MRI Systems II – B_1

Lecture #3 – January 15th, 2018

Lecture 2 - Summary

$\vec{\tau} = \vec{\mu} \times \vec{B}$ $\vec{S} = \vec{r} \times \vec{p}$

$\vec{\tau} = \frac{d\vec{S}}{dt}$ $\vec{\mu} = \gamma \vec{S}$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

Equation of Motion for a Magnetic Dipole

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of Motion for the bulk magnetization.

$$\vec{B}_0 = B_0 \vec{k}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \left(\vec{B}_0 \right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$
$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$
$$M_z(t) = M_z^0$$

How do we perturb the equilibrium?

Lecture 2 - Summary

- Free Precession in the Laboratory Frame
- **Forced Precession in the Laboratory Frame**
  - Coordinate system anchored to scanner
- **Free Precession in the Rotating Frame**
- **Forced Precession in the Rotating Frame**
  - Coordinate system anchored to spin system
- **...all without relaxation.**
  - a) Relaxation time constants are “really” long
  - b) Time scale of event is << relaxation time constant

Dipoles to Images

$\vec{\mu}$	Magnetic Moment	
$\downarrow$		B_0
$\vec{M}$	Bulk Magnetization	
$\downarrow$		B_1
$\vec{M}_{xy}(t)$	Transverse Magnetization	
$\downarrow$		$G_{Encoding}$
$\vec{M}_{xy}(\vec{r}, t)$	Spatially Encoded Magnetization	
$\downarrow$		Coil
$V(t)$	Received Voltage	
$\downarrow$		PSD
$S(t)$	Complex Signal	
$\downarrow$		$G_{Decoding}$
$S\left(\vec{k}\right)$	k-space signal	
$\downarrow$		Reconstruction
$I(\vec{r})$	Image	

MRI Systems II – B_1

Lecture #3 – January 15th, 2018

Lecture #3 Learning Objectives

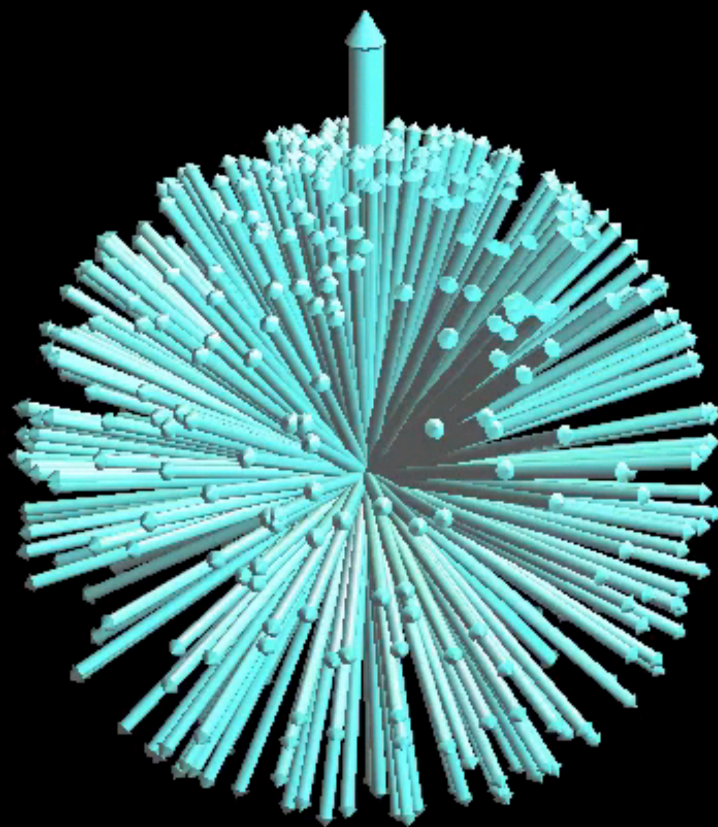
- Distinguish spin, precession, and nutation.
- Appreciate that any B-field acts on the the spin system.
- Understand the advantage of a circularly polarized RF B-field.
- Differentiate the lab and rotating frames.
- Define the equation of motion in the lab and rotating frames.
- Know how to compute the flip angle from the B1-envelope function.
- Understand how to apply the RF hard pulse matrix operator.

B_1 Field - RF Pulse

- B_1 is a
  - radiofrequency (RF)
    - 42.58MHz/T (63MHz at 1.5T)
  - short duration pulse (~0.1 to 5ms)
  - small amplitude
    - <30 μT
  - circularly polarized
    - rotates at Larmor frequency
  - magnetic field
  - perpendicular to B_0

Resonance

Ensemble of Precessing Spins



"The equilibrium magnetization is stationary, so even though the individual spins are precessing, there is no net emission of radio waves in equilibrium."

David Geffen School of Medicine

Hanson, L. G. (2008). "Is Quantum Mechanics Necessary for Understanding Magnetic Resonance?" Concepts in Magnetic Resonance Part A 32(A): 329-340.

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Resonance

- Quantum Physics
  - Electromagnetic radiation of frequency ω_{RF} carries energy that induces a coherent transition of spins from $N_\uparrow$ to $N_\downarrow$.
- Classical Physics
  - $\vec{B}_1(t)$rotates in the same manner as the precessing spins.
  - Coherently “pushes” on bulk magnetization.

Resonance Condition (Quantum)

$$\Delta E = E_{\downarrow} - E_{\uparrow} = \hbar \gamma B_0$$

Zeeman Splitting

$$E_{RF} = \hbar \omega_{RF}$$

Planck's Law

$$\hbar \gamma B_0 = \hbar \omega_{RF}$$

$$\omega_{RF} = \gamma B_0 = \omega_0$$

Resonance Condition

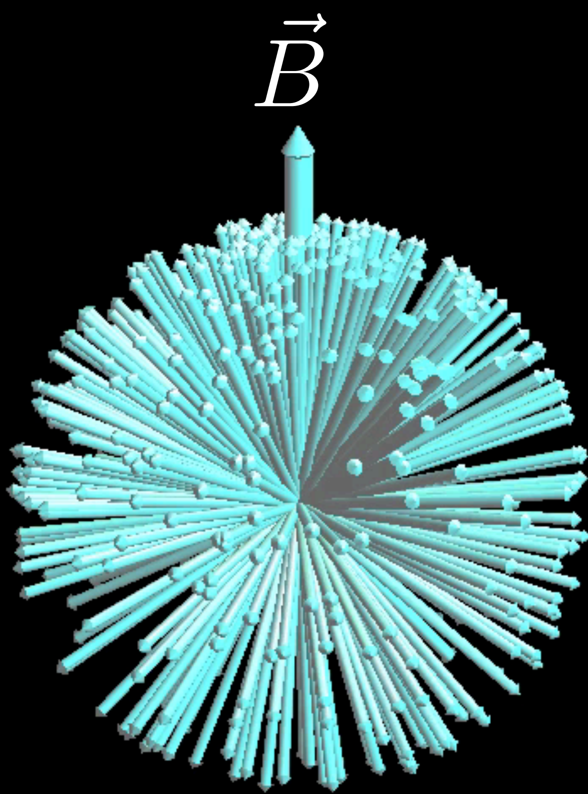
Resonance requires that the frequency of the RF energy (ω_{RF}) match the frequency of precession (ω_0).

Resonance Condition (Classical)

"Establishment of a phase coherence among these 'randomly' precessing spins in a magnetized spin system is referred to as *resonance*."

– Liang & Lauterbur p.69

$$N_\uparrow \approx N_{total} \times (1 + 2.25 \times 10^{-6})$$



$$N_\downarrow \approx N_{total} \times (1 - 2.25 \times 10^{-6})$$

RF Birdcage Coil

MRI Hardware

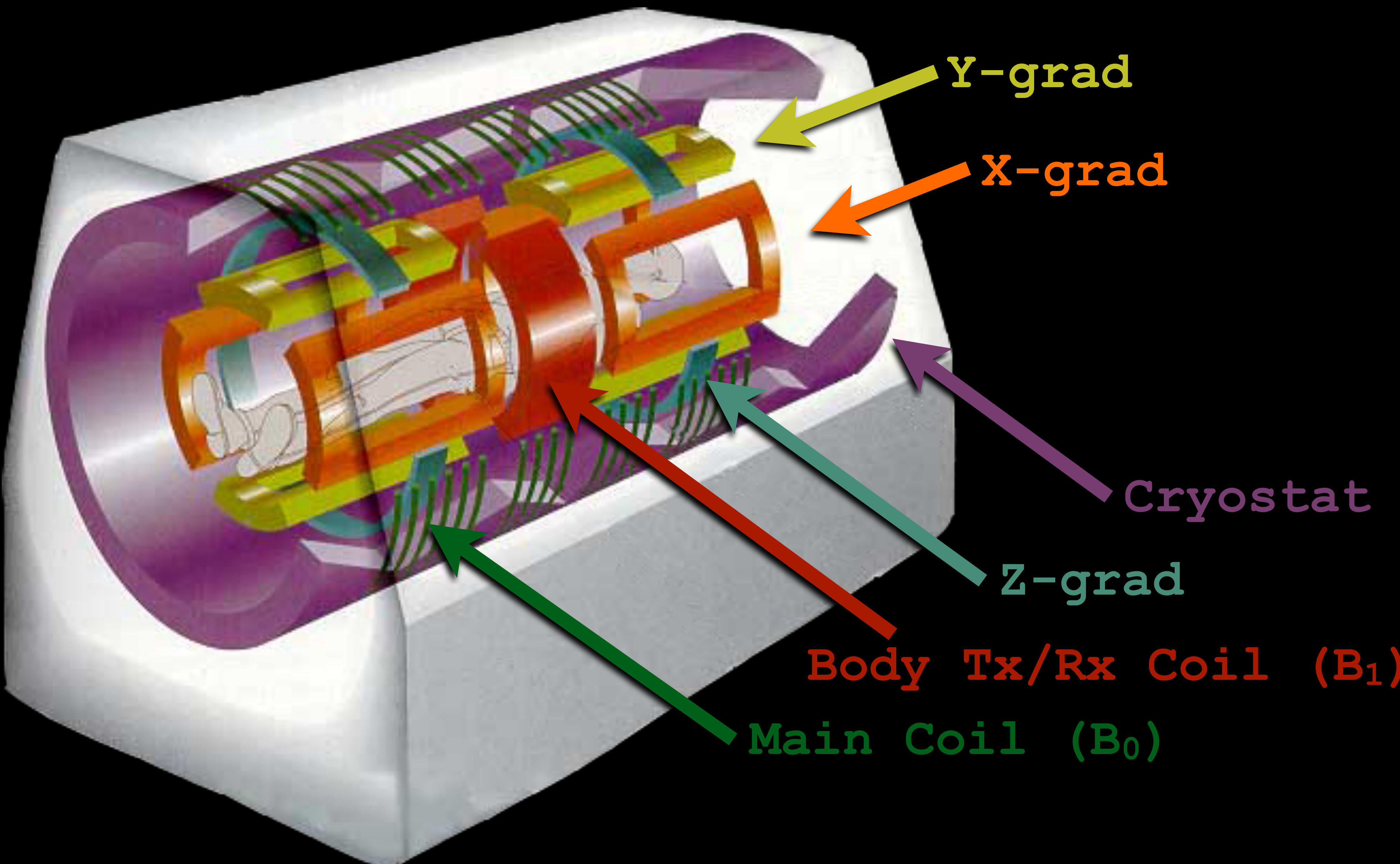
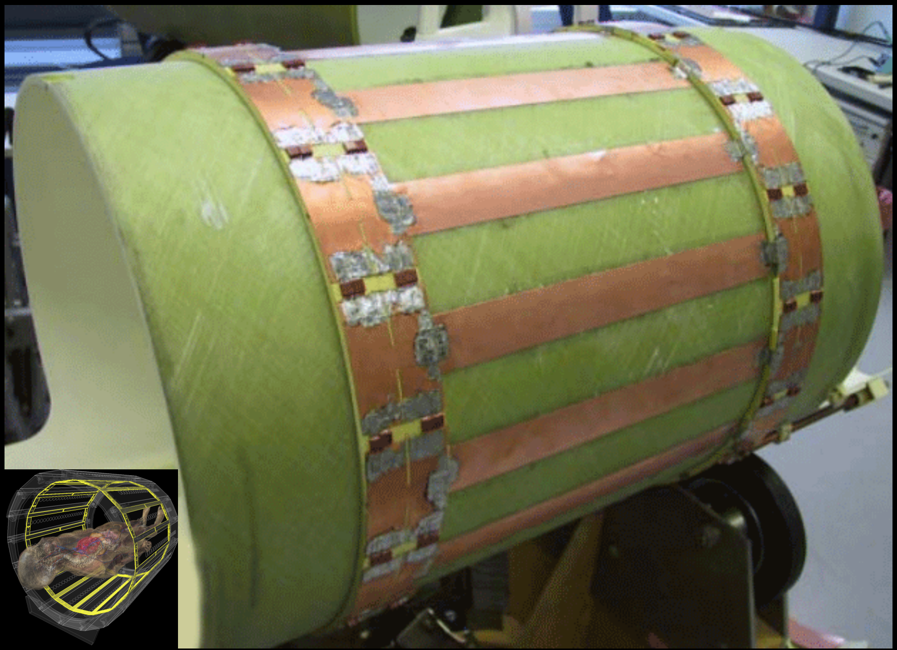


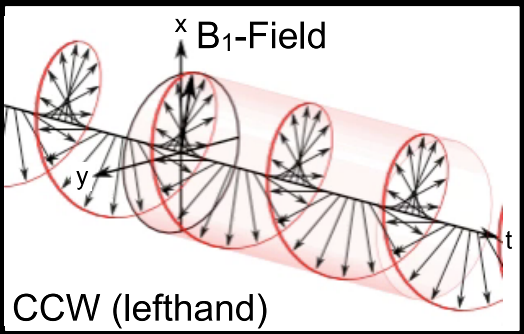
Image Adapted From: http://www.ee.duke.edu/~jshorey

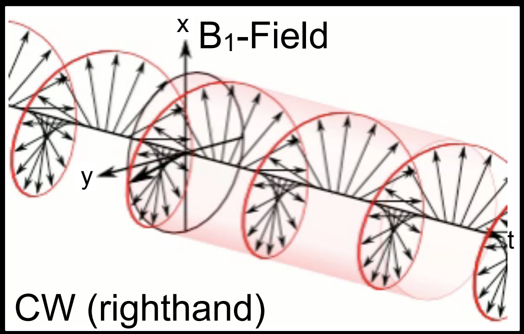
RF Birdcage Coil

- **Most common design**
- **Highly efficient**
  - Nearly all of the fields produced contribute to imaging
- **Very uniform field**
  - Especially radially
  - Decays axially
  - Uniform sphere if L≈D
- **Generates a “quadrature” field**
  - Circular polarization

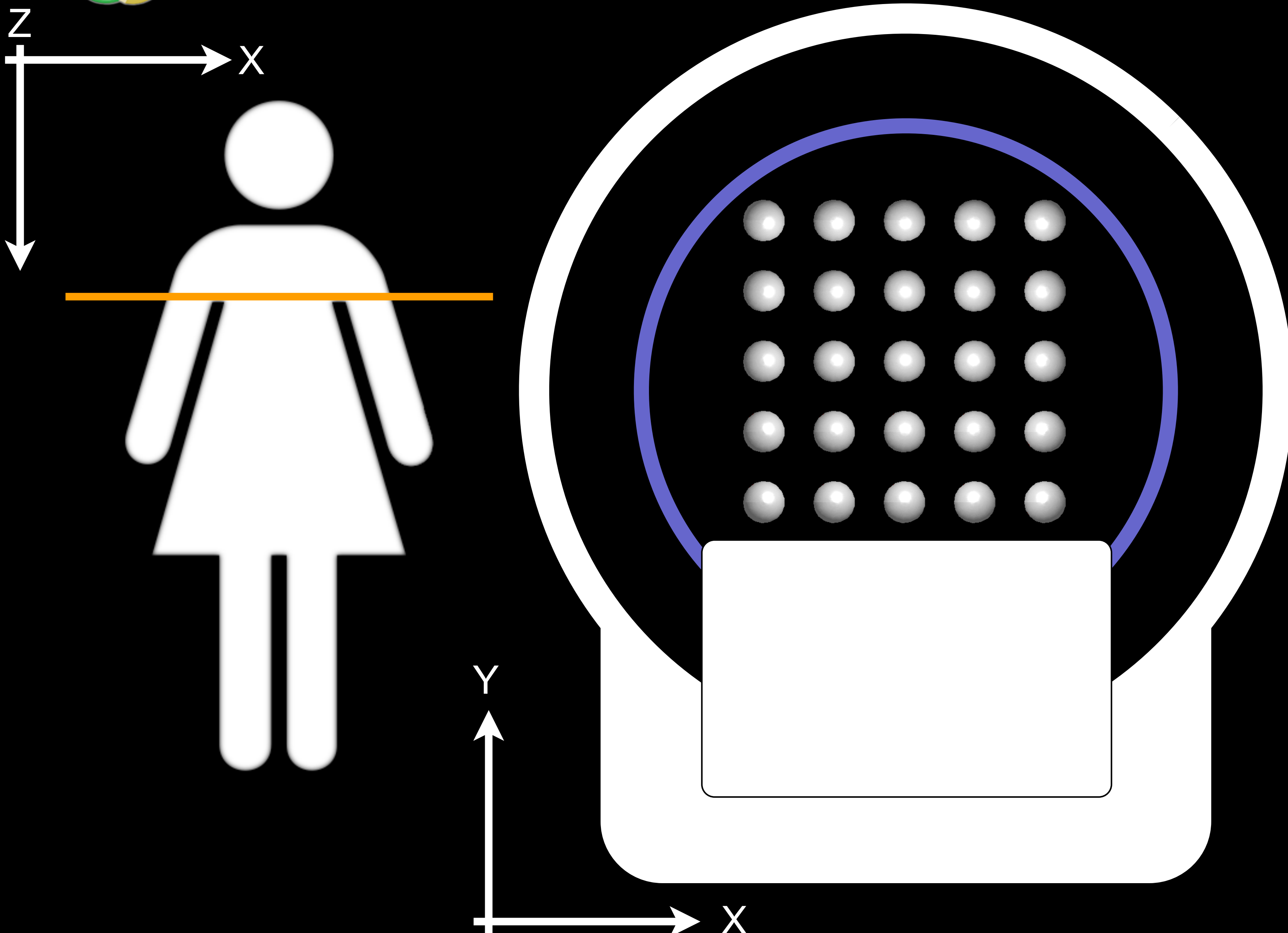
Body Tx/Rx Coil (B_1)







RF Excitation - Lab Frame



RF pulses can generate transverse magnetization (M_{xy}).

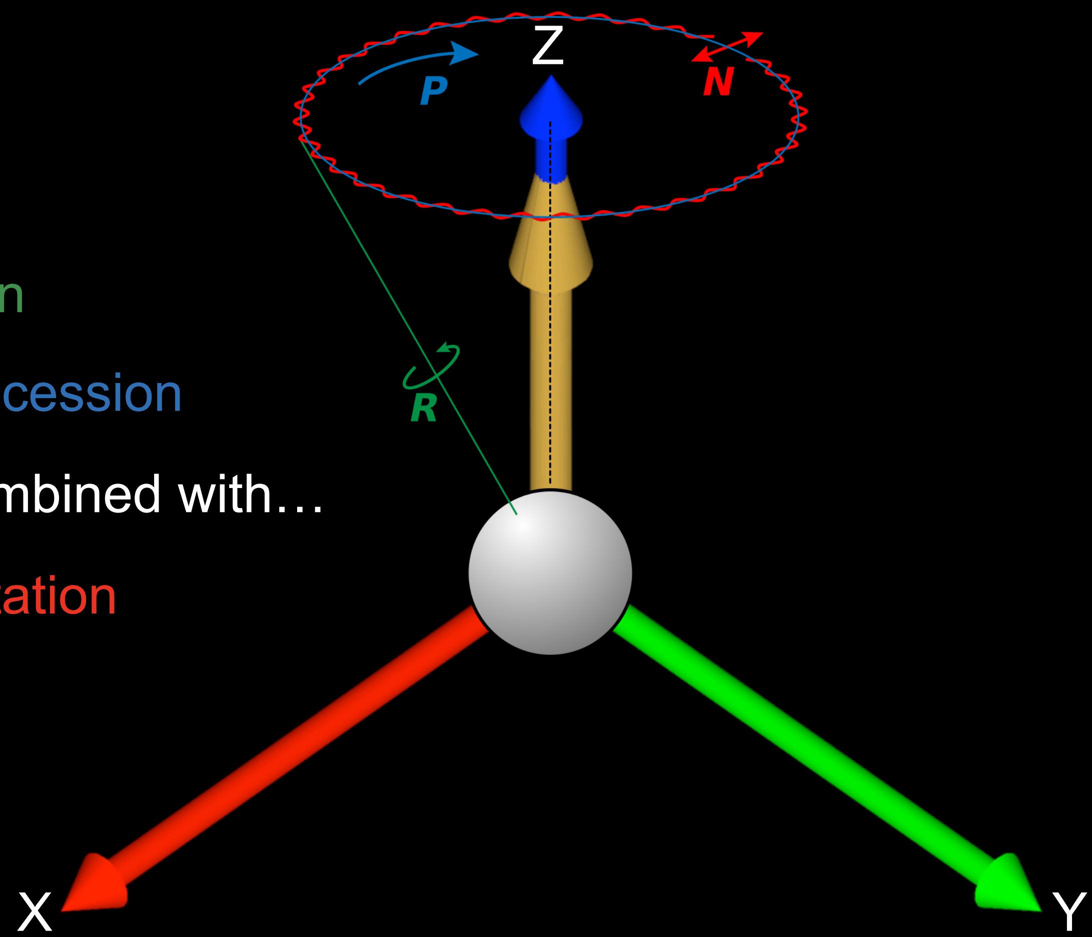
RF Excitation - Lab Frame

^{1}H has intrinsic Spin

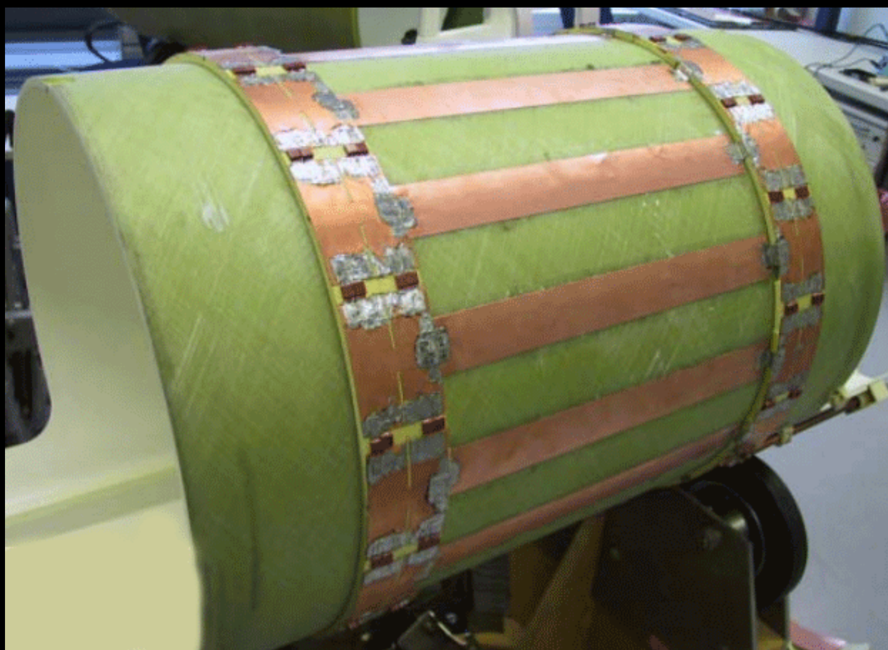
$\omega_0 = \gamma B_0$ Precession

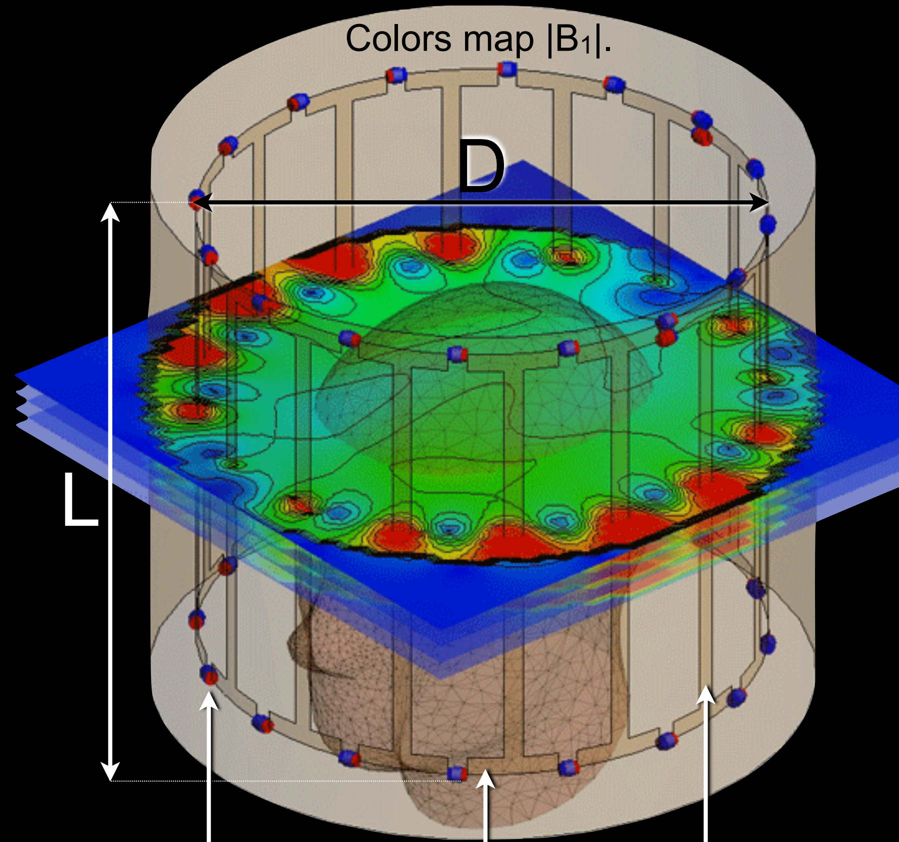
Combined with…

$\omega_1 = \gamma B_1$ Nutation



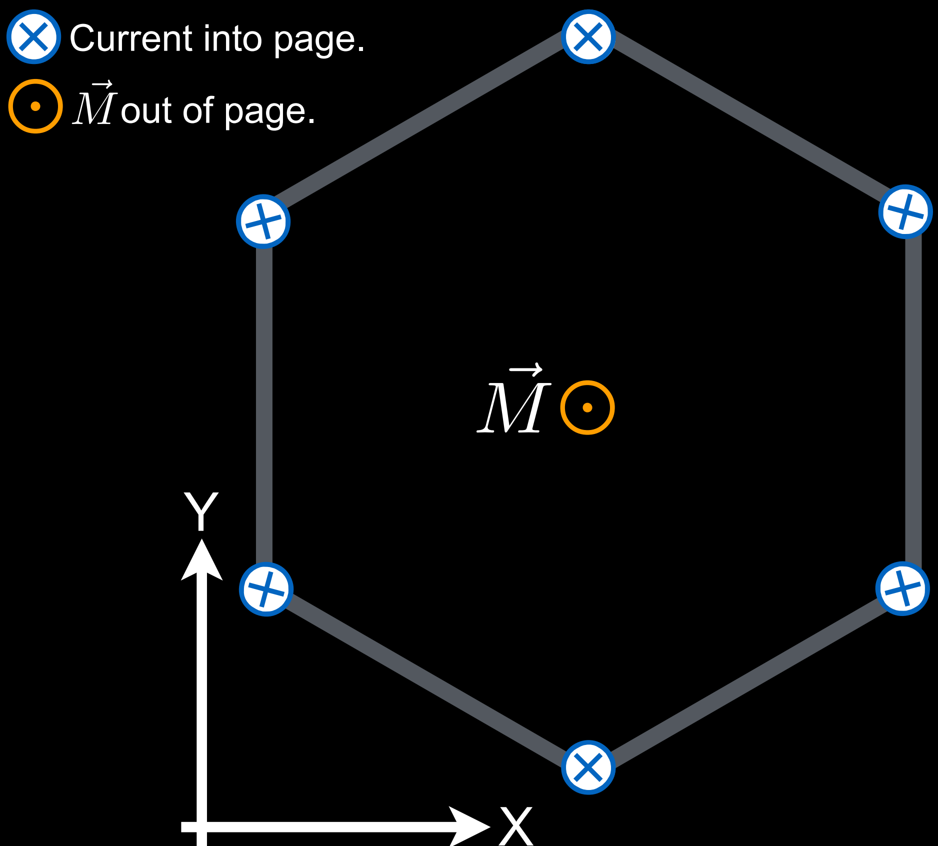
RF Birdcage Coil

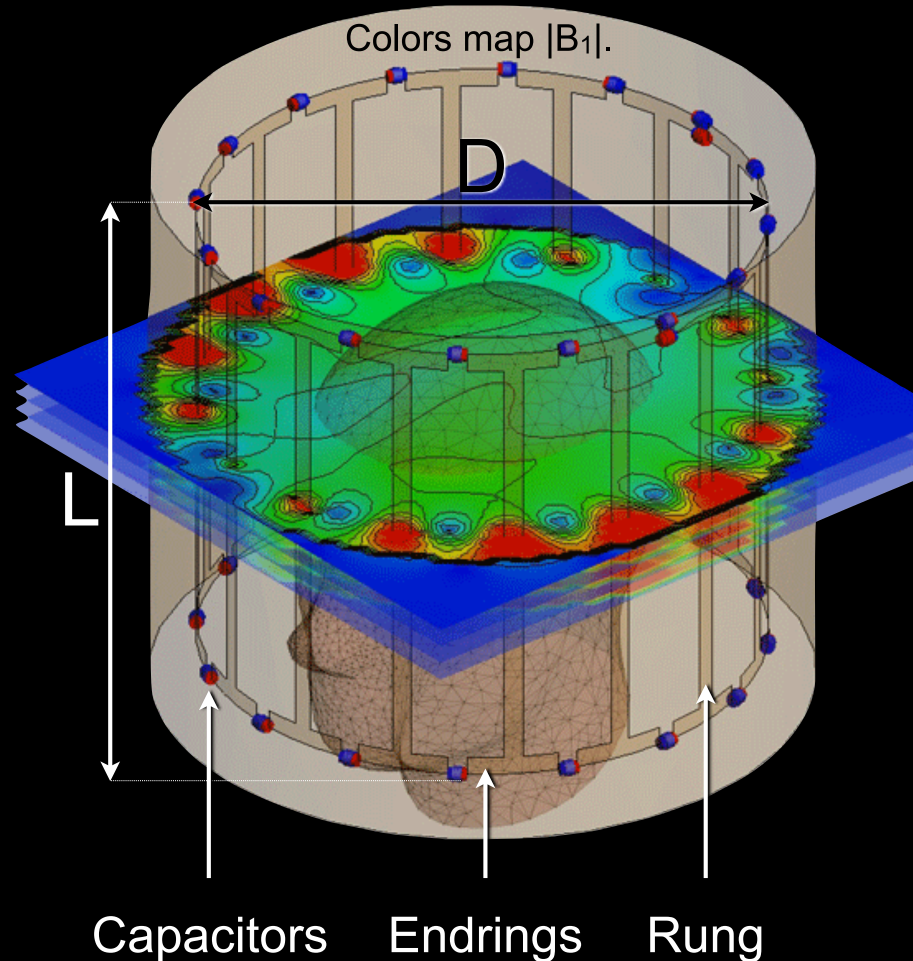




Birdcage coils are used to generate **low SAR** [W/kg] **circularly polarized** RF B_1-fields.

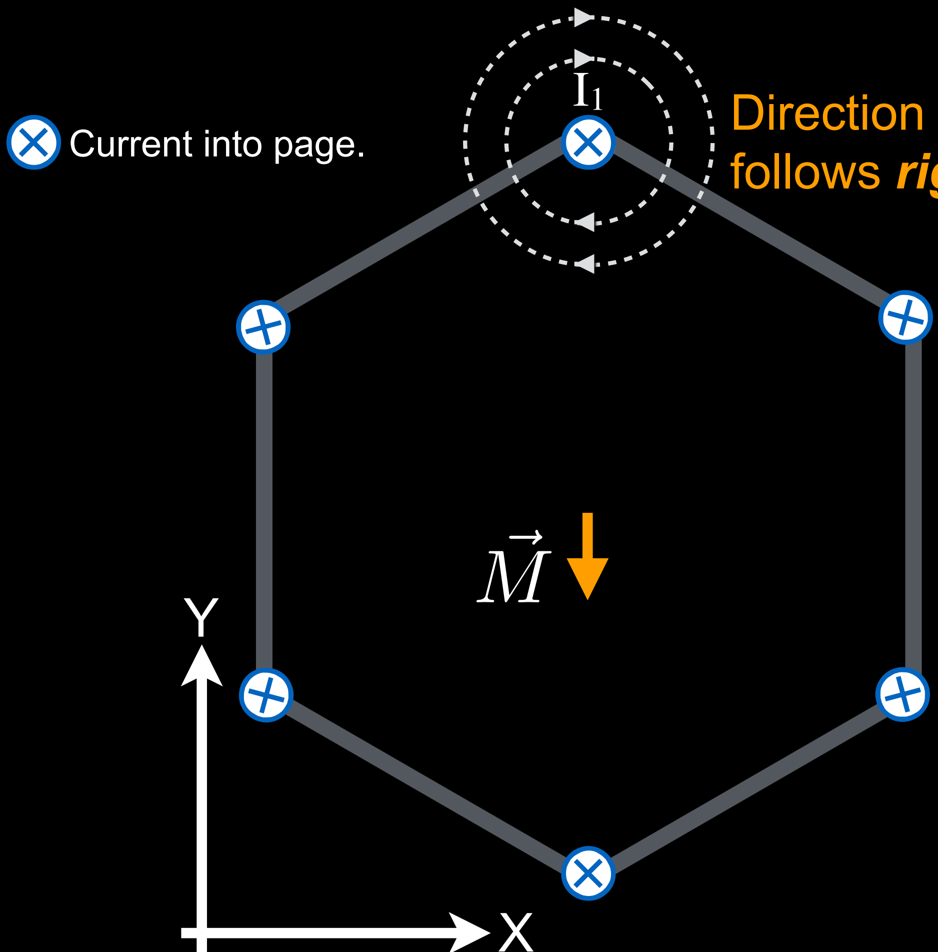
RF Birdcage Coil



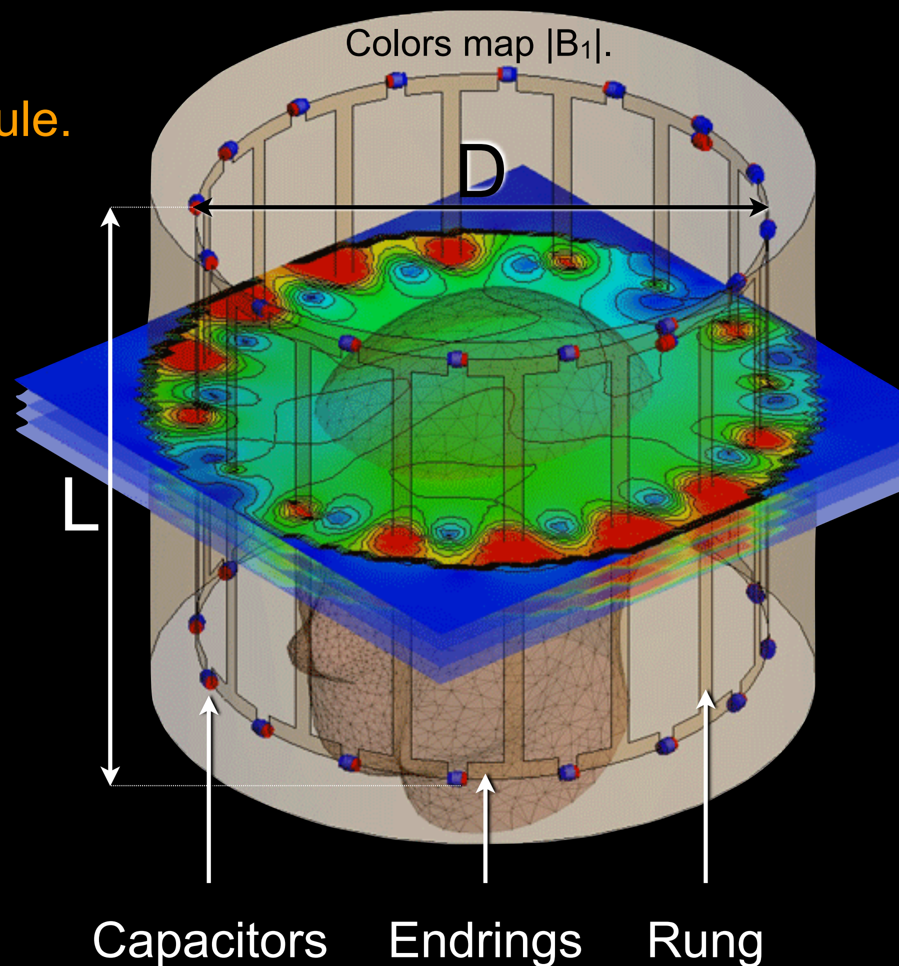


In the absence of any applied RF the bulk magnetization is oriented along the z-axis.

RF Birdcage Coil

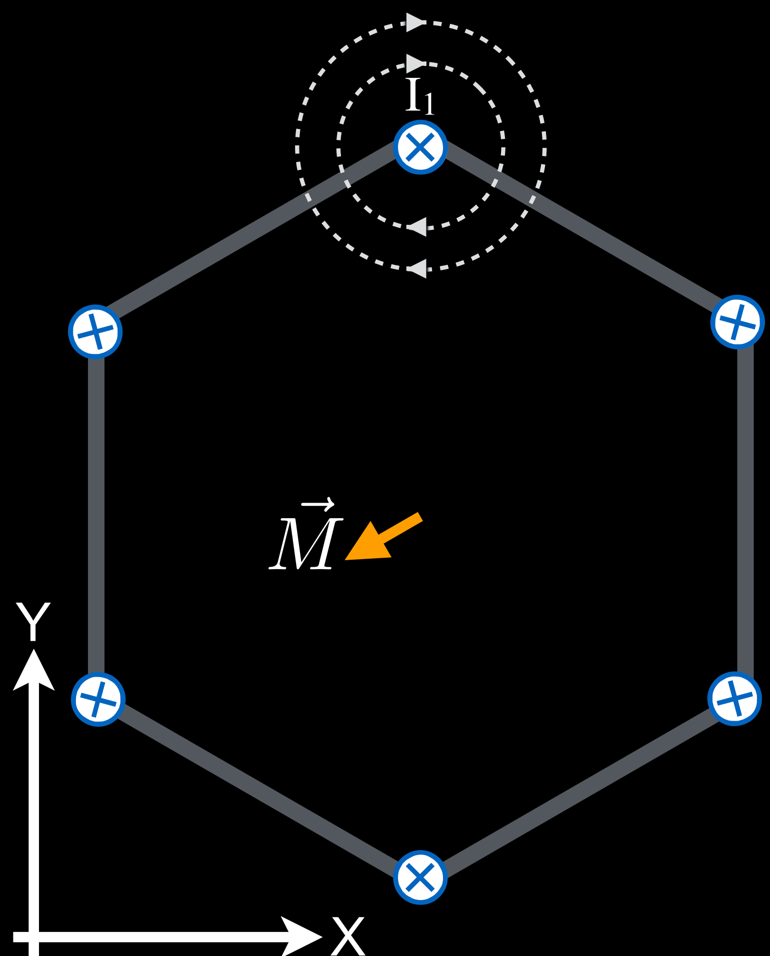


Direction of B_1-field follows ***right hand*** rule.

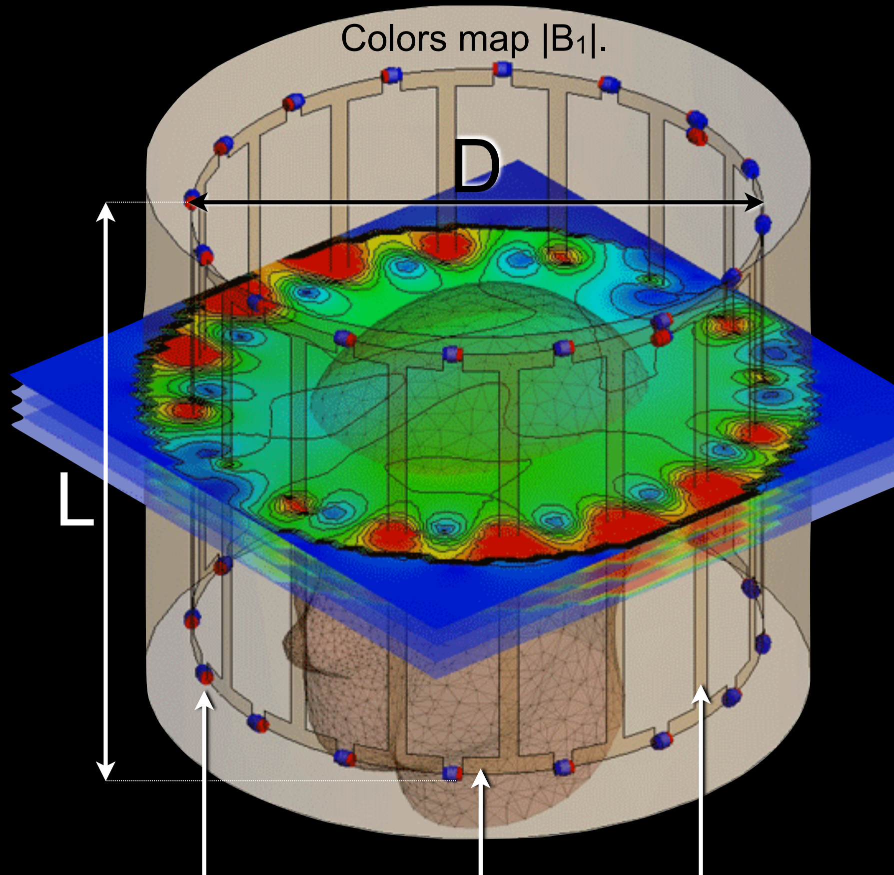


A current (I_1) induces a ***left-handed*** nutation about the B_1-field.

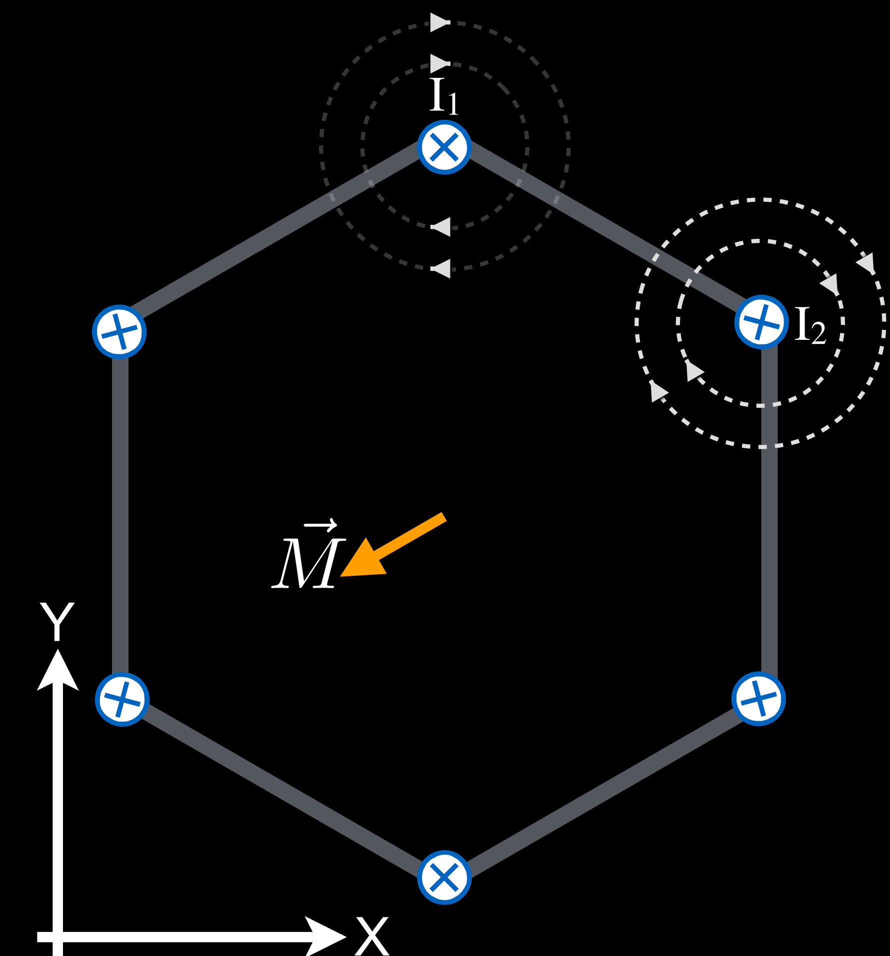
RF Birdcage Coil



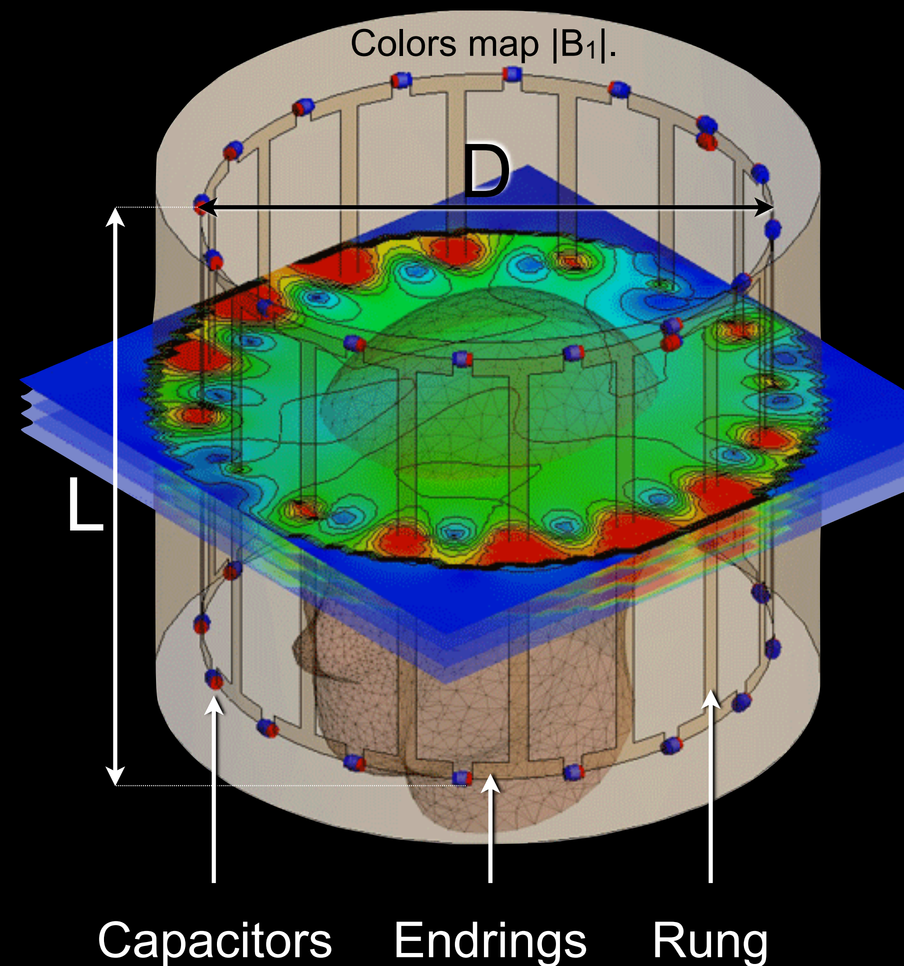
Precession from B_0 advances the spin clockwise (***left hand rule***).



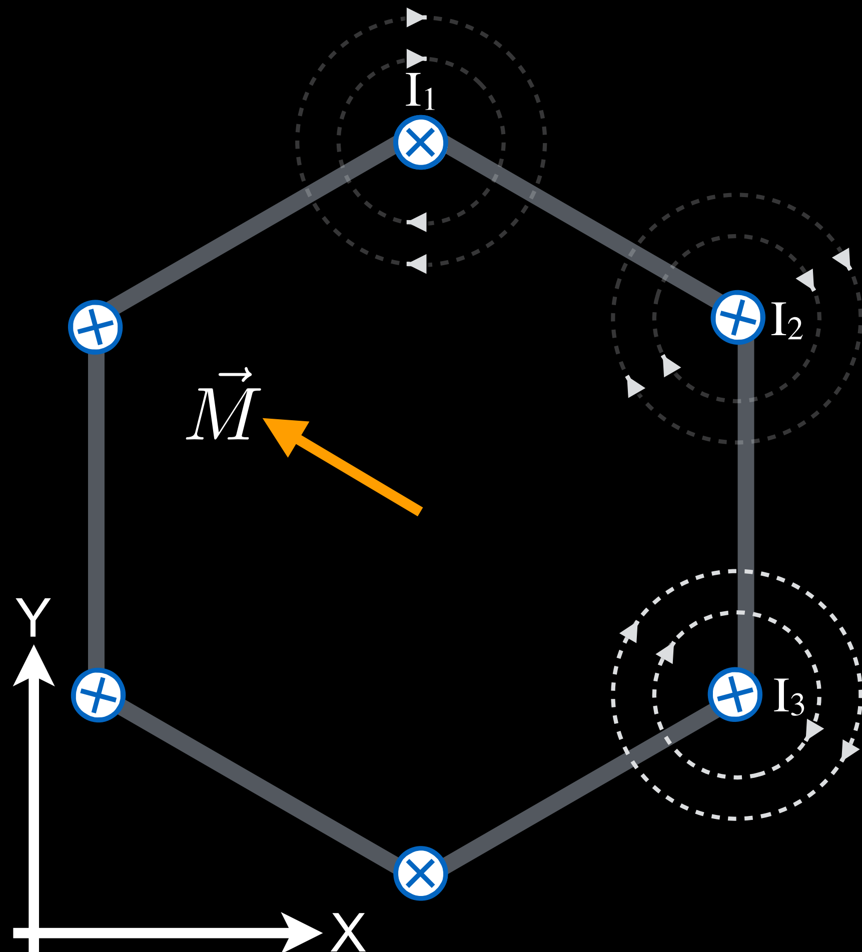
RF Birdcage Coil

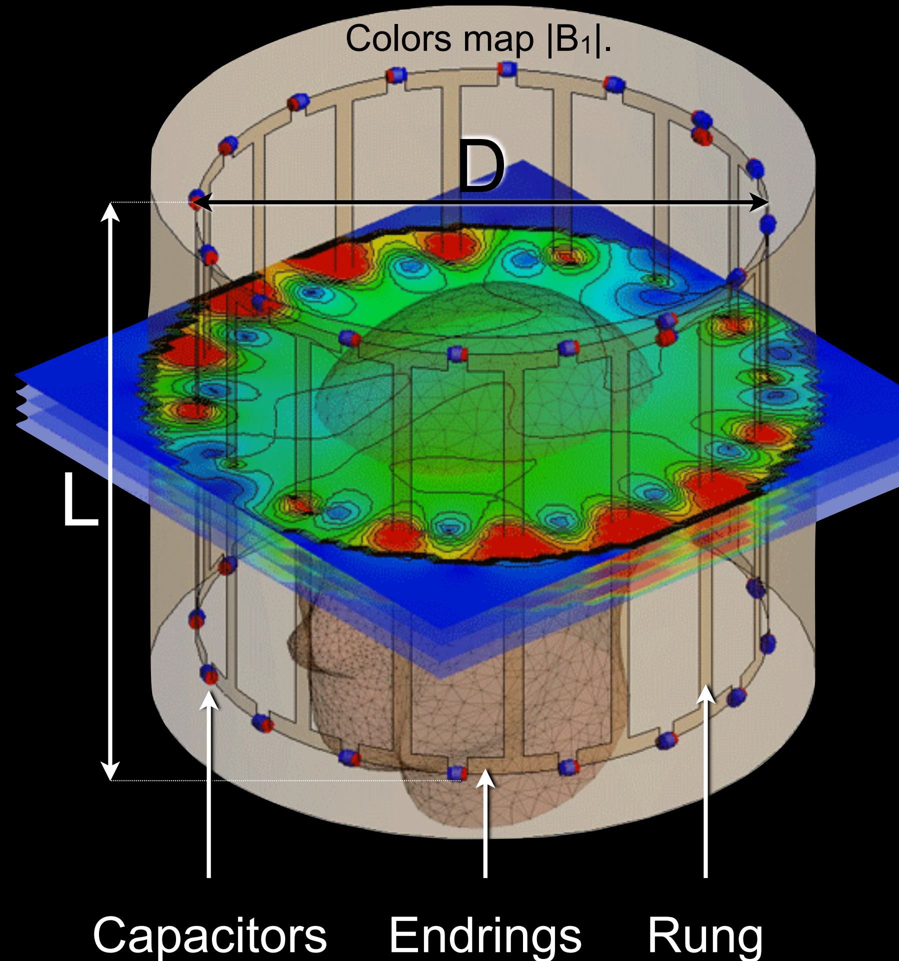


B_1 nutation from I_2 generates more M_{xy}.



RF Birdcage Coil

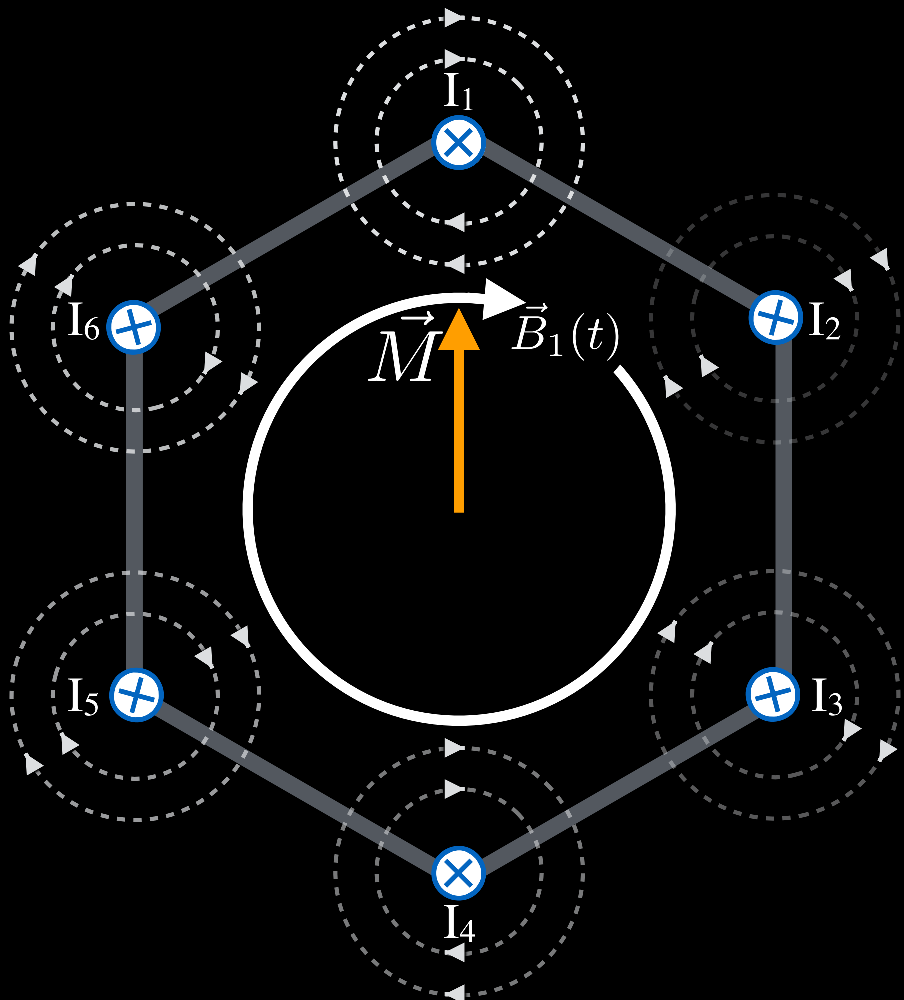


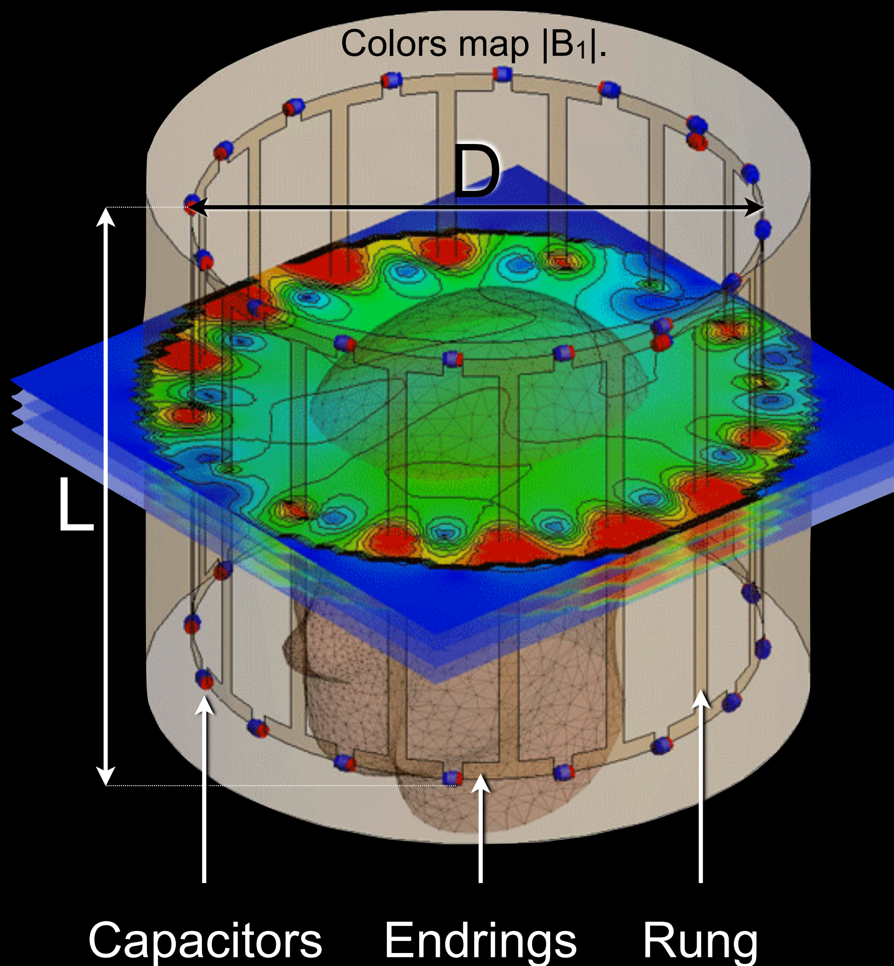


$$I_n(t) = I_0 \sin\left(\omega_{RF} t - \frac{2\pi(n-1)}{N_{Rungs}}\right)$$

Current in the n^{th} rung.
Creates a CW B_1-field.

RF Birdcage Coil





$$I_n(t) = I_0 \sin\left(\omega_{RF} t - \frac{2\pi(n-1)}{N_{Rungs}}\right)$$

Current in the n^{th} rung.
Creates a CW B_1-field.

Consider reading Chp. 16.3 in Haacke.

B_1 Inhomogeneity

B_1 Inhomogeneity: Imperfect B_1 amplitude as a function of spatial position.

Sources:

- Hardware imperfections.
- Conductivity & permittivity of subject/object [1].
- Wavelength effects.

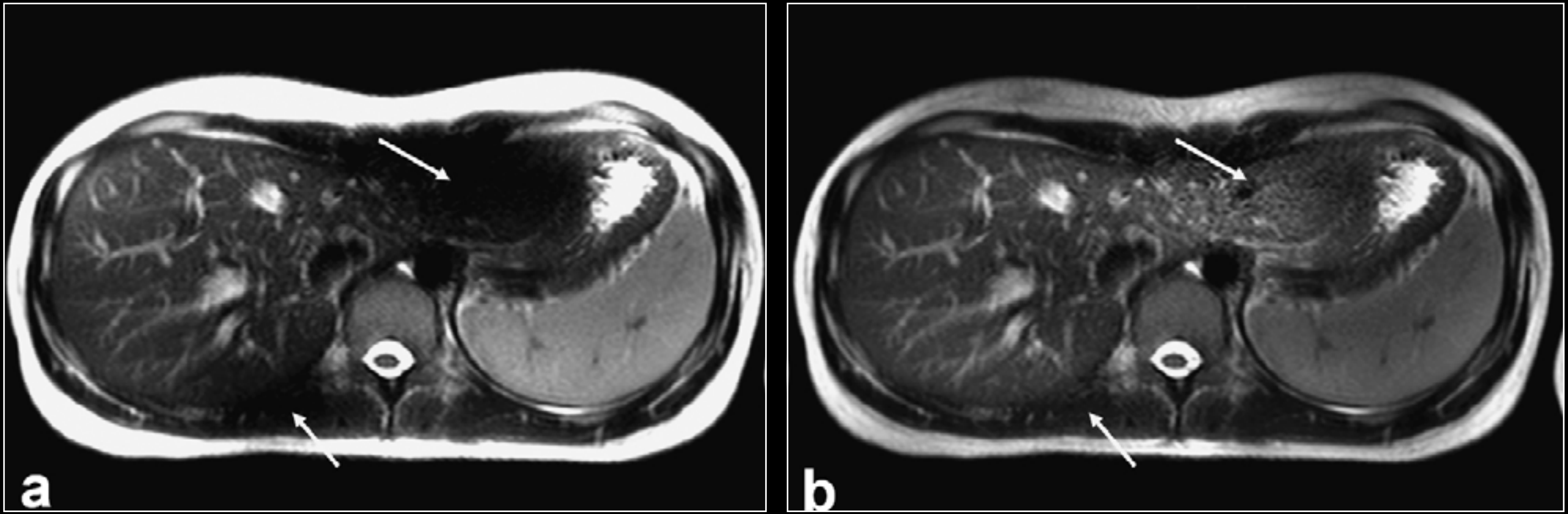


Fig. 5. Signal loss due to ***inhomogeneous flip-angle distribution*** at 3T. (a) Wavelength effects result in reduced signal intensity in the abdomen (arrows). (b) This effect can in some cases be reduced by manually increasing the RF-transmitter amplitude (here by 50%) and by applying image post-processing filters to obtain more uniform image intensities. Images courtesy of W. Horger, Siemens Medical Solutions, Germany [2]

[1] Franklin KM *et al*. Improvement in B1-inhomogeneity artifacts in the abdomen at 3T MR imaging using a RF cushion. JMRI. 2008;27(6):1443-7.
[2] Artifacts in 3-T MRI: Physical background and reduction strategies, Dietrich O. *et al*. *European J. of Radiology*, v65, Issue 1, Jan 2008, p29–35

SAR, Polarization, and B_1 Safety

SAR Limitations

- **Specific Absorption Rate**
  - Measure of the rate of energy absorption during exposure to a RF electromagnetic field
  - Measured in units of [**W/kg**]
- High-field (>1.5T) imaging with high flip angles (>45-90°) can be challenging.

$$\mathrm{SAR} \propto \omega_0^2 B_1^2 \propto B_0^2 \alpha^2$$

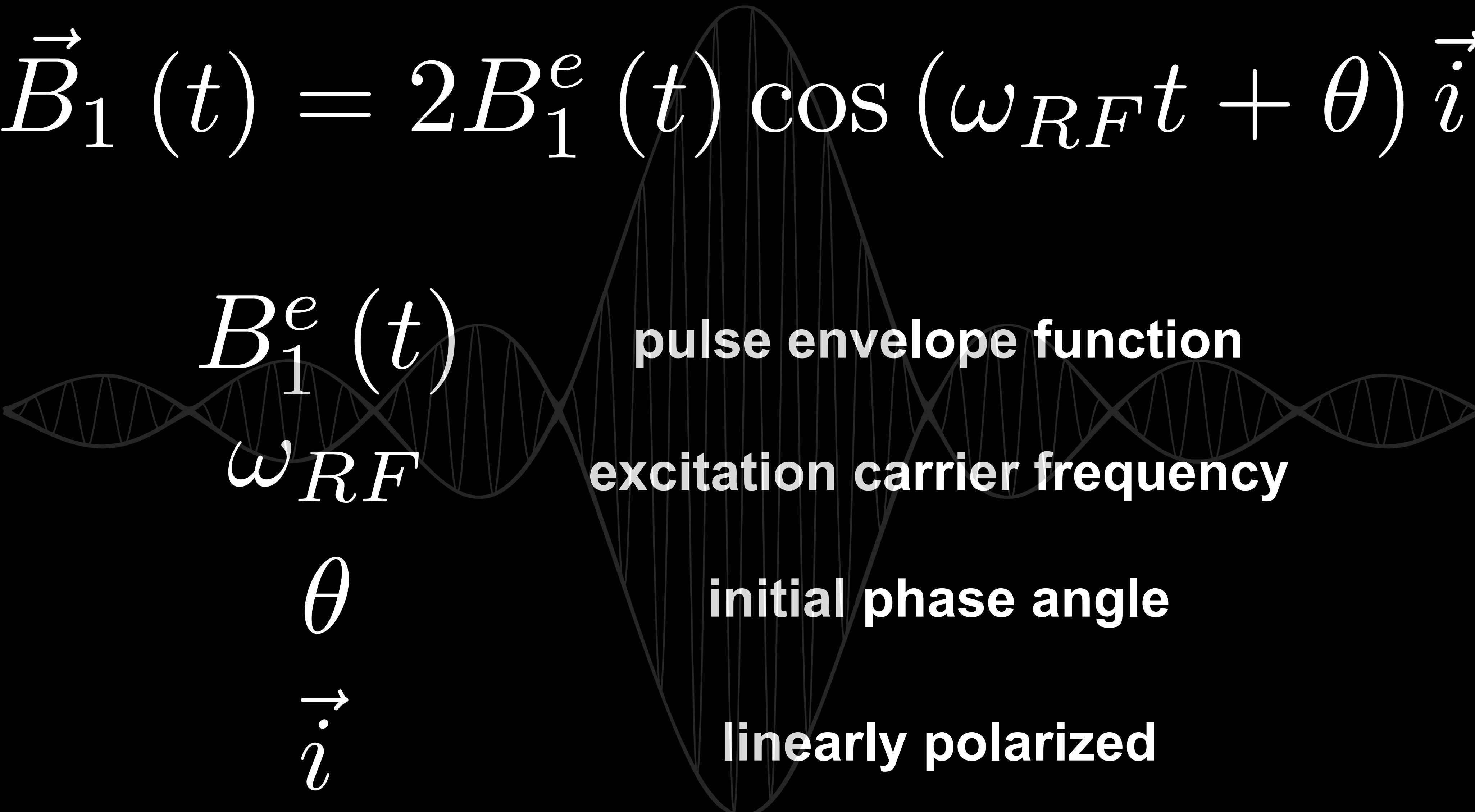
SAR Limits

Limit	Whole-Body Average	Head Average	Head, Trunk Local SAR	Extremities Local
IEC (6-minute average)				
Normal (all patients)	2 W/kg (0.5°C)	3.2 W/kg	10 W/kg	20 W/kg
First level (supervised)	4 W/kg (1°C)	3.2 W/kg	10 W/kg	20 W/kg
Second level (IRB approval)	4 W/kg (>1°C)	>3.2 W/kg	>10 W/kg	>20 W/kg
Localized heating limit	39°C in 10 g	38°C in 10 g		40°C in 10 g
FDA	4 W/kg for 15 min	3 W/kg for 10 min	8 W/kg in 1g for 10 min	12 W/kg in 1g for 5 min

Bottomley PA. Turning up the heat on MRI. J Am Coll Radiol 2008;5(7):853-855.

Basic RF Pulse - Linear Polarized

$$\vec{B}_1(t) = 2B_1^e(t)\cos(\omega_{RF}t + \theta)\,\vec{i}$$

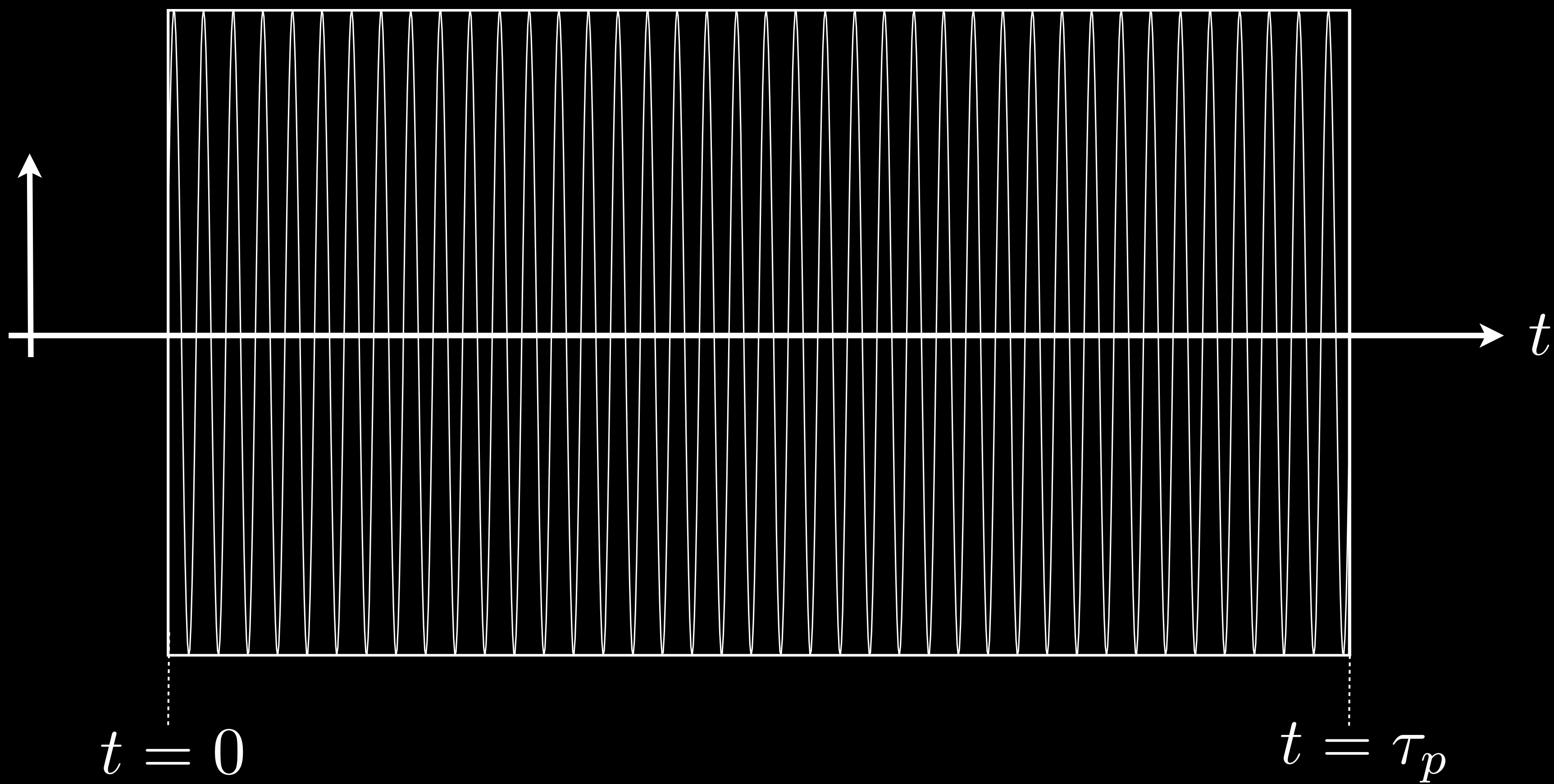


$B_1^e(t)$	**pulse envelope function**
ω_{RF}	**excitation carrier frequency**
θ	**initial phase angle**
$\vec{i}$	**linearly polarized**

B_1 is perpendicular to B_0.

Rect Envelope Function

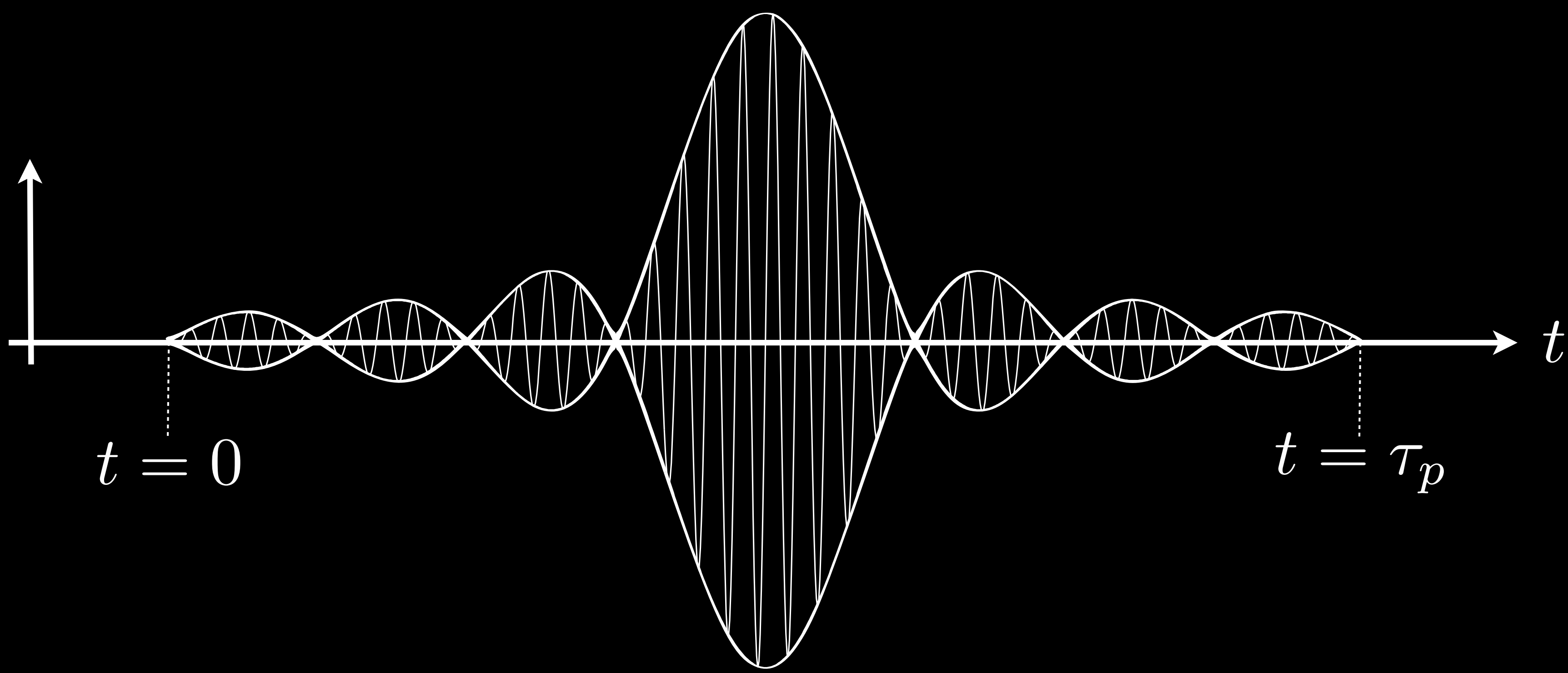
$$B_1^e(t) = B_1 \sqcap\left(\frac{t-\tau_p/2}{\tau_p}\right) = \begin{cases} B_1, & 0 \le t \le \tau_p \\ 0, & otherwise \end{cases}$$



RECT functions are used to excite a “single” frequency.

Sinc Envelope Function

$$B_1^e(t) = \begin{cases} B_1 \text{sinc}\left[\pi f_\omega \left(t - \tau_p/2\right)\right], & 0 \le t \le \tau_p \\ 0, & otherwise \end{cases}$$



SINC functions are used to excite a narrow band of frequencies.

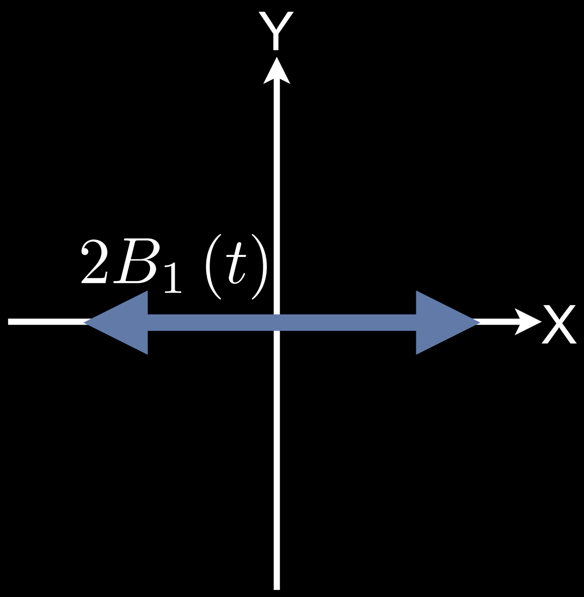
Circular vs. Linear Polarization

- **Linear Polarization**
  - **Simple, cheap**
  - **Higher RF power**
- **Circular Polarization**
  - **Generated with a quadrature RF transmitter coil**
    - **More complex & more expensive**
  - **Reduced RF power deposition**

Linearly Polarized Fields

Linear Polarization

$$2B_1^e(t)\cos(\omega_{RF}t)\,\hat{i}$$

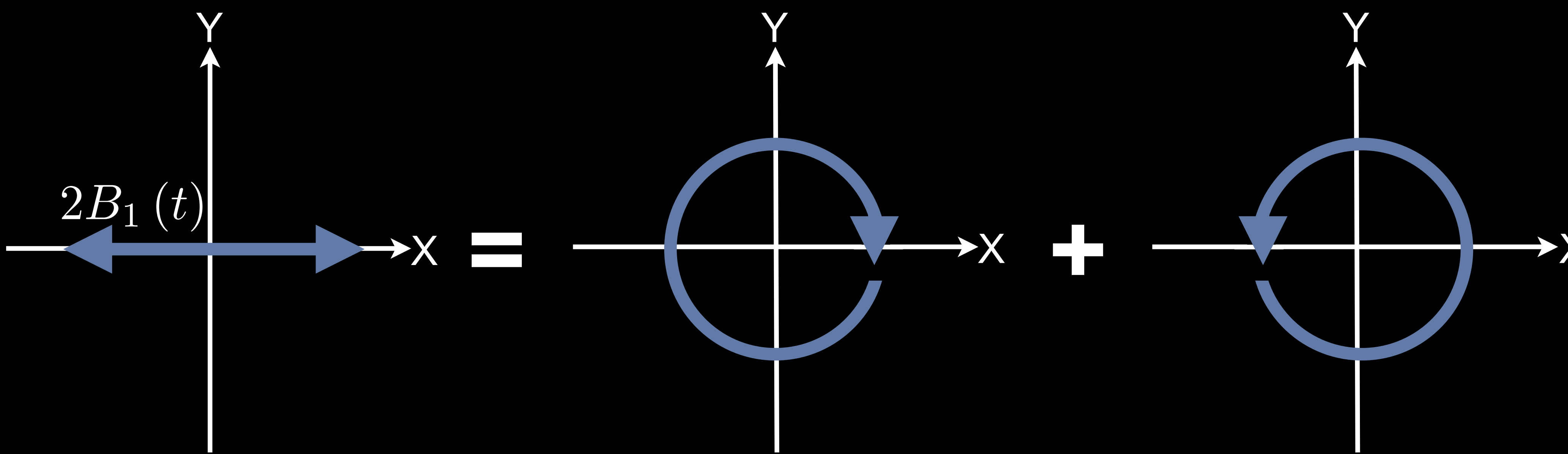


Arrow indicates direction of B-field.

Circularly Polarized Fields

Linear Polarization | CW Circular Polarization | CCW Circular Polarization

$$2B_1^e(t)\cos(\omega_{RF}t)\,\hat{i} = B_1^e(t)\left[\cos(\omega_{RF}t)\,\hat{i} - \sin(\omega_{RF}t)\,\hat{j}\right] + B_1^e(t)\left[\cos(\omega_{RF}t)\,\hat{i} + \sin(\omega_{RF}t)\,\hat{j}\right]$$



On-Resonance
Excitation+Heating

Very Off-Resonance
Heating

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Arrow indicates direction of B-field.

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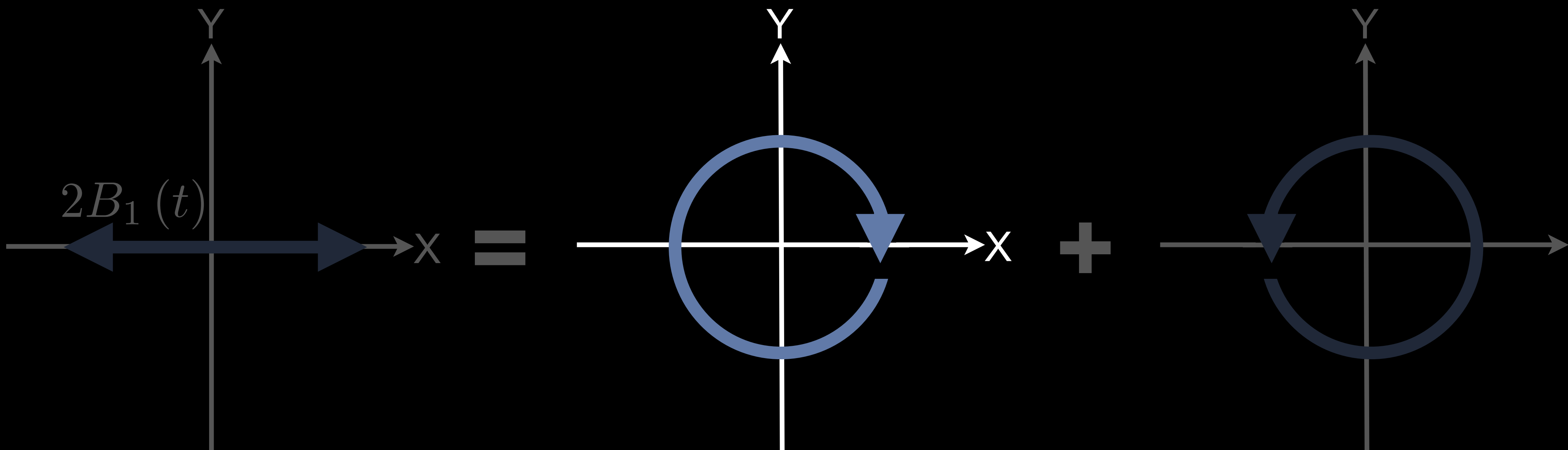
Circularly Polarized Fields

Linear Polarization		CW Circular Polarization		CCW Circular Polarization
$2B_1^e(t)\cos(\omega_{RF}t)\,\hat{i}$	=	$B_1^e(t)\left[\cos(\omega_{RF}t)\,\hat{i}-\sin(\omega_{RF}t)\,\hat{j}\right]$	+	$B_1^e(t)\left[\cos(\omega_{RF}t)\,\hat{i}+\sin(\omega_{RF}t)\,\hat{j}\right]$



	On-Resonance Excitation+Heating	Very Off-Resonance Heating
First Generation MRI Systems Used Linear Polarization	Modern MRI Systems Only Use CW Circular Polarization	Modern MRI Systems Don't Apply The CCW Field

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Arrow indicates direction of B-field.

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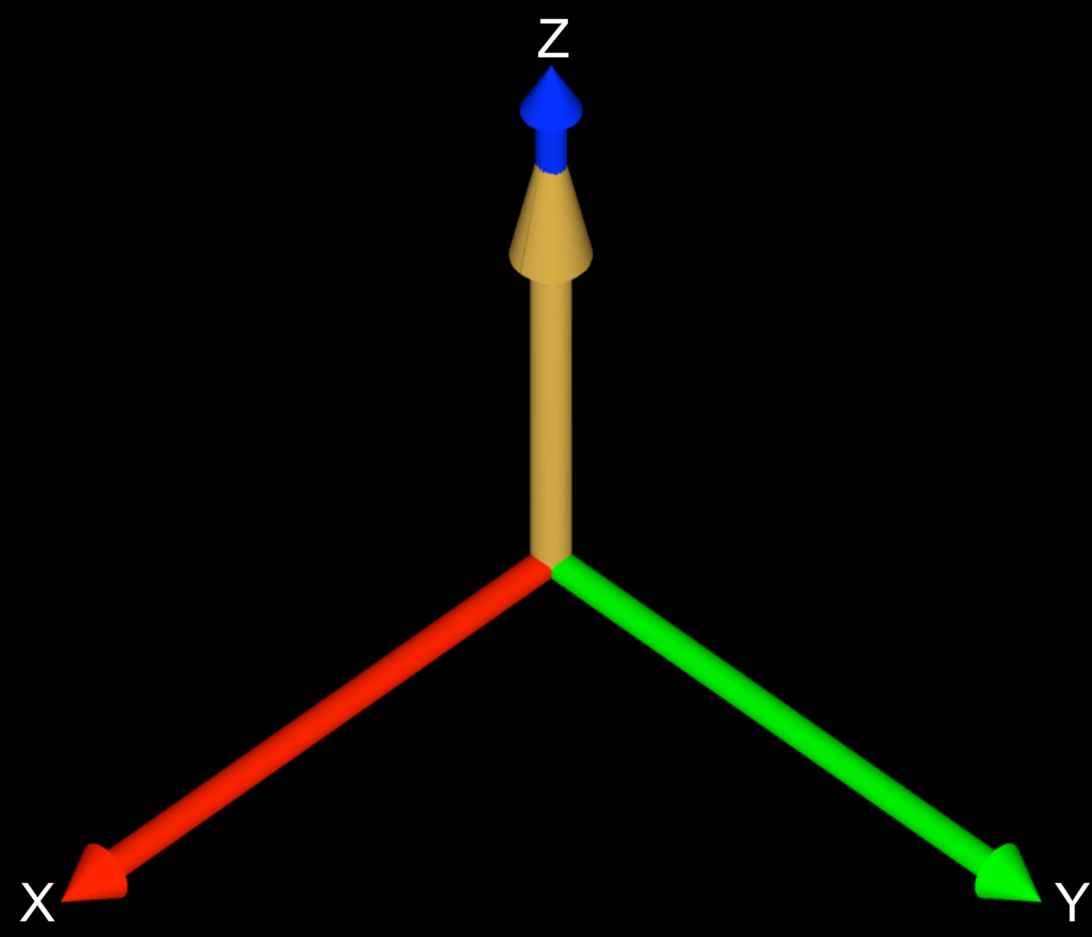
Forced Precession in the Laboratory Frame without Relaxation

Four Special Cases…

- **Free Precession in the Laboratory Frame**
- **Forced Precession in the Laboratory Frame**
  - Coordinate system anchored to scanner
- **Free Precession in the Rotating Frame**
- **Forced Precession in the Rotating Frame**
  - Coordinate system anchored to spin system
- **...all without relaxation.**
  - a) Relaxation time constants are “really” long
  - b) Time scale of event is << relaxation time constant

Forced Precession - Lab Frame



Forced Precession in the Laboratory Frame without Relaxation

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma\left(\vec{B}_0 + \vec{B}_1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \gamma B^e_{1,x}(t) & \gamma B^e_{1,y}(t) & \gamma B_0 \end{vmatrix}$$

$\Downarrow$

$$\frac{dM_x}{dt} = \gamma B_0 M_y - \gamma B^e_{1,y}(t) M_z$$

$$\frac{dM_y}{dt} = -\gamma B_0 M_x + \gamma B^e_{1,x}(t) M_z$$

$$\frac{dM_z}{dt} = \gamma B^e_{1,y}(t) M_x - \gamma B^e_{1,x}(t) M_y$$

Complex Coupling

$\Downarrow$

Forced Precession in the Laboratory Frame without Relaxation

$$\frac{dM_x}{dt} = \gamma B_0 M_y - \gamma B_{1,y}^e(t) M_z$$

$$\frac{dM_y}{dt} = -\gamma B_0 M_x + \gamma B_{1,x}^e(t) M_z$$

$$\frac{dM_z}{dt} = \gamma B_{1,y}^e(t) M_x - \gamma B_{1,x}^e(t) M_y$$

Complex Coupling

$$\Downarrow$$

$$B_1(t) = B_1^e(t)\left[\cos(\omega_{RF} t + \theta)\,\hat{i} - \sin(\omega_{RF} t + \theta)\,\hat{j}\right]$$

Forced Precession in the Laboratory Frame without Relaxation

$$\left.\begin{aligned}\frac{dM_x}{dt} &= \gamma B_0 M_y - \gamma B_{1,y}^e(t) M_z \\ \frac{dM_y}{dt} &= -\gamma B_0 M_x + \gamma B_{1,x}^e(t) M_z \\ \frac{dM_z}{dt} &= \gamma B_{1,y}^e(t) M_x - \gamma B_{1,x}^e(t) M_y\end{aligned}\right\}$$

Complex Coupling

$$\Downarrow$$

$$B_1(t) = B_1^e(t)\left[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}\right]$$

$$\Downarrow$$

$$\frac{dM_x}{dt} = \gamma B_0 M_y + \gamma B_1^e(t)\sin(\omega_{RF}t + \theta)M_z$$

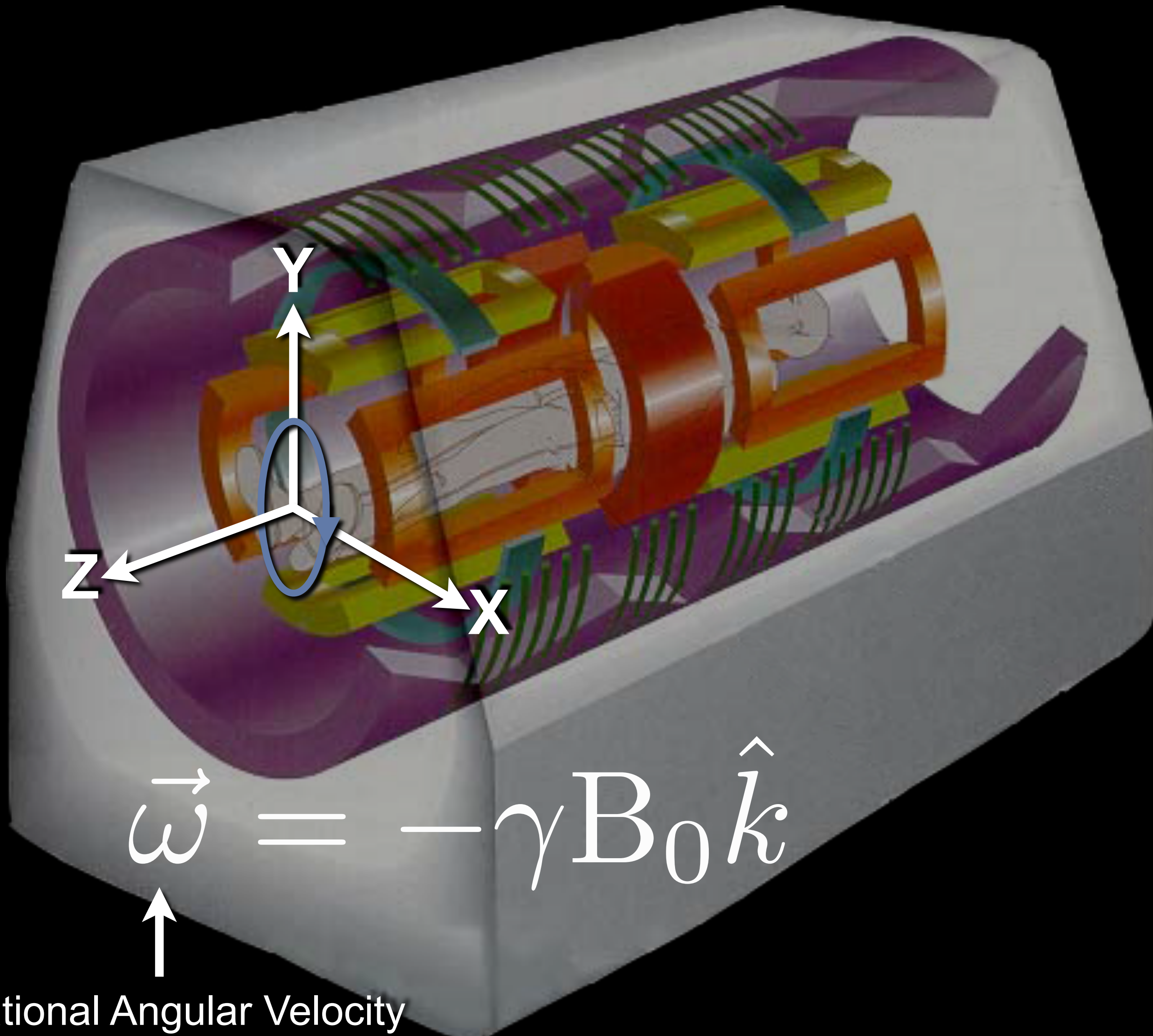
$$\frac{dM_y}{dt} = -\gamma B_0 M_x + \gamma B_1^e(t)\cos(\omega_{RF}t + \theta)M_z$$

$$\frac{dM_z}{dt} = -\gamma B_1^e(t)\sin(\omega_{RF}t + \theta)M_x - \gamma B_1^e(t)\cos(\omega_{RF}t + \theta)M_y$$

This is a relatively complicated set of coupled ODEs!

Rotating Coordinate Frame

Laboratory Coordinates



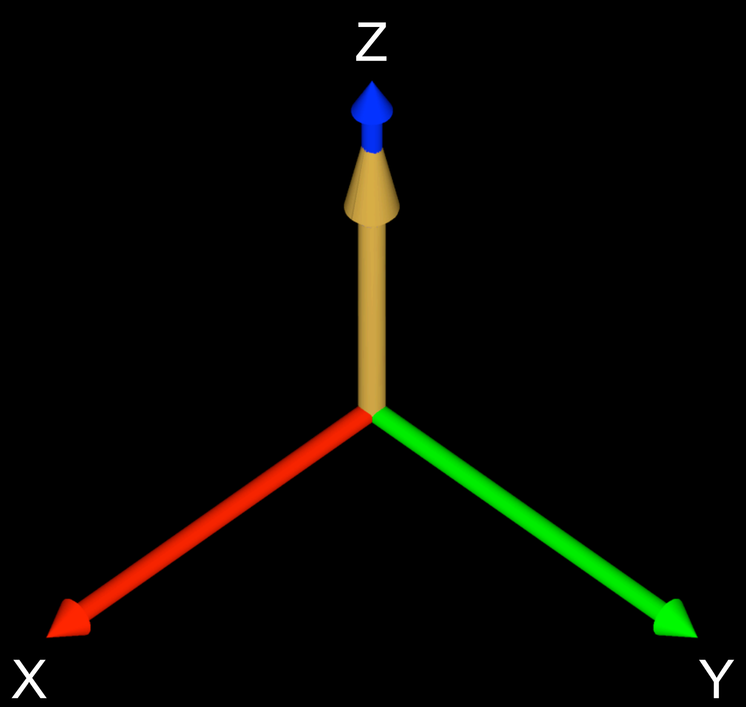
$$\vec{\omega} = -\gamma B_0 \hat{k}$$

Rotational Angular Velocity

Lab vs. Rotating Frame

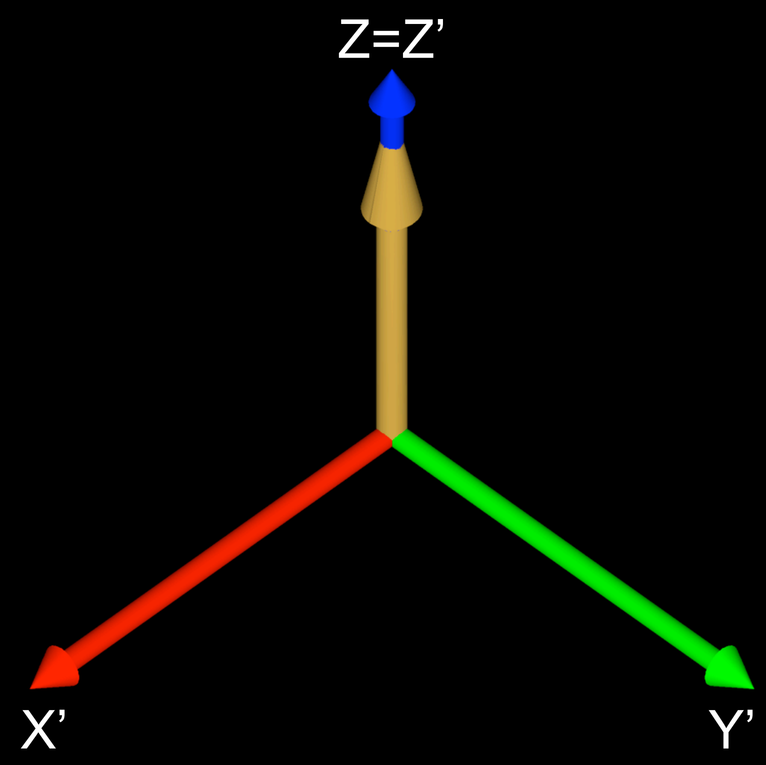
- **The rotating frame simplifies the mathematics and permits more intuitive understanding.**

Laboratory Frame



Spins Precess

Rotating Frame

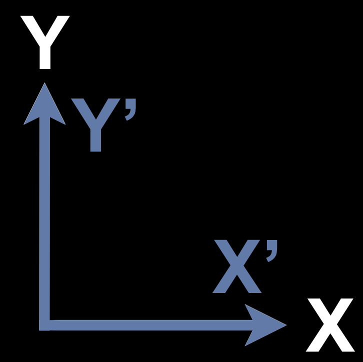


Observer Precesses

Note: Both coordinate frames share the same z-axis.

Rotating Frame Coordinates

- Simplifies the mathematics of MRI
- If the rotational frequency of the rotating frame (x'-y') is matched to the bulk magnetization's precessional frequency, then rotational motion of the bulk magnetization is "removed" or demodulated.
- The rotating frame's transverse (x'y') plane rotates clockwise (left-handed) at frequency ω.



Relationship Between Lab and Rotating Frames

Rotating Frame

$$\hat{i}' \equiv \cos(\omega t)\,\hat{i} - \sin(\omega t)\,\hat{j}$$
$$\hat{j}' \equiv \sin(\omega t)\,\hat{i} + \cos(\omega t)\,\hat{j}$$
$$\hat{k}' \equiv \hat{k}$$

Laboratory Frame

$$\hat{i} \equiv \cos(\omega t)\,\hat{i}' + \sin(\omega t)\,\hat{j}'$$
$$\hat{j} \equiv -\sin(\omega t)\,\hat{i}' + \cos(\omega t)\,\hat{j}'$$
$$\hat{k} \equiv \hat{k}'$$

Note: Both coordinate frames share the same z-axis.

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix}$$

Bulk magnetization components in the rotating frame.

$$\vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix}$$

Applied B-field components in the rotating frame.

$$B_{z'} \equiv B_z$$
$$M_{z'} \equiv M_z$$

Note: B-field and bulk magnetization z-components are equivalent in the two frames.

Equation of Motion

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats). [Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \right)$$

Equation of motion for an ensemble of spins (isochromats). [Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Four Special Cases…

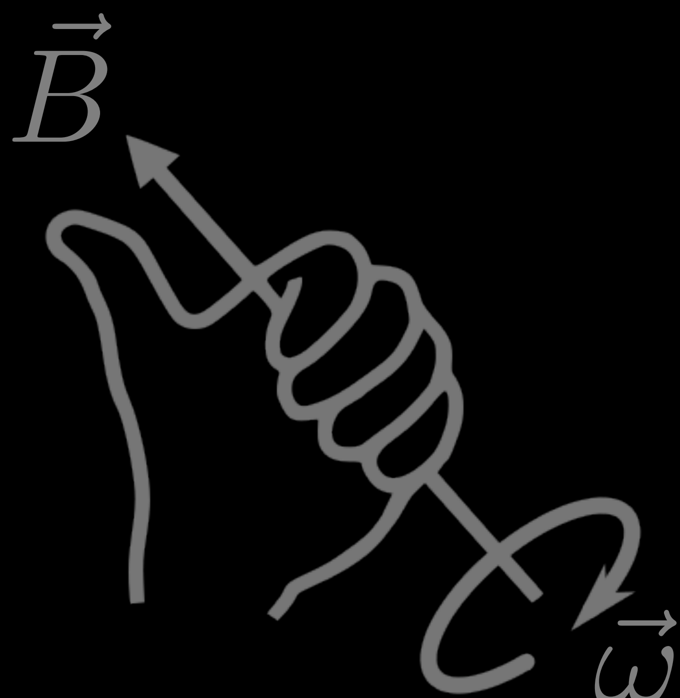
- **Free Precession in the Laboratory Frame**
- **Forced Precession in the Laboratory Frame**
  - Coordinate system anchored to scanner
- **Free Precession in the Rotating Frame**
- **Forced Precession in the Rotating Frame**
  - Coordinate system anchored to spin system
- **...all without relaxation.**
  - a) Relaxation time constants are “really” long
  - b) Time scale of event is << relaxation time constant

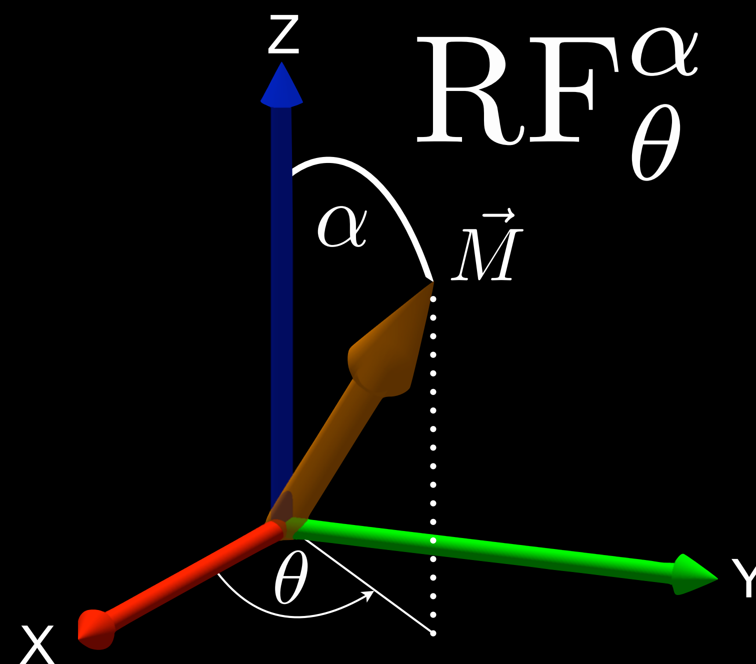
To The Board…

Mathematics of Hard RF Pulses

Parameters & Rules for RF Pulses

- RF pulses have a "**flip angle**" (**α**)
  - RF fields induce **left-hand** rotations
    - All B-fields do this for positive γ
- RF pulses have a "**phase**" (**θ**)
  - Phase of 0° is about the x-axis
  - Phase of 90° is about the y-axis

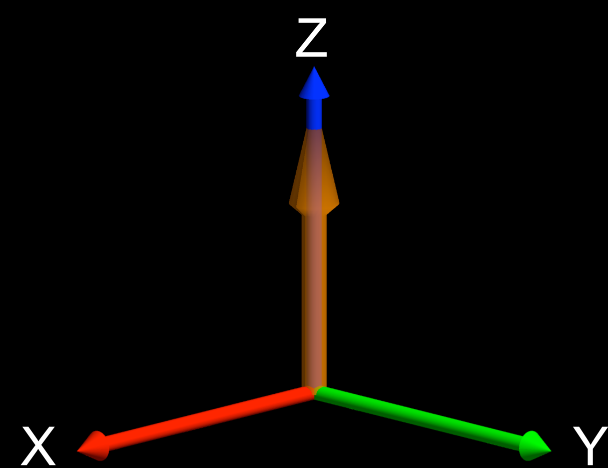


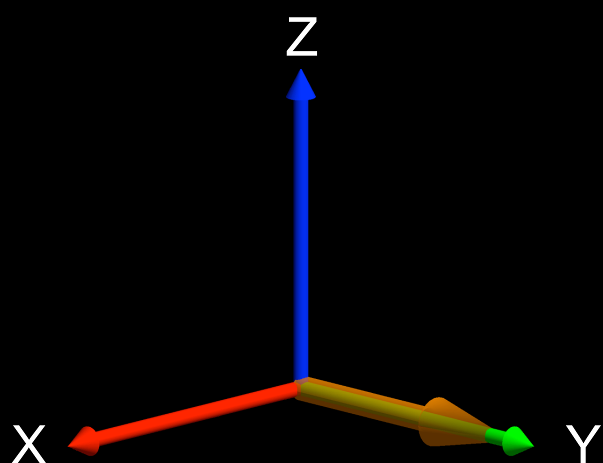


Rules for RF Pulses

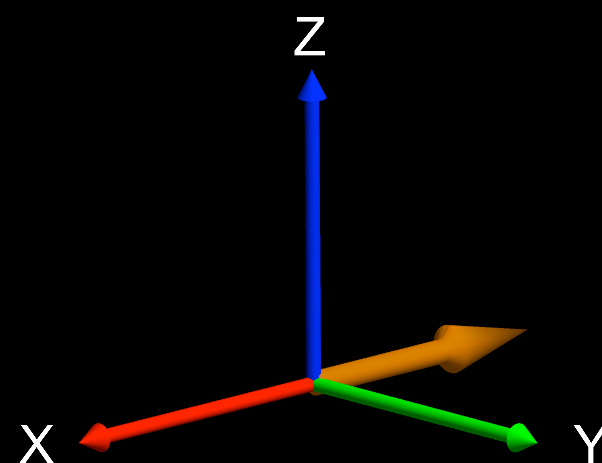
$$\mathbf{RF}_{\theta}^{\alpha}$$

α → Flip Angle
θ → Phase





$$\mathbf{RF}_{0^\circ}^{90^\circ}$$



$$\mathbf{RF}_{90^\circ}^{90^\circ}$$

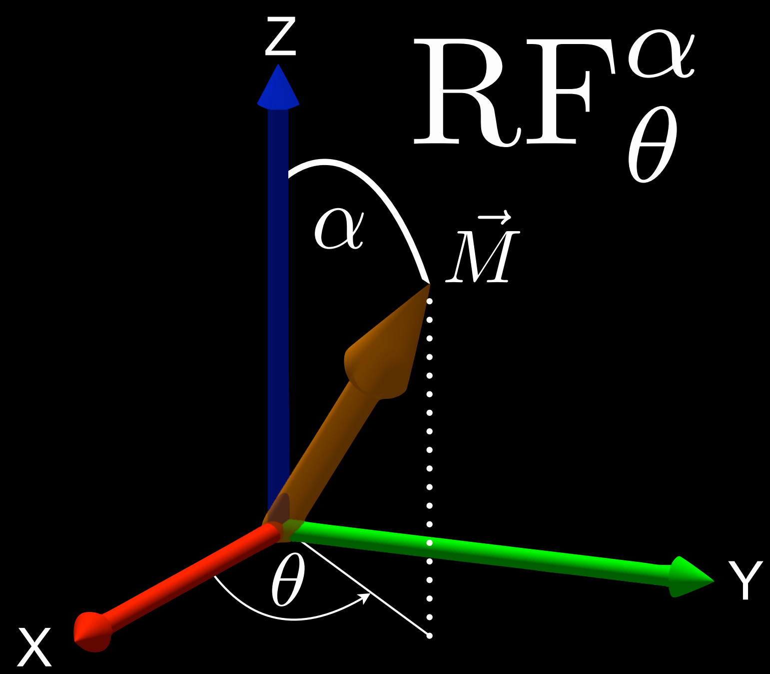
David Geffen School of Medicine

B-fields induce left-handed nutation!

UCLA Radiology

Flip Angle

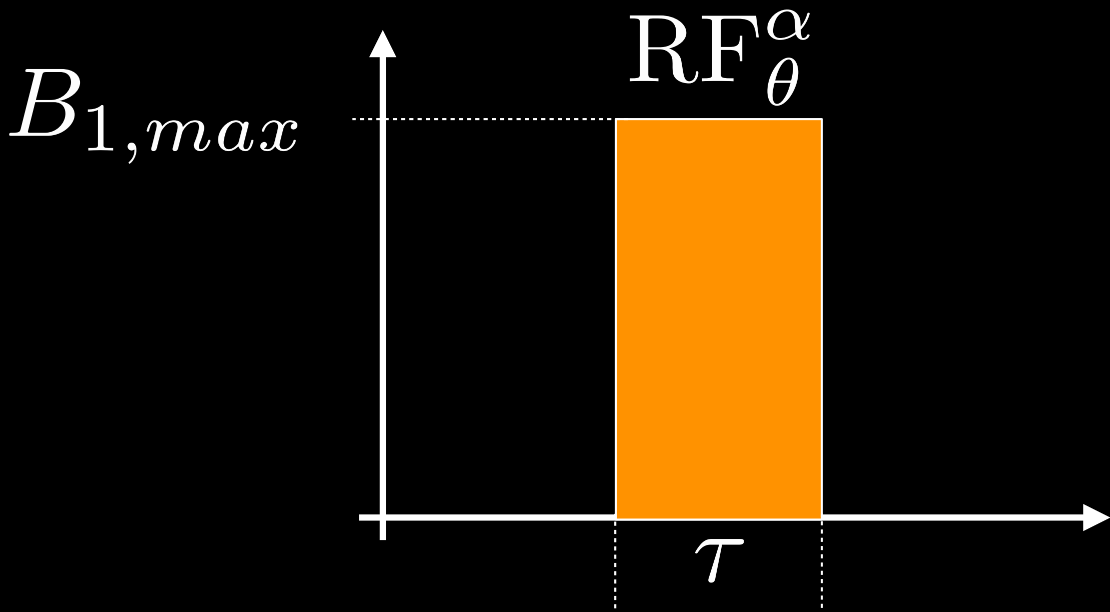
- **"Amount of rotation of the bulk magnetization vector produced by an RF pulse, with respect to the direction of the static magnetic field."**
  - Liang & Lauterbur, p. 374



$$\omega_1 = \gamma B_1$$

B-fields induce nutation!

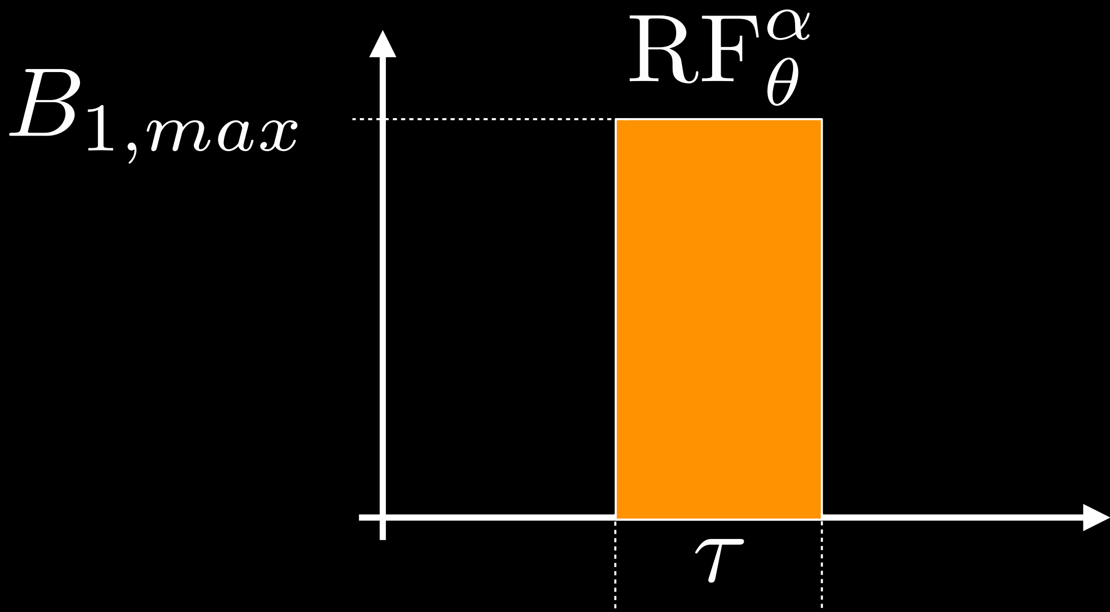
How to determine α?



$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)\, dt$$

Rules: 1) Specify α [radians]
2) Use $B_{1,max}$ if we can
3) Shortest duration pulse

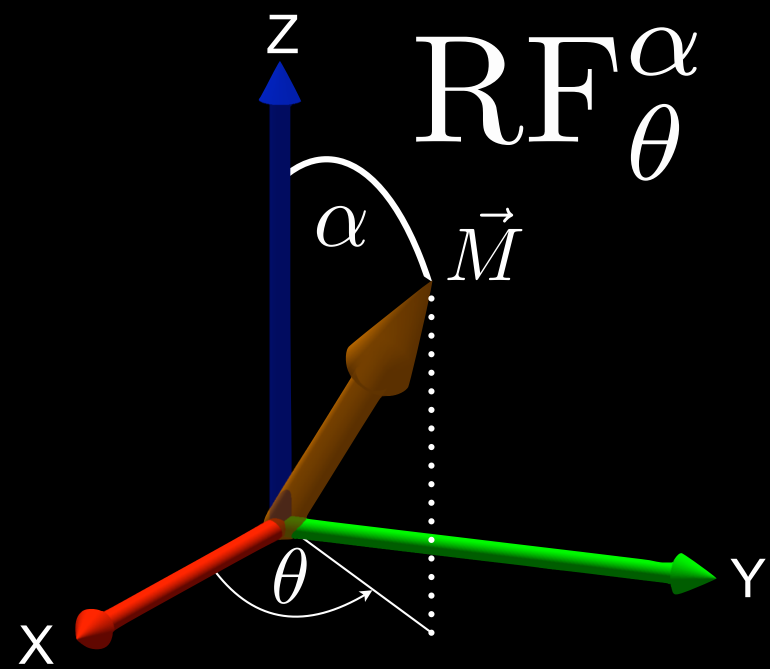
How to determine α?



$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)\, dt$$

$$\tau = \frac{\alpha}{\gamma B_{1,max}} = \frac{\pi/2}{2\pi \cdot 42.57Hz/\mu T \cdot 60\mu T} = 0.098ms$$

Bulk Magnetization in the Lab Frame

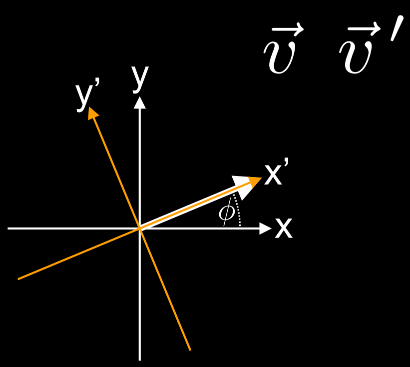


How do we mathematically account for α and θ?
Use a composite of three operators.

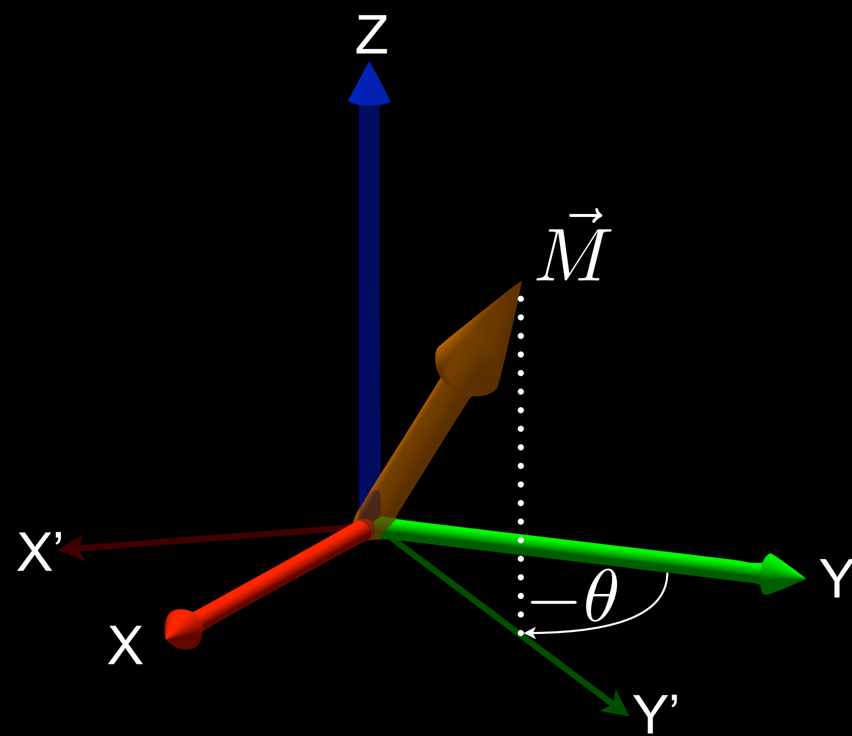
Change of Basis

$$\mathrm{R}_z^{\phi} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{v}\,' = \mathrm{R}\vec{v}$$



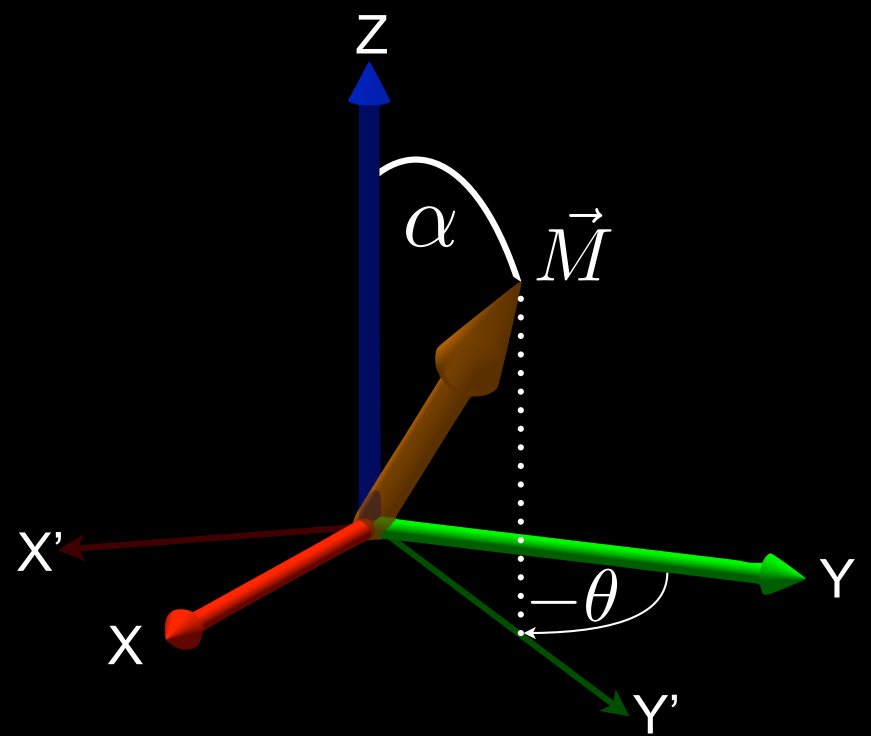
Change of Basis by -θ



$$\mathbf{R}_z\left(\theta\right) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate into a coordinate system where ***M*** falls along the y'-axis.

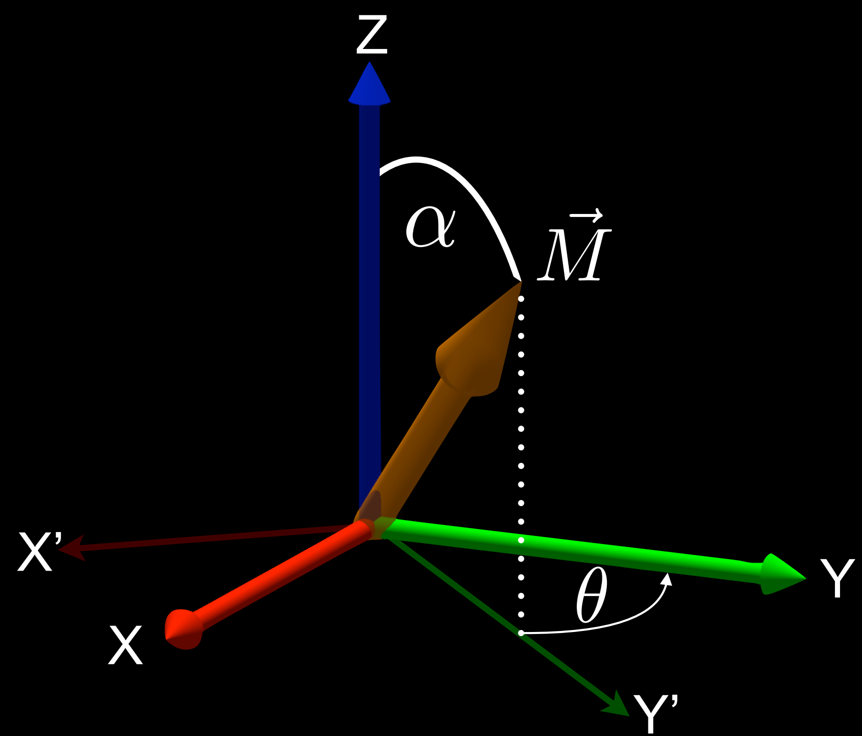
Rotation by Alpha about X'-axis



$$\mathbf{R}_{x'}(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

Rotate ***M*** by α about x'-axis.

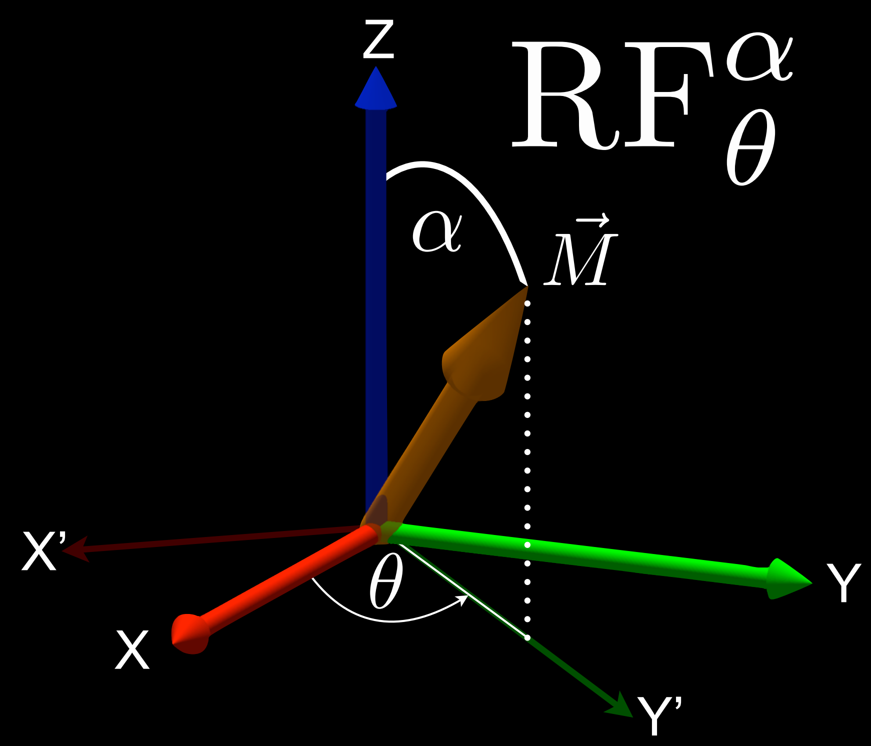
Reverse the Change of Basis by θ



$$\mathbf{R}_z\left(\theta\right) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate back to the lab frame's x-axis and y-axis.

RF Pulse Operator



$$\mathbf{R}_\theta^\alpha = \mathbf{R}_z(\theta)\mathbf{R}_{x'}(\alpha)\mathbf{R}_z(-\theta)$$

$$= \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha \end{bmatrix}$$

This is the composite matrix operator for a hard RF pulse.

Homework #1 & Matlab

M219, Winter 2018

Homework Assignment #1 (15 Points)

Due via E-mail on Tuesday, January 23rd by 9pm

To submit the assignment, e-mail `DEnnis@mednet.ucla.edu` a PDF entitled `M219_HW01_[First Initial]_[Last Name].pdf` (*e.g.* `M219_HW01_D_Ennis.pdf`). Please only submit neat and clear solutions. Late assignments will be discounted by $e^{-t/\tau}$, where $\tau = 72$ hours.

For all problems – Clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

If your assignments are hard to read, poorly commented, or sloppy, then points may be deducted. As appropriate, each solution should be obtained using Matlab. Please comment and submit your code as individual files that run for each problem.

Problem #1 (5 points, plus 1 *Extra Credit* point) – Design the Main (B_0) Field. For this problem you will design the main (B_0) magnet that meets the following specifications: 1) 1.5T field strength (at isocenter); 2) 70cm bore; 3) Length $< 2m$; 4) Field variation $< 100,000ppm$ for $50cm$ along the z-axis.

A. Modify the `PAM_Lec01_Bz_Uniformity.m` function to design the length and current needed to meet these design specifications. This Matlab function use the following expression:

$$B_z(z) = \frac{\mu_0 NI}{2L}(\cos\alpha_2 - \cos\alpha_1) \quad (1)$$

Note, that according to this expression there is an axial (z), but no radial (x or y) dependence on the magnetic field strength and the field remains z-oriented. Make a plot of $B_z(z)$ for the length and current you have designed. [2 points]

B. What is the magnetic field variation (maximum, minimum, mean) for $50cm$? Calculate and report the field homogeneity $[(B_{0,max} - B_{0,min})/B_{0,mean}]$ in PPM for $50cm$. What is the vRMS error for $50cm$ relative to the target field strength of 1.5T? [2 points]

C. How would you improve the design of your magnet to improve the field homogeneity to $< 1,000ppm$? [1 point]

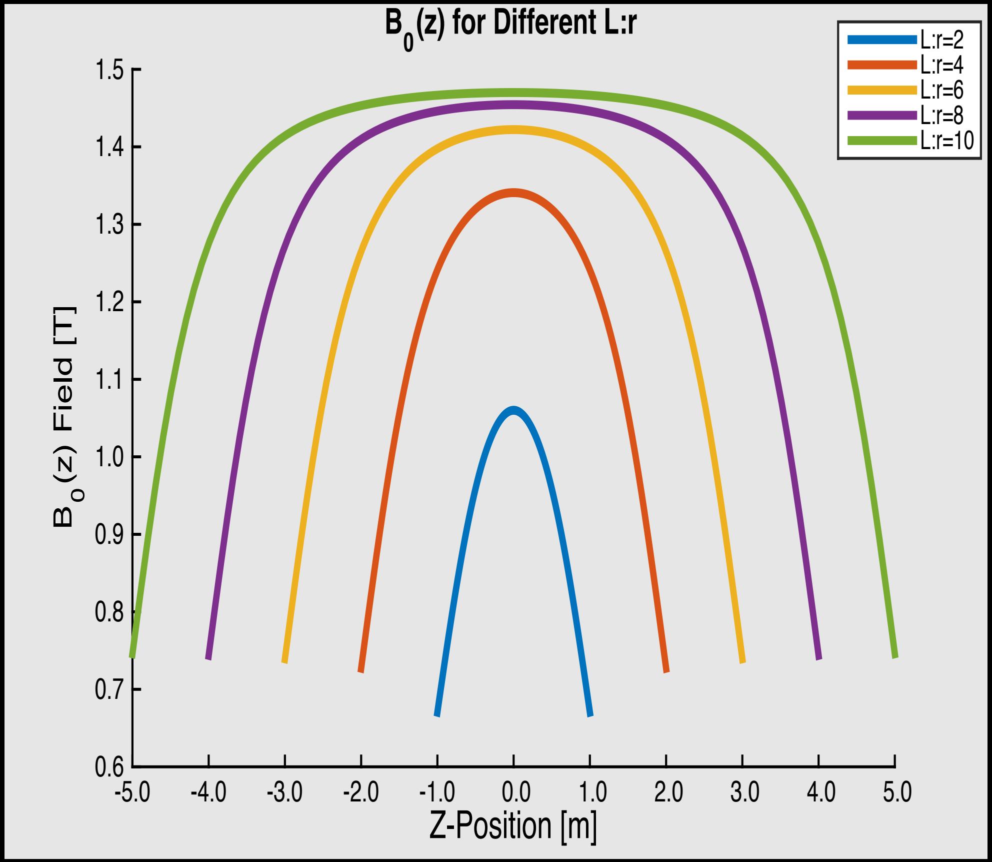
D. *Extra Credit*: Use the principle of superposition and Eqn. 1 to improve the field homogeneity to $< 1,000ppm$. [2 points]

B-fields

$$B_z(z) = \frac{\mu I N}{2L}\left(\cos\alpha_2 - \cos\alpha_1\right)$$

Haacke p. 834

```
% Filename: PAM_Lec01_Bz_Uniformity.m
%
% Demonstrate the axial uniformity of the B-field for a solenoid.
%
% DBE@UCLA 2014.12.12

%%  Define some constants
mu=4*pi*10e-7;  % Air [T.m.A-1]
I=508.25;       % Current [amps]
L=2:2:10;       % Length [meters]
N=2350;         % Number of windings [#]
r=1;            % Radius [m]

%%  Calculate Bz(z)
figure; hold on;
for ind=1:numel(L)                              % Loop over designs
  z=linspace(0,L(ind),100);                     % Z-distance to span
  alpha1=atan2(r,z);                            % Calculate alpha1
  alpha2=pi-atan2(r,L(ind)-z);                  % Calculate alpha2
  Bz=(mu*I*N/2) * (cos(alpha1)-cos(alpha2));    % Calculate Bz
  p(ind)=plot(z-L(ind)/2,Bz);                   % Plot each Bz(z)
end

%% Plot the results
set(p,'LineWidth',3);                          % Increase plot thickness
xlabel('Z-Position [m]');
ylabel('B_0(z) Field [T]');
title('B_0(z) for Different L:r');
l=legend('L:r=2','L:r=4','L:r=6','L:r=8','L:r=10');
```



- **L:r=2 to 10m**
- **$\mu=4\pi\times10^{-7}$ T•m•A^{-1}**

B_z only “uniform” when L>>r.

Problem #2 (5 points, plus 1 *Extra Credit* point) – B_0 **vs.** B_1 **fields.** Assume a hard RF pulse with a flip angle of $\alpha = \pi/2$, phase of $\pi/4$, and $B_{1,max} = 20$ gauss for ^{31}P at $B_0 = 0.15T$.

A. What is the duration (τ_{RF}) of the RF pulse? [1/2 point]

B. Find ω_0, the frequency of *precession* in MHz for the B_0 field. [1/2 point]

C. Find ω_1, the frequency of *nutation* in MHz for the B_1 field. [1/2 point]

D. How many cycles of precession does the bulk magnetization go through during the RF pulse? How does this compare to the number of cycles of nutation? [1/2 point]

E. Use `PAM_B1_op.m` to generate the M_x, M_y, and M_z components for this RF pulse from 0 to τ_{RF} in the *rotating* frame using MATLAB. This can be done with a *for-loop*. Use 1,000 points for your simulation. Plot the results; label the axes. [1 point]

F. Now incorporate the use of `PAM_B0_op.m` to generate the M_x, M_y, and M_z components in the *laboratory* frame using MATLAB. Plot the results; label the axes. *Hint*: The RF phase is constant in the rotating frame, but not the laboratory frame. [2 points]

G. *Extra Credit*: Explain how B_1 field can be effective at perturbing the spin system when B_0 is so much larger in magnitude. [1 point]

Problem #3 (5 points) – T_1 **and** T_2 **relaxation**

A. In lecture we learned that T_1 and T_2 relaxation are tissue dependent characteristics. Using the equations for relaxation during free precession in the rotating frame, find a general expressions for T_1 contrast after an *inversion* pulse. [1/2 point]

B. Derive an analytic expression for the time that maximizes the image contrast (signal difference) between white matter ($790ms$) and gray matter ($925ms$). Assume that the proton densities are the same. [1 point]

C. Plot the T_1 relaxation results for white matter ($790ms$) and gray matter ($925ms$). Prove that your solution in (A) produces the same result as simply taking the difference between the two curves. Label the axes. [1 point]

D. Using the equations for relaxation during free precession in the rotating frame, find a general expressions for T_2 contrast after an *saturation* pulse. [1/2 point]

E. Repeat the process and derive an analytic expression for T_2 contrast after a *saturation* pulse. Assume that the proton densities are the same. [1 point]

F. Plot the T_2 relaxation results for white matter ($92ms$) and gray matter ($100ms$). Prove that your solution in (C) produces the same result as simply taking the difference between the two curves. Label the axes. [1 point]

Problem #4 (1 *Extra Credit* point) Create your own three-part question using the concepts from the first four lectures. Provide an answer. Your question may be chosen to appear on the final exam (and you'll already know the answer!).

B_1 Operator - Nutation

$$\mathbf{RF}^{\alpha}_{\theta,H} = \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha & 0 \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha & 0 \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vec{\mathrm{M}}^{+}_{H} = \mathrm{RF}^{\alpha}_{\theta,H}\vec{\mathrm{M}}^{-}_{H}$$

$$\begin{bmatrix} M_x^+ \\ M_y^+ \\ M_z^+ \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha & 0 \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha & 0 \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x^- \\ M_y^- \\ M_z^- \\ 1 \end{bmatrix}$$

B_1 Operator - Nutation

```
% This function returns the 4x4 homogenous coordinate expression for an RF
% pulse with a particular gyromagnetic ratio (gamma), B1 amplitude, time
% step (dt), and phase (theta).  THETA=0 is phased about the X-axis and
% THETA=90 is phased about the Y-axis.
%
% SYNTAX: dB1=PAM_B1_op(gamma,B1,dt,theta)
%
% INPUTS: gamma - gyromagnetic ratio [Hz/T]
%         B1    - B1 amplitude       [T]
%         dt    - time step          [s]
%         theta - phase angle        [radians]
%
% OUTPUT: RF - RF pulse operator [4x4]
%
% DBE@UCLA 2015.01.21

function dB1=PAM_B1_op(gamma,B1,dt,theta)

% Define the incremental flip angle in time dt
alpha=2*pi*gamma*B1*dt;

% Change of basis
R_theta=[cos(-theta)  -sin(-theta) 0 0;
         sin(-theta)   cos(-theta) 0 0;
         0             0           1 0;
         0             0           0 1];

% Flip angle rotation
R_alpha=[1  0           0          0;
         0  cos(alpha)  sin(alpha) 0;
         0 -sin(alpha)  cos(alpha) 0;
         0  0           0          1];

% Homogeneous expression for RF MATRIX
dB1=R_theta.'*R_alpha*R_theta;

return
```

B_0 Operator - Precession

$$B_{0,H} = \begin{bmatrix} \cos\gamma B_0 t & \sin\gamma B_0 t & 0 & 0 \\ -\sin\gamma B_0 t & \cos\gamma B_0 t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} M_x(0_+) \\ M_y(0_+) \\ M_z(0_+) \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\gamma B_0 t & \sin\gamma B_0 t & 0 & 0 \\ -\sin\gamma B_0 t & \cos\gamma B_0 t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x(0_-) \\ M_y(0_-) \\ M_z(0_-) \\ 1 \end{bmatrix}$$

Homogeneous coordinate expression for precession.

B_0 Operator - Precession

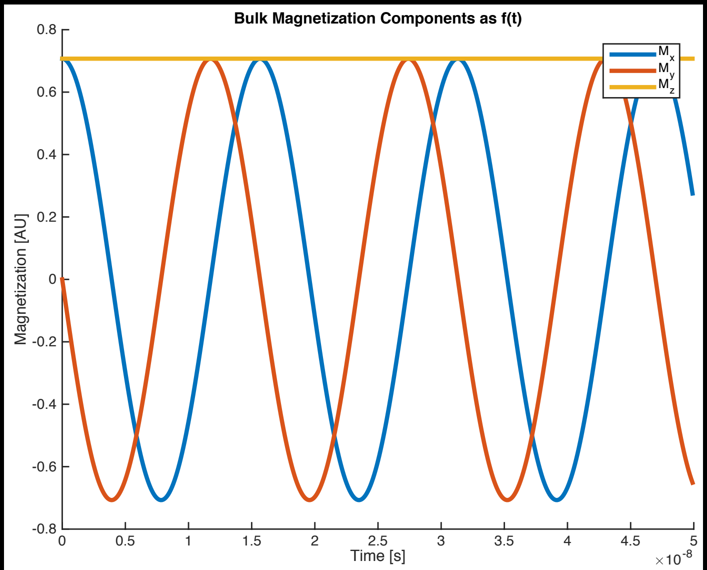
```
% This function returns the 4x4 homogenous coordinate expression for
% precession for a particular gyromagnetic ratio (gamma), external
% field (B0), and time step (dt).
%
% SYNTAX:  dB0=PAM_B0_op(gamma,B0,dt)
%
% INPUTS:  gamma - Gyromagnetic ratio [Hz/T]
%          B0    - Main magnetic field [T]
%          dt    - Time step or vector [s]
%
% OUTPUTS: dB0   - Precessional operator [4x4]
%
% DBE@UCLA 01.21.2015

function dB0=PAM_B0_op(gamma,B0,dt)

if nargin==0
  gamma=42.57e6;         % Gyromagnetic ratio for 1H
  B0=1.5;                % Typical B0 field strength
  dt=ones(1,100)*1e-6;   % 100 1μs time steps
end

dB0=zeros(4,4,numel(dt));  % Initialize the array

for n=1:numel(dt)
  dw=2*pi*gamma*B0*dt(n);  % Incremental precession (rotation angle)

  % Precessional Operator (left handed)
  dB0(:,:,n)=[ cos(dw)  sin(dw)   0  0;
              -sin(dw)  cos(dw)   0  0;
                 0         0      1  0;
                 0         0      0  1];

end
return
```

Matlab Example - Free Precession

```
%% Filename: PAM_Lec02_B0_Free_Precession.m
%
% Demonstrate the precession of the bulk magnetization vector.
%
% DBE@UCLA 2015.01.06

%% Define some constants
gamma=42.57e6;          % Gyromagnetic ratio for 1H [MHz/T]
B0=1.5;                 % B0 magnetic field strength [T]
dt=0.01e-8;             % Time step [s]
nt=500;                 % Number of time points to simulate
t=(0:nt-1)*0.01e-8;     % Time vector [s]

M0=[sqrt(2)/2 0 sqrt(2)/2 1]';  % Initial condition (I.C.)

M=zeros(4,nt);          % Initialize the magnetization array
M(:,1)=M0;              % Define the first time point as the I.C.

%% Simulate precession of the bulk magnetization vector
dB0=PAM_B0_op(gamma,B0,dt); % Calculate the homogenous coordinate transform

for n=2:nt
  M(:,n)=dB0*M(:,n-1);
end
```

$$\vec{M}(t) = \mathrm{R}_z(\gamma B_0 t)\vec{M}^0$$

```
%% Plot the results
figure; hold on;
  p(1)=plot(t,M(1,:));      % Plot the Mx component
  p(2)=plot(t,M(2,:));      % Plot the My component
  p(3)=plot(t,M(3,:));      % Plot the Mz component
    set(p,'LineWidth',3);   % Increase plot thickness
  ylabel('Magnetization [AU]');
  xlabel('Time [s]');
  legend('M_x','M_y','M_z');
  title('Bulk Magnetization Components as f(t)');
```



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A. What is the duration (τ_{RF}) of the RF pulse? [1/2 point]

B. Find ω_0, the frequency of *precession* in MHz for the B_0 field. [1/2 point]

C. Find ω_1, the frequency of *nutation* in MHz for the B_1 field. [1/2 point]

D. How many cycles of precession does the bulk magnetization go through during the RF pulse? How does this compare to the number of cycles of nutation? [1/2 point]

E. Use `PAM_B1_op.m` to generate the M_x, M_y, and M_z components for this RF pulse from 0 to τ_{RF} in the *rotating* frame using MATLAB. This can be done with a *for-loop*. Use 1,000 points for your simulation. Plot the results; label the axes. [1 point]

F. Now incorporate the use of `PAM_B0_op.m` to generate the M_x, M_y, and M_z components in the *laboratory* frame using MATLAB. Plot the results; label the axes. *Hint*: The RF phase is constant in the rotating frame, but not the laboratory frame. [2 points]

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B. Derive an analytic expression for the time that maximizes the image contrast (signal difference) between white matter ($790ms$) and gray matter ($925ms$). Assume that the proton densities are the same. [1 point]

C. Plot the T_1 relaxation results for white matter ($790ms$) and gray matter ($925ms$). Prove that your solution in (A) produces the same result as simply taking the difference between the two curves. Label the axes. [1 point]

D. Using the equations for relaxation during free precession in the rotating frame, find a general expressions for T_2 contrast after an *saturation* pulse. [1/2 point]

E. Repeat the process and derive an analytic expression for T_2 contrast after a *saturation* pulse. Assume that the proton densities are the same. [1 point]

F. Plot the T_2 relaxation results for white matter ($92ms$) and gray matter ($100ms$). Prove that your solution in (C) produces the same result as simply taking the difference between the two curves. Label the axes. [1 point]

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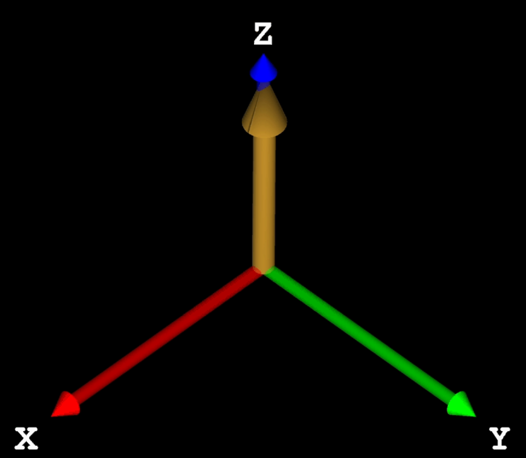
Types of RF Pulses

Types of RF Pulses

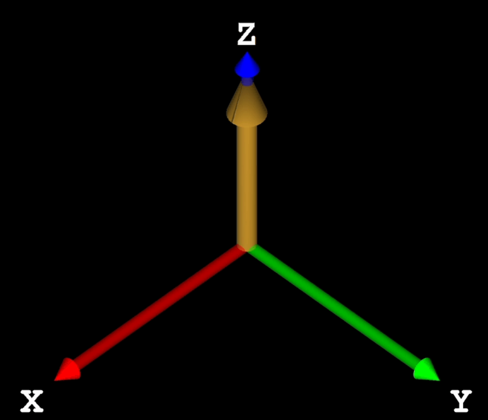
- Excitation Pulses
- Inversion Pulses
- Refocusing Pulses
- Saturation Pulses
- Spectrally Selective Pulses
- Spectral-spatial Pulses
- Adiabatic Pulses

Excitation Pulses

- **Tip M_z into the transverse plane**
- **Typically 200μs to 5ms**
- **Non-uniform across slice thickness**
  - Imperfect slice profile
- **Non-uniform within slice**
  - Termed B_1 inhomogeneity
  - Non-uniform signal intensity across FOV



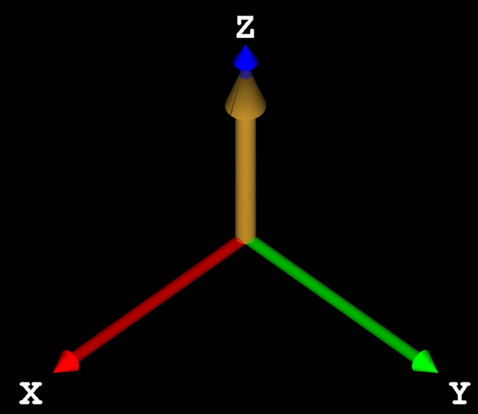
90° Excitation Pulse



Small Flip Angle Pulse

Inversion Pulses

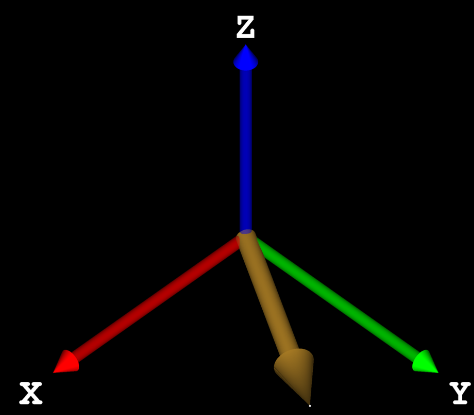
- **Typically, 180° RF Pulse**
  - non-180° that still results in $-M_Z$
- **Invert M_Z to $-M_Z$**
  - Ideally produces no M_{XY}
- **Hard Pulse**
  - Constant RF amplitude
  - Typically non-selective
- **Soft (Amplitude Modulated) Pulse**
  - **Frequency/spatially/spectrally selective**
- **Typically followed by a crusher gradient**



180° Inversion Pulse

Refocusing Pulses

- **Typically, 180° RF Pulse**
  - Provides optimally refocused M_{XY}
  - Largest spin echo signal
- **Refocus spin dephasing due to**
  - imaging gradients
  - local magnetic field inhomogeneity
  - magnetic susceptibility variation
  - chemical shift
- **Typically followed by a crusher gradient**



180° Refocusing Pulse

Thanks

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