Spatial Localization I

M219 - Principles and Applications of MRI

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2/6/2023

Course Overview

- 2023 course schedule
  - https://mrrl.ucla.edu/pages/m219_2023
- Assignments
  - Homework #2 is due on 2/15
- Office hours, Fridays 10-12pm
  - In-person (Ueberroth, 1417B)
  - Zoom is also available (https://uclahs.zoom.us/j/98066349714?pwd=cnVmV1J5QjR1d3I3cmJkQnVLSFZVZz09)

3 Types of Magnetic Fields

B_0 - Large static field

e.g., 1.5 Tesla or 3 Tesla

B_1 - Radiofrequency field

e.g., 0.16 G

$G_{x,y,z}$ - Gradient fields

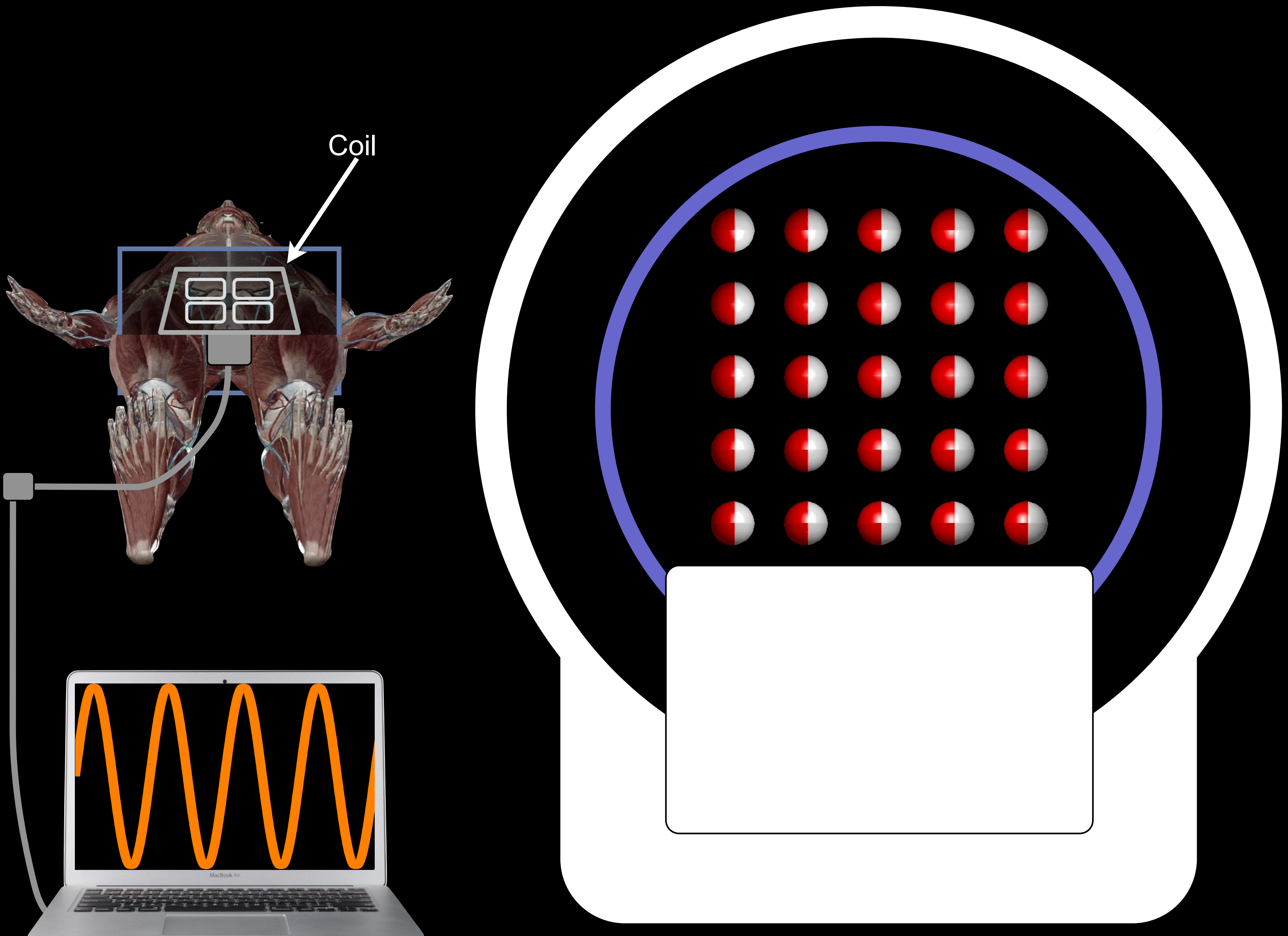
e.g., 4 G/cm

Selective Excitation

Frequency and Phase Encoding

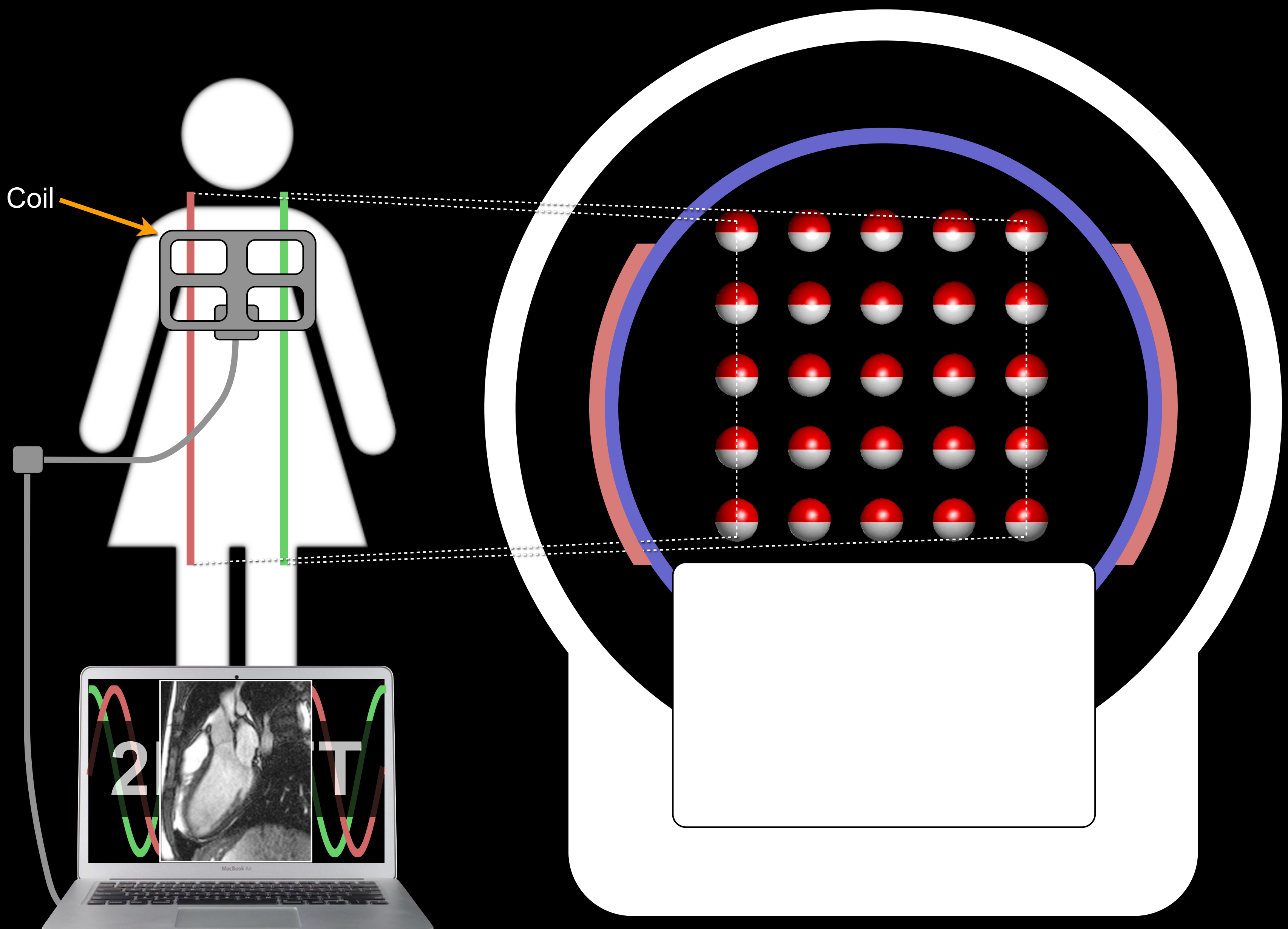
How do we measure M_{xy}?

Faraday's Law of Induction



Precessing spins *induce* a current in a nearby coil.

Faraday's Law of Induction



The trick is to encode spatial information and image contrast in the echo.

Basic Detection Principles

$$S(t) = \int_{\text{object}} M_{xy}(\boldsymbol{r},0)e^{-i\gamma\Delta B(\boldsymbol{r})t}d\boldsymbol{r}$$

Observations

Detected signal is the vector sum of all transverse magnetizations in the “rotating frame” within the imaging volume.

The Larmor frequency precession (Lab frame rotation) is necessary for detection, although only the baseband signal matters for imaging

Basic Detection Principles

Magnetic Flux Through The Coil – *Reciprocity*

$$\int_{vol} d\epsilon = s_r(t) = -\int_{vol} \frac{\partial}{\partial t}[B_r(\vec{r}) \cdot M(\vec{r},0)]dV$$

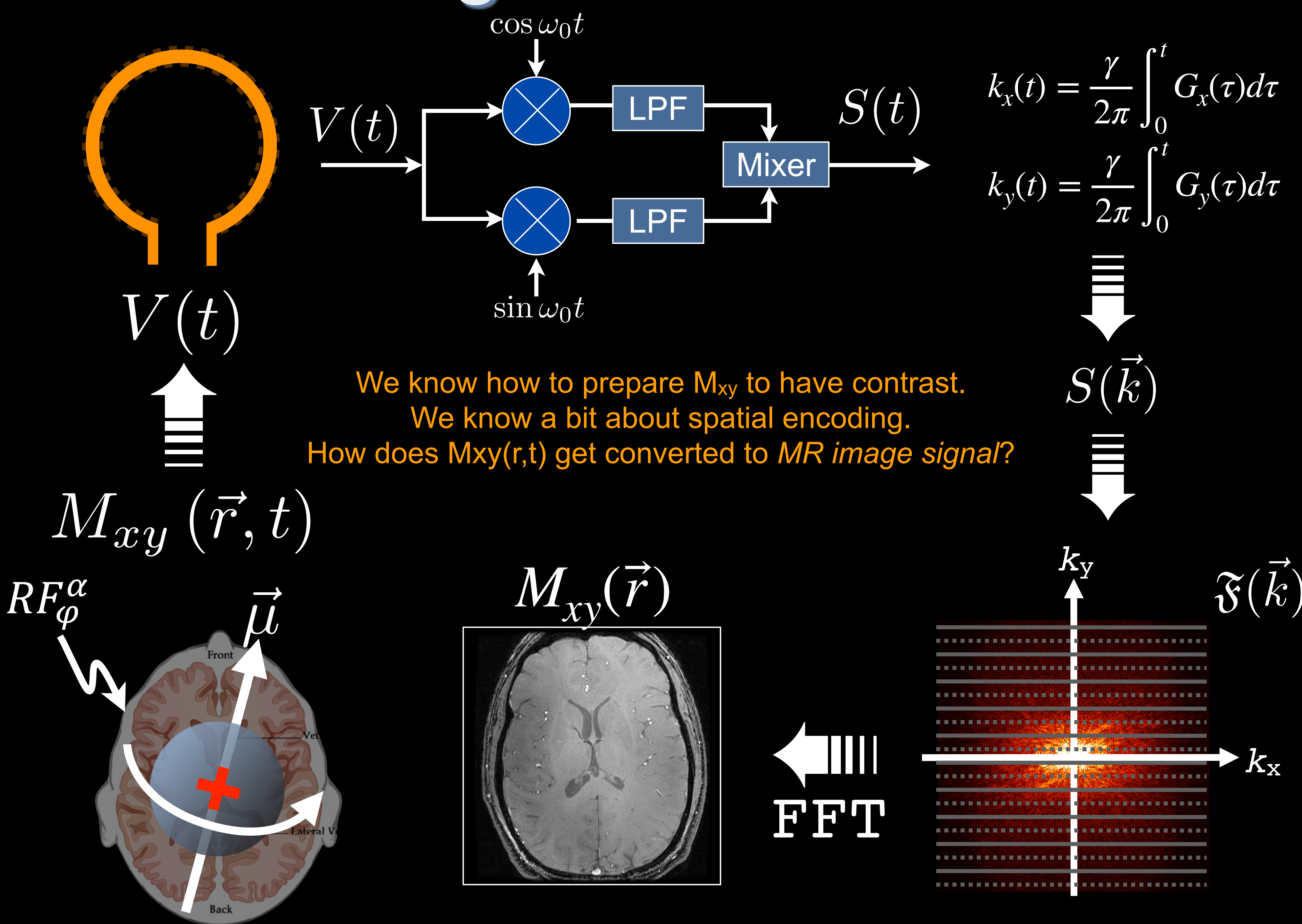
$$s_r(t) = i\omega_0 B_r(\vec{r}) \int_{vol} M(\vec{r},0) e^{-i\omega_0 t} e^{-i\gamma \int_0^t \vec{G}(\tau)\vec{r} d\tau} dV$$

With Simplifications…

$$s(t) = \int_x \int_y M(x,y) e^{-i2\pi(k_x(t)\cdot x + k_y(t)\cdot y)} dx dy = m(k_x(t), k_y(t))$$

Signals in MRI

We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?



Signals in MRI

$V(t)$

$\cos\omega_0 t$, $\sin\omega_0 t$

$V(t)$ → LPF → Mixer → $S(t)$

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

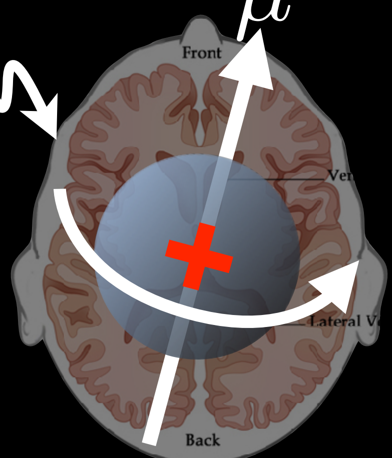
$S(\vec{k})$

We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?

$M_{xy}(\vec{r}, t)$

RF_{φ}^{α}

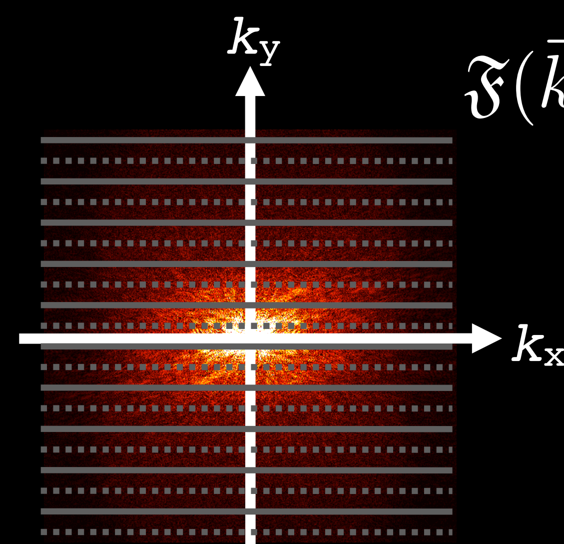
$\vec{\mu}$



$M_{xy}(\vec{r})$

FFT

$\mathfrak{F}(\vec{k})$



To the Board

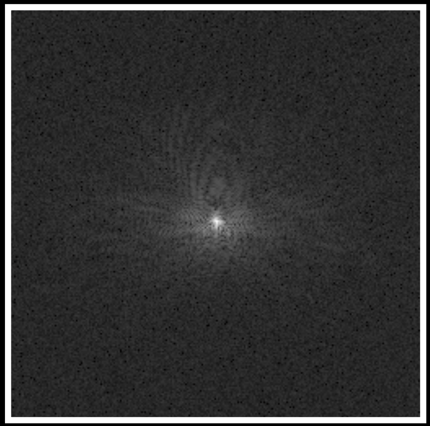
MR Signal Equation

$$s(t) = \int_x \int_y M(x,y) e^{-i2\pi(k_x(t)\cdot x + k_y(t)\cdot y)} dx dy$$

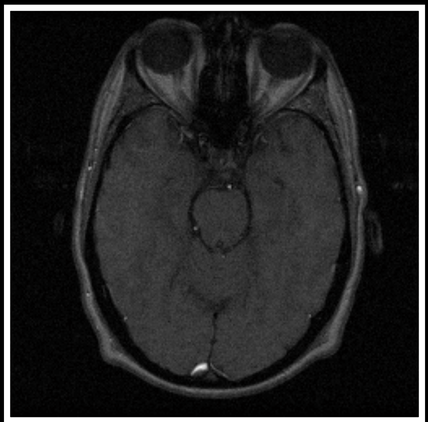
$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau \quad k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

$$s(t) = m(k_x(t), k_y(t))$$

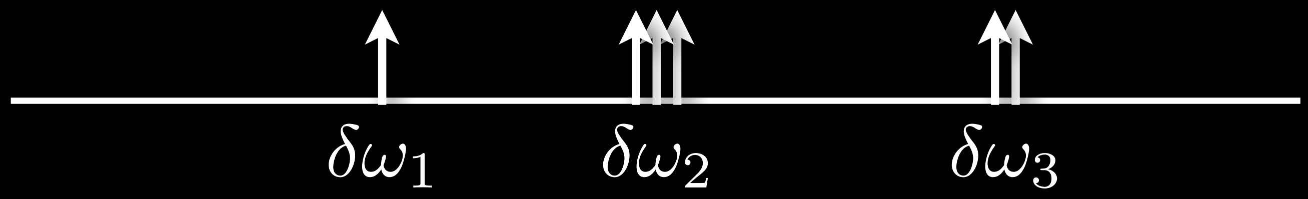
$$m = \mathcal{FT}(\, M(x,y)\,)$$



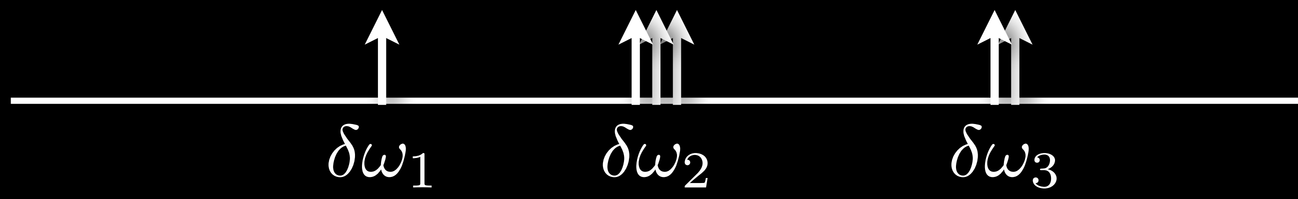
FT



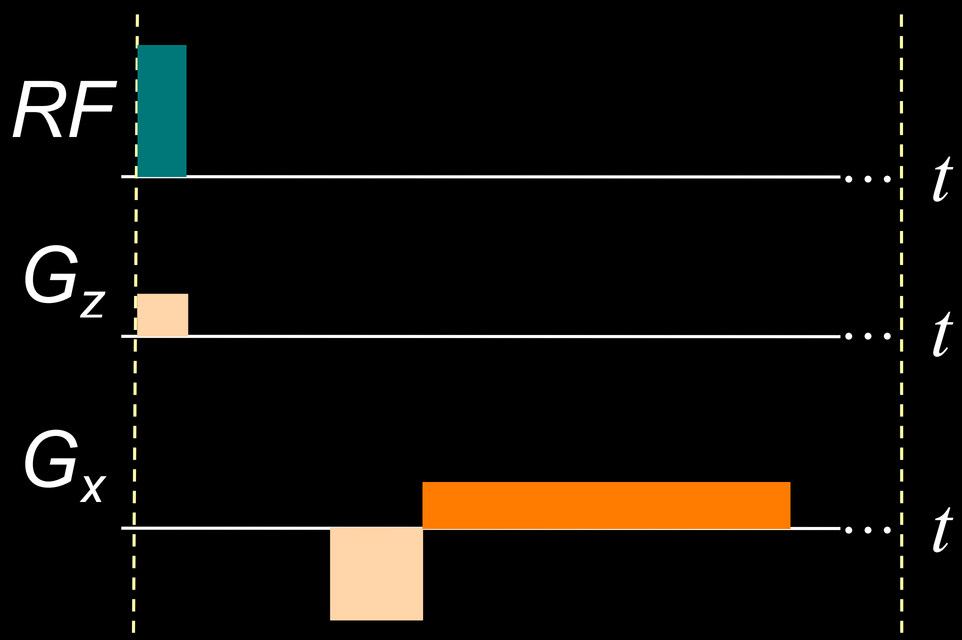
1D Imaging



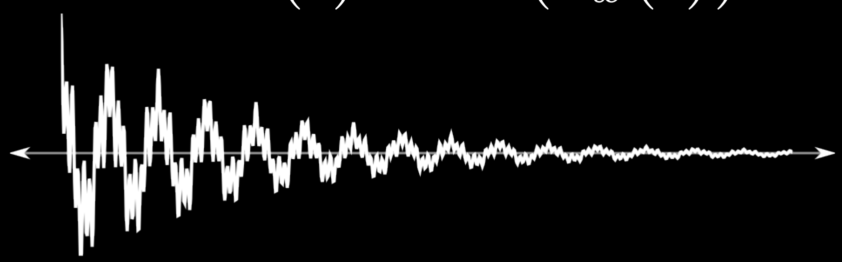
1D Imaging



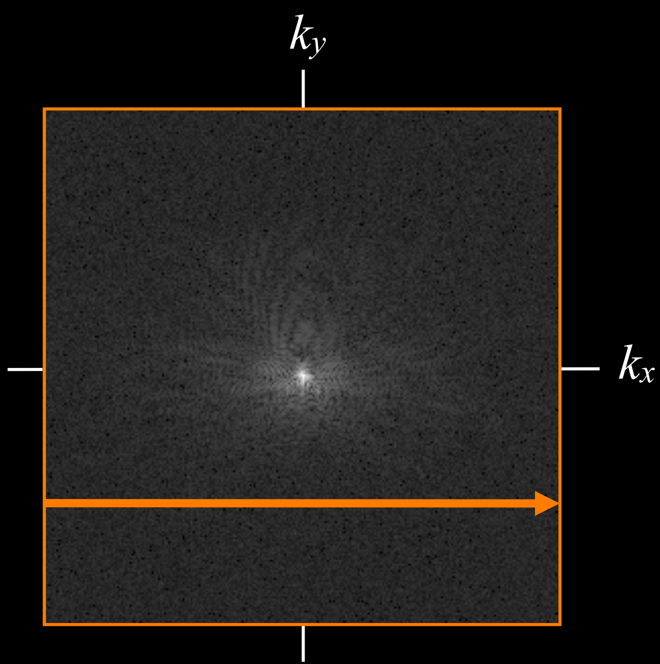
Pulse Sequence Diagram



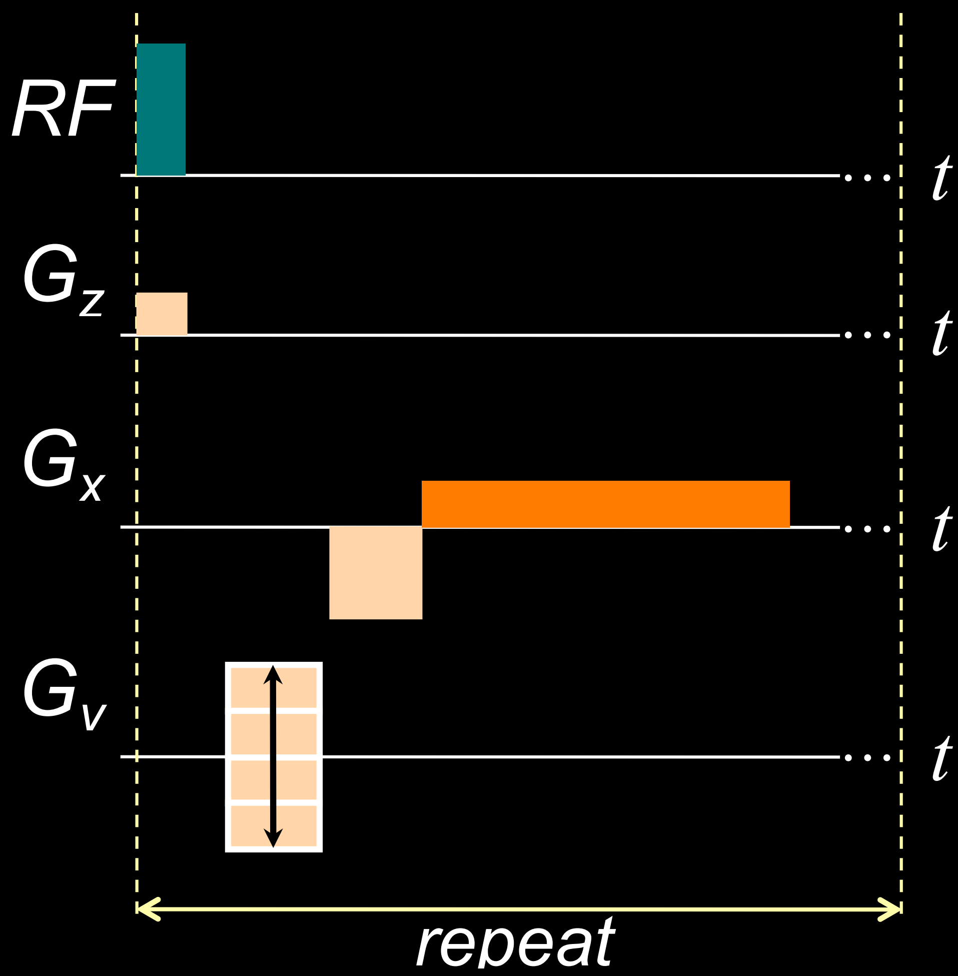
$$s(t) = m(k_x(t))$$



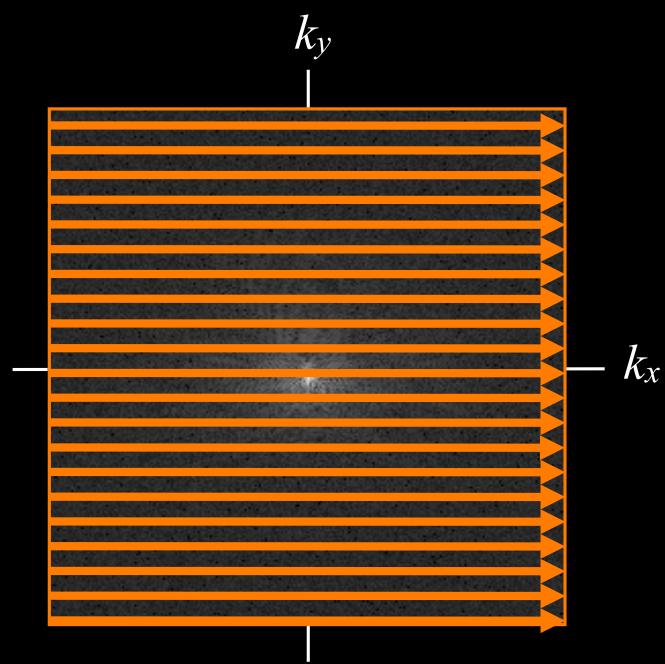
2D Imaging



$$s(t) = m(\, k_x(t), k_y(t) \,)$$

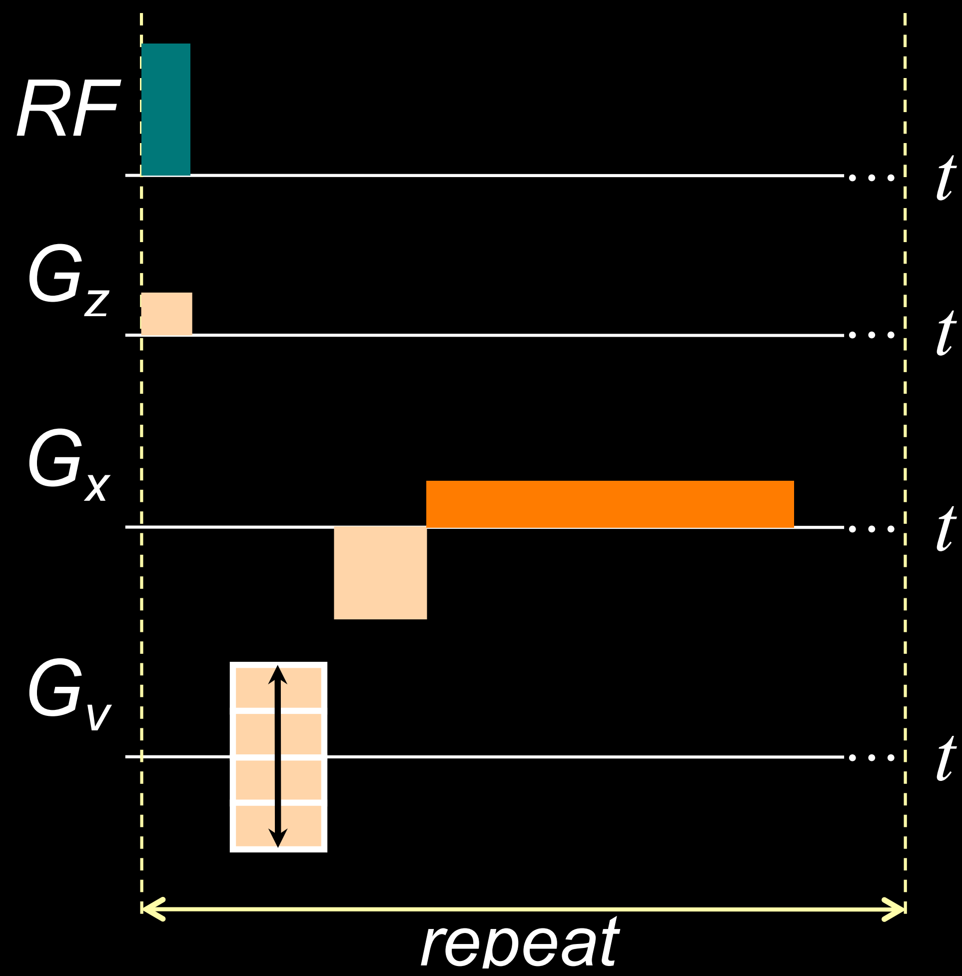


2D Imaging

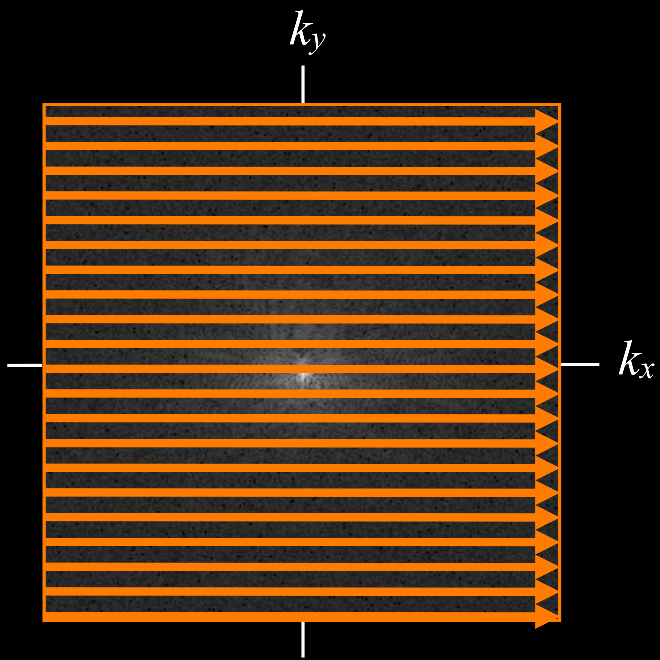


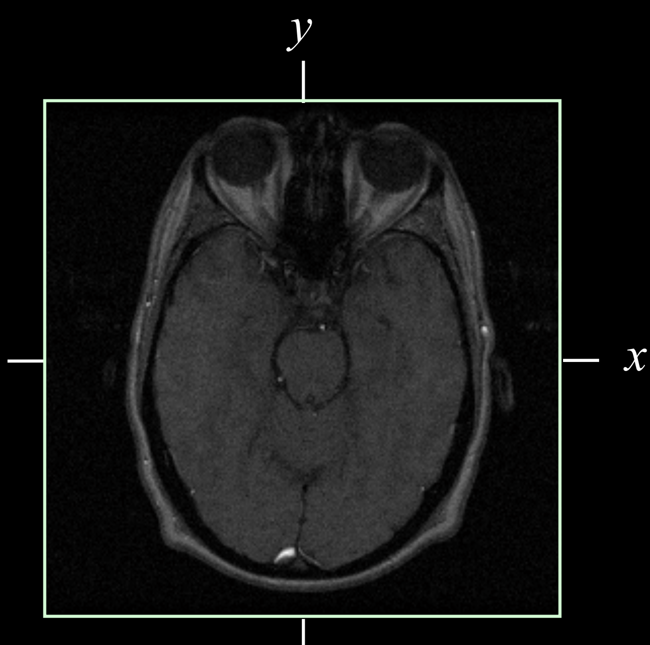
$$s(t) = m(\,k_x(t), k_y(t)\,)$$

Pulse Sequence Diagram

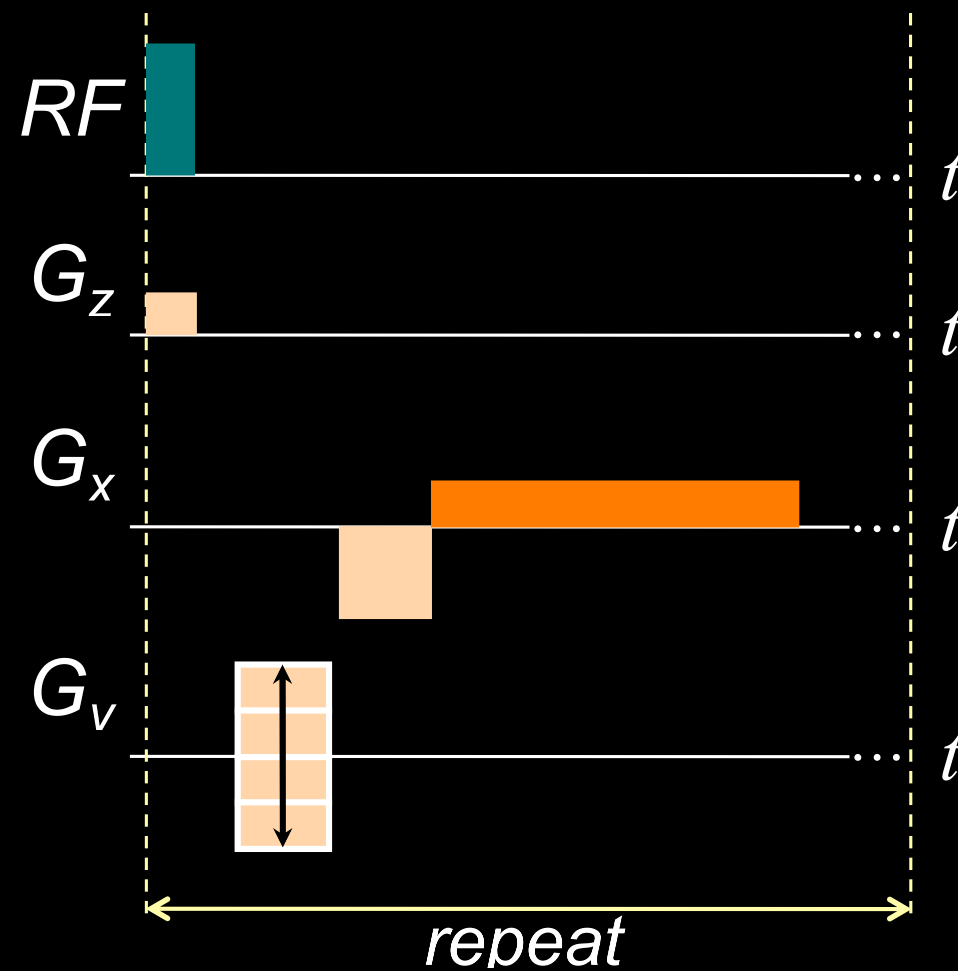


2D Imaging



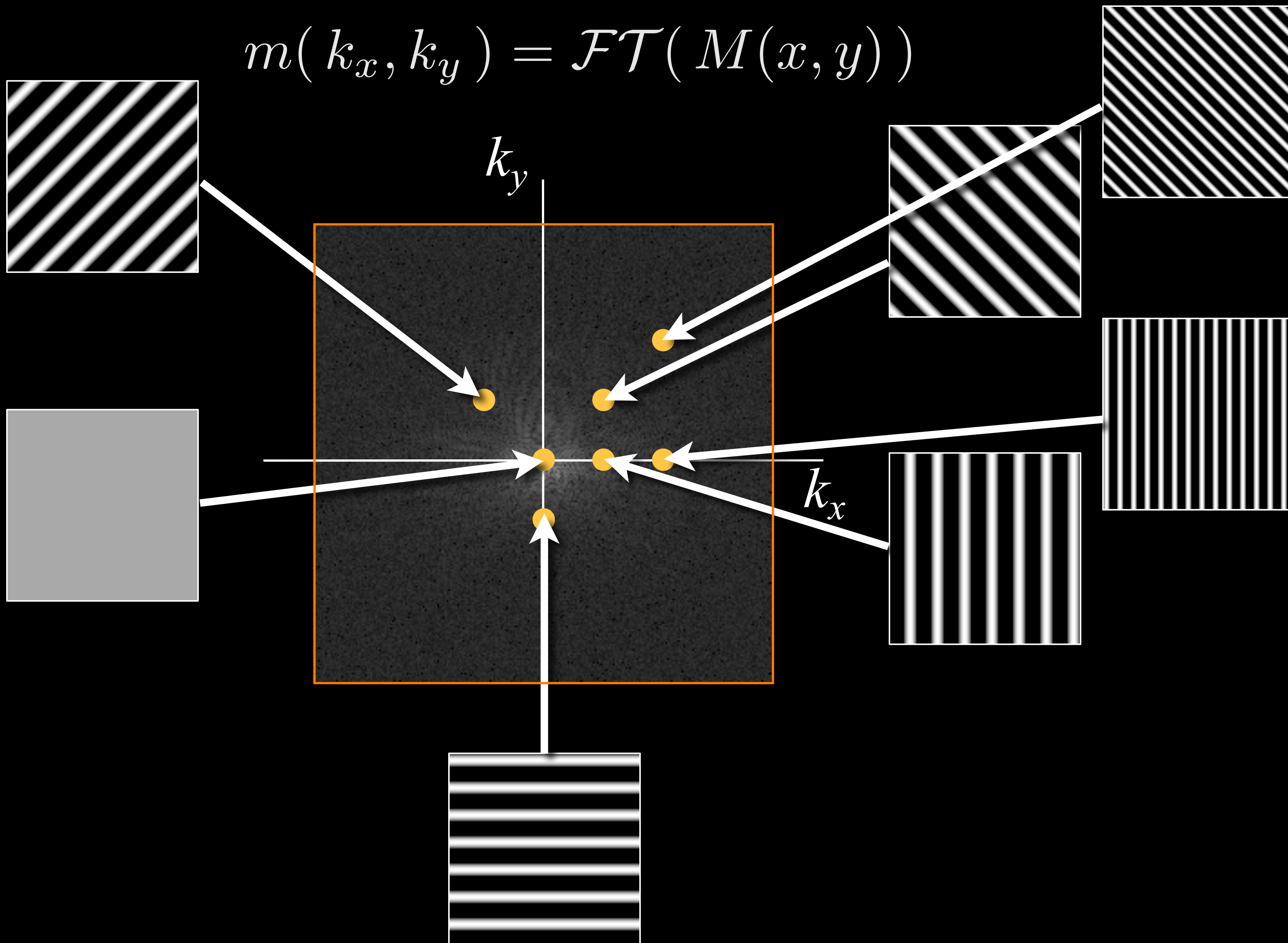


Pulse Sequence Diagram

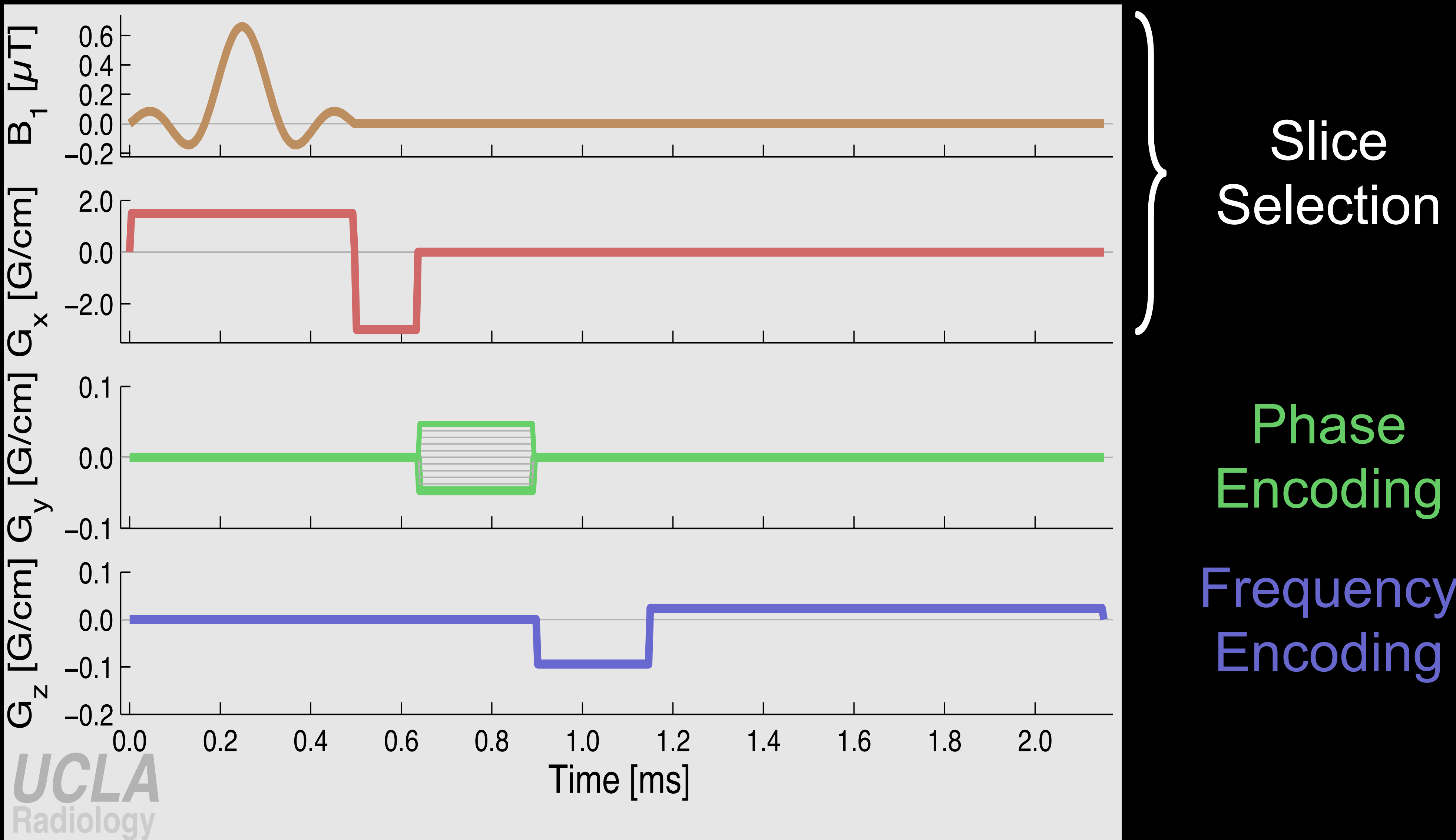


2D k-Space: MRI Data

$$m(k_x, k_y) = \mathcal{FT}(M(x, y))$$



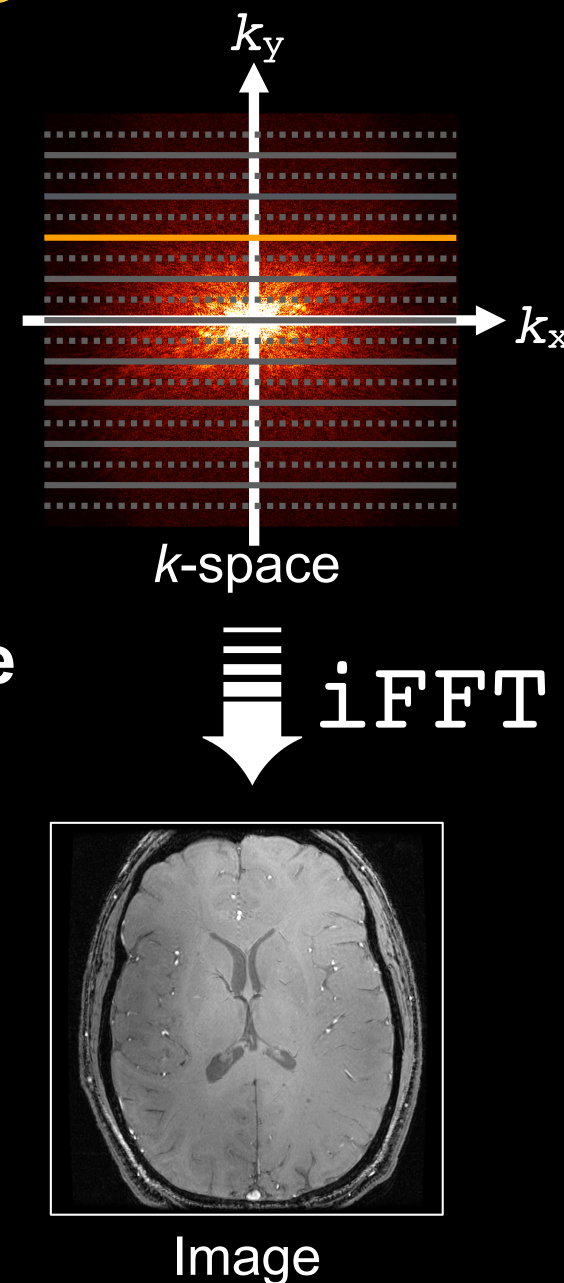
3 Steps for Spatial Localization

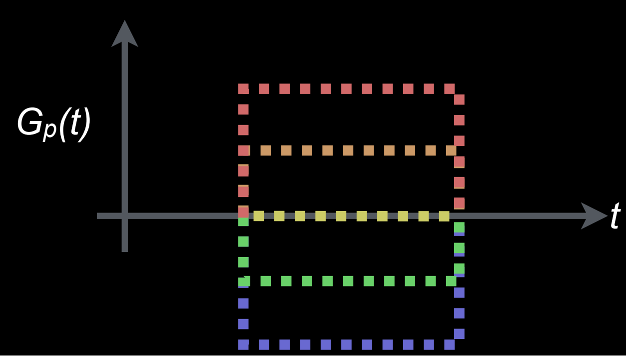


Pulse Sequence Diagram - Timing diagram of the RF and gradient events that comprise an MRI pulse sequence.

Phase Encoding

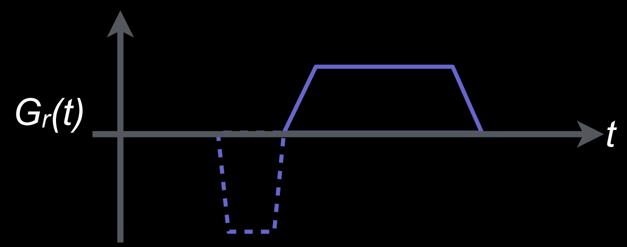
- **Consists of:**
  - **Phase encoding gradient**
    - **Magnitude changes with each TR**
    - **Can be played with other gradients**
      - **Crushers, Slice-selection rephaser, readout dephasing**
- **Used with Cartesian imaging**
- **After excitation, before readout**
- **Adds linear spatial variation of phase**
- **Phase encode in**
  - **one direction for 2D imaging**
  - **two directions for 3D imaging**
- **Only one PE step per echo**



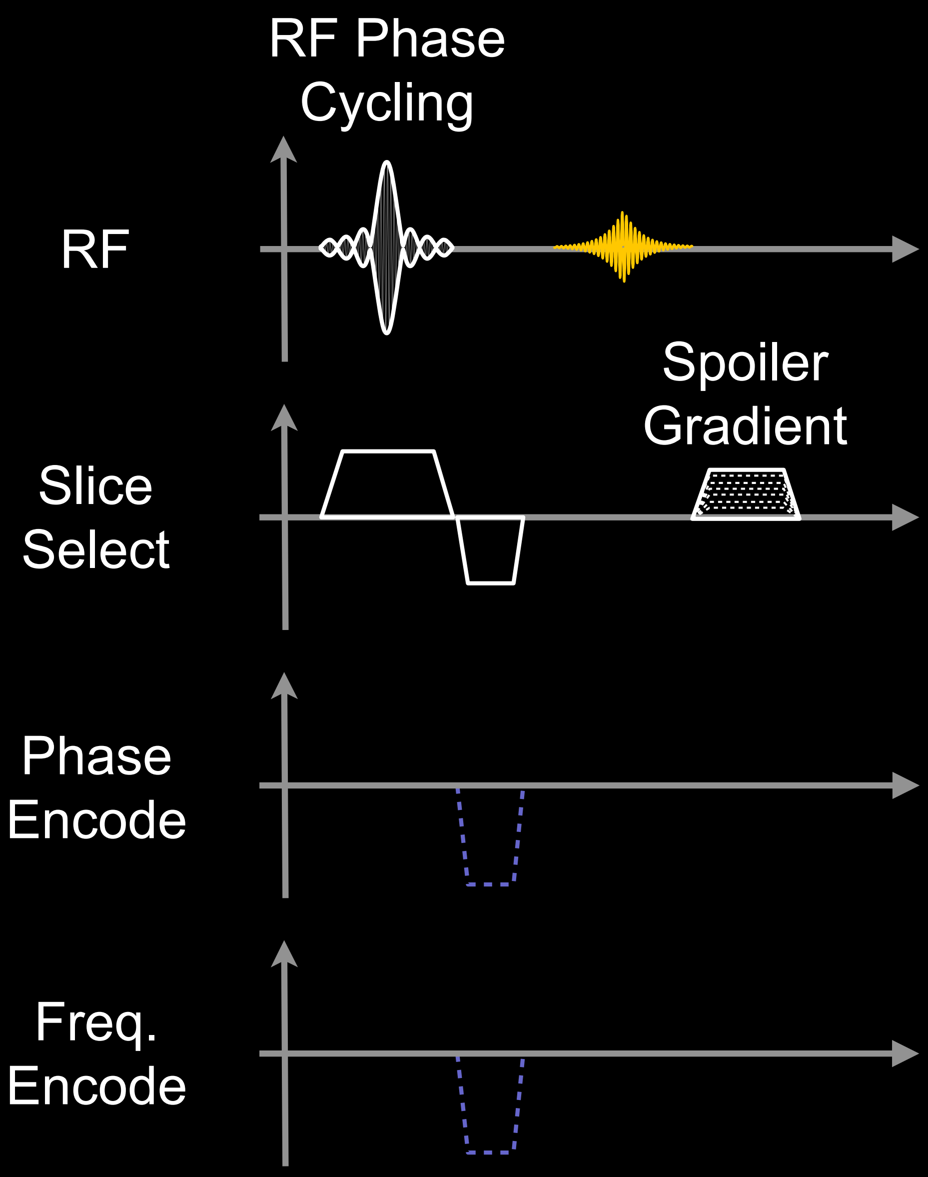


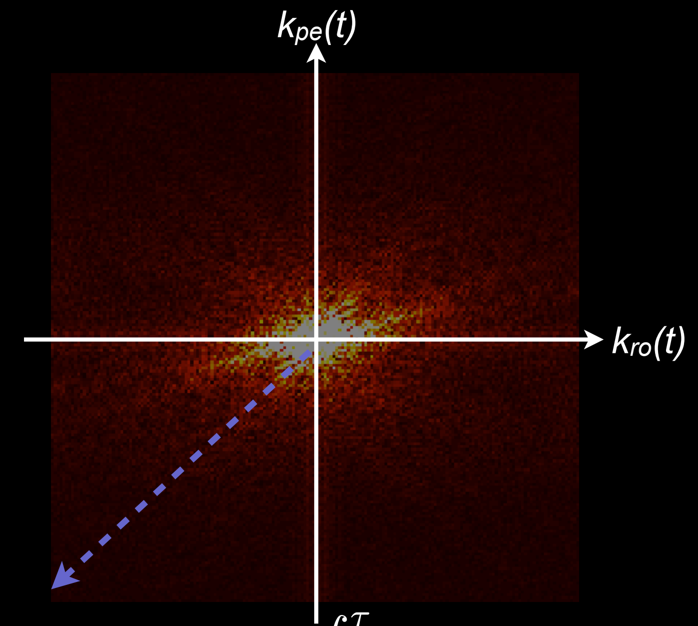
Frequency Encoding

- **Consists of:**
  - **Frequency encoding gradient**
    - **Constant magnitude for Cartesian imaging**
  - **No simultaneous**
    - **RF (B_1)**
    - **Other gradients**
      - **phase encoding, slice encoding, crushers**
  - **Readout pre-phasing gradient**
    - **Prepares spin phase so peak echo amplitude occurs at middle of readout (TE)**
    - **AKA "readout de-phasing gradient"**
- **Adds linear spatial variation of frequency**
- **Helps form an *echo***

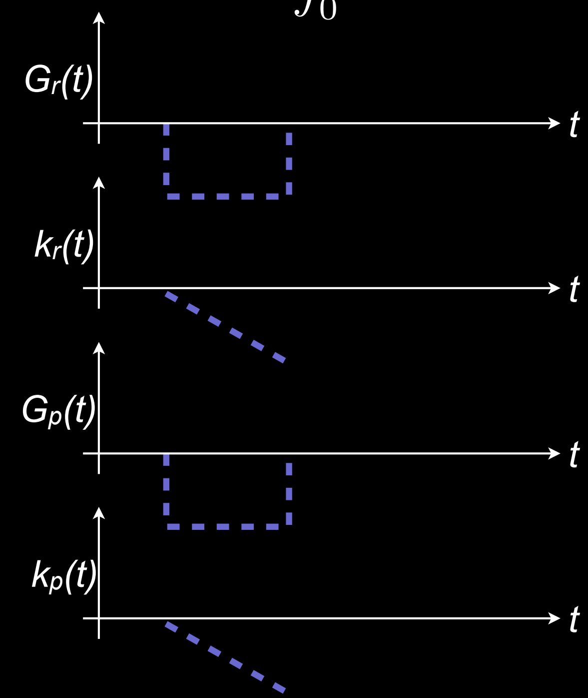


Where am I in *k*-space?



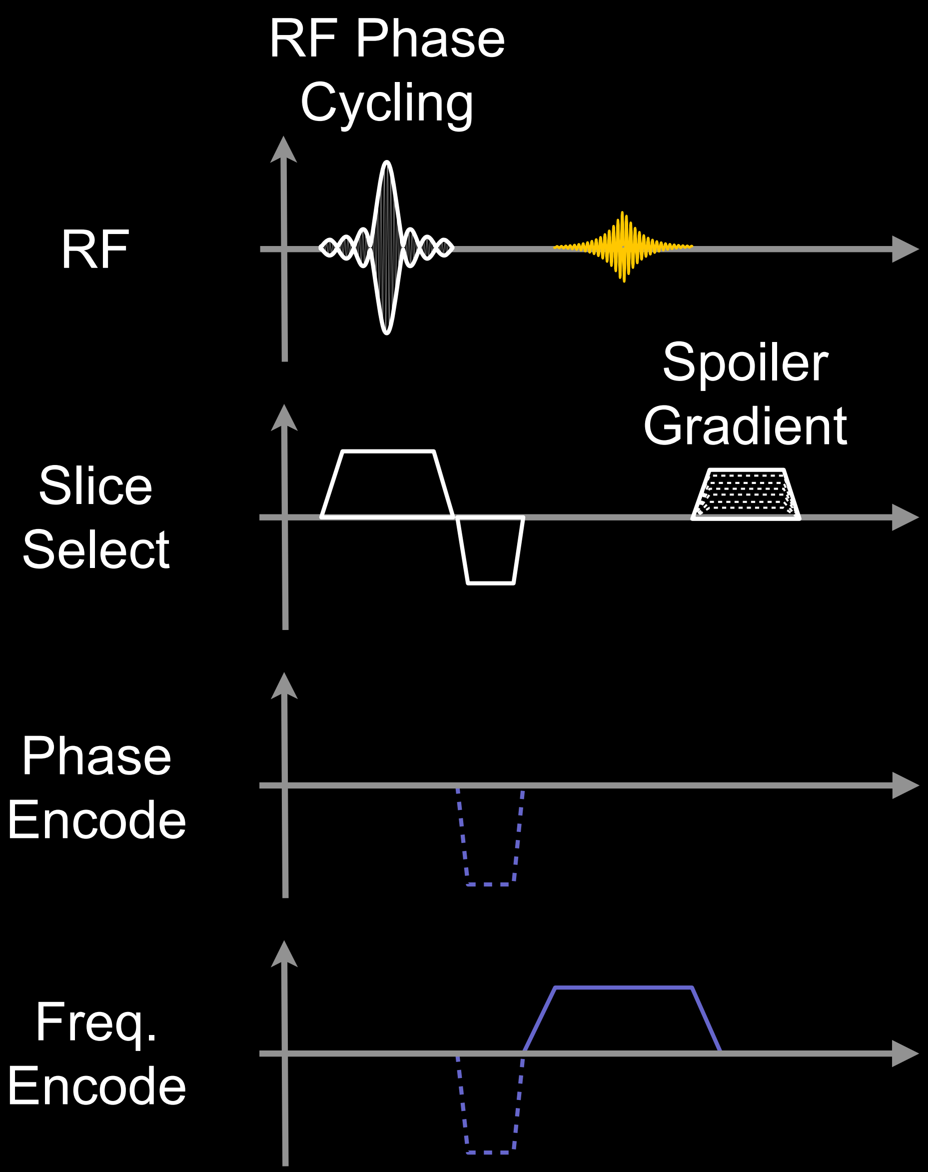


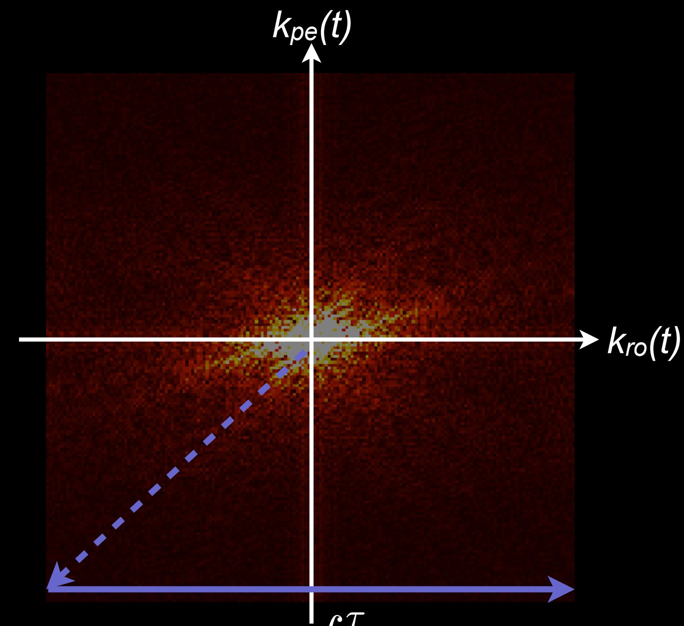
$$\vec{k}(t) = \frac{\gamma}{2\pi} \int_0^{\tau} \vec{G}(t)\, d\tau$$



One phase encoded echo is acquired per TR.

Where am I in *k*-space?





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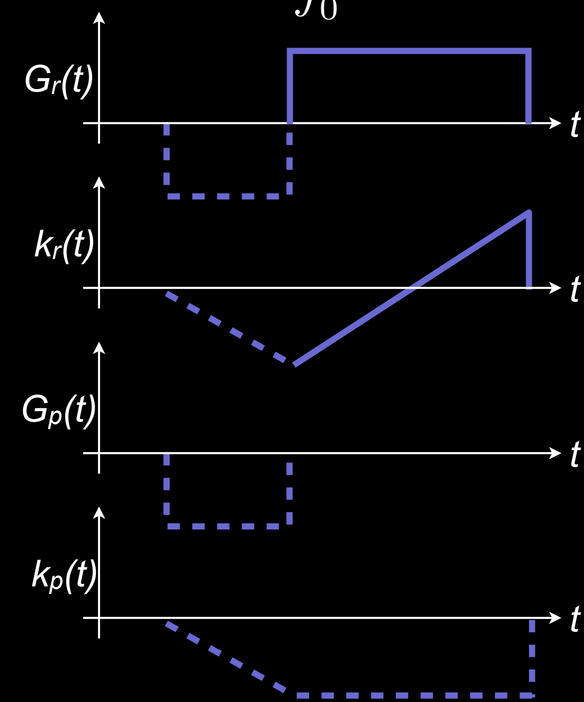

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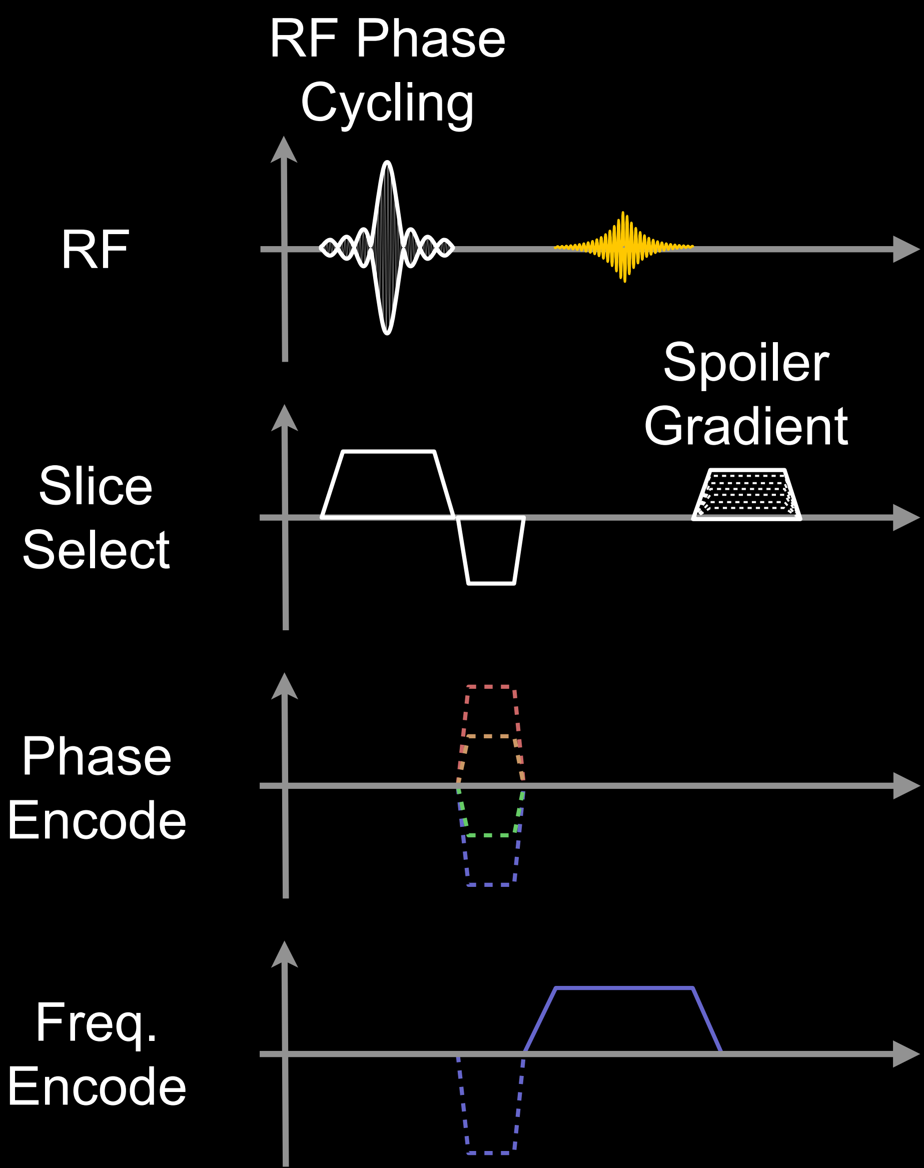
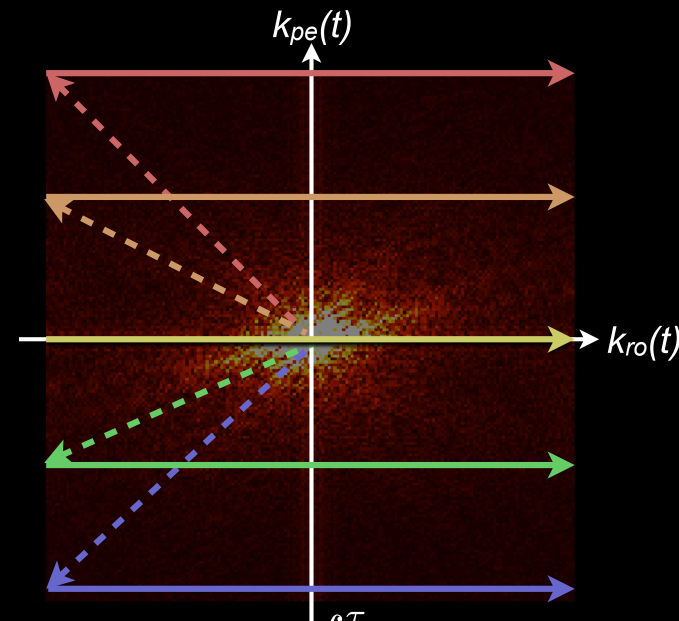
Where am I in *k*-space?





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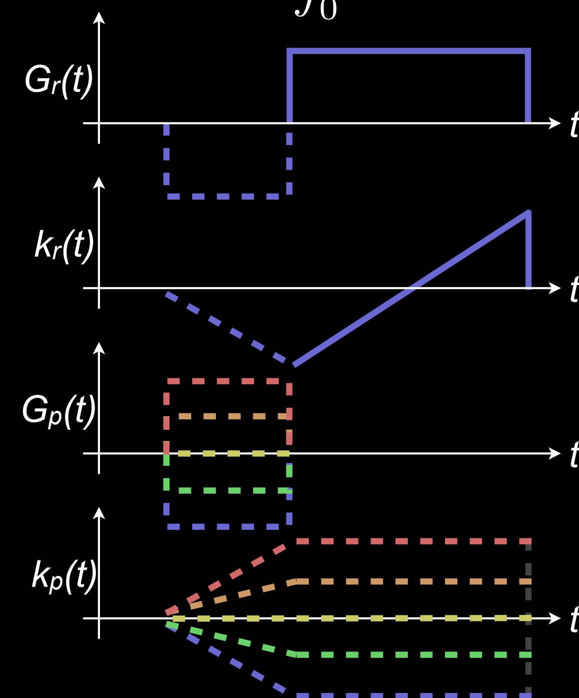

One phase encoded echo is acquired per TR.

MRI Sampling Requirements

k-space Sampling

Remember that the collected data in MRI is discrete

Discrete sampling can lead to artifacts if not careful

Sampling considerations
- Field of View
- Spatial Resolution

Sampling Considerations

$$s(t) = m(\ k_x(t), k_y(t)\ )$$

$$s(n\Delta t) = M(k_x(n\Delta t), k_y(n\Delta t))$$

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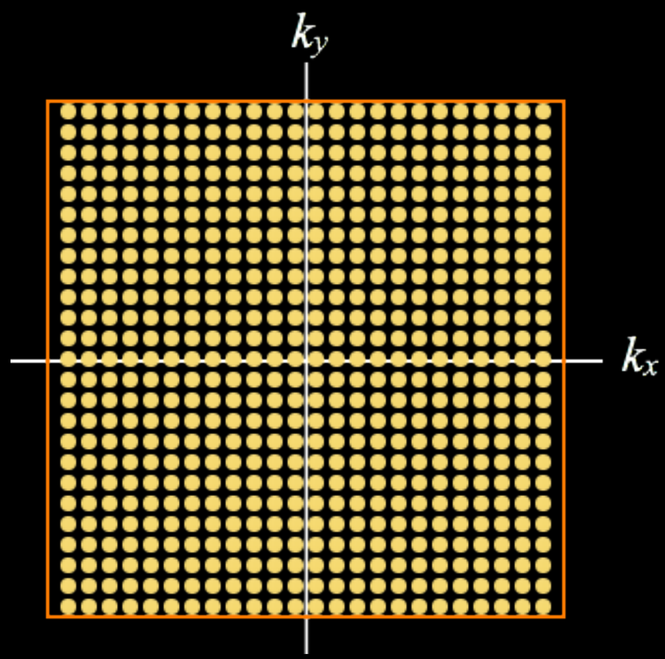
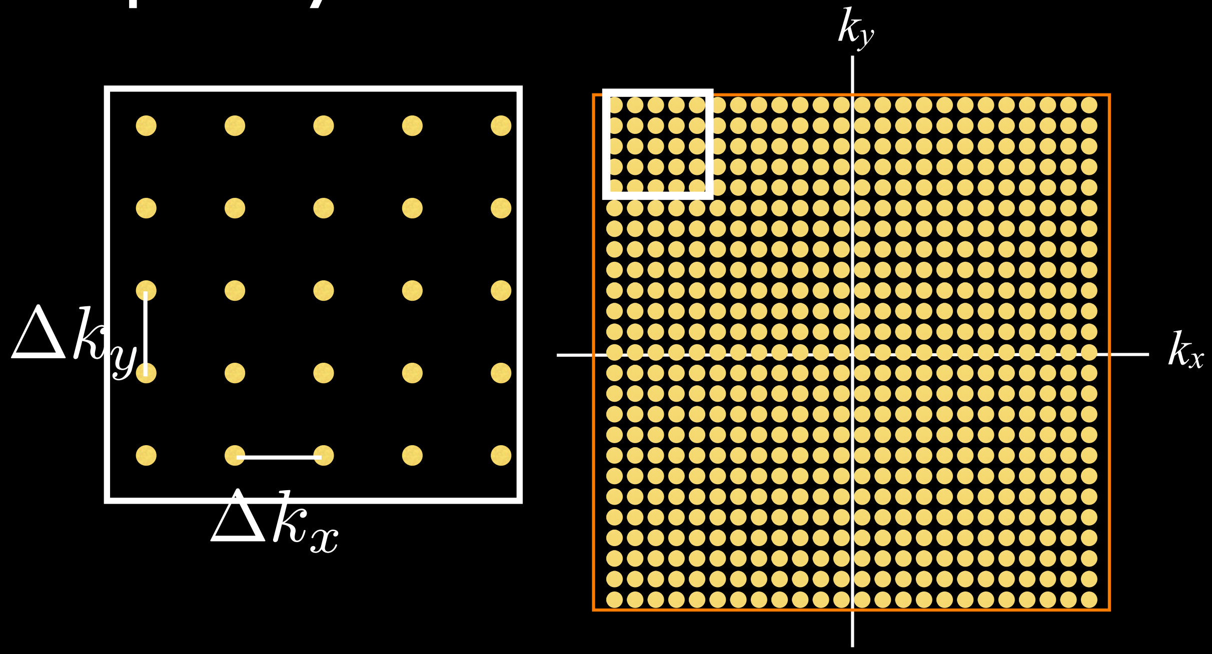
Sampling period

discrete sampling in spatial frequency domain



Sampling Considerations

discrete sampling in spatial frequency domain



$$w_{k_x} = N_{read} \times \Delta k_x$$

$$w_{k_y} = N_{PE} \times \Delta k_y$$

Review: Properties of DFT

Convolution

$$f(x) * h(x) \longleftrightarrow F(k_x)\, H(k_x)$$

Similarity (scaling)

$$f(ax) \longleftrightarrow \frac{1}{|a|} F(\frac{k_x}{a})$$

Shift

$$f(x-a) \longleftrightarrow \exp(-i2\pi(ak_x)) \cdot F(k_x)$$

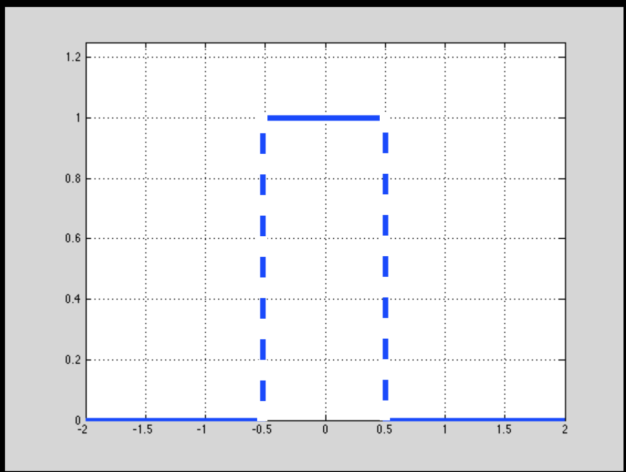
Review: Properties of DFT

comb or “Shah”

$\mathrm{III}(x)$

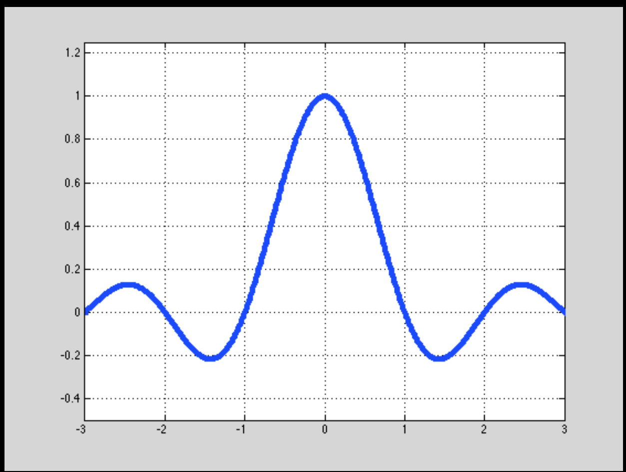
FT
↔

$\mathrm{III}(k_x)$



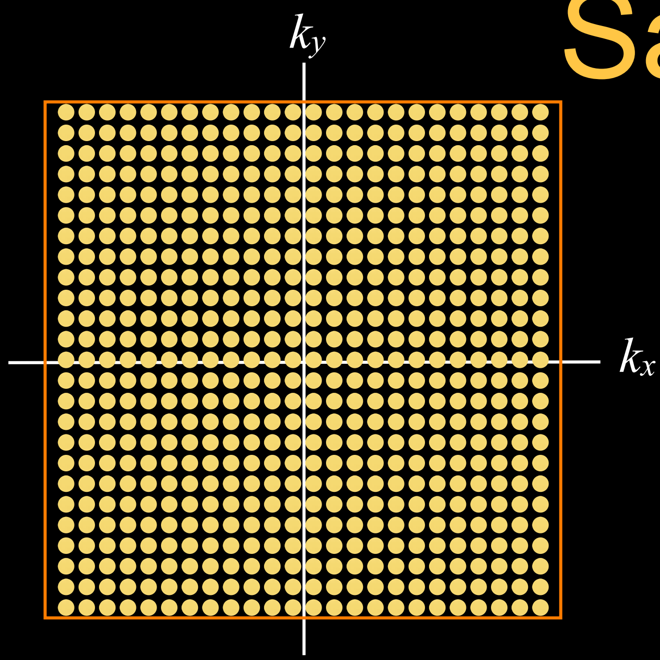
rect

FT
↔



$$\mathrm{sinc}(k_x) = \frac{\sin(\pi k_x)}{\pi k_x}$$

Sampling Model



Sampled Version

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)\frac{1}{\Delta k_x \Delta k_y} \sqcap \left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)$$

Sampling — Extent

FT ↕

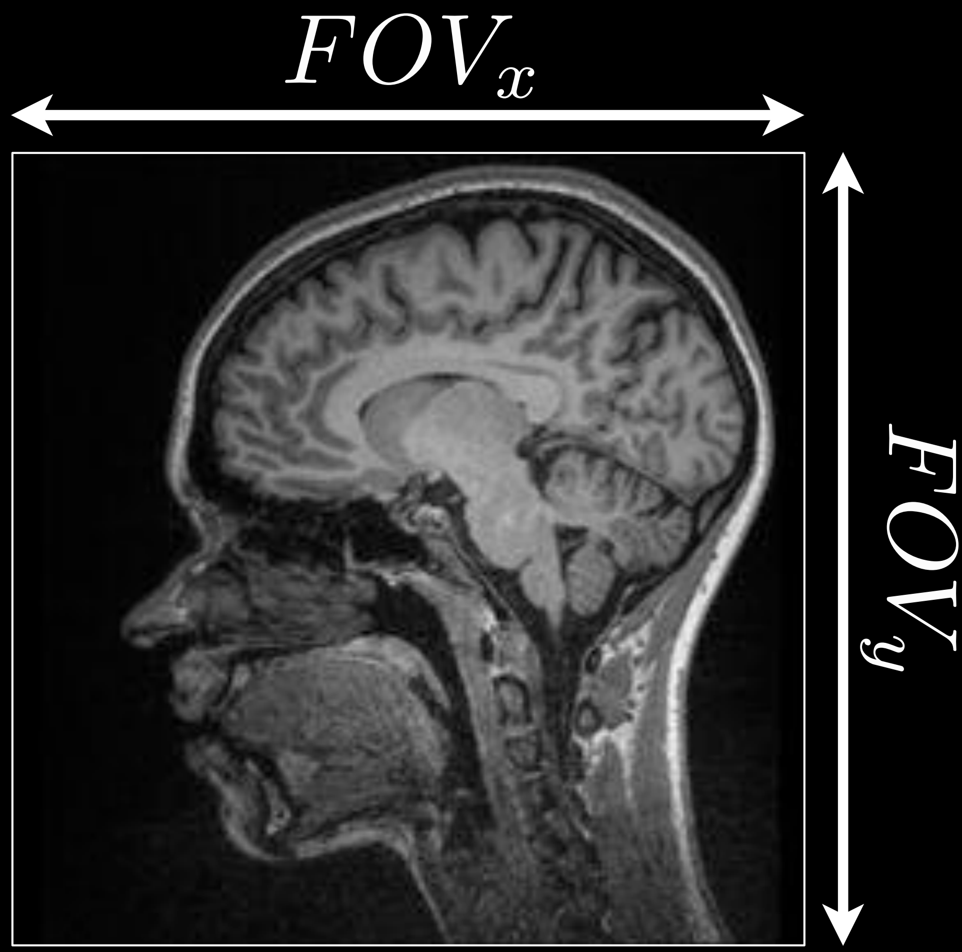
$$\hat{m}(x, y) = m(x, y) * \text{III}(\Delta k_x x, \Delta k_y y) * sinc(w_{k_x} x) sinc(w_{k_y} y)$$

Field of View — Spatial Resolution

Field of View

$$m(x,y) * \mathrm{III}(\Delta k_x x, \Delta k_y y)$$

Field of View



Eq. 5.76

$$\Delta k_x = \frac{1}{FOV_x} = \frac{\gamma}{2\pi} G_{xr} \Delta t$$

$$\Delta k_y = \frac{1}{FOV_y} = \frac{\gamma}{2\pi} G_{yi} \tau_y$$

To the Board

Field of View

To avoid any aliasing artifacts:

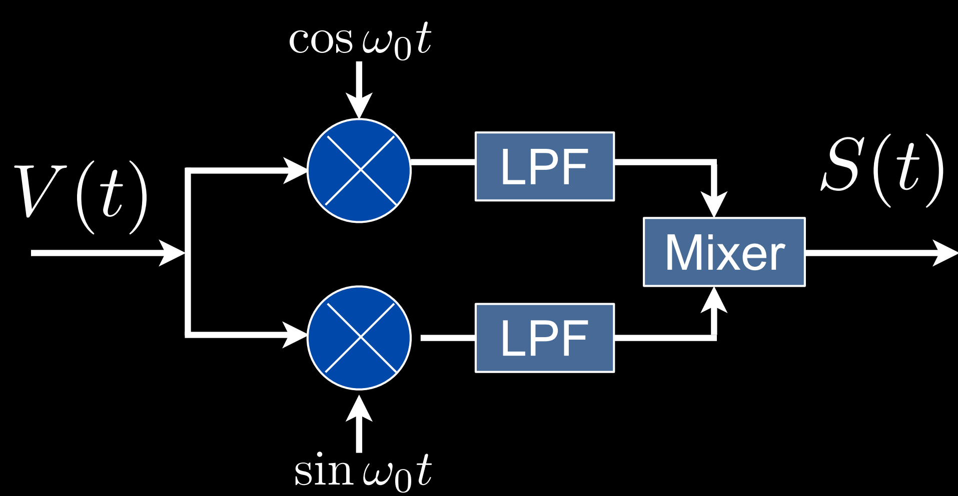
In phase encoding,
- Reduce Δk_y

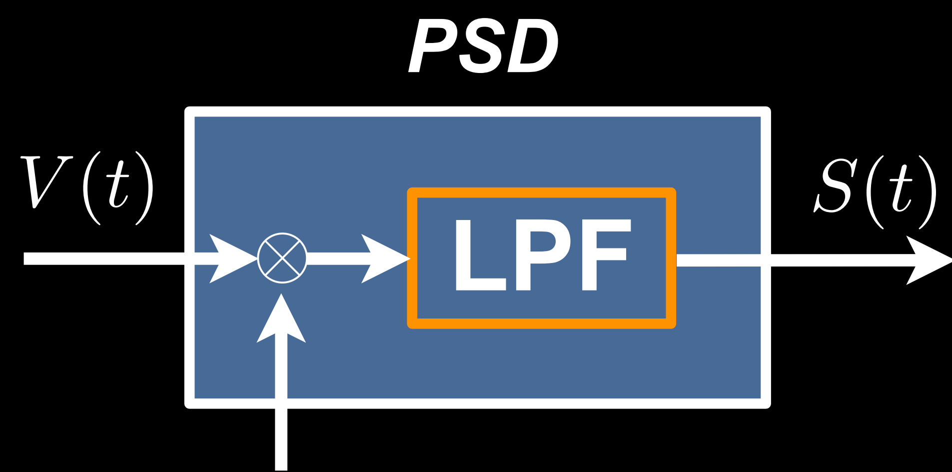
Either lose spatial resolution
or
increase scan time

Field of View

To avoid any aliasing artifacts:

In frequency encoding,
- Reduce Δk_x
- Utilize LPF (low pass filter)

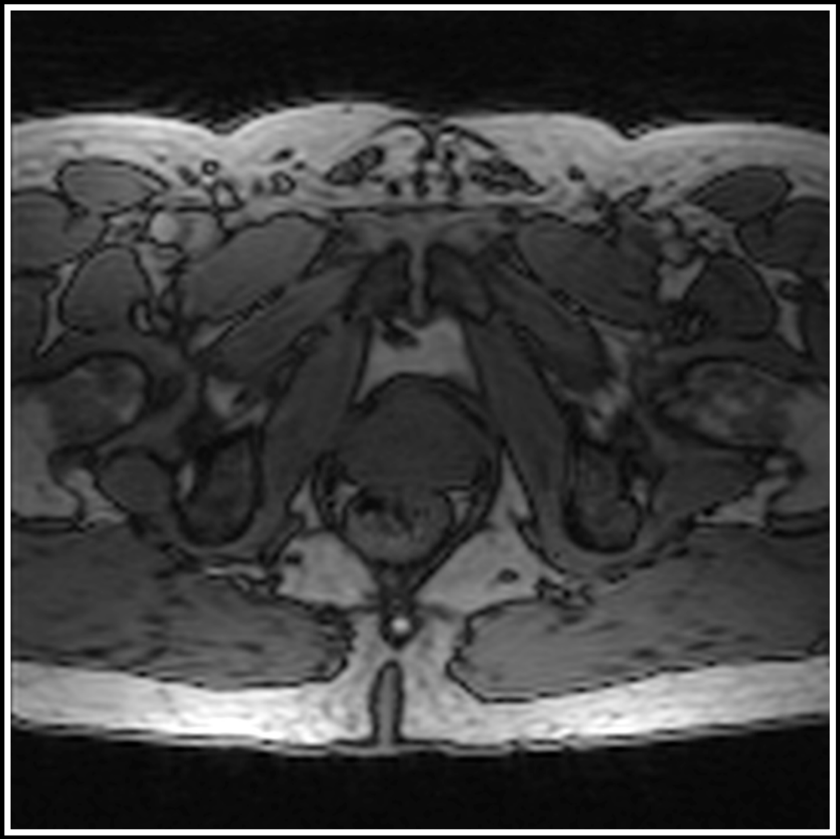




Typically, put long axis of object in readout direction

Field of View

Prostate Imaging Example



Which direction will be readout direction?

Questions?

- Related reading materials
  - Nishimura - Chap 5

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