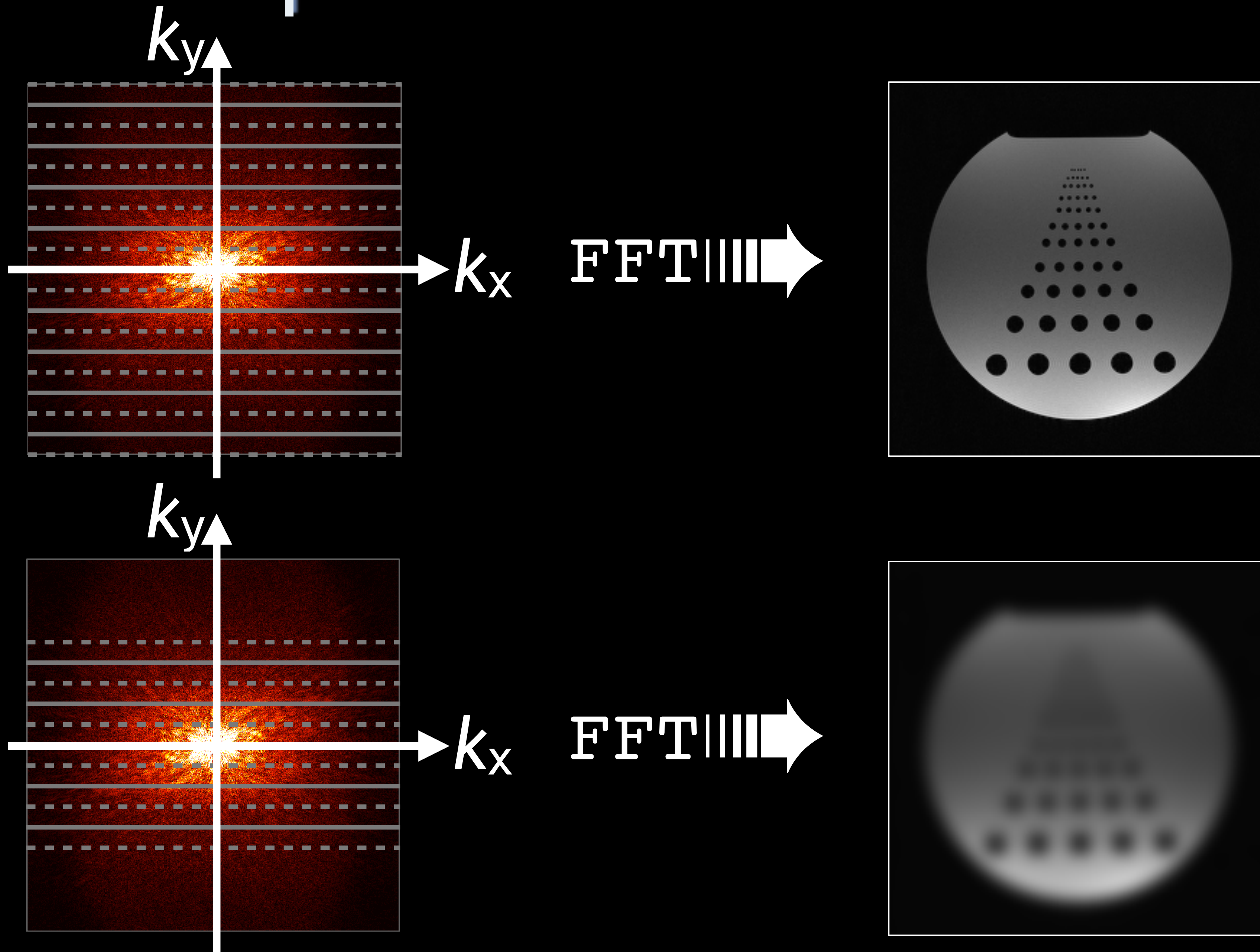


Any signal/image can be decomposed into a summation of sine waves of appropriate amplitude.

Fourier Representation

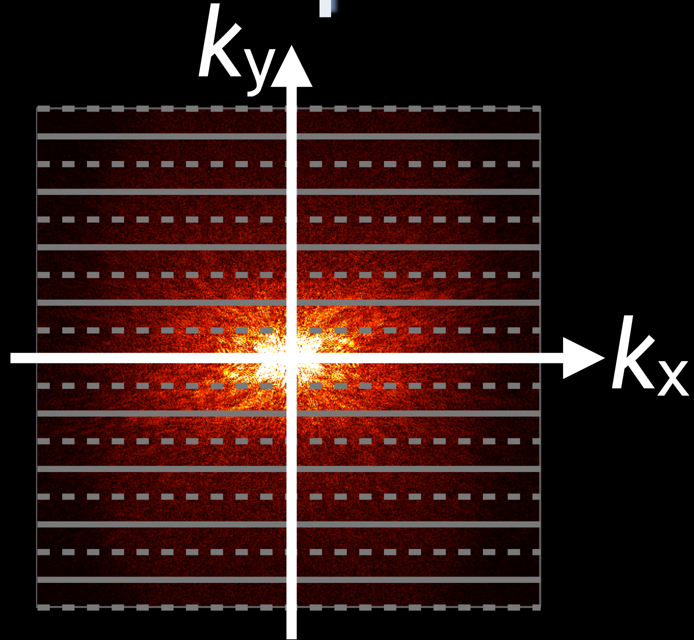


k-space and Resolution

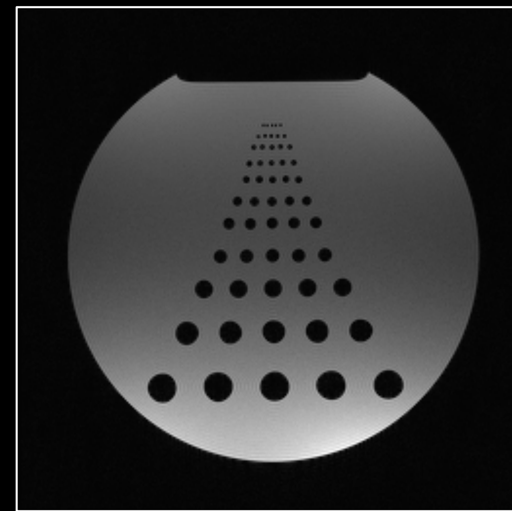


Acquiring fewer high phase encodes decreases resolution.

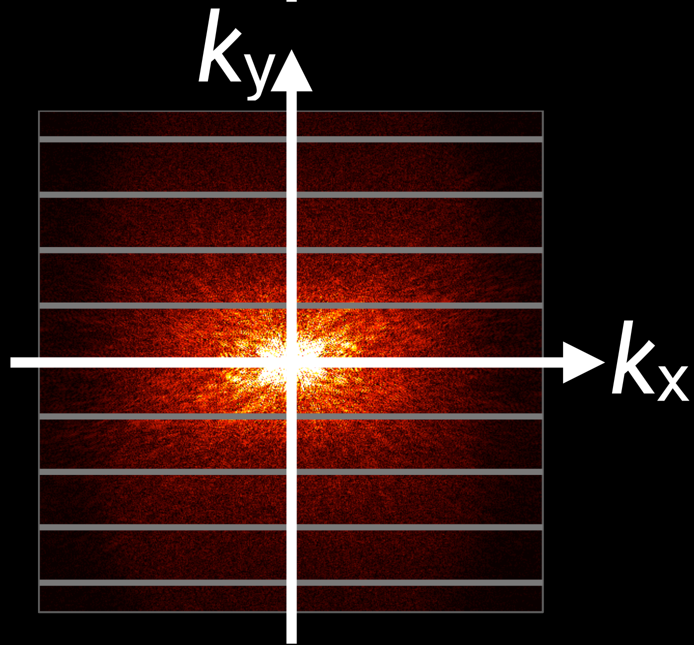
k-space and Field of View



FFT



$$FOV = \frac{1}{\Delta k}$$

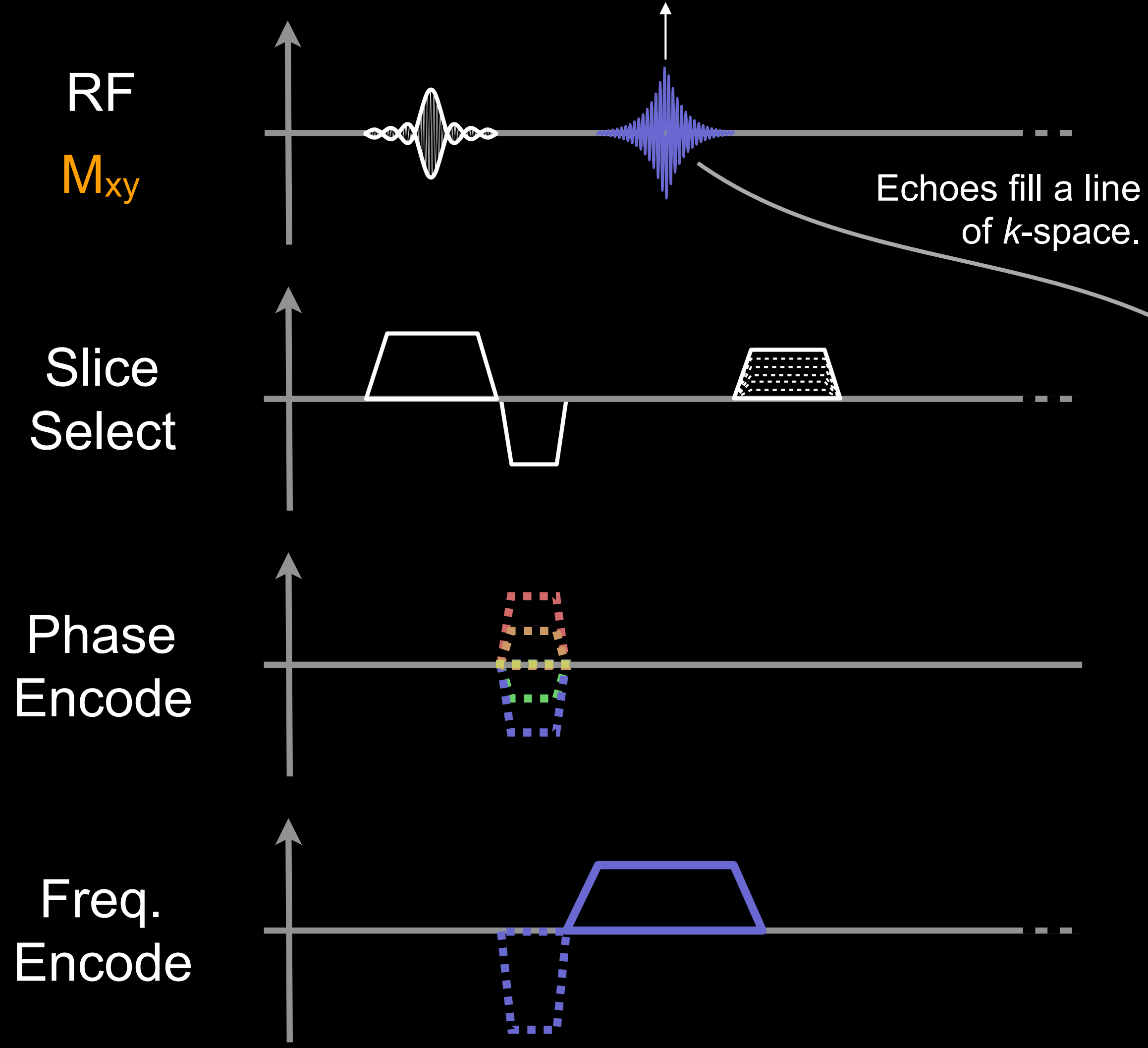


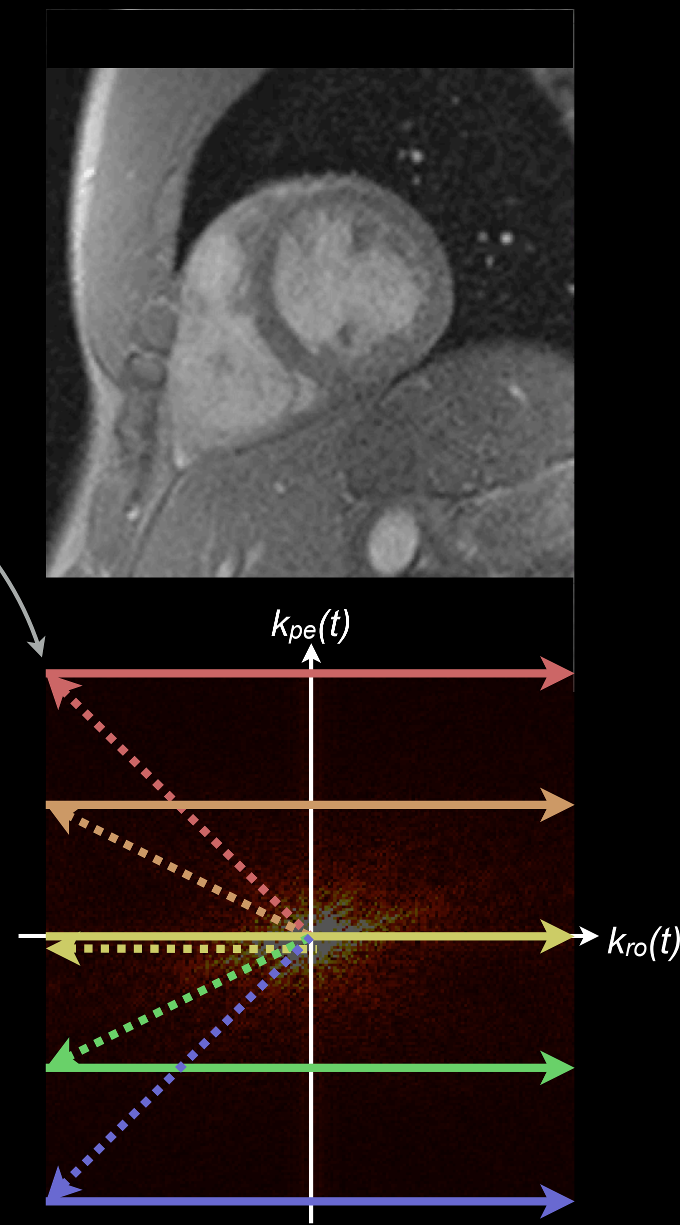
FFT

Uniformly skipping lines in *k*-space causes ***aliasing***.

Spoiled Gradient Echo

$$M_{xy}(\vec{r},0) \propto \frac{\rho\left(1-e^{-TR/T_1}\right)}{1-\cos\alpha e^{-TR/T_1}} \sin\alpha e^{-TE/T_2^*}$$

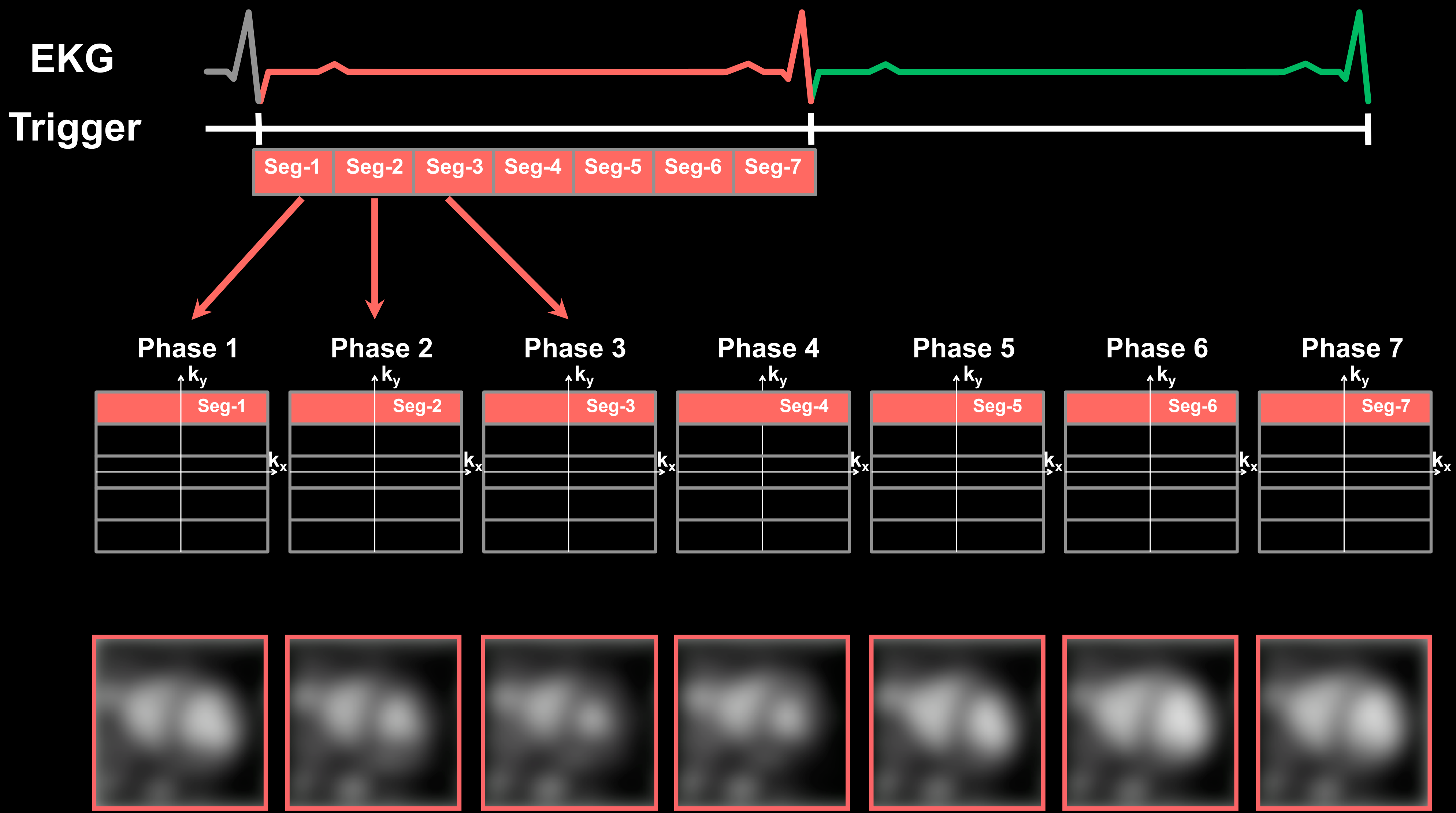




Gradients move us through *k*-space!

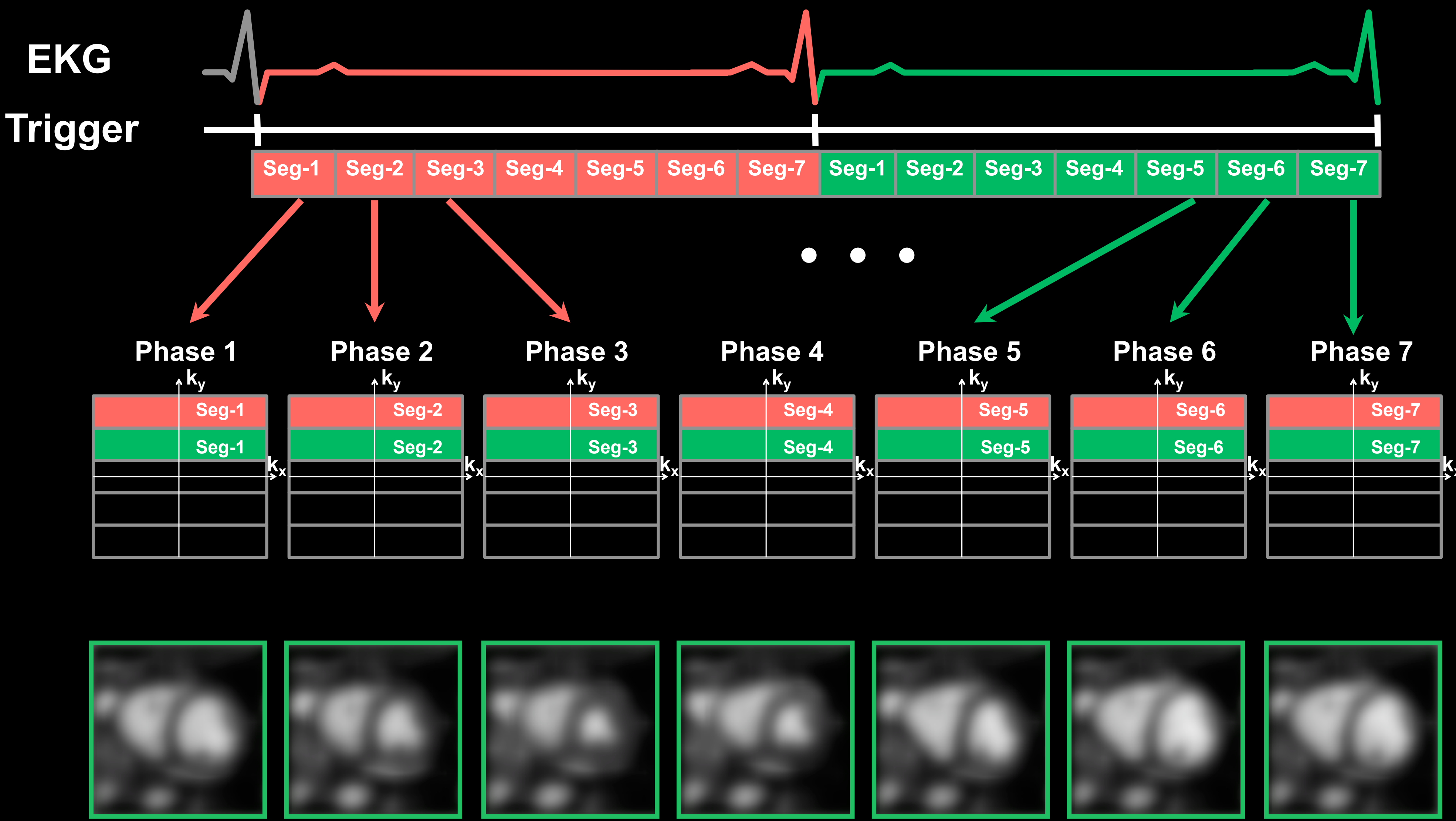
MRI is slow.
How do we make movies?

Segmented Cardiac Imaging



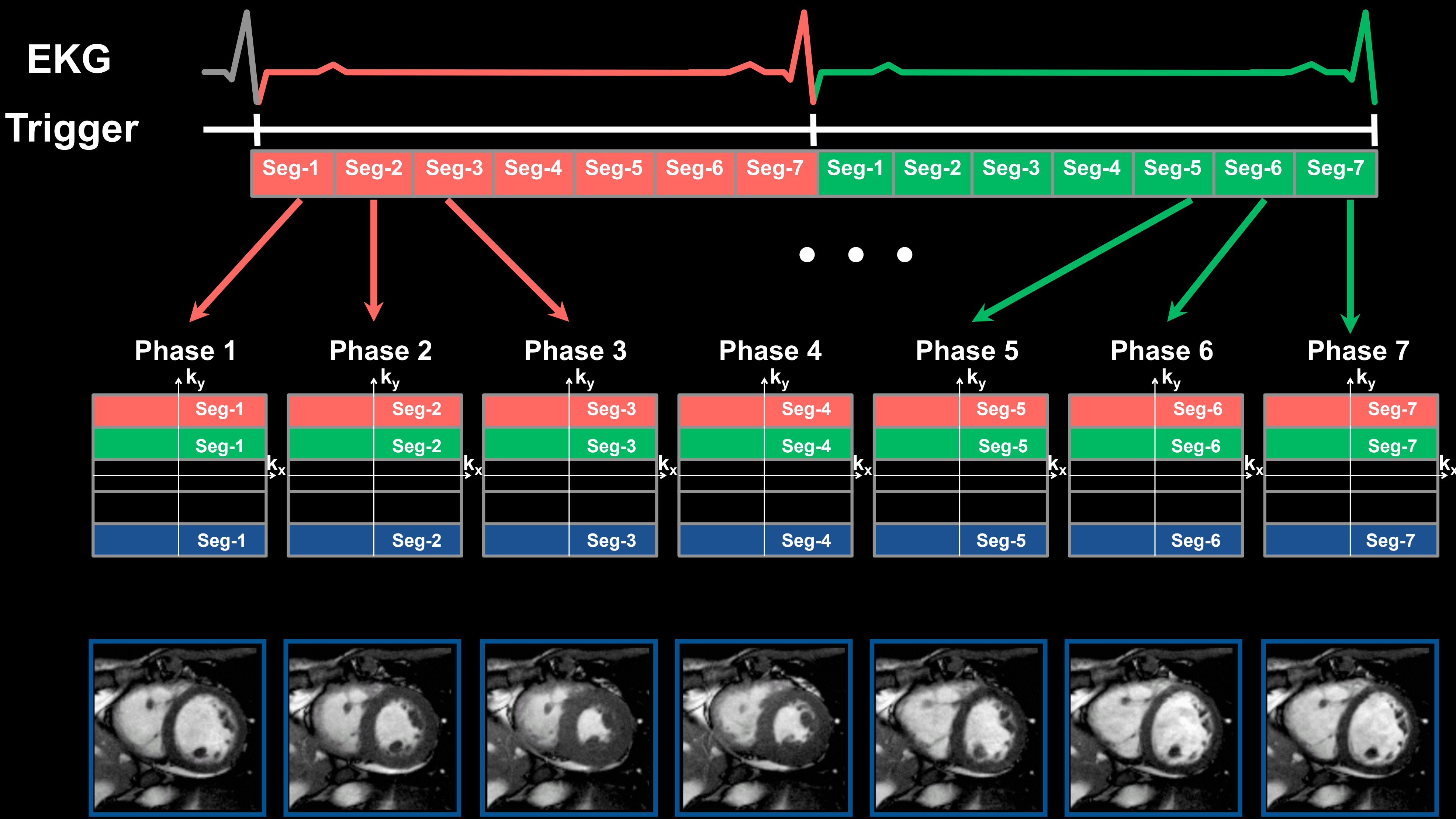
A *k*-space ***segment*** is a few lines of k-space.

Segmented Cardiac Imaging



Each heartbeat acquires a unique *k*-space segment.

Segmented Cardiac Imaging



Once all heartbeats are acquired a movie can be played.

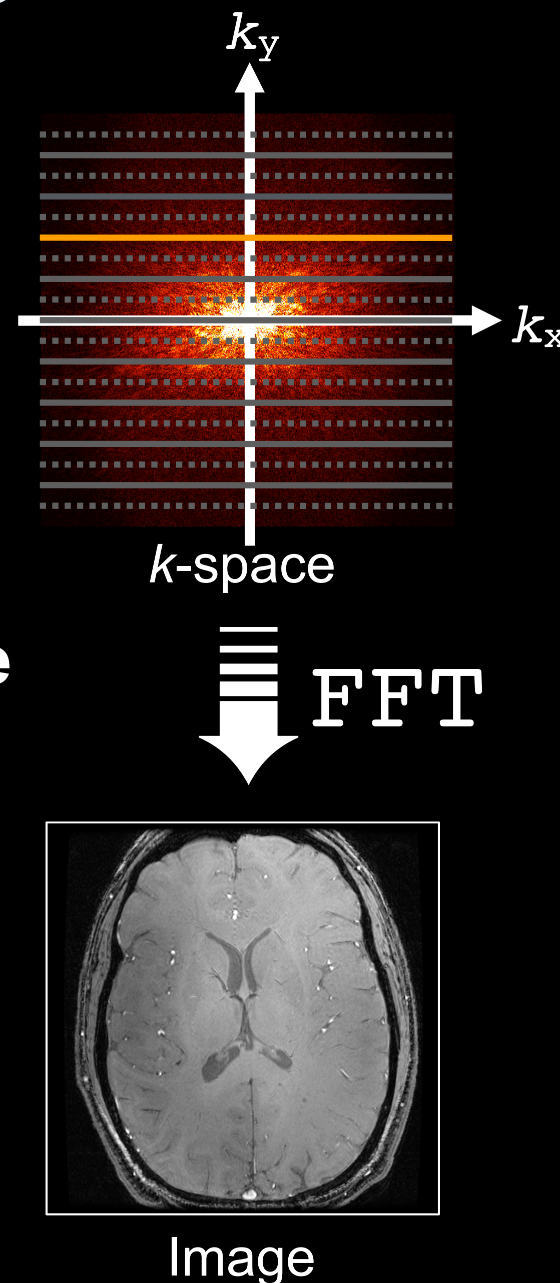
Phase & Frequency Encoding

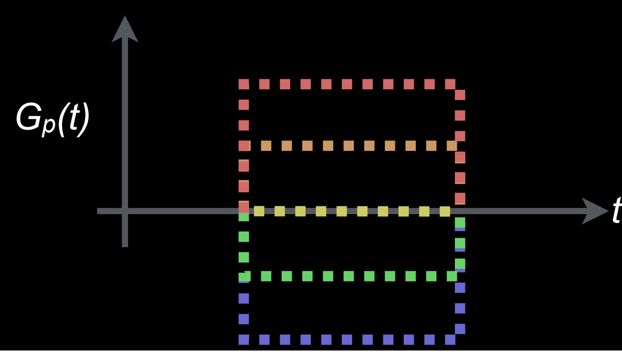
Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.

Phase Encoding

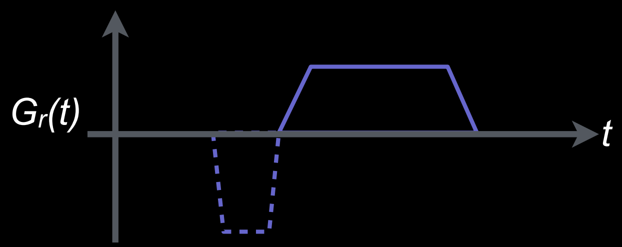
- **Consists of:**
  - **Phase encoding gradient**
    - **Magnitude changes with each TR**
    - **Can be played with other gradients**
      - **Crushers, Slice-selection rephaser, readout dephasing**
- **Used with Cartesian imaging**
- **After excitation, before readout**
- **Adds linear spatial variation of phase**
- **Phase encode in**
  - **one direction for 2D imaging**
  - **two directions for 3D imaging**
- **Only one PE step per echo**



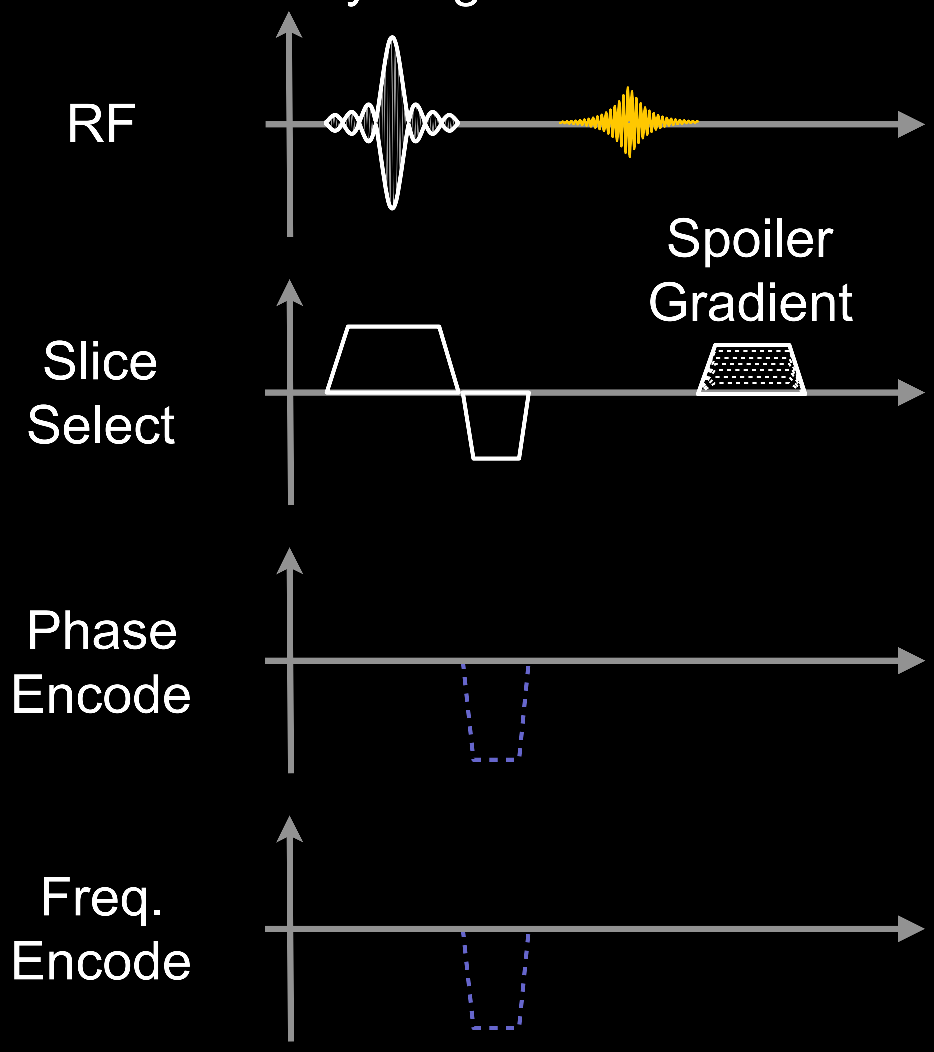


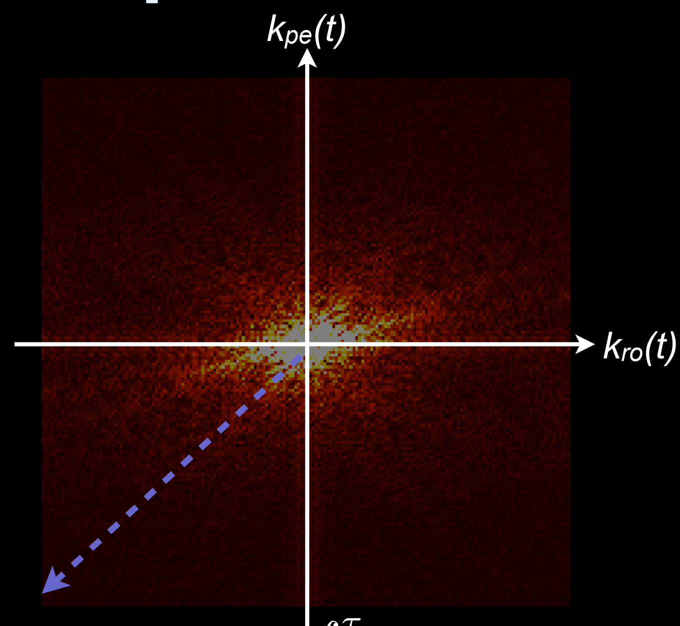
Frequency Encoding

- Consists of:
  - Frequency encoding gradient
    - Constant magnitude for Cartesian imaging
  - No simultaneous
    - RF (B_1)
    - Other gradients
      - phase encoding, slice encoding, crushers
  - Readout pre-phasing gradient
    - Prepares spin phase so peak echo amplitude occurs at middle of readout (TE)
    - AKA “readout de-phasing gradient”
- Adds linear spatial variation of frequency
- Helps form an *echo*

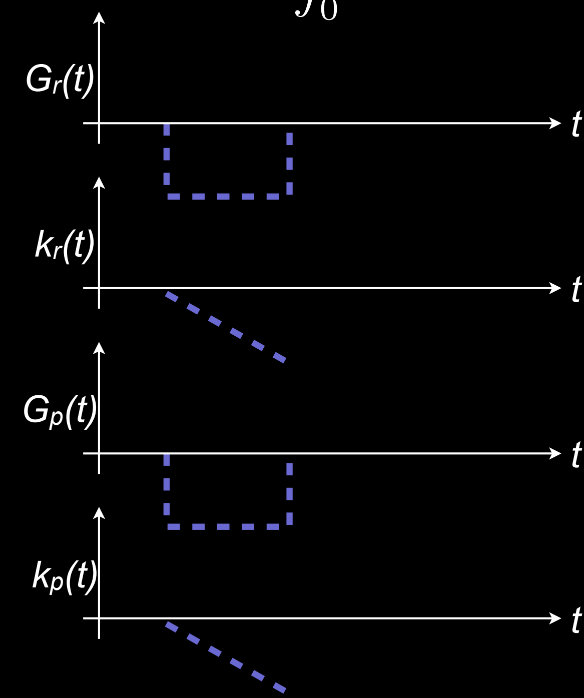


Where am I in *k*-space?



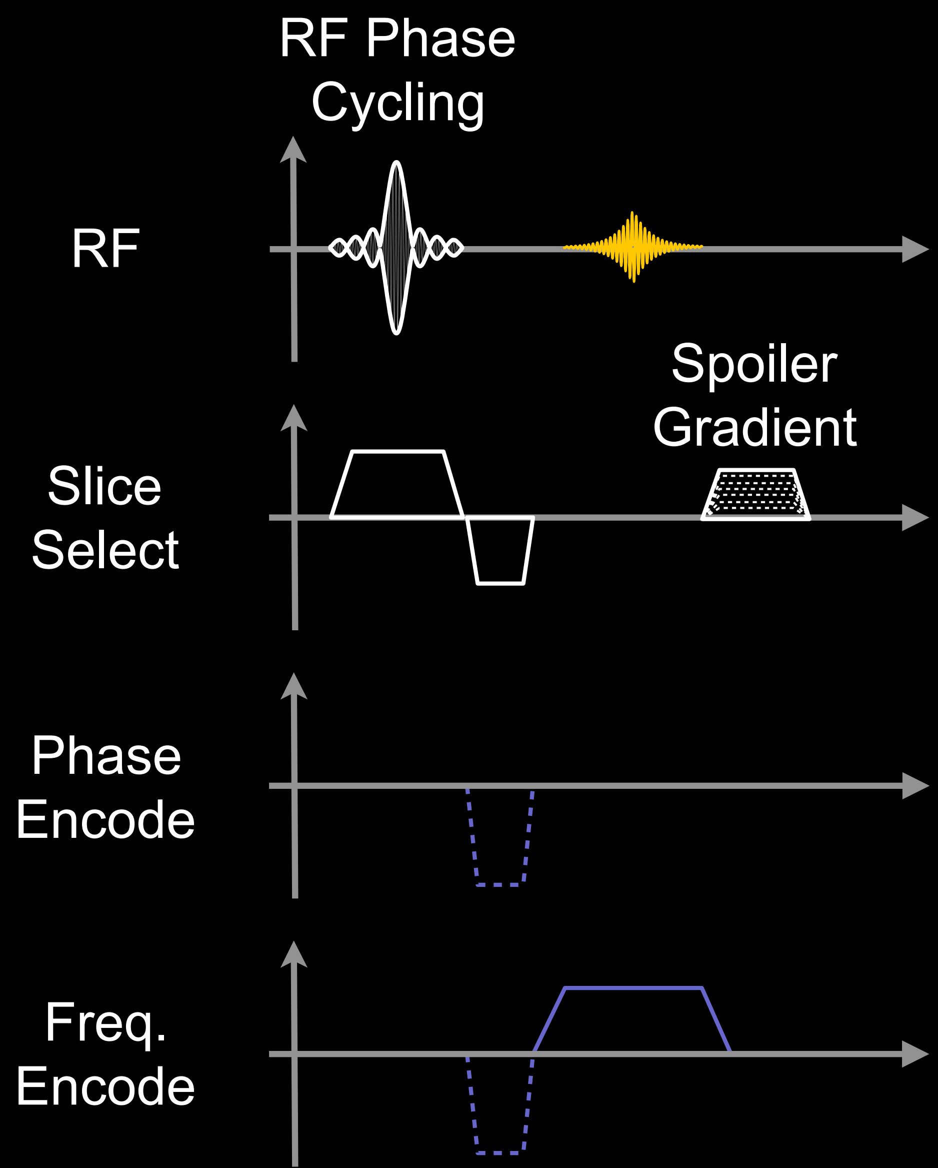


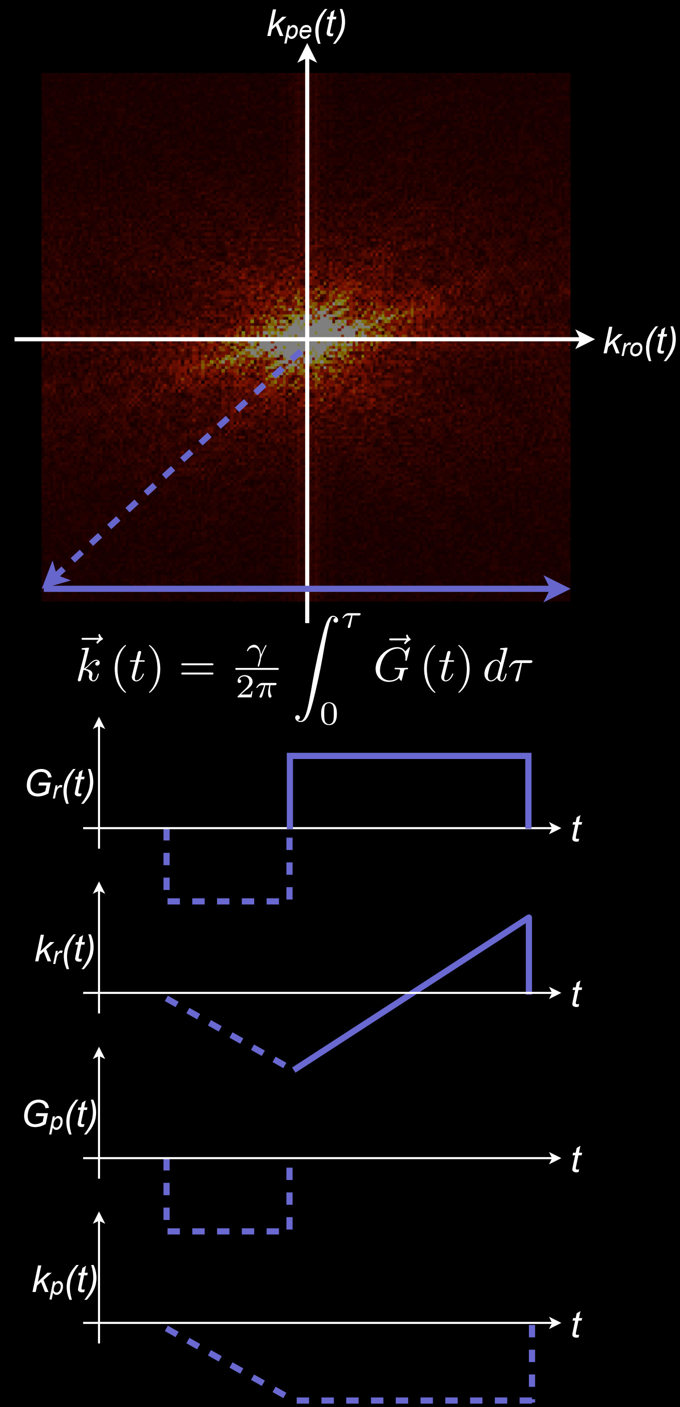
$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau}\vec{G}(t)\,d\tau$$



One phase encoded echo is acquired per TR.

Where am I in *k*-space?

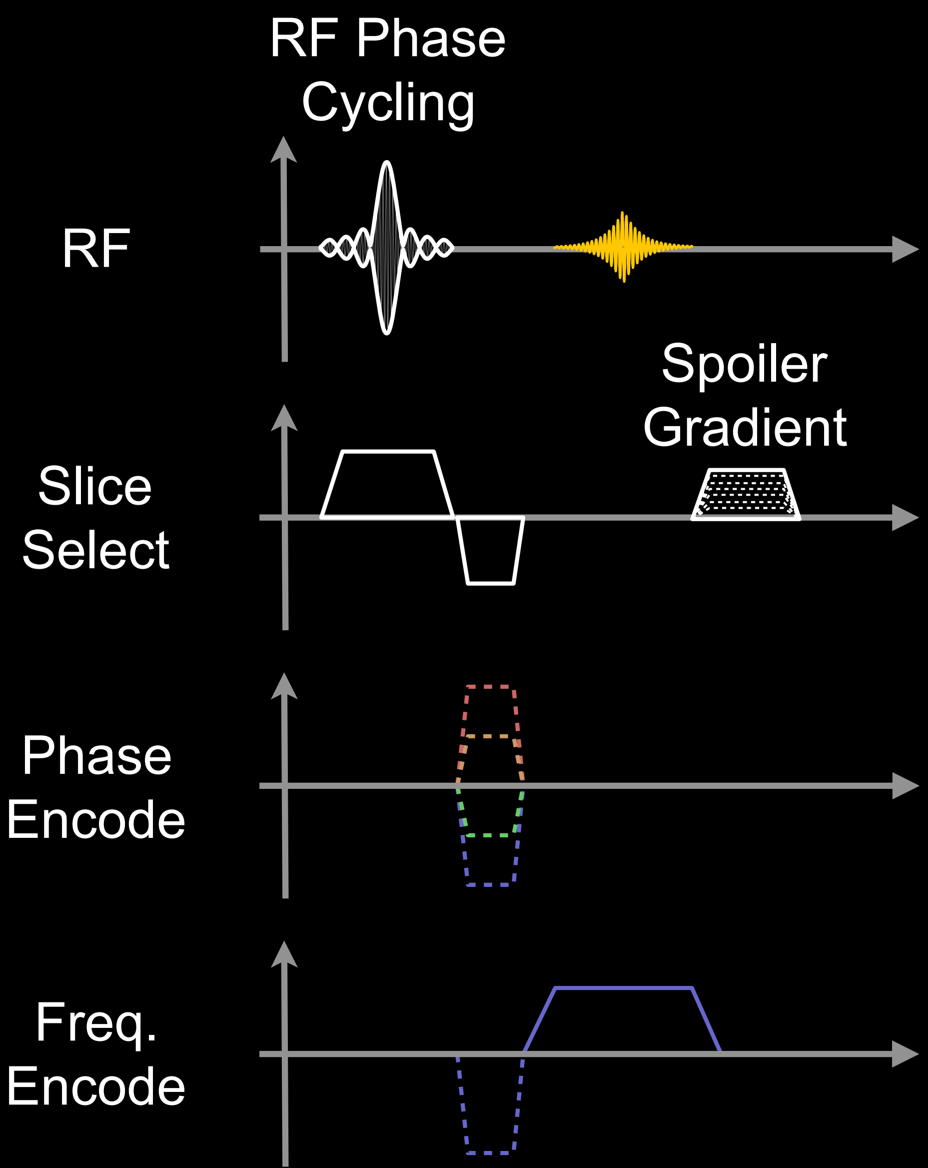


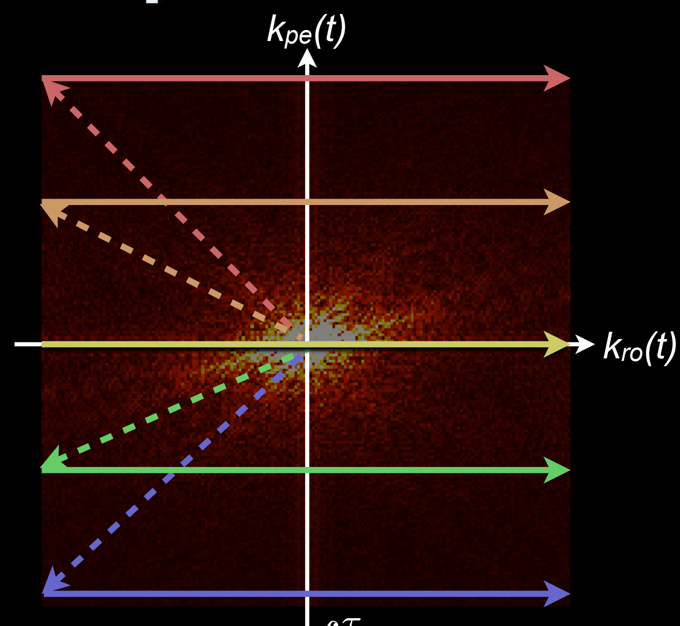


$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$

One phase encoded echo is acquired per TR.

Where am I in *k*-space?





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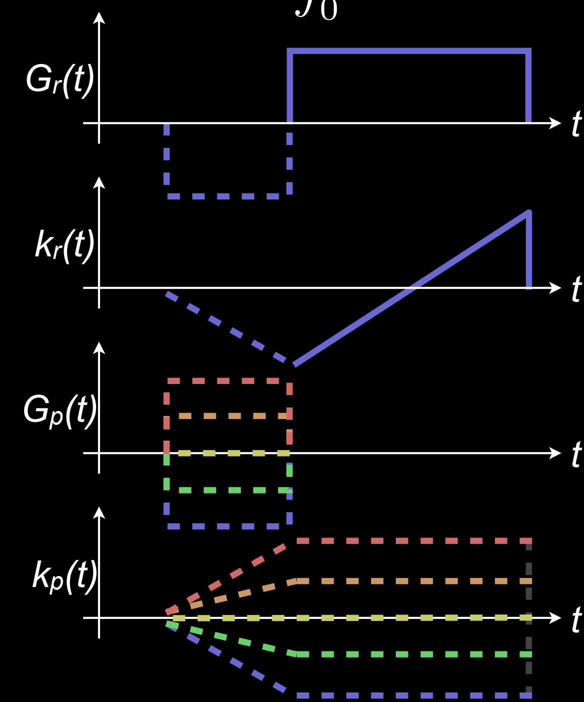

One phase encoded echo is acquired per TR.

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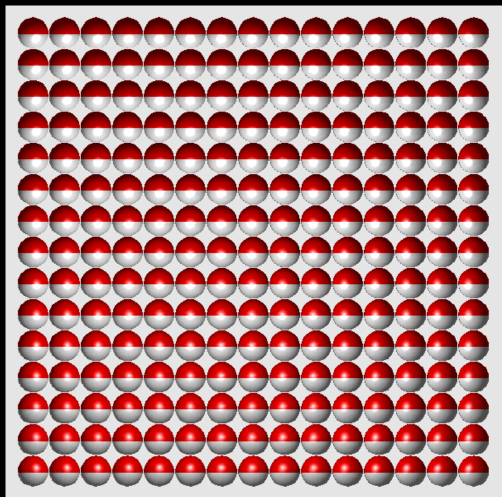
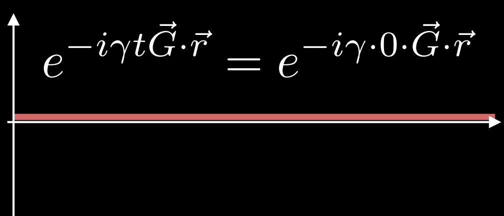
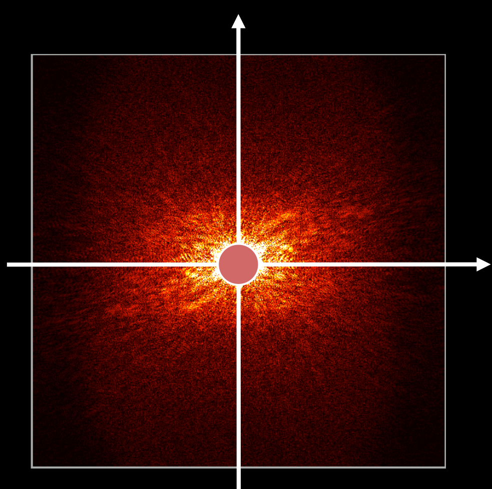
Frequency Encoding



$G_{Freq}=0$



$$e^{-i\gamma t\vec{G}\cdot\vec{r}} = e^{-i\gamma\cdot 0\cdot\vec{G}\cdot\vec{r}}$$

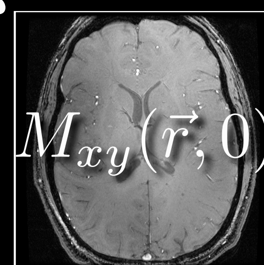


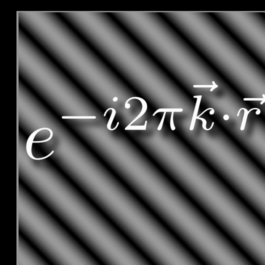
$$\vec{k}\left(t\right) = \frac{\gamma}{2\pi}\int_0^{\tau}\vec{G}\left(t\right)d\tau$$

In general…

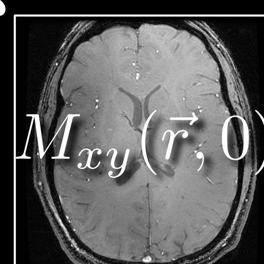
$$2\pi\vec{k}\left(t\right) = \gamma\vec{G}t$$

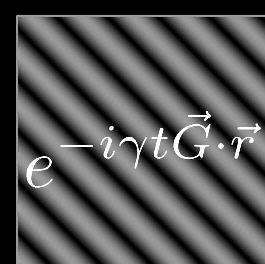
For a constant amplitude gradient…





$$S(\vec{k}) = \int \underbrace{M_{xy}(\vec{r},0)}_{object}\, e^{-i2\pi\vec{k}\cdot\vec{r}}\, d\vec{r}$$





$$\int \underbrace{M_{xy}(\vec{r},0)}_{object}\, e^{-i\gamma t\vec{G}\cdot\vec{r}}\, d\vec{r}$$

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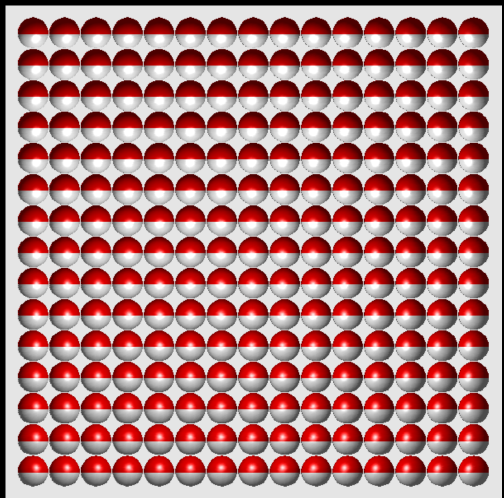
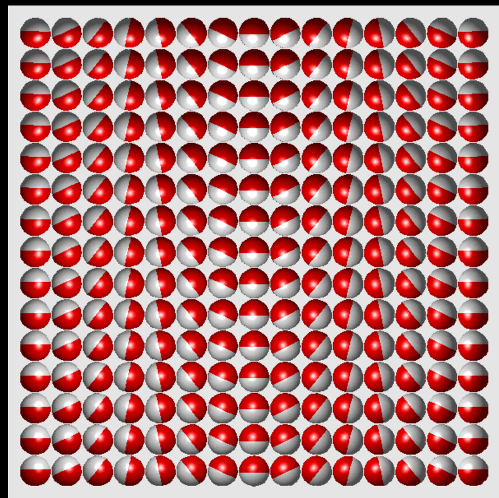
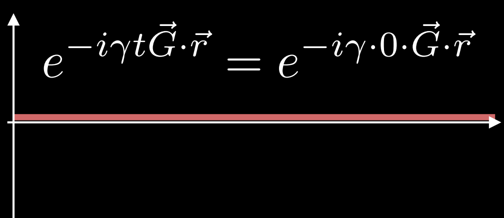
Frequency Encoding



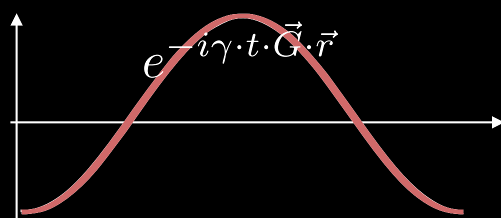
$G_{Freq}=0$



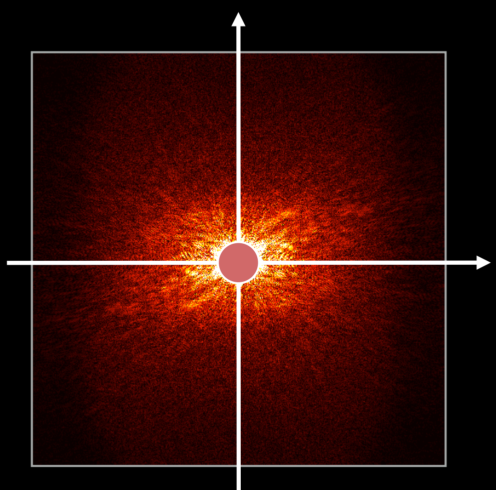
$G_{Freq}=G\cdot t$

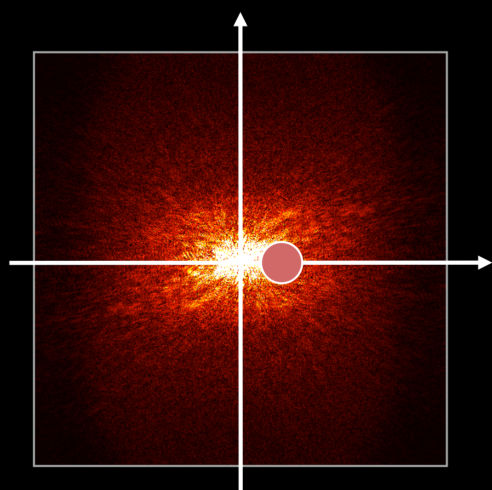


$$e^{-i\gamma t\vec{G}\cdot\vec{r}} = e^{-i\gamma\cdot 0\cdot\vec{G}\cdot\vec{r}}$$

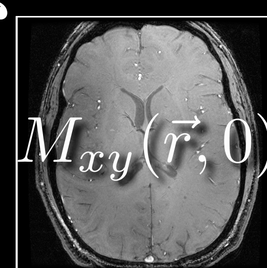


$$e^{-i\gamma\cdot t\cdot\vec{G}\cdot\vec{r}}$$

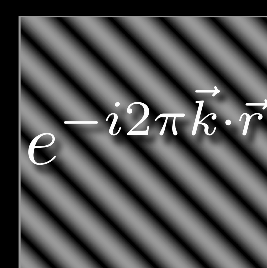




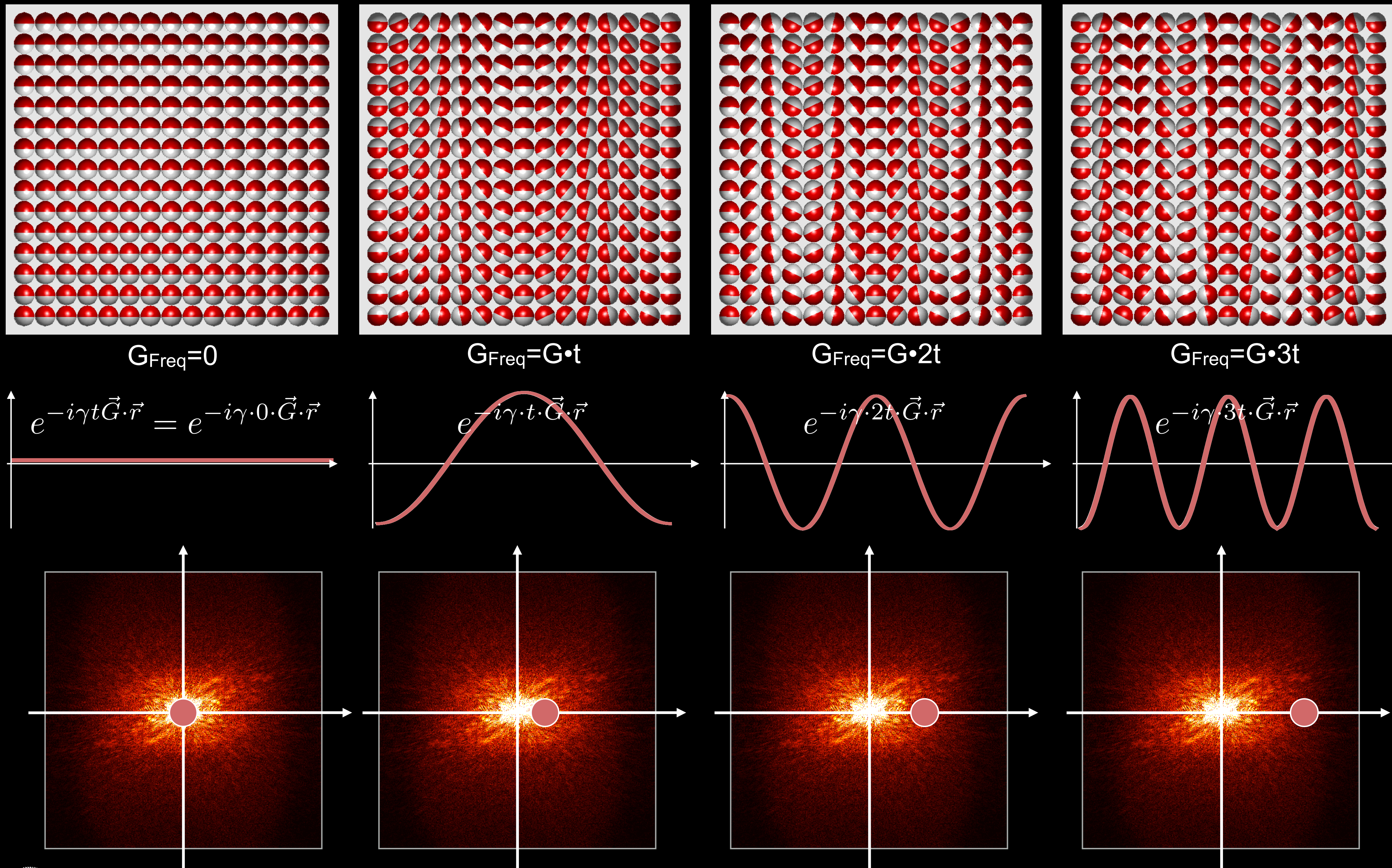
$$S(\vec{k}) = \int M_{xy}(\vec{r},0)\, e^{-i2\pi\vec{k}\cdot\vec{r}}\, d\vec{r}$$



object



Frequency Encoding



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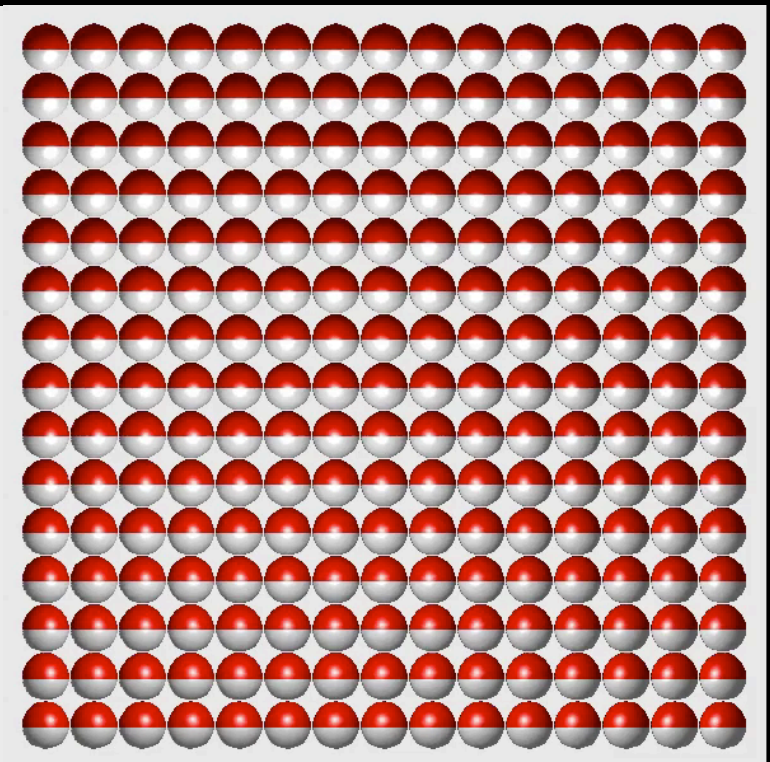
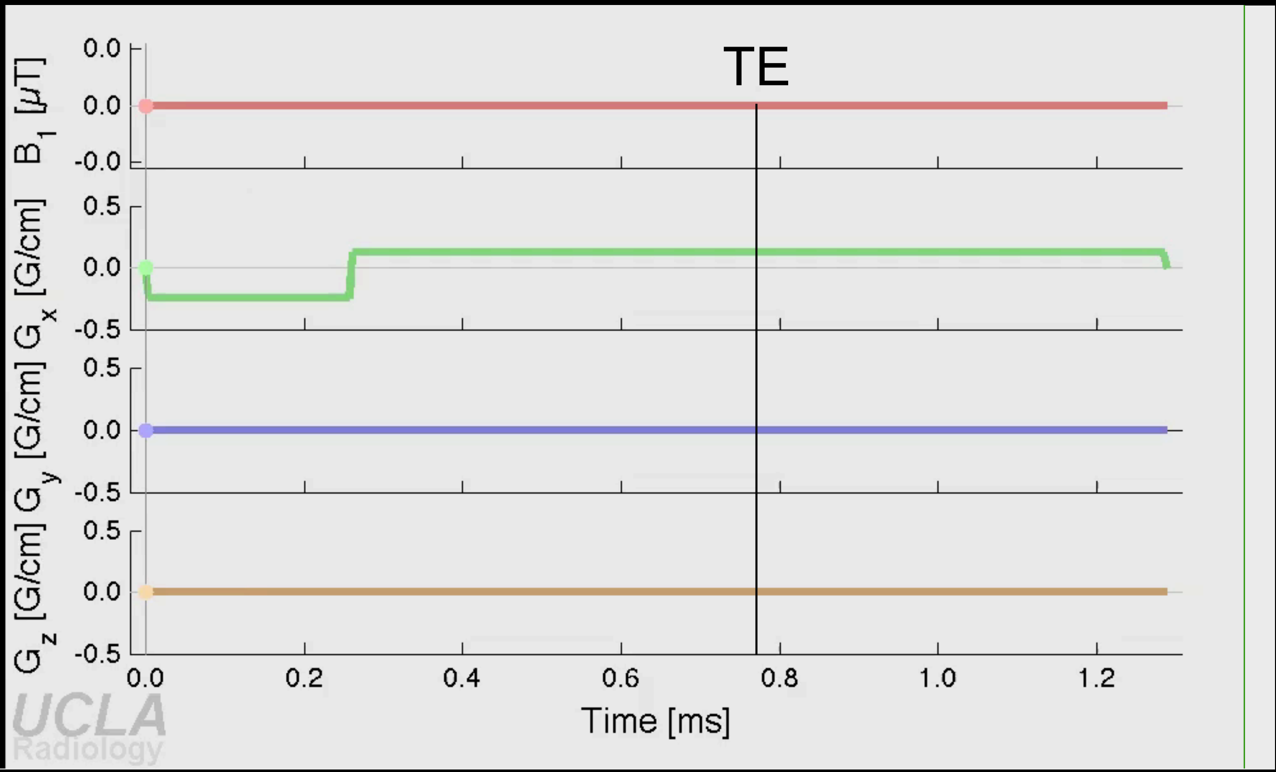
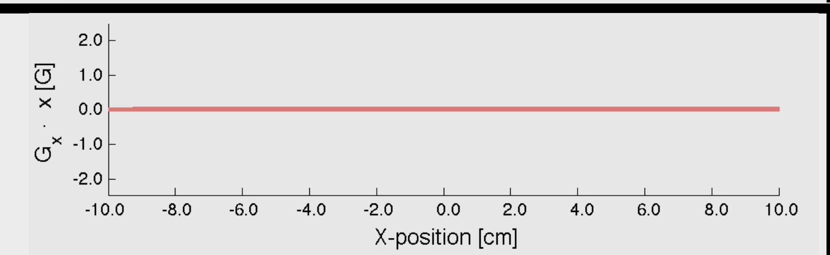
Frequency Encoding

$B_0 - G_x \bullet x$ B_0 $B_0 + G_x \bullet x$



Frequency Encoding





Applied Magnetic Field

N-Dimensional Imaging

MR Signal Equation

$$s(t) = \int\int_{x,y} \vec{M}^0_{xy}(\vec{r}) \cdot e^{-i\Delta\omega(\vec{r})t} \mathrm{d}\vec{r}$$

The MRI Signal Equation is the…

$$s(t) = \int\int_{x,y} \vec{M}^0_{xy}(x,y) \cdot e^{-i\Delta\omega(x,y)t} \mathrm{d}x\mathrm{d}y$$

…2D Fourier Transform!

$$\Delta\omega(x,y) = \gamma G_x \cdot x + \gamma G_y \cdot y$$

Gradients define Δw

$$k_x(t) = \frac{\gamma}{2\pi} G_x t \qquad k_y(t) = \frac{\gamma}{2\pi} G_y t$$

k-space is convenient…

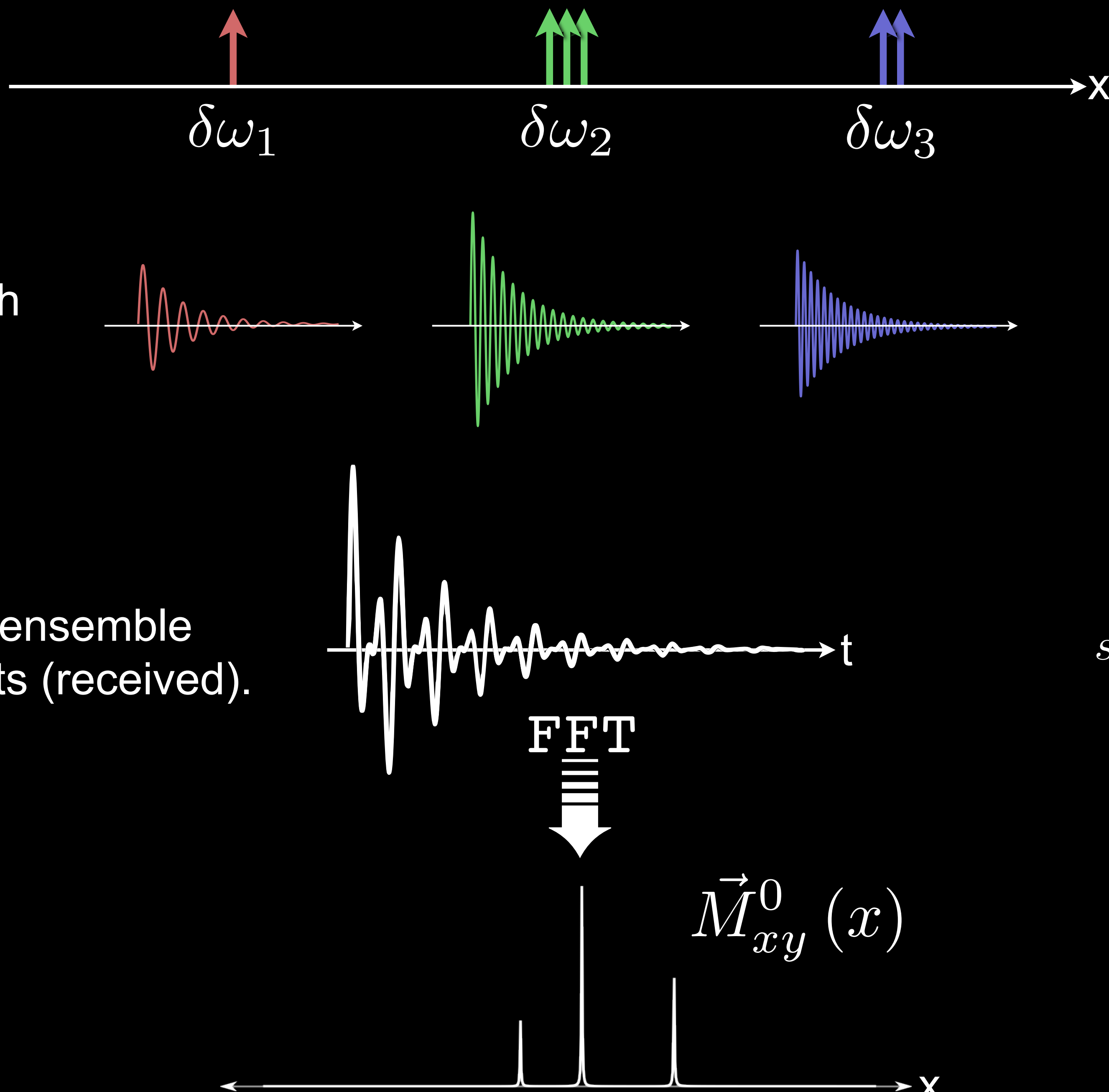
$$s(k_x(t), k_y(t)) = \int\int_{x,y} \vec{M}^0_{xy}(x,y) \cdot e^{-i2\pi[k_x(t)x + k_y(t)y]} \mathrm{d}x\mathrm{d}y$$

1D Imaging

Spins in a gradient.

S(t) from each isochromat.

S(t) from the ensemble of isochromats (received).



$s(t) = m(k_x(t))$

2D Imaging

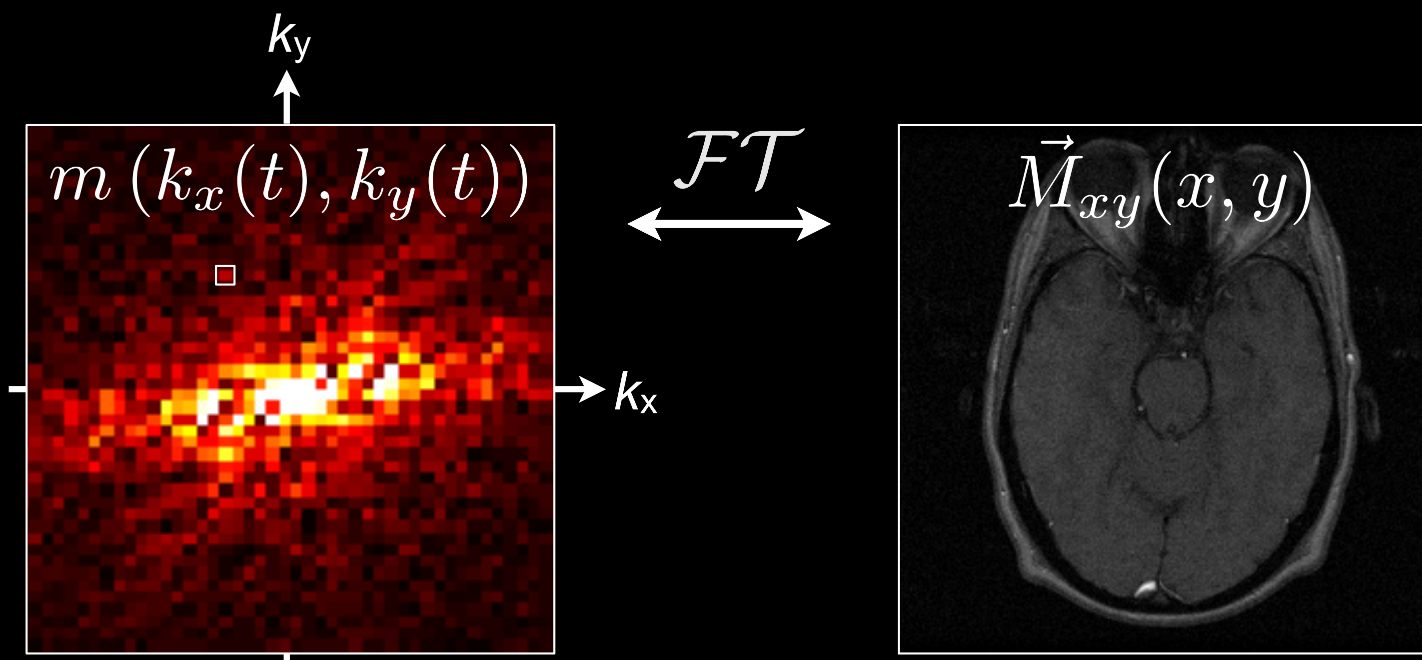
$$s\left(k_x(t), k_y(t)\right) = \int\int_{x,y} \vec{M}^{0}_{xy}\left(x, y\right) \cdot e^{-i2\pi\left[k_x(t)x + k_y(t)y\right]} \mathrm{d}x\mathrm{d}y$$

$$s(t) = m(k_x(t), k_y(t))$$

Traversing *k*-space fills the *k*-space matrix, *m*.

$$m = \mathcal{FT}(\,M(x, y)\,)$$

m is filled with the Fourier coefficients of the underlying M_{xy}.



3D Imaging

$$s(t) = \int_{object} M_{xy}(\vec{r}, 0) \cdot e^{-i\Delta\omega(\vec{r})t}\, \mathrm{d}\vec{r}$$

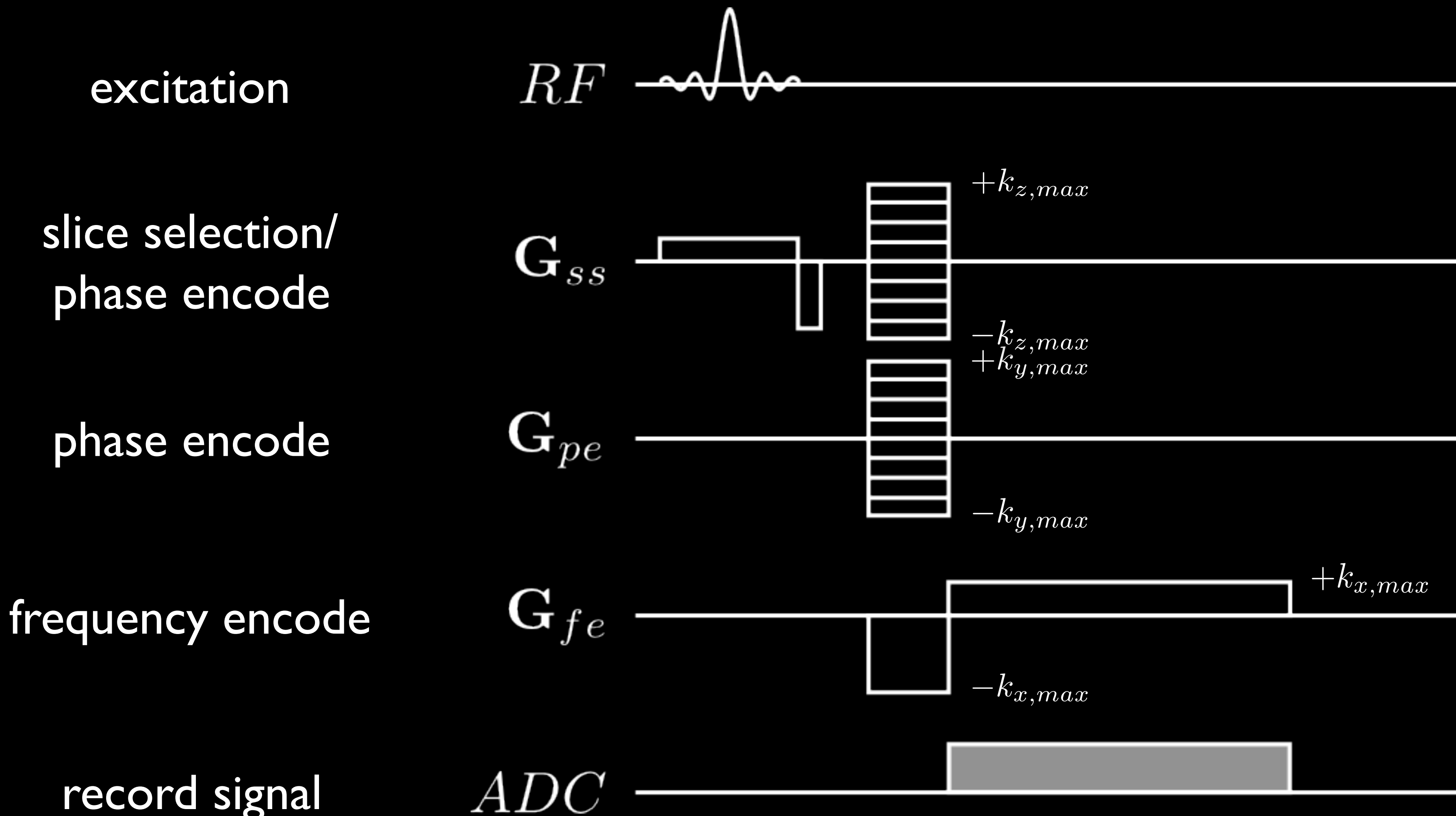
$$s(t) = \iint_{X,Y,Z} M(x, y, z) \cdot e^{-i\,\Delta\omega(x,y,z)\,t}\, \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$\Delta\omega(x, y, z) = \gamma G_x \cdot x + \gamma G_y \cdot y + \gamma G_z \cdot z$$

$$s(t) = \iint_{X,Y,Z} M(x, y, z) \cdot e^{-i2\pi[k_x(t)x + k_y(t)y + k_z(t)z]}\, \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t \qquad k_y(t) = \frac{\gamma}{2\pi} G_y t \qquad k_z(t) = \frac{\gamma}{2\pi} G_z t$$

3D Imaging


excitation
slice selection/
phase encode
phase encode
frequency encode
record signal
RF
$\mathbf{G}_{ss}$
$\mathbf{G}_{pe}$
$\mathbf{G}_{fe}$
ADC
$+k_{z,max}$
$-k_{z,max}$
$+k_{y,max}$
$-k_{y,max}$
$+k_{x,max}$
$-k_{x,max}$

Thanks

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