Bloch Equations and Relaxation / MRI Signal Detection

M219 - Principles and Applications of MRI

Kyung Sung, Ph.D.

1/19/2022

Course Overview

- Course website
  - https://mrrl.ucla.edu/pages/m219
- Course schedule
  - https://mrrl.ucla.edu/pages/m219_2022
- Assignments
  - Homework #1 due on 1/26 by 5pm
  - Homework #2 will be out on 1/26

Course Overview

- Office Hours
  - TA (Ran Yan) - Tuesday 4-5pm
    https://uclahs.zoom.us/j/96870184581?pwd=VkczL0lyRkxsQ3FHcnIxQ1M2U3hPdz09

    Password: 900645

  - Instructor (Kyung Sung) - Friday 2-3pm
    https://uclahs.zoom.us/j/94058312815?pwd=TkI3ajhkamdGTnhqOVNnbk5RMnJGQT09

    Password: 888767

Last Time...

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \vec{S} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \frac{d\vec{S}}{dt} \qquad \vec{\mu} = \gamma \vec{S}$$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

Equation of Motion for a Magnetic Dipole

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of Motion for the bulk magnetization.

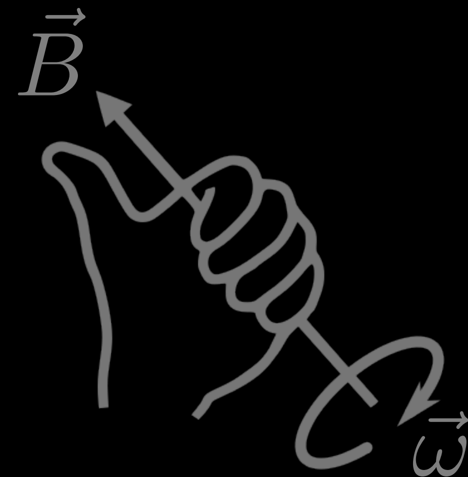
$$\vec{B}_0 = B_0 \vec{k}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \left(\vec{B}_0 \right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$
$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$
$$M_z(t) = M_z^0$$

Free Precession w/o Relaxation

$$\mathbf{R}_z(\omega_0 t) = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Precession is left-handed (clockwise).

$$\vec{M}(t) = \mathbf{R}_z(\omega_0 t)\vec{M}^0$$

$$\Updownarrow$$

$$\omega_0 = -\gamma B_0$$

Basic RF Pulse

$$\overrightarrow{B} = \overrightarrow{B}_0 + \overrightarrow{B}_1(t)$$

$$\overrightarrow{B}_1(t) = B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$B_1^e(t)$ **pulse envelope function**

ω_{RF} **excitation carrier frequency**

θ **initial phase angle**

B_1 is perpendicular to B_0.

$$\overrightarrow{B}_0 = B_0\hat{k}$$

Relationship Between Lab and Rotating Frames

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{matrix} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{matrix}$$

$$\overrightarrow{M}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{M}_{rot}(t)$$

Bulk magnetization components in the rotating frame.

$$\overrightarrow{B}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{B}_{rot}(t)$$

Applied B-field components in the rotating frame.

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \Rightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats).
[Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \right)$$

Equation of motion for an ensemble of spins (isochromats).
[Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\overrightarrow{B}(t) = B_0\hat{k} + B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$$\overrightarrow{B}_{lab}(t) = \begin{pmatrix} B_1^e(t)\cos(\omega_{RF}t + \theta) \\ -B_1^e(t)\sin(\omega_{RF} + \theta) \\ B_0 \end{pmatrix} \qquad \overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t)\cos\theta \\ -B_1^e(t)\sin\theta \\ B_0 \end{pmatrix}$$

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

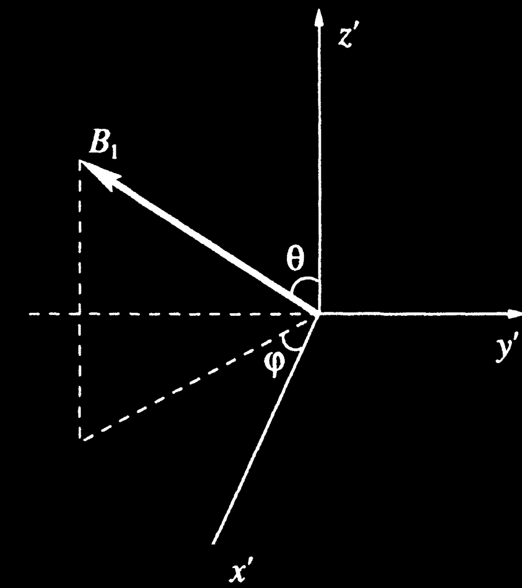
Assume no RF phase (θ = 0)

$$\overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

$$\overrightarrow{B}_{eff}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} \end{pmatrix}$$

Off-Resonance Excitation

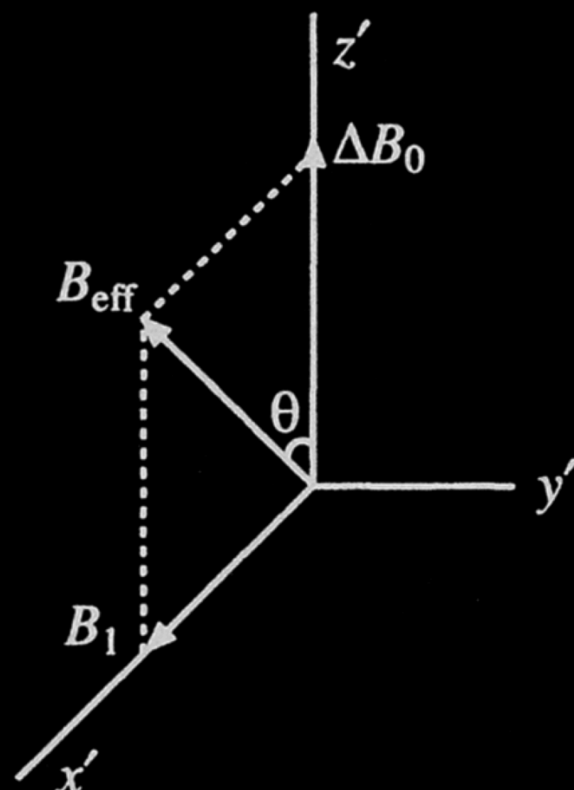
$$\frac{\partial \vec{M}_{rot}}{\partial t} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$



$$\vec{B}_{\text{eff}} = \left(B_0 - \frac{\omega_{\text{rf}}}{\gamma} \right) \vec{k}' + B_1^e(t)\vec{i}'$$

$$= \frac{\Delta\omega_0}{\gamma}\vec{k}' + B_1^e(t)\vec{i}'$$

$$\Delta\omega_0 = \omega_0 - \omega_{\text{rf}}$$

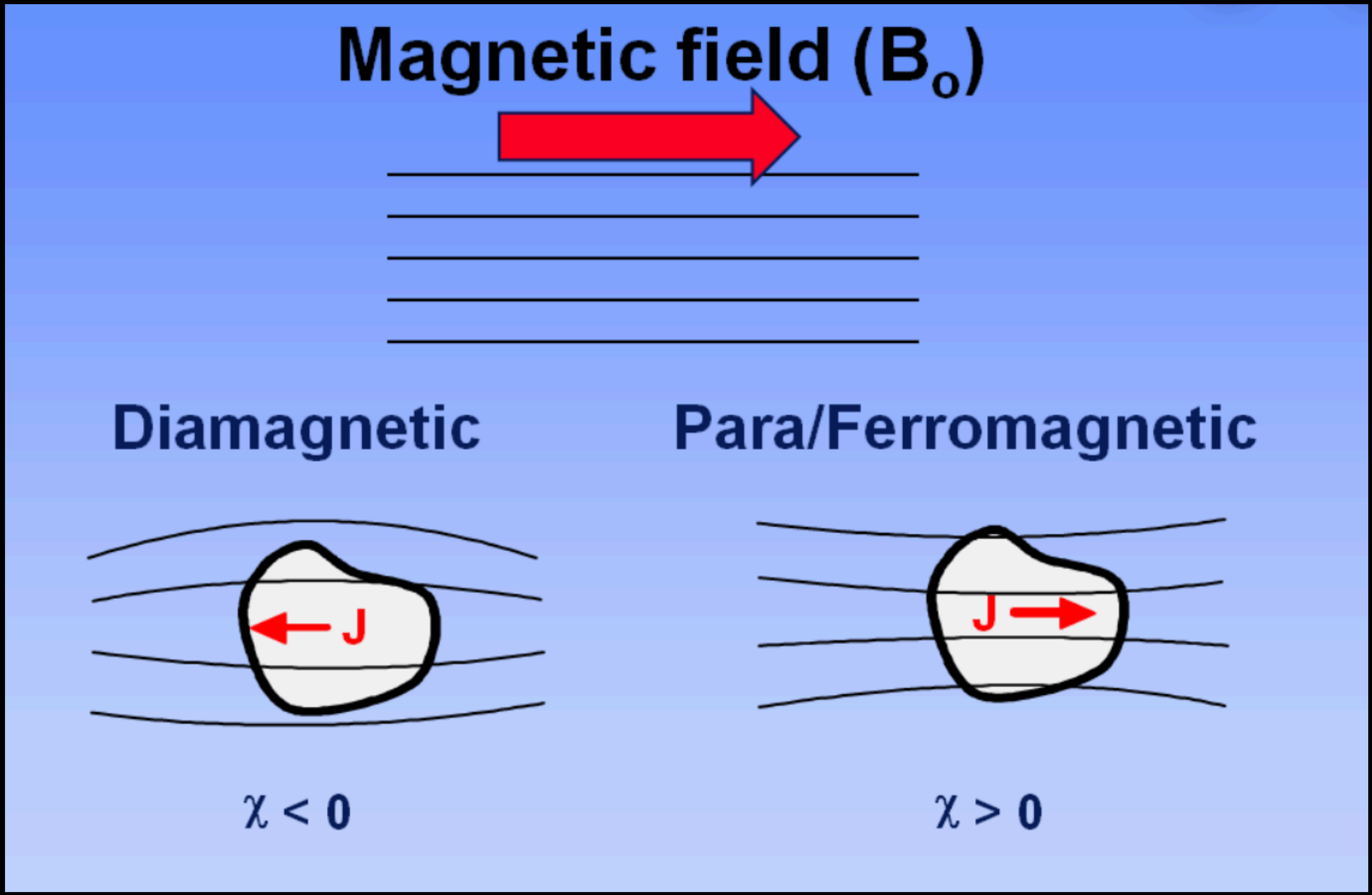


No closed-form solution for generic B1

Sources of Off-Resonance

- **B0 Field Inhomogeneity**
  - **Imperfect shimming**
- **Magnetic susceptibility**
  - **metallic objects**
  - **diamagnetic tissue**
  - **paramagnetic tissue**
  - **air/tissue interface**
- **Eddy Currents**
- **Chemical Shift**

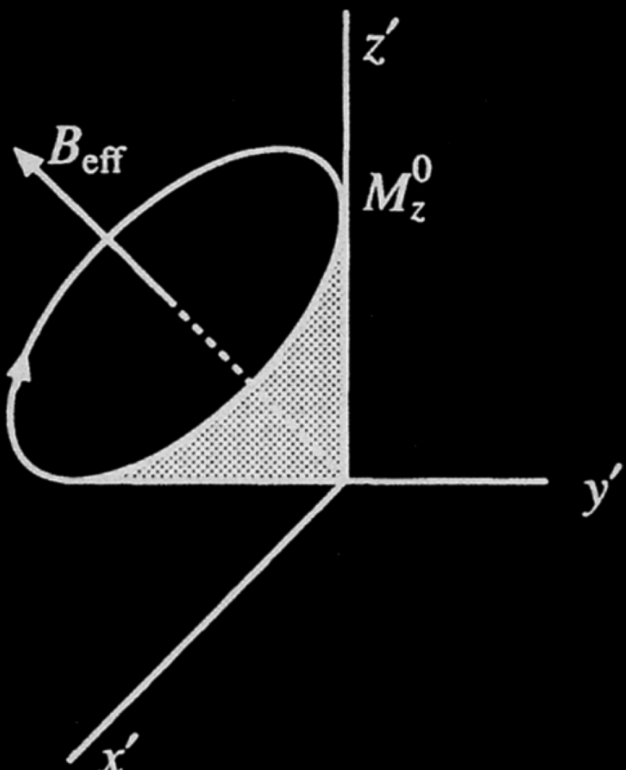
Magnetic Susceptibility



Magnetic Susceptibility

Magnetic Property	Direction of Polarization (I) Relative to External Field	Relative Magnetic Susceptibility (χ) in ppm	Typical Materials
Diamagnetism	Opposite	−10	Water, fat, calcium, most biologic tissues
Paramagnetism	Same	+1	Molecular O_2, simple salts and chelates of metals (Gd, Fe, Mn, Cu), organic free radicals
Superparamagnetism	Same	+5000	Ferritin, hemosiderin, SPIO contrast agents
Ferromagnetism	Same	> 10,000	Iron, steel

Off-Resonance Excitation



$$\vec{B}_{\text{eff}} = \left(B_0 - \frac{\omega_{\text{rf}}}{\gamma}\right)\vec{k}' + B_1^e(t)\vec{i}'$$

$$= \frac{\Delta\omega_0}{\gamma}\vec{k}' + B_1^e(t)\vec{i}'$$

Important Observations

For non-constant B1, the actual axis of rotation changes!

M will not rotate to the target location due to off-resonance

Effective flip angle and signal phase vary depending on off-resonance $\Delta\omega_0$

David Geffen School of Medicine

UCLA Radiology

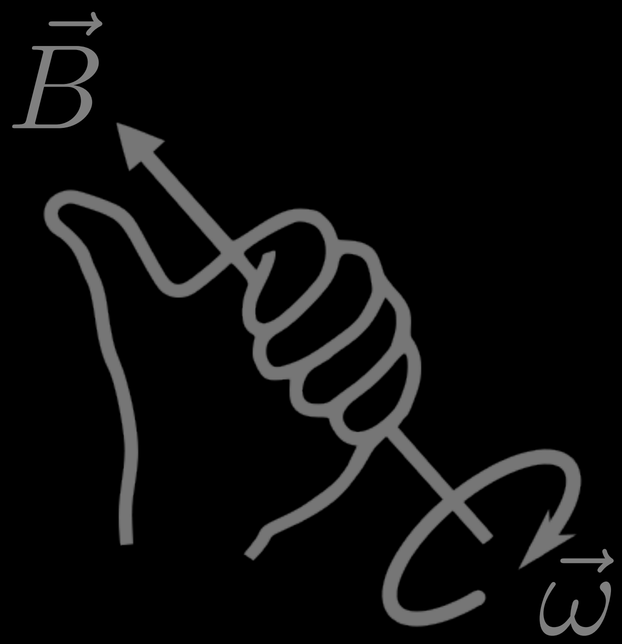
Frequency Selectivity of RF Pulses

Matlab Demo

Mathematics of Hard RF Pulses

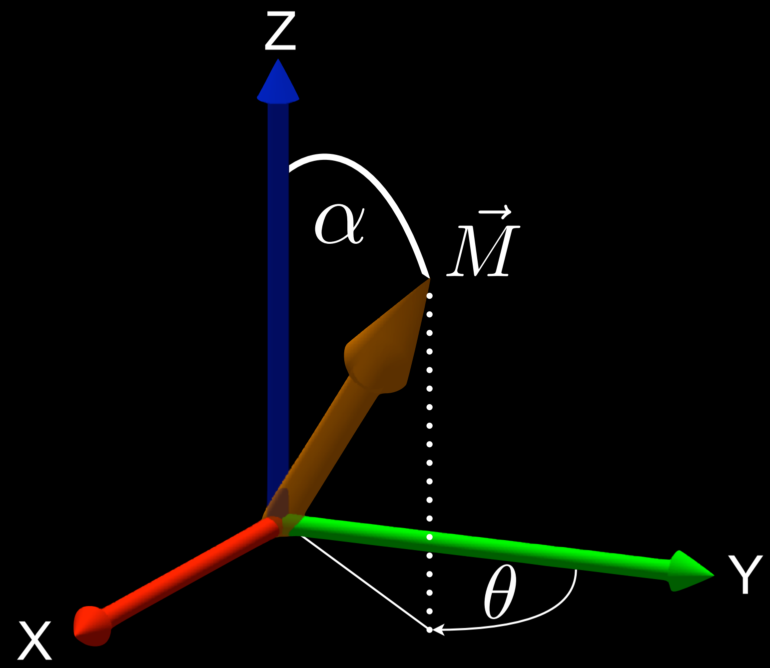
Rules for RF Pulses

- RF fields induce left-hand rotations
- Phase of 0° is about the x-axis
- Phase of 90° is about the y-axis



Flip Angle - α

- **"Amount of rotation of the bulk magnetization vector produced by an RF pulse, with respect to the direction of the static magnetic field."**
  – Liang & Lauterbur, p. 374



$$\omega_1 = \gamma B_1$$

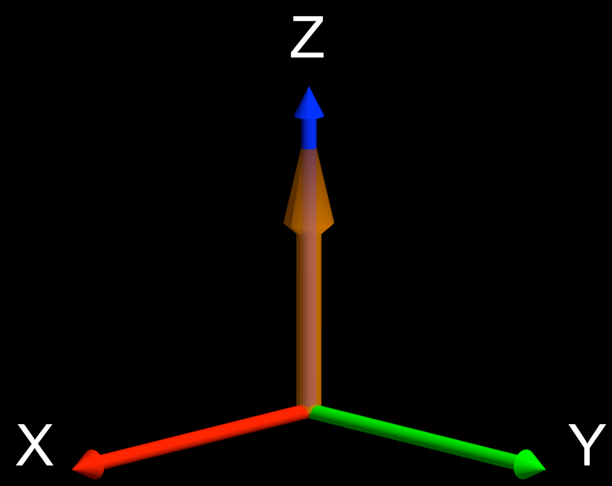
B-fields induce precession!

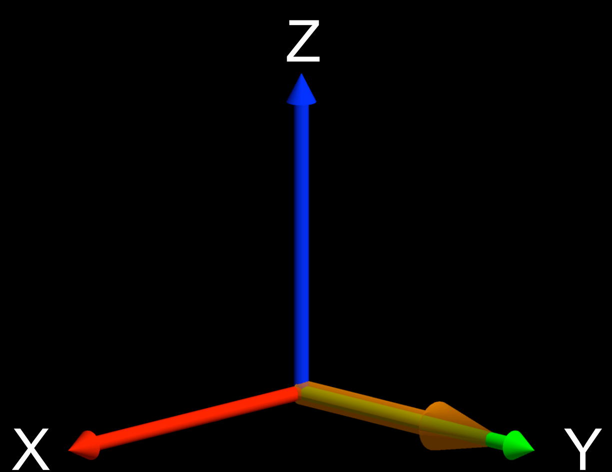
Rules for RF Pulses

$$R_{\theta}^{\alpha}$$

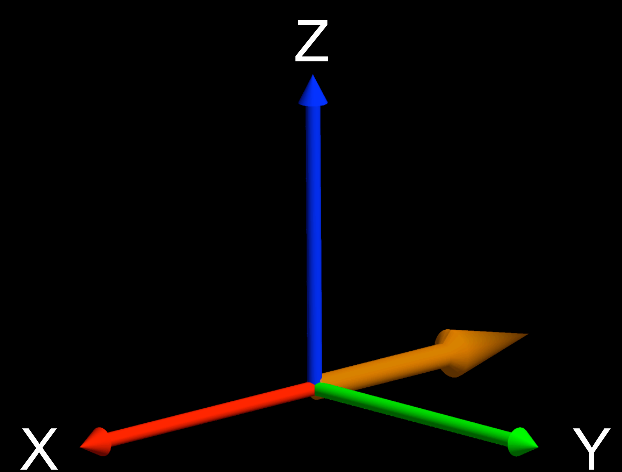
→ Flip Angle

→ Phase



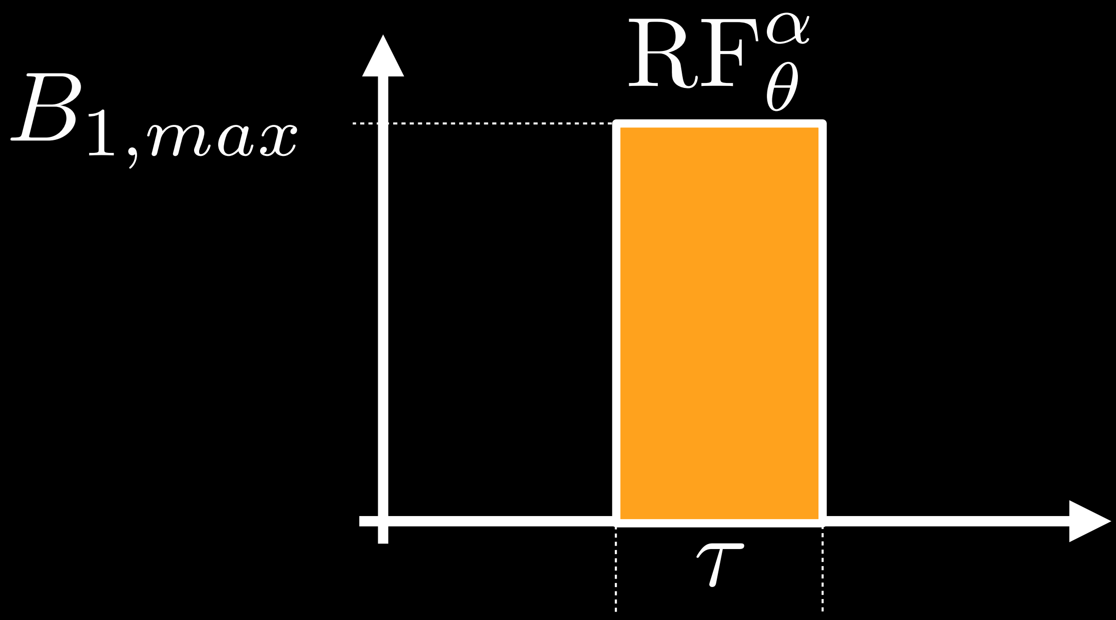


$R_{0^\circ}^{90^\circ}$



$R_{90^\circ}^{90^\circ}$

How to determine α?



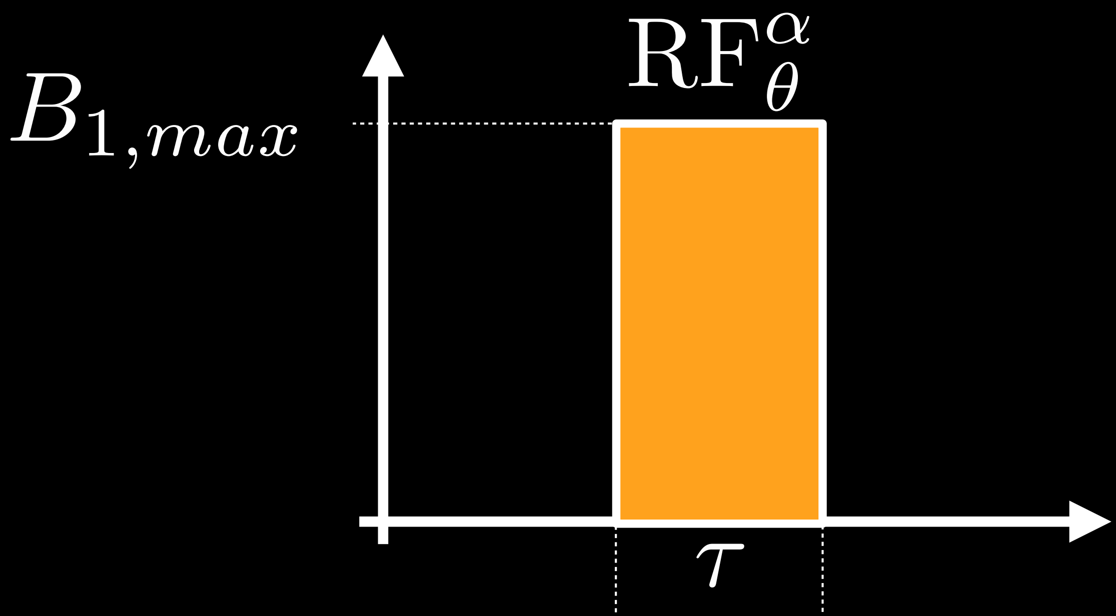
$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)\, dt$$

Rules: 1) Specify α

2) Use $B_{1,max}$ if we can

3) Shortest duration pulse

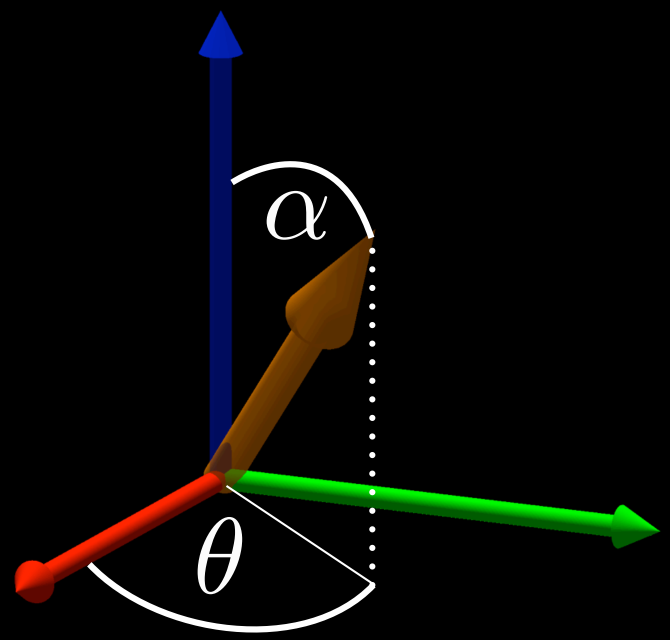
How to determine α?



$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)\, dt$$

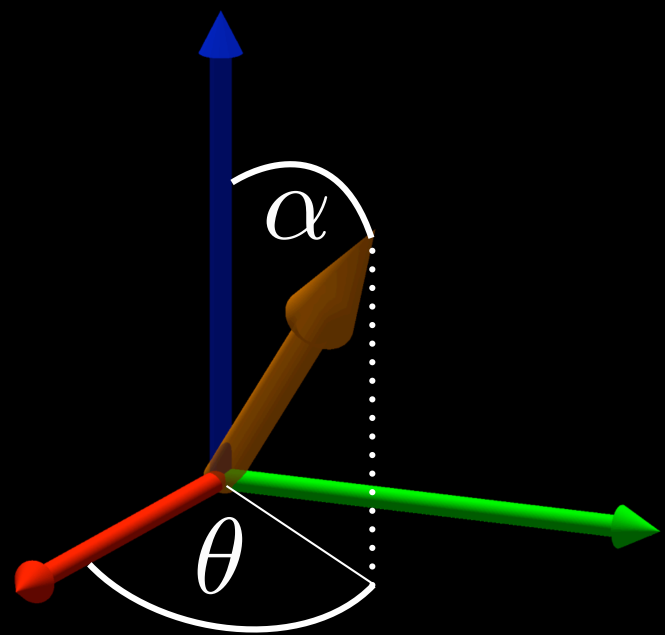
$$\tau = \frac{\alpha}{\gamma B_{1,max}} = \frac{\pi/2}{2\pi \cdot 42.57Hz/\mu T \cdot 60\mu T} = 0.098ms$$

Change of Basis (θ)



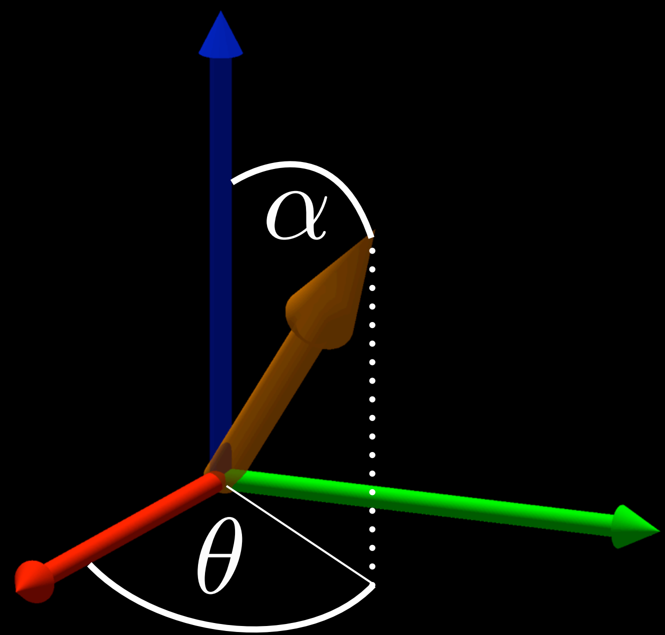
$$\mathbf{R}_Z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation by Alpha



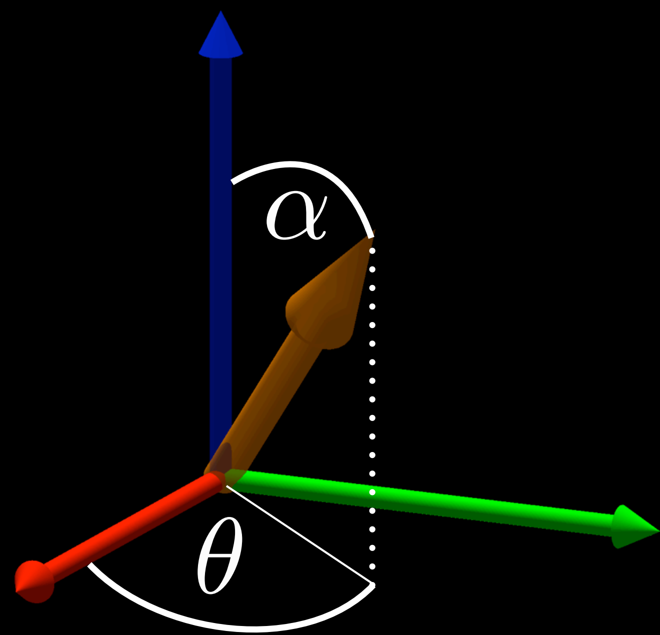
$$\mathbf{R}_X(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Change of Basis (-θ)



$$\mathbf{R}_Z(-\theta) = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 0 \\ -\sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

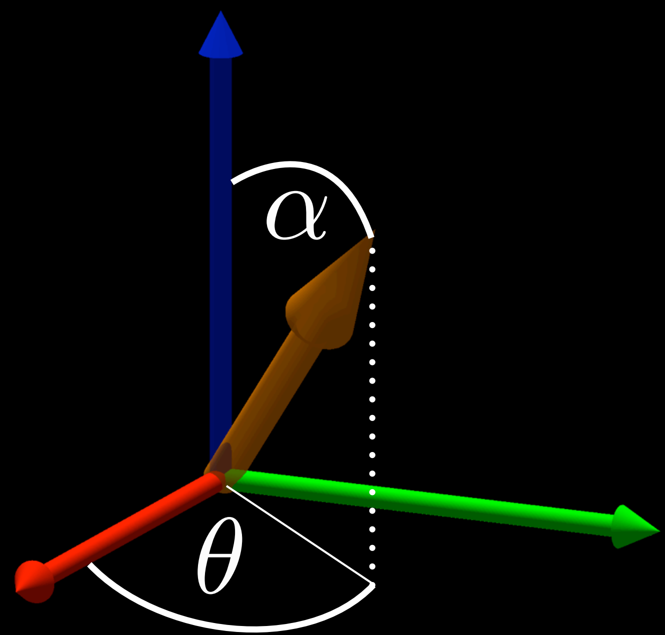
RF Pulse Operator



$$\mathbf{R}_\theta^\alpha = \mathbf{R}_Z(-\theta)\,\mathbf{R}_X(\alpha)\,\mathbf{R}_Z(\theta)$$

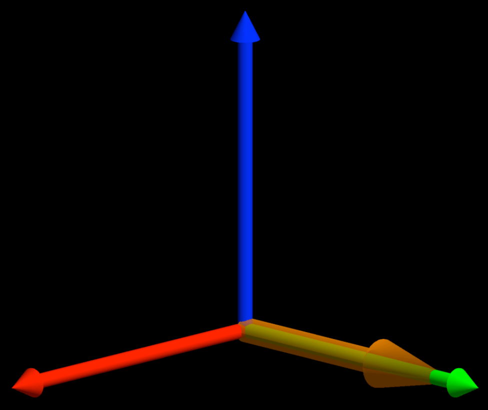
$$= \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha \end{bmatrix}$$

RF Pulse Operator

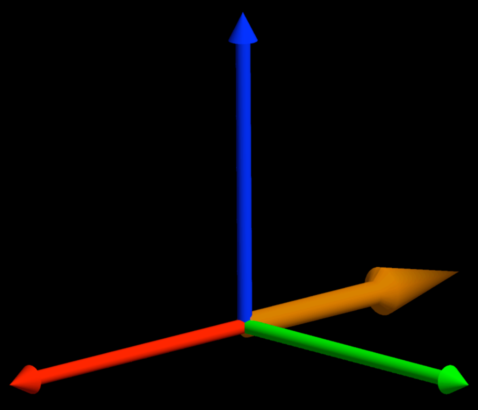


$$\vec{\mathrm{M}}\left(0_{+}\right) = \mathrm{RF}_{\theta}^{\alpha}\vec{\mathrm{M}}\left(0_{-}\right)$$

Hard RF Pulses



$\mathrm{R}_{0^\circ}^{90^\circ}$



$\mathrm{R}_{90^\circ}^{90^\circ}$

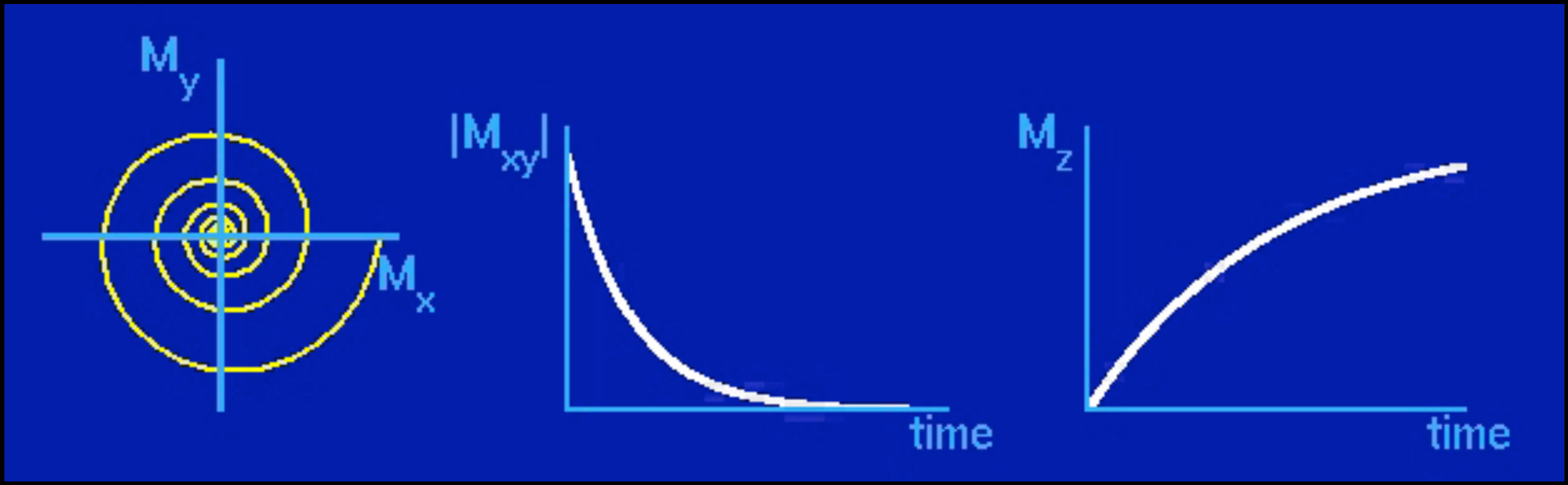
$$\mathbf{R}_{0^\circ}^{90^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{R}_{90^\circ}^{90^\circ} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

T_1 & T_2 Relaxation

Relaxation

- Magnetization returns exponentially to equilibrium:
  - Longitudinal recovery time constant is T1
  - Transverse decay time constant is T2
- Relaxation and precession are independent



T_1 Relaxation

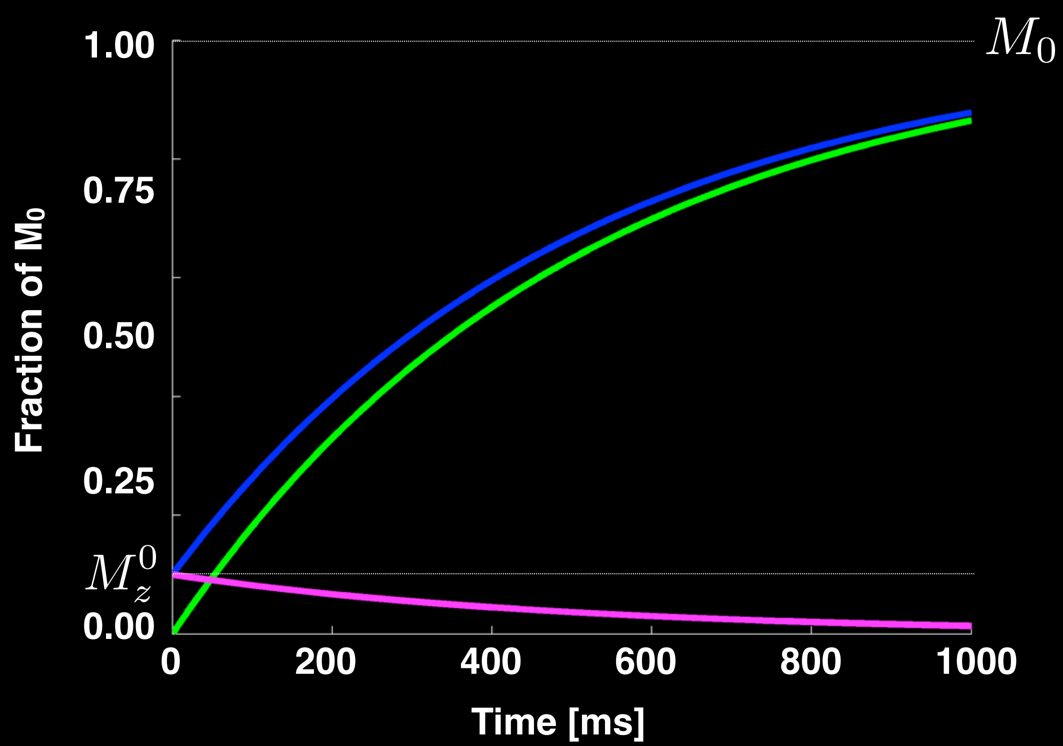
- Longitudinal or spin-lattice relaxation
  - Typically, (10s ms) < T1 < (100s ms)
- T1 is long for
  - Small molecules (water)
  - Large molecules (proteins)
- T1 is short for
  - Fats and intermediate-sized molecules
- T1 increases with increasing B0
- T1 decreases with contrast agents

Short T_1s are bright on T_1-weighted image

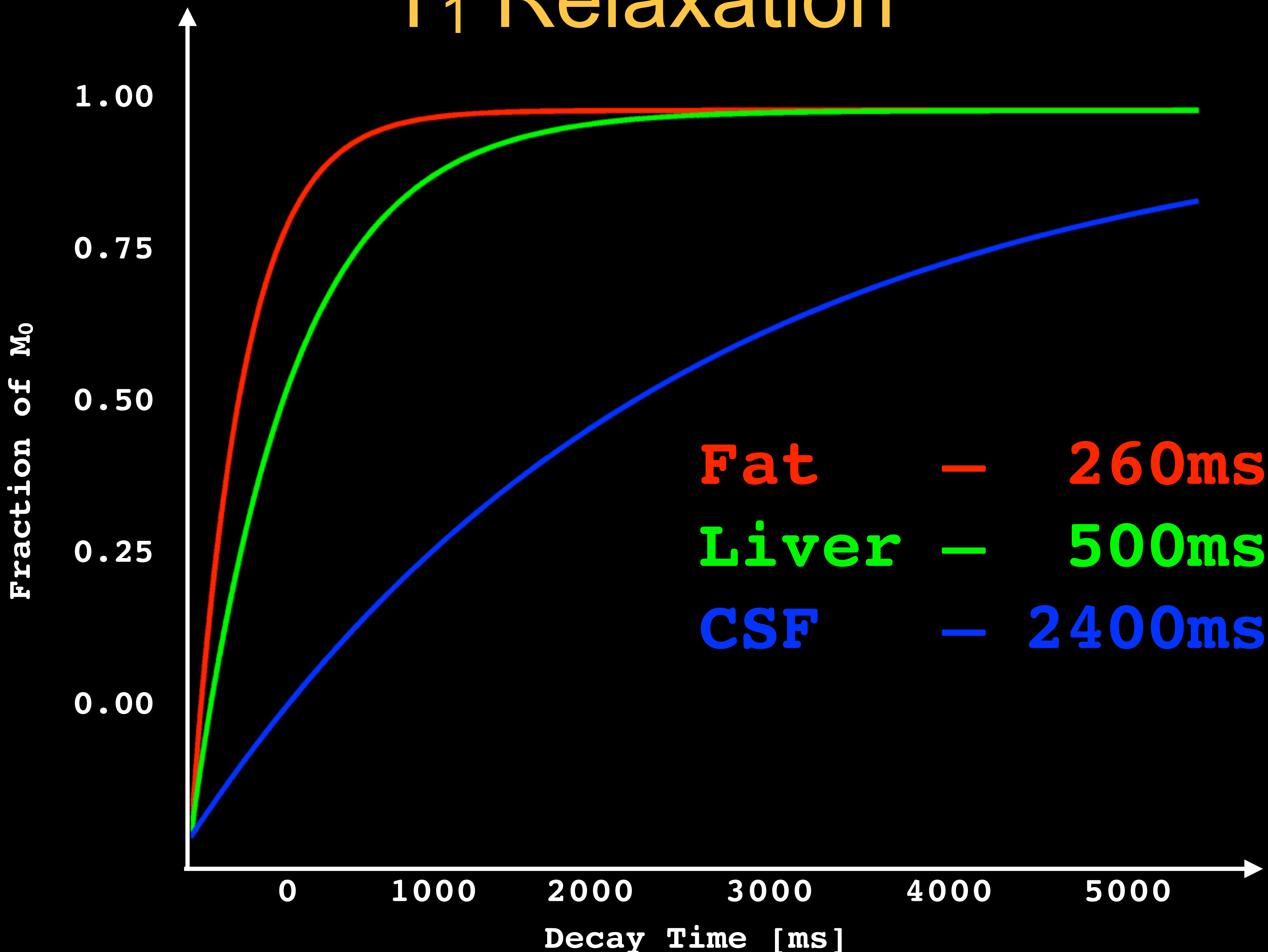
T_1 Relaxation

Free Precession in the Lab ***or*** Rotating Frame with Relaxation

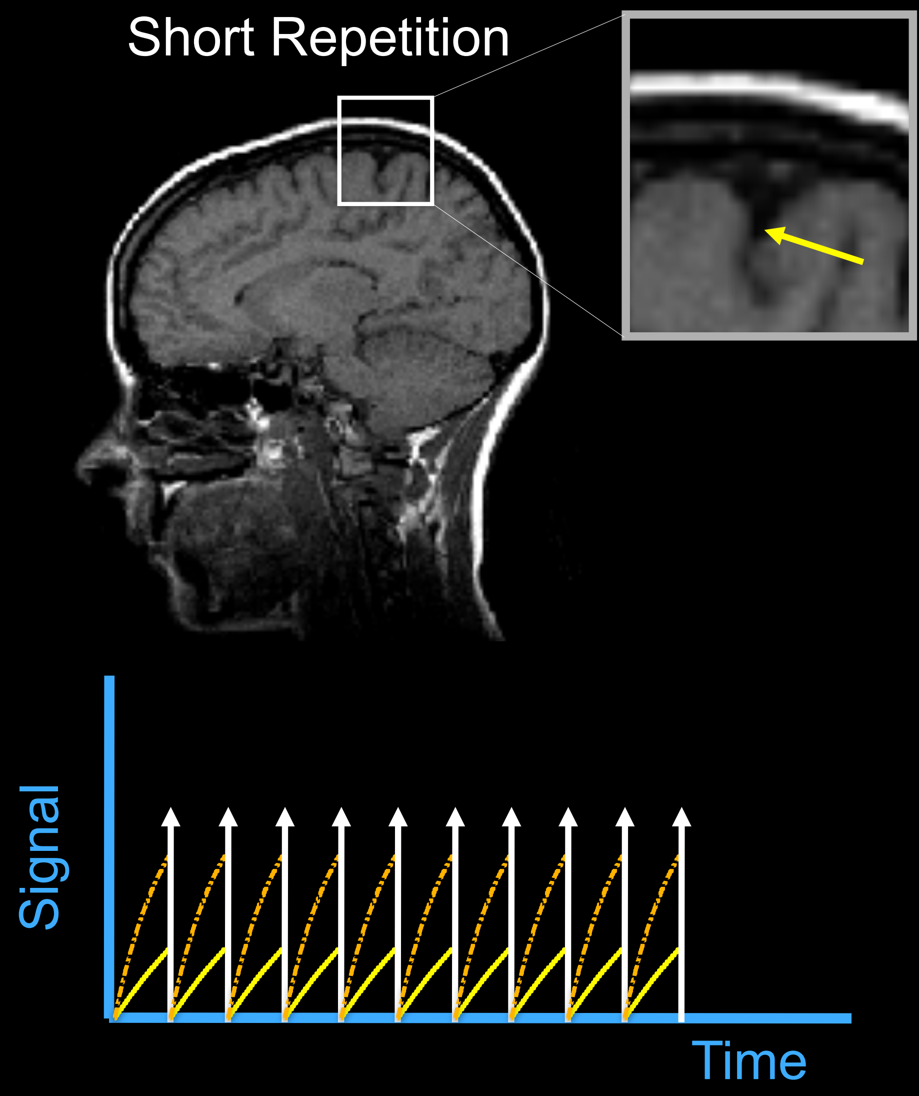
$$\underbrace{M_{z'}(t)}_{\text{Net Magnetization}} = \underbrace{M_z^0 e^{-t/T_1}}_{\text{Prepared Magnetization Decays } (M_z^0)} + \underbrace{M_0\left(1 - e^{-t/T_1}\right)}_{\text{Return to Thermal Equilibrium } (M_0)}$$



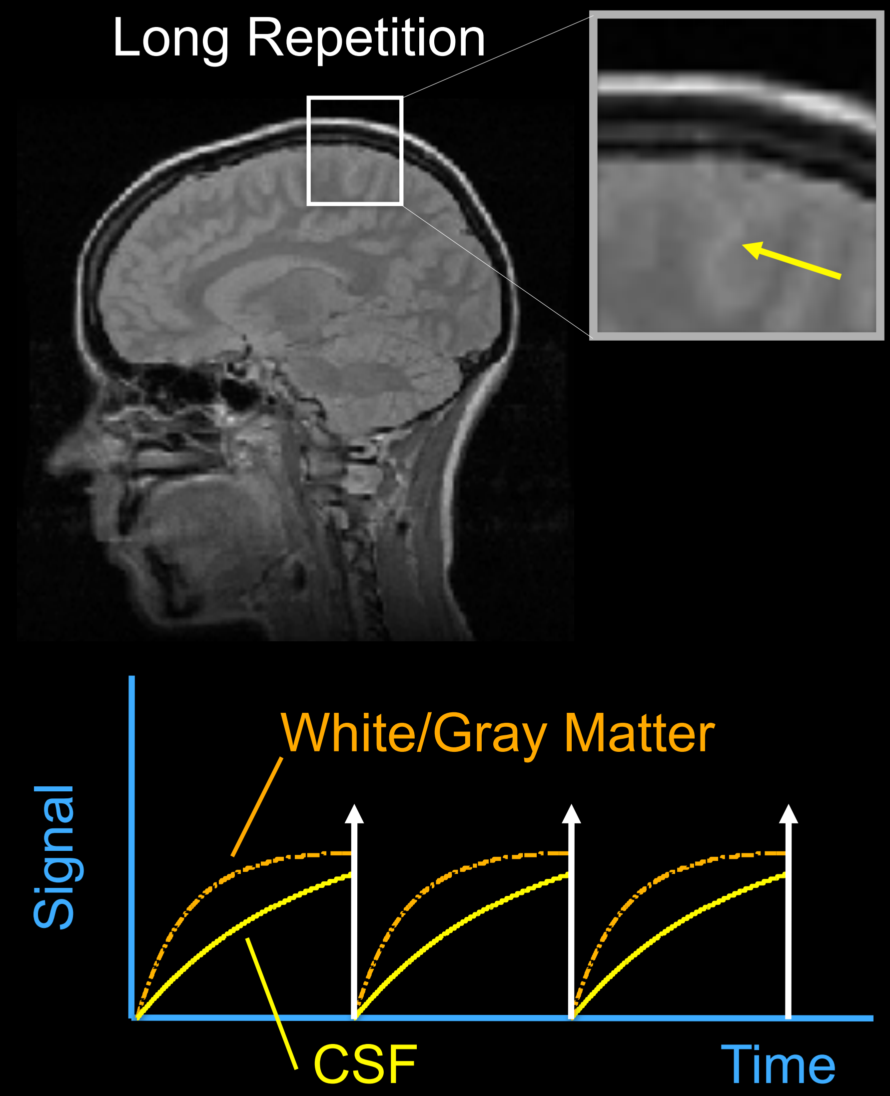
T_1 Relaxation



T_1 Contrast



Short Repetition
Signal
Time



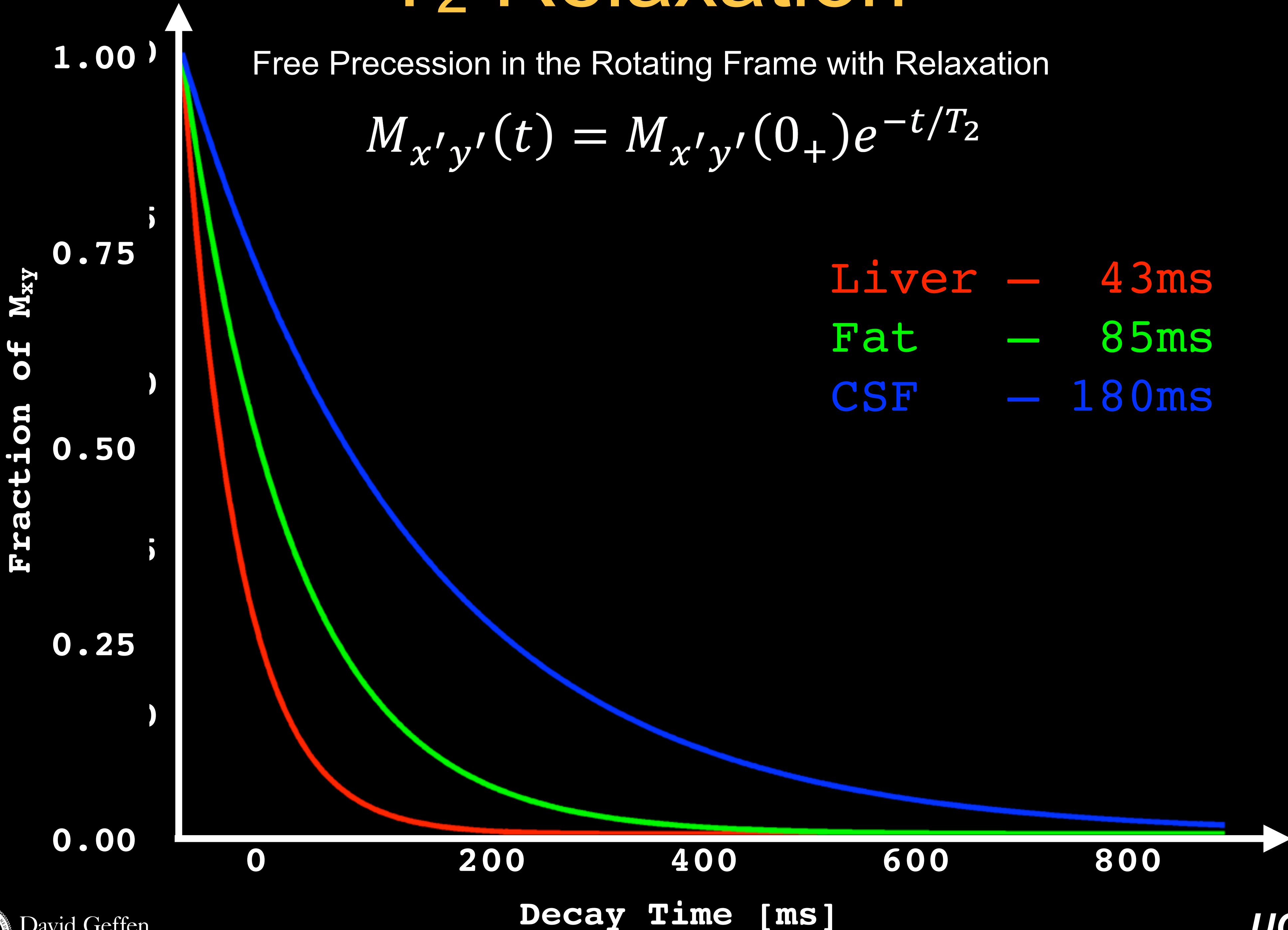
Long Repetition
White/Gray Matter
Signal
CSF
Time

T_2 Relaxation

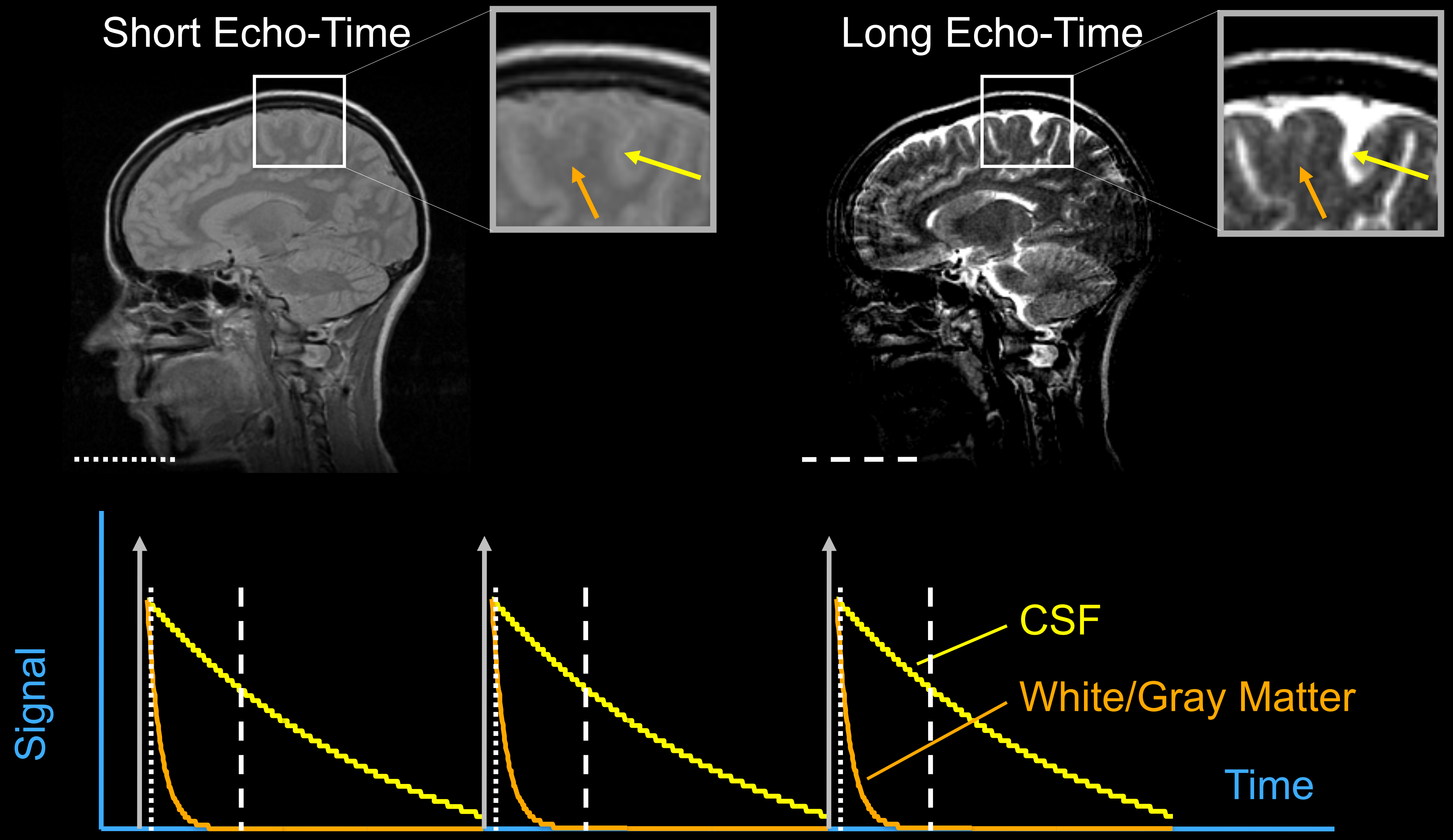
- Transverse or spin-spin relaxation
  - Molecular interaction causes spin dephasing
  - Typically, T2 < (10s ms)
- Increasing molecular size, decrease T2
  - Fat has a short T2
- Increasing molecular mobility, increases T2
  - Liquids (CSF, edema) have long T2s
- Increasing molecular interactions, decreases T2
  - Solids have short T2s
- T2 relatively independent of B0

Long T_2 is bright on T_2 weighted image

T_2 Relaxation



T2 Contrast


Short Echo-Time
Long Echo-Time
CSF
White/Gray Matter
Signal
Time

T_1 and T_2 Values @ 1.5T

Tissue	**T_1 [ms]**	**T_2 [ms]**
gray matter	925	100
white matter	790	92
muscle	875	47
fat	260	85
kidney	650	58
liver	500	43
CSF	2400	180

Each tissue has “unique” relaxation properties, which enables “soft tissue contrast”.

Bloch Equations with Relaxation

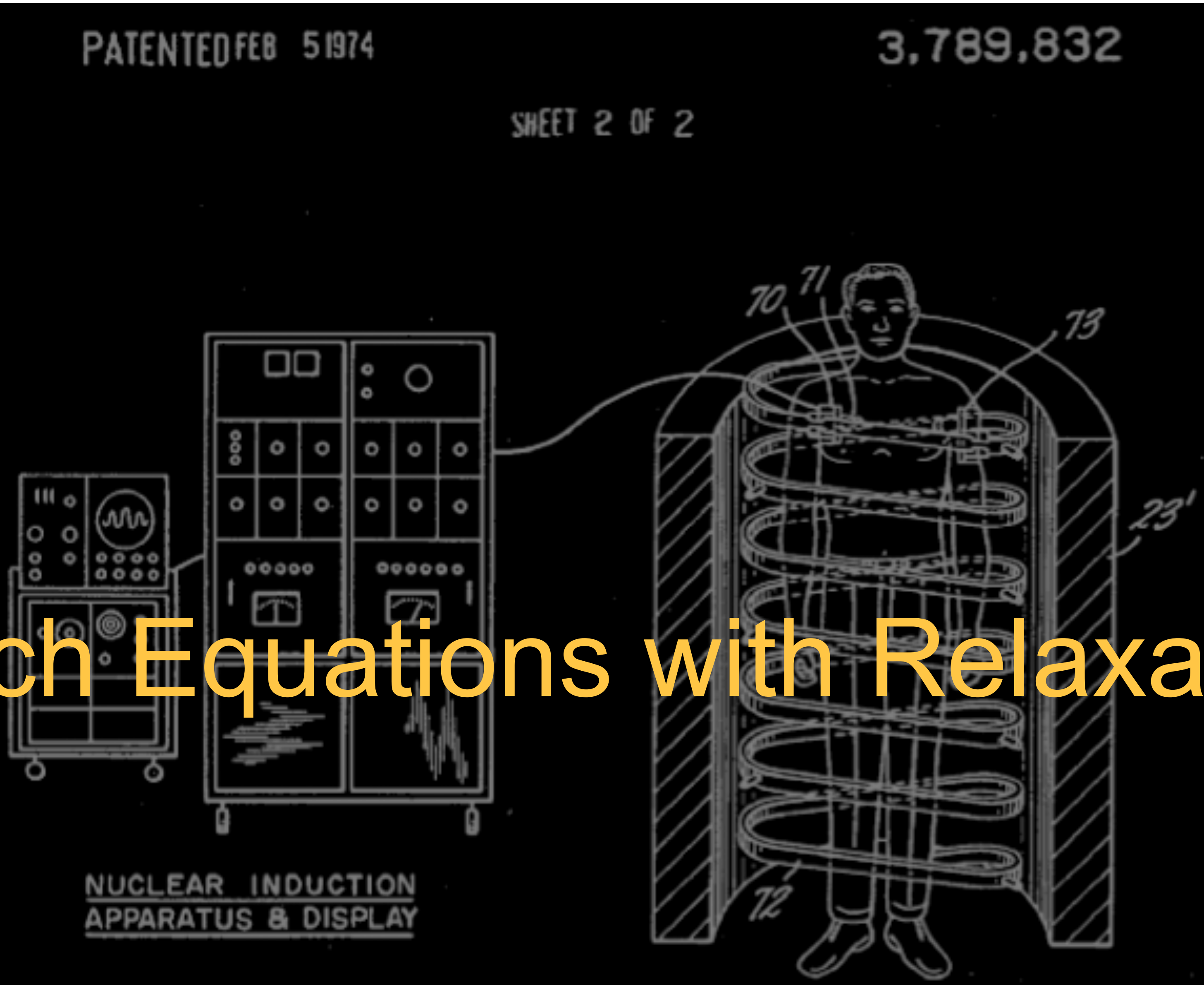


FIG. 2

Bloch Equations with Relaxation

$$\frac{d\vec{\mathrm{M}}}{dt} = \vec{\mathrm{M}} \times \gamma\vec{\mathrm{B}} - \frac{M_x\hat{\mathrm{i}} + M_y\hat{\mathrm{j}}}{T_2} - \frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}$$

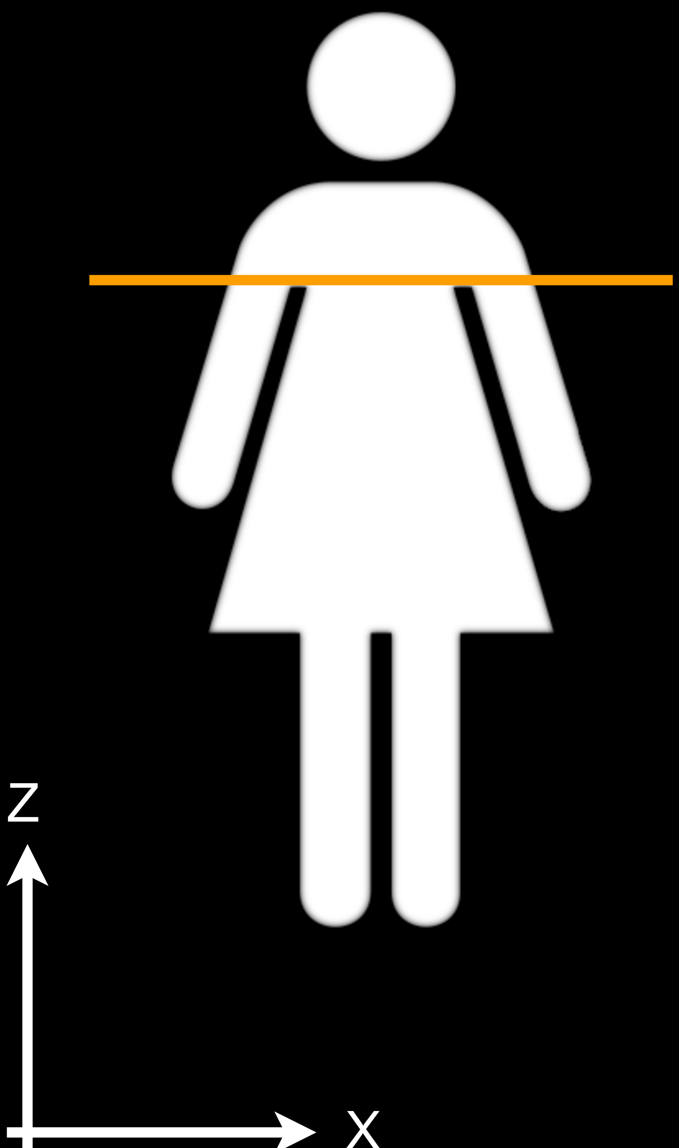
- **Differential Equation**
  - Ordinary, Coupled, Non-linear
- **No analytic solution, in general.**
  - Analytic solutions for simple cases.
  - Numerical solutions for all cases.
- **Phenomenological**
  - Exponential behavior is an approximation.

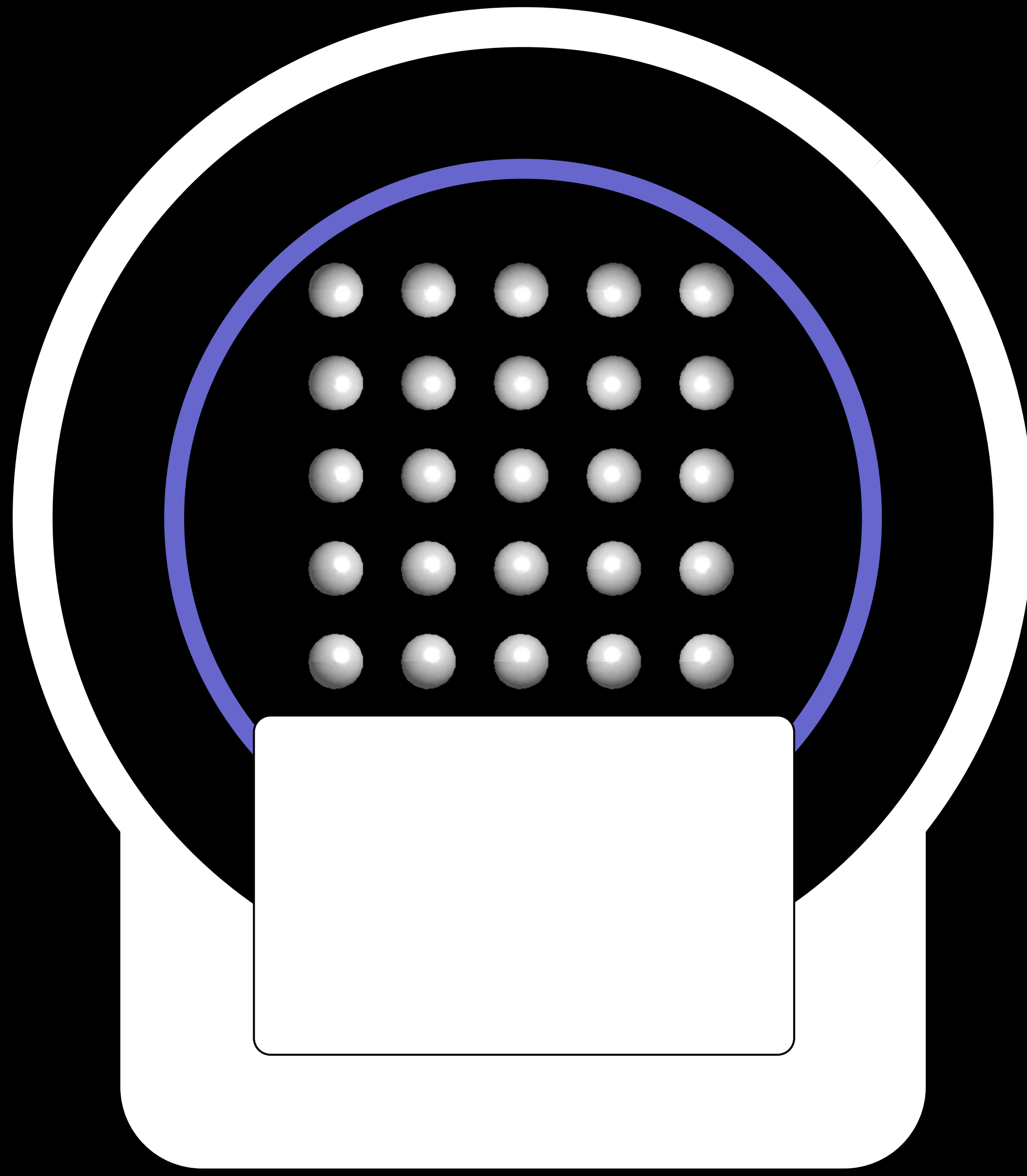
Bloch Equations - Lab Frame

$$\frac{d\vec{\mathrm{M}}}{dt} = \underbrace{\vec{\mathrm{M}} \times \gamma \vec{\mathrm{B}}}_{\text{Precession}} - \underbrace{\frac{M_x \hat{\mathrm{i}} + M_y \hat{\mathrm{j}}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}}_{\text{Longitudinal Relaxation}}$$

- Precession
  - Magnitude of M unchanged
  - Phase (rotation) of M changes due to B
- Relaxation
  - T_1 changes are slow O(100ms)
  - T_2 changes are fast O(10ms)
  - Magnitude of M can be ZERO
- Diffusion
  - Spins are thermodynamically driven to exchange positions.
  - Bloch-Torrey Equations

Excitation and Relaxation





The magnetization relaxes after excitation (forced precession).

Bloch Equations – Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

"Precession" | Transverse Relaxation | Longitudinal Relaxation

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

Effective B-field that M experiences in the rotating frame

Fictitious field created by the rotating frame that demodulates the apparent effect of B_0

The applied B_0 and B_1 field in the rotating frame

Free Precession in the Rotating Frame with Relaxation

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{\mathrm{B}}_{rot} = \mathrm{B}_0 \hat{k}$$

$$\vec{B}_{eff} = \vec{0}$$

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

- **No precession**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, separable ODEs!**

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

Transverse Relaxation (first term); Longitudinal Relaxation (second term)

Solution:

$$M_{z'}(t) = M_z^0 e^{-t/T_1} + M_0\left(1 - e^{-t/T_1}\right)$$

$$M_{x'y'}(t) = M_{x'y'}(0_+)e^{-t/T_2}$$

The precessional term drops out in the rotating frame.

Forced Precession in the Rotating Frame with Relaxation

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{B}_{rot} = B_0 \hat{k} + B_1^e(t)\hat{i}'$$

$$\vec{B}_{eff} = B_1^e(t)\hat{i}'$$

The precessional term ***does not*** drop out in the rotating frame.

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} = B_1^e(t)\hat{\vec{i}}'$$

- **B1 induced nutation**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, coupled PDEs!**
- **When does this equation apply?**

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The precessional term ***does not*** drop out in the rotating frame.

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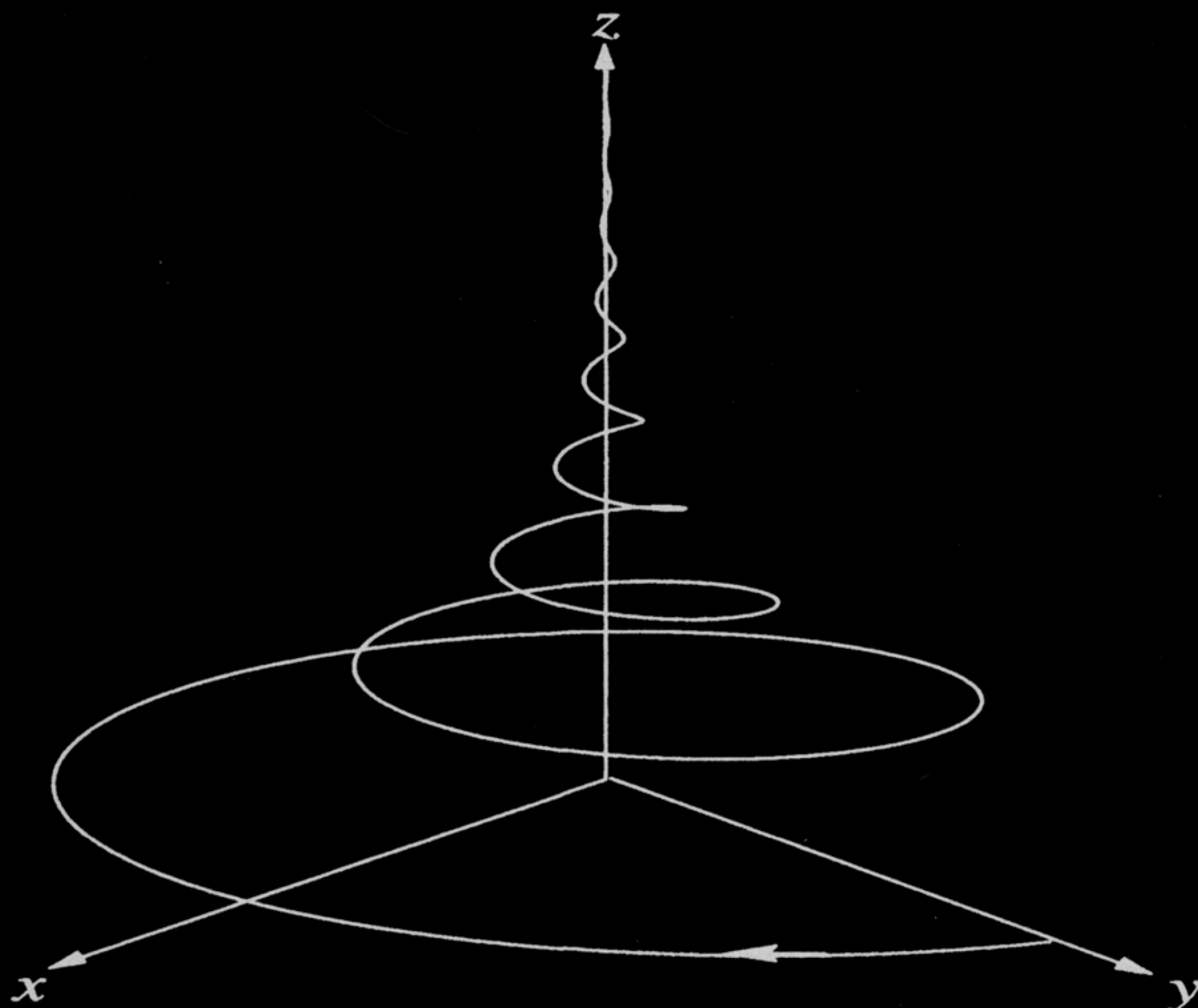
Forced Precession in the Rotating Frame with Relaxation

- RF pulses are short
  - 100μs to 5ms
- Relaxation time constants are long
  - T_1 O(100s) ms
  - T_2 O(10s) ms
- Complicated Coupling
- Best suited for simulation

Free? Forced? Relaxation?

- We've considered all combinations of:
  - Free and forced precession
  - With and without relaxation
  - Laboratory and rotating frames
- Which one's concern M219 the most?
  - Free precession in the rotating frame with relaxation
  - Forced precession in the rotating frame without relaxation.
- We can, in fact, simulate all of them…

Spin Gymnastics - Lab Frame



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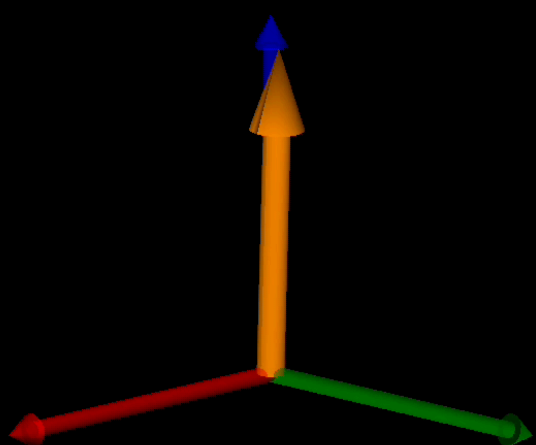
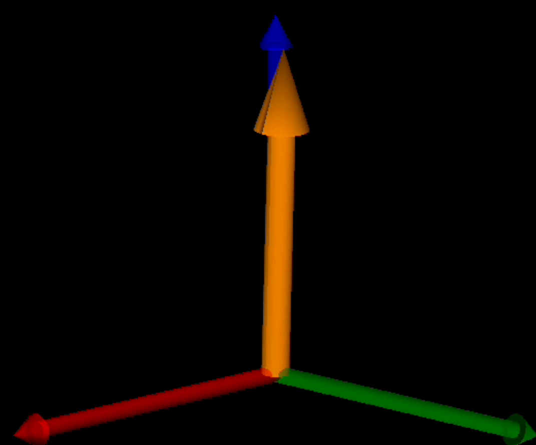
Spin Gymnastics - Rotating Frame

$$M_Z(t) = M_Z^0 e^{-\frac{t}{T_1}} + M_0\left(1 - e^{-\frac{t}{T_1}}\right)$$

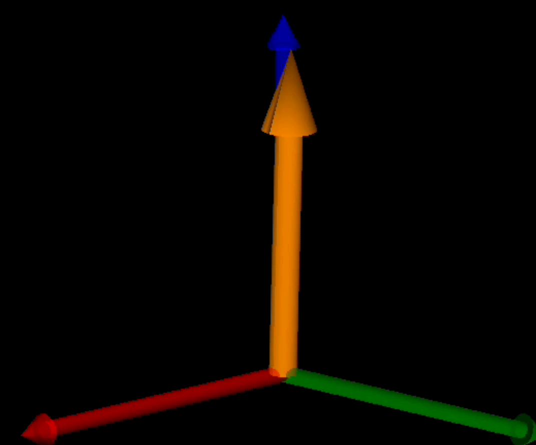
$$M_{xy}(t) = M_{xy}^0 e^{-t/T_2}$$



90° RF



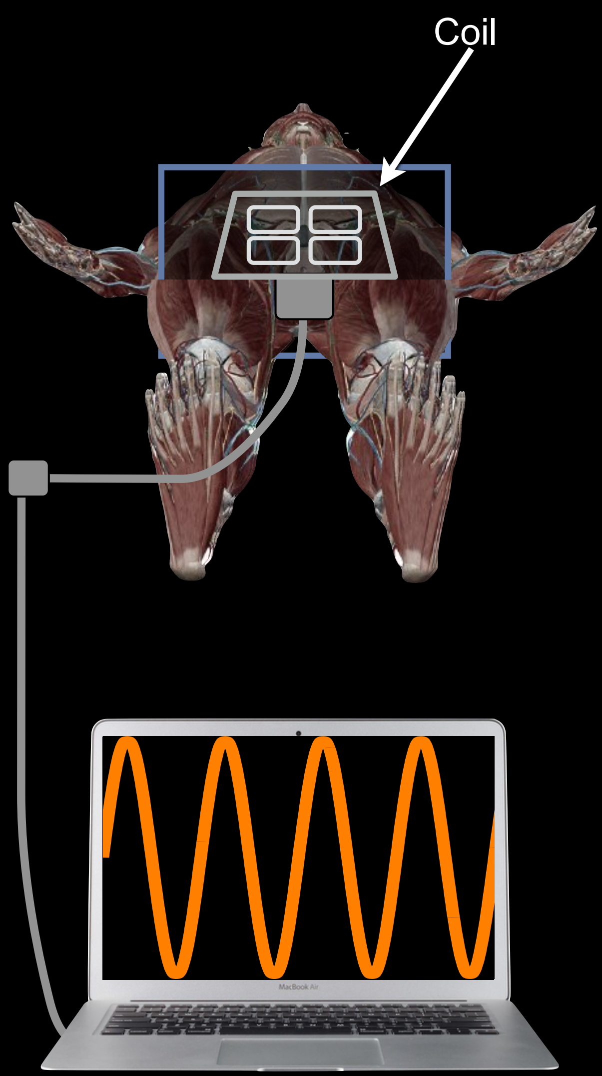
135° RF

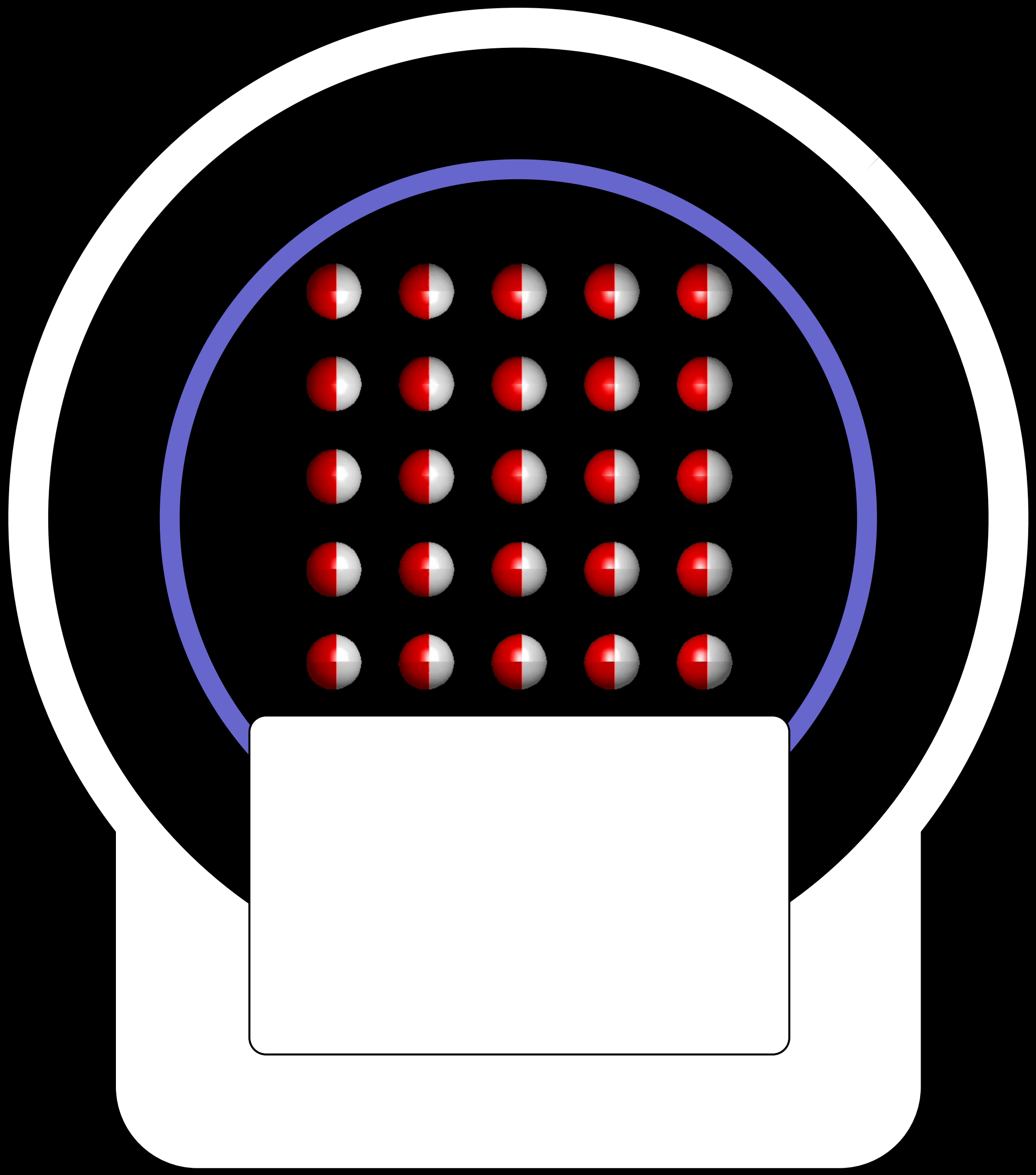


180° RF

How do we measure M_{xy}?

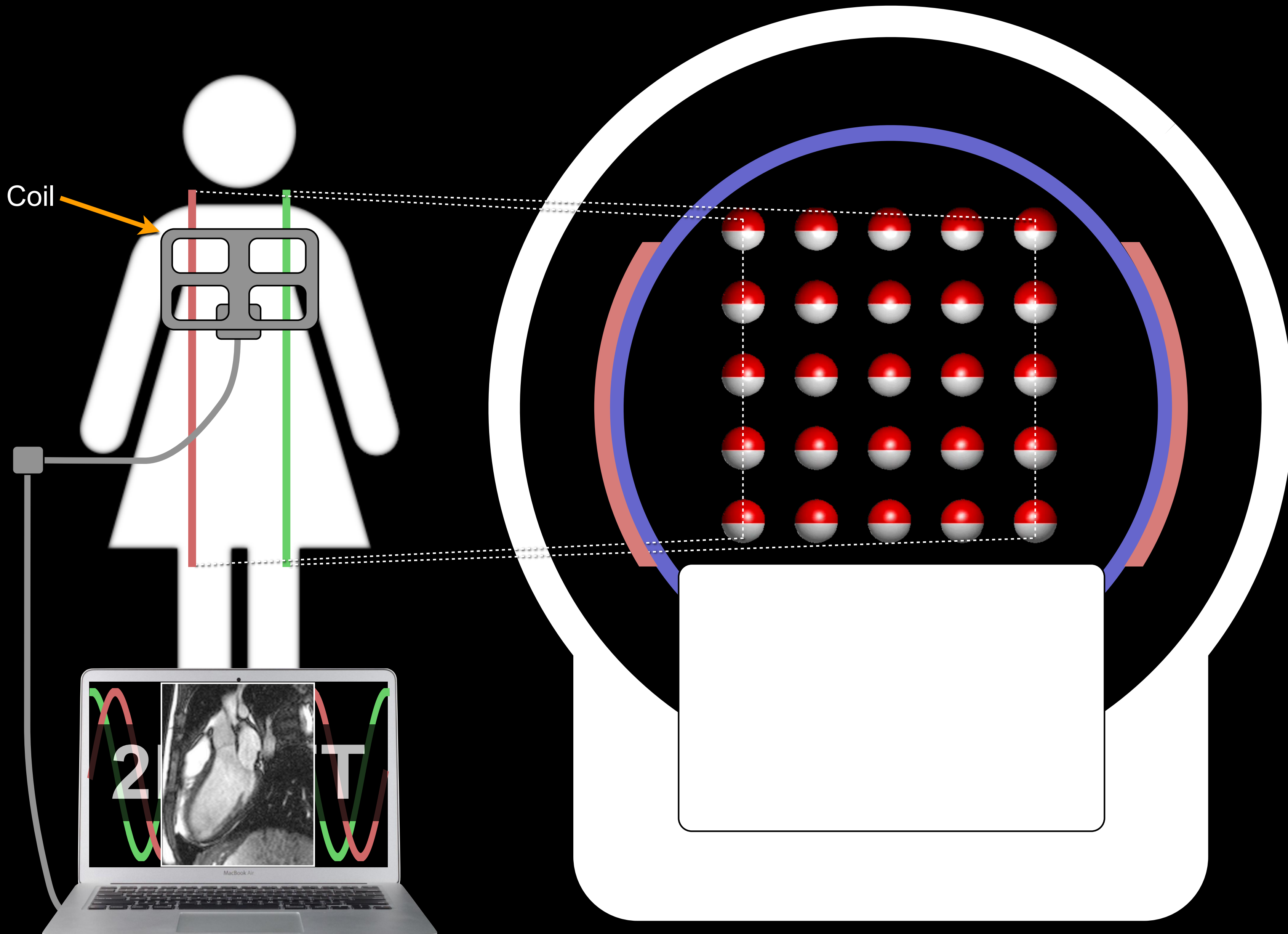
Faraday's Law of Induction





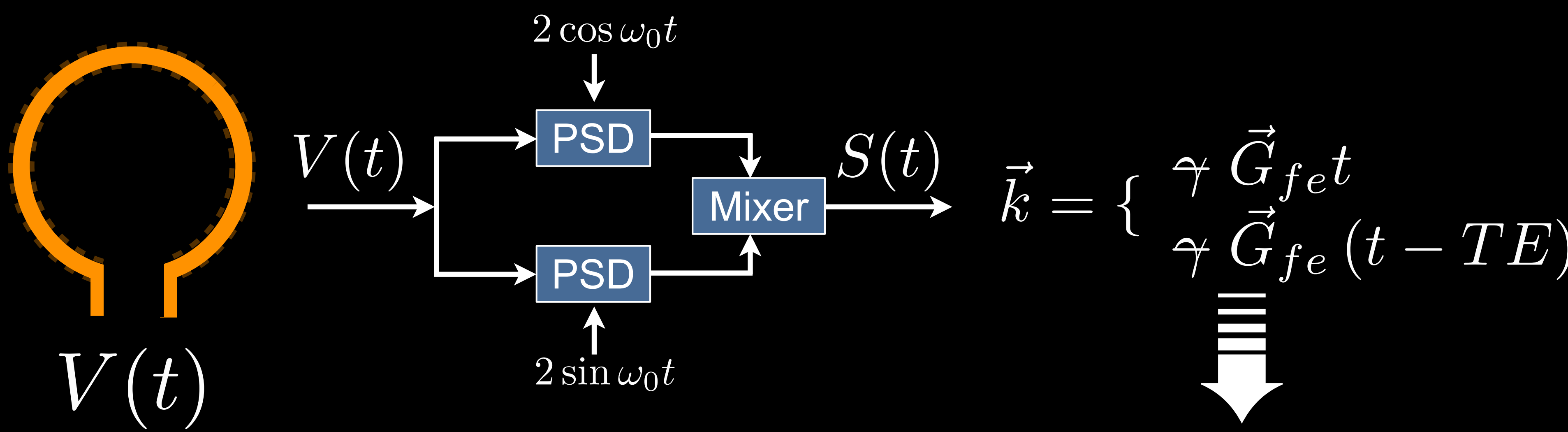
Precessing spins *induce* a current in a nearby coil.

Faraday's Law of Induction

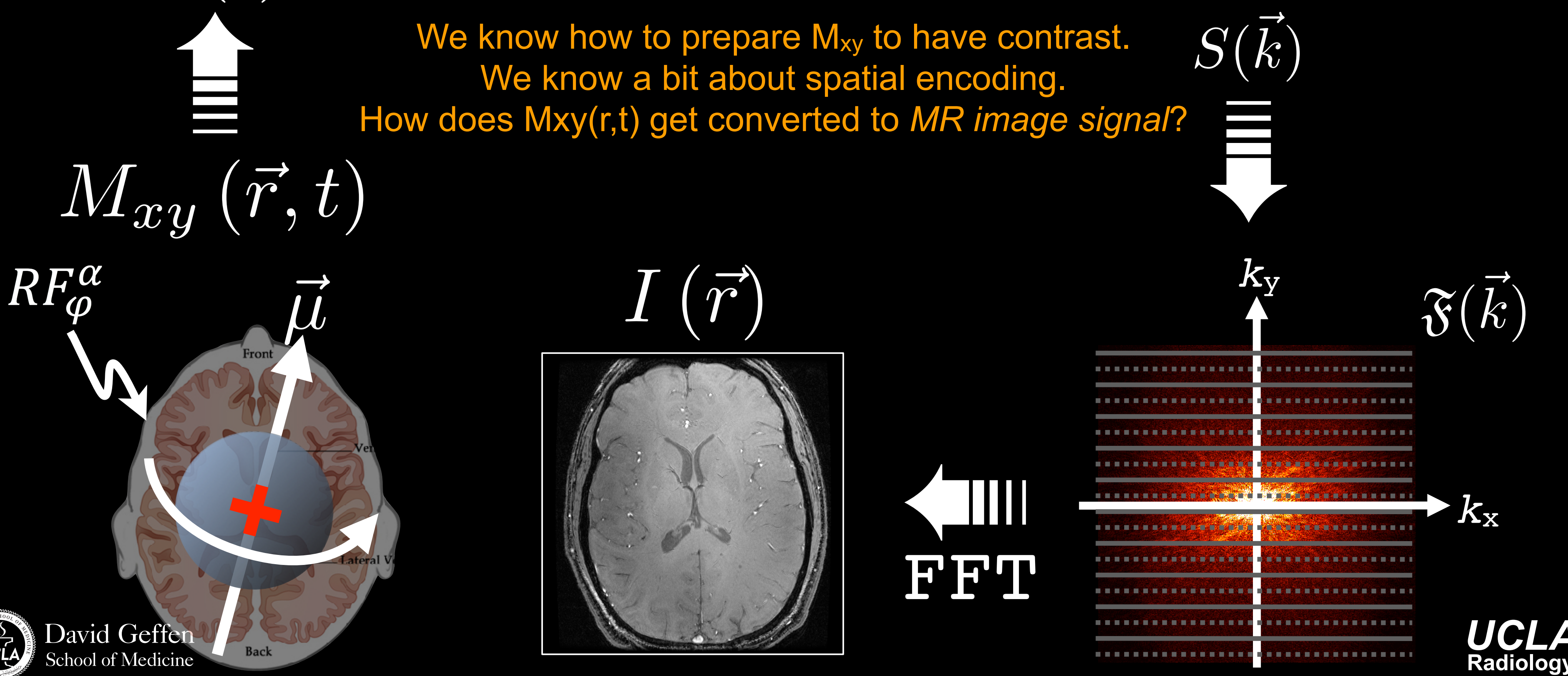


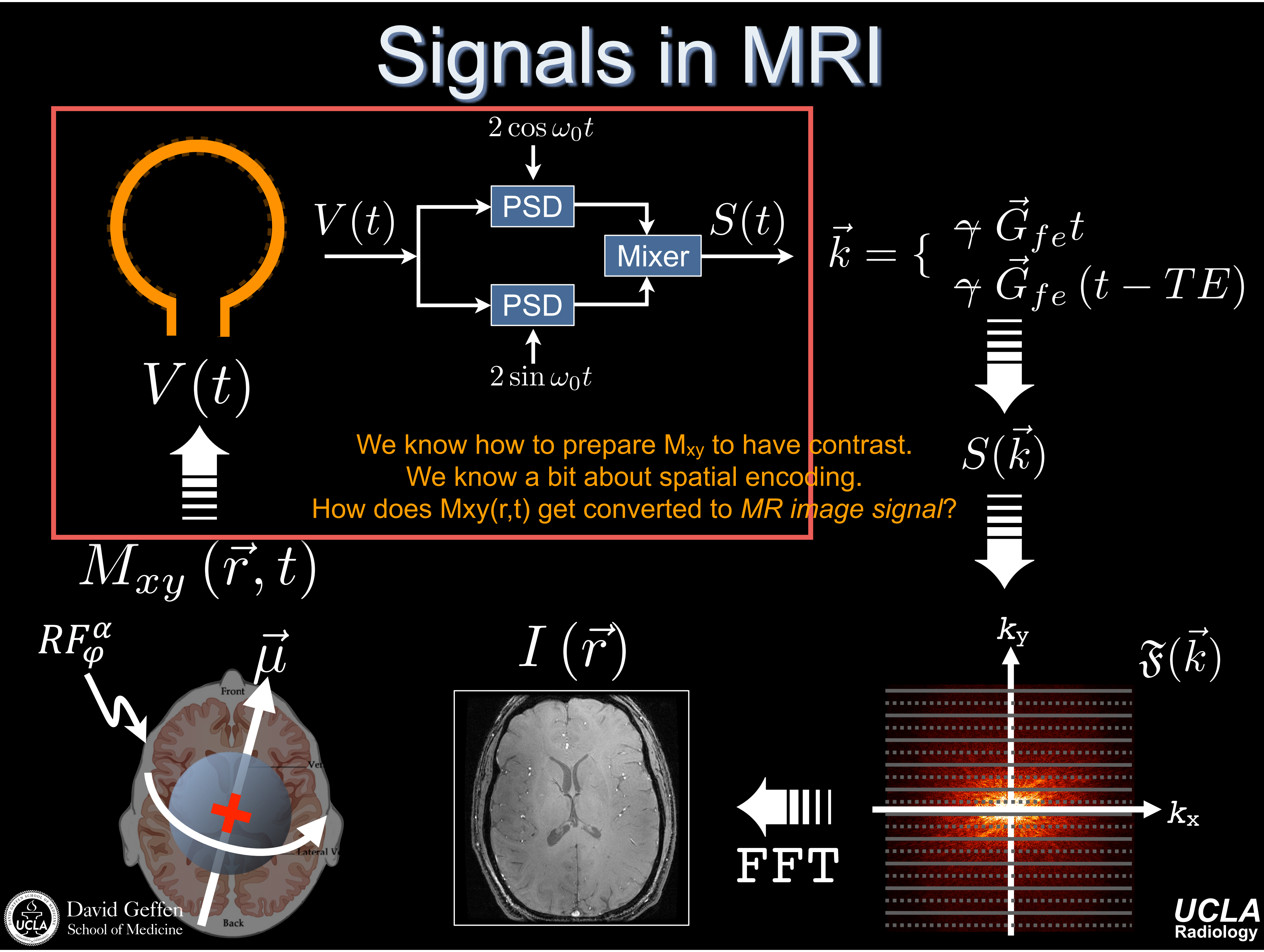
The trick is to encode spatial information and image contrast in the echo.

Signals in MRI



We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?




Signals in MRI
$2\cos\omega_0 t$
PSD
Mixer
PSD
$2\sin\omega_0 t$
$V(t)$
$S(t)$
$V(t)$
$\vec{k} = \{ \begin{array}{l} \gamma \vec{G}_{fe} t \\ \gamma \vec{G}_{fe}(t - TE) \end{array}$
$S(\vec{k})$
We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?
$M_{xy}(\vec{r}, t)$
RF^{α}_{φ}
$\vec{\mu}$
Front
Back
$I(\vec{r})$
FFT
k_y
k_x
$\mathfrak{F}(\vec{k})$
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Basic Detection Principles

Magnetic Flux Through The Coil – *Reciprocity*

$$\Phi\left(t\right) = \int_{object} \vec{B}_r\left(\vec{r}\right) \cdot \vec{M}\left(\vec{r}, t\right) d\vec{r}$$ Eqn. 3.126

Magnetic Flux (arrow to $\Phi(t)$)

Coil Sensitivity (arrow to $\vec{B}_r(\vec{r})$)

Bulk Magnetization (arrow to $\vec{M}(\vec{r}, t)$)

What happens if the coil has poor sensitivity?

What happens if the coil's sensitivity is perpendicular to the bulk magnetization? How would that happen?

Basic Detection Principles

We get here

$$S(t) = \int_{\text{object}} M_{xy}(\mathbf{r}, 0) e^{-i\gamma \Delta B(\mathbf{r}) t} d\mathbf{r}$$

From Here

$$V(t) = -\frac{\partial \Phi(t)}{\partial t} = -\frac{\partial}{\partial t} \int_{object} \vec{B}(\vec{r}) \cdot \vec{M}(\vec{r}, t)\, d\vec{r}$$

with 25 pages of Math!

Basic Detection Principles

$$S(t) = \int_{\text{object}} M_{xy}(\boldsymbol{r}, 0) e^{-i\gamma \Delta B(\boldsymbol{r}) t} d\boldsymbol{r}$$

Observations

Detected signal is the vector sum of all transverse magnetizations in the "rotating frame" within the imaging volume.

The Larmor frequency precession (Lab frame rotation) is necessary for detection, although only the baseband signal matters for imaging

Basic Detection Examples

TO THE BOARD

1. **Signal of a voxel (T2* decay)**
2. **Signal of a 1D homogeneous object**
3. **Signal of a 1D inhomogeneous object**

Questions?

- Related reading materials
  - Liang/Lauterbur - Chap 3.3
  - Nishimura - Chap 5.2, 5.3

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