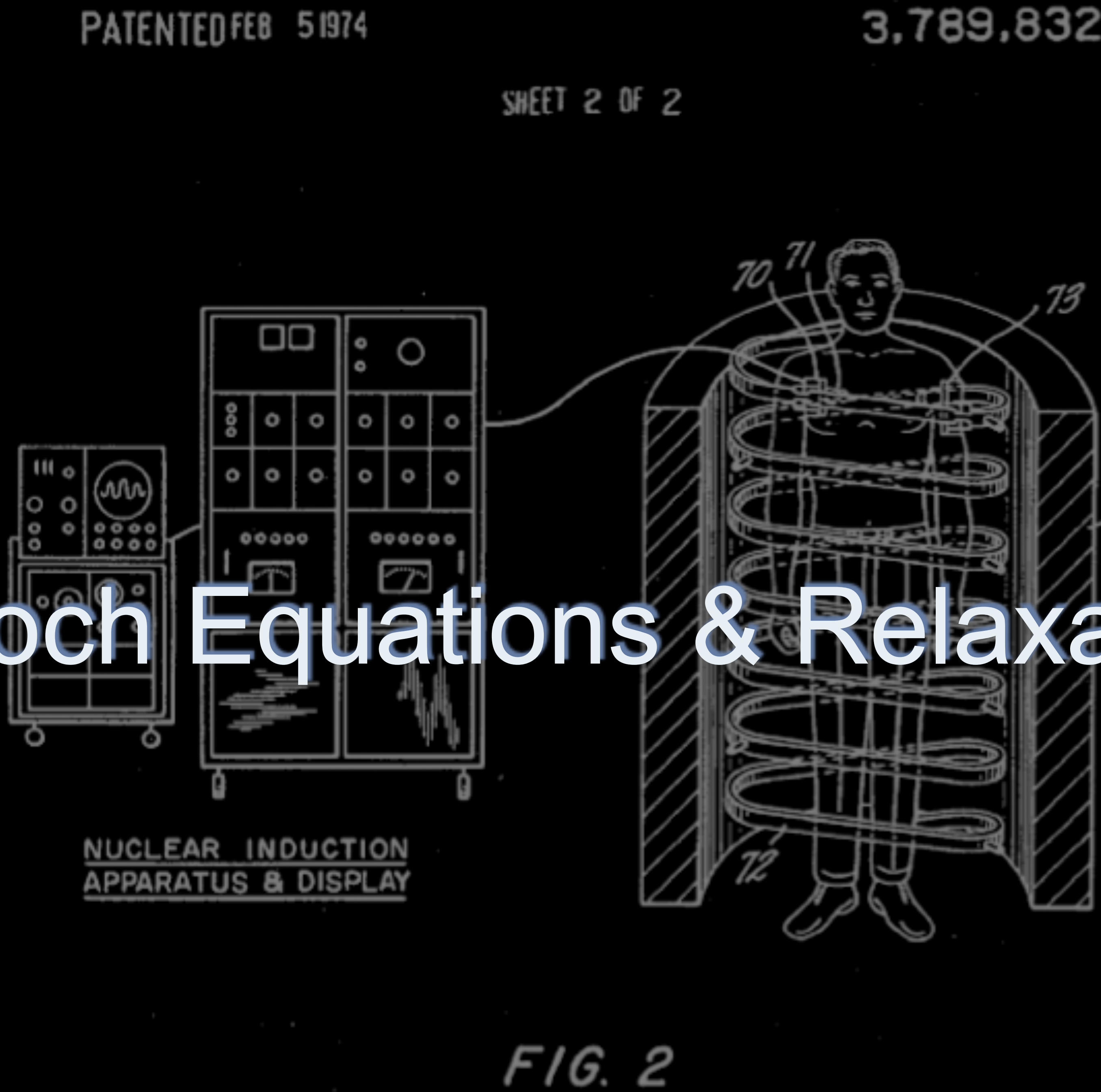

Bloch Equations & Relaxation

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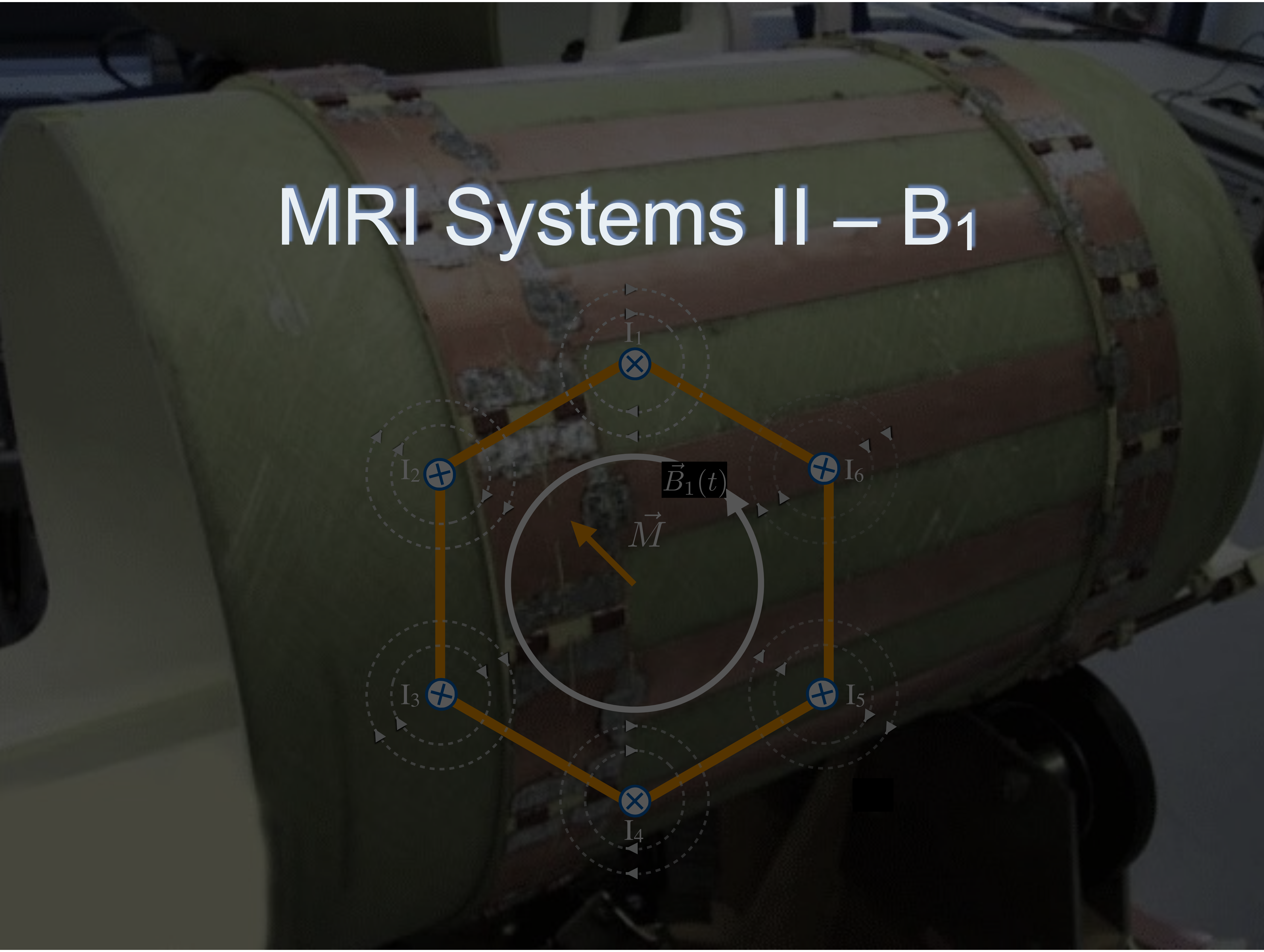
Radiology

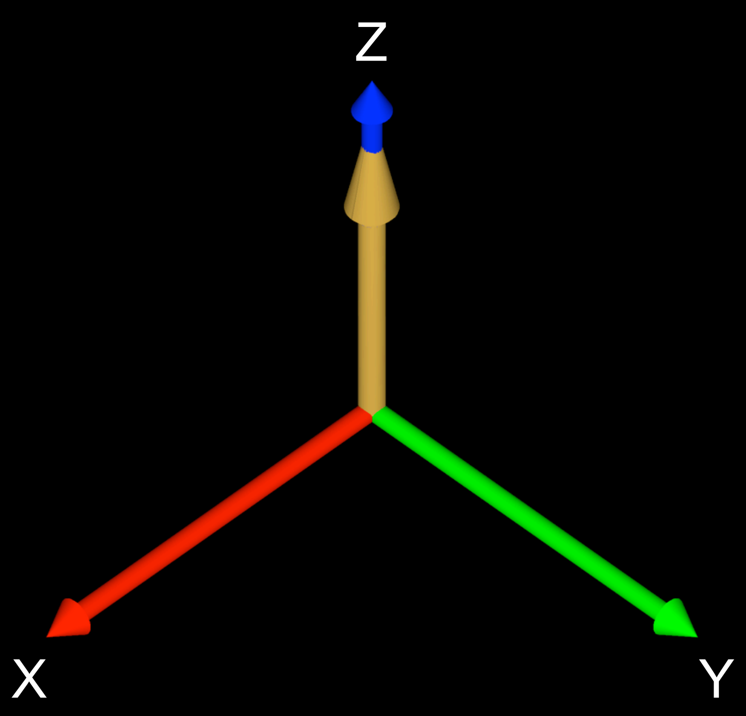
MRI Systems II – B_1



Lecture #3 Summary - RF Pulses

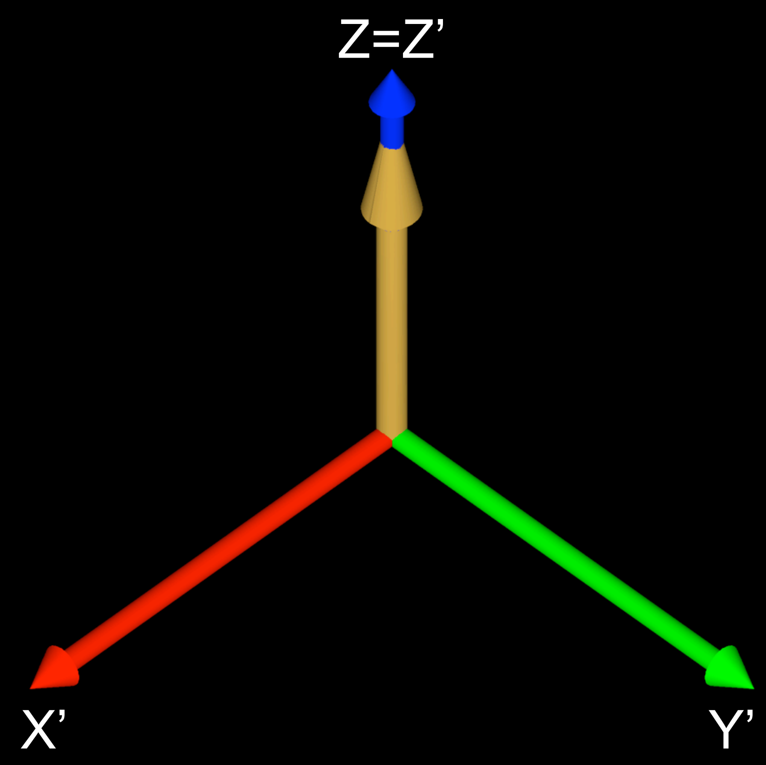
- **The rotating frame simplifies the mathematics and permits more intuitive understanding.**

Laboratory Frame



Spins Precess

Rotating Frame



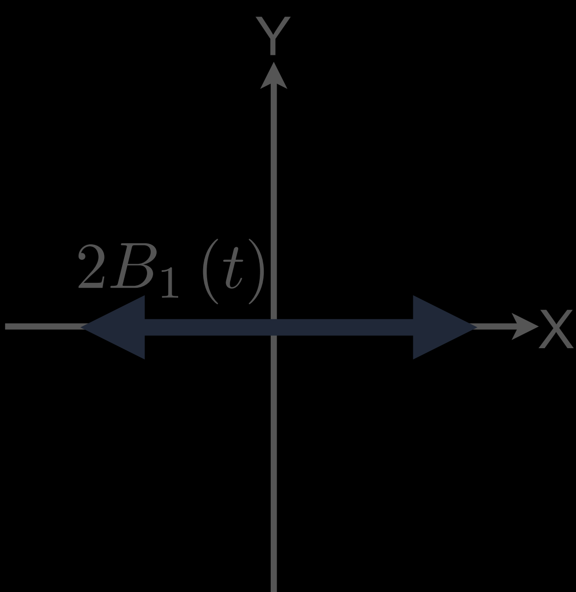
Observer Precesses

Note: Both coordinate frames share the same z-axis.

Circularly Polarized Fields

Linear Polarization | CW Circular Polarization | CCW Circular Polarization

$$2B_1^e(t)\cos(\omega_{RF}t)\,\hat{i} = B_1^e(t)\left[\cos(\omega_{RF}t)\,\hat{i} - \sin(\omega_{RF}t)\,\hat{j}\right] + B_1^e(t)\left[\cos(\omega_{RF}t)\,\hat{i} + \sin(\omega_{RF}t)\,\hat{j}\right]$$



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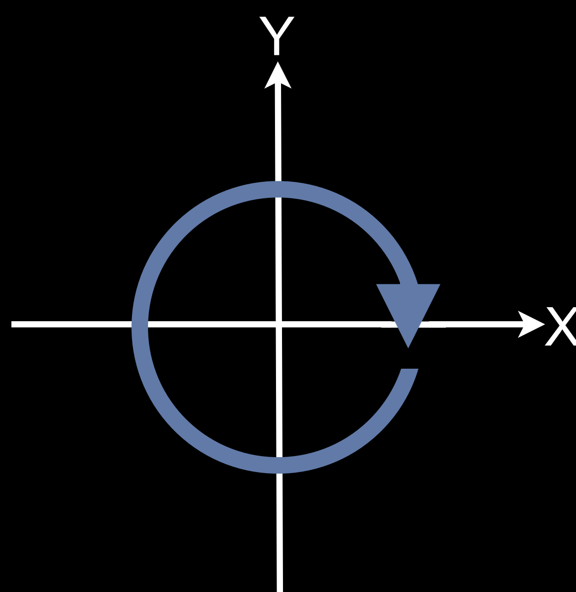

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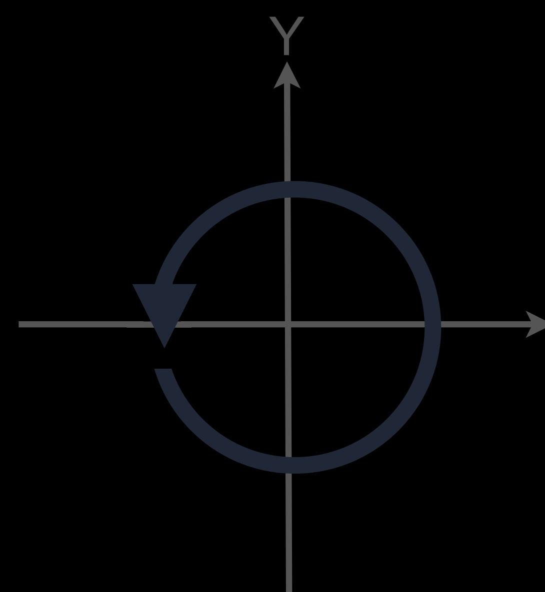


On-Resonance
Excitation+Heating

Very Off-Resonance
Heating

First Generation MRI Systems Used Linear Polarization

Modern MRI Systems Only Use CW Circular Polarization

Modern MRI Systems Don't Apply The CCW Field

Arrow indicates direction of B-field.

Equation of Motion

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats). [Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \right)$$

Equation of motion for an ensemble of spins (isochromats). [Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Free Precession in the Rotating Frame without Relaxation

$$\begin{aligned}\vec{B}_{eff} &= \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \\ &= \frac{-\gamma B_0 \hat{k}'}{\gamma} + B_0 \hat{k}' \\ &= 0\end{aligned}$$

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\frac{d\vec{M}_{rot}}{dt} = \begin{vmatrix} \hat{i}' & \hat{j}' & \hat{k}' \\ M_{x'} & M_{y'} & M_{z'} \\ 0 & 0 & 0 \end{vmatrix}$$

$$\frac{dM_{x'}}{dt} = 0$$

$$\frac{dM_{y'}}{dt} = 0$$

$$\frac{dM_{z'}}{dt} = 0$$

Forced Precession in the Rotating Frame without Relaxation

$$\vec{B}_{eff} = \frac{\vec{\omega_{rot}}}{\gamma} + \vec{B}_{rot}$$

$$= \frac{\vec{\omega_{rot}}}{\gamma} + B_0\hat{k}' + B_1^e(t)\hat{i}'$$

$$= B_1^e(t)\hat{i}'$$

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma\vec{B}_{eff}$$

$$\frac{dM_{x'}}{dt} = 0$$

$$\frac{dM_{y'}}{dt} = \gamma B_1^e(t)M_{z'}$$

$$\frac{dM_{z'}}{dt} = -\gamma B_1^e(t)M_{y'}$$

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma\vec{B}_{eff}$$

$$= \vec{M}_{rot} \times \gamma B_1^e(t)\hat{i}'$$

$$= \begin{vmatrix} \hat{i}' & \hat{j}' & \hat{k}' \\ \vec{M}_{x'} & \vec{M}_{y'} & \vec{M}_{z'} \\ \gamma B_1^e(t) & 0 & 0 \end{vmatrix}$$

Forced Precession in the Rotating Frame without Relaxation

$$M_{x'} = M_x^0 = 0$$

$$M_{y'} = M_z^0 \sin\left(\int_0^t B_1^e(\tau)d\tau\right)$$

$$M_{z'} = M_z^0 \cos\left(\underbrace{\int_0^t B_1^e(\tau)d\tau}\right)$$

This is the flip angle.

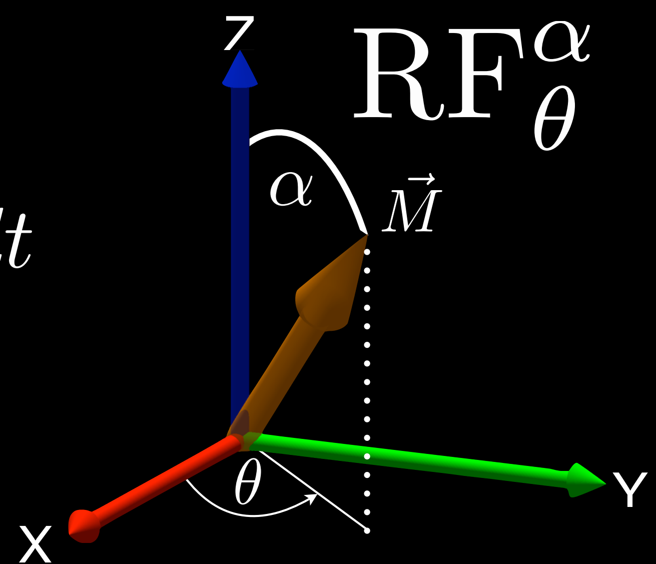
Lecture #3 Summary - RF Pulses

$$\mathbf{R}_\theta^\alpha = \mathbf{R}_Z(-\theta)\,\mathbf{R}_X(\alpha)\,\mathbf{R}_Z(\theta)$$
$$= \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha \end{bmatrix}$$

RF Pulse Operator

$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)\,dt$$

$$\mathrm{RF}_\theta^\alpha$$



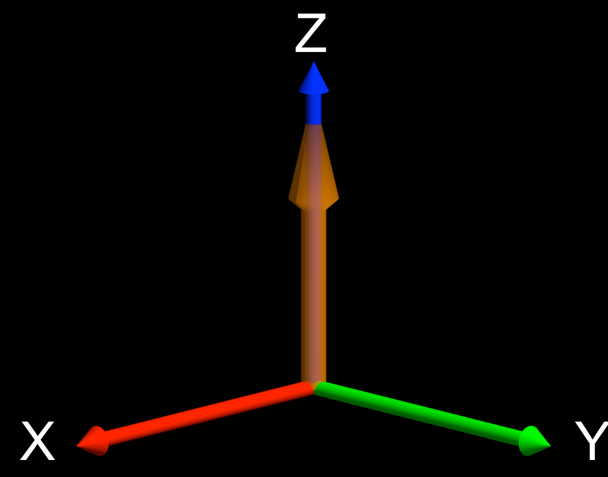
Choosing the flip angle.

Lecture #3 Summary - RF Pulses

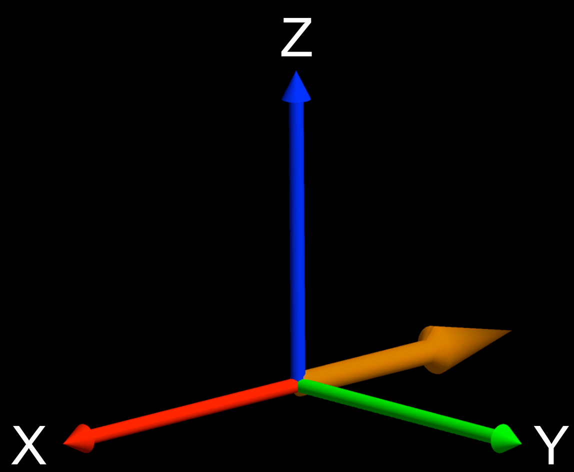
$$\mathrm{RF}^{\alpha}_{\theta}$$

α → Flip Angle

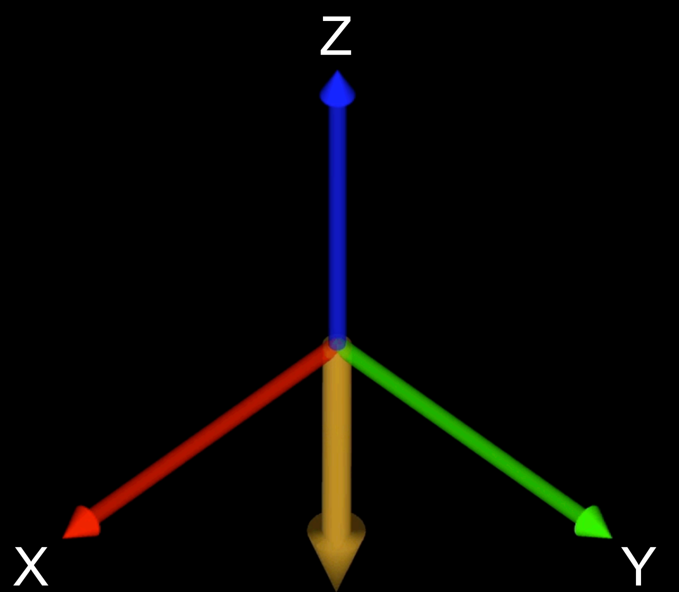
θ → Phase



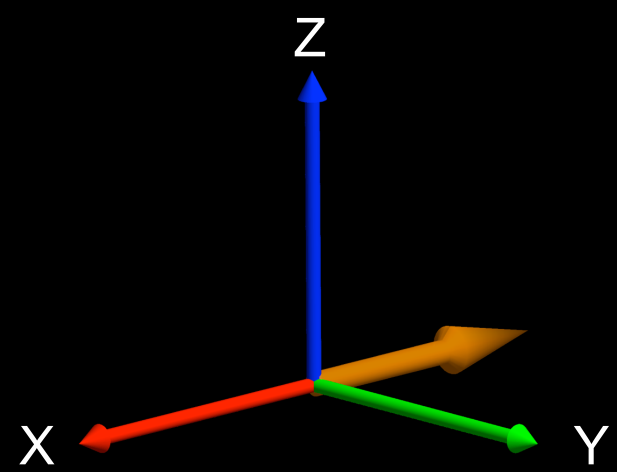
Initial Condition



Condition “A”



Condition “B”



Condition “C”

B-fields induce left-handed nutation!

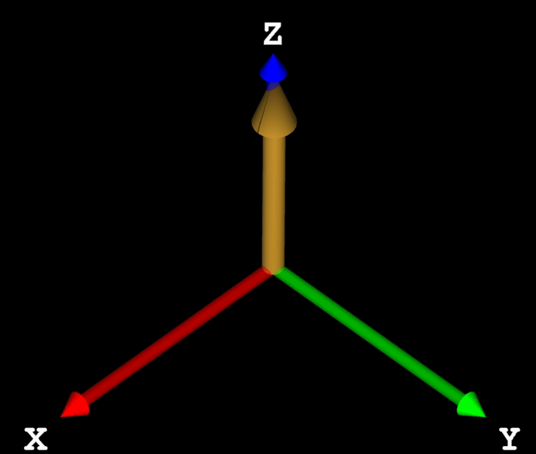
Types of RF Pulses

Types of RF Pulses

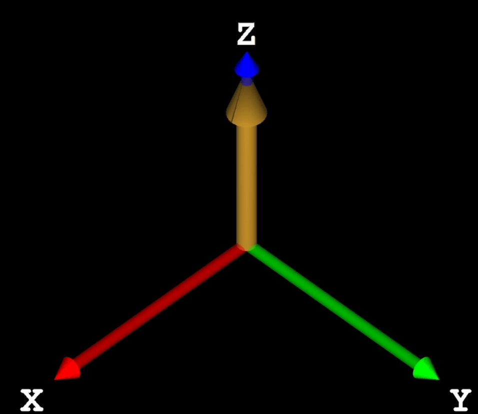
- Excitation Pulses
- Inversion Pulses
- Refocusing Pulses
- Saturation Pulses
- Spectrally Selective Pulses
- Spectral-spatial Pulses
- Adiabatic Pulses

Excitation Pulses

- **Tip M_z into the transverse plane**
- **Typically 200μs to 5ms**
- **Non-uniform across slice thickness**
  - Imperfect slice profile
- **Non-uniform within slice**
  - Termed B_1 inhomogeneity
  - Non-uniform signal intensity across FOV



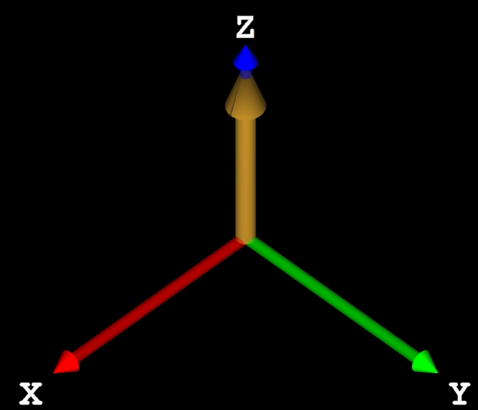
90° Excitation Pulse



Small Flip Angle Pulse

Inversion Pulses

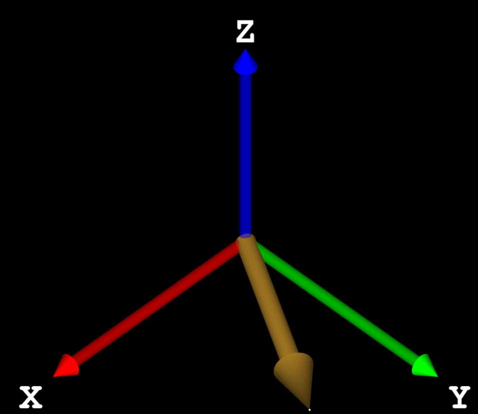
- **Typically, 180° RF Pulse**
  - non-180° that still results in $-M_Z$
- **Invert M_Z to $-M_Z$**
  - Ideally produces no M_{XY}
- **Hard Pulse**
  - Constant RF amplitude
  - Typically non-selective
- **Soft (Amplitude Modulated) Pulse**
  - **Frequency/spatially/spectrally selective**
- **Typically followed by a crusher gradient**



180° Inversion Pulse

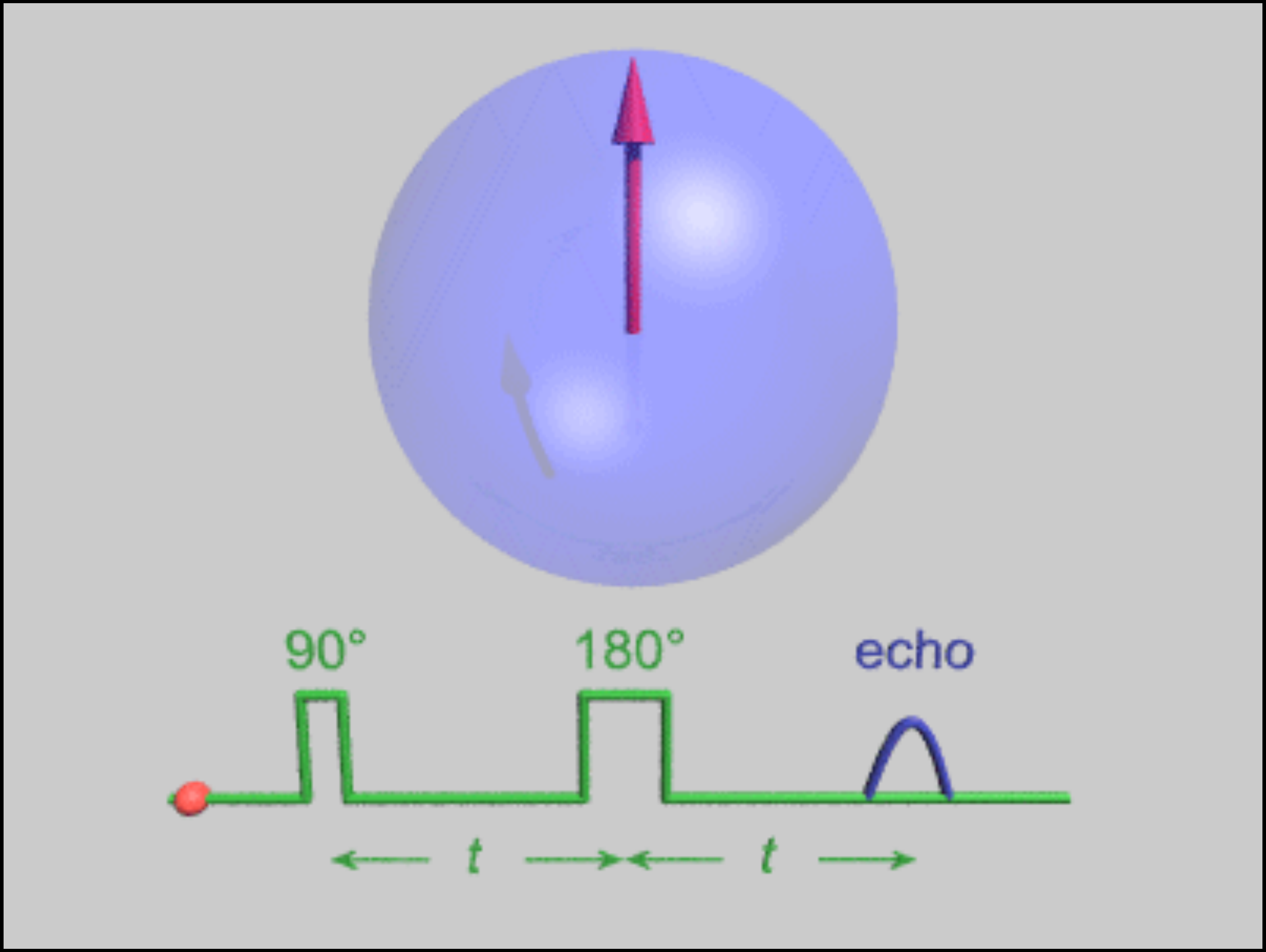
Refocusing Pulses

- **Typically, 180° RF Pulse**
  - Provides optimally refocused M_{XY}
  - Largest spin echo signal
- **Refocus spin dephasing due to**
  - imaging gradients
  - local magnetic field inhomogeneity
  - magnetic susceptibility variation
  - chemical shift
- **Typically followed by a crusher gradient**



180° Refocusing Pulse

Spin Echo - Refocusing



http://en.wikipedia.org/wiki/File:HahnEcho_GWM.gif

To The Board…

Bloch Equations & Relaxation

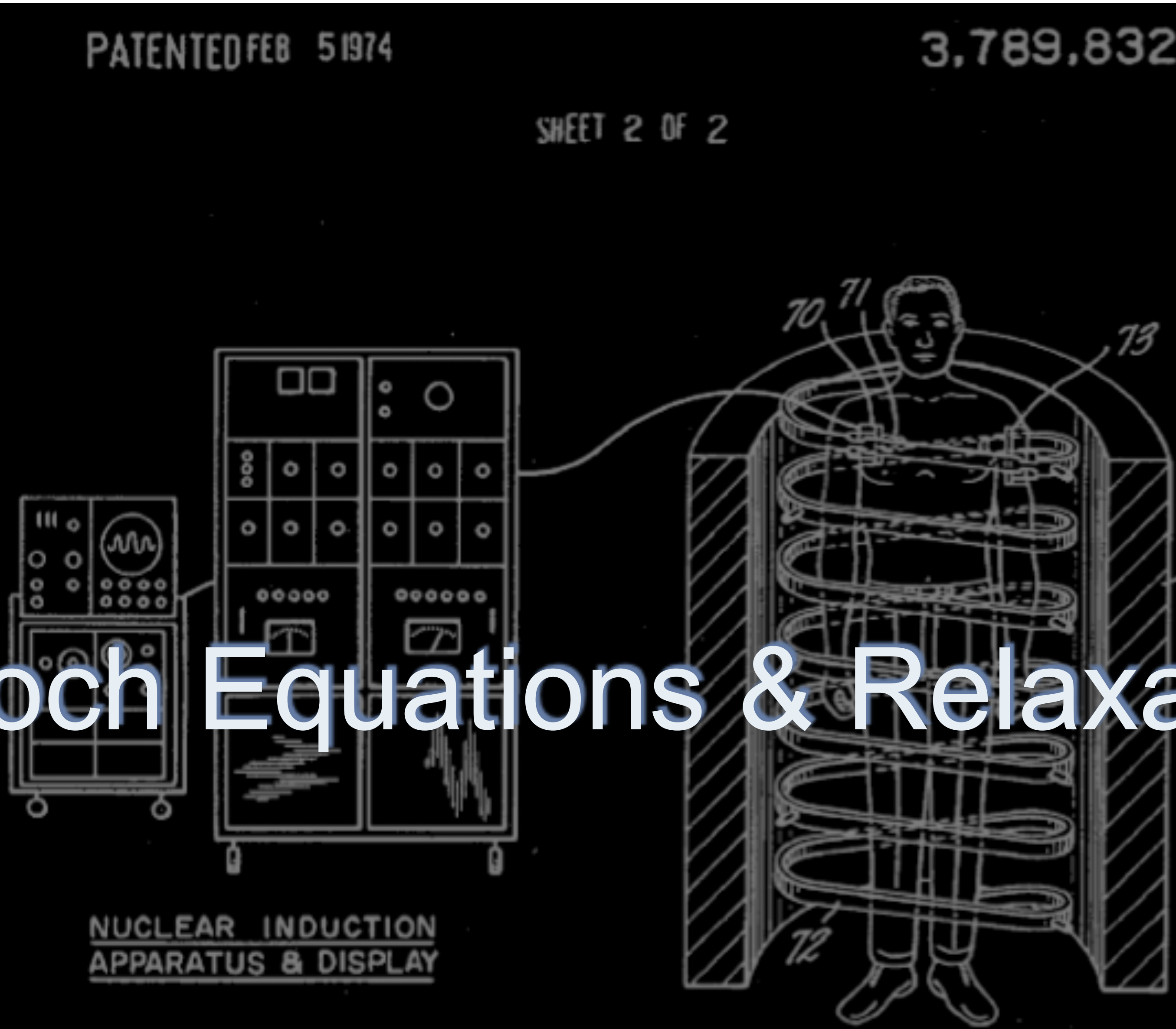


FIG. 2

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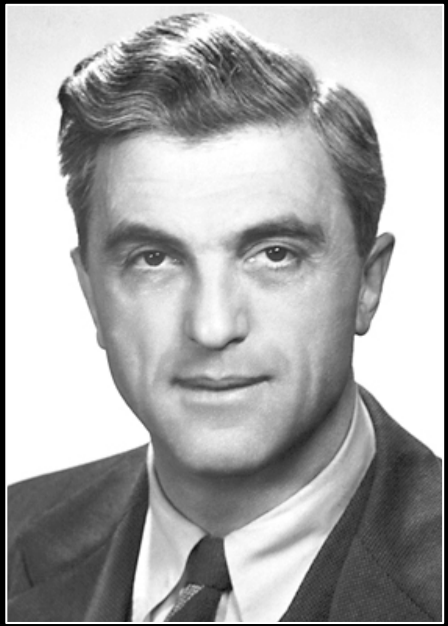
Lecture #4 Learning Objectives

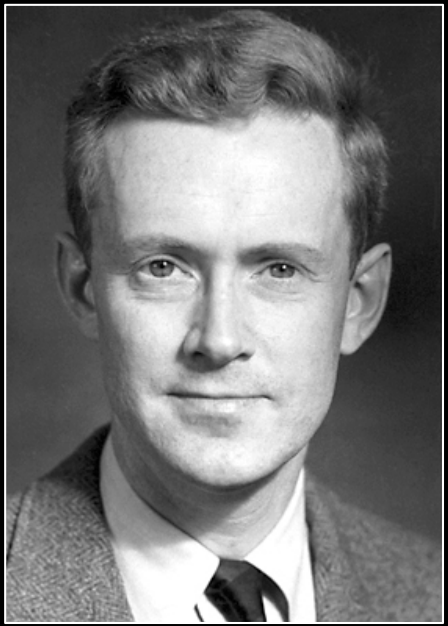
- Understand the terms in the phenomenological Bloch equations.
- Remember a few specific T1 and T2 values and understand what T1 and T2 govern.
- Distinguish between free and forced precession.
- Distinguish between the lab and rotating frames.
- Understand the importance of:
  - Free precession with relaxation in the rotating frame.
  - Forced precession without relaxation in the rotating frame.
- Appreciate the advantage of homogeneous coordinates and how to use them.

1952 Nobel Prize in Physics

"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"



Felix Bloch
b. 23 Oct 1905
d. 10 Sep 1983



Edward Purcell
b. 30 Sep 1912
d. 07 Mar 1997

Bloch Equations with Relaxation

$$\frac{d\vec{\mathrm{M}}}{dt} = \vec{\mathrm{M}} \times \gamma\vec{\mathrm{B}} - \frac{M_x\hat{\mathrm{i}} + M_y\hat{\mathrm{j}}}{T_2} - \frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}$$

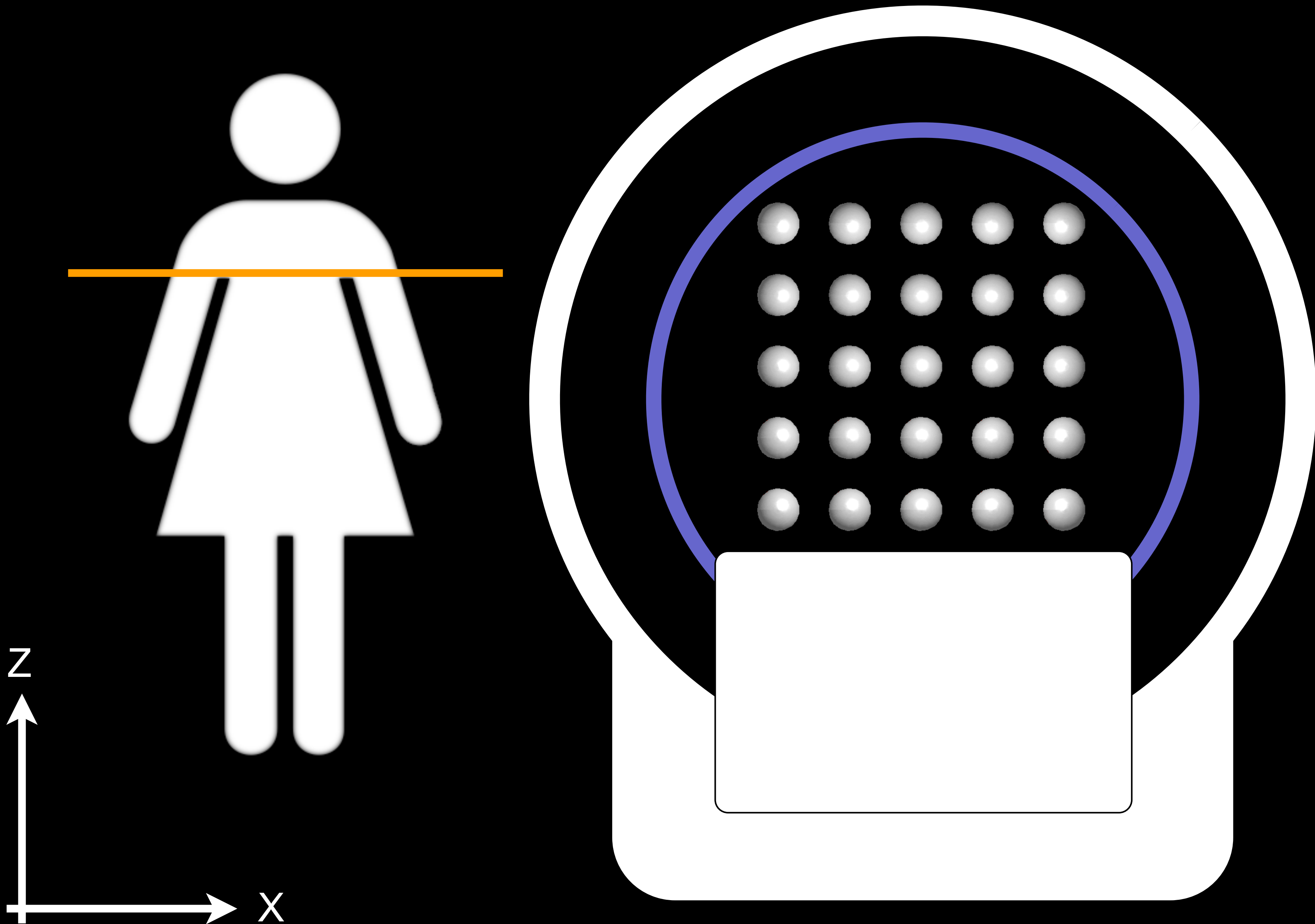
- **Differential Equation**
  - Ordinary, Coupled, Non-linear
- **No analytic solution, in general.**
  - Analytic solutions for simple cases.
  - Numerical solutions for all cases.
- **Phenomenological**
  - Exponential behavior is an approximation.

Bloch Equations - Lab Frame

$$\frac{d\vec{\mathrm{M}}}{dt} = \underbrace{\vec{\mathrm{M}} \times \gamma\vec{\mathrm{B}}}_{\text{Precession}} - \underbrace{\frac{M_x\hat{\mathrm{i}} + M_y\hat{\mathrm{j}}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}}_{\text{Longitudinal Relaxation}}$$

- Precession
  - Magnitude of M unchanged
  - Phase (rotation) of M changes due to B
- Relaxation
  - T_1 changes are slow O(100ms)
  - T_2 changes are fast O(10ms)
  - Magnitude of M can be ZERO
- Diffusion
  - Spins are thermodynamically driven to exchange positions.
  - Bloch-Torrey Equations

Excitation and Relaxation



The magnetization relaxes after excitation (forced precession).

Bloch Equations – Rotating Frame

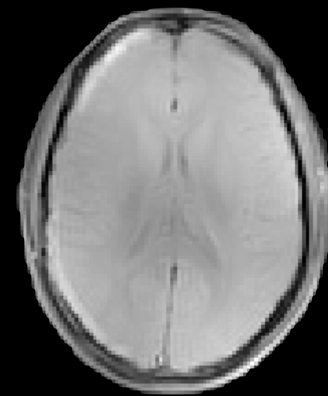
$$\frac{\partial \vec{M}_{rot}}{\partial t} = \underbrace{\gamma \vec{M}_{rot} \times \vec{B}_{eff}}_{\text{“Precession”}} - \underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

- $\vec{B}_{eff}$: Effective B-field that *M* experiences in the rotating frame.
- $\frac{\vec{\omega}_{rot}}{\gamma}$: Fictitious field that demodulates the apparent effect of B_0
- $\vec{B}_{rot}$: Applied B-field in the rotating frame.

T1 and T2 Values

Tissue	T1 [ms]	T2 [ms]
gray matter	925	100
white matter	790	92
muscle	875	47
fat	260	85
kidney	650	58
liver	500	43
CSF	2400	180



TI=25ms
TE=12ms

TI=200ms
TE=12ms

TI=500ms
TE=12ms

TI=1000ms
TE=12ms

Each tissue as "unique" relaxation properties.

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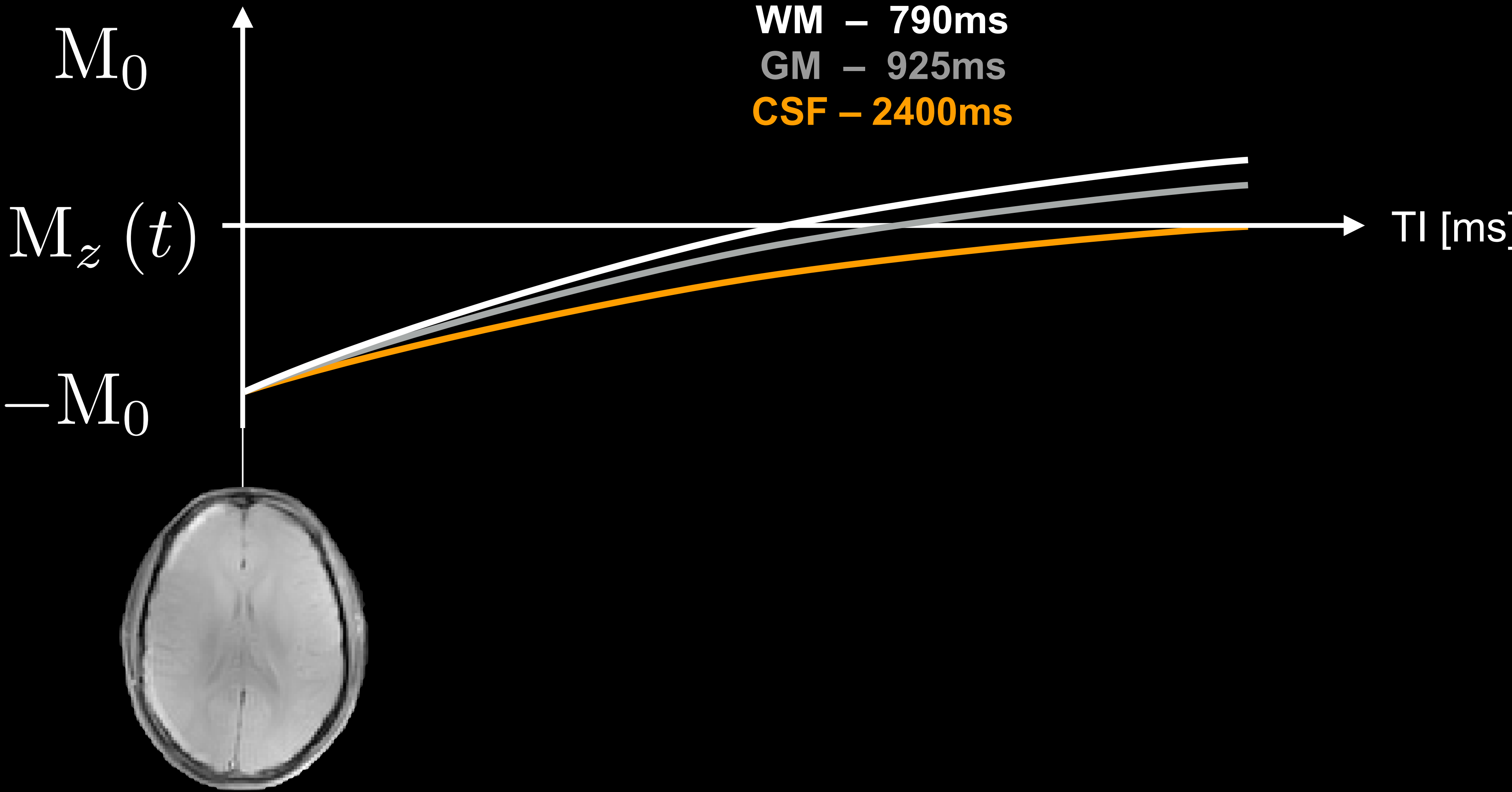
T_1-weighted MRI with Inversion Recovery



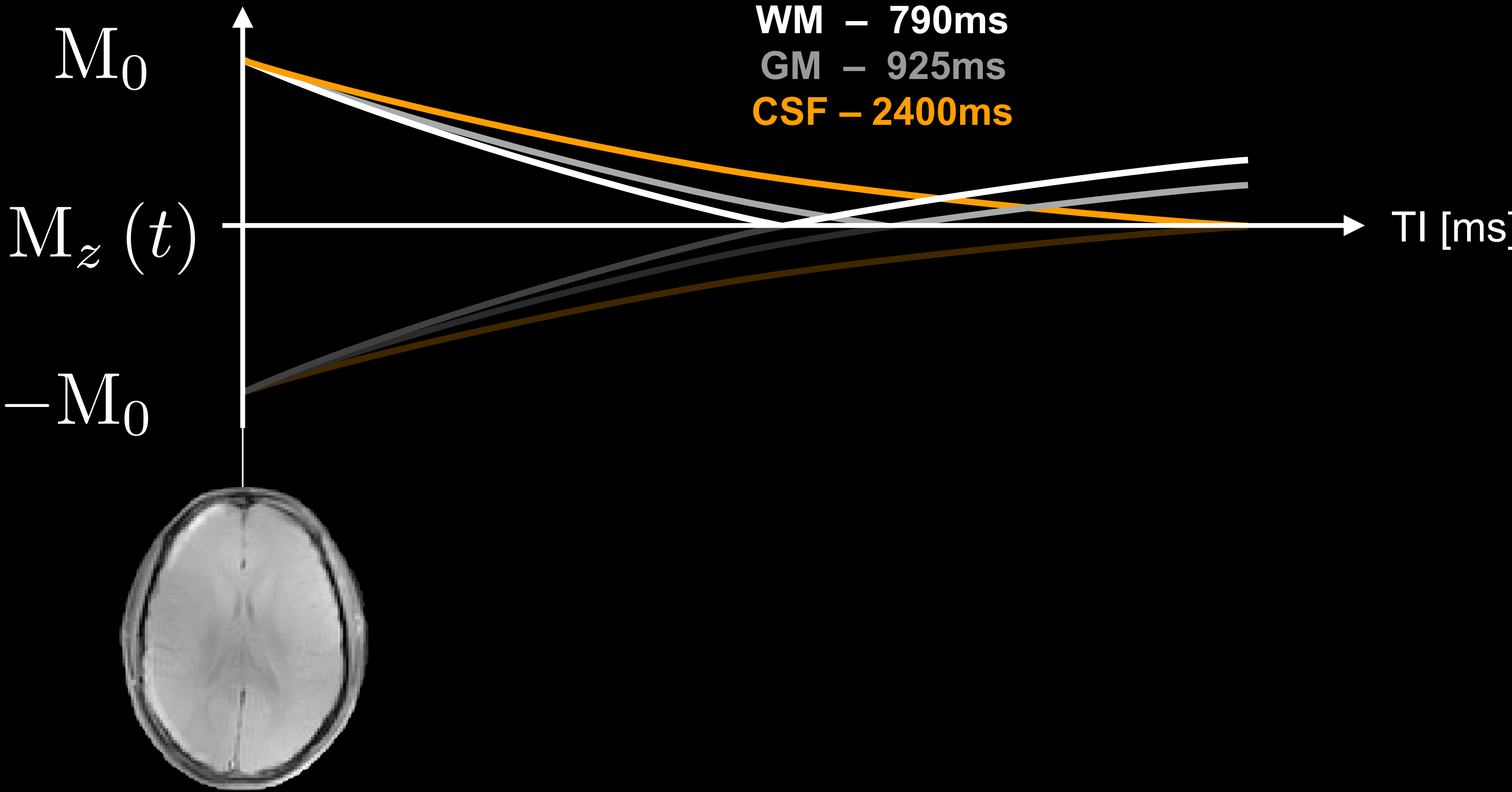
TI=25ms

David Geffen School of Medicine

M_z recovery after an inversion pulse (180° RF).

UCLA Radiology

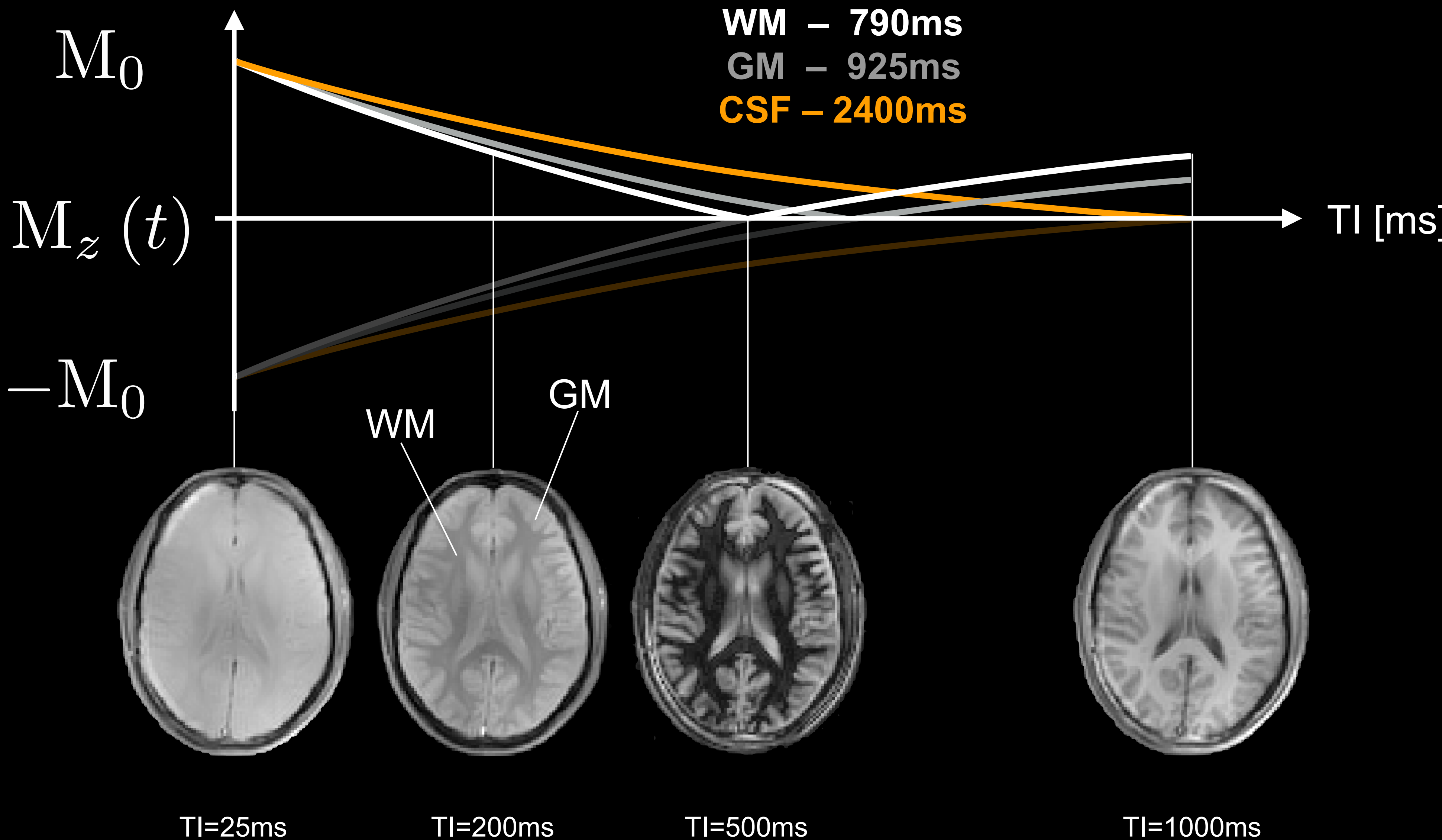
T_1-weighted MRI with Inversion Recovery



TI=25ms

MRI measures the magnitude of the magnetization.

T_1-weighted MRI with Inversion Recovery



MRI measures the magnitude of the magnetization.

T_1 & T_2 Relaxation

Tissue	T_1 [ms]	T_2 [ms]
gray matter	925	100
white matter	790	92

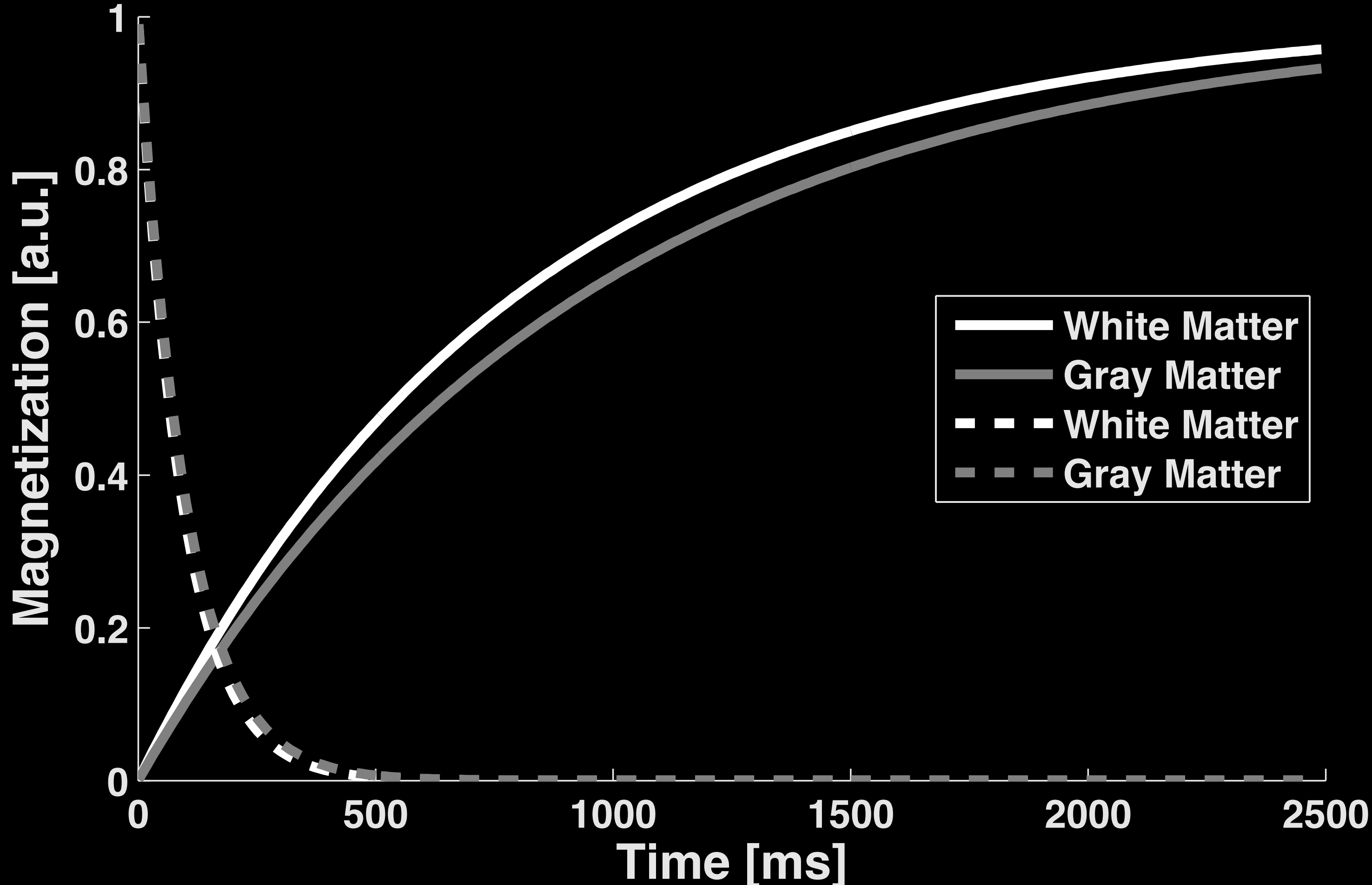
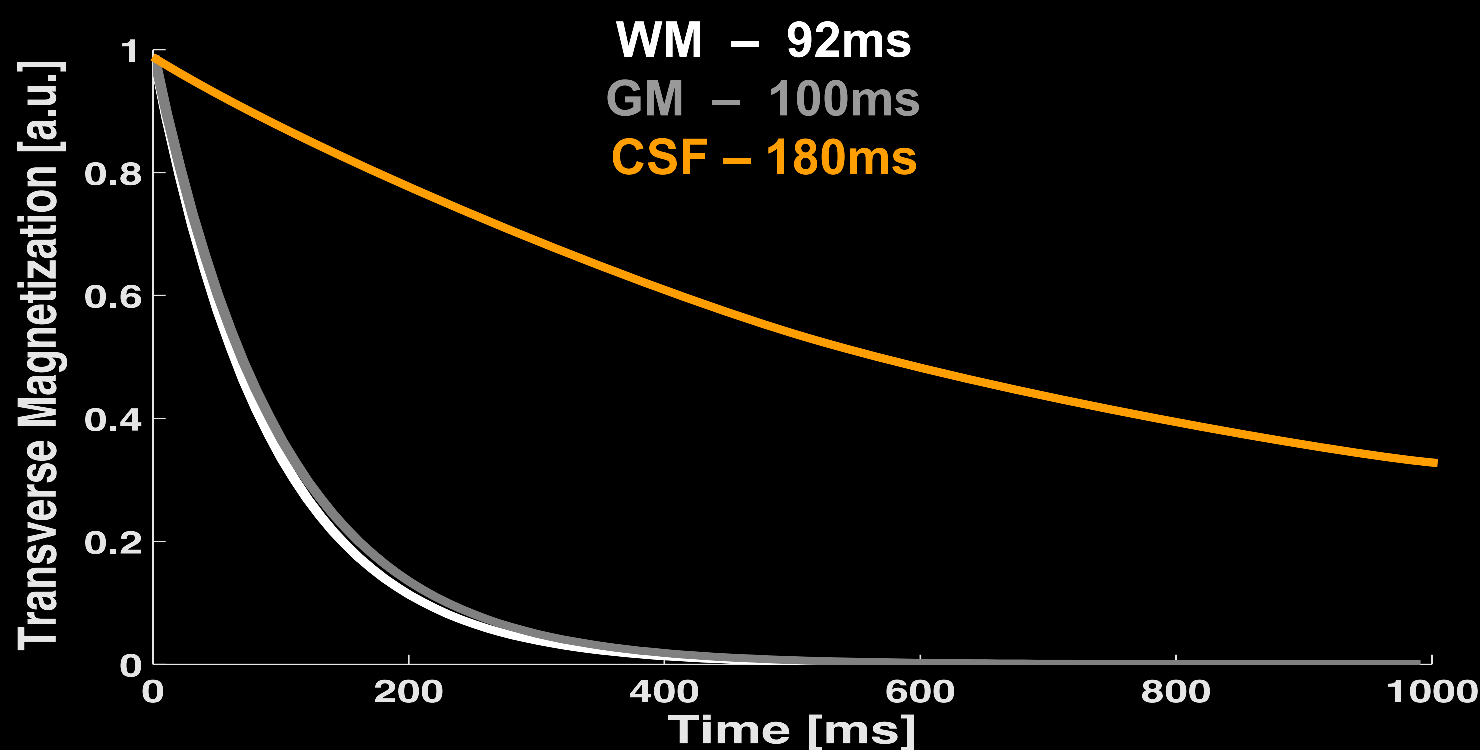


Image contrast is all about taking a "snapshot" at the right time.

T_2-weighted MRI



TE=12ms TE=47ms TE=106ms TE=153ms TE=235ms

Long T_2 is bright on T_2-weighted (long TE) images.

Free Precession
in the Rotating Frame
with Relaxation

Free Precession in the Rotating Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{\mathrm{B}}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{\mathrm{B}}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k}$$

Conventional MRI Systems

$$\vec{\mathrm{B}}_{rot} = \mathrm{B}_0 \hat{k}$$

Free Precession

$$\vec{\mathrm{B}}_{eff} = 0$$

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

- **No precession**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, separable ODEs!**
  - Homogeneous in x and y.
  - Inhomogeneous in z.

The precessional term drops out in the rotating frame.

To The Board...

Forced Precession
in the Rotating Frame
with Relaxation

Forced Precession in the Rotating Frame With Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{\mathrm{B}}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{\mathrm{B}}_{rot} \qquad \vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k}$$

$$\vec{B}_{rot} = B_0 \hat{k}' + B_1^e(t)(\cos\theta \hat{i}' + \sin\theta \hat{j}')$$

Forced Precession

$$\vec{B}_{eff} = B_1^e(t)(\cos\theta \hat{i}' + \sin\theta \hat{j}')$$

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

The precessional term drops out in the rotating frame.

To The Board…

Forced Precession in the Rotating Frame with Relaxation

- RF pulses are short
  - 100μs to 5ms
- Relaxation time constants are long
  - T_1 O(100s) ms
  - T_2 O(10s) ms
- Complicated Coupling
- Best suited for simulation

Free? Forced? Relaxation?

- **We've considered all combinations of:**
  - **Free and forced precession**
  - **With and without relaxation**
  - **Laboratory and rotating frames**
- **Which one's concern M219 the most?**
  - **Free precession in the rotating frame with relaxation**
  - **Forced precession in the rotating frame without relaxation.**
- **We can, in fact, simulate all of them…**

Matlab

Bloch Equation Simulations

Bloch Equations (Rotating Frame, Free Precession)

$$\frac{d\vec{\mathrm{M}}}{dt} = -\frac{\mathrm{M_x}\hat{i} + \mathrm{M_y}\hat{j}}{T_2} - \frac{(\mathrm{M_z} - \mathrm{M_0})\,\hat{k}}{T_1}$$

Bloch Equations (Rotating Frame, Free Precession)

$$\frac{d\vec{\mathrm{M}}}{dt} = -\frac{\mathrm{M_x}\hat{i} + \mathrm{M_y}\hat{j}}{T_2} - \frac{(\mathrm{M_z} - \mathrm{M_0})\,\hat{k}}{T_1}$$

⇓

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_2} & 0 & 0 \\ 0 & -\frac{1}{T_2} & 0 \\ 0 & 0 & -\frac{1}{T_1} \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{M_0}{T_1} \end{bmatrix}$$

Bloch Equations (Rotating Frame, Free Precession)

$$\frac{d\vec{\mathrm{M}}}{dt} = -\frac{\mathrm{M_x}\hat{i} + \mathrm{M_y}\hat{j}}{T_2} - \frac{(\mathrm{M_z} - \mathrm{M_0})\,\hat{k}}{T_1}$$

⇓

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_2} & 0 & 0 \\ 0 & -\frac{1}{T_2} & 0 \\ 0 & 0 & -\frac{1}{T_1} \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{M_0}{T_1} \end{bmatrix}$$

⇓

$$\frac{d\vec{\mathrm{M}}}{dt} = \alpha\vec{\mathrm{M}} + \beta$$

An *affine transformation* between two vector spaces consists of a translation followed by a linear transformation.

http://en.wikipedia.org/wiki/Affine_transformation

Homogenous Coordinates

Homogenous coordinates allow us to transform an affine (non-linear) equation in 3D to a linear equation in 4D.

Affine | **Linear**

$$\frac{d\vec{\mathrm{M}}}{dt} = \alpha\vec{\mathrm{M}} + \beta \quad \longleftrightarrow \quad \frac{d\vec{\mathrm{M}}_H}{dt} = \mathbf{T}_H\vec{\mathrm{M}}_H$$

Now we can use the machinery of linear algebra for writing out the Bloch Equation mechanics.

Homogenous Coordinate Expressions

Cartesian Coordinates

$$\vec{\mathrm{M}} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Homogeneous Coordinates

$$\vec{\mathrm{M}}_H = \begin{bmatrix} M_x \\ M_y \\ M_z \\ 1 \end{bmatrix}$$

$$\xrightarrow{Augment}$$

$$\xleftarrow[Reduce]{}$$

$$\mathbf{T} = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix}$$

$$\mathbf{T}_H = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} & T_{xt} \\ T_{yx} & T_{yy} & T_{yz} & T_{yt} \\ T_{zx} & T_{zy} & T_{zz} & T_{zt} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotating Frame Bloch Equations (Free Precession)

$$\frac{d\vec{\mathrm{M}}}{dt} = -\frac{\mathrm{M_x}\hat{i} + \mathrm{M_y}\hat{j}}{T_2} - \frac{(\mathrm{M_z} - \mathrm{M_0})\,\hat{k}}{T_1}$$

⇓

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_2} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_1} & \frac{M_0}{T_1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \\ 1 \end{bmatrix}$$

⇓

$$\frac{d\vec{\mathrm{M}}_H}{dt} = \mathbf{T}_H \vec{\mathrm{M}}_H$$

Advantages/Disadvantages

+ 1:1 Correlation with pulse diagram
+ Simple to implement (Matlab!)
+ Not *ad hoc*
+ Provides understanding in complex systems

- Masks understanding in simple systems
- Reduction to algebraic expression is cumbersome
- Discrete (not continuous)
- Perfect simulations are very difficult
    - Must consider assumptions
- Image Prep vs. Imaging

Bulk Magnetization - Precession

$$B_{0,H} = \begin{bmatrix} \cos\gamma B_0 t & \sin\gamma B_0 t & 0 & 0 \\ -\sin\gamma B_0 t & \cos\gamma B_0 t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} M_x(0_+) \\ M_y(0_+) \\ M_z(0_+) \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\gamma B_0 t & \sin\gamma B_0 t & 0 & 0 \\ -\sin\gamma B_0 t & \cos\gamma B_0 t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x(0_-) \\ M_y(0_-) \\ M_z(0_-) \\ 1 \end{bmatrix}$$

Homogeneous coordinate expression for precession.

RF Pulse Homogeneous Operator

$$\mathbf{RF}^{\alpha}_{\theta,H} = \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha & 0 \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha & 0 \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vec{\mathrm{M}}^{+}_{H} = \mathrm{RF}^{\alpha}_{\theta,H}\vec{\mathrm{M}}^{-}_{H}$$

$$\begin{bmatrix} M_x^+ \\ M_y^+ \\ M_z^+ \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha & 0 \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha & 0 \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x^- \\ M_y^- \\ M_z^- \\ 1 \end{bmatrix}$$

Relaxation Operator

$$\begin{bmatrix} M_x(0_+) \\ M_y(0_+) \\ M_z(0_+) \end{bmatrix} = \begin{bmatrix} e^{-\frac{t}{T_2}} & 0 & 0 \\ 0 & e^{-\frac{t}{T_2}} & 0 \\ 0 & 0 & e^{-\frac{t}{T_1}} \end{bmatrix} \begin{bmatrix} M_x(0_-) \\ M_y(0_-) \\ M_z(0_-) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0(1 - e^{-\frac{t}{T_1}}) \end{bmatrix}$$

$$\begin{bmatrix} M_x(0_+) \\ M_y(0_+) \\ M_z(0_+) \\ 1 \end{bmatrix} = \begin{bmatrix} e^{-\frac{t}{T_2}} & 0 & 0 & 0 \\ 0 & e^{-\frac{t}{T_2}} & 0 & 0 \\ 0 & 0 & e^{-\frac{t}{T_1}} & M_0(1 - e^{-\frac{t}{T_1}}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x(0_-) \\ M_y(0_-) \\ M_z(0_-) \\ 1 \end{bmatrix}$$

Relaxation Operator

$$\mathbf{E}\left(T_1, T_2, t, \mathrm{M}_0\right) = \begin{bmatrix} E_2 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_1 & \mathrm{M}_0\left(1 - E_1\right) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_1 = e^{-t/T_1} \qquad E_2 = e^{-t/T_2}$$

$$\vec{\mathrm{M}}^{+} = \mathrm{E}\left(T_1, T_2, t, \mathrm{M}_0\right) \vec{\mathrm{M}}^{-}$$

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*In general we drop the sub-scripted *H*

UCLA Radiology

B_0, RF Pulse, & Relaxation Operators

$$\vec{\mathrm{M}}^{+} = B_{0,H}\vec{\mathrm{M}}^{-}$$

$$\vec{\mathrm{M}}^{+} = \mathrm{RF}_{\theta}^{\alpha}\vec{\mathrm{M}}^{-}$$

$$\vec{\mathrm{M}}^{+} = \mathrm{E}\left(T_1, T_2, t, \mathrm{M}_0\right)\vec{\mathrm{M}}^{-}$$

Thanks

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