* Transverse $M_{xy}(\vec{r}, t) \Rightarrow$ FID $S_r(t)$

$$M = M_x + i M_y$$

$$\frac{dM}{dt} = \frac{dM_x}{dt} + i \frac{dM_y}{dt}$$

$$\Longrightarrow M(\vec{r}, t) = \underbrace{M(\vec{r}, 0)\, e^{-i\omega_0 t}\, e^{-t/T_2(\vec{r})}}_{M_a(\vec{r}, t)}\, e^{-i\int_0^t \Delta\omega(\vec{r}, \tau)\, d\tau}$$

* $\vec{B}(\vec{r}, t) = [B_0 + \Delta B(\vec{r}, t)]\hat{k}$

$$\Delta\omega(\vec{r}, t) = \gamma\, \Delta B(\vec{r}, t)$$

1) linear gradient

$$\Delta\omega(\vec{r}, t) = \gamma \cdot \vec{G}\, \vec{r}$$

$$= \gamma\,(G_x \cdot x + G_y \cdot y + G_z \cdot z)$$

$$\therefore M(\vec{r}, t) = M_a \cdot e^{-i 2\pi\left(\frac{\gamma}{2\pi}\vec{G}\cdot\vec{r}\right)t}$$

2) time varying

$$\Delta\omega(\vec{r}, t) = \gamma\, \vec{G}(t) \cdot \vec{r}$$

$$\therefore M(\vec{r}, t) = M_a \cdot \underbrace{e^{-i 2\pi \frac{\gamma}{2\pi}\int_0^t \vec{G}(\tau)\cdot\vec{r}\, d\tau}}_{\text{spatially vary phase due to } \vec{G}(\vec{r}, t)}$$

* Faraday's law of Induction

electromotive force ($\mathcal{E}$)

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t}$$

$$d\mathcal{E} = -\frac{d M(\vec{r},t)}{dt} \cdot dV \qquad \text{(see Eq. 5.38)}$$

$$\int_V d\mathcal{E}\, dV = S_r(t) = -k \int_V \frac{d}{dt} M(\vec{r},t)\, dV$$

$$= -k \int_V M(\vec{r},0) \left[-i\left(\omega_0 + \gamma \vec{G}(t)\cdot\vec{r}\right)\right] \cdot e^{-i\omega_0 t} e^{-i\gamma \int_0^t \vec{G}(\tau)\vec{r}\, d\tau}$$

(ignore T_2 decay, $\frac{d}{dt} e^{at} = a e^{at}$)

in general, $\omega_0 \gg \gamma \vec{G}\cdot\vec{r}$

$$S_r(t) = k\, i\, \omega_0 \int_V M(\vec{r},0)\, e^{-i\omega_0 t} e^{-i\gamma \int_0^t \vec{G}(\tau)\vec{r}\, d\tau}$$

3 simplifications

1) 2D imaging $\quad \Delta z$

def. $m(x,y) = \int_{z-\frac{\Delta z}{2}}^{z+\frac{\Delta z}{2}} m(x,y,z)$

2) ignore T_2 decay

3) demodulate by ω_0

Def. $s(t) = s_r(t) \cdot e^{i\omega_0 t}$

"baseband"

Signal Equation

$$s(t) = \iint_{x\,y} \underbrace{m(x,y)}_{\text{desire}} e^{-i\gamma \int_0^t \bar{G}(\tau)\cdot r\, d\tau}\, dx\, dy$$

$$= \iint_{x\,y} m(x,y)\, e^{-i\gamma\left[\left(\int_0^t G_x(\tau)\,d\tau\right)\cdot x + \left(\int_0^t G_y(\tau)\,d\tau\right) y\right]} dx\,dy$$

$$= \iint_{x\,y} m(x,y)\, e^{-i2\pi\left[\underbrace{\left(\frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,d\tau\right)}_{\triangleq k_x(t)} x + \underbrace{\left(\frac{\gamma}{2\pi}\int_0^t G_y(\tau)\,d\tau\right)}_{\triangleq k_y(t)} y\right]} dx\,dy$$

$$s(t) = \iint_{x\,y} m(x,y)\, e^{-i2\pi(k_x(t)\cdot x + k_y(t)\, y)}\, dx\,dy$$

2D FT of $m(x,y)$

$= M(k_x(t), k_y(t))$

- $s(t)$ equals values of M along trajectory in FT space "K space"
- $G_x G_y$ control path in K-space
- to image $m(x,y)$ acquire set samples $\{s(t)\}$ to cover k-space " sufficiently

Complex $s(t)$ demodulation :

$$s_r(t) = d(t)\, e^{-i(\omega_0 t + \phi(t))}$$

$\boxed{\text{Single surface receive coil}}$ (sensitive to the rate of change of magnetization only along one axis)

physical $\quad s_p(t) = \mathrm{Re}\{\ \text{"}\ \}$

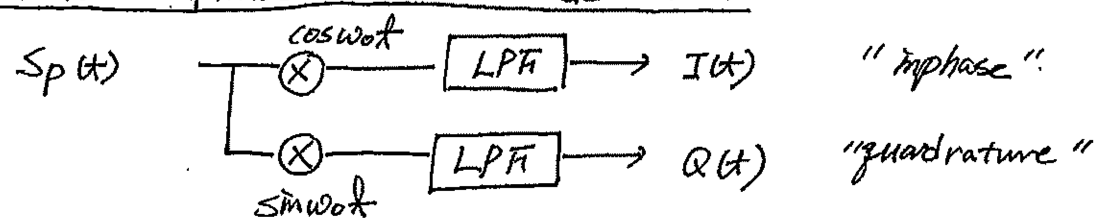
$$= d(t) \cos(\omega_0 t + \phi(t))$$

$$= d(t) \cos(\phi(t))\, \underline{\cos \omega_0 t} - d(t) \sin(\phi(t))\, \underline{\sin \omega_0 t}$$

Want : $\quad s(t) = s_r(t)\, e^{+i\omega_0 t} = d(t)\, e^{-i\phi(t)}$

$$= d(t) \cos \phi(t) - i\, d(t) \sin \phi(t)$$

<u>Quadrature phase sensitive detection</u>



$I(t)\ //\ \left(d(t) \cos(\omega_0 t + \phi(t)) \cos \omega_0 t \right) * \boxed{\text{LPF}}$

$\Rightarrow d(t) \cos \phi(t)$

$Q(t)\ //\ \Rightarrow -d(t) \sin \phi(t)$ $\qquad$ derive at home.!!

⇒ We'll be receiving complex values even though we have a coil only sensitive to one axis (direction)