M219, Winter 2018

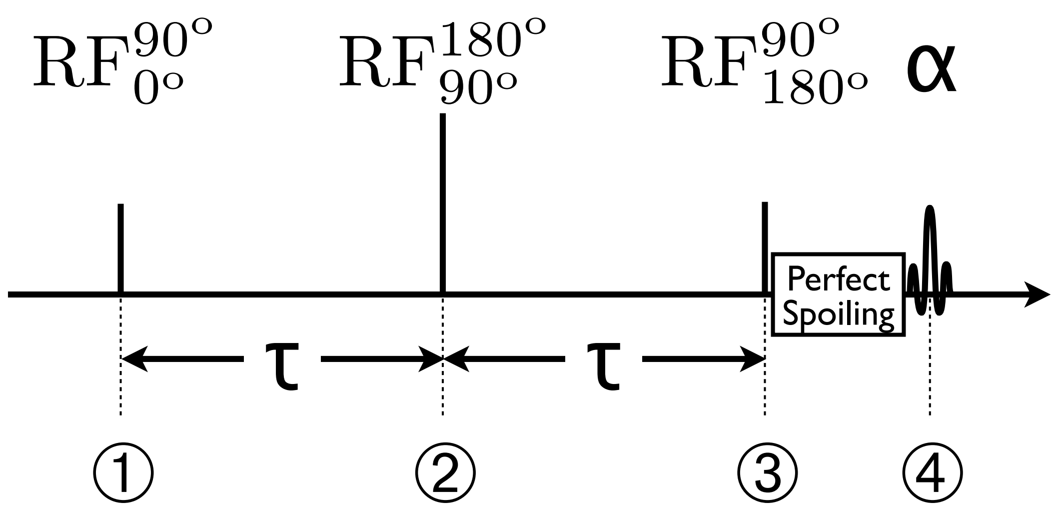
Homework Assignment #2 (15 Points)

Due Tuesday, February 6th by 9pm

To submit the assignment, e-mail `DEnnis@mednet.ucla.edu` a PDF entitled `M219_HW02_[First Initial]_[Last Name].pdf` (*e.g.* `M219_HW02_D_Ennis.pdf`). Please only submit neat and clear solutions. Late assignments will be discounted by $e^{-t/\tau}$, where $\tau = 72$ hours.

For all problems – Clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually. As appropriate, each solution should be obtained using Matlab. Please comment and submit your code using the `publish.m` function for each problem.

1 Pulse Sequence Diagrams (5 points)



A. The pulse sequence above can be used to prepare the magnetization prior to a gradient-echo pulse sequence. Derive M_z and M_{xy} immediately before and after each of the events shown. Assume the "perfect" spoiling takes no time (i.e. $t_{3+} \approx t_{4-}$) and results in zero transverse magnetiztion. Include off-resonance.(4 points)

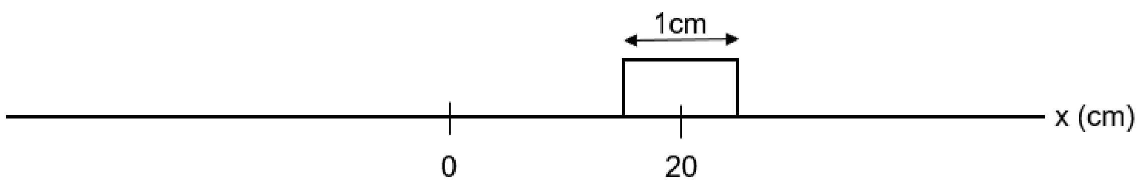
B. What is the effect of this preparation sequence on the image contrast? How does adjusting τ alter image contrast? (1/2 point)

C. C. How is the image contrast changed if the 180^o RF-pulse is removed? (1/2 point)

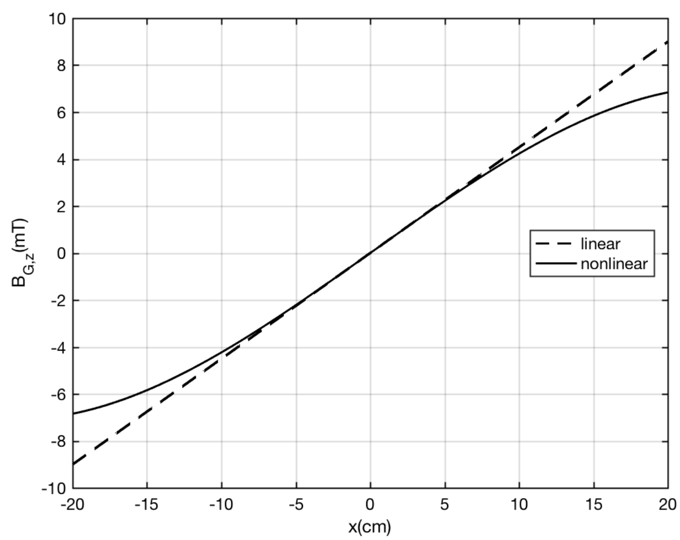
2 Gradient Nonlinearity (5 points)

A. Given a gradient field G_x $[\frac{mT}{m}]$, derive the frequency of a spin as a function of its position, $\omega(x)$, and similarly the position of a spin as a function of its frequency, $x(\omega)$. [2 points]

B. Give the range of frequencies for the following object, given that $B_0 = 1.5\text{T}$ and $G_x = 80\frac{mT}{m}$ [1 point]:

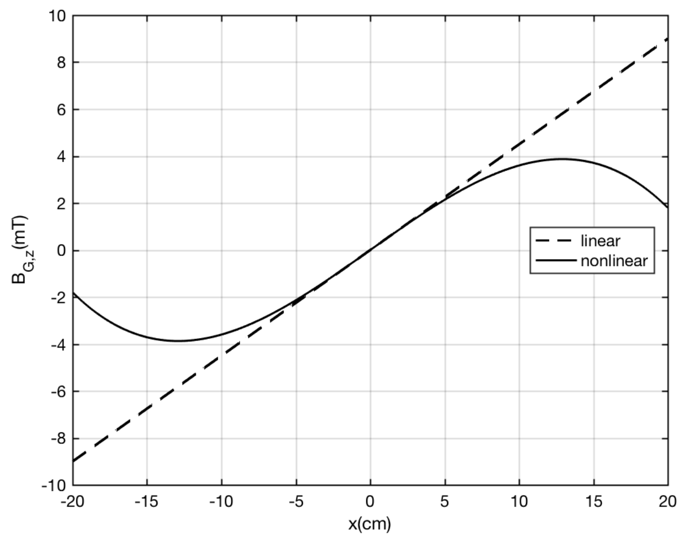


C. In a real MRI scanner, gradient coils are not completely linear. Redo part (B) with the following non-linear gradient: $B_{G,z}(x) = G_x x + Ax^3$ where $A = -2.7x10^2\frac{mT}{m^3}$ [1 point]:



D. Imagine we acquire our object frequencies using the non-linear gradient. We then use those frequencies to determine where the object is, but we assume a linear gradient for this calculation. Calculate how the object differs from the actual object (in size and position). [1 point]

E. Based on what was seen in parts (A)-(D), what might be a potential problem with the following non-linear gradient? [1 point]



3 Spin Echo Simulation and Validation (5 points)

A. In class we derived the state of the magnetization at several time points during the spin echo pulse sequence. Now, derive a similar result, but for the second TR. The goal is to define the analytic expressions for the for the magnetization immediate before and after each event from right before the second 90° pulse to right before the third 90° pulse. Include off-resonance and assume perfect spoiling.

B. Plot the analytic result for the first two complete TRs, for Tissue-1 (T_1=1200ms, T_2= 150ms) and Tissue-2 (T_1=400ms, T_2=50ms). Use TE=75ms, TR=5000ms. What kind of image contrast does this produce? Explain.

C. Simulate the same pulse sequence in Matlab (rotating frame) and compare the differences between the analytic and simulated signals. The differences should be small, but explain any discrepancy. No need to account for off-resonance.