Bulk Magnetization and Nuclear Precession

M219 - Principles and Applications of MRI

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1/10/2022

Course Overview

- Course website
  - https://mrrl.ucla.edu/pages/m219
- Course schedule
  - https://mrrl.ucla.edu/pages/m219_2022
- Assignments
  - Homework #1 due on 1/26 by 5pm
- Office hours
  - TBD

	Date	Lecture Topic	Reading and Assignments
#1	1/3 Mon	Introduction slides	
#2	1/5 Wed	MRI Systems I: B0 slides	• Advances in whole-body MRI magnet • Superconducting systems for MRI • MR safe practices
#3	1/10 Mon	Bulk Magnetization and Nuclear Precession	Homework #1 out
#4	1/12 Wed	MRI Systems II: B1	
#5	1/17 Mon	No Lecture - MLK Holiday	
#6	1/19 Wed	Bloch Equations and Relaxation / MRI Signal Detection	
#7	1/24 Mon	MRI Systems III: Gradients	
#8	1/26 Wed	Fundamental Math of MRI	Homework #1 due, Homework #2 out
#9	1/31 Mon	Spatial Localization I	
#10	2/2 Wed	Spatial Localization II	

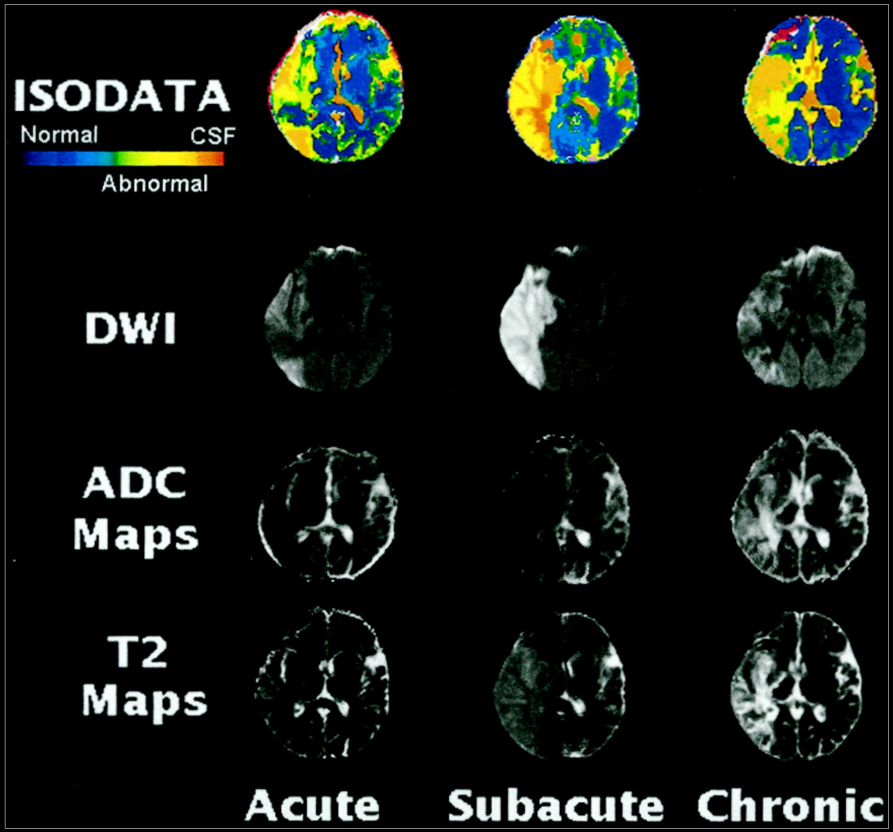
Requirements for MRI

- NMR Active Nuclei
  - e.g. ^{1}H in $H_2 0$
- Magnetic Field (B_0): Polarizer
- RF System (B_1): Exciter
- Coil: Receiver
- Gradients (G_X, G_Y, G_Z): Spatial Encoding

MRI Advantages

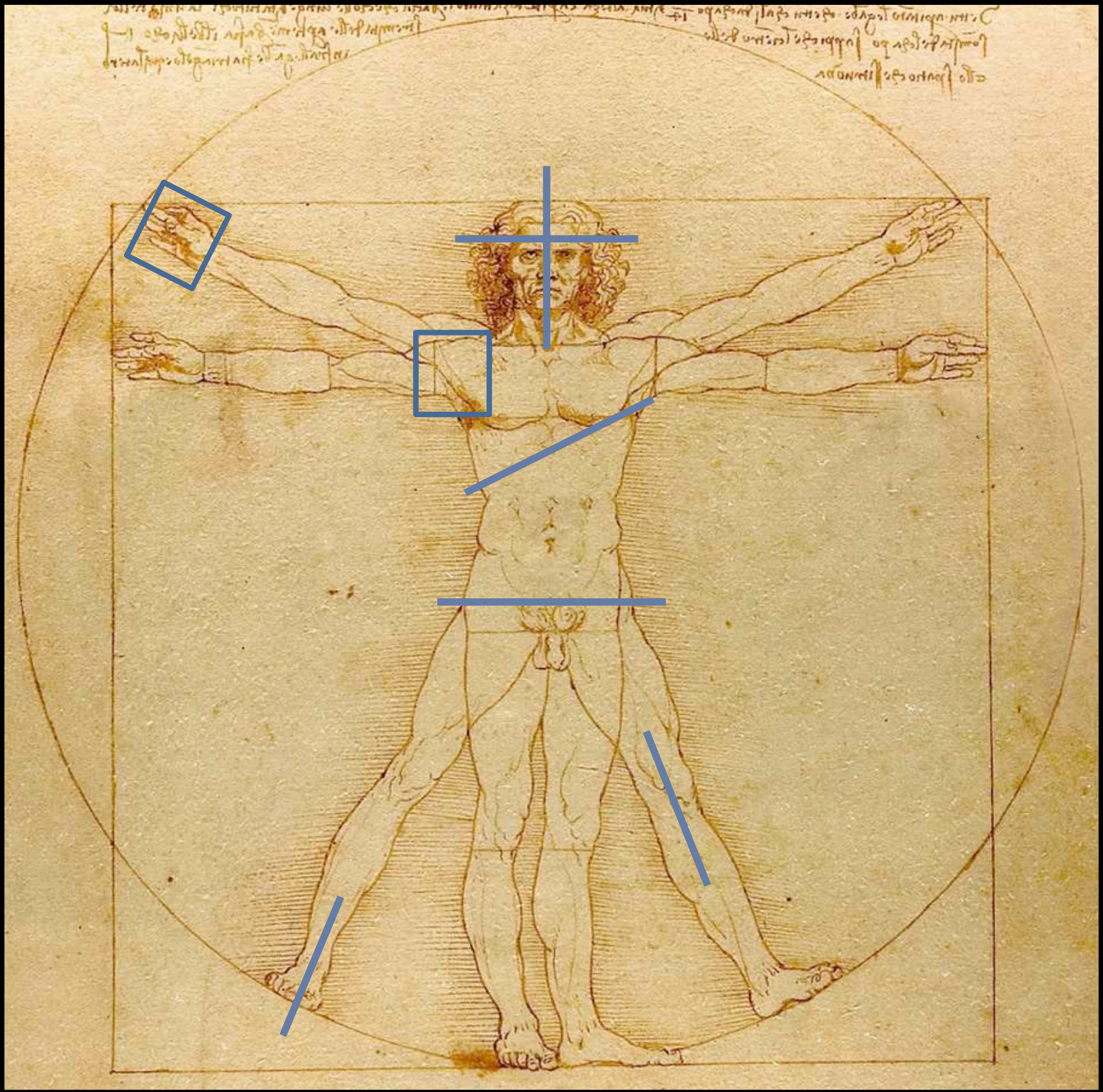
Tissue Characterization

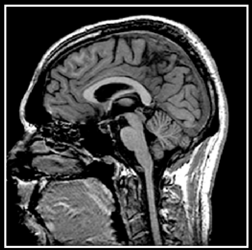
- **Routine**
  - T_1, T_2, T_2^*, proton weighted
  - Perfusion
  - Diffusion
  - Contrast enhancement
    - Tumor evaluation
- **Advanced**
  - T1- and T2-mapping
  - Fat/Water & Iron quantification
  - Spectroscopy (molecular)
  - Susceptibility weighted imaging (SWI) for blood products and calcium
  - Non-contrast angiography

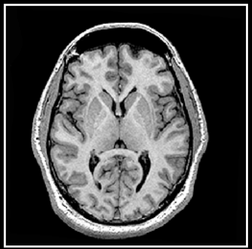


Demonstration of the multiparametric ISODATA segmentation methodology and corresponding DWI (b=1000 s/mm2), ADC map, and T2 map at different times after stroke. *Jacobs M A et al. Stroke. 2001;32:950-957*

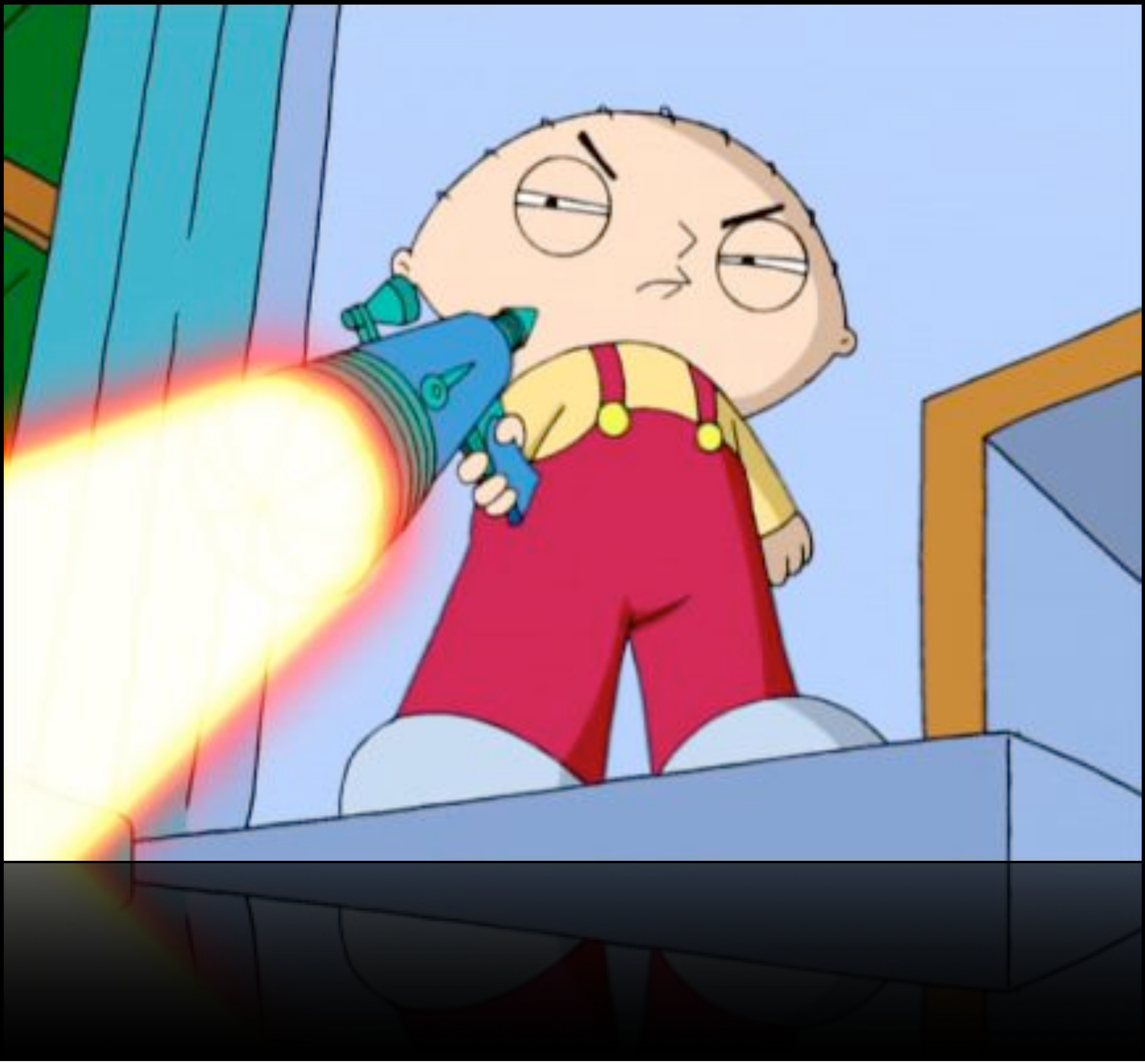
Arbitrary Imaging Planes







No Ionizing Radiation



MRI Disadvantages

MRI - Disadvantages

- **Safety**
  - Main Field (B_0)
  - Radiofrequency Field (B_1)
  - Gradients (G_x, G_y, and G_z)
- **Slow**
- **Expensive**
- **Technically challenging**

MRI Safety Designations

MR Safe: “An item that poses no known hazards in all MR environments.” (e.g. a plastic Petri dish)

MR Conditional: “An item that has been demonstrated to pose no known hazards in a specified MR environment with specified conditions of use. Field conditions that define the specified MR environment include field strength, spatial gradient, dB/dt (time rate of change of the magnetic field), radio frequency fields, and specific absorption rate. Additional conditions, including specific configurations of the item, may be required.” (e.g. a Patient Monitor)

MR Unsafe: “An item that is known to pose hazards in all MR environments.” (e.g. Floor Buffer)

“MRI Compatible” is not an FDA term.

RF (B_1) Safety - SAR Limits

- **RF pulses deposit energy in the body.**
- **Specific Absorption Rate [W/kg]**
  - Rate of energy absorption during exposure to RF
- High-field (>1.5T) imaging with high flip angles (>45-90°) can be challenging. $\mathrm{SAR} \propto \omega_0^2 B_1^2 \propto B_0^2 \alpha^2$

Limit	Whole-Body Average
Normal (all patients)	2 W/kg (0.5°C)
First level (supervised)	4 W/kg (1°C)

The scanner (FDA!) limits SAR, which in turn limits the max. flip angle.

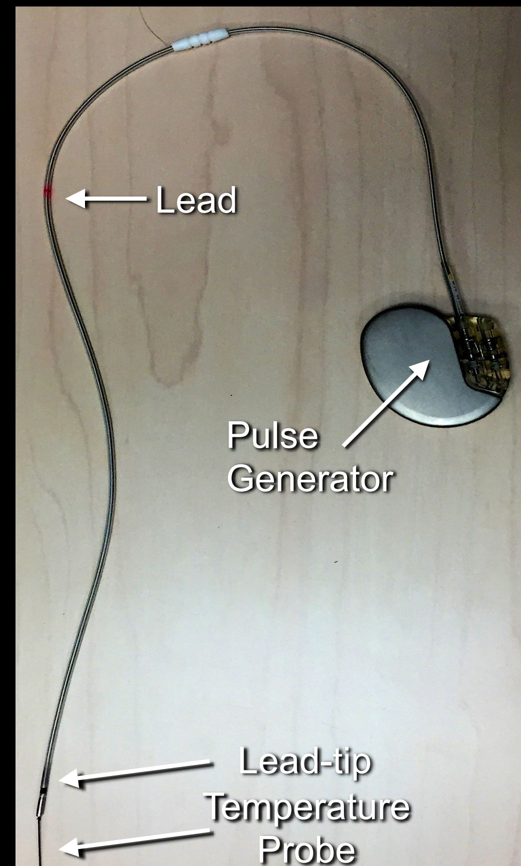
Bottomley PA. Turning up the heat on MRI. J Am Coll Radiol 2008;5(7):853-855.

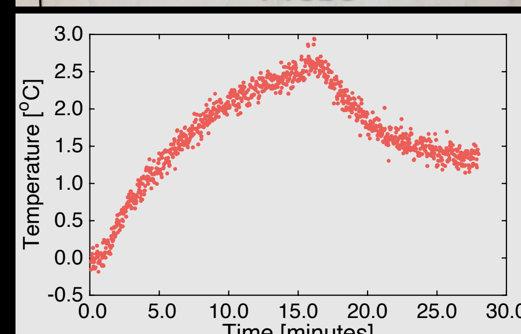
RF (B_1) Safety - Burns & Heating

- **Tissue burns**
- **RF induced heating of implanted devices**

Eising EG et al. J. Clin. Imaging 2010;34(4):293-297.

Solution: Avoid skin-to-skin loops; avoid arms directly touching scanner bore.

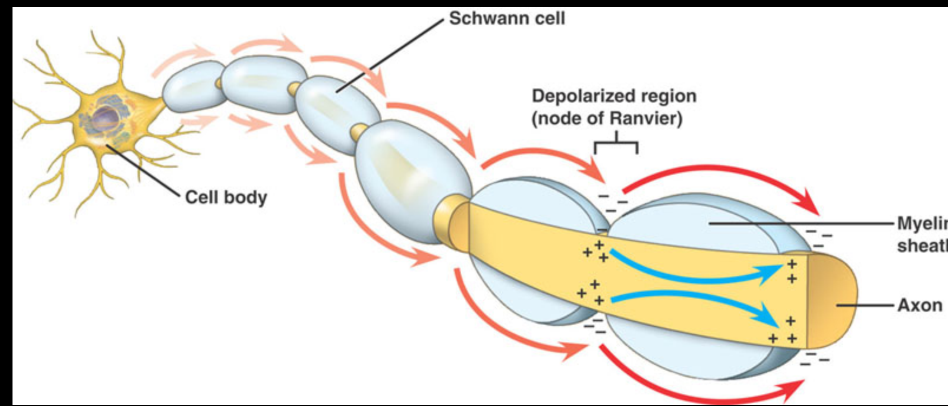


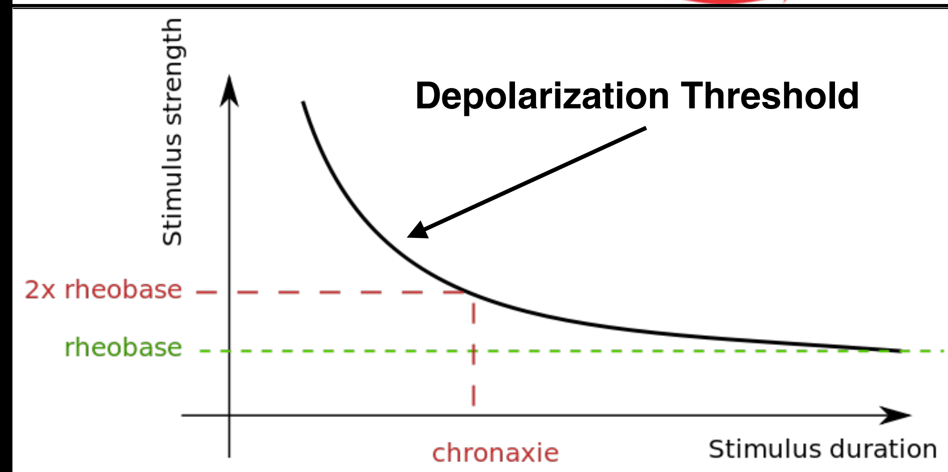


RF energy contributes to patient and device heating (or burns!).

Gradient Safety

- **Noise**
- **Peripheral nerve stimulation (PNS)**





Solution: De-rate gradient slew rates, but this increases scan time.

Solution: Ear plugs Head phones

Time-varying gradients induce mechanical vibrations and PNS.

MRI is Expensive

- **Purchase**
  - **$1-3 million**
- **Site**
  - **$0.5-1.0 million**
- **Maintain (Service Contract)**
  - **$100,000 per year**
- **Operate**
  - **$500-1000/hour**

Technically Challenging

- **Numerous scan parameters**
  - Dependent upon clinical question
  - Spin Echo vs Gradient Echo
  - TE, TR, TI, Flip Angle, Bandwidth
- **Physiologic Monitoring**
  - ECG
  - Respiration
  - Blood Pressure
  - General anesthesia/Sedation
- **Breath holding**
- **Contrast agents**
- **Coil Selection**
- **Anatomic Localization**

Quiz: NMR - True or False?

1. Electron spin is the key to NMR
2. MRI is *nothing* without spin, charge, and mass
3. All atomic nuclei are NMR active.
4. Spin and precession are the same.
5. Higher fields lead to faster precession

Quiz: Main Field - True or False?

1. B_0 is rare earth permanent magnet.
2. 1 Tesla=1000 Gauss.
3. Higher fields increase polarization, which contributes to better image quality
4. Exams at higher fields have lower SAR.
5. ^1H always precesses at the same Larmor frequency.

Main Field (B_0) - Principles

- B_0 is a strong magnetic field
  - >1.5T
  - Z-oriented

$$\vec{B}_0 = B_0\vec{k}$$

- B_0 generates bulk magnetization ($\vec{M}$)
  - More B_0, more

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

- B_0 forces $\vec{M}$ to precess
  - Larmor Equation

$$\omega = \gamma B$$

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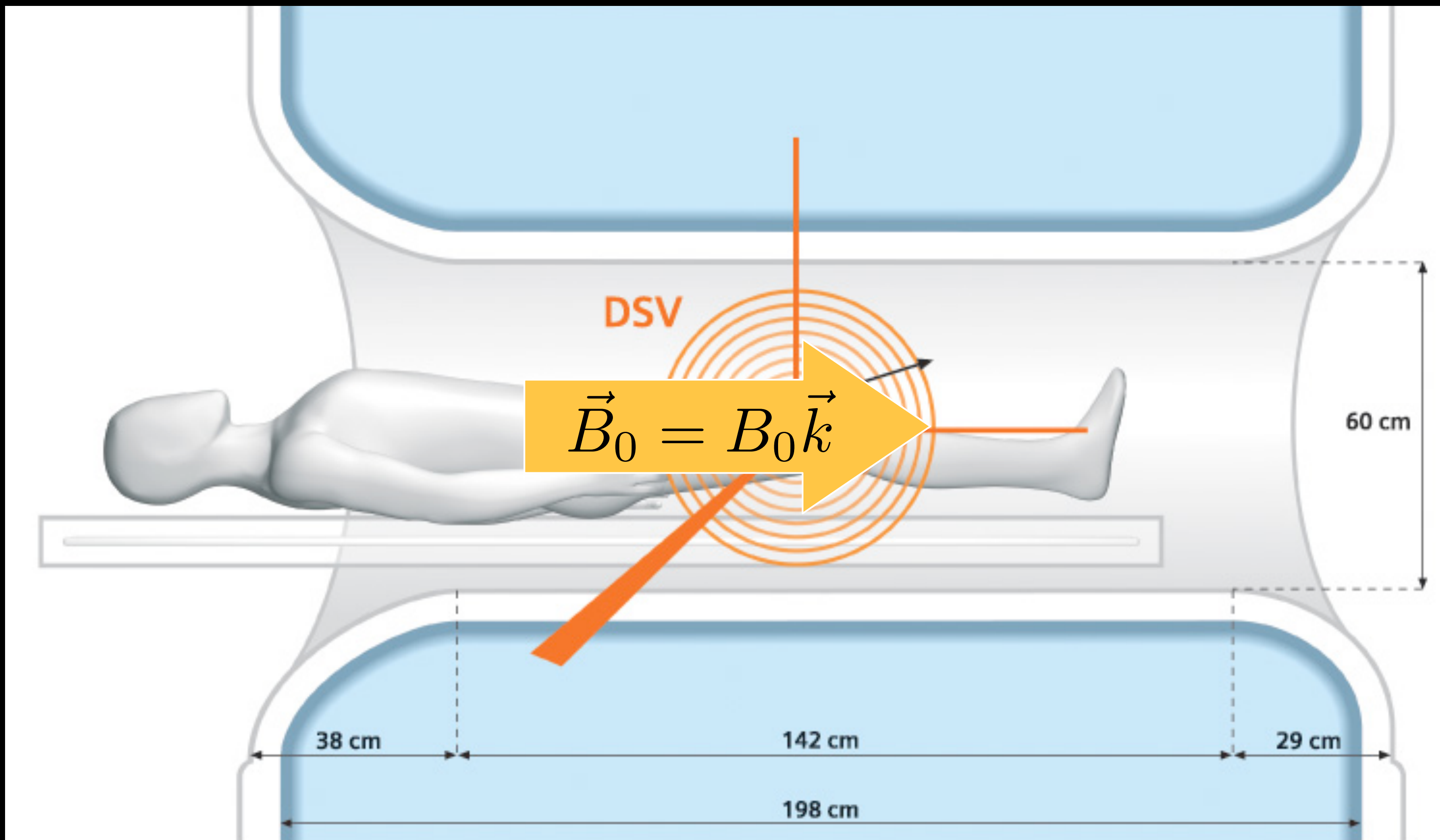
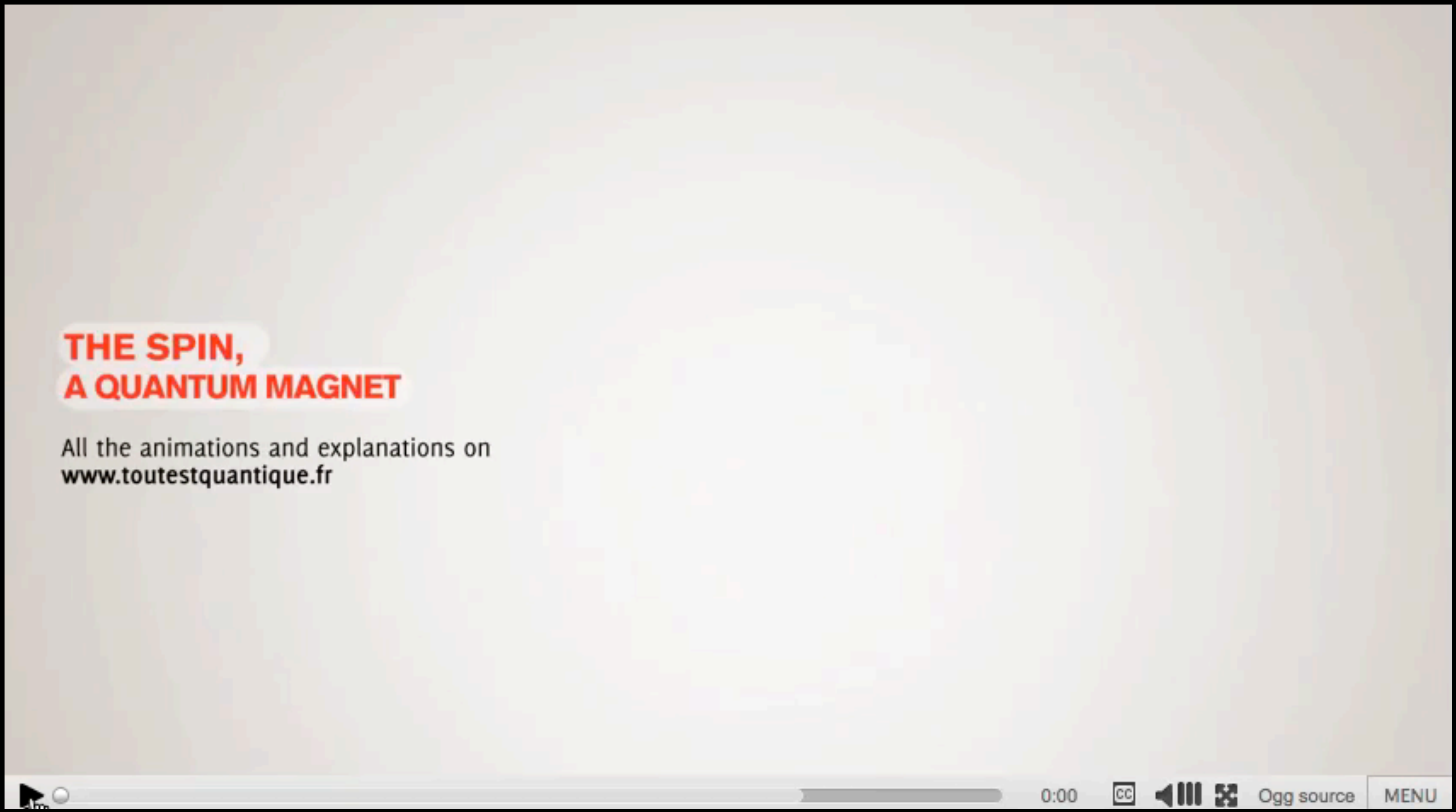
B_0 Field



Nuclear Spin

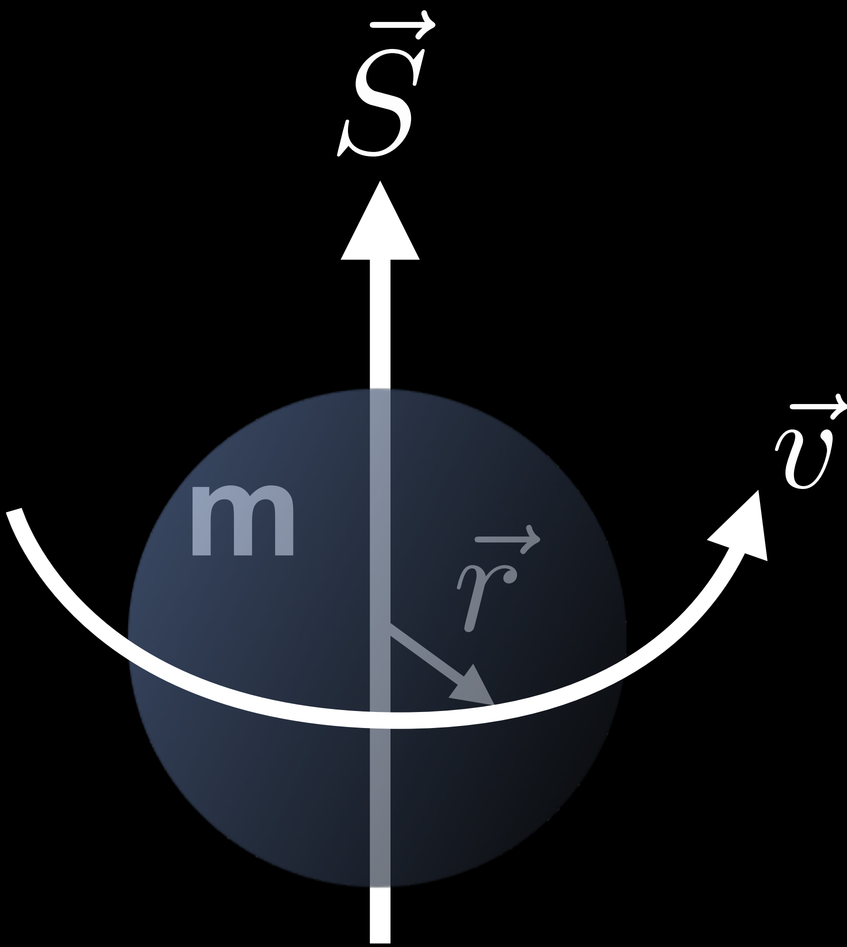
How was spin first observed?

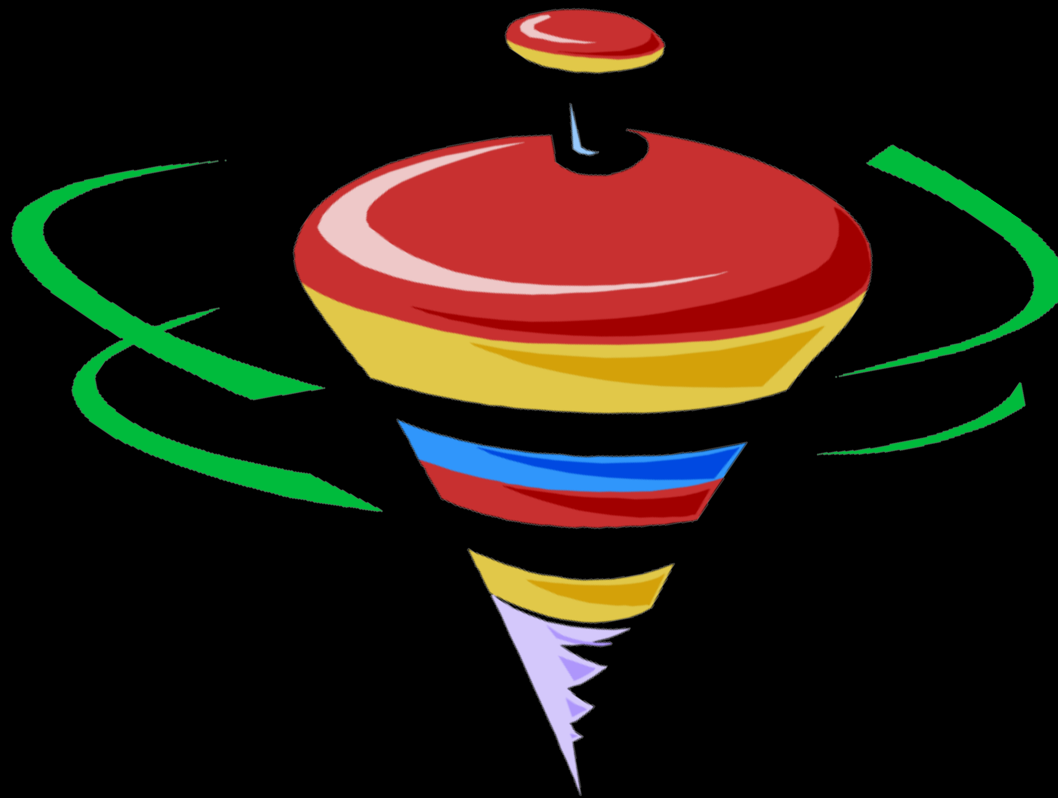


Otto Stern and Walther Gerlach performed the Stern–Gerlach experiment in Frankfurt, Germany in 1922.

Spin Angular Momentum

Spin + Mass ➠ Spin Angular Momentum ➠ $\vec{S}$ [kg·m²s⁻¹]

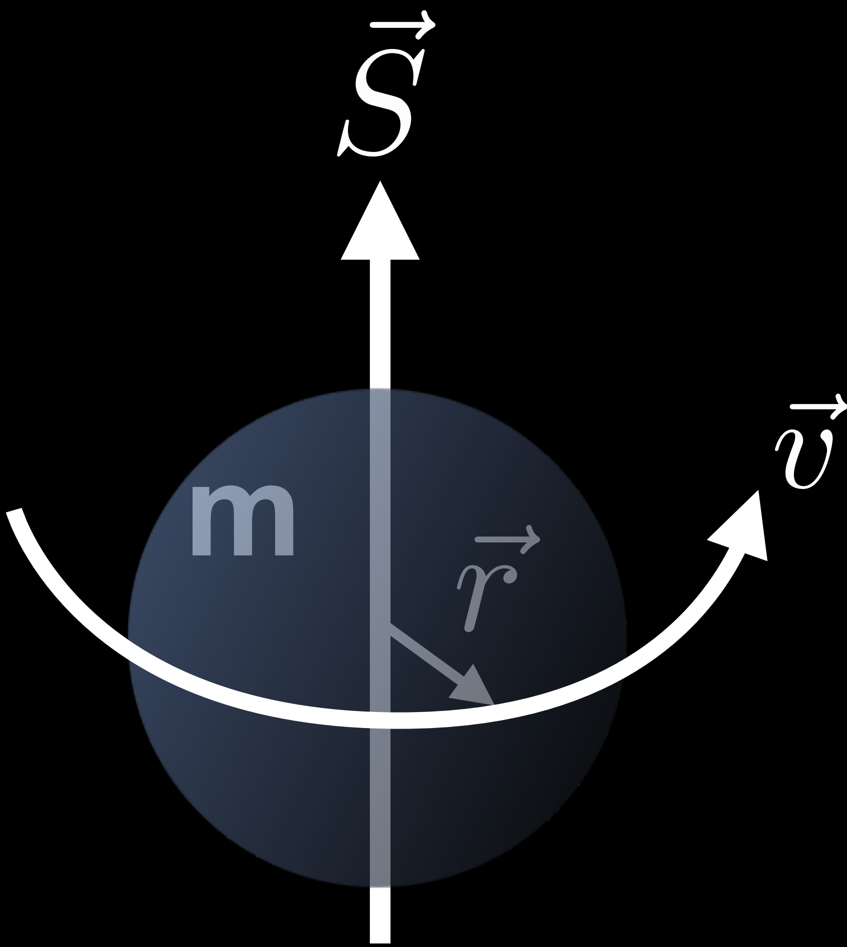




Hydrogen nuclei have spin angular momentum.

Spin Angular Momentum

Spin + Mass ➠ Spin Angular Momentum ➠ $\vec{S}$ [kg·m²s⁻¹]



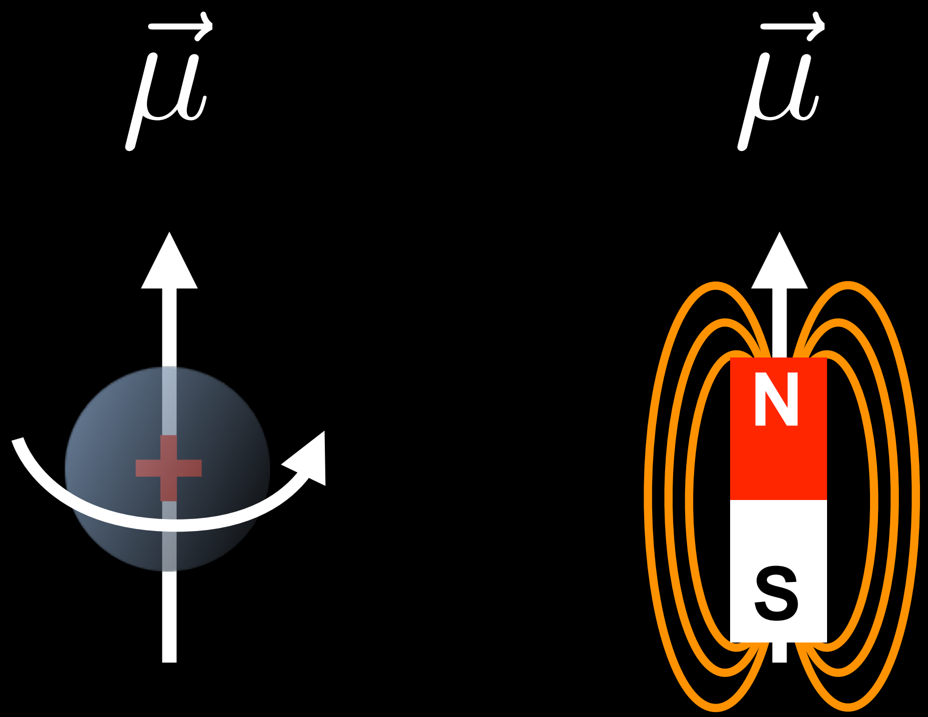
$$\vec{S} = \vec{r} \times \vec{p}$$
$$= \vec{r} \times m\vec{v}$$

Hydrogen nuclei have spin angular momentum.

Magnetic Dipole Moments

Spin + Charge ➟ Magnetic Moment ➟ $\vec{\mu}$ [J•T^{-1} *or* kg•m^2/s^2/T]

"a measure of the strength of the system's net magnetic source"
--http://en.wikipedia.org/wiki/Magnetic_moment



Hydrogen nuclei have magnetic dipole moments.

Gyromagnetic Ratio

- Gyromagnetic Ratio
  - Physical constant
  - Unique for each NMR active nuclei
  - Ratio of the magnetic moment to the angular momentum

$$\vec{\mu} = \gamma \vec{S}$$

  - Governs the frequency of ***precession***
  - Gamma vs. Gamma-bar

$$\bar{\gamma} = \gamma / 2\pi$$

NMR Active Nuclei

Isotope	*Spin [I]*	Natural Abundance	Gyromagnetic Ratio [MHz/T]	Relative Sensitivity	Absolute Sensitivity
^{1}H	**1/2**	**0.9980**	**42.57**	**1**	**9.98E-01**
^{2}H	1	0.0160	6.54	0.015	2.40E-04
^{12}C	0	0.9890	---	---	---
^{13}C	**1/2**	**0.0110**	**10.71**	**0.016**	**1.76E-04**
^{14}N	1	0.9960	3.08	0.001	9.96E-04
^{15}N	**1/2**	**0.0040**	**-4.32**	**0.001**	**4.00E-06**
^{16}O	0	0.9890	---	---	---
^{17}O	5/2	0.0004	-5.77	0.029	1.16E-05
^{19}F	**1/2**	**1.0000**	**40.05**	**0.83**	**8.30E-01**
^{23}Na	**3/2**	**1.0000**	**11.26**	**0.093**	**9.30E-02**
^{31}P	**1/2**	**1.0000**	**17.24**	**0.066**	**6.60E-02**

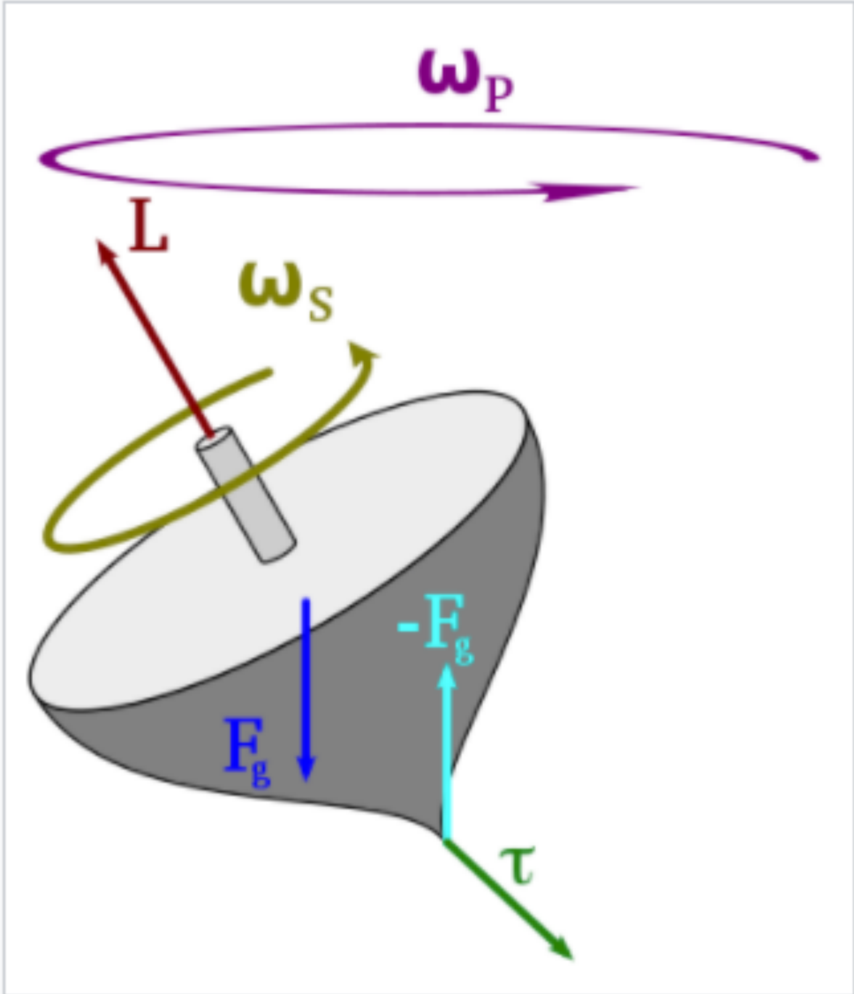
The ***relative*** sensitivity is at constant magnetic field and equal number of nuclei.

– Using a factor of $\gamma^{\frac{11}{4}} I\left(I+1\right)$; ^{1}H is the reference standard.

The ***absolute*** sensitivity is the relative sensitivity multiplied by natural abundance.

P. Callaghan & http://www.cryst.bbk.ac.uk/PPS2/projects/schirra/html/nuclei.htm

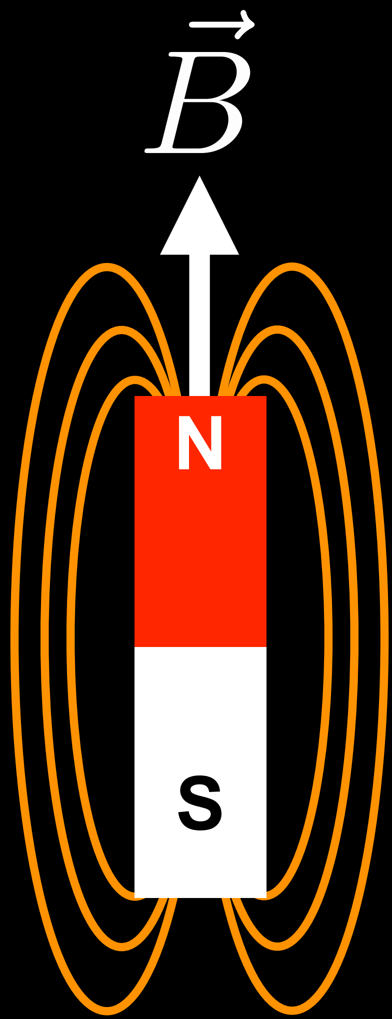
Precession



The torque caused by the normal force $-\mathbf{F}_g$ and the weight of the top causes a change in the angular momentum $\mathbf{L}$ in the direction of that torque. This causes the top to precess.

https://en.wikipedia.org/wiki/Precession

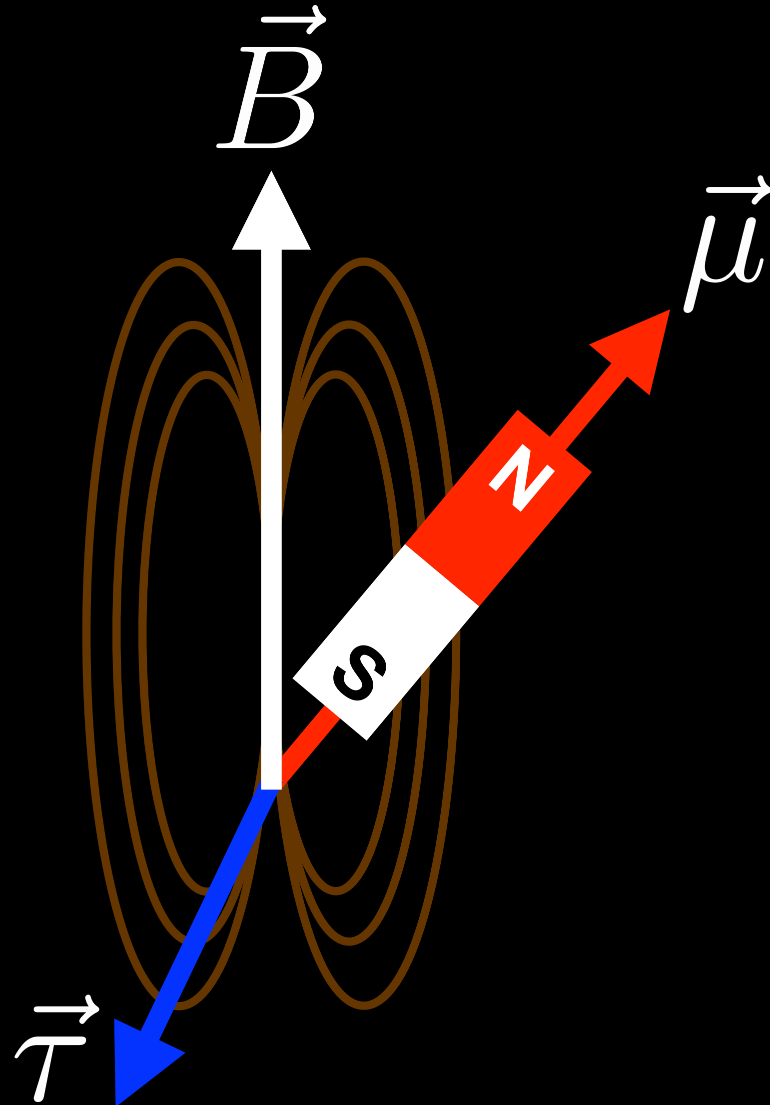
B-Field



"vector field which can exert a magnetic force on moving electric charges and on magnetic dipoles"
--http://en.wikipedia.org/wiki/Magnetic_field

Magnetic Dipole in a B-Field

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



B_0 exerts a torque on the [1]H magnetic dipole moment.

To The Board…

Main Field (B_0) - Principles

- B_0 is a strong magnetic field
  - >1.5T
  - Z-oriented

$$\vec{B}_0 = B_0\vec{k}$$ Eqn. 3.5

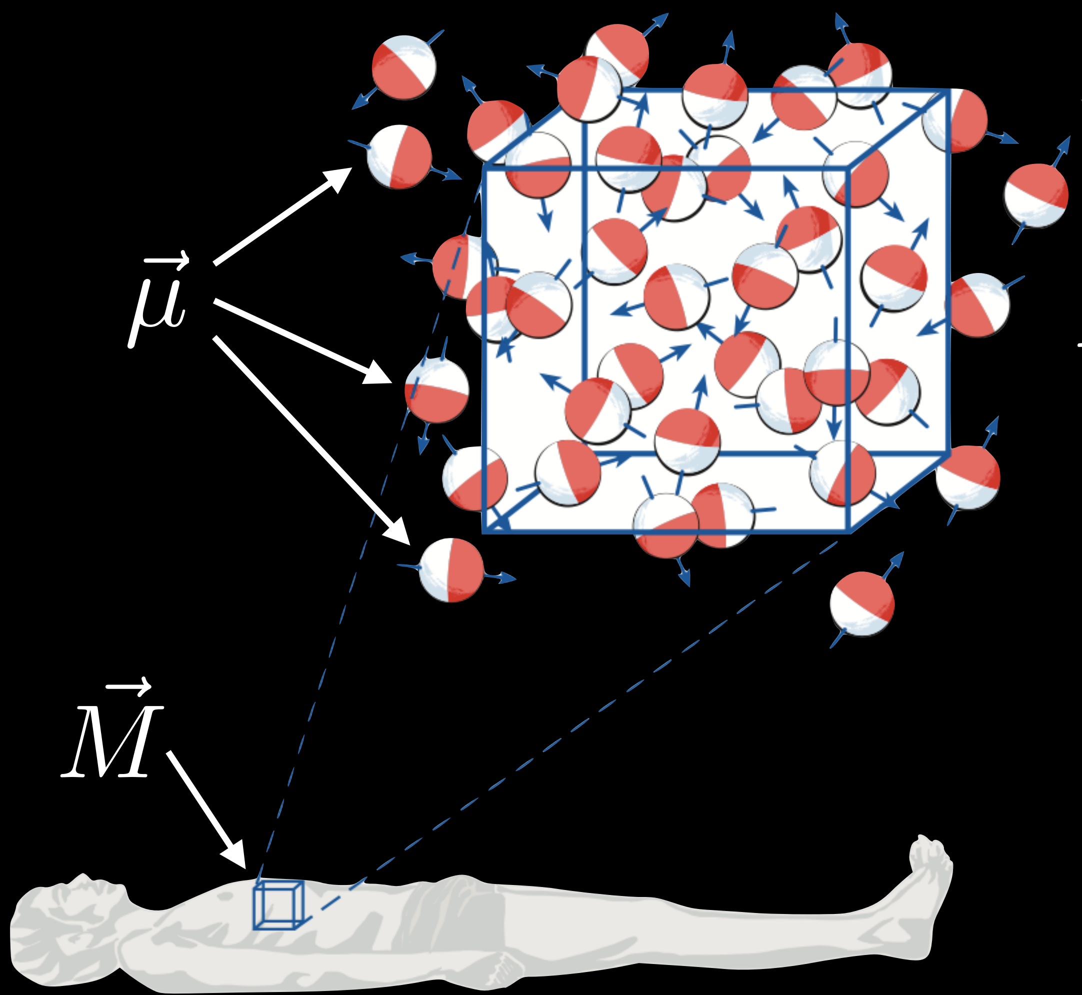
- B_0 generates bulk magnetization ($\vec{M}$)
  - More B_0, more

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$ Eqn. 3.26

- B_0 forces $\vec{M}$ to precess
  - Larmor Equation

$$\omega = \gamma B$$ Eqn. 3.18

Bulk Magnetization



$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$ Eqn. 3.26

$N_{total}=0.24\times10^{23}$ spins in a 2x2x10mm voxel

But not all spins contribute to our measured signal...

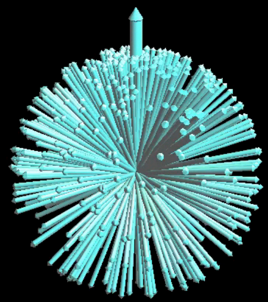
Equilibrium Bulk Magnetization

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

$$\vec{M} = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

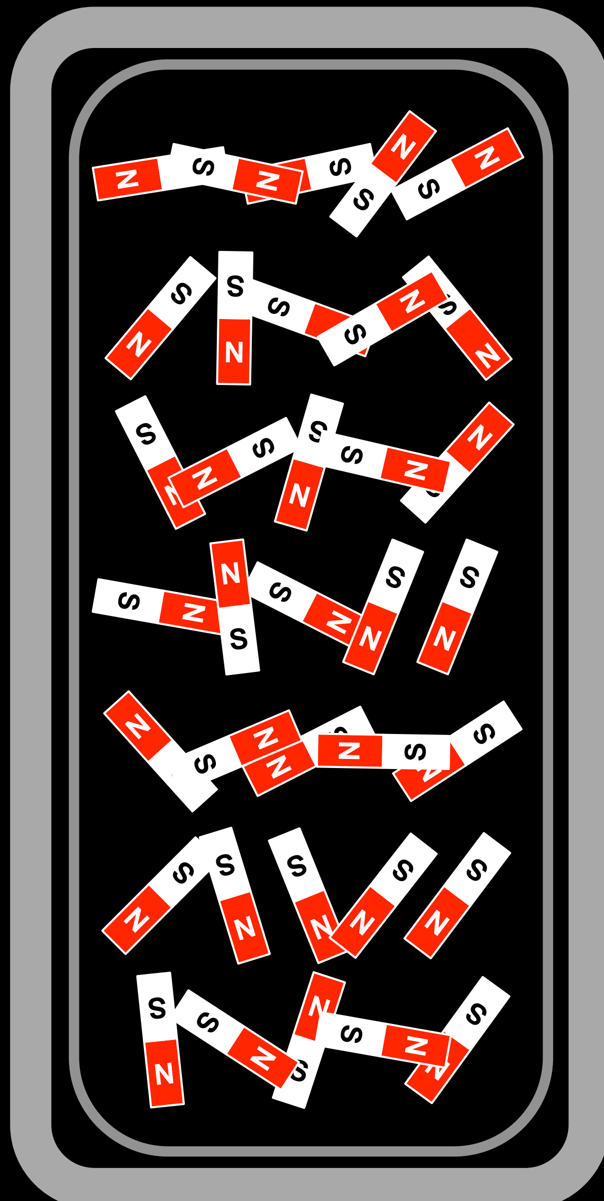
$$\vec{M}_z^0 = |\vec{M}| = \frac{\gamma^2 \hbar^2 B_0 N_s}{4KT_s}$$

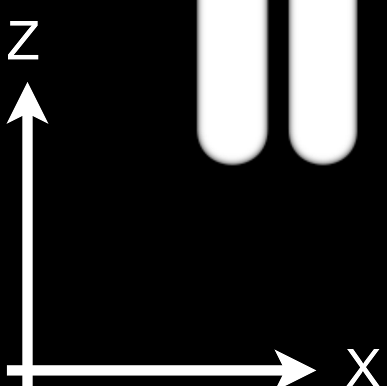
$$\vec{M}_x^0 = \vec{M}_y^0 = 0$$



Bulk magnetization at equilibrium in a B_0 field.

B_0 Field OFF

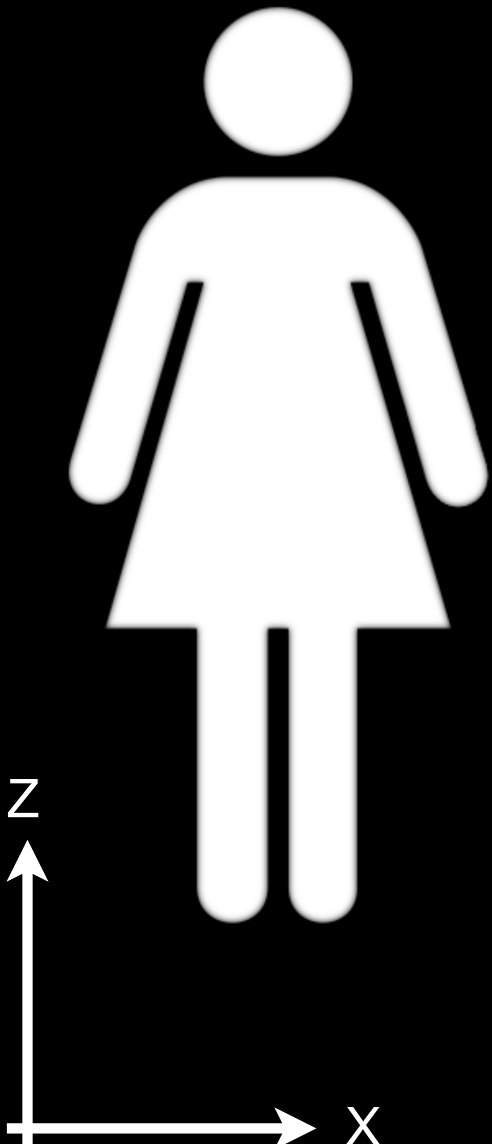


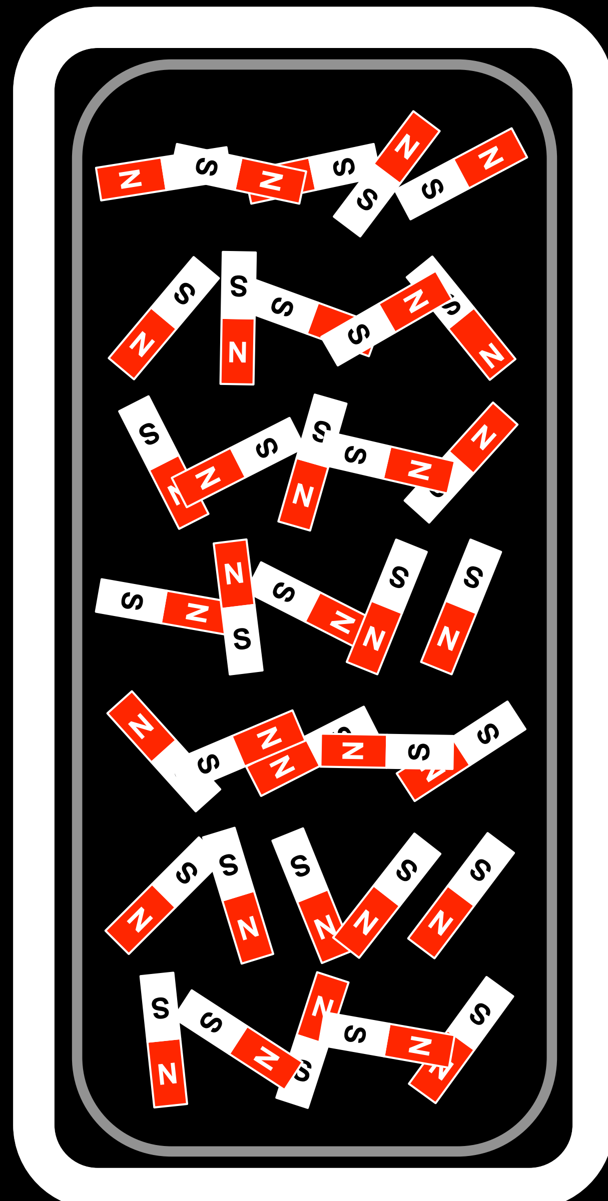


$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = 0$$

Spins point in all directions.

B_0 Field ON

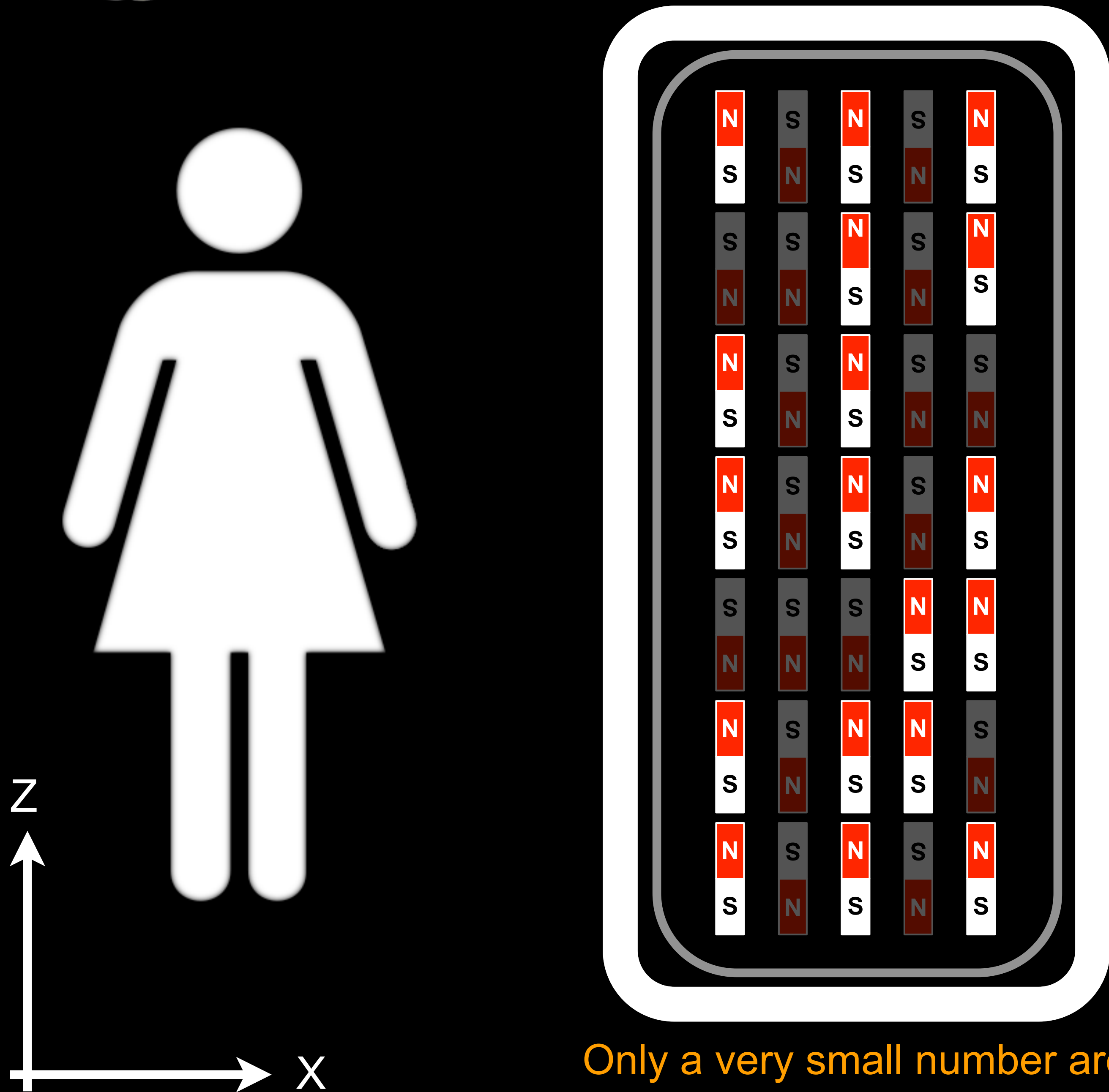




$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = M_z$$

B_0 polarizes the spins and generates bulk magnetization.

B_0 Field ON



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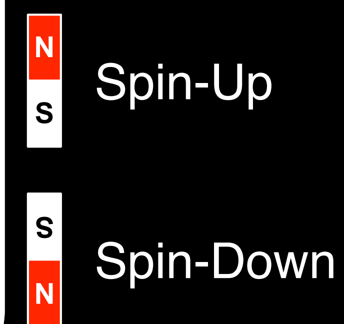
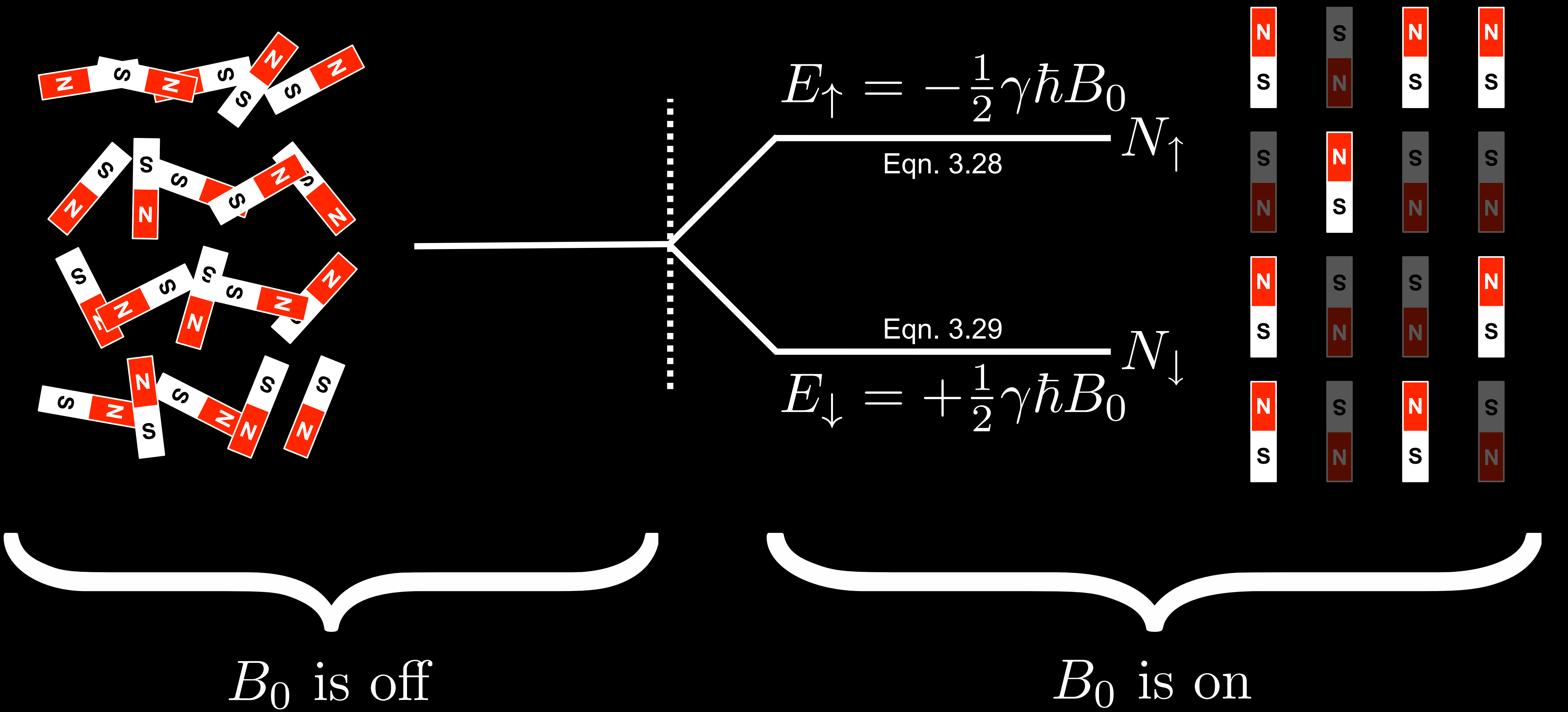


Only a very small number are spin-up relative to spin-down.

David Geffen
School of Medicine

UCLA
Radiology

Zeeman Splitting



$N_\uparrow$ = Spin-Up State, Low Energy

$N_\downarrow$ = Spin-Down State, High Energy

Zeeman Splitting

The spin population difference in the two spin states is related to their energy difference. According to the well-known Boltzmann relationship, we have

$$\frac{N_\uparrow}{N_\downarrow} = \exp\left(\frac{\Delta E}{KT_s}\right) \tag{3.31}$$

where

$N_\uparrow$:	number of pointing-up spins
$N_\downarrow$:	number of pointing-down spins
T_s :	absolute temperature of the spin system
K :	Boltzmann constant (1.38×10^{-23} J/K)

After simplification…

$$N_\uparrow - N_\downarrow \approx N_s \frac{\gamma \hbar B_0}{2KT_s} \tag{3.35}$$

Zeeman Splitting

$$\frac{N_\uparrow - N_\downarrow}{N_{total}} \approx \frac{\gamma h B_0}{2KT} \qquad \text{Eqn. 3.35}$$

$$\begin{aligned}
\gamma &= 42.58 \times 10^6 \text{ Hz/T} \\
h &= 6.6 \times 10^-34 \text{ J} \cdot \text{s [Planck' Constant]} \\
T &= 300\text{K (room temperature)} \\
K &= 1.38 \times 10^{-23} \text{ J/K [Boltzmann Constant]} \\
B_0 &= 1.5\text{T}
\end{aligned}$$

$$\frac{N_\uparrow - N_\downarrow}{N_{total}} \approx \frac{42.58 \times 10^6 \cdot 6.6 \times 10^{-34} \cdot 1.5}{2 \cdot 1.38 \times 10^{-23} \cdot 300} \approx 4.5 \times 10^{-6}$$

$$\vec{M}_z^0 = |\vec{M}| = \frac{\gamma^2 \hbar^2 B_0 N_s}{4KT_s} \qquad \text{Eqn. 3.39}$$

Main Field (B_0) - Principles

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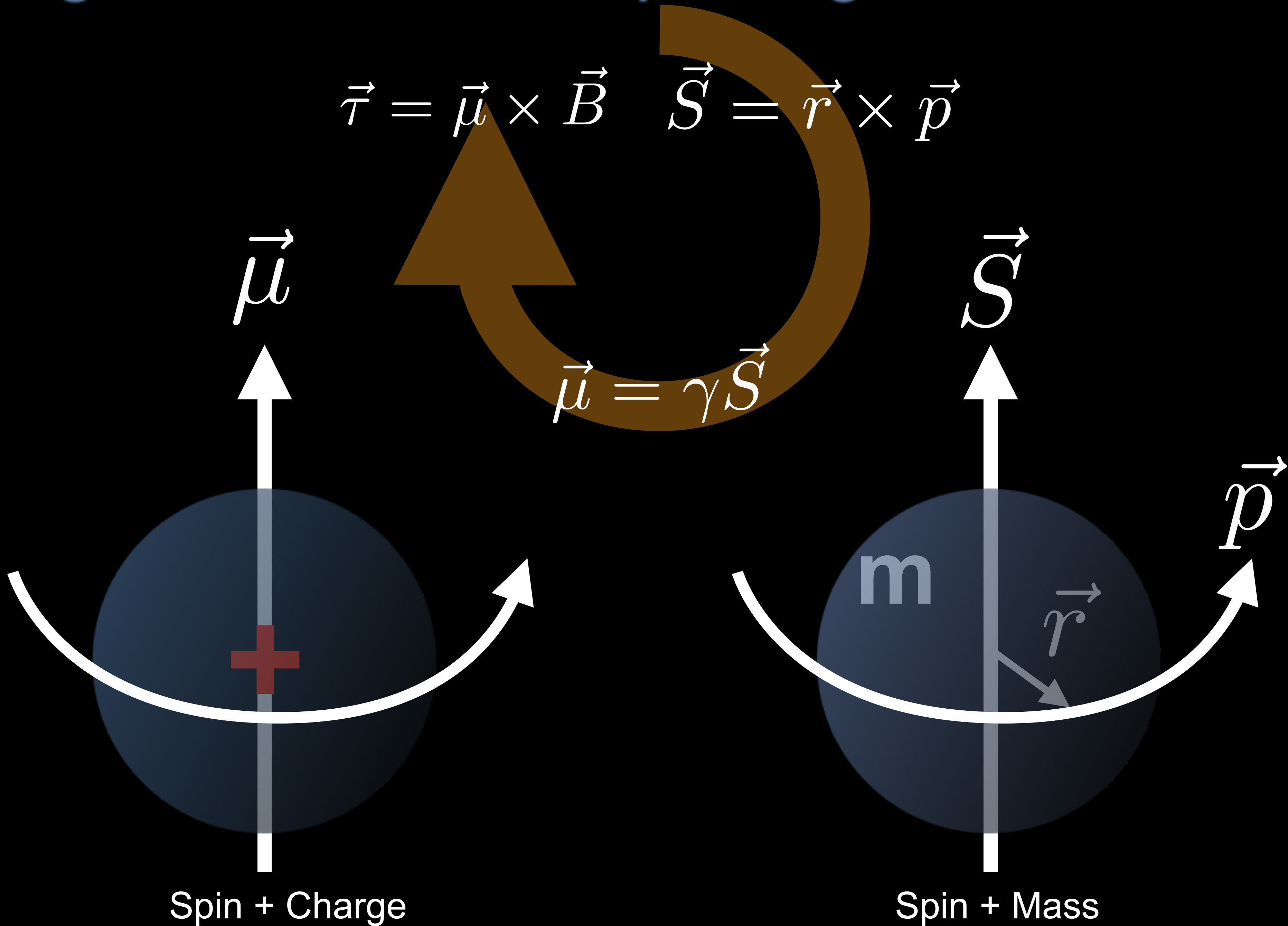
$$\omega = \gamma B$$ Eqn. 3.18

Spin vs. Precession

- **Spin**
  - Intrinsic form of angular momentum
  - Quantum mechanical phenomena
  - No classical physics counterpart
    - Except by hand-waving analogy…

- **Precession**
  - Spin+Mass+Charge give rise to precession

Magnetic Moment & Spin Angular Momentum



Spin + Mass and Spin + Charge ➠ NMR

To the board

Equation of Motion for the Bulk Magnetization

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats) [Classical Description]

What is a general solution?

The *equation of motion* describes the bulk magnetization "behavior" in the presence of a B-field.

To the board

Rotations & Euler’s Formula

Vectors

- A ***vector*** ($\vec{v}$) describes a physical quantity (e.g. bulk magnetization or velocity) at a point in space and time and has a magnitude (positive real number), a direction, and physical units.

- To define a vector we need a ***basis***:

$$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

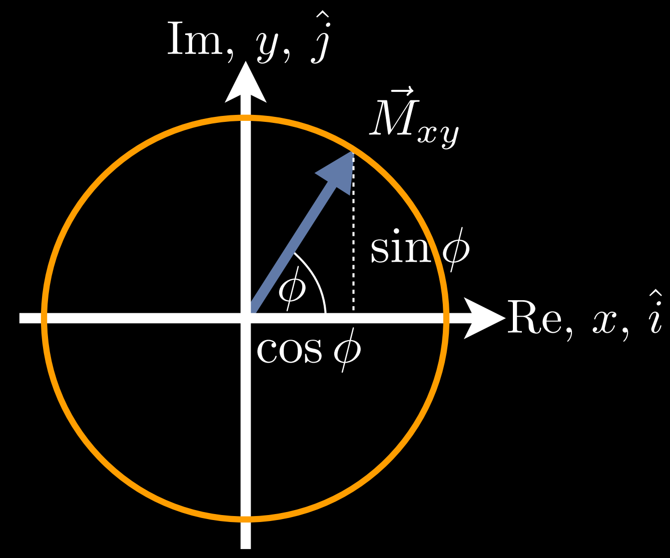
- A 3D ***vector*** has components:

$$\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

2D Vectors - Euler's Formula

- Euler's formula provides a compact representation of a 2D vector using a complex exponential:

$$e^{i\phi} = \cos\phi + i\sin\phi$$



$$\begin{aligned}
\vec{M}_{xy} &= M_x\hat{i} + M_y\hat{j} && \text{Vector components}\\
&= M_x + iM_y && \text{Complex components}\\
&= |\vec{M}_{xy}|\cos\phi\hat{i} + |\vec{M}_{xy}|\sin\phi\hat{j} && \text{Trigonometric components}\\
&= |\vec{M}_{xy}|\cos\phi + i|\vec{M}_{xy}|\sin\phi && \text{Complex trigonometric components}\\
&= |\vec{M}_{xy}|e^{i\phi} && \text{Euler's notation}\\
&= |\vec{M}_{xy}|\cos\phi + i|\vec{M}_{xy}|\sin\phi\\
&= |\vec{M}_{xy}|e^{i\phi}
\end{aligned}$$

Euler's formula is mathematically convenient.
There is nothing explicitly *imaginary* about M_{xy}.

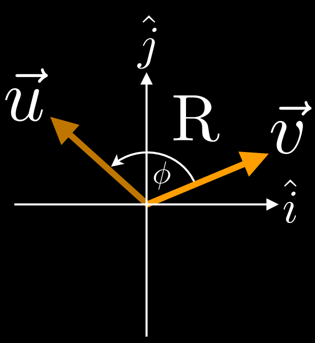
Rotations

- ***Rotations*** (R) are vector valued orthogonal transformations that preserve the magnitude of vectors and the angles between them.
- The simplest rotation matrix is the ***identity*** matrix:

$$\mathrm{R} = \mathrm{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ therefore } \vec{v} = \mathrm{I}\vec{v}$$

- More simply, R transforms (rotates) one vector to another:

$$\vec{u} = \mathrm{R}\vec{v}$$



Rotations

$\hat{i}$ ends up here

$\hat{j}$ ends up here

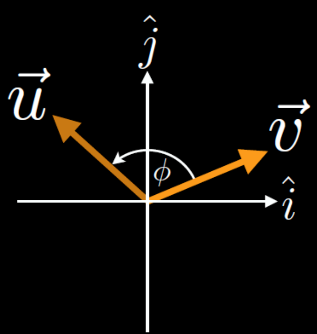
$\hat{k}$ does not change

Magnitude of rotation

Axis (phase) of rotation

$$R_z^{\phi} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{u} = R\vec{v}$$



Note: Positive values of *Φ* produce right-handed (CCW) rotations.

Matlab Demo

RIGHT-HANDED

$$R_z^{\phi} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LEFT-HANDED

$$R_Z(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_Y(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$R_X(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

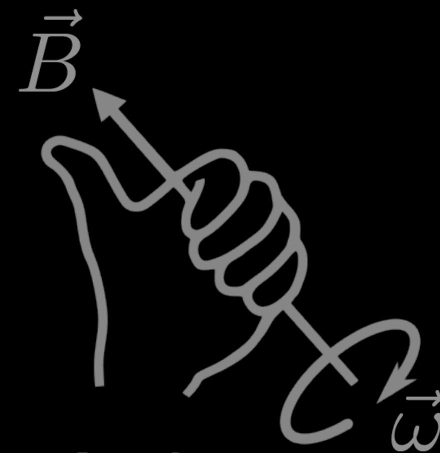
Free Precession In The Laboratory Frame Without Relaxation

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \left(\vec{B_0} \right)$$

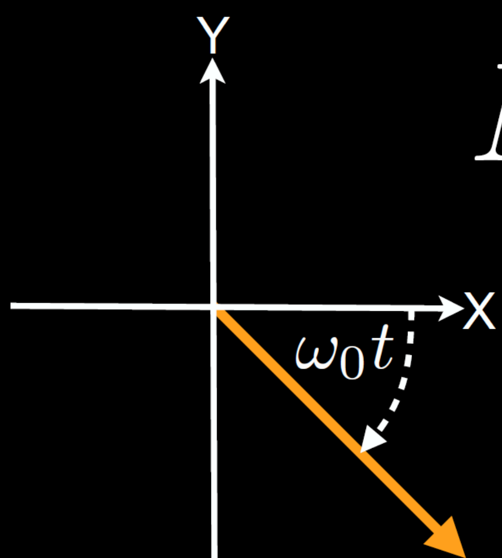
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

Free Precession In The Laboratory Frame Without Relaxation

$$\mathbf{R}_z(\omega_0 t) = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Precession is left-handed (clockwise).



$$\vec{M}(t) = \mathbf{R}_z(\omega_0 t)\vec{M}^0$$

$$\Updownarrow$$

$$\omega_0 = \gamma B_0$$

Matlab Demo

```
%% Define some constants
gamma=42.57e6;          % Gyromagnetic ratio for 1H [MHz/T]
B0=1.5;                 % B0 magnetic field strength [T]
dt=0.01e-8;             % Time step [s]
nt=500;                 % Number of time points to simulate
t=(0:nt-1)*0.01e-8;     % Time vector [s]

M0=[sqrt(2)/2 0 sqrt(2)/2 1]';  % Initial condition (I.C.)

M=zeros(4,nt);                  % Initialize the magnetization array
M(:,1)=M0;                      % Define the first time point as the I.C.

%% Simulate precession of the bulk magnetization vector
dB0=PAM_B0_op(gamma,B0,dt);     % Calculate the homogenous coordinate transform

for n=2:nt
  M(:,n)=dB0*M(:,n-1);
end

%% Plot the results
figure; hold on;
  p(1)=plot(t,M(1,:));          % Plot the Mx component
  p(2)=plot(t,M(2,:));          % Plot the My component
  p(3)=plot(t,M(3,:));          % Plot the Mz component
    set(p,'LineWidth',3);       % Increase plot thickness
  ylabel('Magnetization [AU]');
  xlabel('Time [s]');
  legend('M_x','M_y','M_z');
  title('Bulk Magnetization Components as f(t)');

%% "Print" the figure
% PAM_fig_style(gcf,'rect');
% PAM_UCLA_Logo;
% print(gcf,'~/PAM_Lec02_B0_Precession.eps','-depsc2');
```

Summary

$\vec{\tau} = \vec{\mu} \times \vec{B}$ $\quad \vec{S} = \vec{r} \times \vec{p}$

$\vec{\tau} = \frac{d\vec{S}}{dt}$ $\quad \vec{\mu} = \gamma \vec{S}$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

Equation of Motion for a Magnetic Dipole

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of Motion for the bulk magnetization.

$$\vec{B}_0 = B_0 \vec{k}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \left(\vec{B}_0 \right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$

$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$

$$M_z(t) = M_z^0$$

$$\vec{M}(t) = \mathbf{R}_z(\omega_0 t) \vec{M}^0$$

Next time…

MRI Systems II – B_1

Questions?

- Related reading materials
  - Liang/Lauterbur - Chap 3.1
  - Nishimura - Chap 4.1, 4.2

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