# Problem 1

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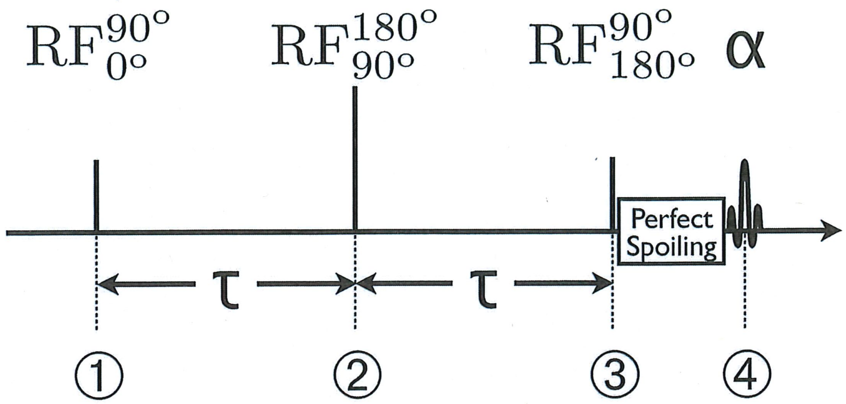
III. PULSE SEQUENCE DIAGRAMS (5-POINTS)




A. The pulse sequence above is used to prepare the magnetization prior to a gradient-echo pulse sequence. Derive $M_z$ and $M_{xy}$ immediately before and after each of the events shown. Assume the "perfect" spoiling takes no time (i.e. $t_{3+} \approx t_{4-}$).

*

+1/4 $M_z^{(1)}(0-) = M_0$ ; $M_z^{(1)}(0+) = 0$

+1/4 $M_{xy}^{(1)}(0-) = 0$ ; $M_{xy}^{(1)}(0+) = M_0$

+1/4 $M_z^{(2)}(0-) = M_0(1-e^{-\tau/T_1})$ ; $M_z^{(2)}(0+) = -M_0(1-e^{-\tau/T_1})$

+1/4 $M_{xy}^{(2)}(0-) = M_0 e^{-\tau/T_2^*}$ or $= M_0 e^{-\tau/T_2} e^{-i\phi_{off}}$ ; $M_{xy}^{(2)}(0+) = M_0 e^{-\tau/T_2} e^{+i\phi_{off}}$

+1/4 $M_z^{(3)}(0-) = -M_0(1-e^{-\tau/T_1})e^{-\tau/T_1} + M_0(1-e^{-\tau/T_1})$

$= -M_0 e^{-\tau/T_1} + M_0 e^{-2\tau/T_1} + M_0 - M_0 e^{-\tau/T_1}$

$= M_0 - 2M_0 e^{-\tau/T_1} + M_0 e^{-2\tau/T_1}$

+1/4 $M_{xy}^{(3)}(0-) = M_0 e^{-\tau/T_2} e^{+i\phi_{off}} e^{-\tau/T_2} e^{-i\phi_{off}}$

$= M_0 e^{-2\tau/T_2}$

+1/4 $M_z^{(3)}(0+) = M_0 e^{-2\tau/T_2}$

+1/4 $M_{xy}^{(3)}(0+) = M_0 - 2M_0 e^{-\tau/T_1} + M_0 e^{-2\tau/T_1}$

+1/2 $M_z^{(4)}(0-) = M_0 e^{-2\tau/T_2}$

+1/2 $M_{xy}^{(4)}(0-) = 0$

+1/2 $M_z^{(4)}(0+) = \cos\alpha \, M_0 e^{-2\tau/T_2}$

+1/2 $M_{xy}^{(4)}(0+) = \sin\alpha \, M_0 e^{-2\tau/T_2}$

# Problem 1

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A (continued).

B. What is the effect of this preparation sequence on the image contrast? How does adjusting $\tau$ alter image contrast?

+1/2

Adds pure $T_2$ (not $T_2^*$) weighting to the signal.

A longer $\tau$ adds more $T_2$- weighting

C. How is the image contrast changed if the 180° RF-pulse is removed?

+1/2

The sequence becomes $T_2^*$ weighted.

# Problem 2

## Table of Contents



```
clear all; clc; close all

gamma=42.58e6;
G=80e-3;
B0=1.5;
```

## Part A.

```
charlambda=char(947);
disp(['x=[w-gamma*B0]/(','gamma*G)']);
```

*x=[w-gamma*B0]/(gamma*G)*

## Part B.

```
x_cent_1=0.2;                                   %[m]
l1=0.01;                                        %[m]
x1(1,1)=x_cent_1-(l1/2);                        %[m]
x1(1,2)=x_cent_1+(l1/2);                        %[m]


Gl=G*x1(1,:);
w1(1,:)=gamma*(Gl+B0)./1E6;                      %[MHz]
disp(['The range of frequencies will be: w0= ',num2str(w1(1,1),5),'MHz
 & w1= ',num2str(w1(1,2),5),'MHz'])
```

*The range of frequencies will be: w0= 64.534MHz & w1= 64.568MHz*

## Part C.

```
A=-2.7e-1;

Gnl(1,:)=G*x1(1,:)+A*(x1(1,:).^3);

w2(1,:)=gamma*(Gnl(1,:)+B0)./1E6;                %[MHz]
disp(['The range of frequencies will be: w0''=
 ',num2str(w2(1,1),5),'MHz & w1''= ',num2str(w2(1,2),5),'MHz'])
```

*The range of frequencies will be: w0'= 64.449MHz & w1'= 64.469MHz*

# Problem 2

## Part D.

```
x2(1,:)=(1/(gamma*G))*(w2(1,:)*1E6)-B0/G;

disp(['The new positions will be: xo''=
 ',num2str(x2(1,1)*100,4),'cm',' & x1''=
 ',num2str(x2(1,2)*100,4),'cm'])

l2=x2(1,2)-x2(1,1);                                   %[m]
x_cent_2=(x2(1,2)+x2(1,1))/2;                         %[m]

Dx=(x_cent_1-x_cent_2)*100;                           %[cm]

disp(['The new lenght will be: ',num2str(l2*100,2),'cm',' and it will
 be shifted ',num2str(Dx,2),'cm'])
```

```
The new positions will be: xo'= 17cm & x1'= 17.59cm
The new lenght will be: 0.59cm and it will be shifted 2.7cm
```

## Part E.

```
disp('Same gradient values at multiple locations.Those frequencies
 cannot be solved for a single location')
```

```
Same gradient values at multiple locations.Those frequencies cannot be
 solved for a single location
```

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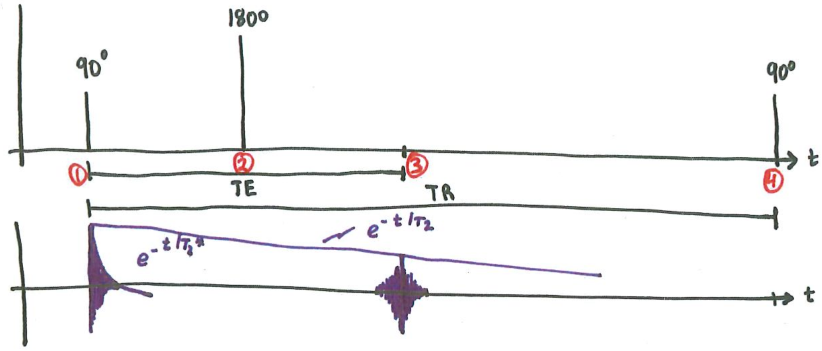
# Problem 3

## Part A.

Lecture #5




$M_{z'}^{(1)}(0_-) = M_z^0 = M_0$

$M_{x'y'}^{(1)}(0_-) = 0$

$M_{z'}^{(1)}(0_+) = 0$

$M_{x'y'}^{(1)}(0_+) = M_0$

$M_{z'}^{(2)}(0_-) = M_0(1 - e^{-TE/2T_1})$

$M_{x'y'}^{(2)}(0_-) = M_0 e^{-TE/2T_2} e^{-i\phi_{off}}$

$\phi_{off} = \omega_{off} t = \gamma \delta B_{off} t$

$M_{z'}^{(2)}(0_+) = -M_0(1 - e^{-TE/2T_1})$

$M_{x'y'}^{(2)}(0_+) = M_0 e^{-TE/2T_2} e^{+i\phi_{off}}$

$$M_{z'}^{(3)}(0_-) = M_z^0 e^{-t/T_1} + M_0(1 - e^{-t/T_1})$$

$$= -M_0(1 - e^{-TE/2T_1})e^{-TE/2T_1} + M_0(1 - e^{-TE/2T_1})$$

$$= -M_0 e^{-TE/2T_1} + M_0 e^{-TE/2T_1} e^{-TE/2T_1} + M_0 - M_0 e^{-TE/2T_1}$$

$$= M_0 (1 - 2e^{-TE/2T_1} + e^{-TE/T_1})$$

1: Steady state value; $2e^{-TE/2T_1}$: Inverted Recovery; $e^{-TE/T_1}$: Recovery since 90°

$M_{z'}^{(3)}(0_+) = M_{z'}^{(3)}(0_-)$

$$M_{x'y'}^{(3)}(0_-) = M_0 e^{-TE/2T_2} e^{+i\phi_{off}} e^{-i\phi_{off}} e^{-TE/2T_2}$$

$$= M_0 e^{-TE/T_2}$$

Not sensitive to off resonance

$$M_{z'}^{(4)}(0_-) = M_0(1 - 2e^{-TE/2T_1} + e^{-TE/T_1})e^{-(TR-TE)/T_1} + M_0(1 - e^{-(TR-TE)/T_1})$$

$$= M_0(1 - 2e^{-(TR-TE/2)/T_1} + e^{-TR/T_1})$$

$M_{x'y'}^{(3)}(0_+) = M_{x'y'}^{(3)}(0_-)$

$$M_{x'y'}^{(4)}(0_-) = M_0 e^{-TE/T_2} e^{-(TR-TE)/T_2}$$

$$= M_0 e^{-TR/T_2}$$

$M_{x'y'}^{(4)}(0_+) =$ ⟸

(3)

# Problem 3

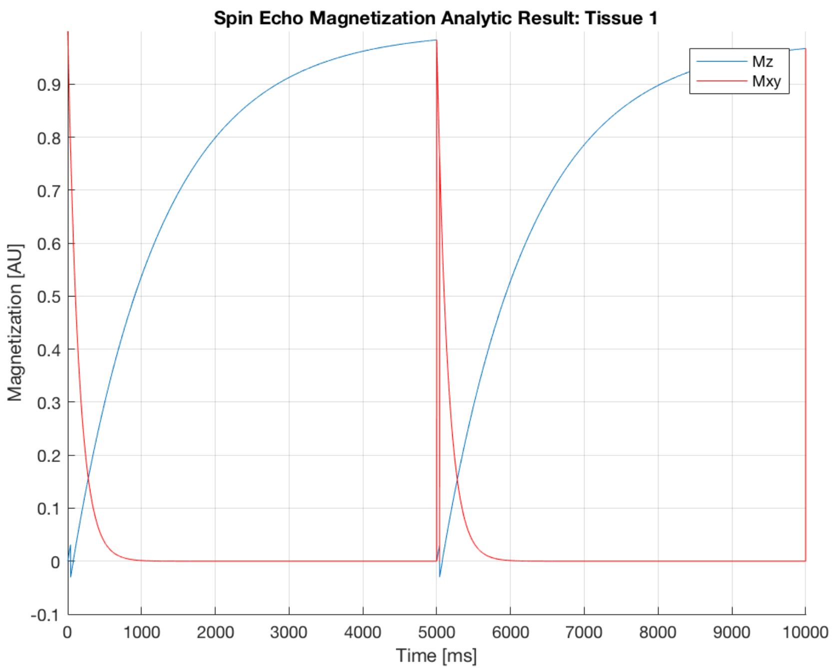
```
clear all, clc, close all

%Variables:

gamma=42.57e6;
TE=75;
TR=5000;
theta=0;

T1=[1200, 400];
T2=[150,50];
```

## Part B.

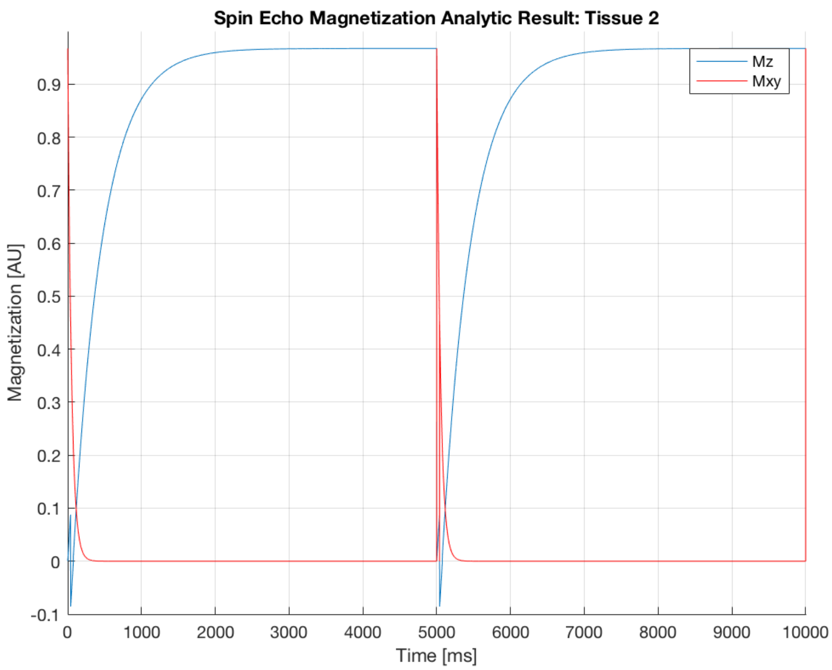
```
M0=1;
Mz(1)=0;
Mxy(1)=M0;
ter=round(TE/2);

for ti=1:2 %types of tissue

     %%after 90 deg 1+
    for i=0:ter%recovery 2-
        Mz(i+1)=M0*(1-exp(-i/(T1(ti))));
        Mxy(i+1)=M0*exp(-i/T2(ti));
    end
   %after 180 deg 2+
   for i=1:ter
        Mz((ter+i+1))=-M0*(1-exp(-((ter-i)/T1(ti))));
       Mxy(ter+i+1)=Mxy(ter)*exp(-(i)/T2(ti));
   end
  for i=1:TR-TE %Before Echo 3-
        Mz(TE+i+1)=M0*(1-2*exp((-TE-2*i)/(2*T1(ti)))+exp((-TE-i)/
T1(ti)));
        Mxy(TE+i+1)=M0*exp((-TE-i)/T2(ti));
  end

  %Second RF pulse:

  M0=Mz(TR+1);
  Mz(TR+1)=0;
  Mxy(TR+1)=M0;

for i=0:(TE/2)%recovery 2-
        Mz(TR+i+1)=M0*(1-exp(-i/(T1(ti))));
        Mxy(TR+i+1)=M0*exp(-i/T2(ti));
    end
   %after 180 deg 2+
   for i=1:ter
        Mz(TR+(ter)+i+1)=-M0*(1-exp(-((ter)-i)/T1(ti)));
        Mxy(TR+(ter)+i+1)=Mxy(TR+(ter))*exp(-(i)/T2(ti));
```

# Problem 3

```
 end
for i=1:TR-TE %Before Echo 3-
        Mz(TR+TE+i+1)=M0*(1-2*exp((-TE-2*i)/(2*T1(ti)))+exp((-TE-i)/
T1(ti)));
        Mxy(TR+TE+i+1)=M0*exp((-TE-i)/T2(ti));
  end
  M0=Mz(2*TR+1);
  Mz(2*TR+1)=0;
  Mxy(2*TR+1)=M0;

figure;hold on
p(1)=plot(Mz);
p(2)=plot(Mxy,'r');
title(['Spin Echo Magnetization Analytic Result: Tissue
 ',num2str(ti)]);
xlabel('Time [ms]');
ylabel('Magnetization [AU]');
ylim([-0.2 1])
xlim([0, 2*TR+2])
legend('Mz','Mxy');
grid on
ylim([-0.1 1])
%
% Long TR, Long TE: T2-weighted
%
```



# Problem 3




## Part C.

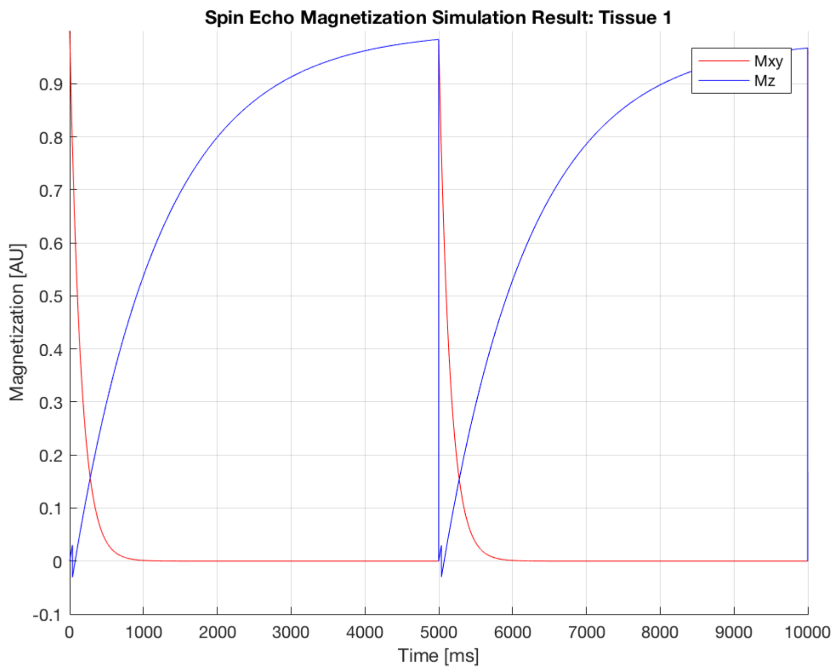
```
alpha=90*pi/180;
M0=1;
%Time

T=2*TR;
N=2*TR;
dt=T/N;


%SATURATION PULSE

ct=cos(theta);
st=sin(theta);
ca=cos(alpha);
sa=sin(alpha);

saturation=[ ct^2+st^2*ca       ct*st-ct*st*ca          -st*sa
   0;
             ct*st-ct*st*ca     st^2+ct^2*ca            ct*sa
   0;
             st*ca              -ct*sa                  ca
   0;
```

# Problem 3

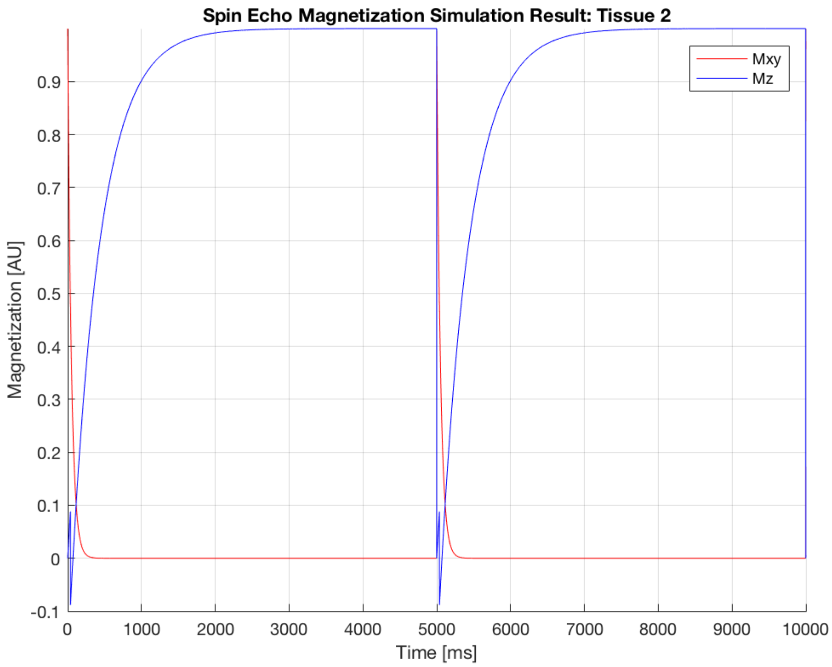
```
        0                       0                    0
   1];

%Refocusing operator:
refocusing=[1    0   0   0;
            0   -1   0   0;
            0    0  -1   0;
            0    0   0   1];

%Creating the magnetization vector:
M0=1;
m=zeros(4,ter);

%Creating relaxation matrix:
relaxation=[exp(-dt/T2(ti))        0               0               0;
    0                  exp(-dt/T2(ti))    0               0;
    0                  0              exp(-dt/T1(ti))     M0*(1-exp(-
dt/T1(ti)));
    0                  0              0               1];
%Time constants
time=linspace(0,T,N+3);
%Free induction decay:
m(:,1)=saturation*[0 0 M0 1]';
i=0;
for n=2:N+3
     m(:,n)=relaxation*m(:,n-1);
    if n>=(i+ter) && n==i+1+ter
      m(:,n)=refocusing*m(:,n-1);
    end
    if n==i+TR+1

        m(3,n)=0;
        M0=m(3,i+TR);
        m(2,n)=M0;
        relaxation=[exp(-dt/T2(ti))       0               0
     0;
    0                  exp(-dt/T2(ti))    0               0;
    0                  0              exp(-dt/T1(ti))     M0*(1-exp(-
dt/T1(ti)));
    0                  0              0               1];
        i=TR;
    end
end
Mxy(1)=0;

figure;hold on
p(2)=plot(time,abs(m(2,:)),'r');
p(3)=plot(time,m(3,:),'b');
title(['Spin Echo Magnetization Simulation Result: Tissue
 ',num2str(ti)]);
xlabel('Time [ms]');
ylabel('Magnetization [AU]');
grid on
legend('Mxy','Mz');
```

# Problem 3

```
ylim([-0.1 1])
```

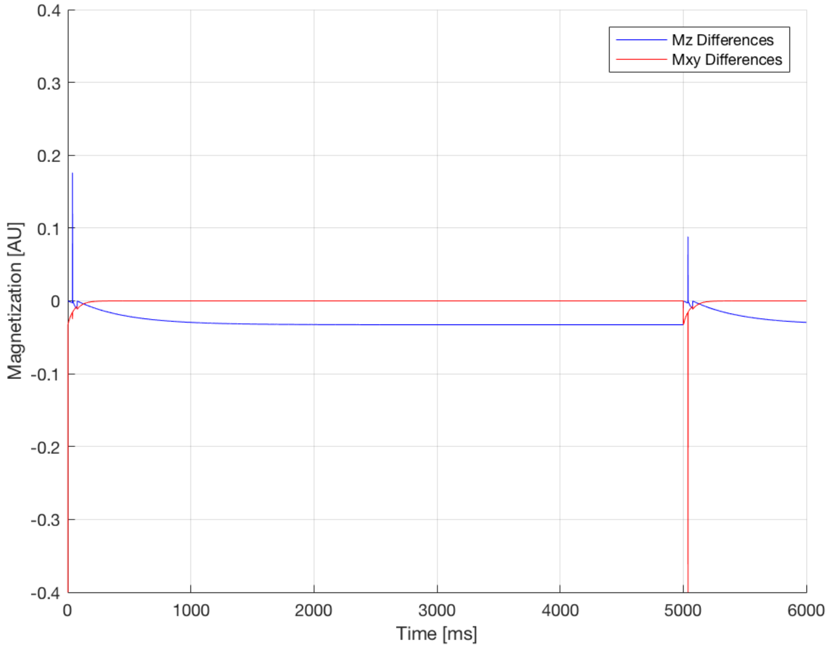


# Problem 3




```
end

figure; hold on
p(1)=plot(Mz-m(3,1:N+1),'b');
p(1)=plot((Mxy-abs(m(2,1:N+1))),'r');
ylim([-0.4,0.4])
xlim([0,6000])
legend('Mz Differences','Mxy Differences')
xlabel('Time [ms]');
ylabel('Magnetization [AU]');
grid on
```

# Problem 3




Main difference is that the RF pulse has some duration. The magnetization rotates through different planes before setting at the rigth values.

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