Pulse Sequences: Rapid Gradient Echo

M229 Advanced Topics in MRI

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2025.04.03

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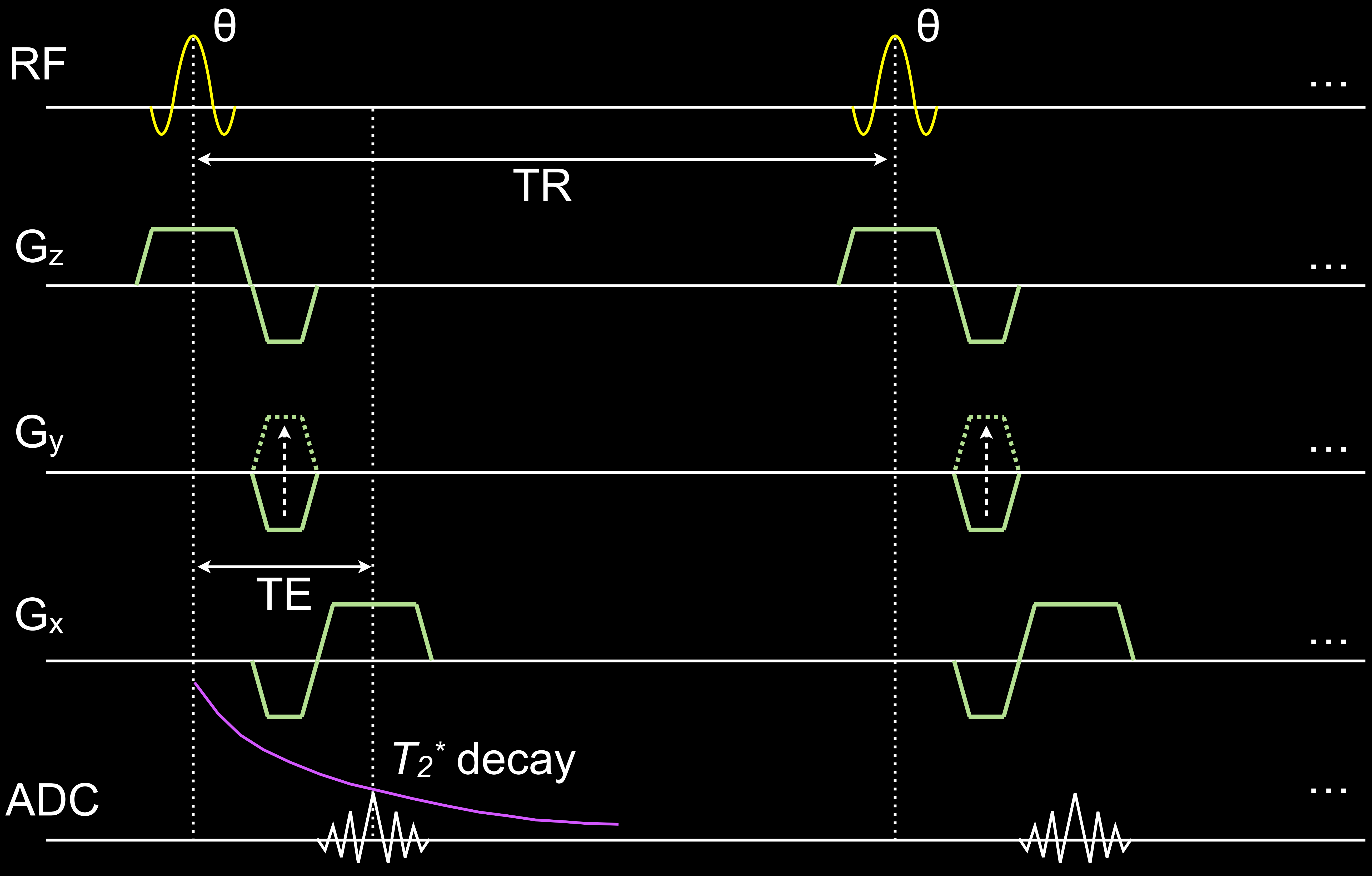
Class Business

- Office hours
  - Holden: by appointment
  - Wenqi (TA for HW1): TBD
  - Timo (TA for HW2): TBD
  - Email beforehand would be helpful
- Follow Brian's Bloch sim tutorial
  - *Archived version will be shared via email.*
- Homework 1 is due on 4/21 (Monday)

Outline

- Gradient Echo (GRE)
- Rapid Gradient Echo
  - Balanced SSFP
  - Gradient-spoiled GRE
  - RF-spoiled GRE
- Comparison
- Extensions and Variations
- Applications

Gradient Echo



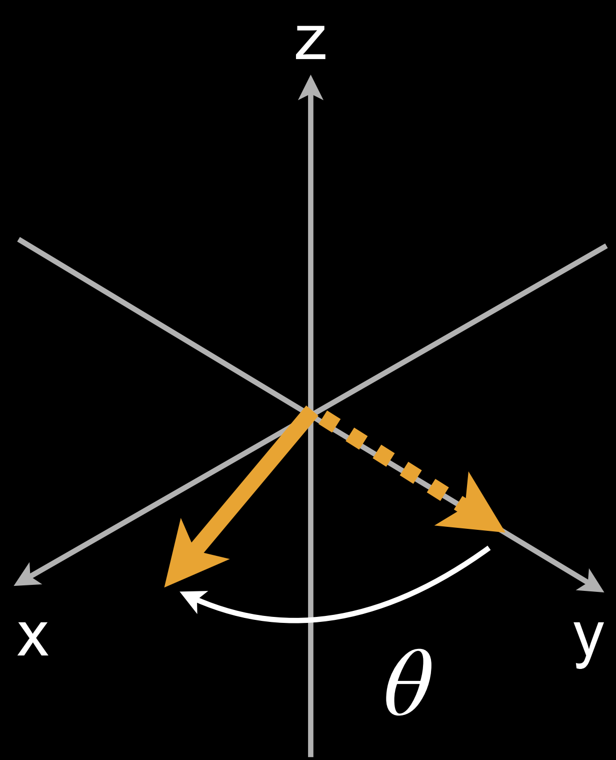
Gradient Echo

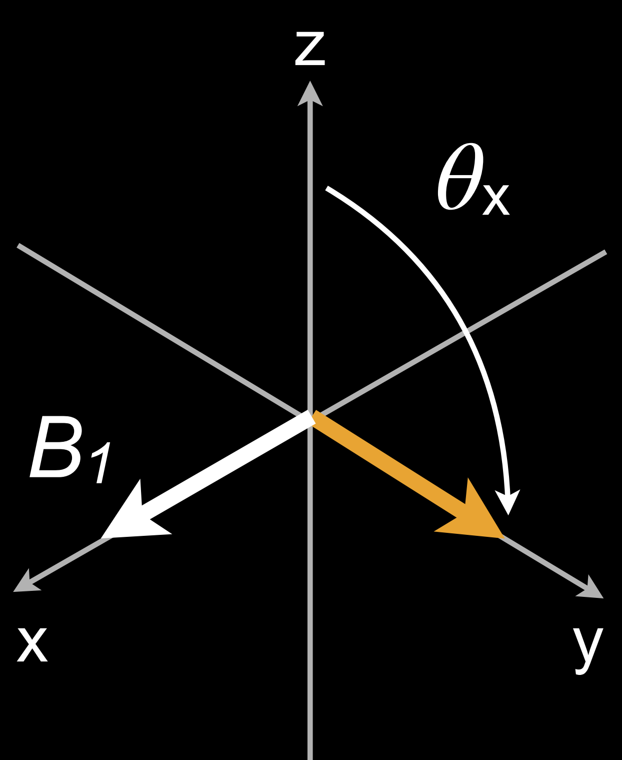
- Gradient reversal on the readout axis forms the echo (vs. RF spin echo)
- A.k.a. gradient-recalled echo, gradient-refocused echo, field echo
- Flip angle θ typically < 90^o
- M_{xy} has T_2* instead of T_2 decay
- Advantageous for fast 3D imaging

Gradient Echo

- Basic steps
  - RF excitation (flip angle θ and phase ϕ)
  - Free precession (from G and ΔB)
  - T_1 and T_2 (or T_2*) relaxation
- Steady state
  - "Dynamic equilibrium"
  - Established after initial transient state
  - M_z and M_{xy} remain the same, TR to TR
  - Need to meet certain conditions

Notations





rotating frame at ω_0

Free Precession (FP)

$$R_z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Excitation

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

Notations

Relaxation + Free Precession:

$$M(t) = \begin{bmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T_1} \end{bmatrix} R_z(\Delta\omega t) M(0) + \begin{bmatrix} 0 \\ 0 \\ M_0(1 - e^{-t/T_1}) \end{bmatrix}$$

$$= \mathrm{A} M(0) + \mathrm{B}$$

Gradient Echo

- When TR>5·T_2*, M_{xy} naturally "spoiled"
  - *i.e.*, consider M_{xy}=0 at the end of each TR

Gradient Echo

Steady-state signal equation:

$$M_{xy,ss}(\mathrm{TE}) = \frac{M_0 \sin\theta(1 - E_1)}{1 - \cos\theta E_1} e^{-\mathrm{TE}/T_2^*}$$

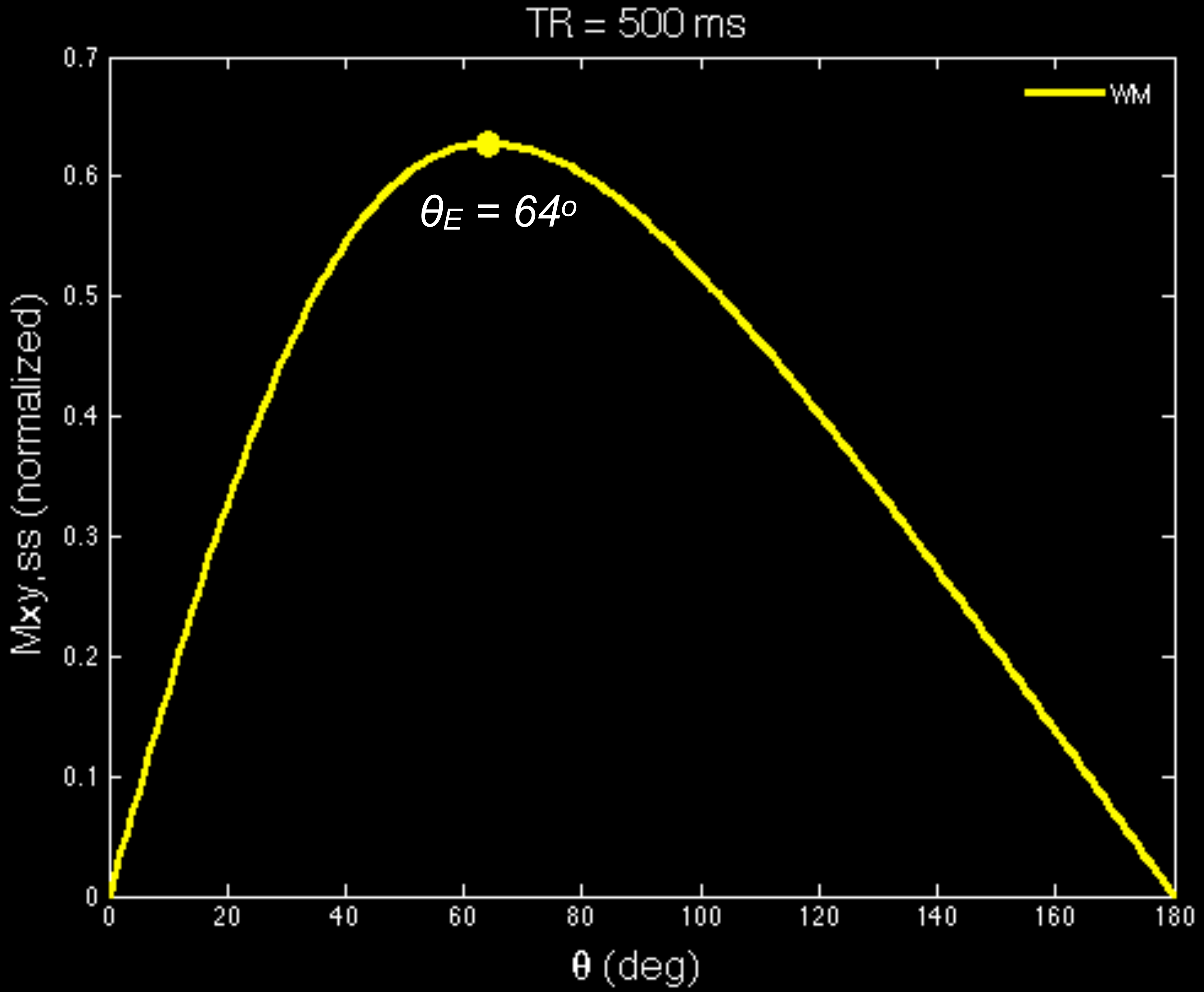
Ernst angle:

$$\theta_E = \cos^{-1}(E_1)$$

$$E_1 = e^{-\mathrm{TR}/T_1}$$

Gradient Echo

Ernst angle:



WM T_1 = 600 ms, T_2 = 80 ms

Gradient Echo

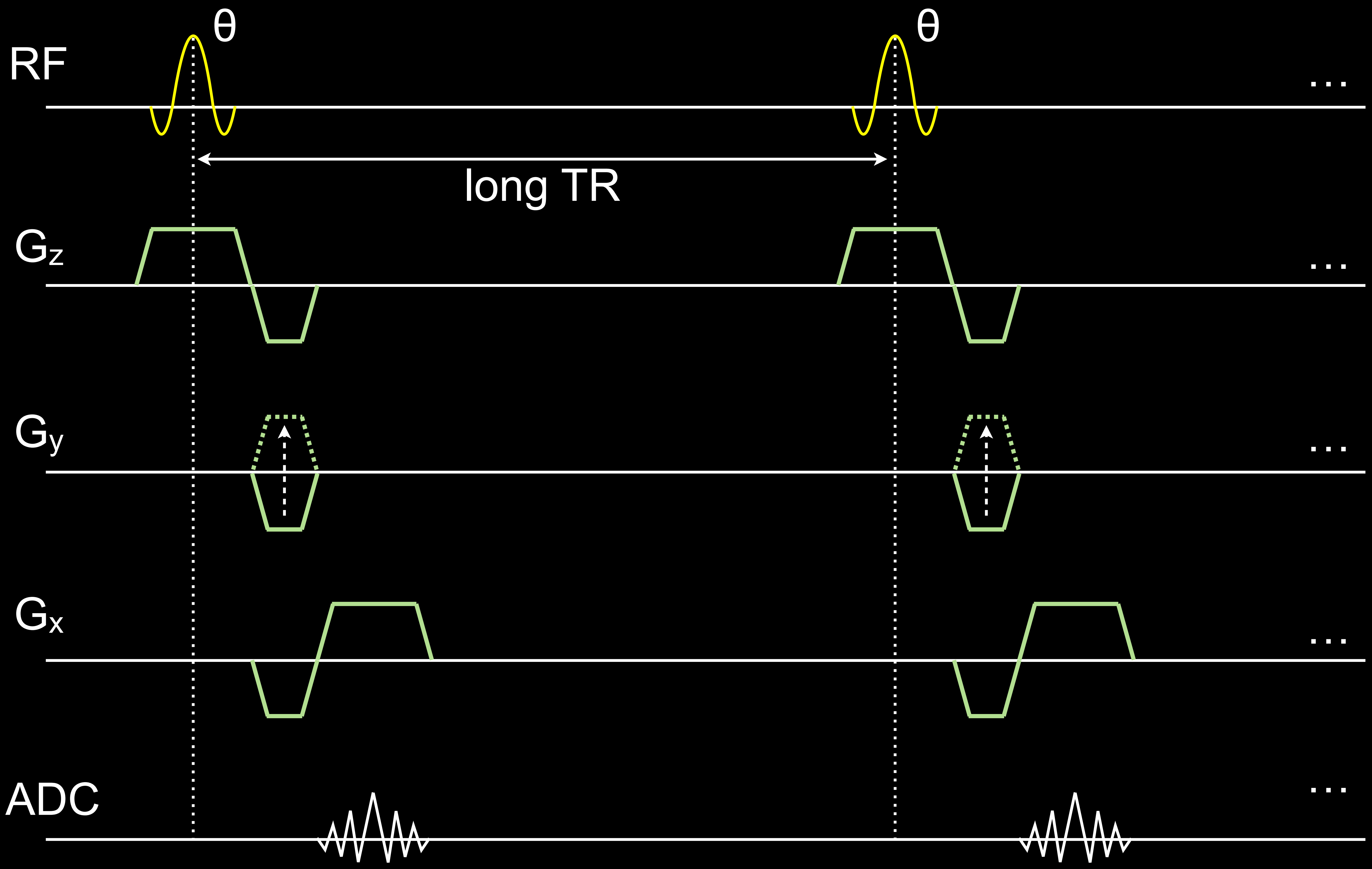
- T_1-weighted image contrast
  - M_{xy} gone at end of each TR
  - TE controls T_2* weighting
- Typical T_2* ~ 50 ms
  - need TR ~ 250 ms for "natural" spoiling
- Reduce TR and maintain T1w contrast?
  - rapid GRE with appropriate spoiling

Rapid Gradient Echo

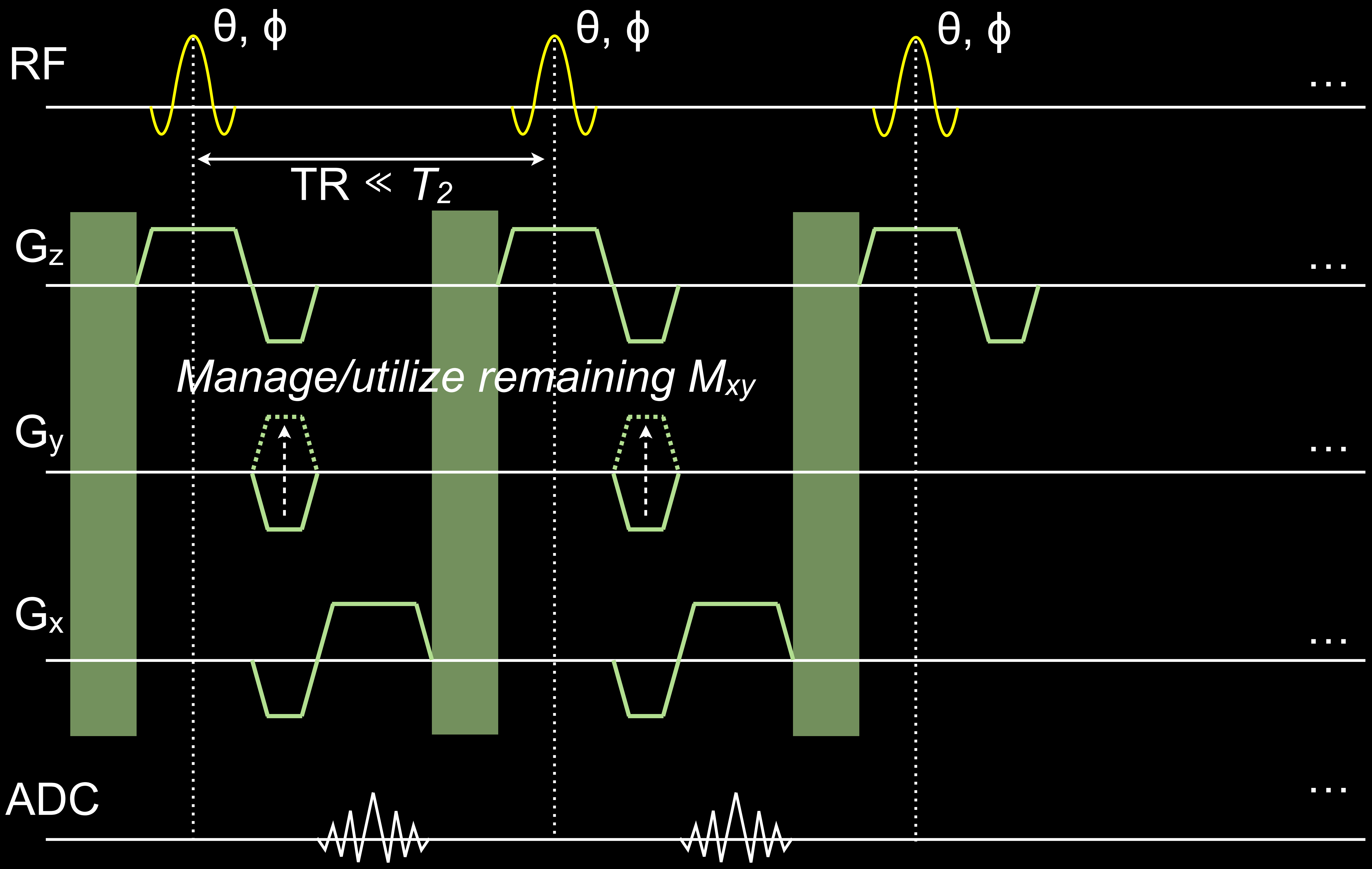
- Rapid imaging with TR $\ll T_2 < T_1$
- Steady state
  - Involves a mixture of M_z and M_{xy}
  - Necessary and sufficient conditions:
    1. Constant RF flip angle θ
    2. Constant TR
    3. Constant dephasing β between RF pulses
    4. RF phase $\phi_n = a + bn + cn^2$

Scheffler K, Con Magn Reson 1999; 11:291-304

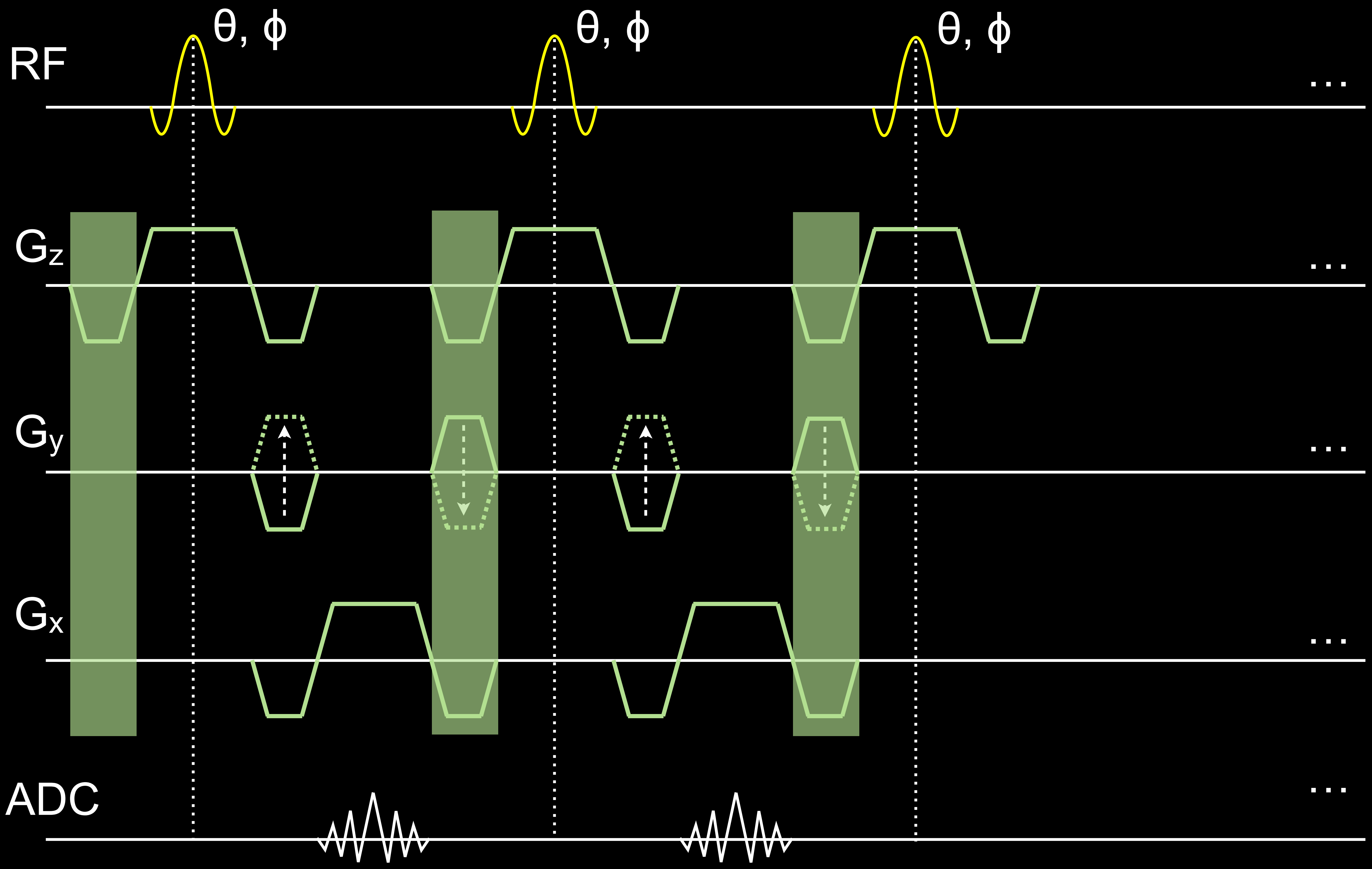
Gradient Echo



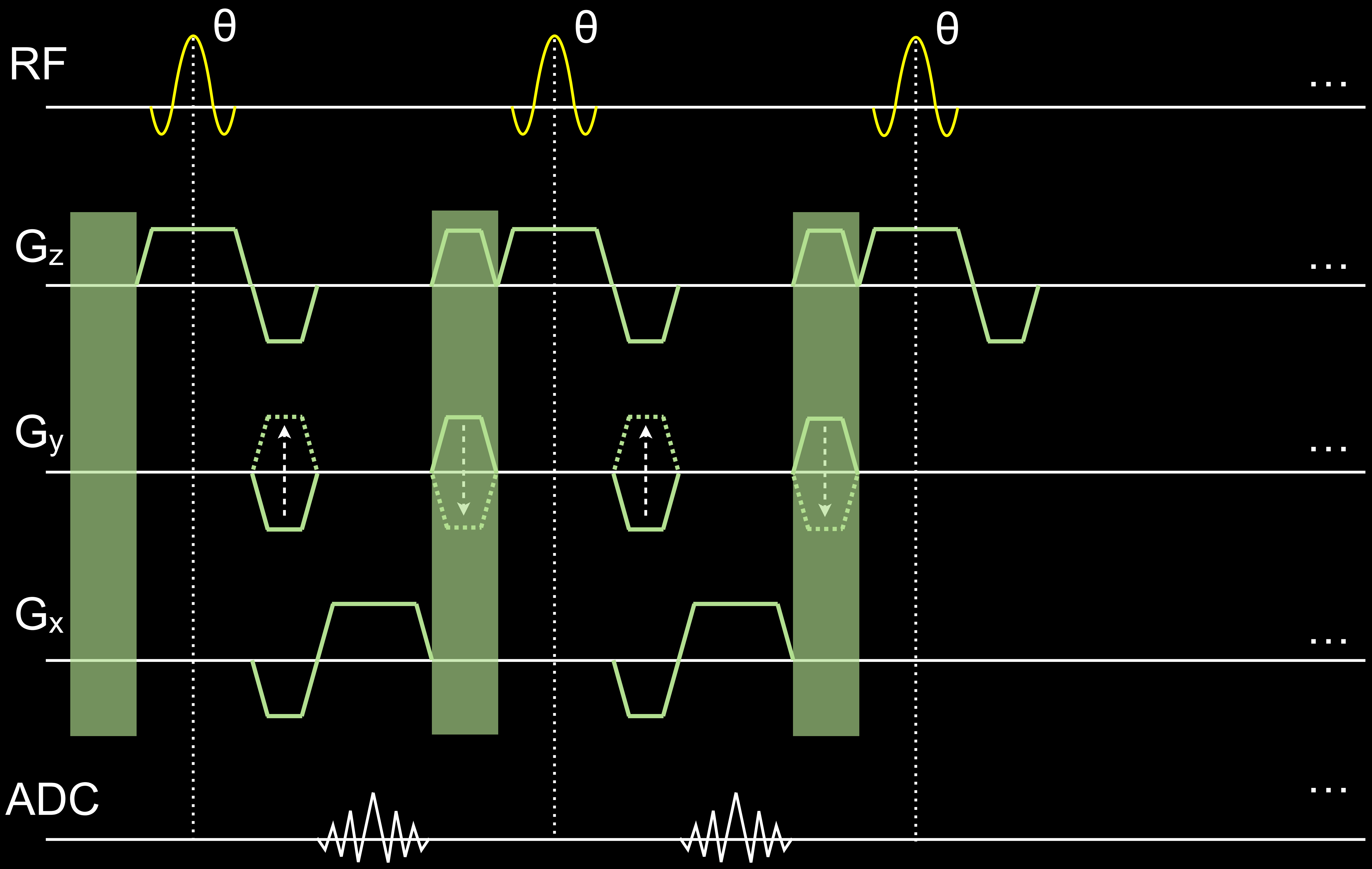
Rapid Gradient Echo



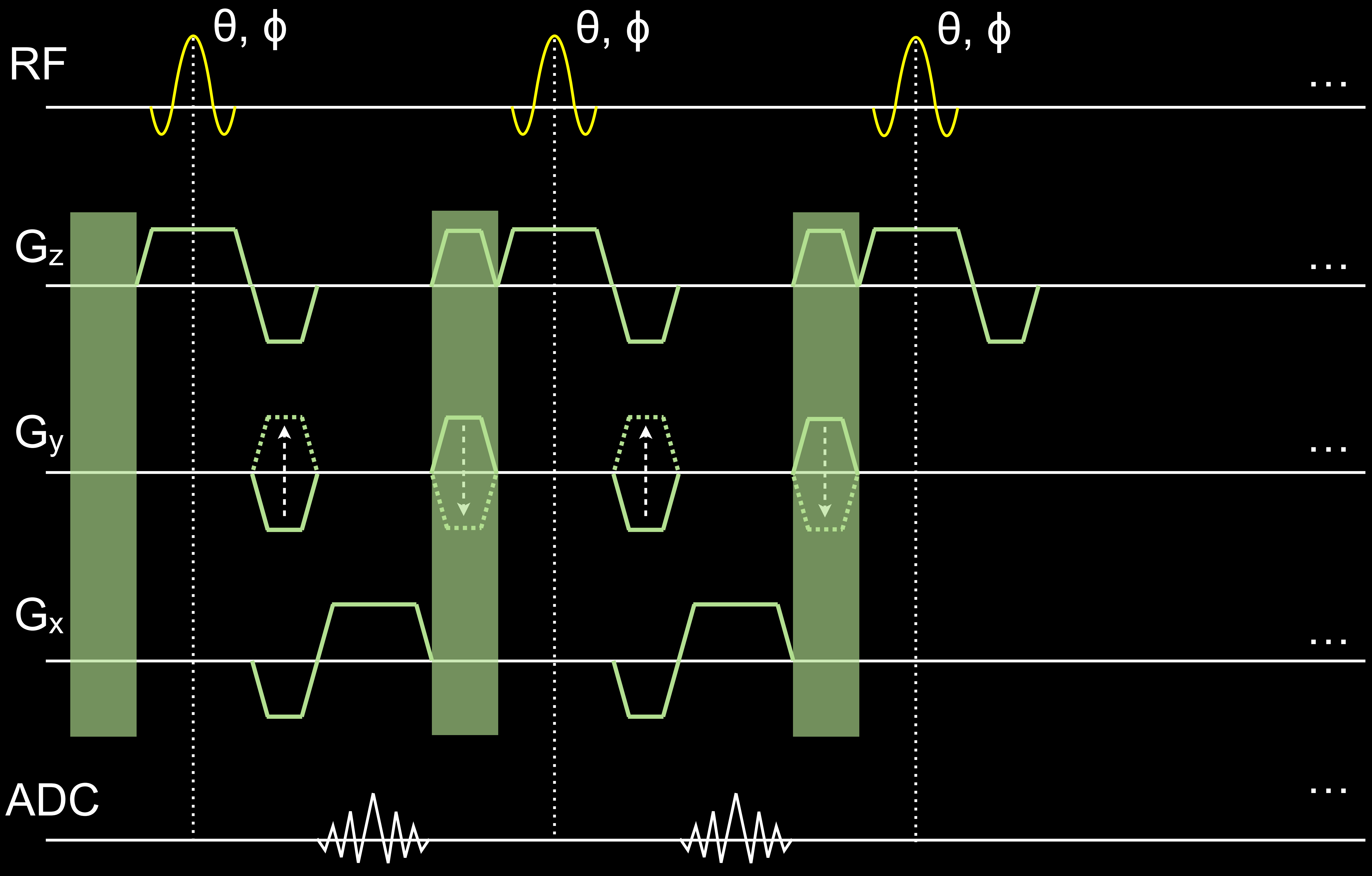
Balanced SSFP



Gradient-spoiled GRE



Gradient & RF-spoiled GRE



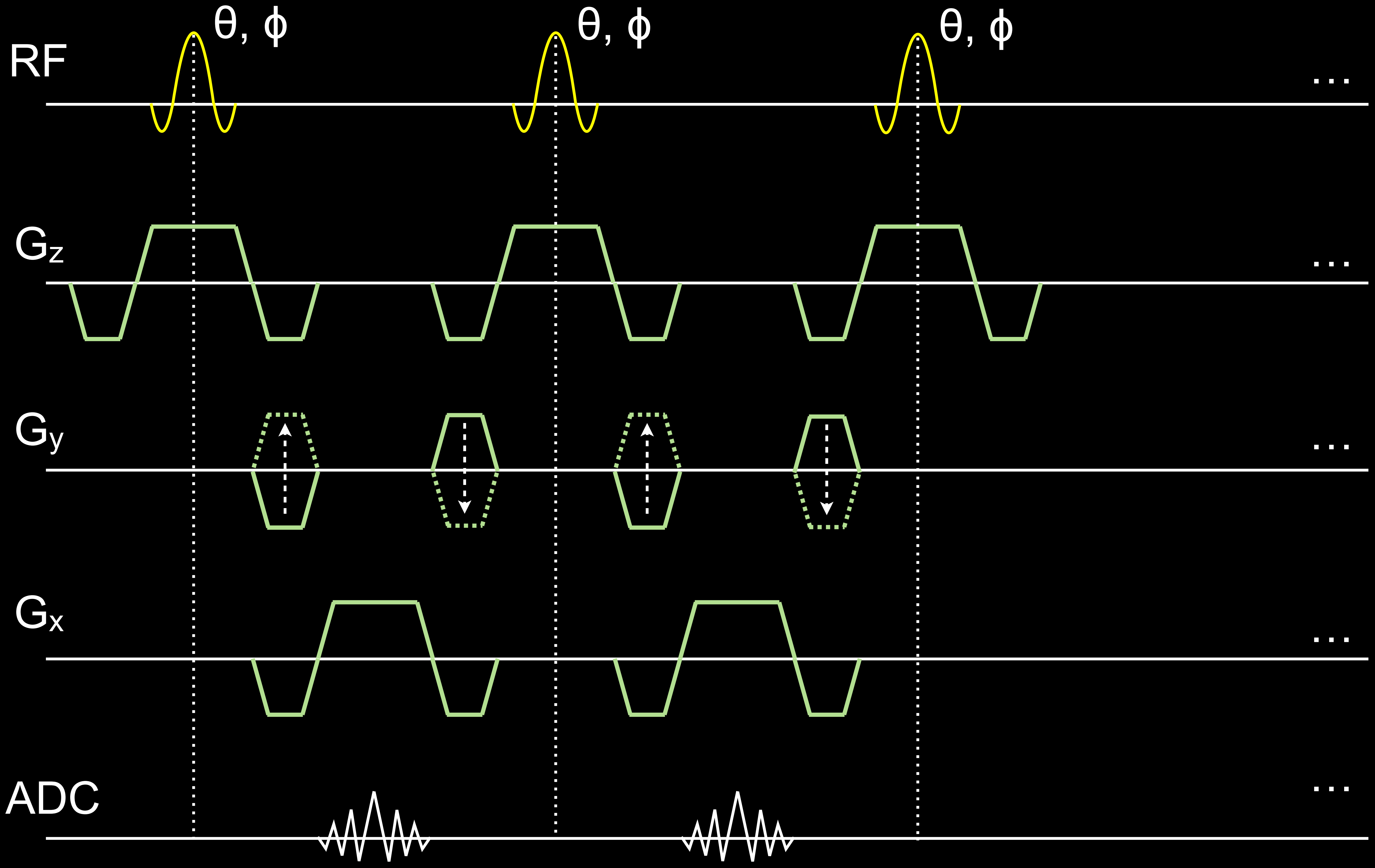
Rapid Gradient Echo

General terminology		Siemens	GE	Philips
Balanced SSFP	bSSFP	TrueFISP	FIESTA	Balanced FFE
Gradient-spoiled GRE	SSFP-FID	FISP	GRASS	FFE
	SSFP-Echo	PSIF	SSFP	T2-FFE
Gradient and RF-spoiled GRE	Spoiled GRE	FLASH	SPGR	T1-FFE

cf. Table 14.1, Handbook of MRI Pulse Sequences

cf. "MRI Acronyms", Siemens Healthcare

Balanced SSFP



Balanced SSFP

- All gradients are balanced
  - Accrued phase β from G_x, G_y, G_z = 0
  - Accrued phase β only comes from ΔB
- Typically use $\phi_n = n \cdot \pi$ ($\Delta\phi = \pi$)
- Typically use TE = TR/2
  - M_{xy} actually has T_2 (not T_2*) decay[1]
- Contrast depends on T_1 and T_2

[1] *Ganter C, MRM 2006; 56:687-691*

Balanced SSFP

Steady-state signal equation (β = 0):

$$M_{xy,ss}(\mathrm{TE}) = M_0 \sin\theta \frac{1 - E_1}{1 - (E_1 - E_2)\cos\theta - E_1 E_2} \sqrt{E_2}$$

$$E_1 = e^{-\mathrm{TR}/T_1}$$

$$E_2 = e^{-\mathrm{TR}/T_2}$$

$$\sqrt{E_2} = e^{-\mathrm{TE}/T_2}$$

Balanced SSFP

Steady-state signal equation (β = 0):

If TR (3-5 ms) $\ll T_2$, $E_1 \sim 1\text{-TR}/T_1$ and $E_2 \sim 1\text{-TR}/T_2$:

$$M_{xy,ss}(\mathrm{TE}) = \frac{M_0 \sin\theta}{(T_1/T_2)(1-\cos\theta) + (1+\cos\theta)}\sqrt{E_2}$$

T_2/T_1 contrast weighting

$$\theta_{max} = \arccos\left(\frac{T_1 - T_2}{T_1 + T_2}\right) \qquad M_{xy,ss}(\theta_{max}) \sim \frac{M_0}{2}\sqrt{\frac{T_2}{T_1}}$$

When $T_1 = T_2$, $\theta_{max} = 90^o$, $M_{xy,ss} \sim 0.5\, M_0$!

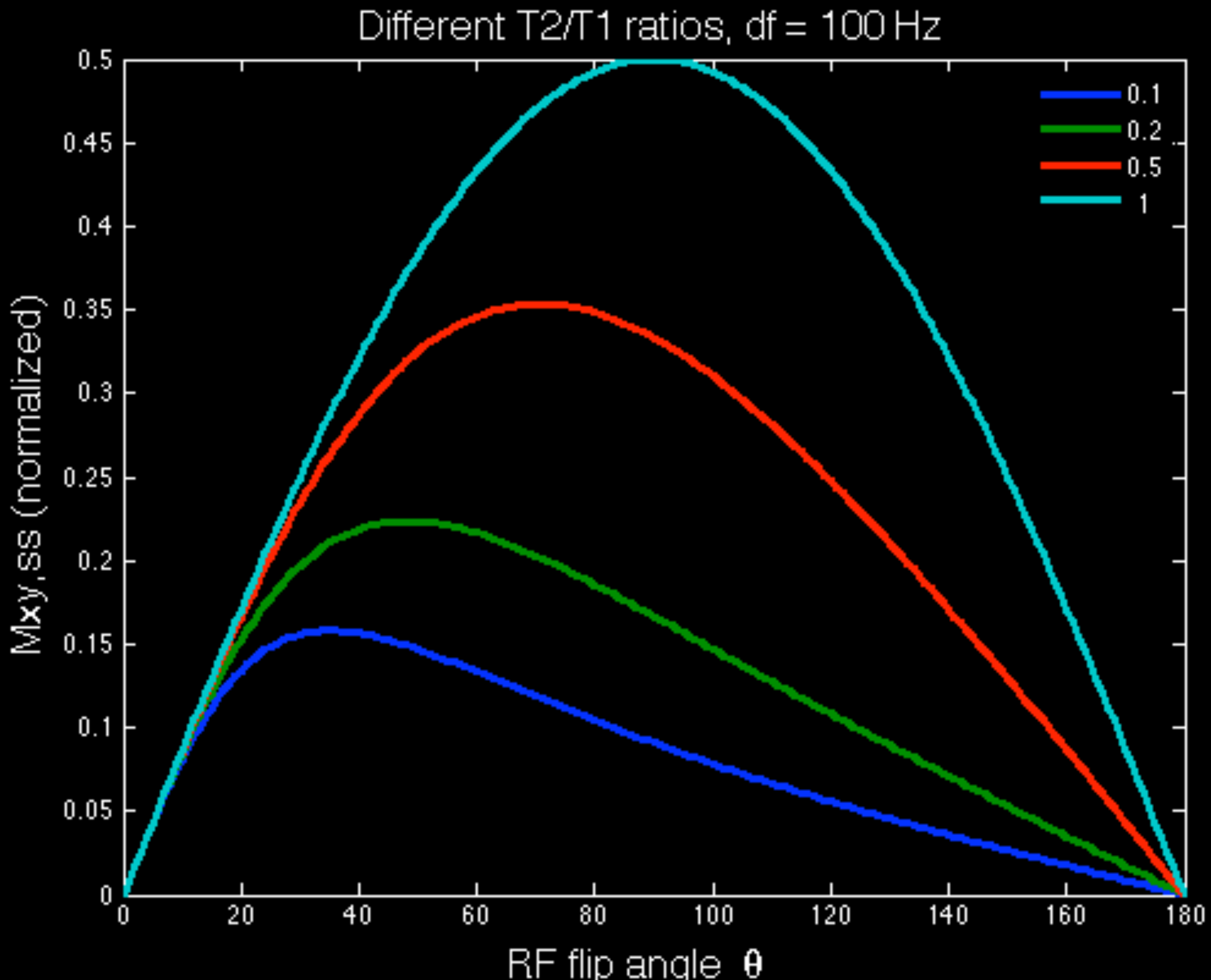
Balanced SSFP

SS signal as a function of flip angle:

TR = 5 ms

$\Delta\phi = 0$

$\beta = \pi$



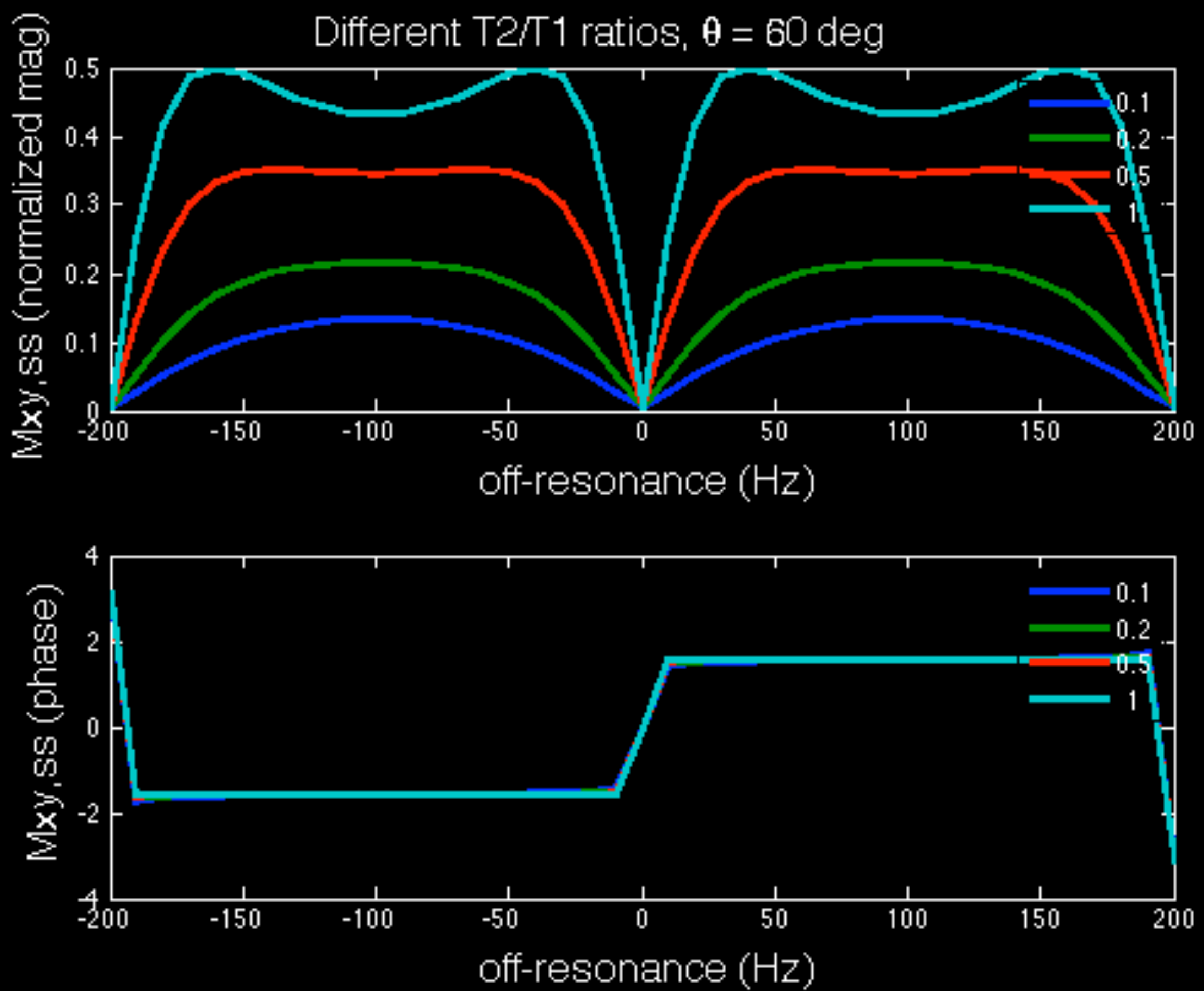
T_1 = 1000 ms, T_2 = 100,200,500,1000 ms

Balanced SSFP

SS signal as a function of off-resonance:

TR = 5 ms

$\Delta\phi = 0$



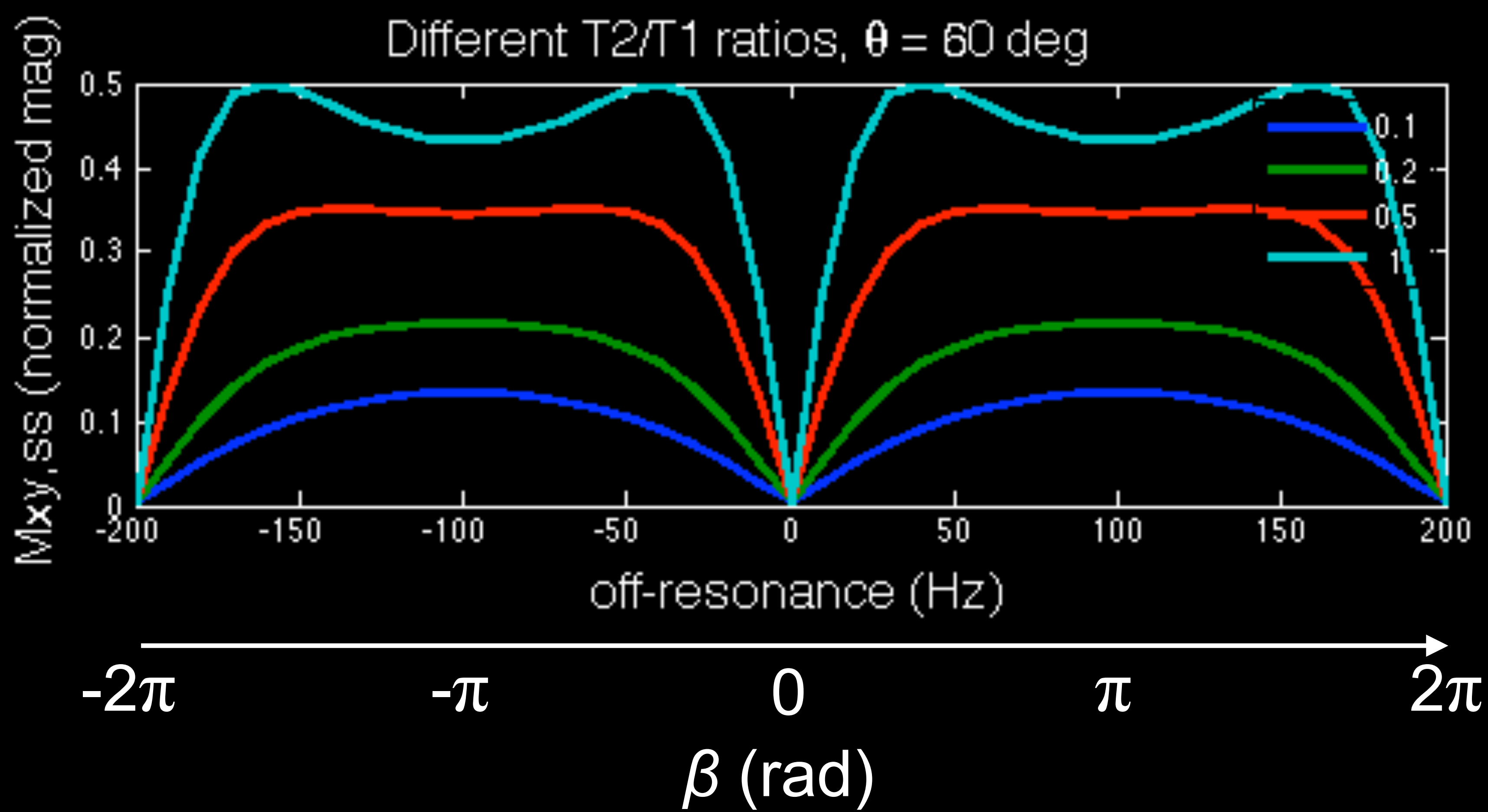
T_1 = 1000 ms, T_2 = 100,200,500,1000 ms

Balanced SSFP

SS signal as a function of off-resonance:

TR = 5 ms

$\Delta\phi = 0$



Recall $\beta = 2\pi\,\Delta f \times \mathrm{TR}$ and $\Delta f = \gamma B / 2\pi$

$\beta = \pm\pi$ corresponds to $\Delta f = \pm 1/(2\ \mathrm{TR})$ Hz

TR = 5 ms: $\Delta f = \pm 100$ Hz

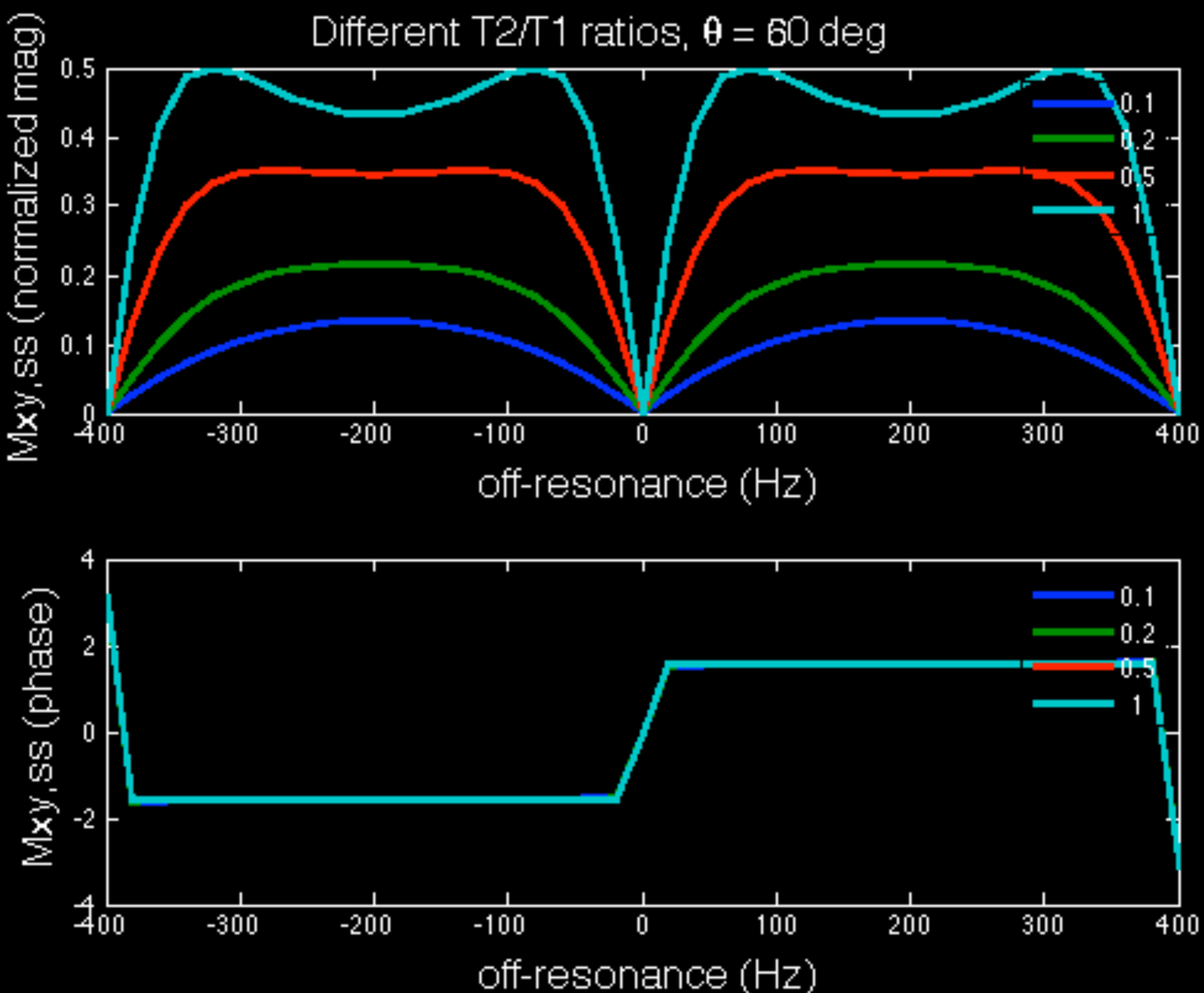
TR = 2.5 ms: $\Delta f = \pm 200$ Hz

Balanced SSFP

SS signal as a function of off-resonance:

TR = 2.5 ms

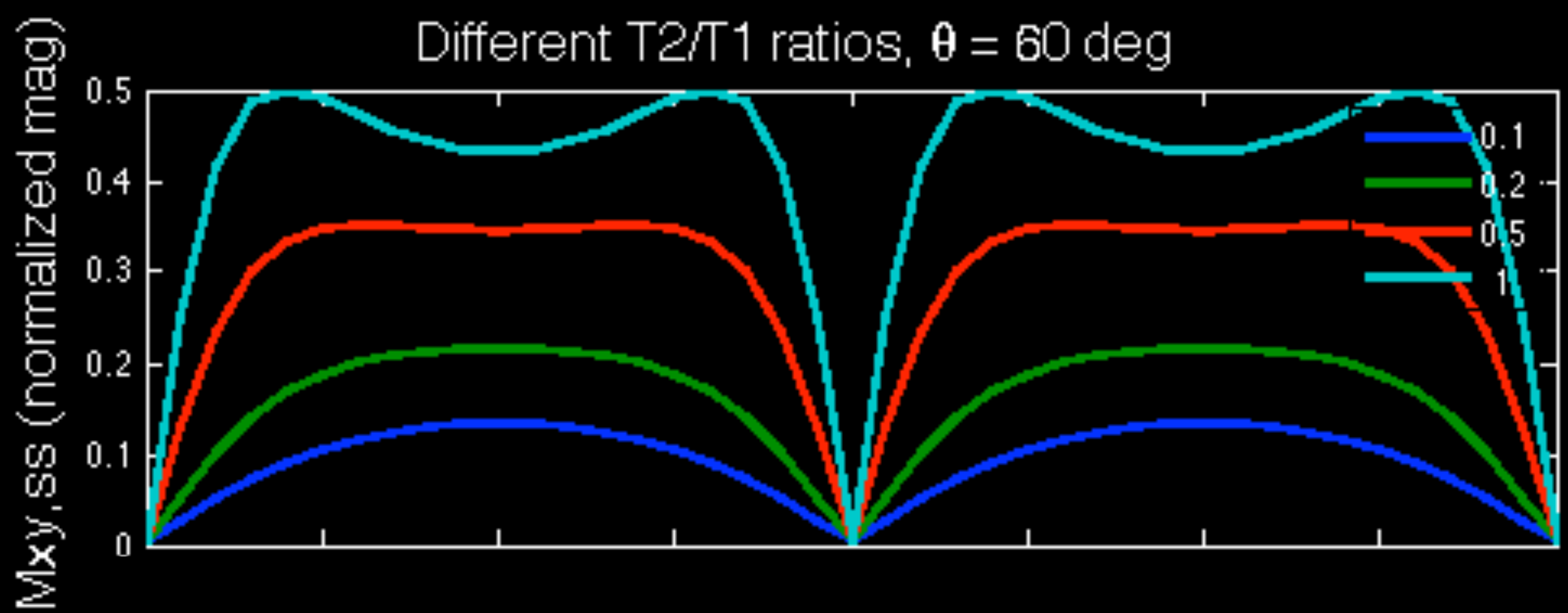
$\Delta\phi = 0$



T_1 = 1000 ms, T_2 = 100,200,500,1000 ms

Balanced SSFP

SS signal as a function of off-resonance:



$\Delta\phi = \pi$

-π 0 π 2π 3π

β (rad)

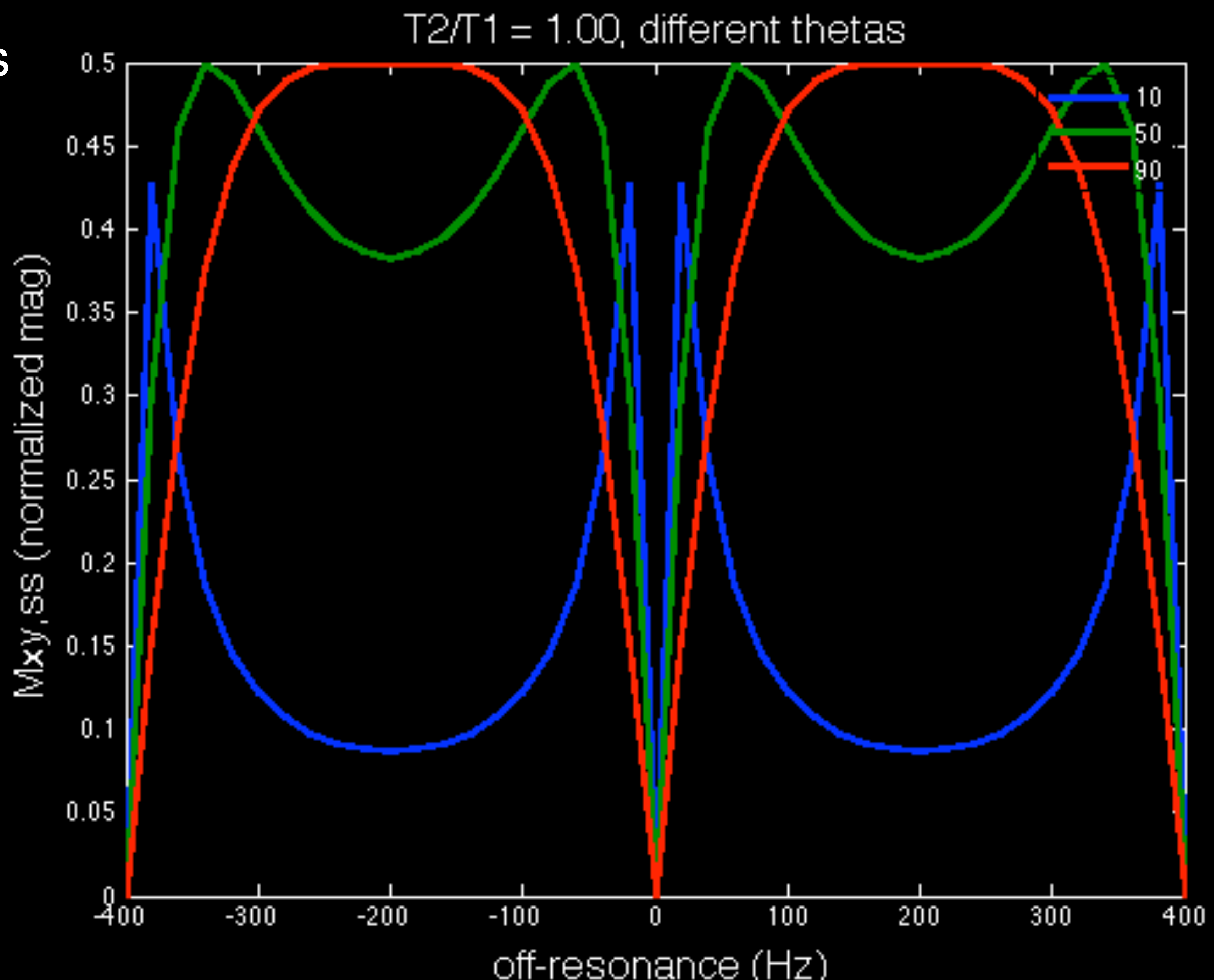
$\Delta\phi$ can shift the off-resonance response

Balanced SSFP

SS signal as a function of off-resonance:

TR = 2.5 ms

$\Delta\phi = 0$



T_1 = 1000 ms, T_2 = 1000 ms

Balanced SSFP

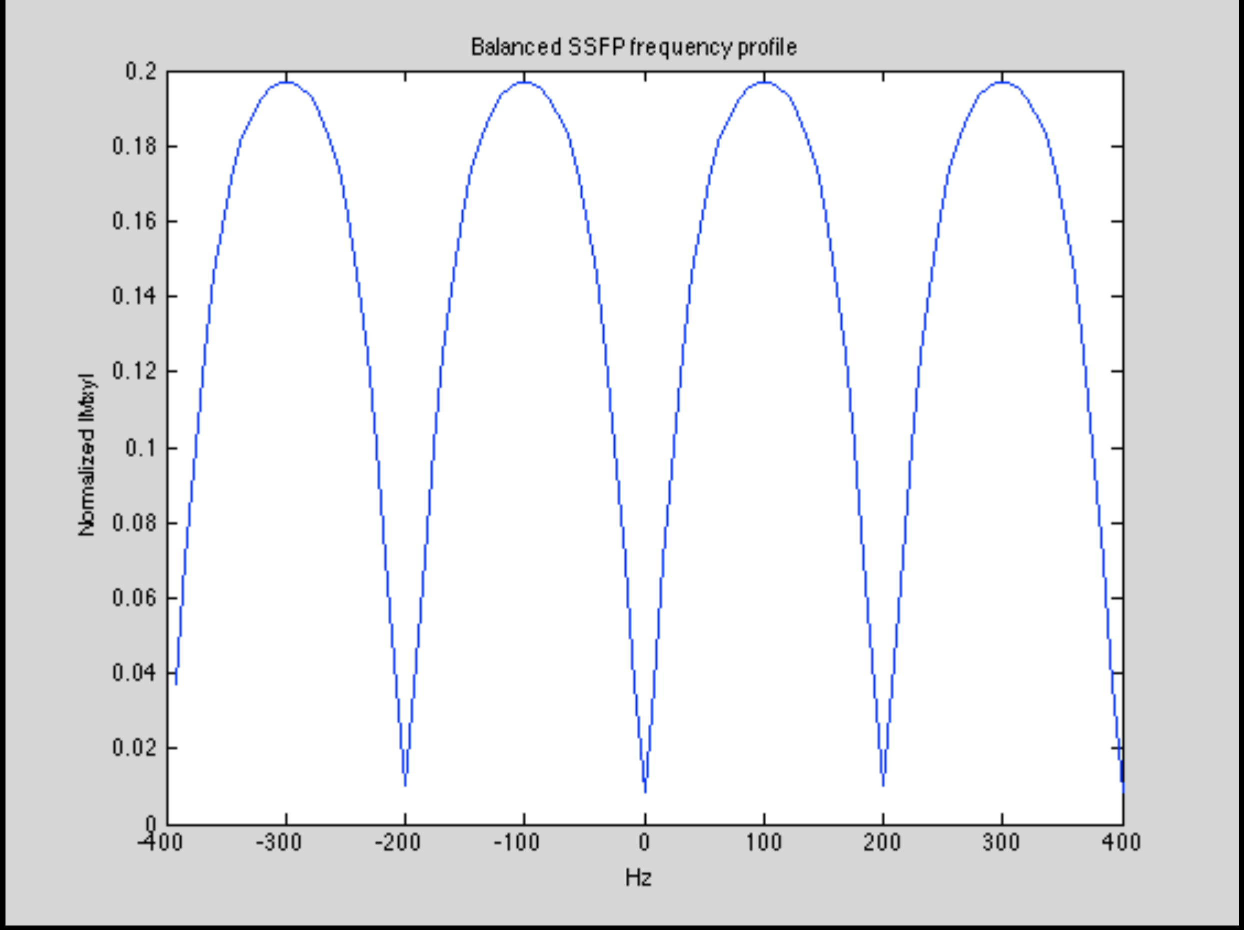
Banding artifacts at 3 T:

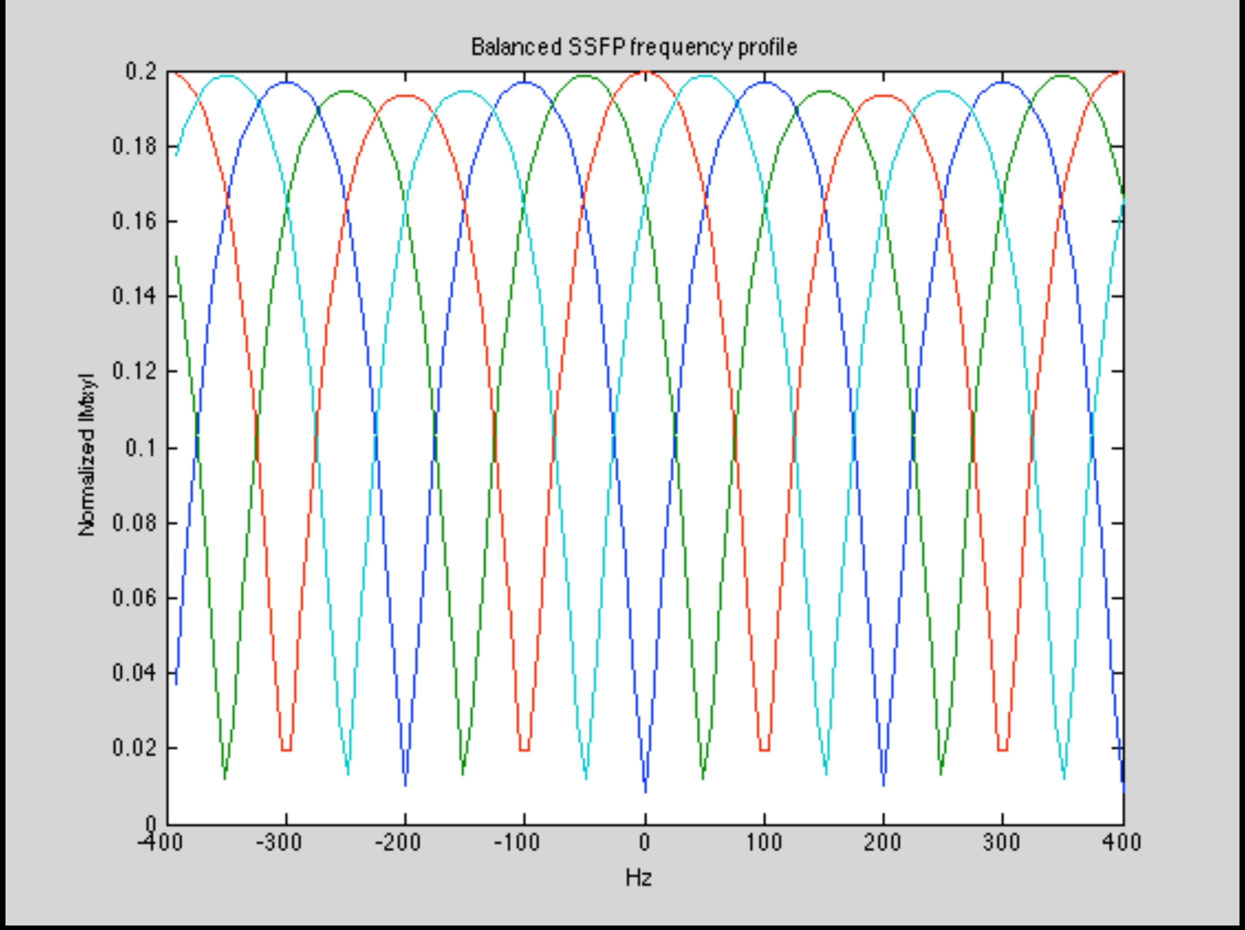
Balanced SSFP

- Banding artifacts
  - bSSFP has freq-dep null bands
  - spatially varying field inhomogeneity
  - shim not perfect
  - worse at high field (e.g., 3 T vs 1.5 T)
- Mitigating banding artifacts
  - reduce TR
  - custom shim; shift center freq
  - phase cycling

Balanced SSFP

- Removing banding artifacts
  - Multi-acquisition bSSFP (phase cycled)
  - Image reconstruction (rSoS, MIP, etc.)





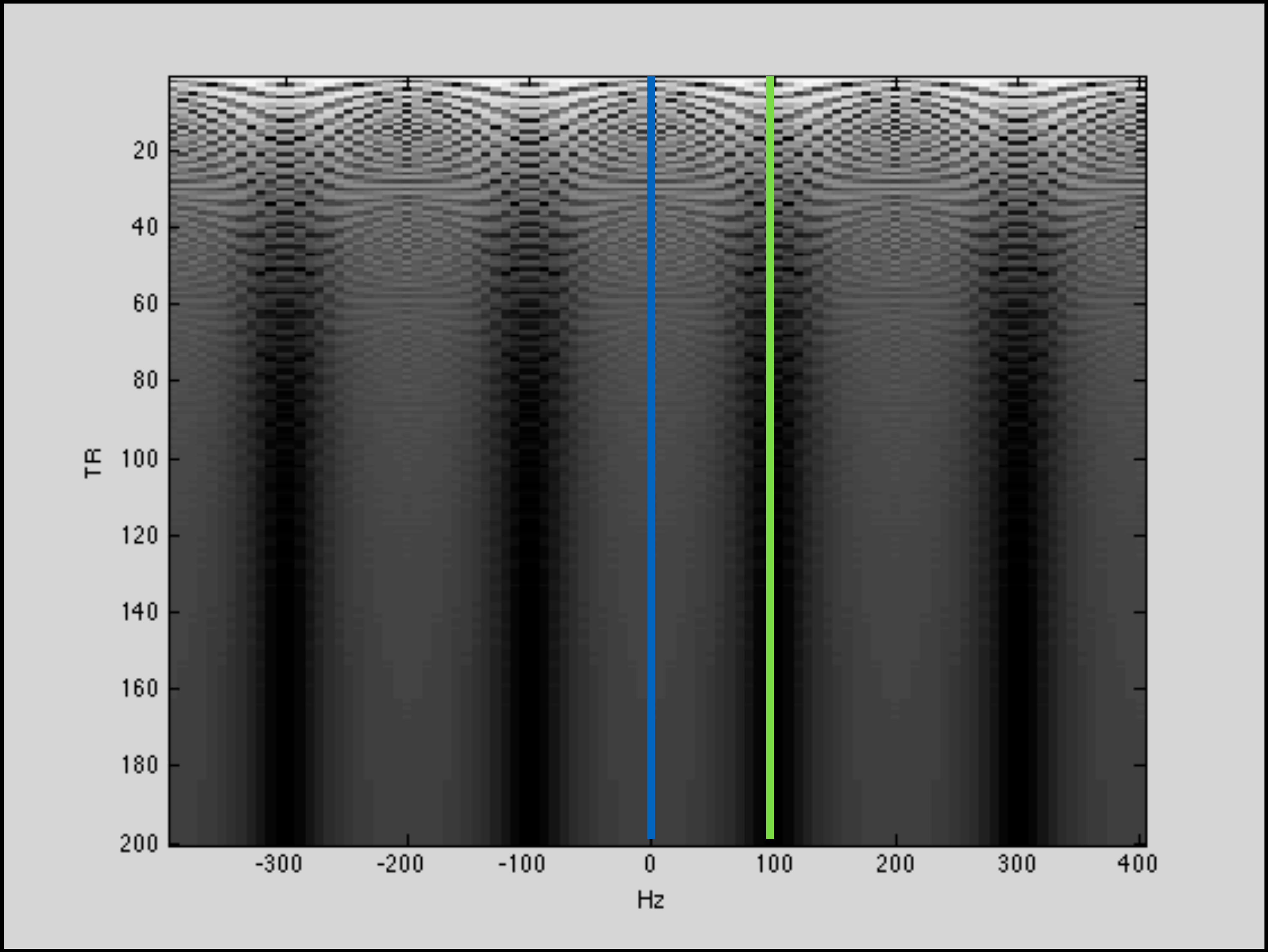
Balanced SSFP

Transition to steady state:

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^o$



T_1 = 600 ms, T_2 = 100 ms

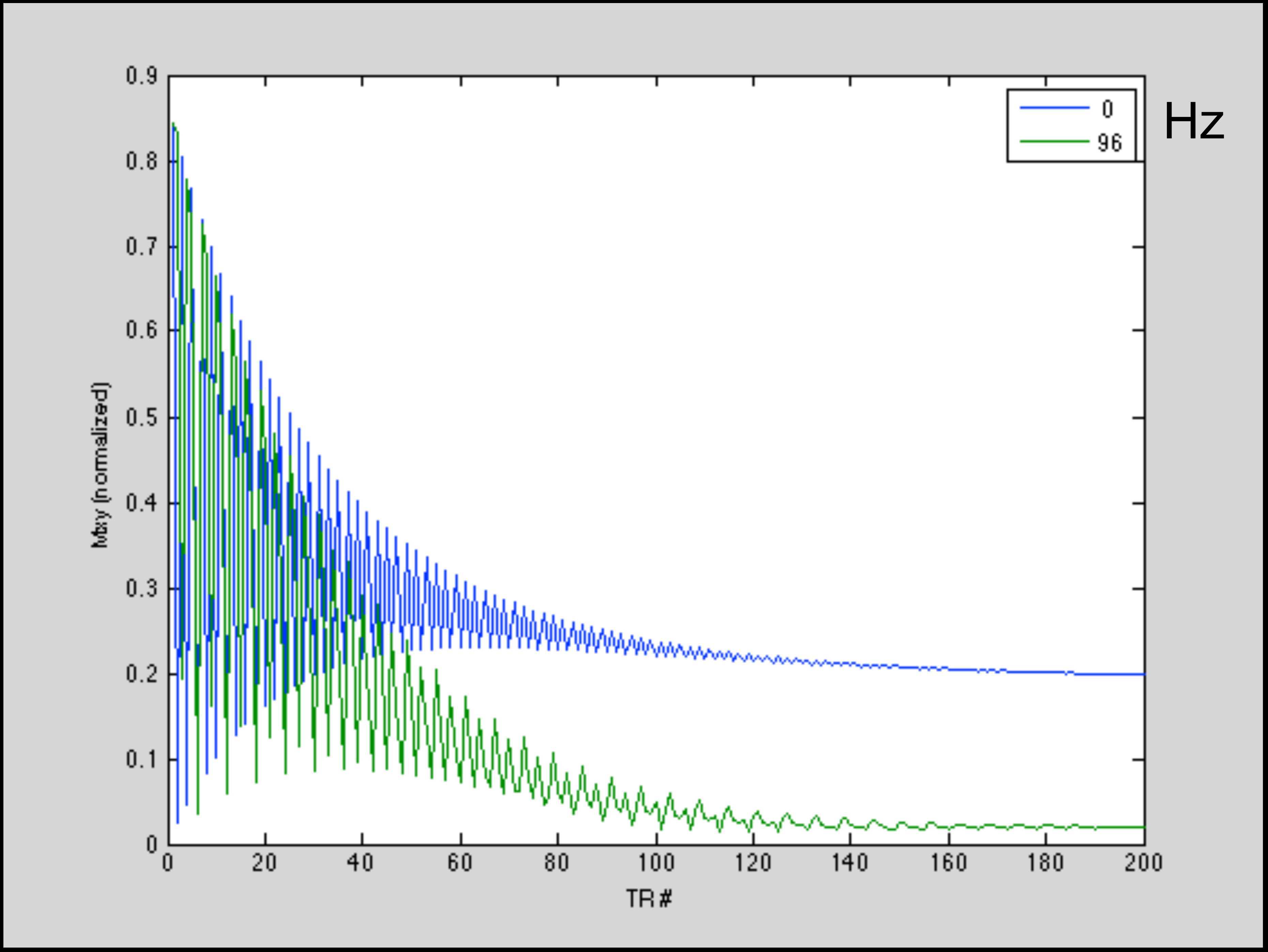
Balanced SSFP

Transition to steady state:

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^o$



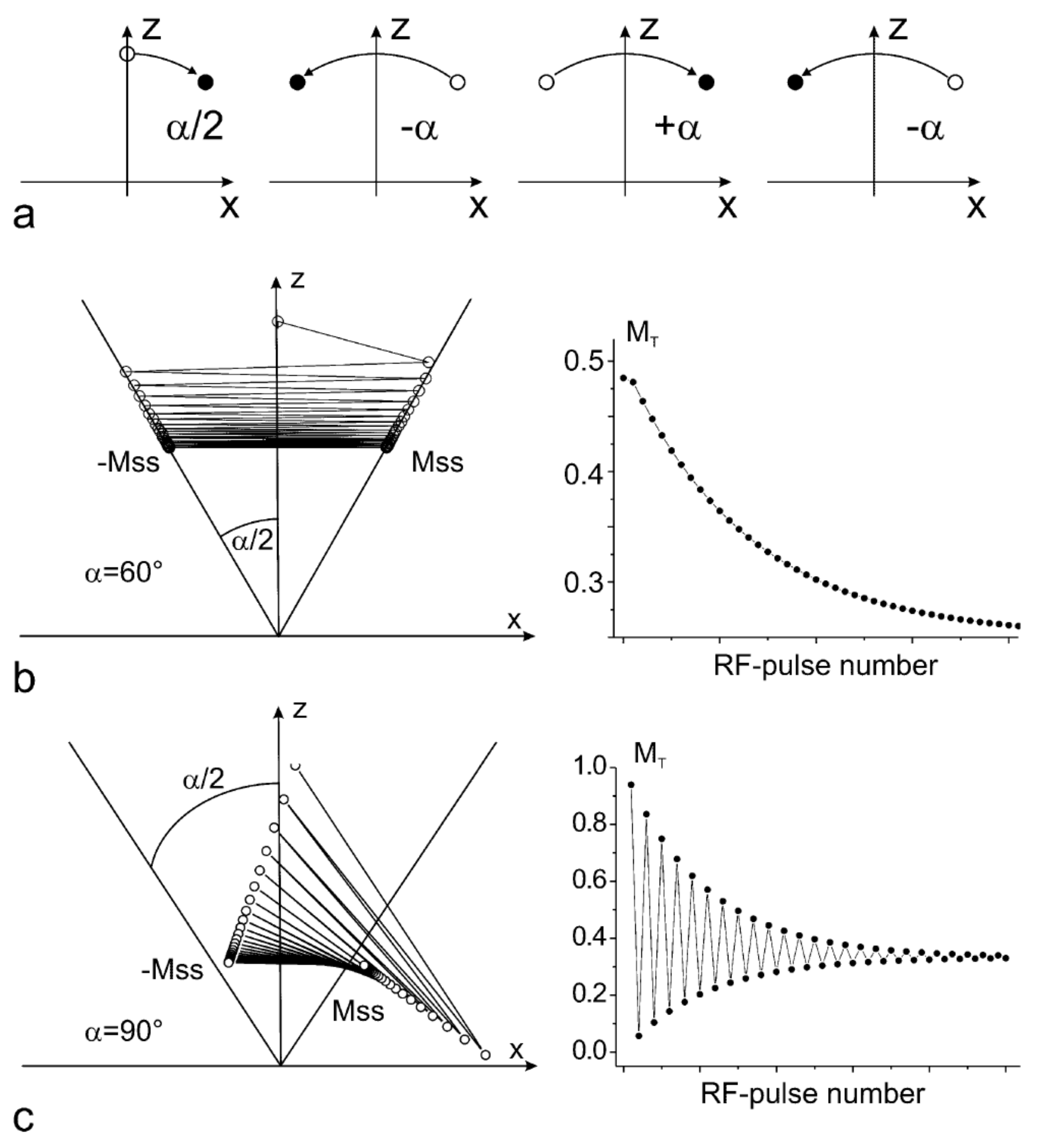
T_1 = 600 ms, T_2 = 100 ms

Balanced SSFP

- Transient state
  - approach to steady state can take $5 \cdot T_1$
  - depends on sequence and tissue params
  - longer transition for larger θ
  - artifacts and variable image contrast
- Catalyzation pulses
  - achieve smoother transition to steady state
  - simple approach: $\theta/2$ - TR/2 preparation
  - other sophisticated designs

Balanced SSFP

Transition to steady state ($\theta/2$ -TR/2 prep):



Scheffler et al., Eur Radiol; 13:2409-2418

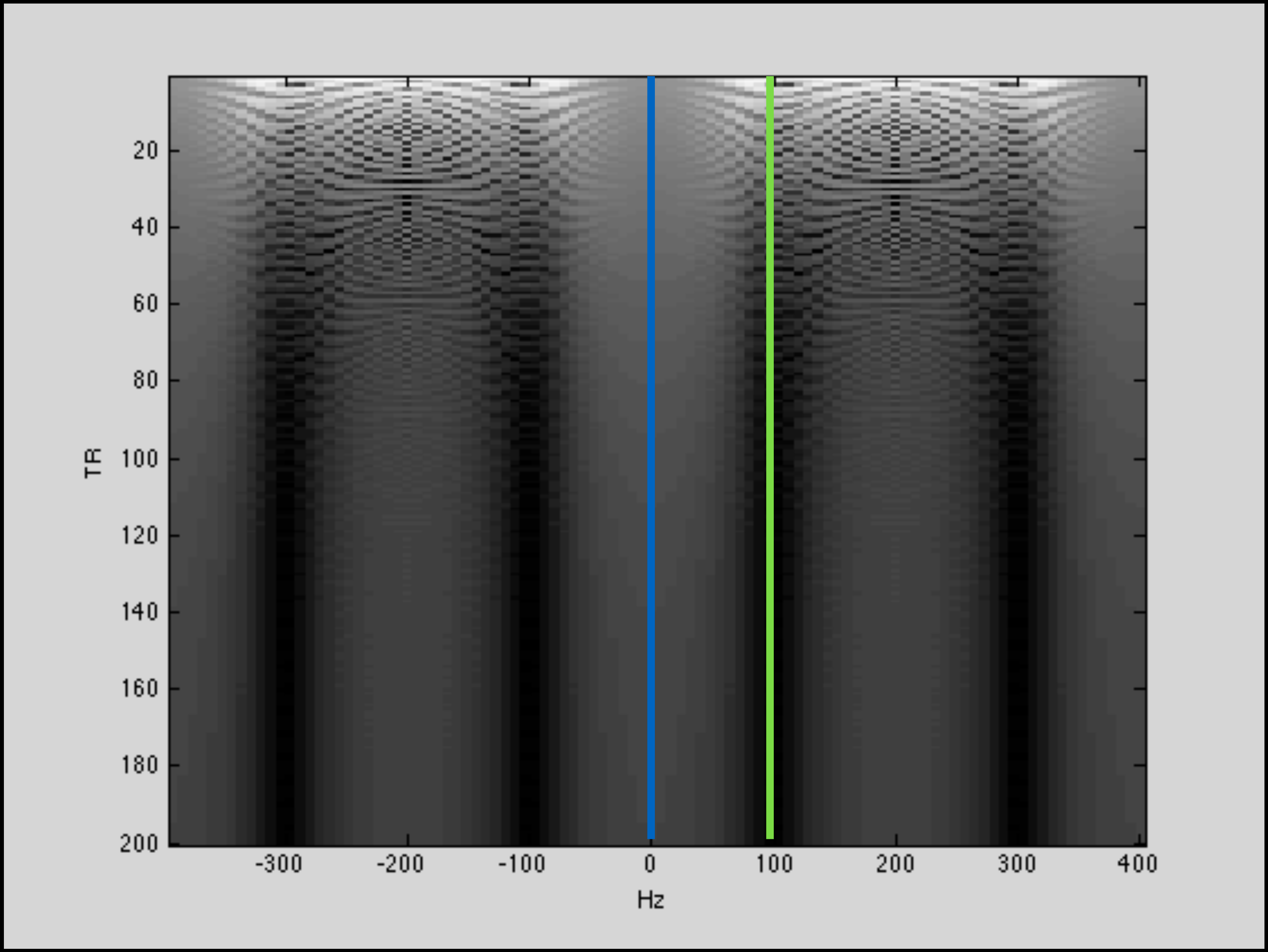
Balanced SSFP

Transition to steady state ($\theta/2$ -TR/2 prep):

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^o$



T_1 = 600 ms, T_2 = 100 ms

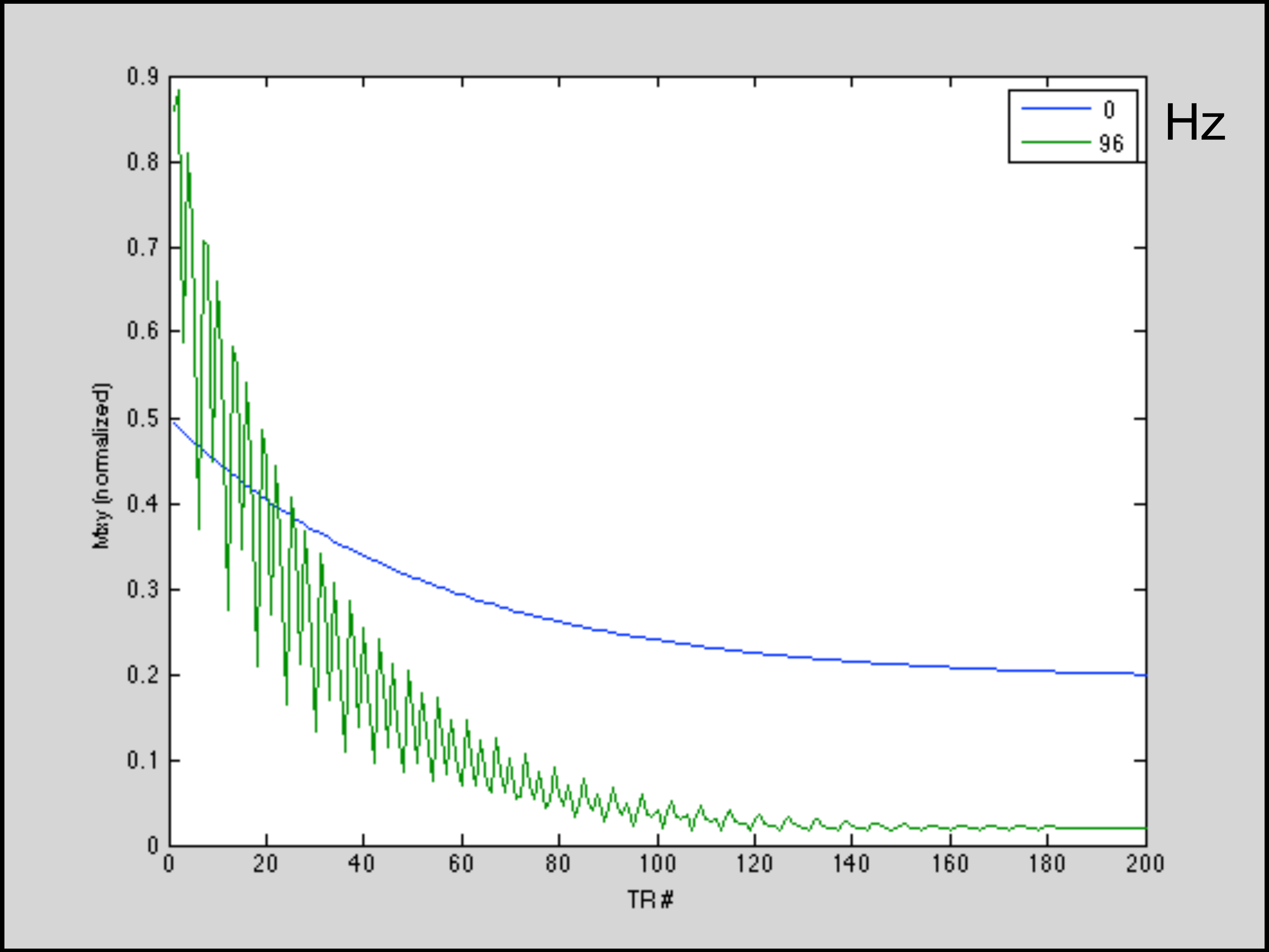
Balanced SSFP

Transition to steady state ($\theta/2$ -TR/2 prep):

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^o$

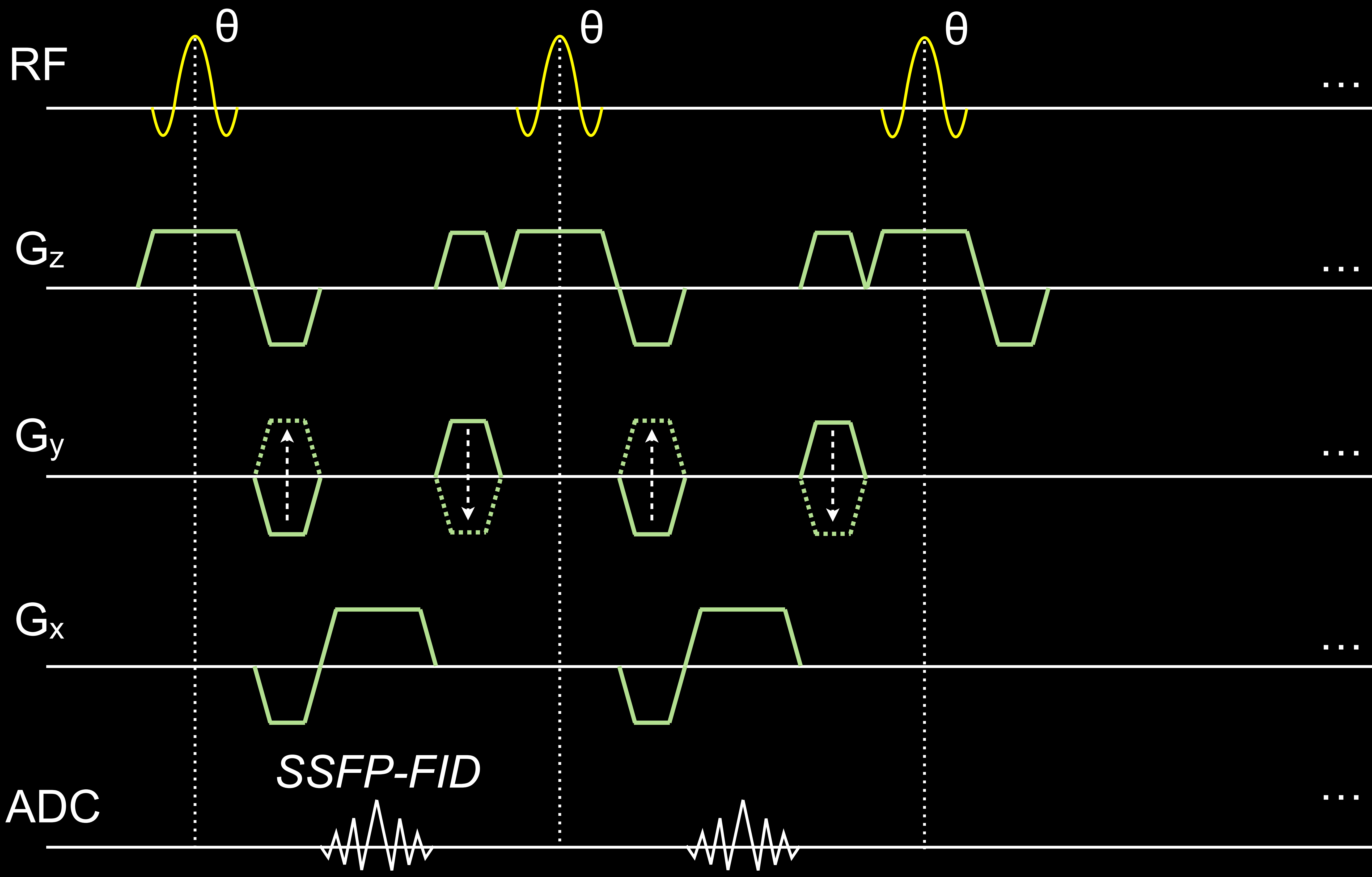


T_1 = 600 ms, T_2 = 100 ms

Balanced SSFP

- Advantages
  - High SNR efficiency
  - G_x and G_z first moments nulled
- Challenges
  - Field homogeneity
  - TR
  - SAR
  - Catalyzation
  - Bright fat

Gradient-spoiled GRE



Gradient-spoiled GRE

- End-of-TR gradient spoiler
  - typically on G_x and/or G_z
  - Range of β within each voxel
  - M_{xy} is a complex sum of all spins
- Contrast depends on T_1 and T_2

Gradient-spoiled GRE

Steady-state signal equation:

$$\mathrm{SSFP}_{\mathrm{FID}} = M_0 \frac{\sin\theta}{1+\cos\theta}\left(1-(E_1-\cos\theta)f(E_1,E_2,\theta)\right)$$

$$f(E_1,E_2,\theta) = \sqrt{\frac{1-E_2^2}{(1-E_1\cos\theta)^2 - E_2^2(E_1-\cos\theta)^2}}$$

When TR≫T_2:

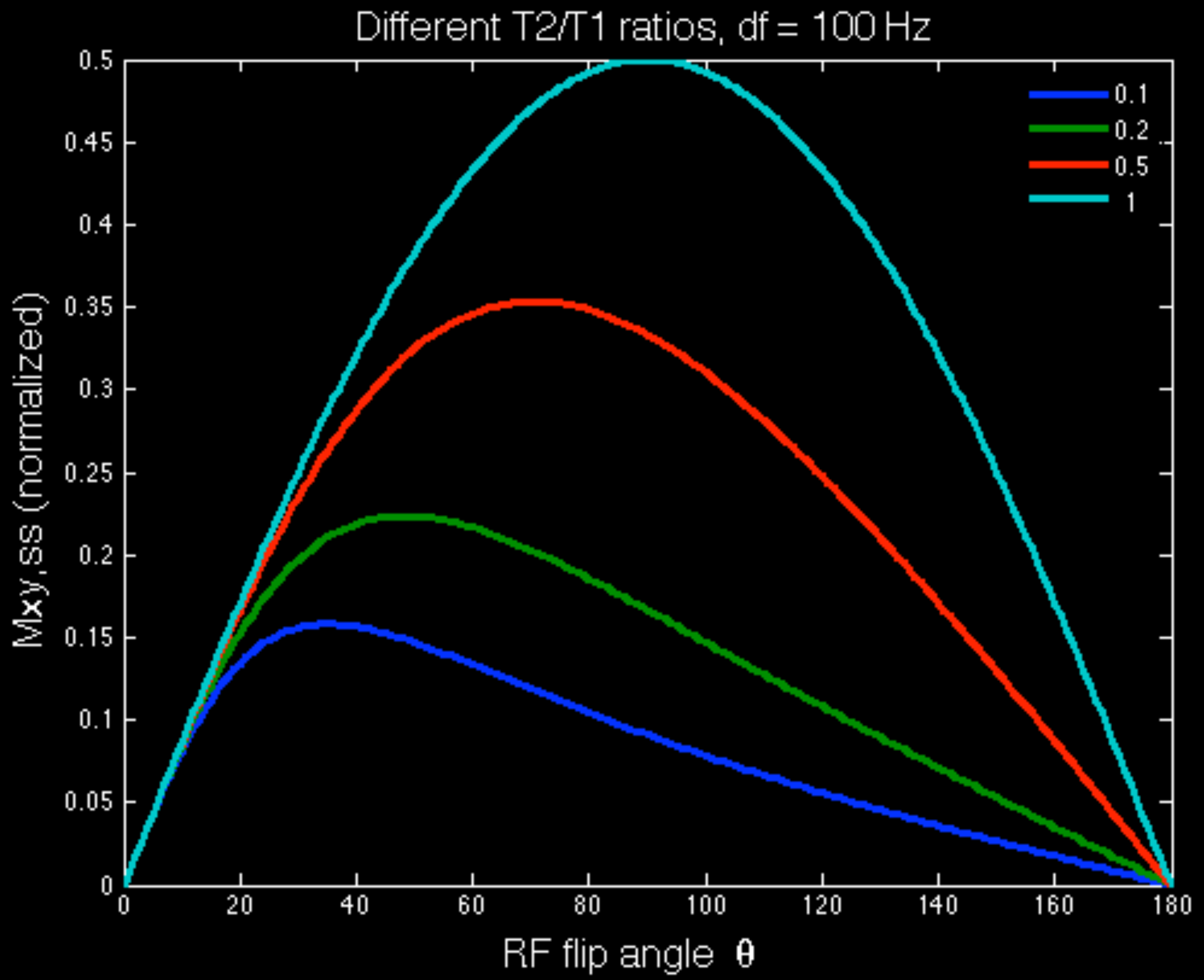
$$\mathrm{SSFP}_{\mathrm{FID}} \to M_0 \sin\theta \frac{1-E_1}{1-E_1\cos\theta}$$

same as ideally spoiled GRE

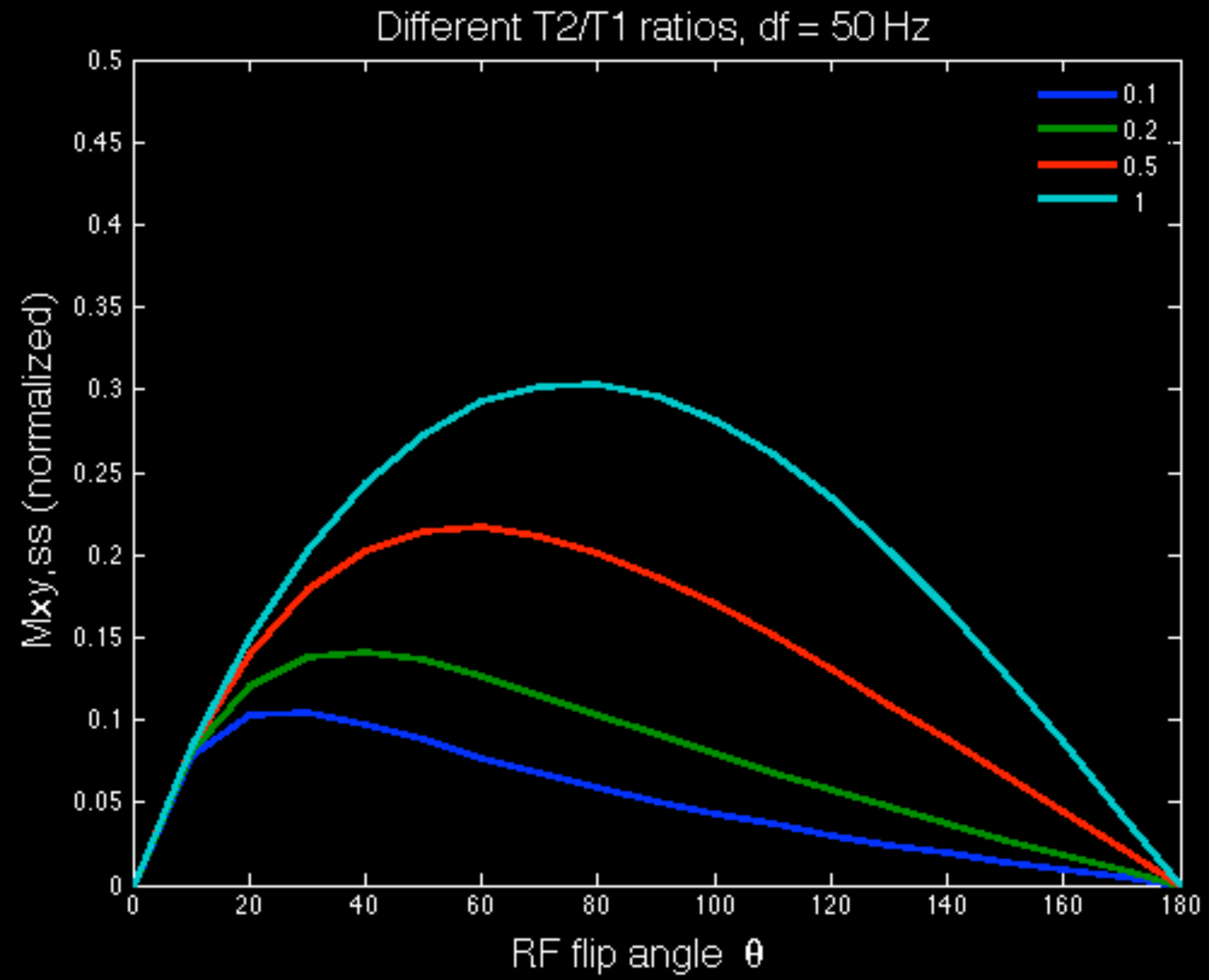
Gradient-spoiled GRE

SS signal as a function of flip angle:

bSSFP



GRE (SSFP-FID)

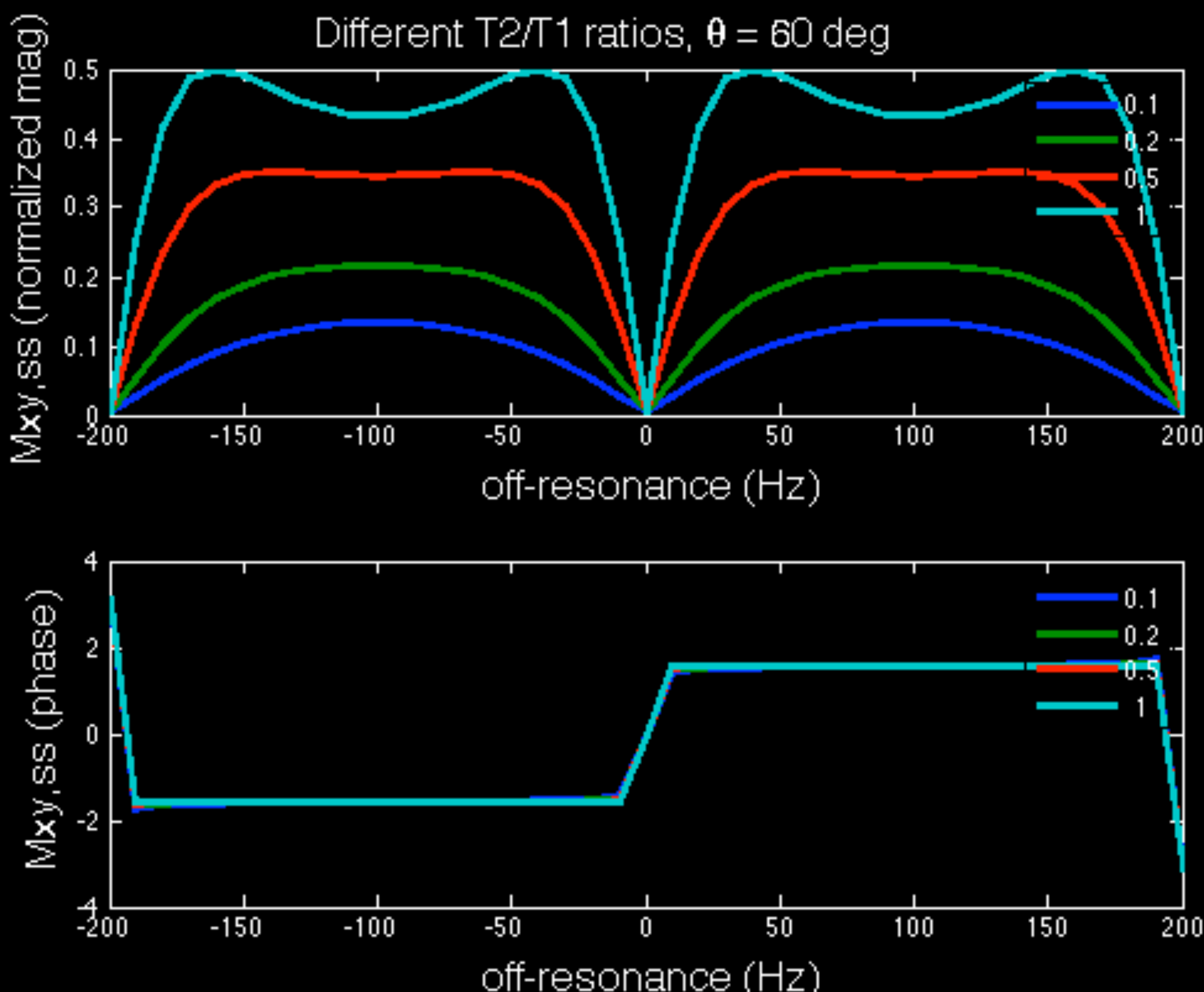


T_1 = 1000 ms, T_2 = 100,200,500,1000 ms

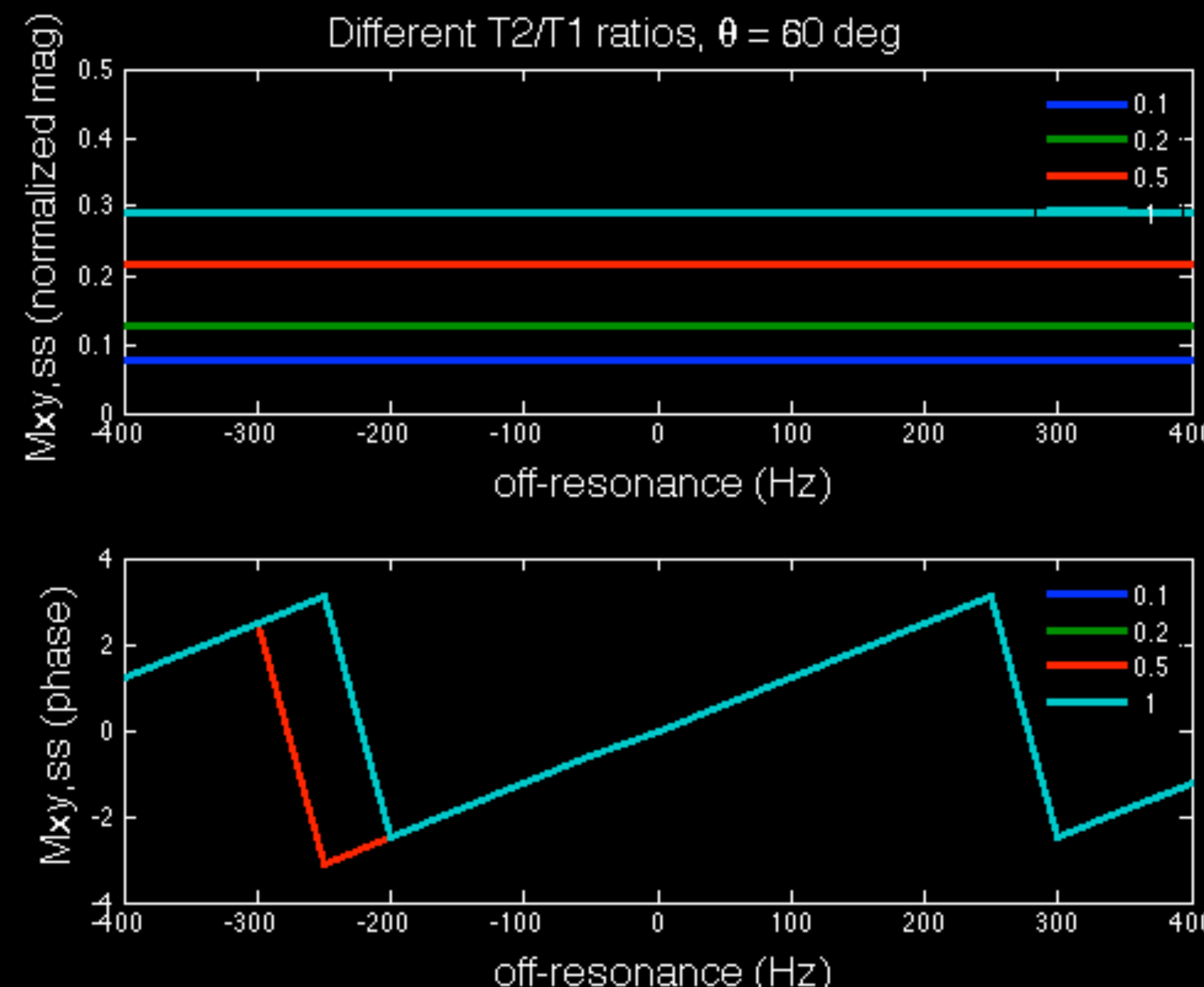
Gradient-spoiled GRE

SS signal as a function of off-resonance:

bSSFP



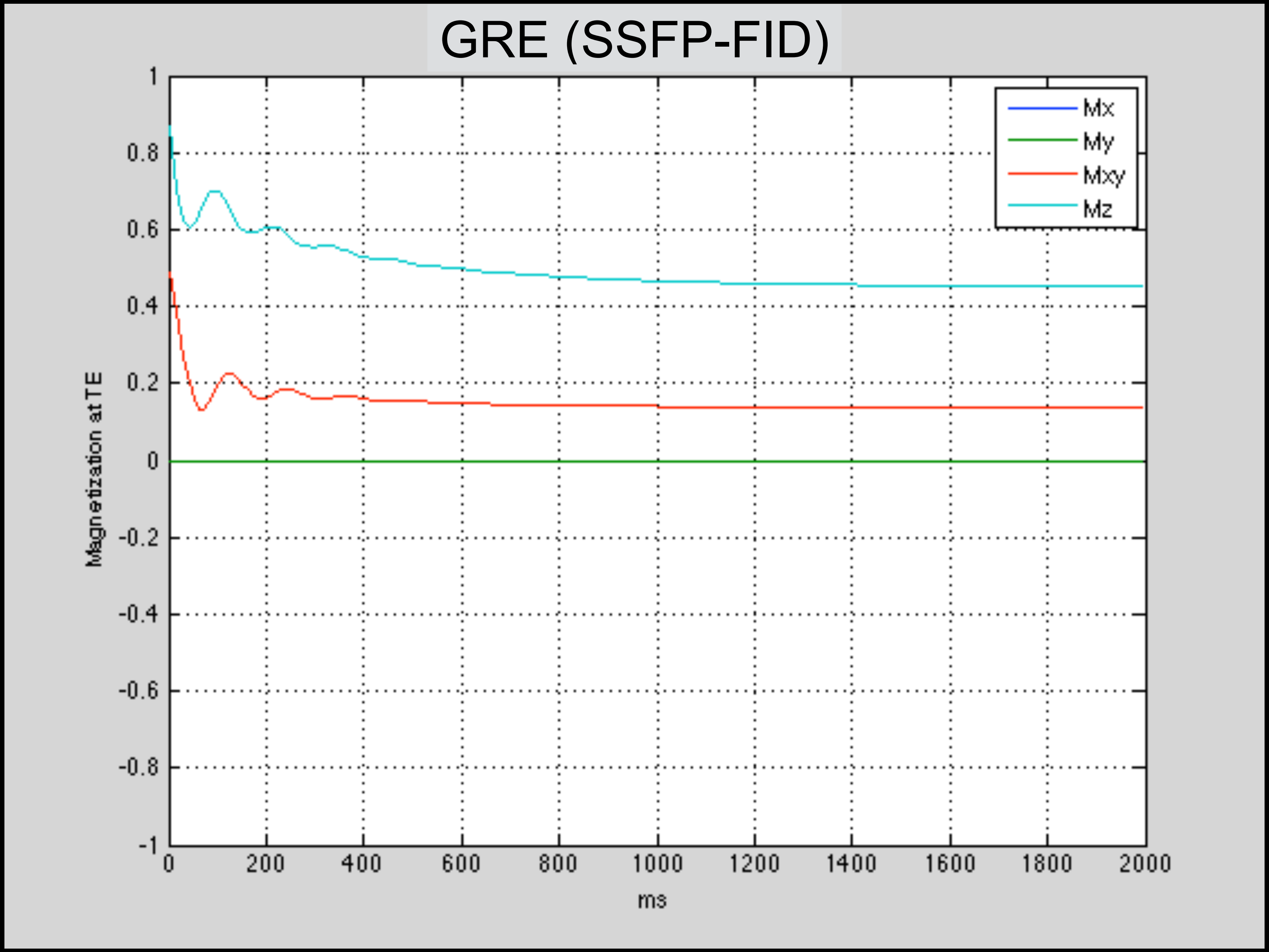
GRE (SSFP-FID)



T_1 = 1000 ms, T_2 = 100,200,500,1000 ms

Gradient-spoiled GRE

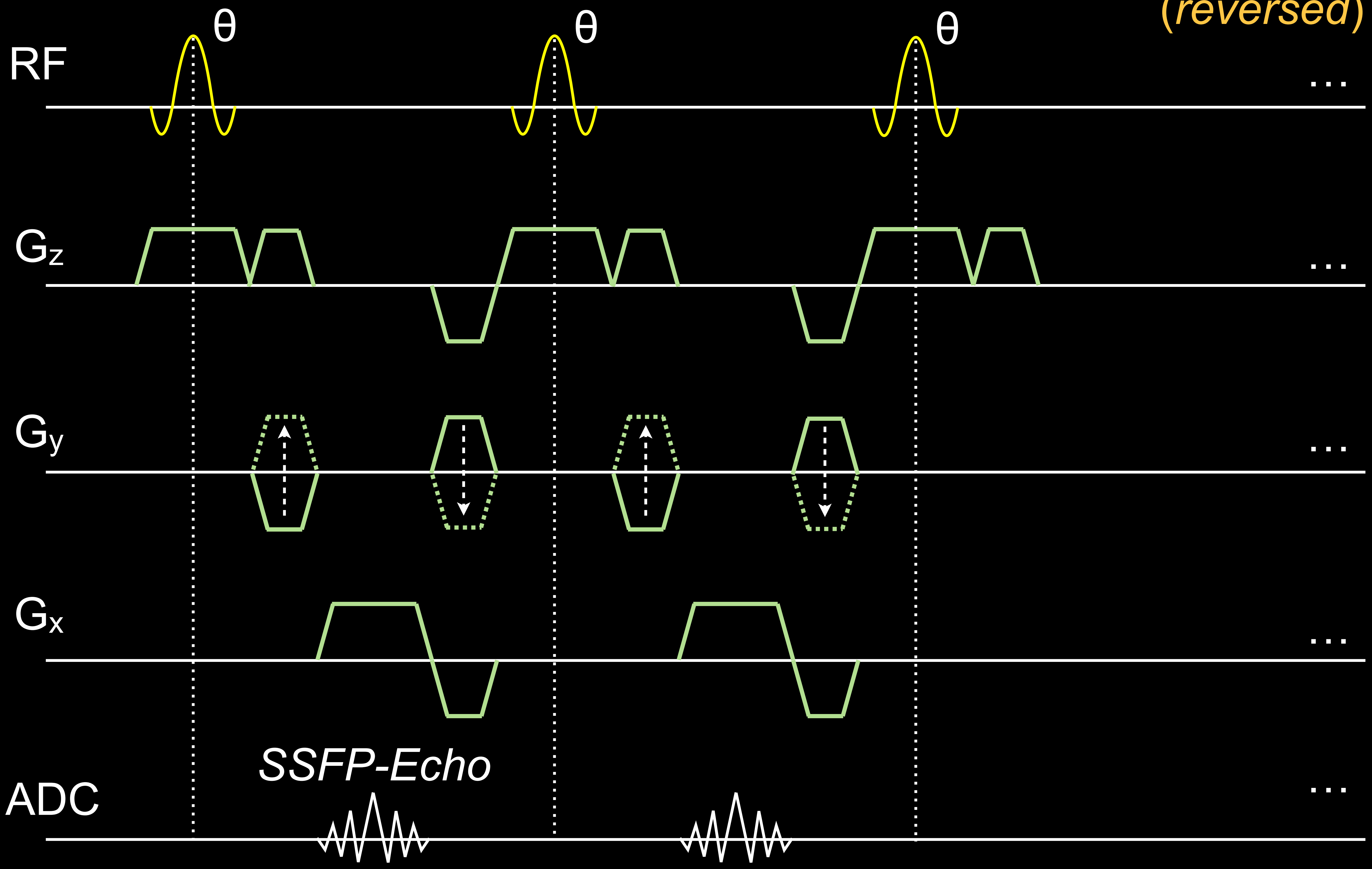
Transition to steady state:



T_1 = 600 ms, T_2 = 100 ms, TE/TR = 2/10 ms, θ = 30^o

Gradient-spoiled GRE

(reversed)



Gradient-spoiled GRE

(*reversed*)

Steady-state signal equation:

$$\mathrm{SSFP}_{\mathrm{Echo}} = M_0 \frac{\sin\theta}{1+\cos\theta}\left(1-(1-E_1\cos\theta)f(E_1,E_2,\theta)\right)$$

$$f(E_1,E_2,\theta) = \sqrt{\frac{1-E_2^2}{(1-E_1\cos\theta)^2 - E_2^2(E_1-\cos\theta)^2}}$$

When TR≪T_1:

$$\frac{\mathrm{SSFP}_{\mathrm{Echo}}}{\mathrm{SSFP}_{\mathrm{FID}}} \sim E_2^2 = e^{-2\mathrm{TR}/T_2}$$

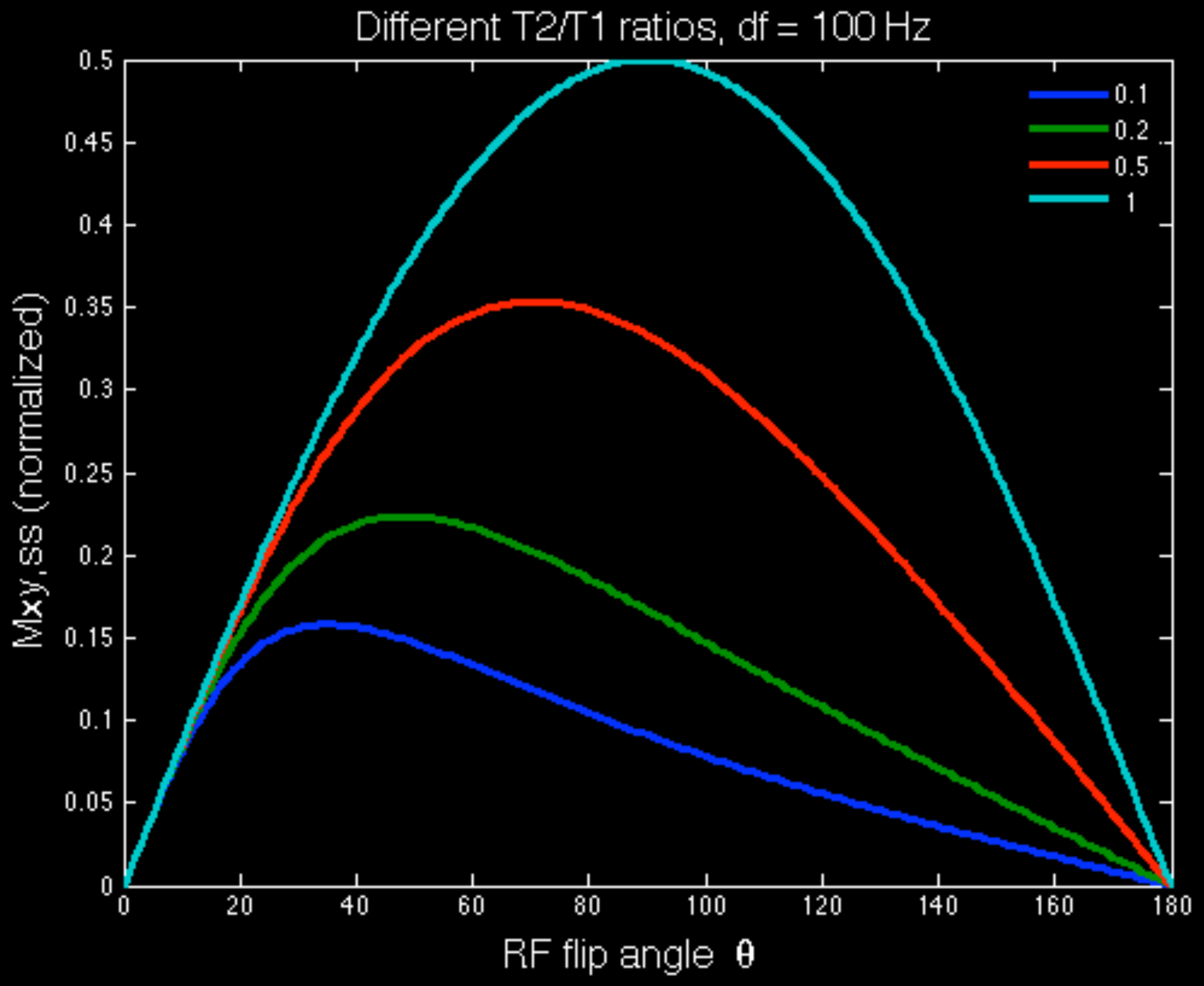
higher T_2 contrast weighting than $SSFP_{FID}$

Gradient-spoiled GRE

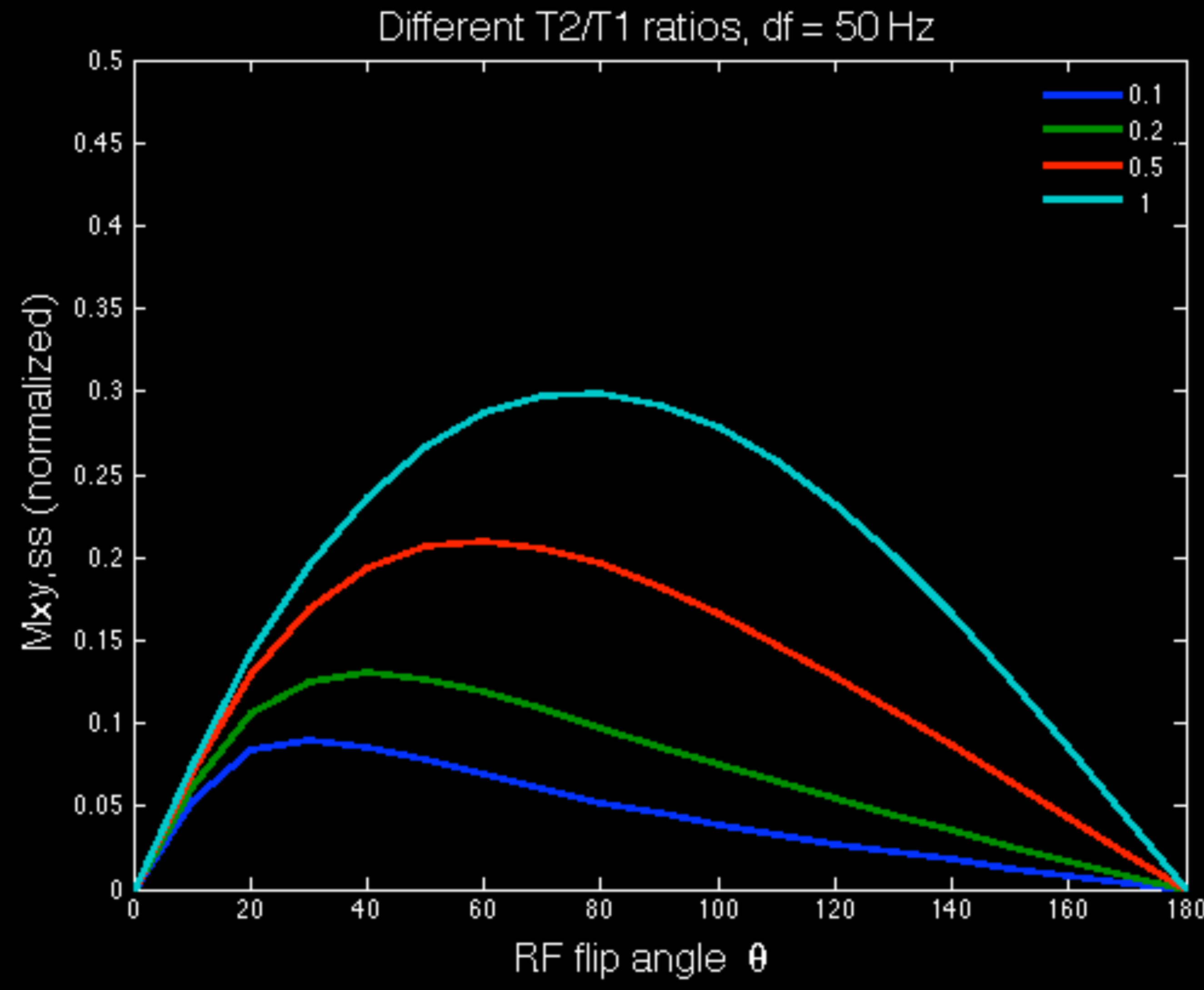
(*reversed*)

SS signal as a function of flip angle:

bSSFP



GRE (SSFP-Echo)

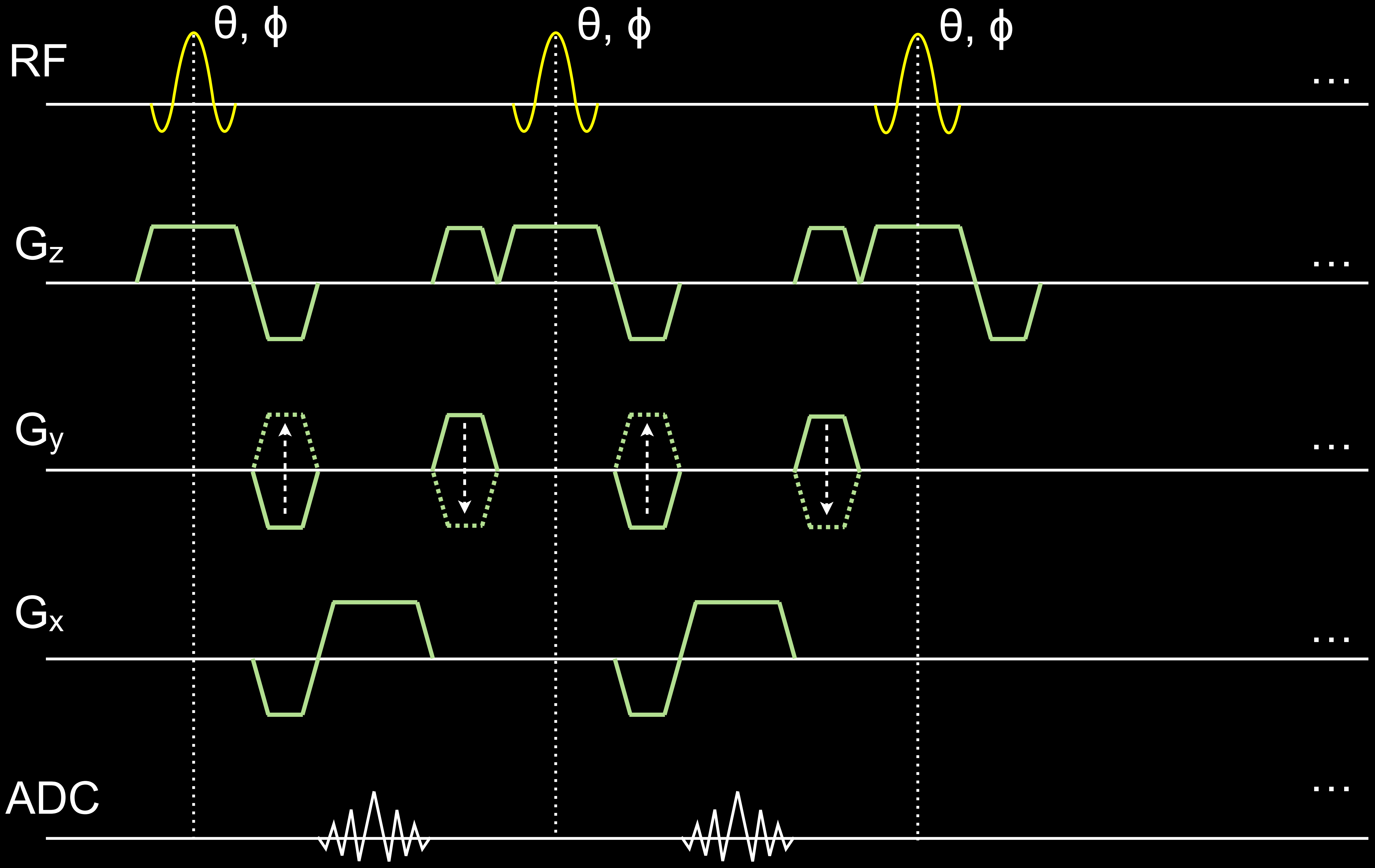


T_1 = 1000 ms, T_2 = 100,200,500,1000 ms

Gradient-spoiled GRE

- Image characteristics
  - no banding (averaged in voxel)
  - SSFP-FID: T_2/T_1 contrast
  - SSFP-Echo: more T_2 contrast
  - sensitive to motion / flow / diffusion
- When all gradients are balanced
  - SSFP-FID and SSFP-Echo coalesce
  - T_2 instead of T_2* weighting
  - Balanced SSFP!

Gradient & RF-spoiled GRE

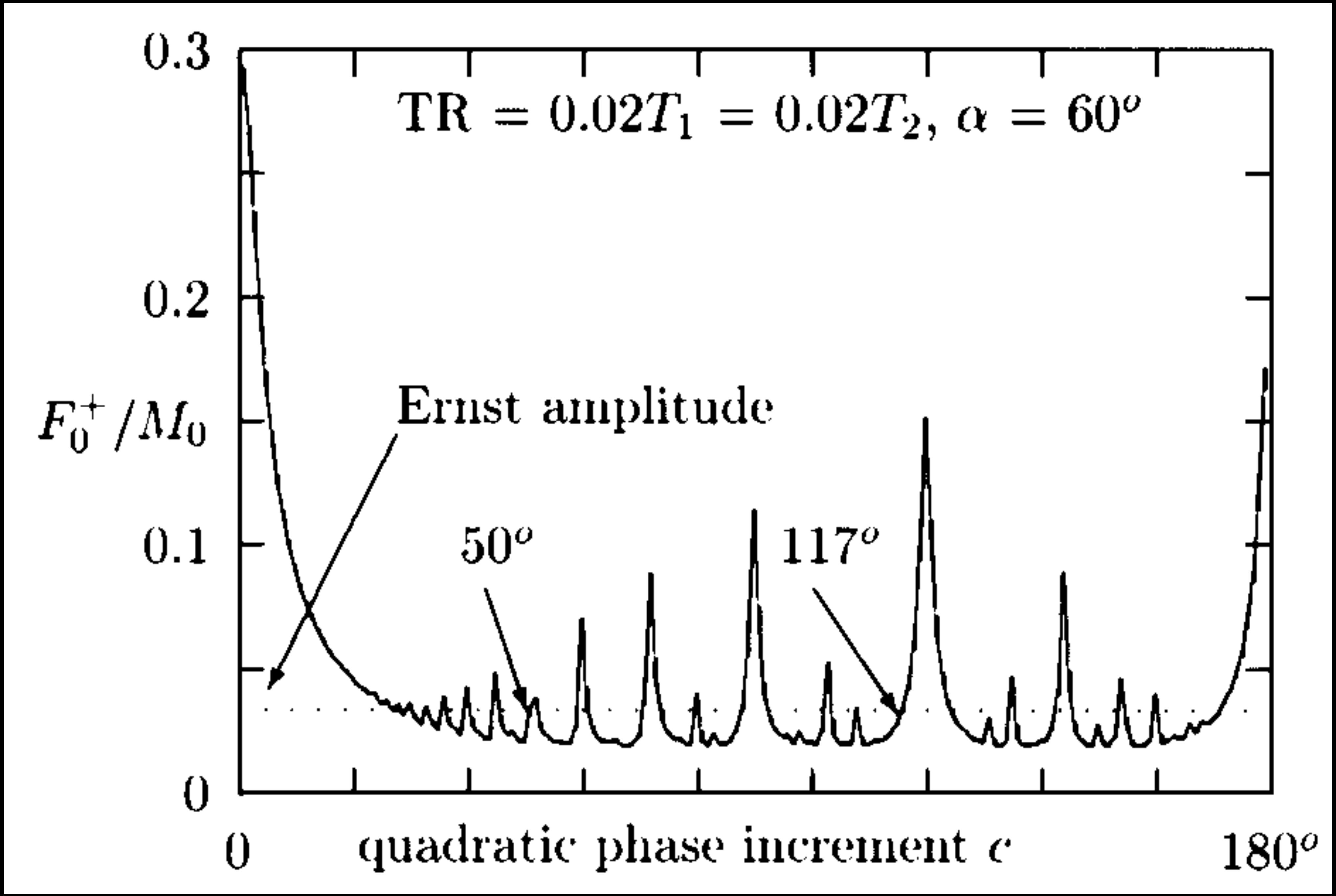


Gradient and RF-spoiled GRE

- RF spoiling (quadratic)
  - $\phi_n = \phi_{n-1} + n\phi_0 = (1/2)\phi_0(n^2 + n + 2)$
  - typically $\phi_0 = 50^o$ or 117^o
  - ADC phase each TR also needs to match ϕ_n
- T_1-weighted contrast
  - approaches contrast of ideally spoiled GRE
  - at expense of reduced SNR (removes T2w contributions)

Gradient and RF-spoiled GRE

Choice of RF phase increment:

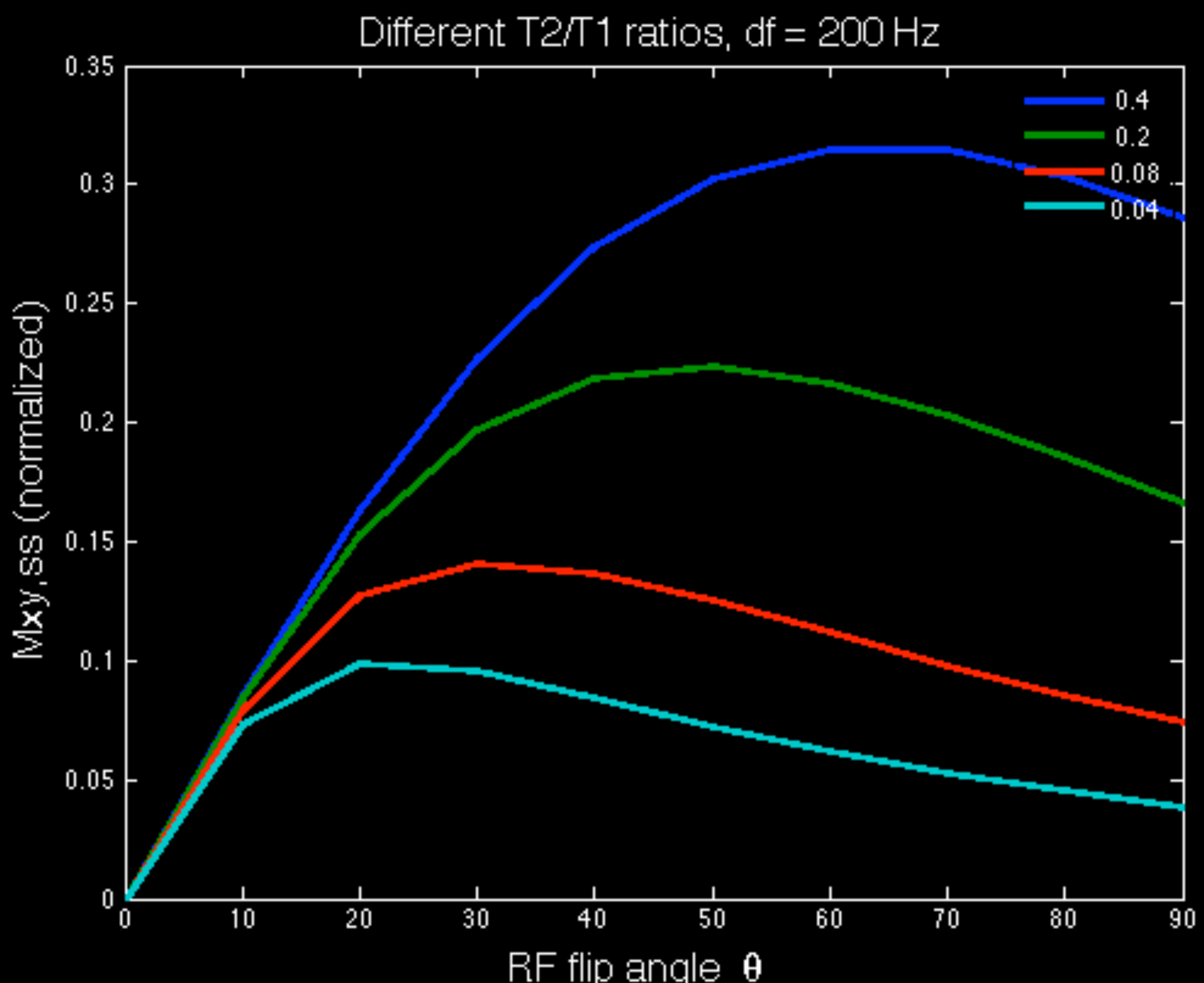


Scheffler K, Con Magn Reson 1999; 11:291-304

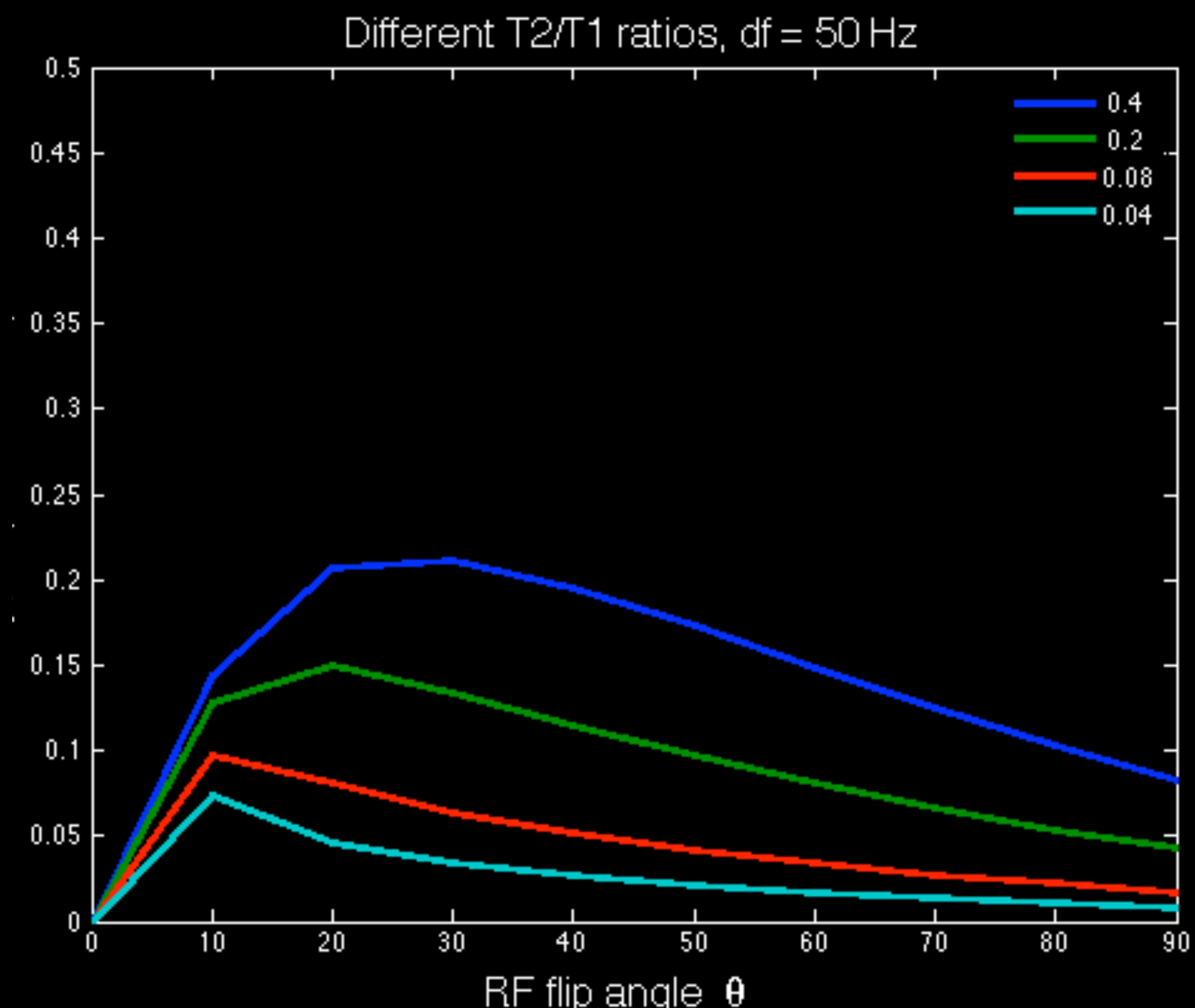
Gradient and RF-spoiled GRE

SS signal as a function of flip angle:

bSSFP



Spoiled GRE

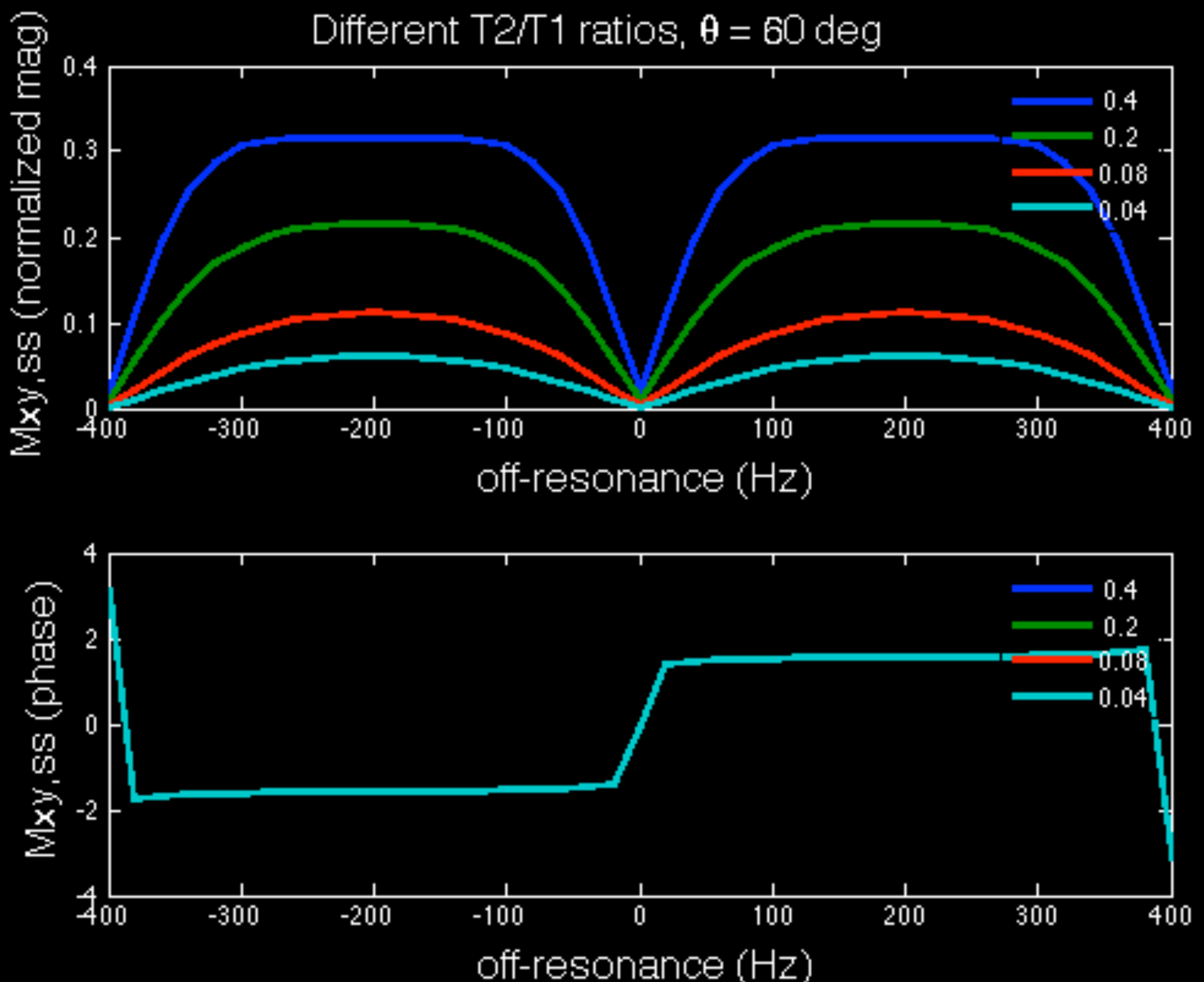


T_1 = 100,200,500,1000 ms, T_2 = 40 ms

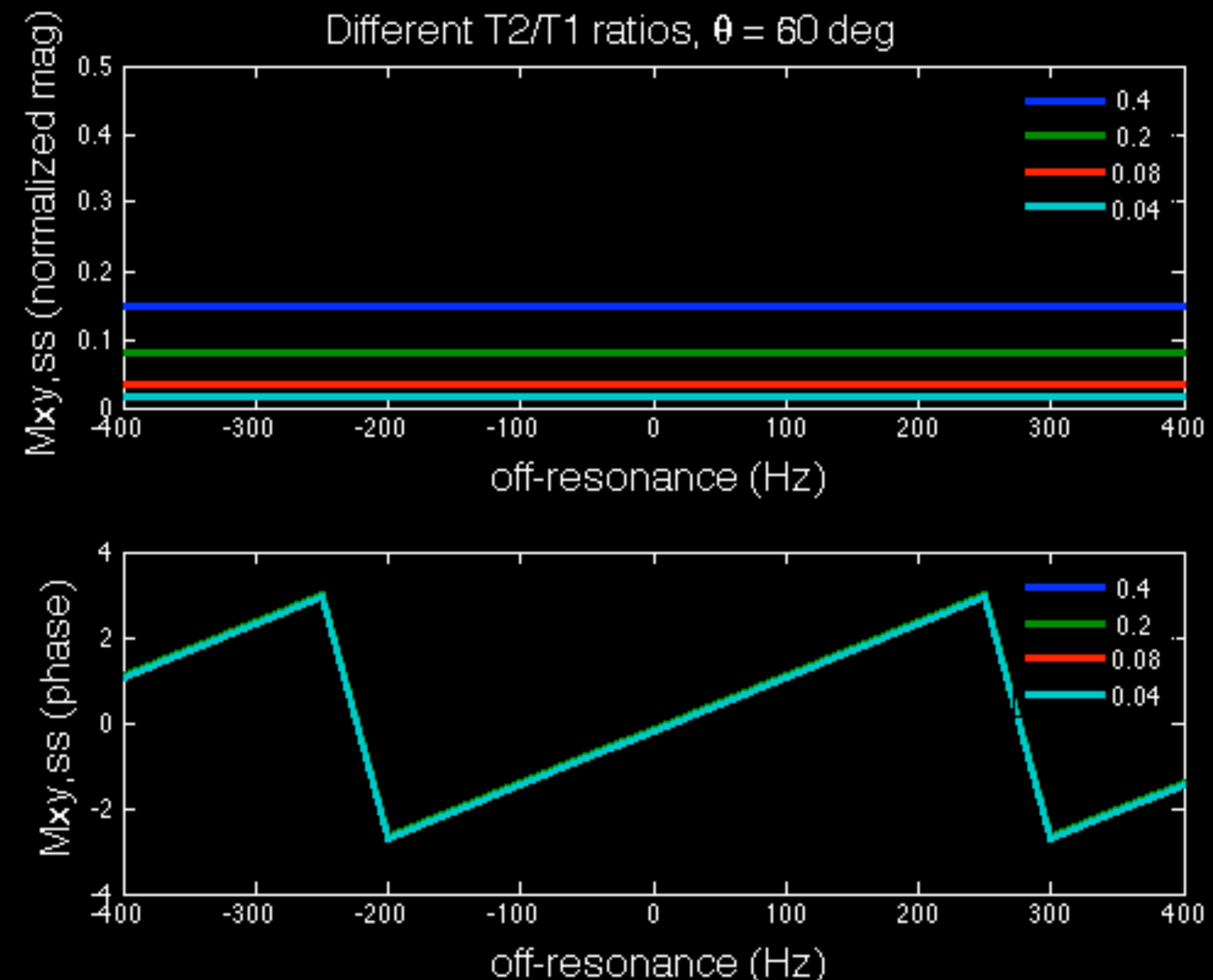
Gradient and RF-spoiled GRE

SS signal as a function of off-resonance:

bSSFP



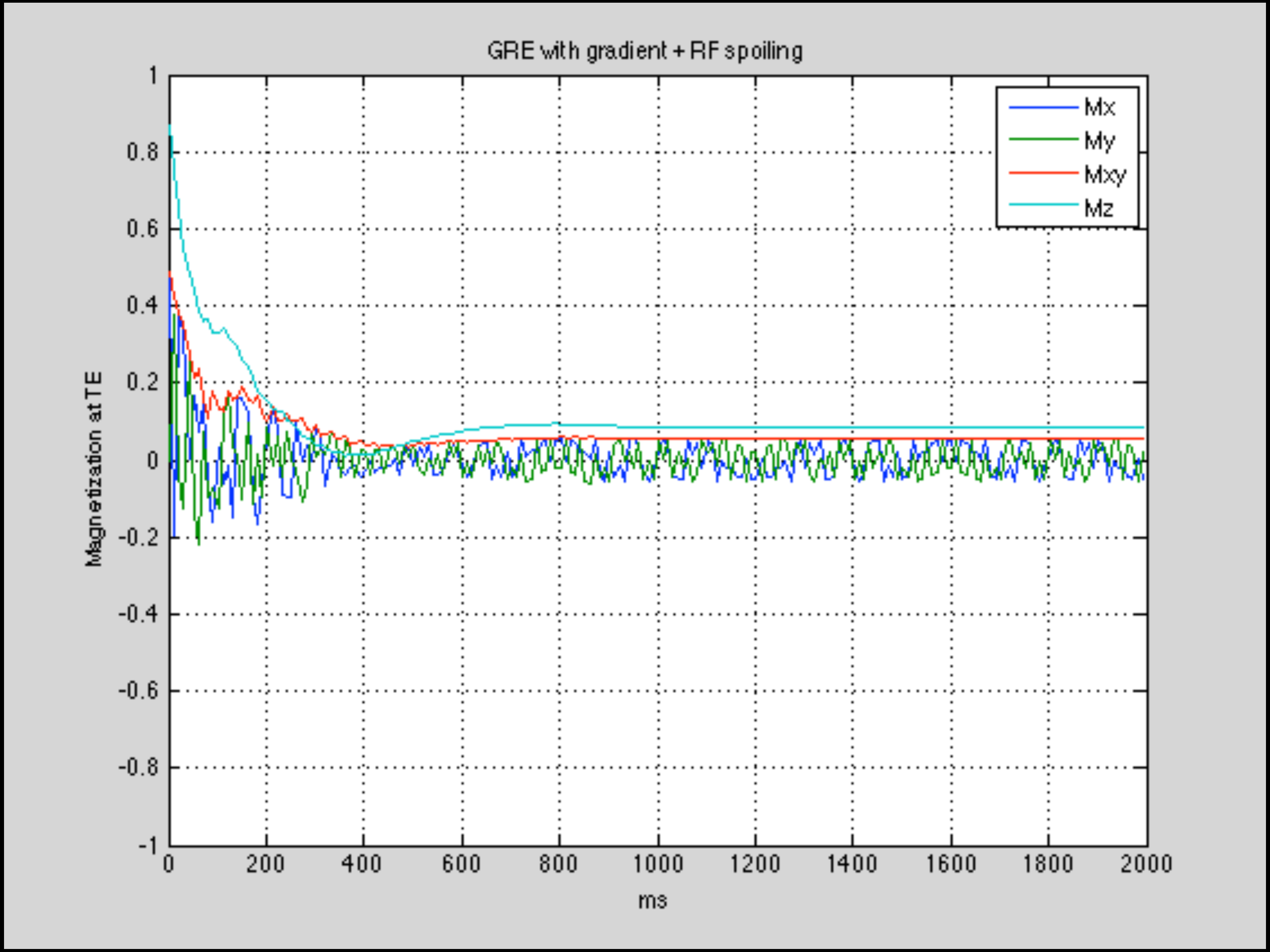
Spoiled GRE



T_1 = 100,200,500,1000 ms, T_2 = 40 ms

Gradient and RF-spoiled GRE

Transition to steady state:

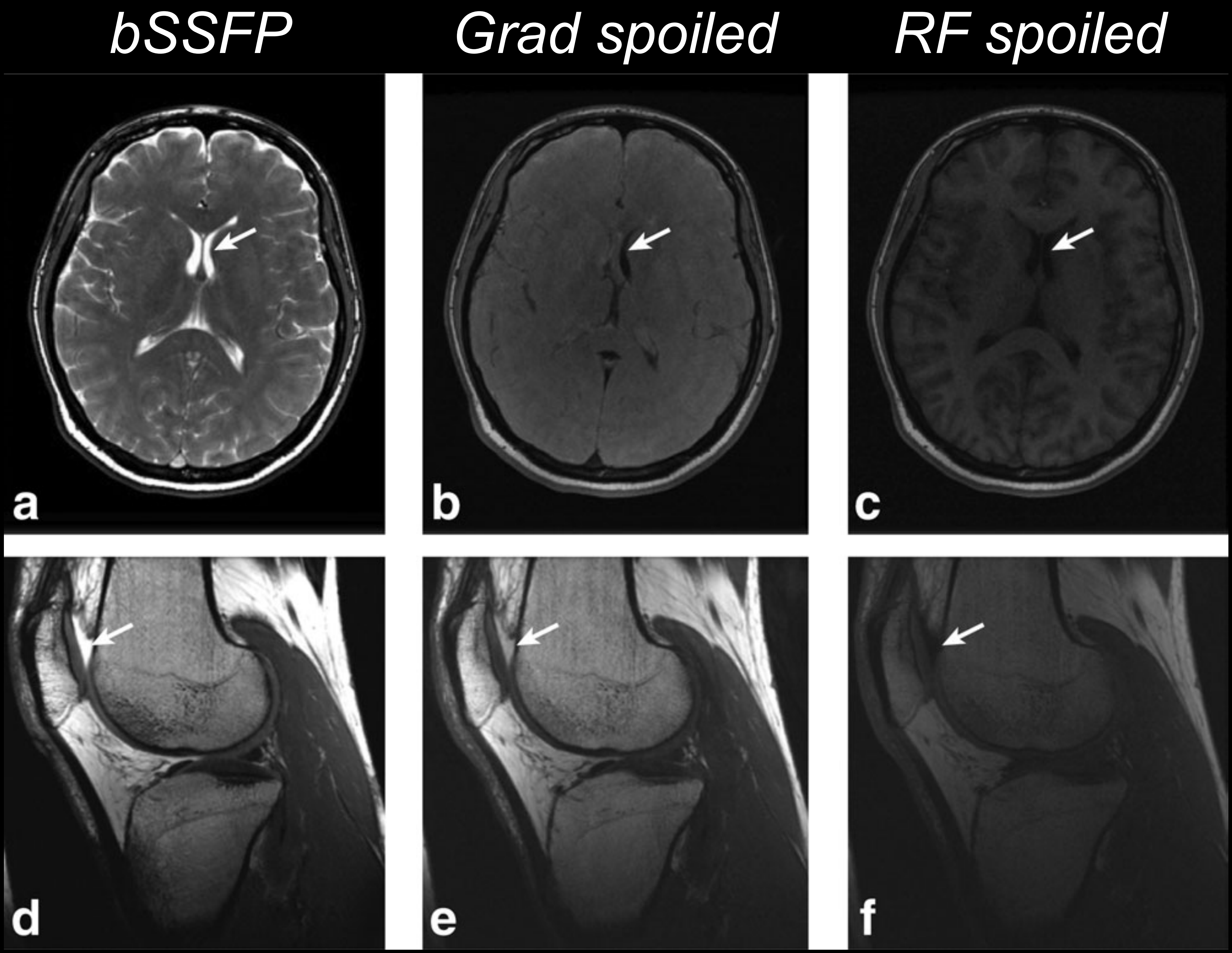


T_1 = 600 ms, T_2 = 100 ms, TE/TR = 2/10 ms, θ = 30^o

Gradient and RF-spoiled GRE

- Image characteristics
  - no banding
  - M_{xy} spoiled before next TR
  - T1w contrast with short TR
  - θ controls degree of T_1 contrast
  - TE controls degree of T_2* contrast
  - robust to motion

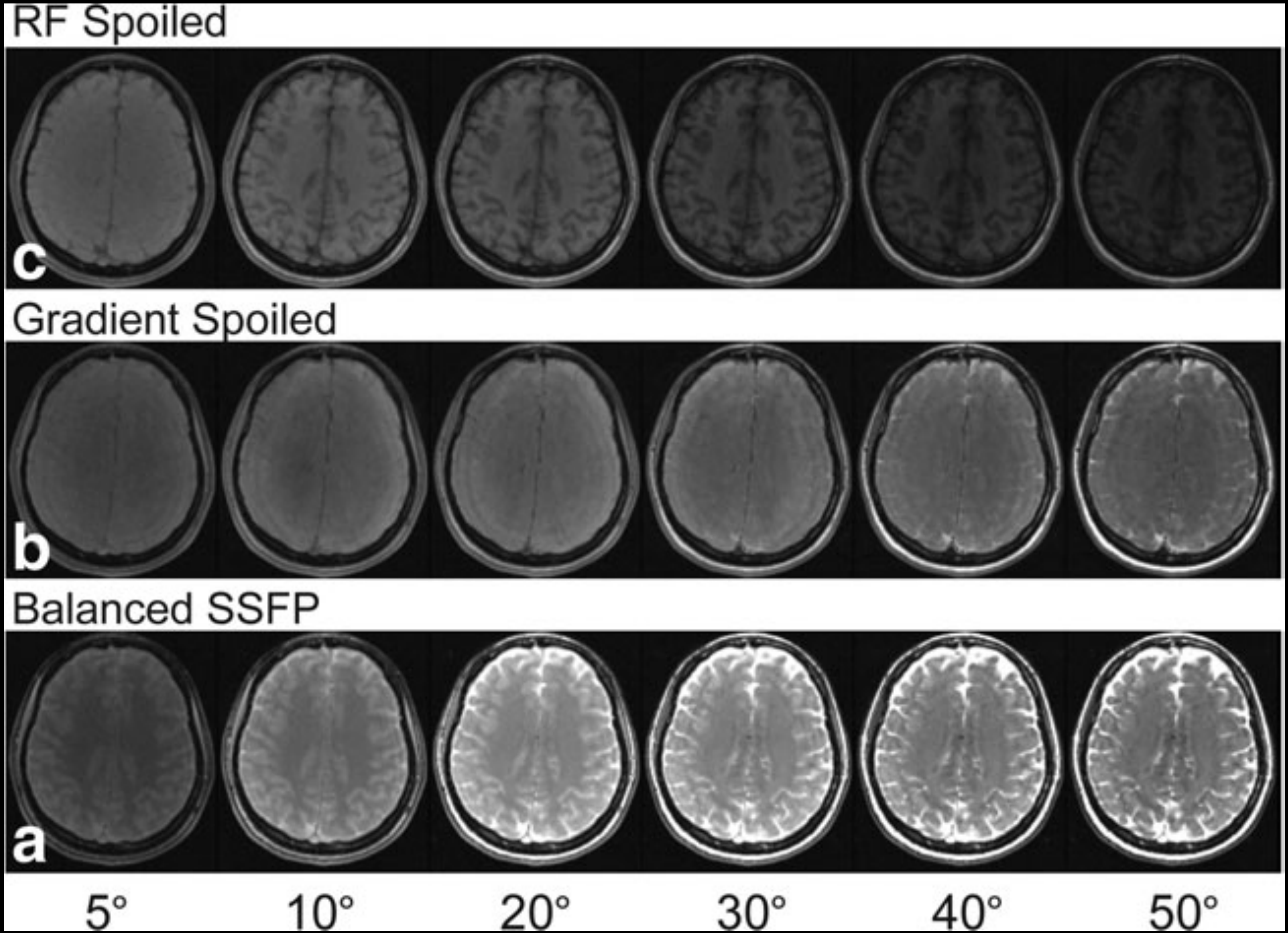
Rapid GRE - Comparison



Hargreaves B, JMRI 2012; 36:1300-1313

Rapid GRE - Comparison

Flip angle



Hargreaves B, JMRI 2012; 36:1300-1313

Rapid GRE - Comparison

Pulse Sequence		Mxy	Contrast	SNR	Artifacts
Balanced SSFP	bSSFP	retained	T_2/T_1	high	banding
Gradient-spoiled GRE	SSFP-FID	averaged	T_2/T_1	mid	motion
	SSFP-Echo	averaged	T_2+T_2/T_1	mid	motion
Gradient and RF-spoiled GRE	Spoiled GRE	cancelled	T_1; T_2*	low	minimal

SS transition

cf. Hargreaves B, JMRI 2012; 36:1300-1313

Considerations

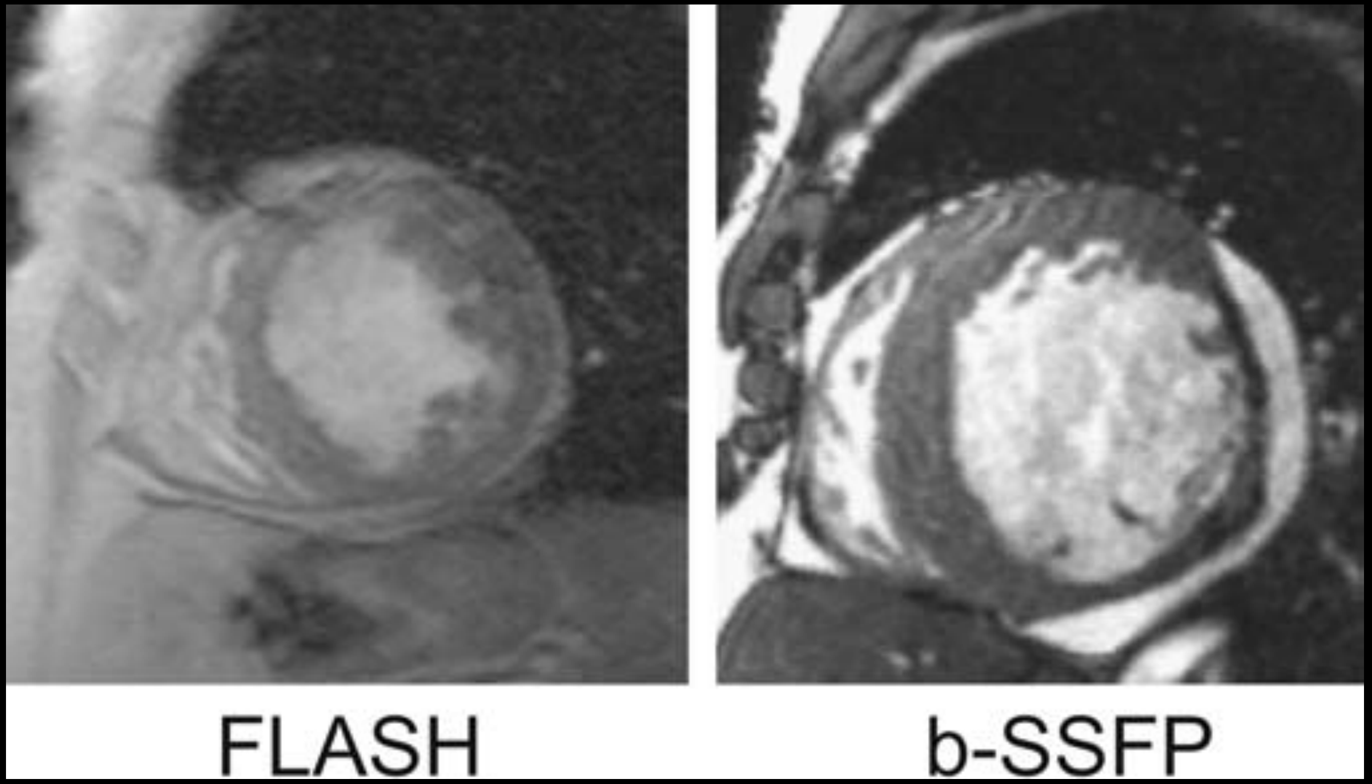
- Chemical shift
- Flow
- Diffusion

Extensions and Variations

- Partial echo
- Multi-echo
- Ultra-short TE
- Magnetization preparation
- Multiple steady states

Applications

- bSSFP
  - Cardiac
  - MRA
  - T_2-like imaging
  - fMRI
  - Phase contrast
  - Mag-prep



Scheffler et al., Eur Radiol; 13:2409-2418

Applications

- SSFP-FID / Echo
  - T_2-like imaging (e.g., cartilage)
  - Bright fluid (bSSFP-like without banding)
  - Diffusion-weighted imaging (SSFP-Echo)

Applications

- Spoiled GRE
  - T1w imaging
  - T_2* BOLD fMRI
  - Susceptibility-weighted imaging (SWI)
  - Phase contrast
  - Thermometry
  - Time-of-flight MRA
  - Contrast-enhanced imaging
  - Mag-prep imaging

Thanks!

- Web resources
  - ISMRM 2010 Edu: Weigel, Bieri, Miller
  - ISMRM 2011 Edu: Weigel, Miller
  - ISMRM 2012 Edu: Miller, Bieri
- Further reading
  - Bernstein et al., Handbook of MRI Sequences
  - Haacke et al., Magnetic Resonance Imaging
  - Nishimura, Principles of MRI
  - pubmed.org

Thanks!

- Acknowledgments
  - Suba's slides from M219 (2014)
  - Brian Hargreaves's Bloch simulator

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