

Basic Pulse Sequences

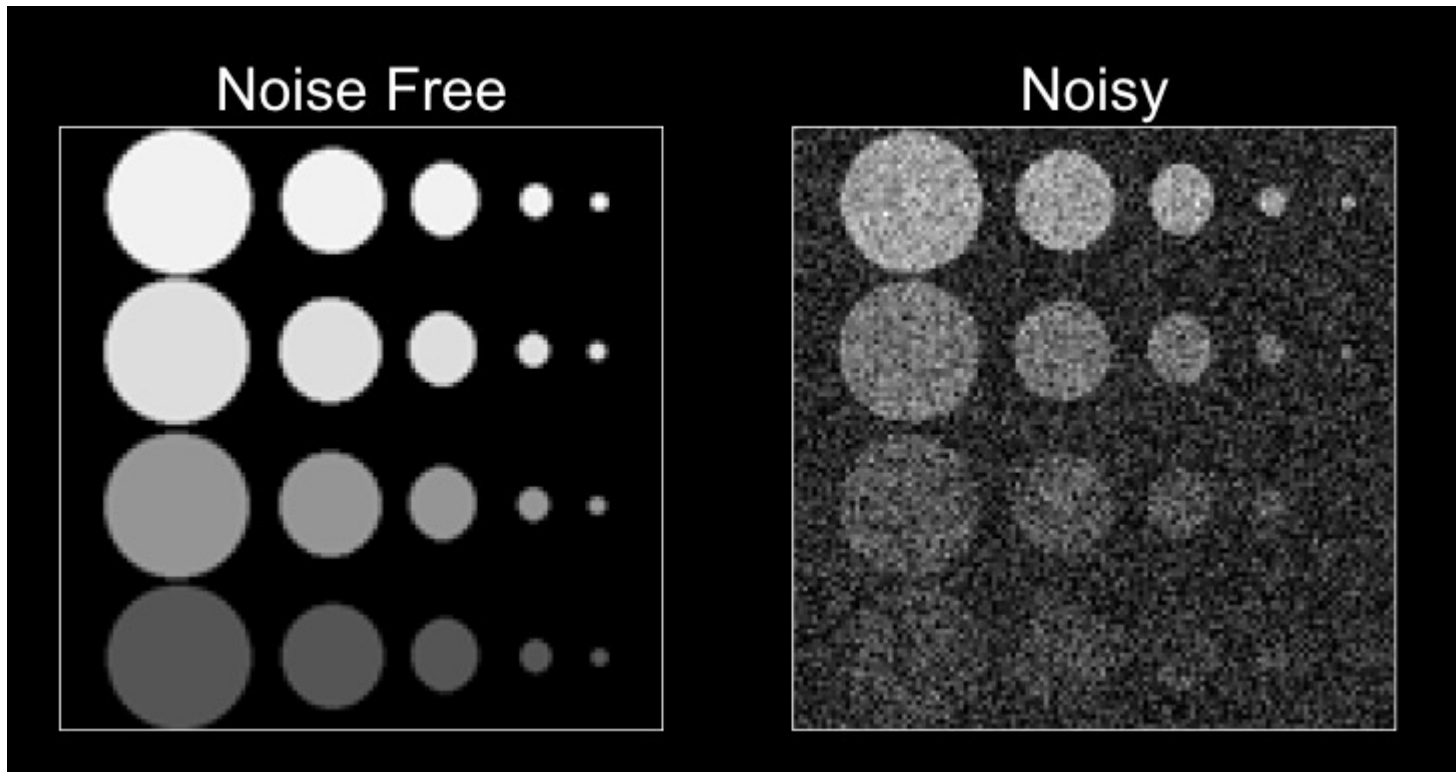
Saturation and Inversion Recovery



Image Contrast



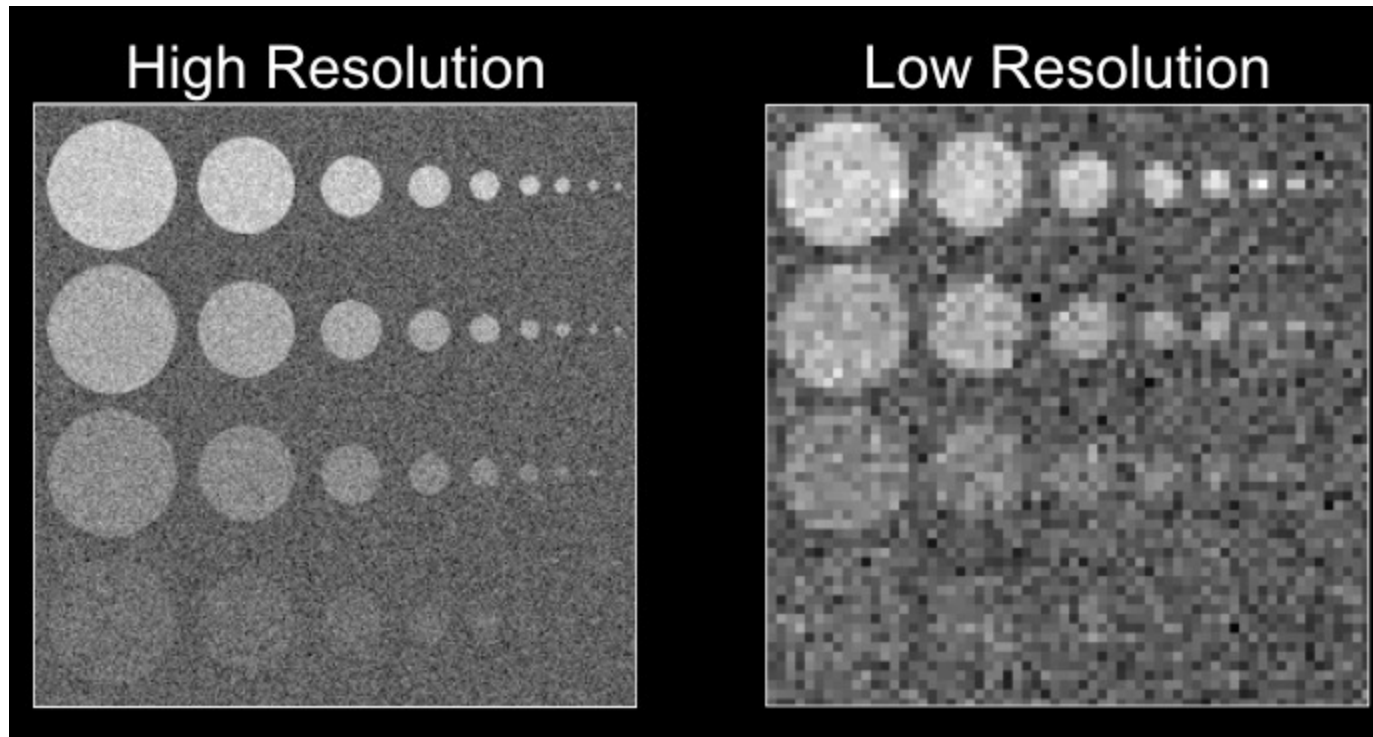
CNR, Object Size, and Noise



- **Large, high-contrast objects are easier to see in the presence of noise**



CNR, Object Size, and Noise



- **Small, low-contrast objects are easier to see with higher resolution**



Image Contrast

$$C_{AB} = \frac{|I_A - I_B|}{I_{ref}}$$

$$C_{AB} = f(\rho, T_1, T_2, T_2^*, D, \dots)$$

$$C_{AB} \approx f(T_1) \quad C_{AB} \approx f(T_2)$$

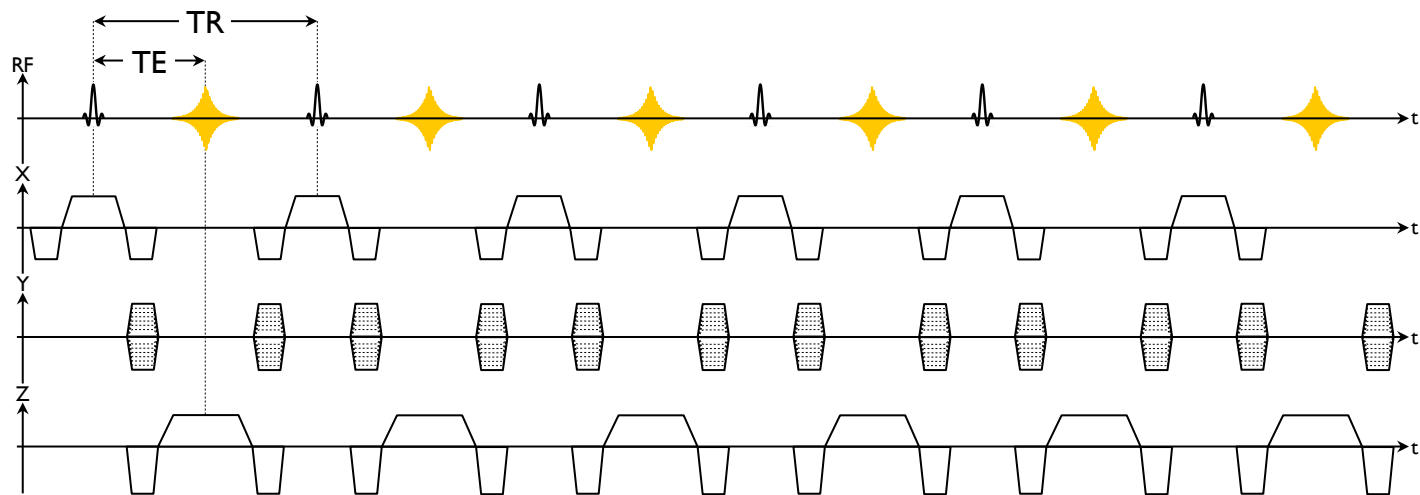
Central goal in MRI is to limit image contrast to a single mechanism



Pulse Sequences

Sheet music for two pianofortes, labeled "Pianoforte I." and "Pianoforte II.", in "Alla breve" time. The music features complex rhythmic patterns, including triplets and sixteenth notes, with dynamic markings like "ff" and "p".

Sheet music is a timing diagram for playing the piano



A pulse sequence is a timing diagram for the MRI scanner...



MR Image Formation (review)

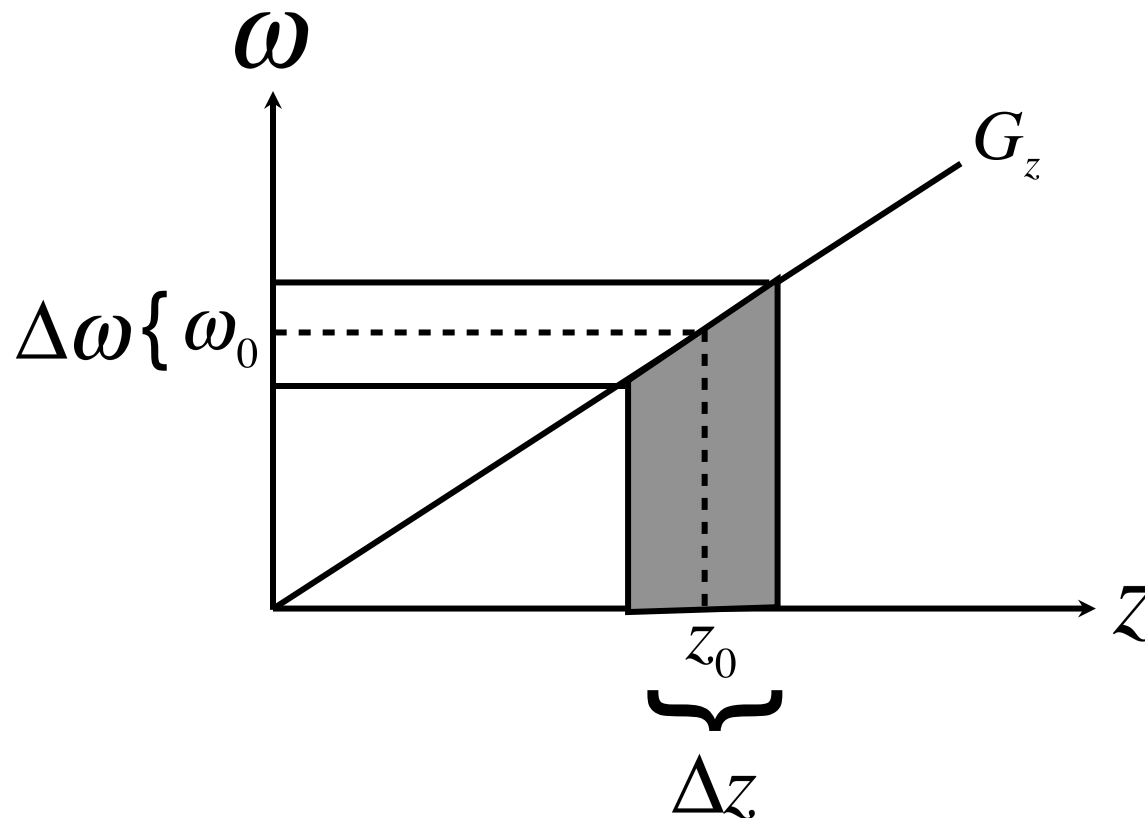
- 2D MR images are formed by
 - Slice selection/excitation
 - Phase Encoding
 - Frequency Encoding

- 3D MR images are formed by
 - Slab selection/excitation
 - Phase encode in z-direction
 - Phase encode in y-direction
 - Frequency encode in x-direction



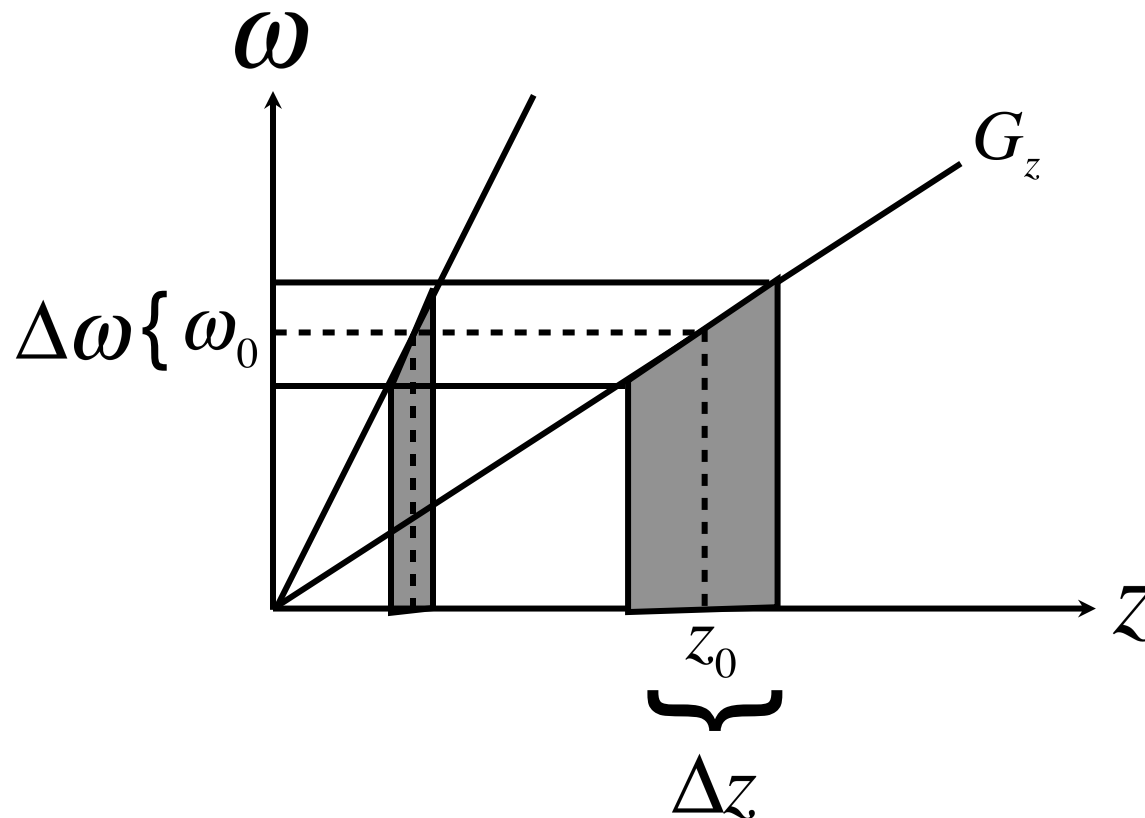
Slice Selection

- Gradient is applied in z-direction (or any direction) during RF excitation
- Only spins within the RF pulse “bandwidth” $\Delta\omega$ will be excited



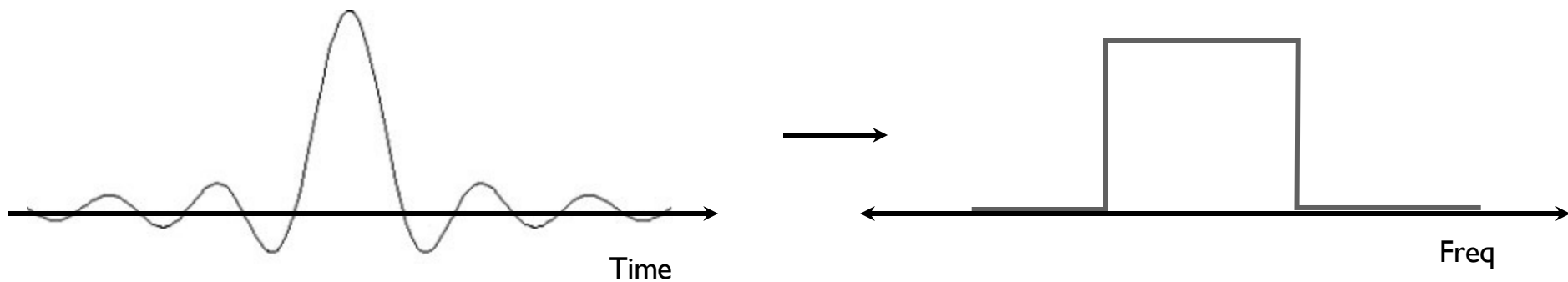
Slice Selection

- Gradient is applied in z-direction (or any direction) during RF excitation
- Only spins within the RF pulse “bandwidth” $\Delta\omega$ will be excited
- Larger gradient amplitude + same bandwidth \rightarrow thinner slice



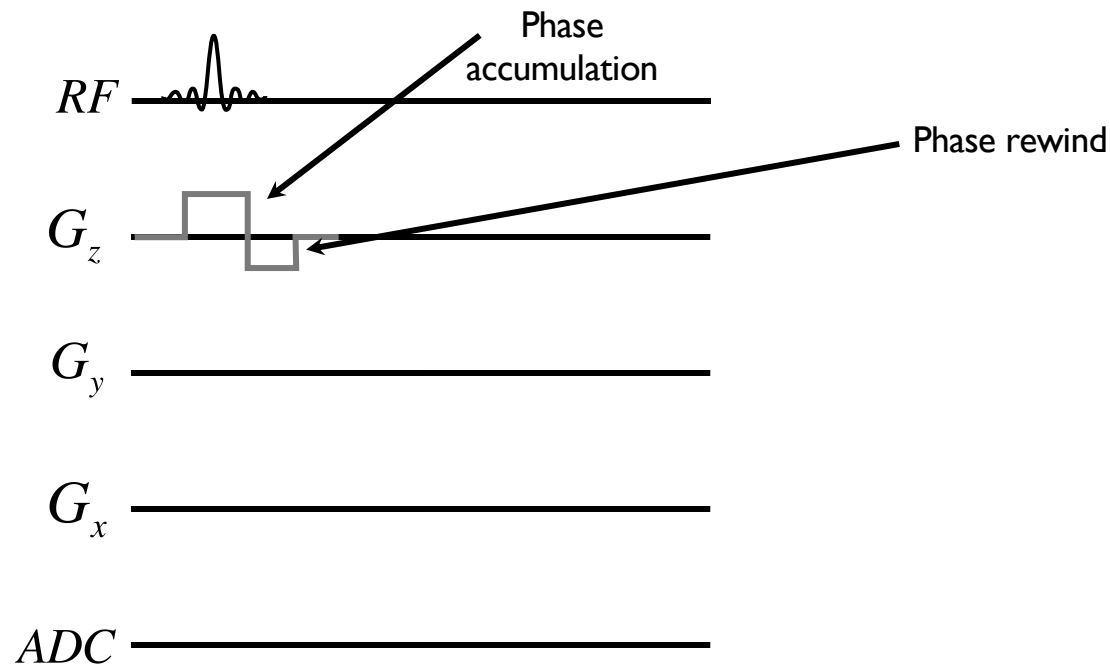
Slice Selection

- Gradient is applied in z-direction (or any direction) during RF excitation
- Only spins within the RF pulse “bandwidth” $\Delta\omega$ will be excited
- Larger gradient amplitude + same bandwidth \rightarrow thinner slice
- A $\text{sinc}(t)$ function is often used to obtain a *rectangular* frequency profile



Slice Selection

- Gradient is applied in z-direction (or any direction) during RF excitation
- Only spins within the RF pulse “bandwidth” $\Delta\omega$ will be excited
- Larger gradient amplitude + same bandwidth \rightarrow thinner slice
- A $\text{sinc}(t)$ function is often used to obtain a *rectangular* frequency profile



Phase Encode

- Pulsing a field gradient for a short period of time results in a phase offset along the direction of the field gradient.

$$\bar{\mathbf{M}}(\bar{\mathbf{r}}, t) = \begin{pmatrix} e^{-t/T_2(\bar{\mathbf{r}})} & 0 & 0 \\ 0 & e^{-t/T_2(\bar{\mathbf{r}})} & 0 \\ 0 & 0 & e^{-t/T_1(\bar{\mathbf{r}})} \end{pmatrix} \begin{pmatrix} \cos\left(\omega_0 t + \gamma \int_0^t \bar{\mathbf{G}}(\tau) \cdot \bar{\mathbf{r}} d\tau\right) & \sin\left(\omega_0 t + \gamma \int_0^t \bar{\mathbf{G}}(\tau) \cdot \bar{\mathbf{r}} d\tau\right) & 0 \\ -\sin\left(\omega_0 t + \gamma \int_0^t \bar{\mathbf{G}}(\tau) \cdot \bar{\mathbf{r}} d\tau\right) & \cos\left(\omega_0 t + \gamma \int_0^t \bar{\mathbf{G}}(\tau) \cdot \bar{\mathbf{r}} d\tau\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} \bar{\mathbf{M}}^0(\bar{\mathbf{r}}, t) + \begin{pmatrix} 0 \\ 0 \\ M_0 \left(1 - e^{-t/T_1(\bar{\mathbf{r}})}\right) \end{pmatrix}$$

- Consider only the *transverse* magnetization $M_{xy}(t) = M_x(t) + j M_y(t)$ for a phase encode gradient in the *y*-orientation:

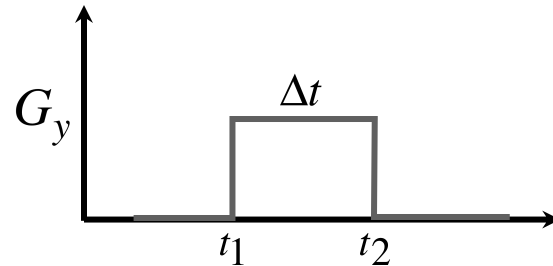
$$M_{xy}(\bar{\mathbf{r}}, t) = \left| \bar{\mathbf{M}}^0(\bar{\mathbf{r}}, t) \right| e^{-t/T_2(\bar{\mathbf{r}})} e^{-j\omega_0 t} e^{-j\gamma y \int_0^t G_y(\tau) d\tau}$$

Transverse Magnetization
Initial (or steady state) magnetization
T2 decay
Precession
Contribution from G_y



Phase Encode

- Consider a square-wave gradient with magnitude G_y pulsed from t_1 to t_2




$$M_{xy}(\bar{\mathbf{r}}, t) = |\bar{\mathbf{M}}^0(\bar{\mathbf{r}}, t)| e^{-t/T_2(\bar{\mathbf{r}})} e^{-j\omega_0 t} e^{-j\gamma y \int_0^t G_y(\tau) d\tau}$$

$$M_{xy}(\bar{\mathbf{r}}, t) = |\bar{\mathbf{M}}^0(\bar{\mathbf{r}}, t)| e^{-t/T_2(\bar{\mathbf{r}})} e^{-j\omega_0 t} e^{-j\gamma y \int_{t_1}^{t_2} G_y(\tau) d\tau}$$

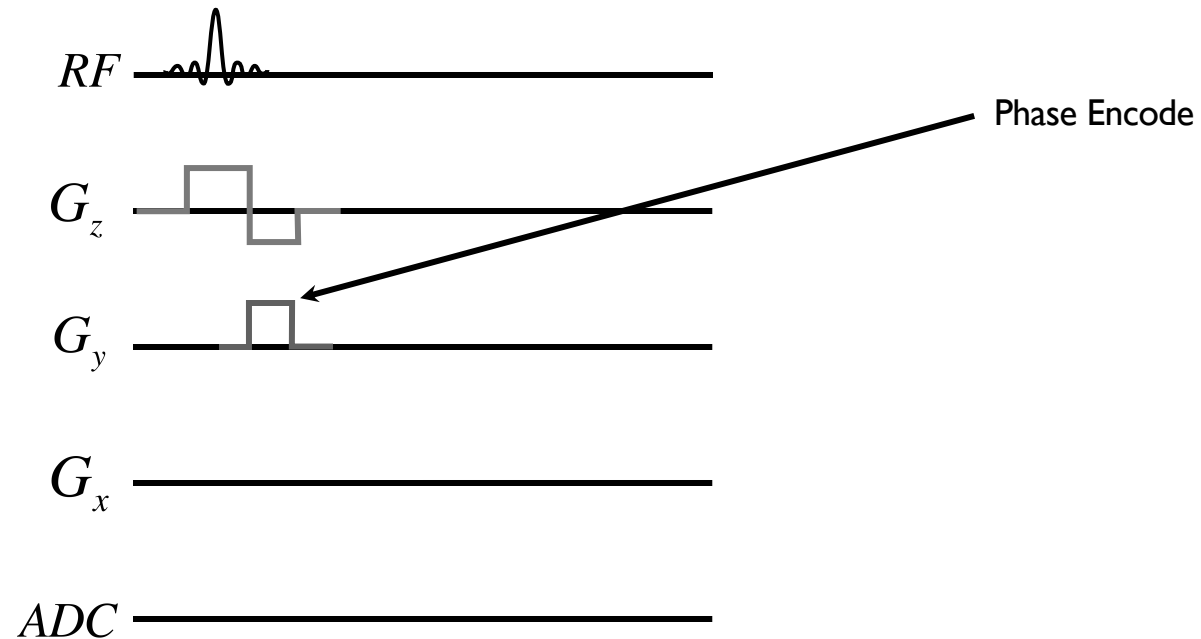
$$M_{xy}(\bar{\mathbf{r}}, t) = |\bar{\mathbf{M}}^0(\bar{\mathbf{r}}, t)| e^{-t/T_2(\bar{\mathbf{r}})} e^{-j\omega_0 t} e^{-j\gamma y G_y (t_2 - t_1)}$$

Linear Phase
offset

$$\theta(y) = y \cdot (\gamma G_y \Delta t)$$


Phase Encode

- Consider a square-wave gradient with magnitude G_y pulsed from t_1 to t_2



Frequency Encode

- Using the same principle, we can apply another gradient in the x-direction, but read out the FID during this time
- Position is then encoded by *frequency*

$$M_{xy}(\bar{\mathbf{r}}, t) = |\bar{\mathbf{M}}^0(\bar{\mathbf{r}}, t)| e^{-t/T_2(\bar{\mathbf{r}})} e^{-j\omega_0 t} e^{-j\gamma x \int_0^t G_x(\tau) d\tau}$$

↓

$$e^{-j\gamma x G_x t} = e^{-j\omega t}$$

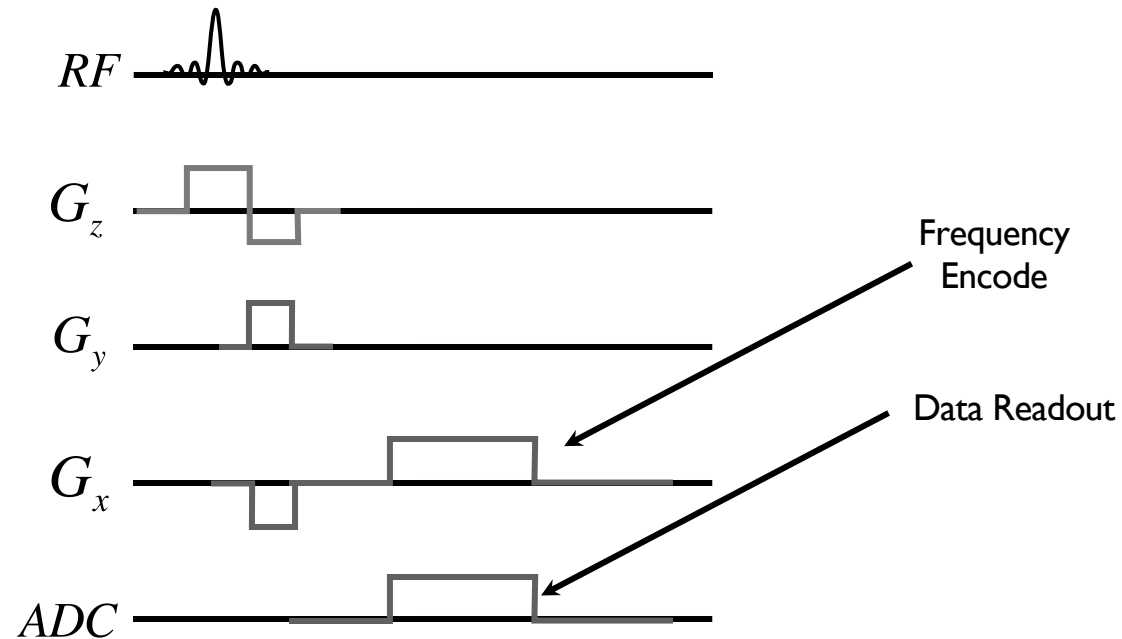
↓

$$\omega(x) = x \cdot (\gamma G_x)$$



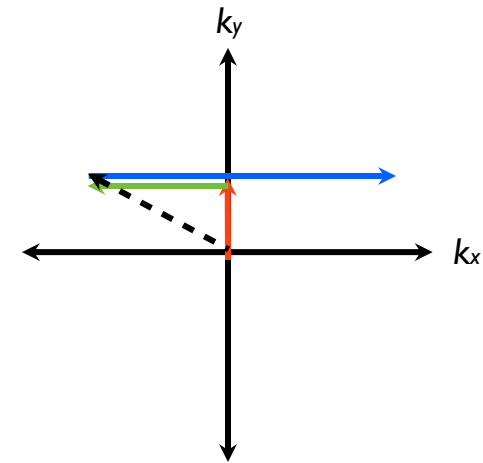
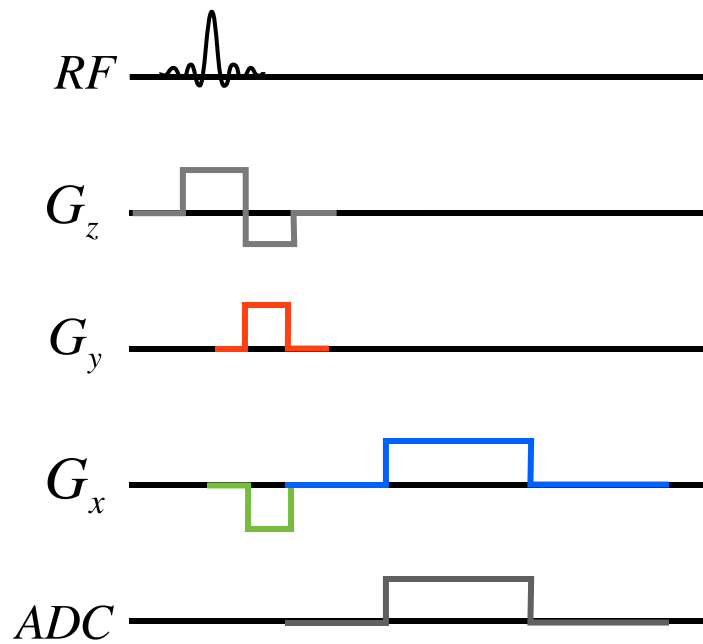
Frequency Encode

- Using the same principle, we can apply another gradient in the x-direction, but read out the FID during this time
- Position is then encoded by *frequency*
- Data is acquired *during* application of the frequency encode gradient



k-Space

- Because all image data is encoded with respect to *spatial frequency* using field gradients, a 2D (or 3D) Fourier transform can be applied to acquired data in order to obtain the original image
- Spatial frequency space = *k*-space
- Application of the gradients at different times results in traversing through *k*-space, while data is only “stored” in *k*-space during data readout.

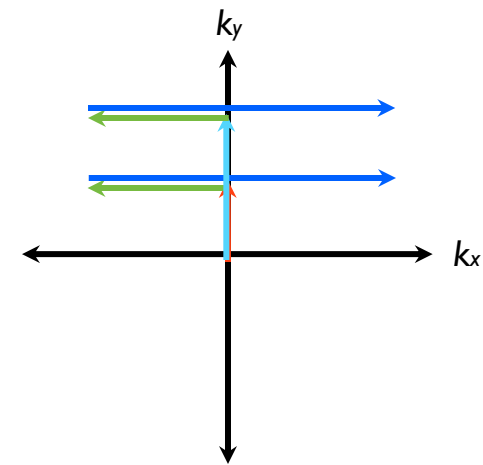
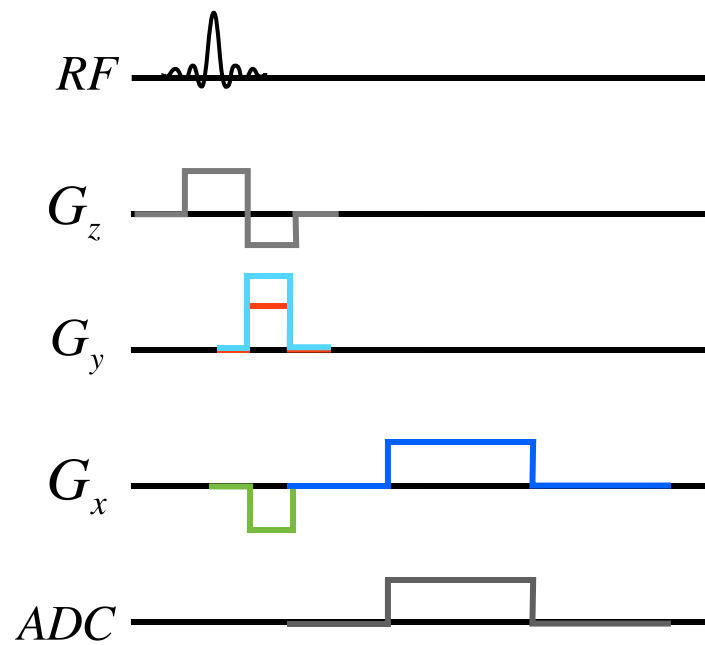


$$\bar{\mathbf{k}}(t) = \gamma \int_0^t \bar{\mathbf{G}}(\tau) d\tau$$



k-Space

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$$\bar{\mathbf{k}}(t) = \gamma \int_0^t \bar{\mathbf{G}}(\tau) d\tau$$

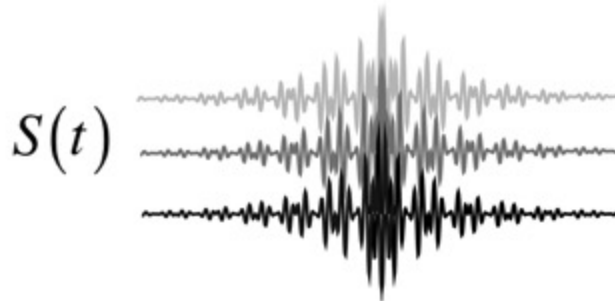


MRI: Dipoles to Images

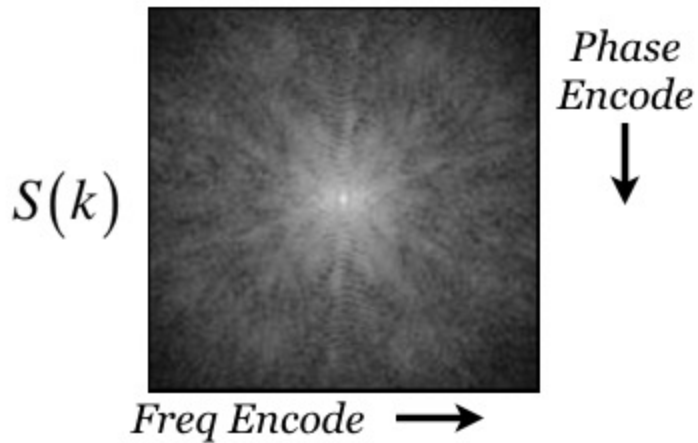
FID: 2 Frequencies



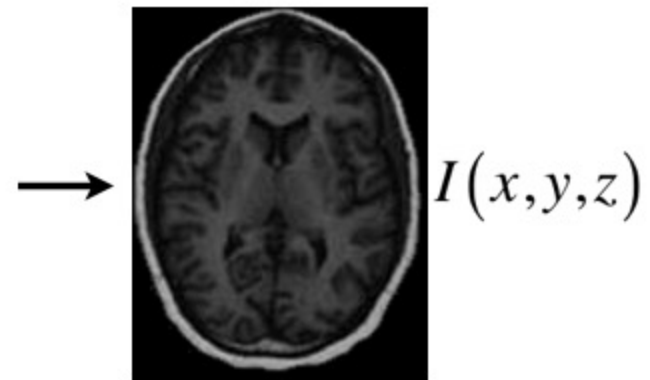
Multiple FIDs: Separated by Phase



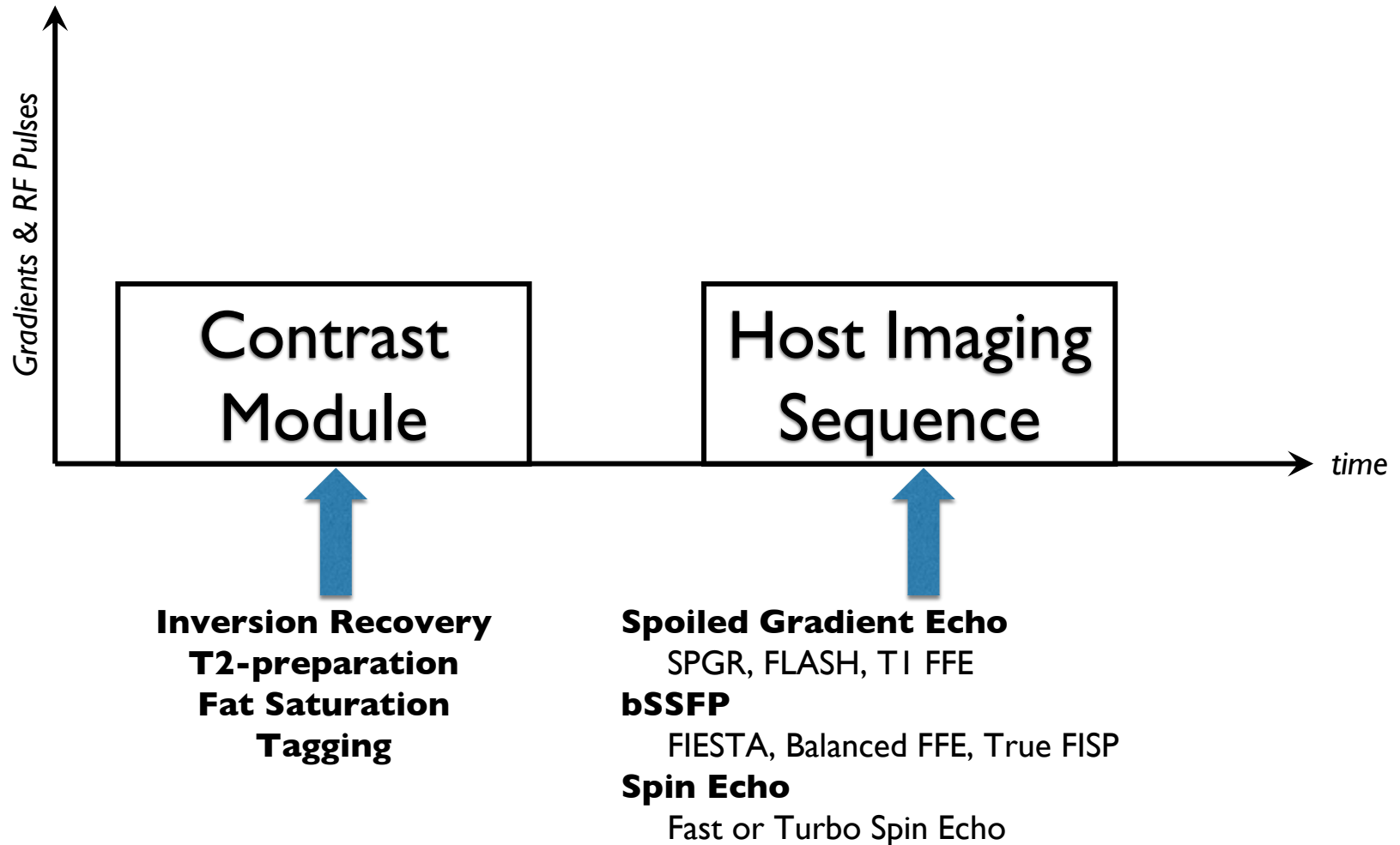
K-Space: 2-D Frequency Domain



MRI Image



Basic Pulse Sequences (Contrast Module → Host Sequence)



Pulse Sequence Definitions

$$M_z^{(n)}(0_-)$$

Longitudinal magnetization
before the n^{th} event.

$$M_z^{(n)}(0_+)$$

Longitudinal magnetization
after the n^{th} event.

$$M_{xy}^{(n)}(0_-)$$

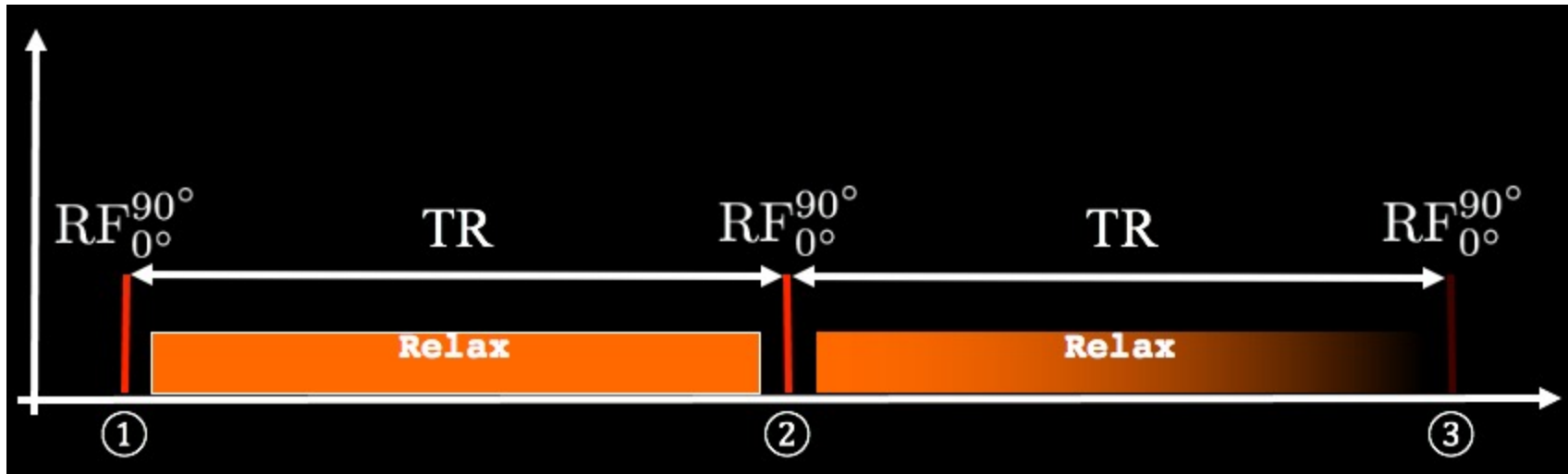
Transverse magnetization
before the n^{th} event.

$$M_{xy}^{(n)}(0_+)$$

Transverse magnetization
after the n^{th} event.



Pulse Sequence Definitions



TR - Repetition Time

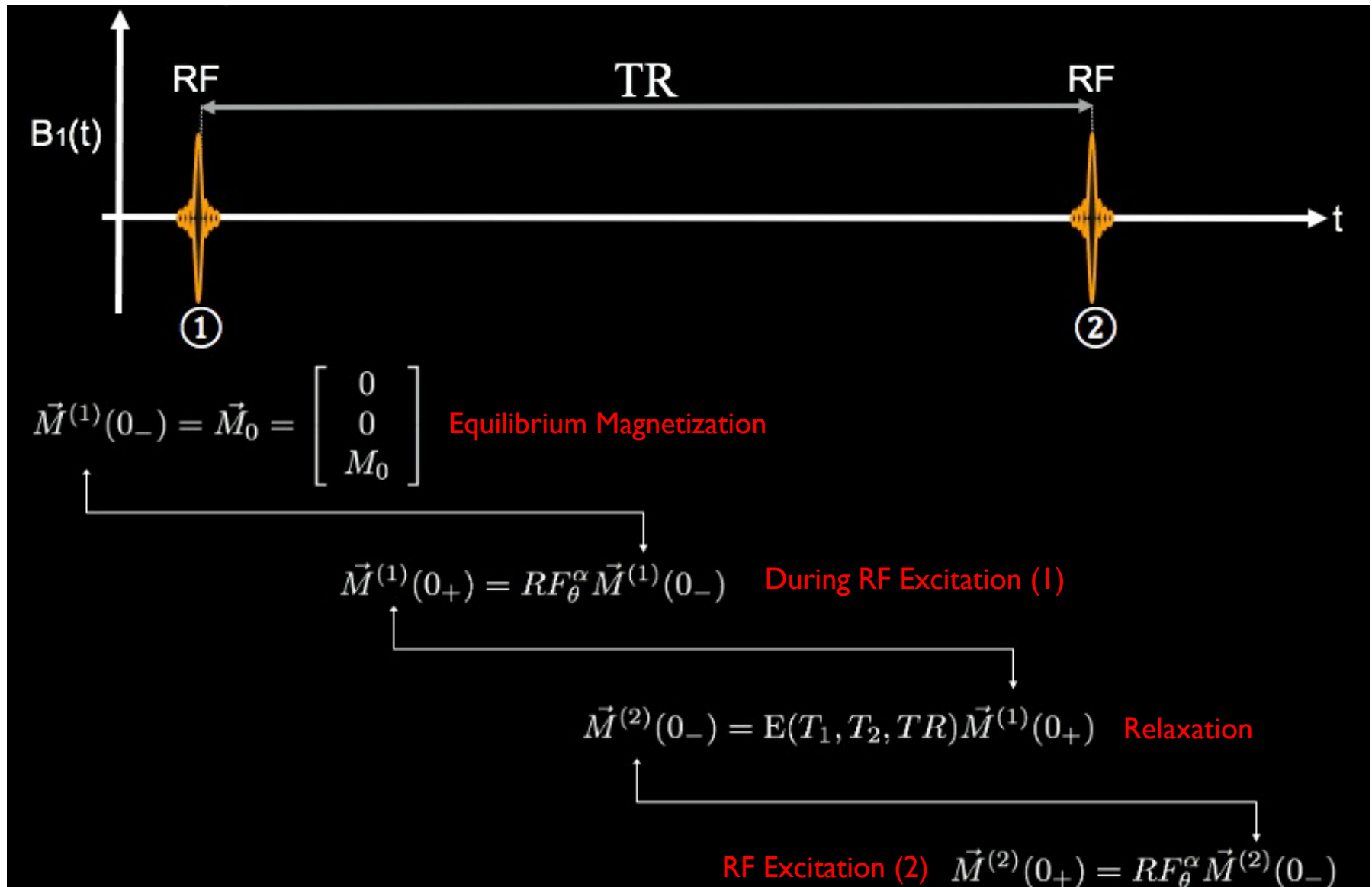
Duration of basic pulse sequence repeating block
At least one echo acquired per TR

TE - Echo Time

Time from excitation to the maximum of the echo
Data is recorded at time TE to form an image
Echo can occur as a result of a *gradient-echo* or *spin-echo*

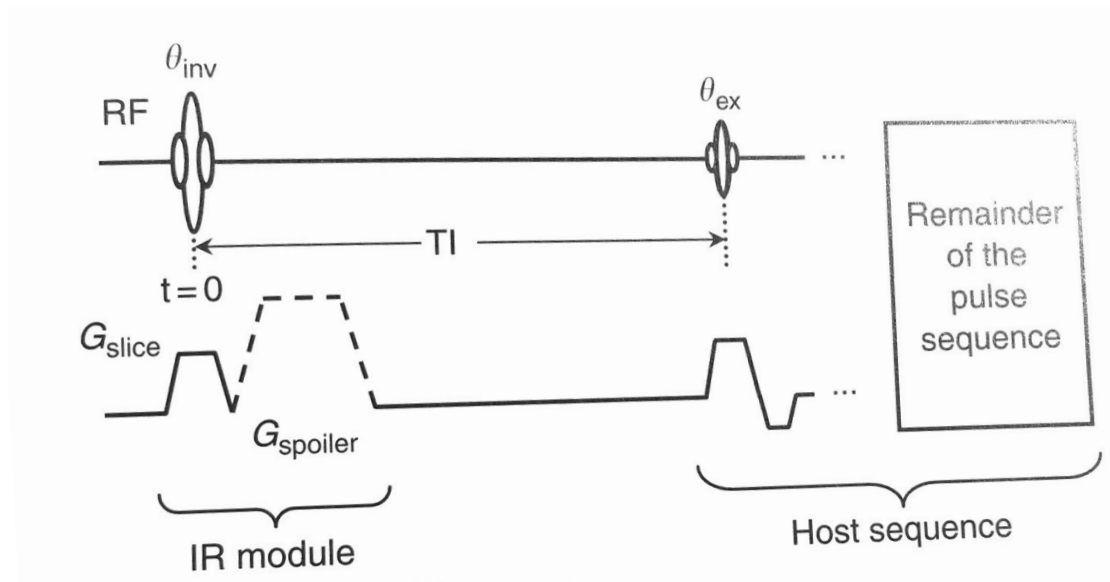


Typical Pulse Sequence

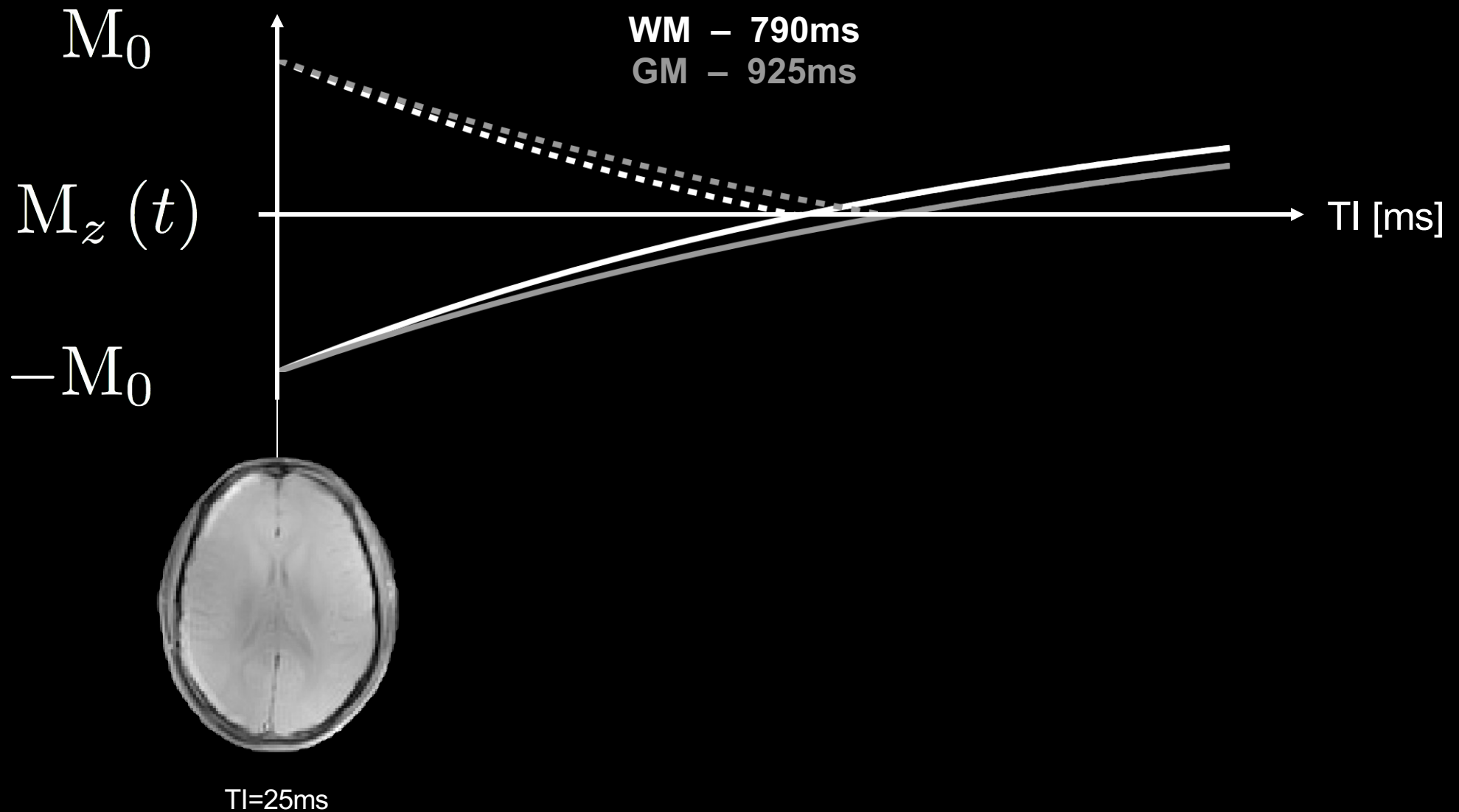


Basic Sequences - Inversion Recovery (IR)

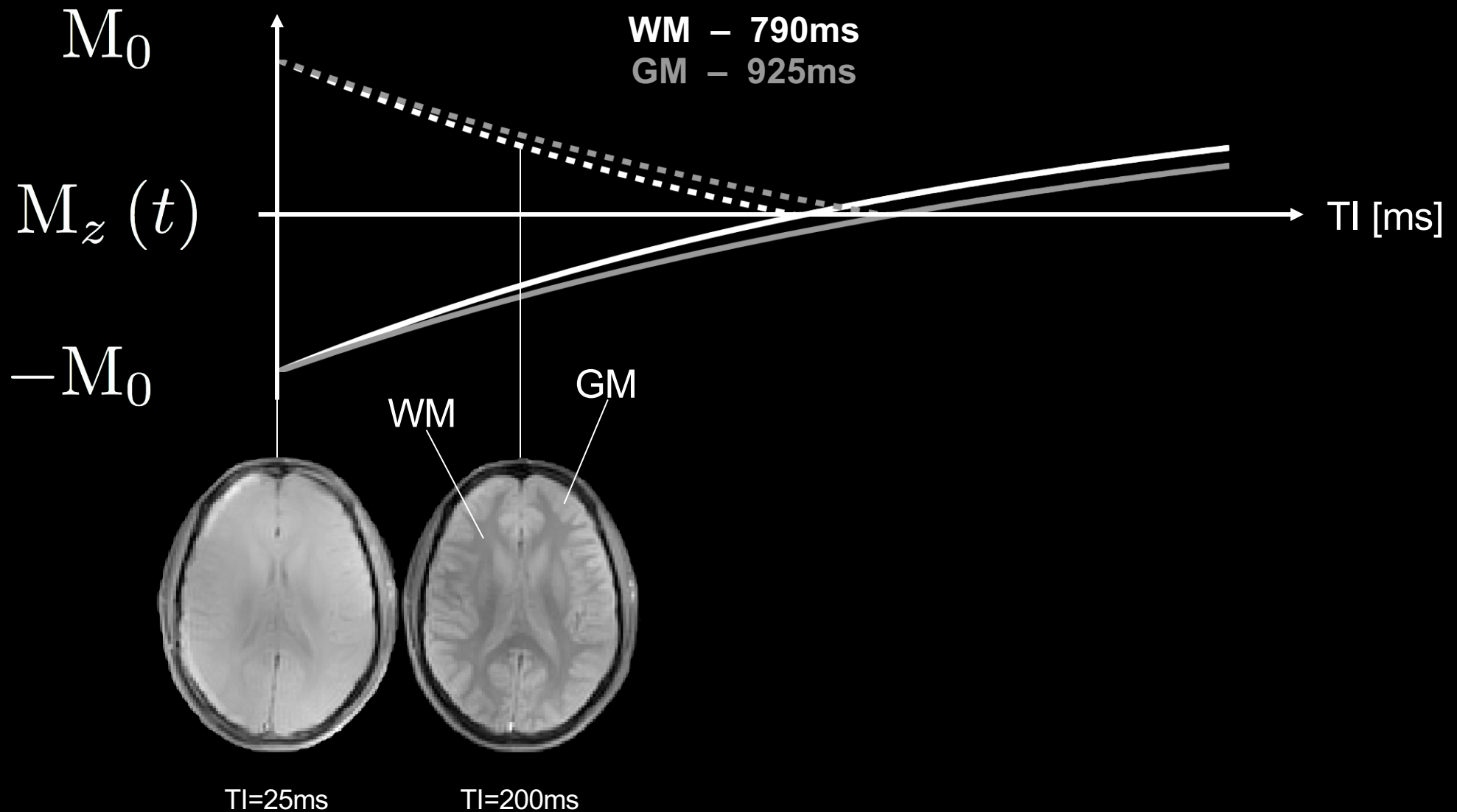
- **Pulse sequences with an inversion pulse followed by a time delay prior to RF excitation are *inversion recovery* pulses**
- **Allow for **T1**-weighting or **T1**-weighted magnetization preparation**
- **Delay between inversion and excitation pulses is known as the *inversion time (TI)***
- **IR module followed by **Host sequence** (e.g. **RARE**, **EPI**, etc)**



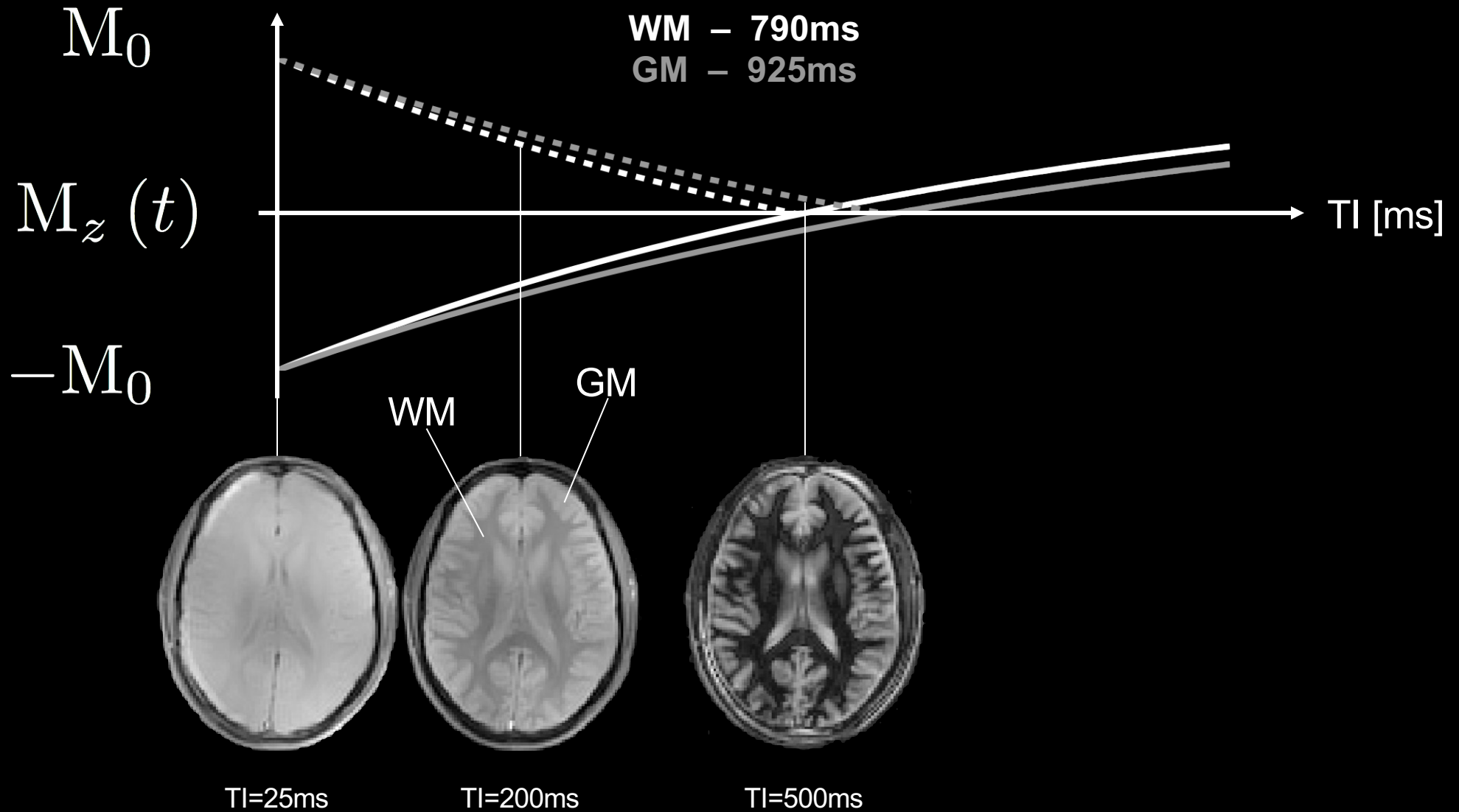
Spin Echo Inversion Recovery



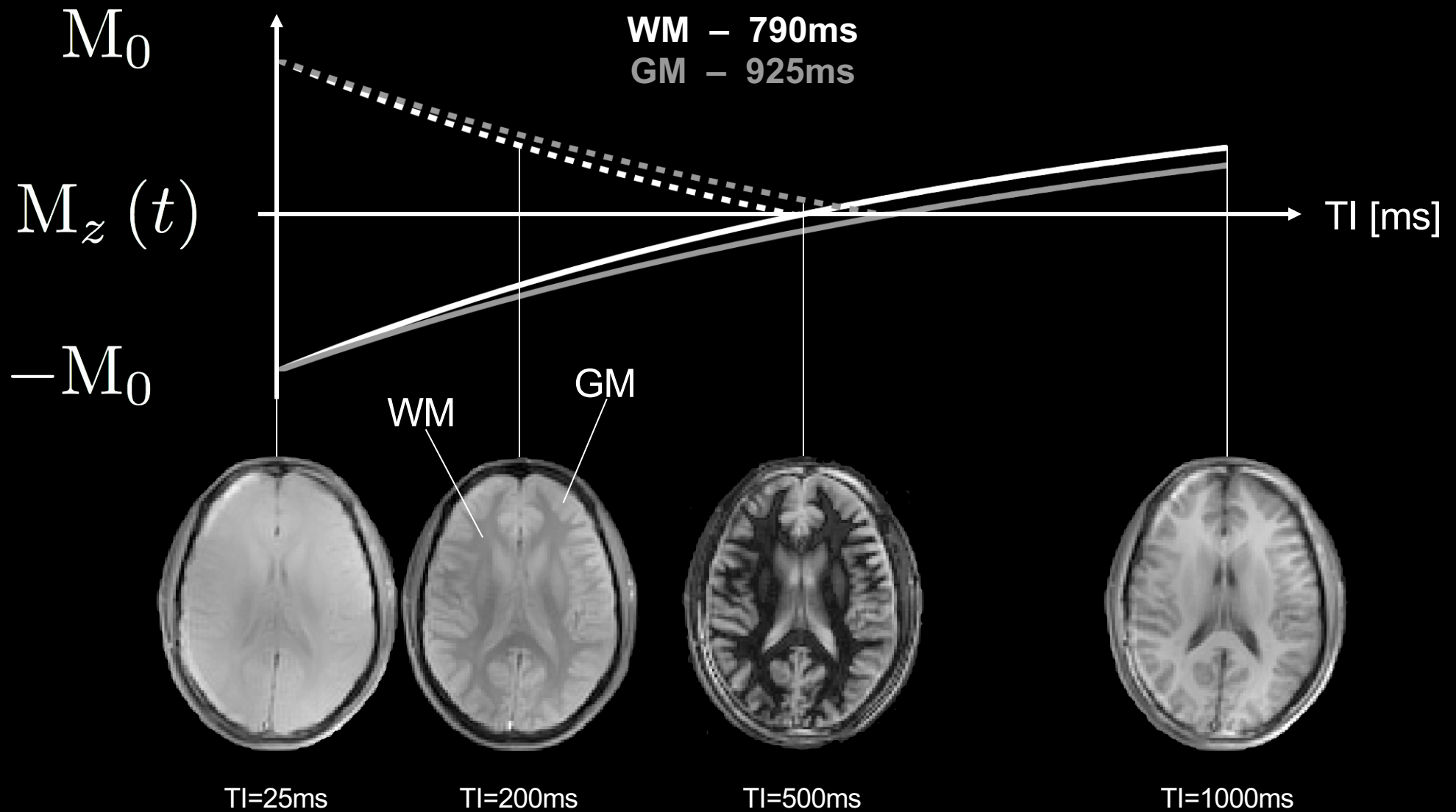
Spin Echo Inversion Recovery



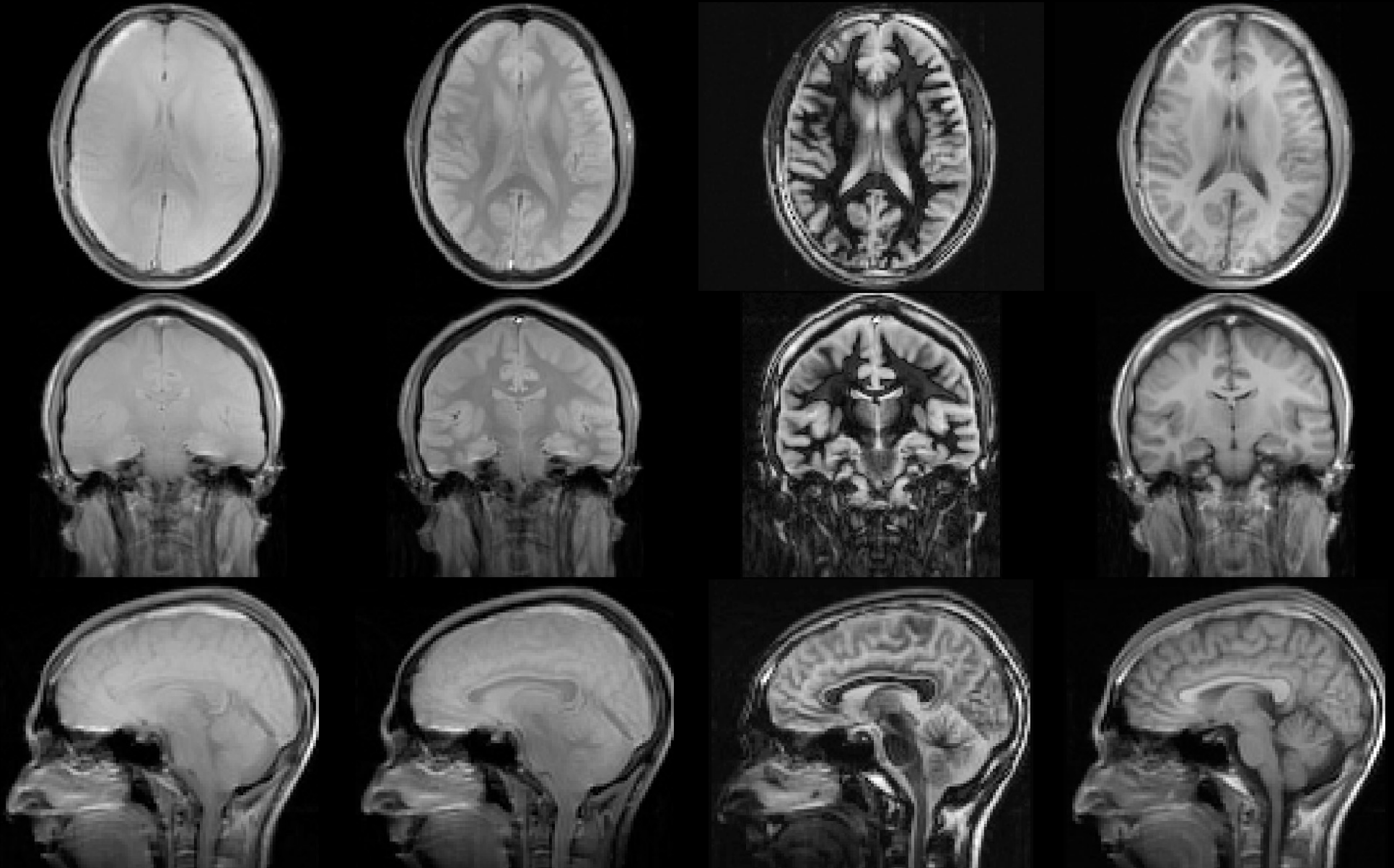
Spin Echo Inversion Recovery



Spin Echo Inversion Recovery



Spin Echo Inversion Recovery



TI=25ms

TI=200ms

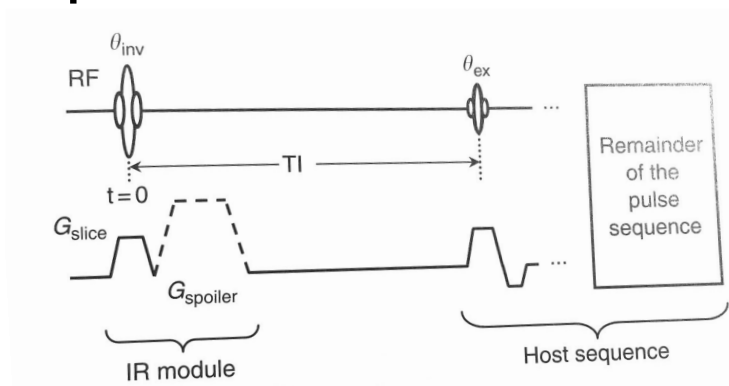
TI=500ms

TI=1000ms

TE=12ms, TR=2000ms



Basic Sequences - Inversion Recovery (IR)



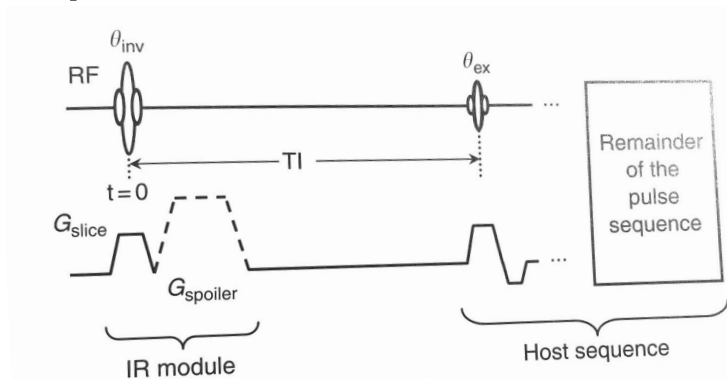
- **Signal Equation for IR:**
- **Defining the transverse and longitudinal magnetization as:**

$$M_{xy} = M_0 \sin \theta_{inv}$$

$$M_z = M_0 \cos \theta_{inv}$$



Basic Sequences - Inversion Recovery (IR)



- **By applying a spoiler gradient after the inversion pulse, $M_{xy} = 0$ (all that is left is longitudinal magnetization)**
- **Bloch equation:**

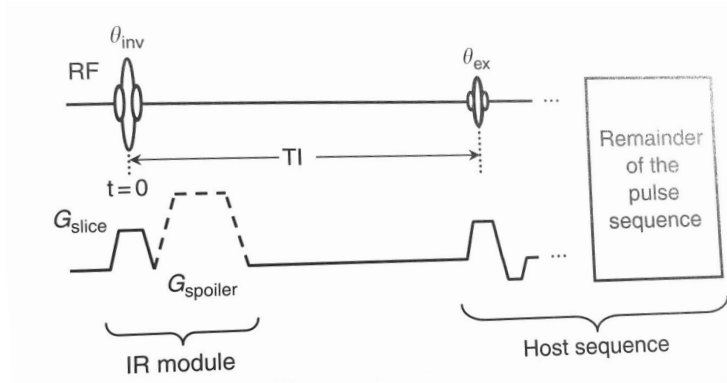
$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1}$$

- **With the solution:**

$$M_z(t) = M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} \right]$$



Basic Sequences - Inversion Recovery (IR)



- **If TR is not infinitely long, then the equations for the spin echo and turbo spin echo are:**
- **Spin Echo (SE)**

$$M_z(t) = M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} + e^{-TR/T_1} \right]$$

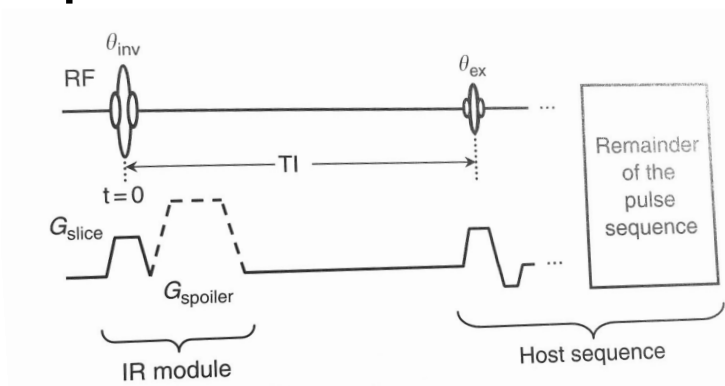
- **Turbo spin echo (TSE) or RARE**

$$M_z(t) = M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} + e^{-(TR-TE_{last})/T_1} \right]$$

TE_{last} = Last echo in echo train



Basic Sequences - Inversion Recovery (IR)

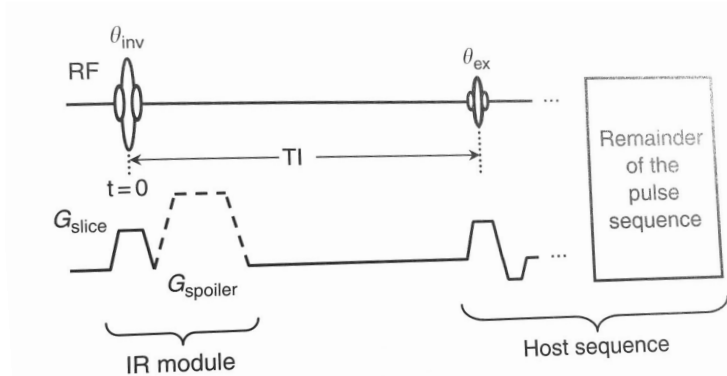


- **Following the IR pulse, the longitudinal magnetization recovers along the z-axis until being nutated by the excitation pulse.**
- **The available magnetization is thus:**

$$M_z(TI) = M_0 \left[1 - 2e^{-TI/T_1} \right]$$



Basic Sequences - Inversion Recovery (IR)



- **Magnetization becomes zero (nulled) when (for infinitely long TR)**

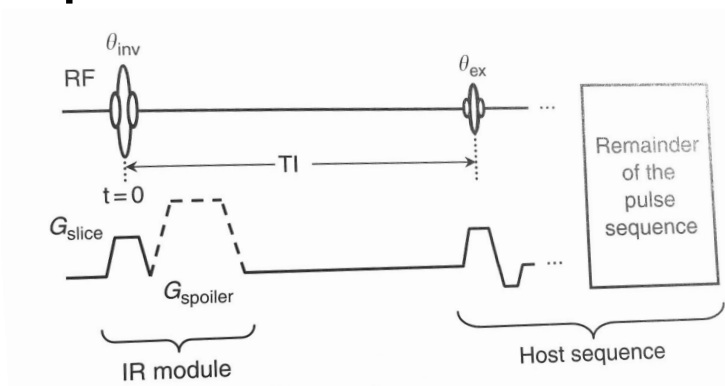
$$TI_{null} = T_1 \ln 2$$

$$TI_{null} = T_1 \left[\ln 2 - \ln \left(1 + e^{-\frac{TR}{T_1}} \right) \right] \quad \text{Spin Echo}$$

$$TI_{null} = T_1 \left[\ln 2 - \ln \left(1 + e^{-\frac{TR - TE_{last}}{T_1}} \right) \right] \quad \text{Turbo Spin Echo}$$



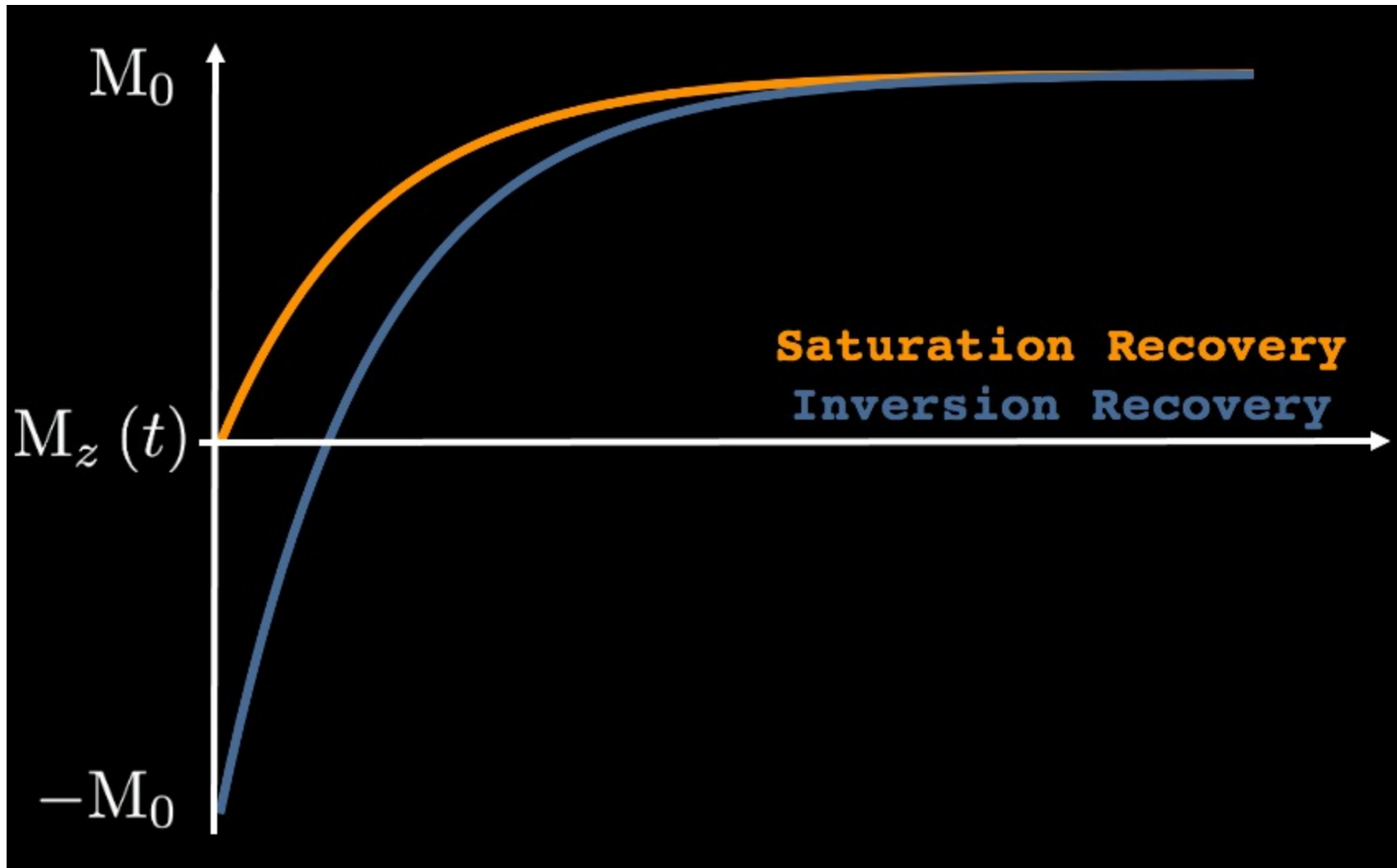
Basic Sequences - Inversion Recovery (IR)



- **If the inversion pulse is 180 degree, this is an inversion recovery sequence**
- **If the inversion pulse is 90 degrees, this is a saturation recovery sequence**



Saturation Recovery (SR) vs. Inversion Recovery (IR)



Saturation Condition

- **The Saturation Condition states:**

$$M_z^{(n)}(0_+) = 0, n \geq 1$$

M_z is ZERO after the event (RF pulse).

This is true if the M_{xy} is “gone” before the next 90° RF-pulse is applied:

No M_{xy} to convert to M_z

How? $TR \gg T_2$

What if $TR < \sim 3T_2$?

M_{xy} can be converted back to M_z

Corrupts/complicates image contrast

Solution? Spoiler gradients to disperse M_{xy}

Steady-state solution arises if the saturation conditions are met/enforced



Saturation Recovery Contrast Optimization

$I(\vec{r})_{TR \rightarrow TR_{opt}} \propto \text{Maximum } T_1 \text{ contrast}$

$$TR_{opt} = \frac{\ln\left(\frac{T_{1,A}}{T_{1,B}}\right)}{\frac{1}{T_{1,B}} - \frac{1}{T_{1,A}}}$$



Inversion Recovery Contrast Optimization

$$I(\vec{r}) \propto \rho(\vec{r}) \left(1 - 2e^{-TI/T_1(\vec{r})} + e^{-TR/T_1(\vec{r})} \right)$$

Image contrast is controlled by TI and TR

Maximum contrast if $TR \gg T_1$ and,

$$TI_{opt} = \frac{\ln\left(\frac{T_{1,A}}{T_{1,B}}\right)}{T_{1,A} - T_{1,B}} T_{1,A} T_{1,B}$$

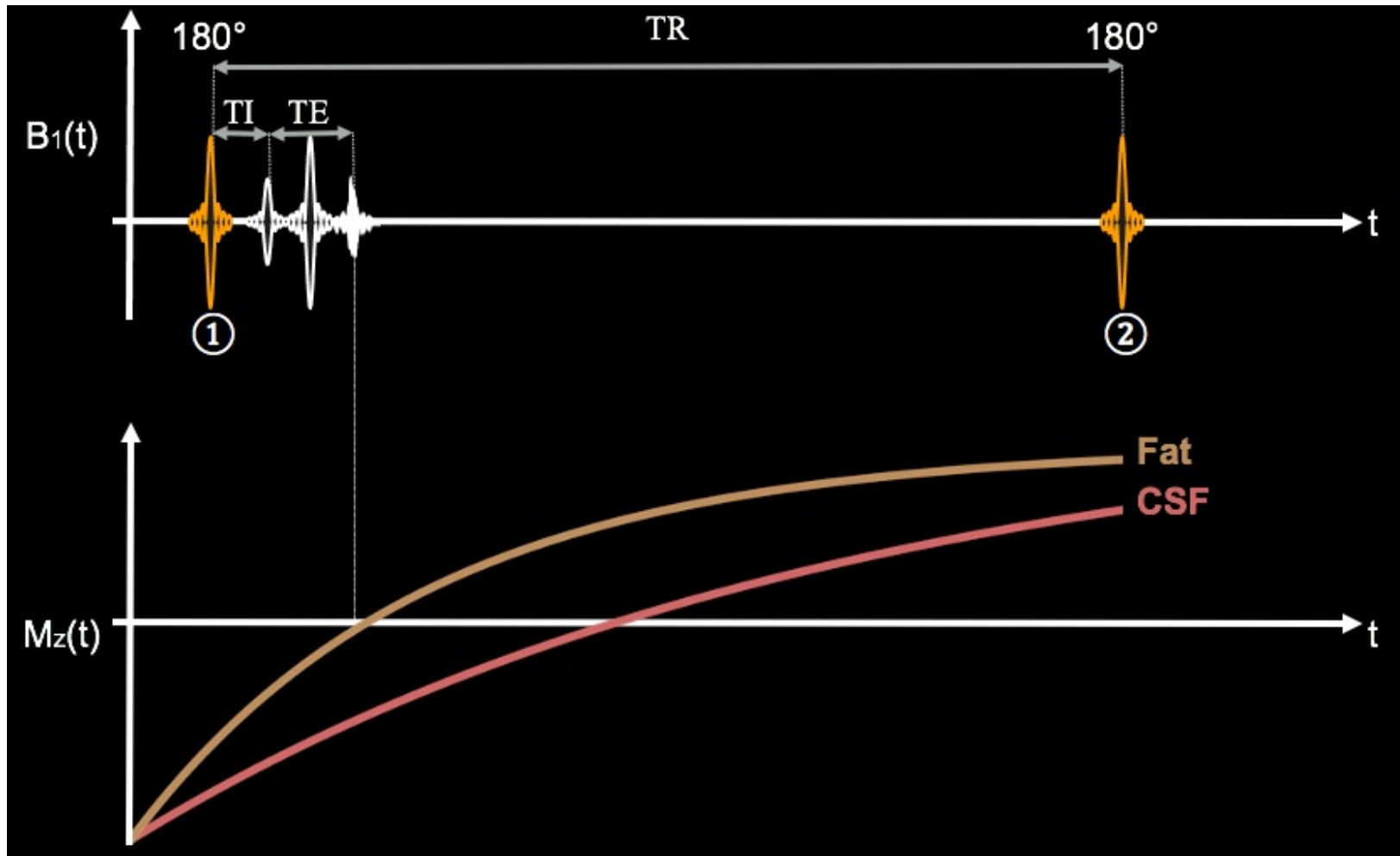


Inversion Recovery

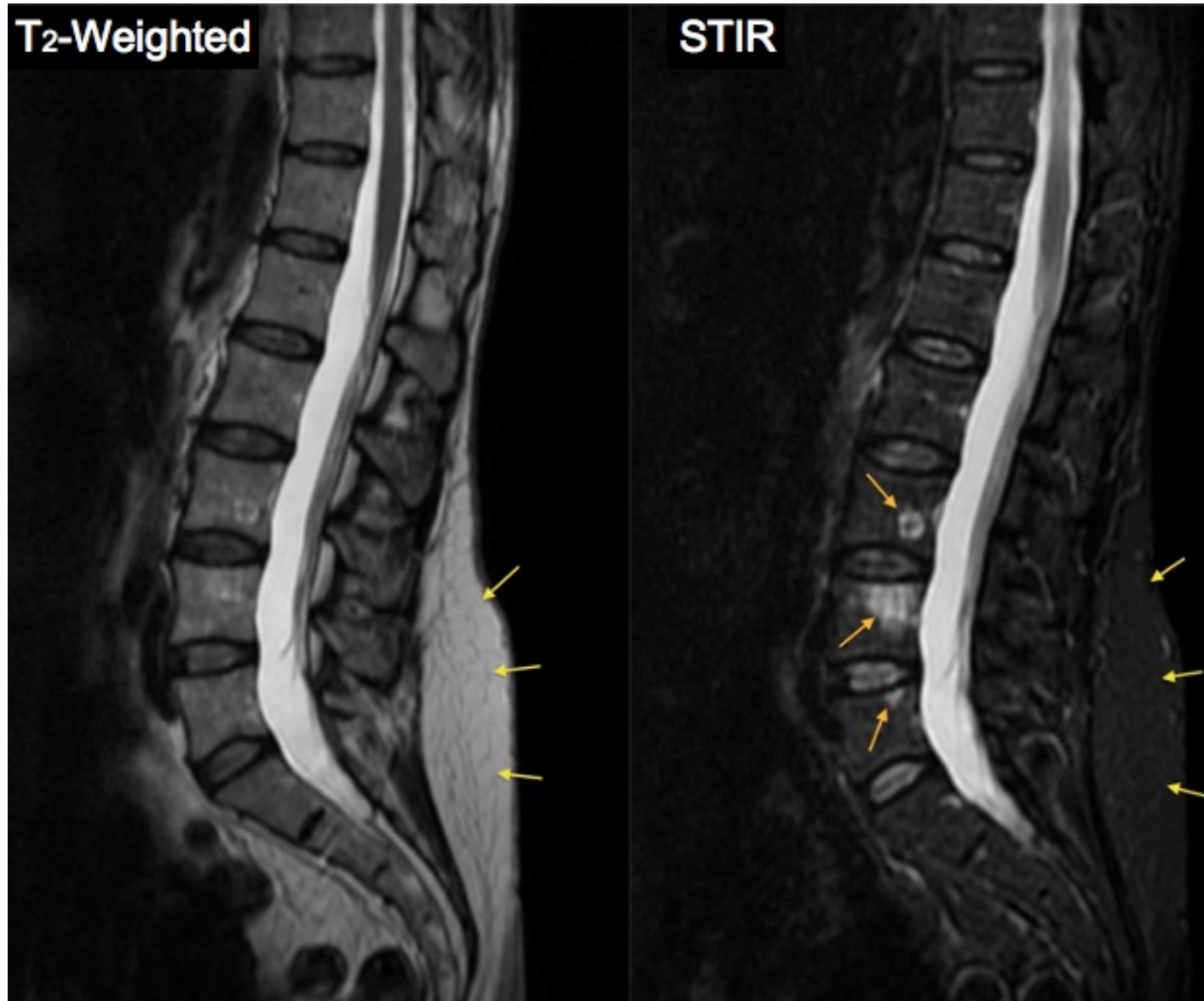
- **Greater T_1 contrast than SR**
- **T_1 species nulling/attenuation**
 - FLAIR (Fluid Attenuated Inversion Recovery)
 - STIR (Short Tau Inversion Recovery)
- **IR is better than SR for generating contrast when:**
 - $\rho(A)=\rho(B)$ and $T_2(A)=T_2(B)$
 - AND
 - $T_1(A)$ and $T_1(B)$ are slightly different
- **Quantitative T_1 mapping**



Short Tau Inversion Recovery (STIR)

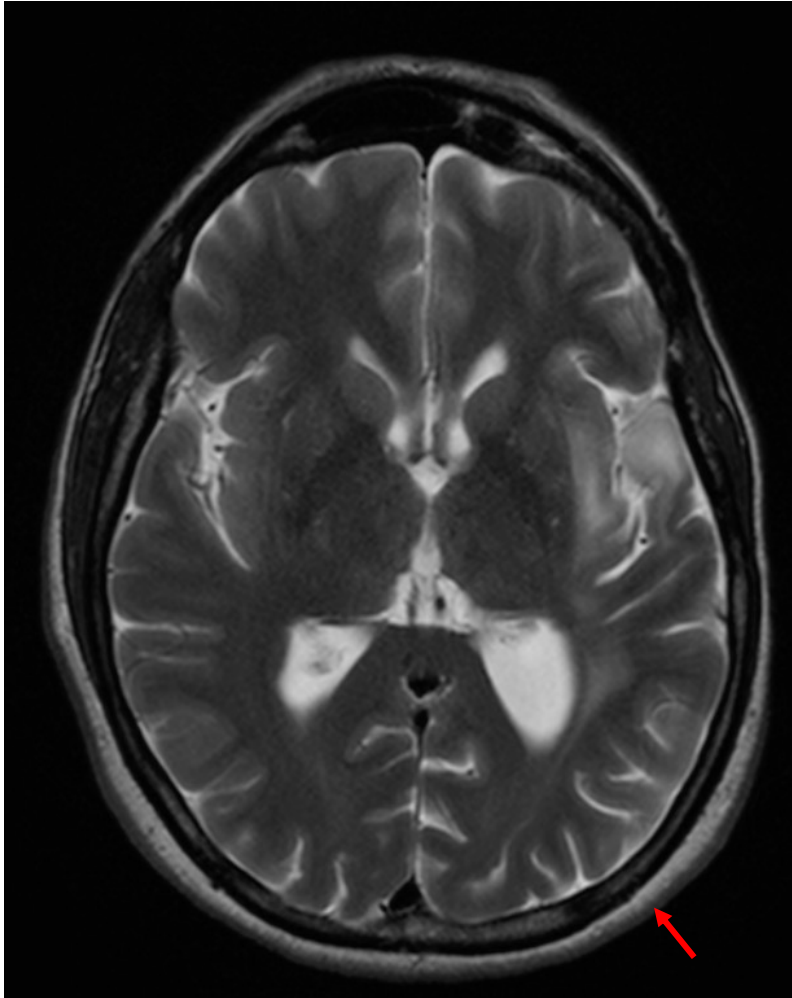


Short Tau Inversion Recovery (STIR)

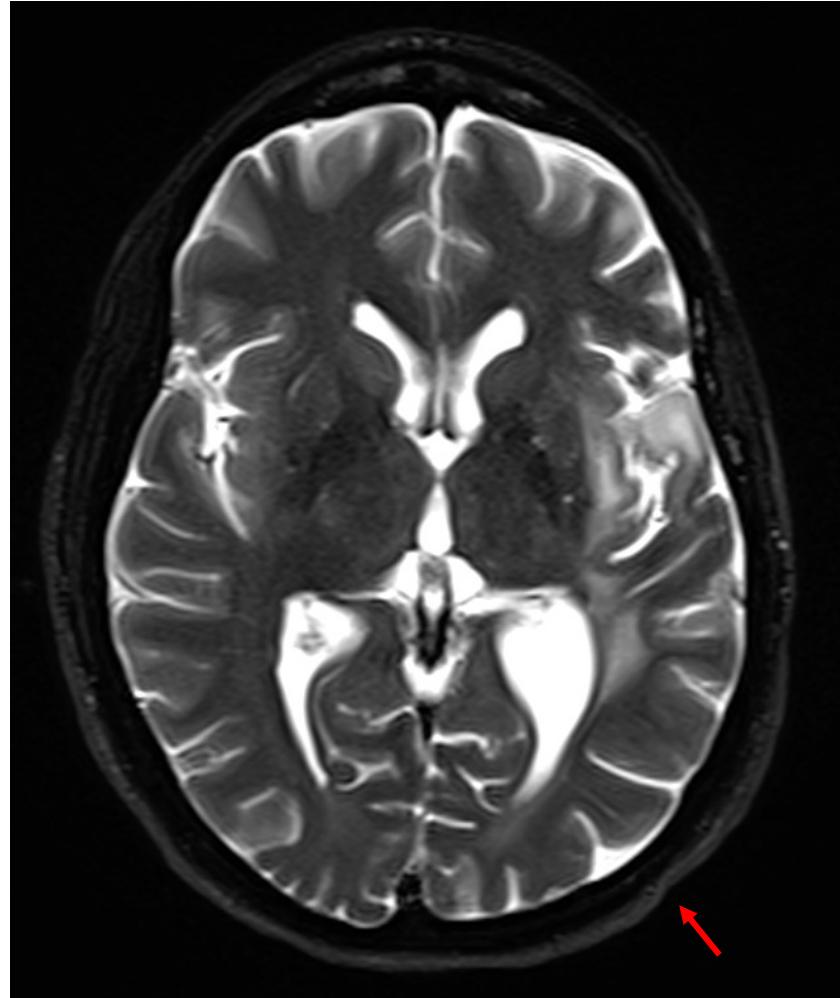


Short Tau Inversion Recovery (STIR)

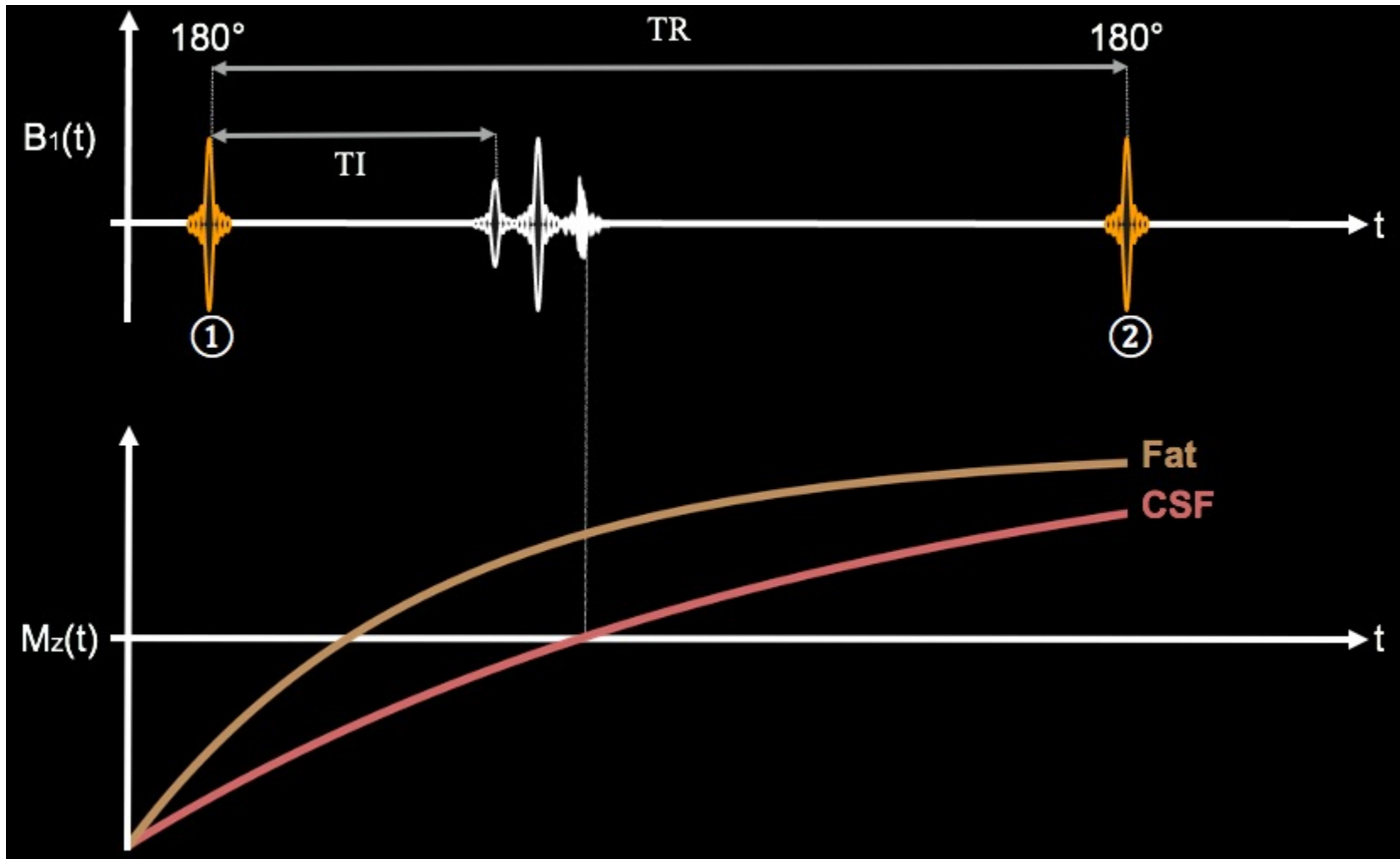
T2-Weighted TSE



T2-Weighted **STIR** TSE

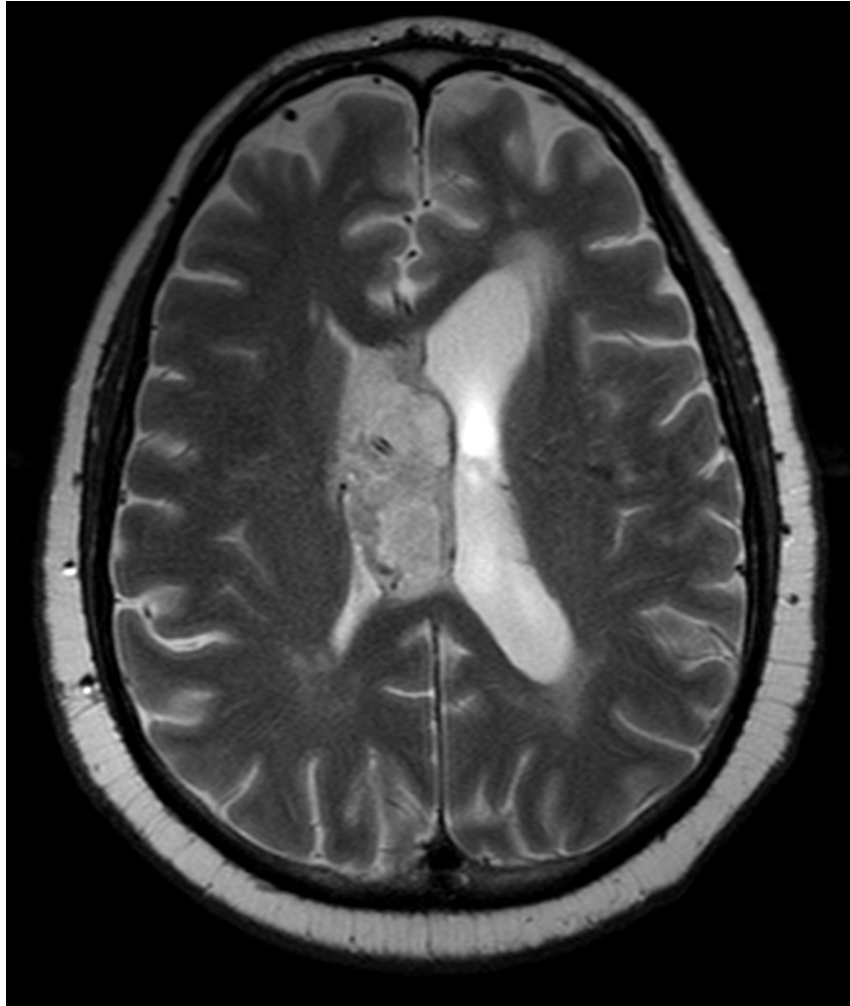


FLuid Attenuated Inversion Recovery (FLAIR)

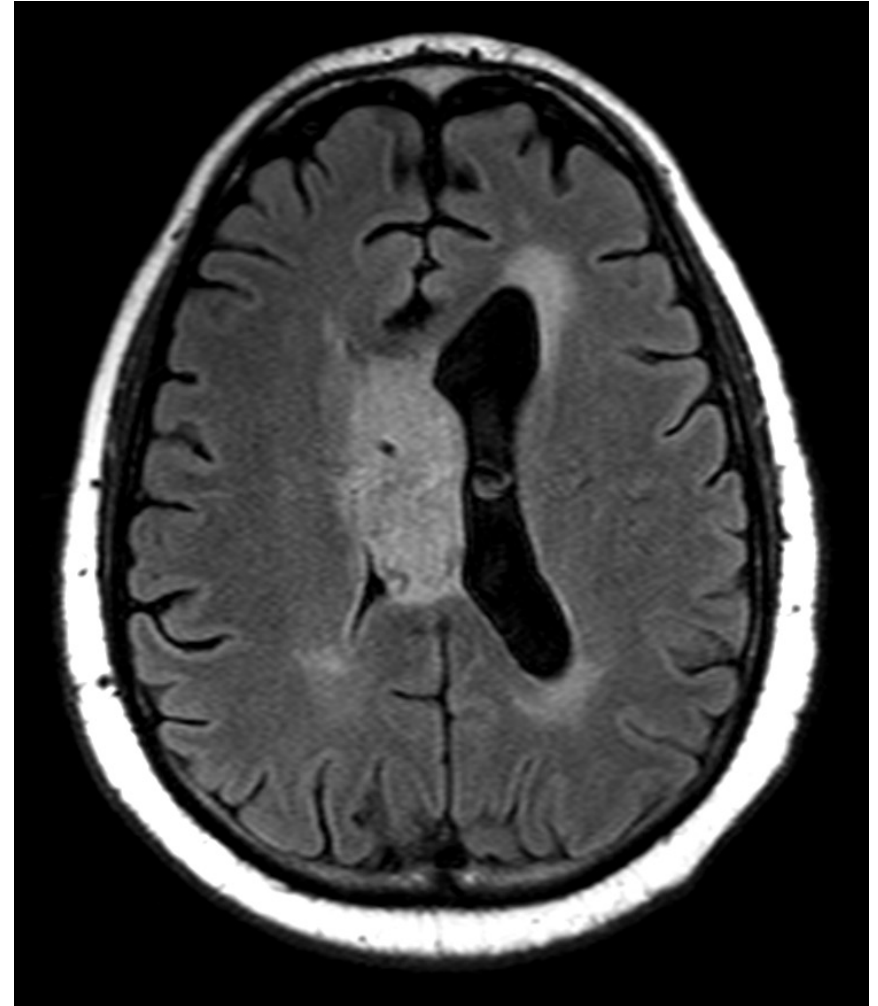


FLuid Attenuated Inversion Recovery (FLAIR)

T2-Weighted TSE

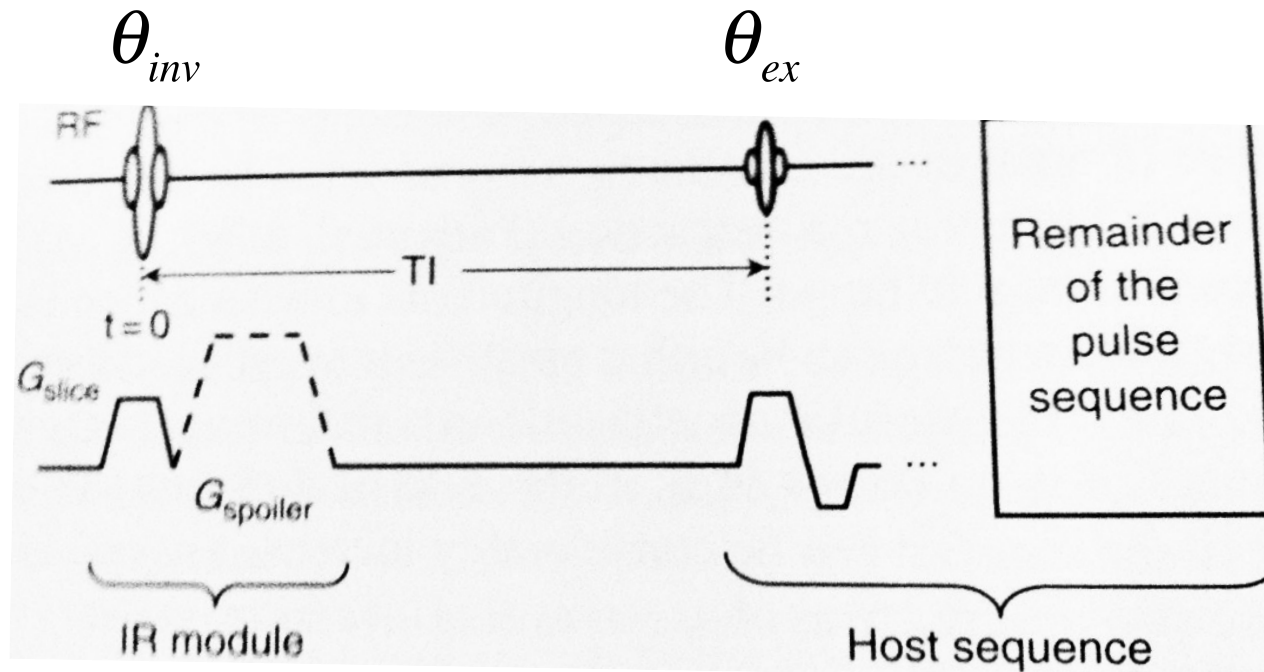


T2-Weighted **FLAIR**



T₁ Measurement with Inversion Recovery (IR)

- The most common method for estimating tissue T₁ is through the use of an inversion recovery sequence
- Involves an IR module or “preparation” prior to a “host sequence”



T₁ Measurement with Inversion Recovery (IR)

- For this sequence, the time-dependent longitudinal magnetization is:

$$M_z(t) = M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} \right]$$

- which assumes an infinitely long TR. With finite TR, $M_z(t)$ depends on details of the host sequence.
- For Spin Echo and Turbo Spin Echo (TSE)/RARE sequences:

$$M_z(t) = \begin{cases} M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} + e^{-TR/T_1} \right] & \text{Spin Echo} \\ M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} + e^{-(TR-TE_{last})/T_1} \right] & \text{TSE / RARE} \end{cases}$$

↖
Last Echo in the
Echo SE Train



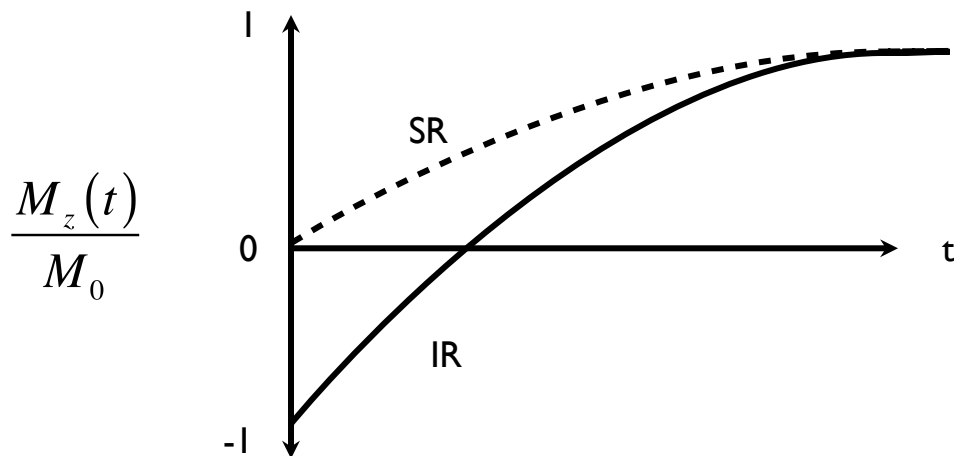
T_1 Measurement with Inversion Recovery (IR)

- For $\theta_{inv} = \pi$ this is complete inversion and the longitudinal magnetization becomes:

$$M_z(t) = M_0 [1 - 2e^{-t/T_1}]$$

- For $\theta_{inv} = \frac{\pi}{2}$ this results in *saturation recovery* (SR)

$$M_z(t) = M_0 [1 - e^{-t/T_1}]$$



T₁ Measurement with Inversion Recovery (IR)

- To quantify T₁, a series of IR images are acquired from the same location, each with a different T₁ while keeping all parameters identical.
- To avoid signal saturation, a long TR must be used (TR > 4T_{1max})
- Note that a “phase sensitive” IR sequence needs to be employed to discriminate negative magnetization (particularly around the null point).

$$M_z(t) = \begin{cases} M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} + e^{-TR/T_1} \right] & \textit{Spin Echo} \\ M_0 \left[1 - (1 - \cos \theta_{inv}) e^{-t/T_1} + e^{-(TR - TE_{last})/T_1} \right] & \textit{TSE / RARE} \end{cases}$$



T₁ Measurement with Inversion Recovery (IR)

- To quantify T₁, a series of IR images are acquired from the same location, each with a different TI while keeping all parameters identical.
- To avoid signal saturation, a long TR must be used (TR > 4T_{1max})
- Note that a “phase sensitive” IR sequence needs to be employed to discriminate negative magnetization (particularly around the null point).

$$R_1 = \frac{1}{T_1} = -\frac{1}{TI} \ln \left(\frac{1 - \frac{M_z}{M_0}}{(1 - \cos \theta_{inv})} \right)$$

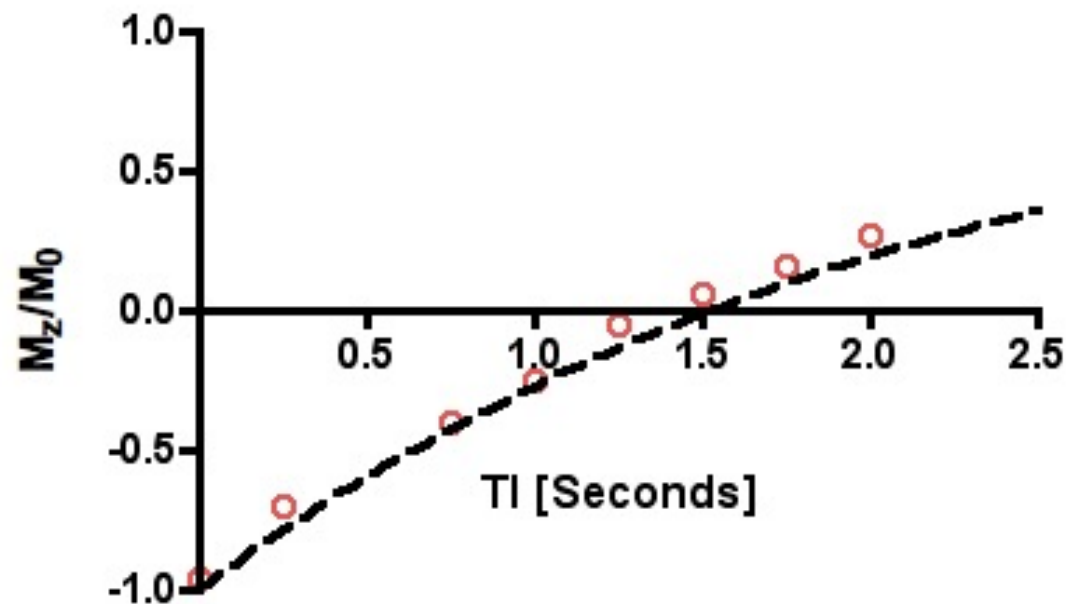
$$R_1 = \frac{1}{T_1} = -\frac{1}{TI} \ln \left(\frac{1}{2} \left(1 - \frac{M_z}{M_0} \right) \right) \quad \text{for } \theta = \pi$$



T₁ Measurement with Inversion Recovery (IR)

- Note that either the inversion time (TI) or the inversion flip angle can be changed and **nonlinear regression** can be used to fit RI or TI

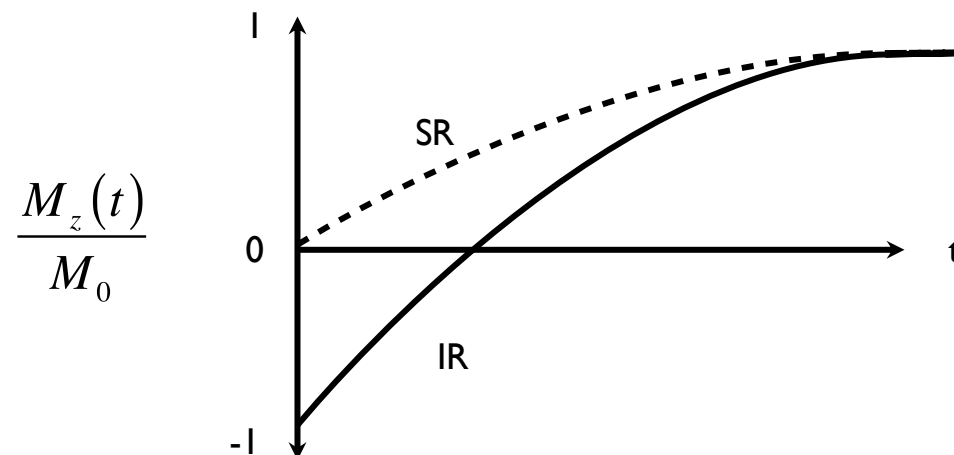
$$R_1 = \frac{1}{T_1} = -\frac{1}{TI} \ln \left(\frac{1 - \frac{M_z}{M_0}}{1 - \cos \theta_{inv}} \right)$$



T₁ Measurement with Saturation Recovery (SR)

- For an SR sequence, the IR prep is obtained by using $\vartheta_{inv} = \pi/2$, followed by a normal excitation $\vartheta_{ex} = \pi/2$.
- This is considered a **saturation recovery** sequence.
- Different “T₁” values can then be plotted and a similar fit can be used to estimate T₁

$$M_z(t) = M_0 \left[1 - e^{-t/T_1} \right]$$



Summary

- SR and IR are “modules” that can be applied to (most) host sequences
- SR/IR provides controllable T1 contrast/weighting
- SR/IR can be used to “null” or zero out tissues of interest to increase anatomical conspicuity
- Can also be used to quantify T1

