* Faraday's law of Induction

electromotive force ($\mathcal{E}$)

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t}$$

$$d\mathcal{E} = \frac{-dM(\vec{r},t)}{dt} \cdot dV \qquad (\text{see Eq. } 5.38)$$

$$\int_V d\mathcal{E}\, dV = S_r(t) = -k \int_V \frac{d}{dt} M(\vec{r},t)\, dV$$

$$= -k \int_V M(\vec{r},0) \left[-i\left(\omega_0 + \gamma \vec{G}(t)\cdot\vec{r}\right)\right] \cdot e^{-i\omega_0 t} e^{-i\gamma\int_0^t \vec{G}(\tau)\vec{r}\,d\tau}$$

(ignore T_2 decay, $\frac{d}{dt} e^{at} = a e^{at}$)

in general, $\omega_0 \gg \gamma \vec{G}\cdot\vec{r}$

$$S_r(t) = k\, i\, \omega_0 \int_V M(\vec{r},0)\, e^{-i\omega_0 t} e^{-i\gamma\int_0^t \vec{G}(\tau)\vec{r}\,d\tau}$$

$\boxed{p. 71}$

$$S_r(t) = i\,\omega_0 K \int_V M(\vec{r},0)\, e^{-i\omega_0 t}\, e^{-i\gamma\int_0^t \vec{G}(\vec{r})\vec{r}\, d\tau}\, dV$$

$$\overset{n}{\cong} i\,\omega_0 K \iint_{xy} M_{xy}(\vec{r})\, e^{-i\omega_0 t}\, e^{-i\gamma\int_0^t \vec{G}(\vec{r})\vec{r}\, d\tau}\, dx\,dy$$

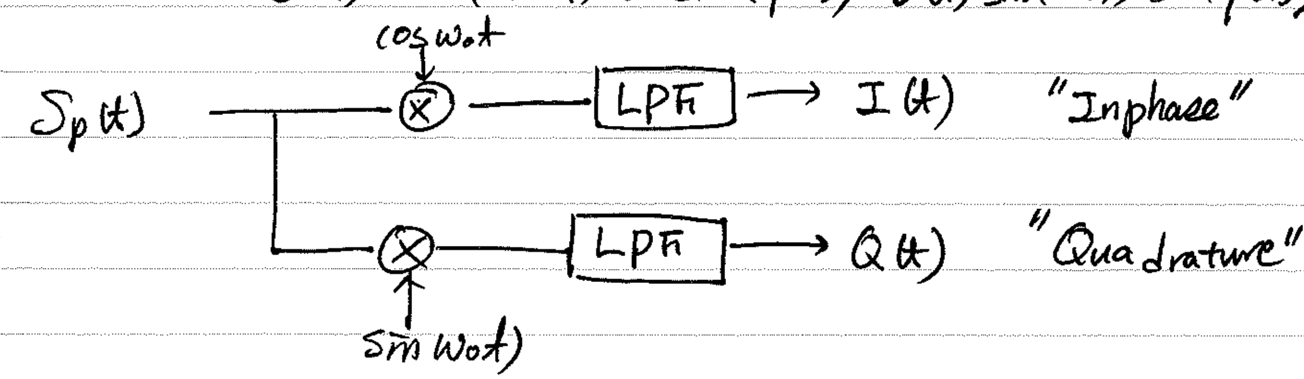
$$S_r(t) = S(t)\, e^{-i\omega_0 t} = a(t)\, e^{-i[\omega_0 t + \phi(t)]}$$

$$S(t) = S_r(t)\, e^{i\omega_0 t} = a(t)\, e^{-i\phi(t)}$$

* Single receive coil ; sensitive to the rate of change of M only along one axis

$$S_p(t) = \mathrm{Re}\{S_r(t)\} = a(t)\cos(\omega_0 t + \phi(t))$$

$$= a(t)\cos(\omega_0 t)\cos t(\phi(t)) - a(t)\sin(\omega_0 t)\sin(\phi(t))$$



$$I(t) = \Big(a(t)\cos(\omega_0 t + \phi(t)) \cdot \cos\omega_0 t\Big) * \boxed{LPF}$$

$$\Longrightarrow a(t)\cos\phi(t)$$

$$Q(t) = -a(t)\sin\phi(t)$$

$$S(t) = I(t) + i\,Q(t) = a(t)e^{-i\phi(t)}$$

3 simplifications

1) 2D imaging $\quad \Delta z$

def. $m(x,y) = \int_{z-\frac{\Delta z}{2}}^{z+\frac{\Delta z}{2}} m(x,y,z)$

2) ignore T_2 decay

3) demodulate by ω_0

Def. $S(t) = S_r(t) \cdot e^{i\omega_0 t}$

"baseband"

Signal Equation

$$S(t) = \iint_{x\,y} \underbrace{m(x,y)}_{\text{desire}} e^{-i\gamma \int_0^t \bar{G}(\tau)\cdot r\, d\tau} \, dx\, dy$$

$$= \iint_{x\,y} m(x,y)\, e^{-i\gamma\left[\left(\int_0^t G_x(\tau)\,d\tau\right)\cdot x + \left(\int_0^t G_y(\tau)\,d\tau\right) y\right]} dx\,dy$$

$$= \iint_{x\,y} m(x,y)\, e^{-i2\pi\left[\underbrace{\left(\frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,d\tau\right)}_{\triangleq K_x(t)} x + \underbrace{\left(\frac{\gamma}{2\pi}\int_0^t G_y(\tau)\,d\tau\right)}_{\triangleq K_y(t)} y\right]} dx\,dy$$

$$S(t) = \iint_{x\,y} m(x,y)\, e^{-i2\pi\left(K_x(t)\cdot x + K_y(t)\, y\right)} dx\,dy$$

2D FT of $m(x,y)$

$= M(K_x(t)\ K_y(t))$

- $s(t)$ equals values of M along trajectory in FT space "K space"
- $G_x G_y$ control path in K-space
- to image $m(x,y)$ acquire set samples $\{s(t)\}$ to cover K-space "sufficiently"