Pulse Sequences: EPG and Simulations

M229 Advanced Topics in MRI

Holden H. Wu, Ph.D.

2025.04.10

Department of Radiological Sciences
David Geffen School of Medicine at UCLA

Class Business

- Office hours
  - Holden: by appointment
  - Wenqi: 10-12 on 4/11 Fri, 4/14 Mon, 4/18 Fri
  - Email beforehand
- Homework 1 due on 4/21 Mon
- Final project
  - Start thinking

Outline

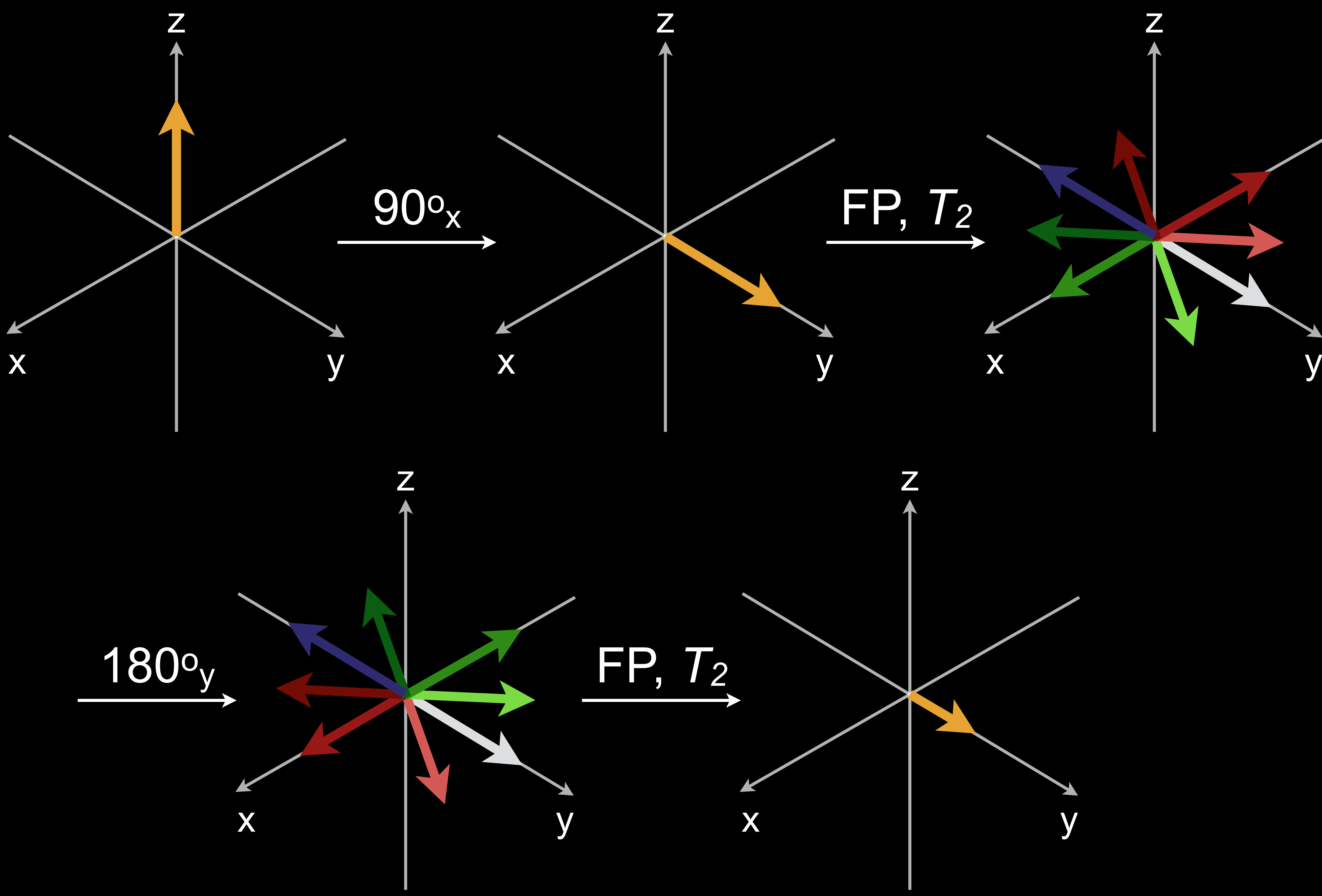
- Multi-Pulse Experiments
- Extended Phase Graphs (EPG)

- EPG Simulations
  - Homework 1

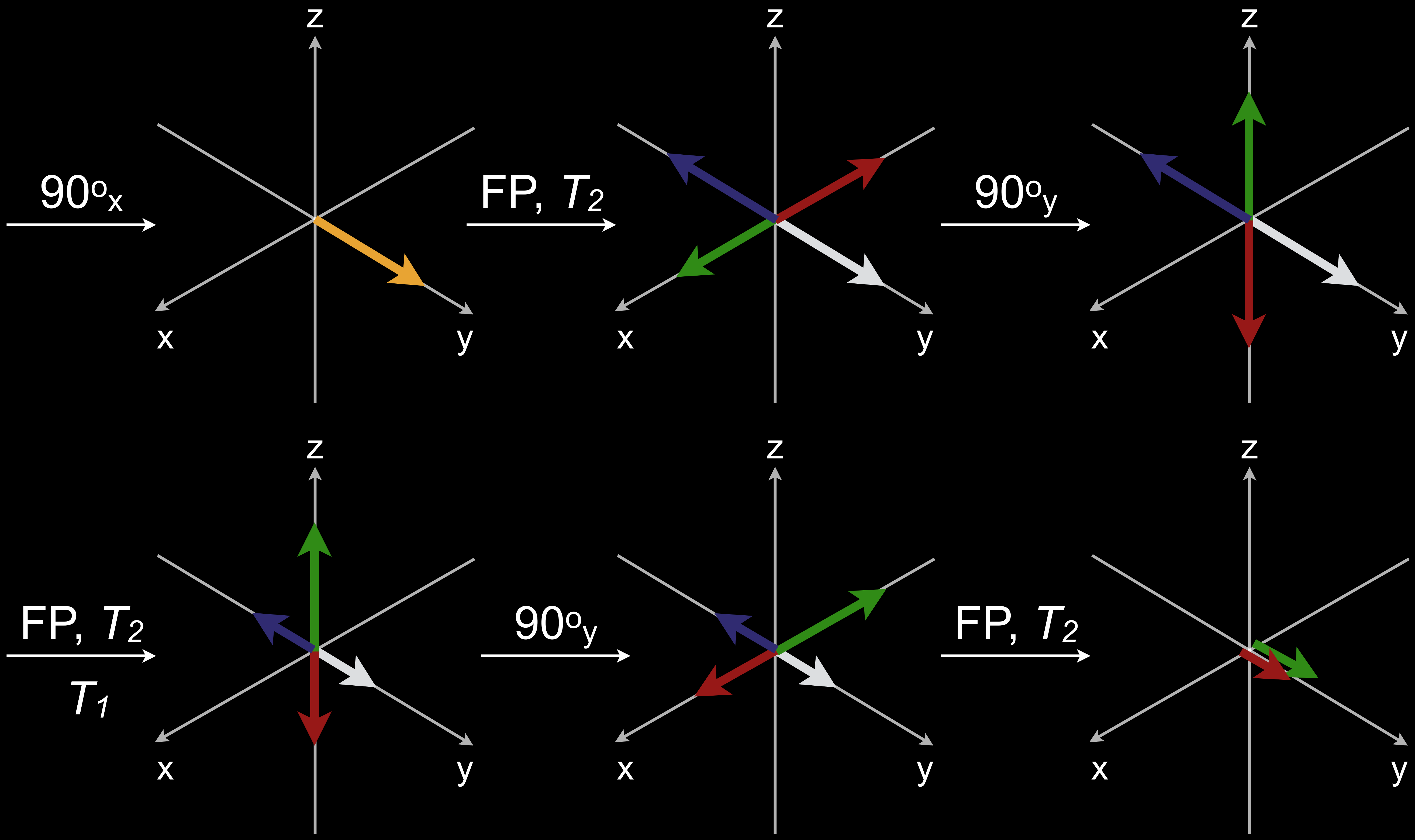
Multi-Pulse Experiments

- Multiple RF pulses -> multiple echoes
  - always have echoes (many types)
  - do not need perfect $90^{o}+180^{o}$ to form SE, etc.
  - generalized view of MR pulse sequences
- Analysis
  - Bloch Equations
  - Extended Phase Graphs (EPG)

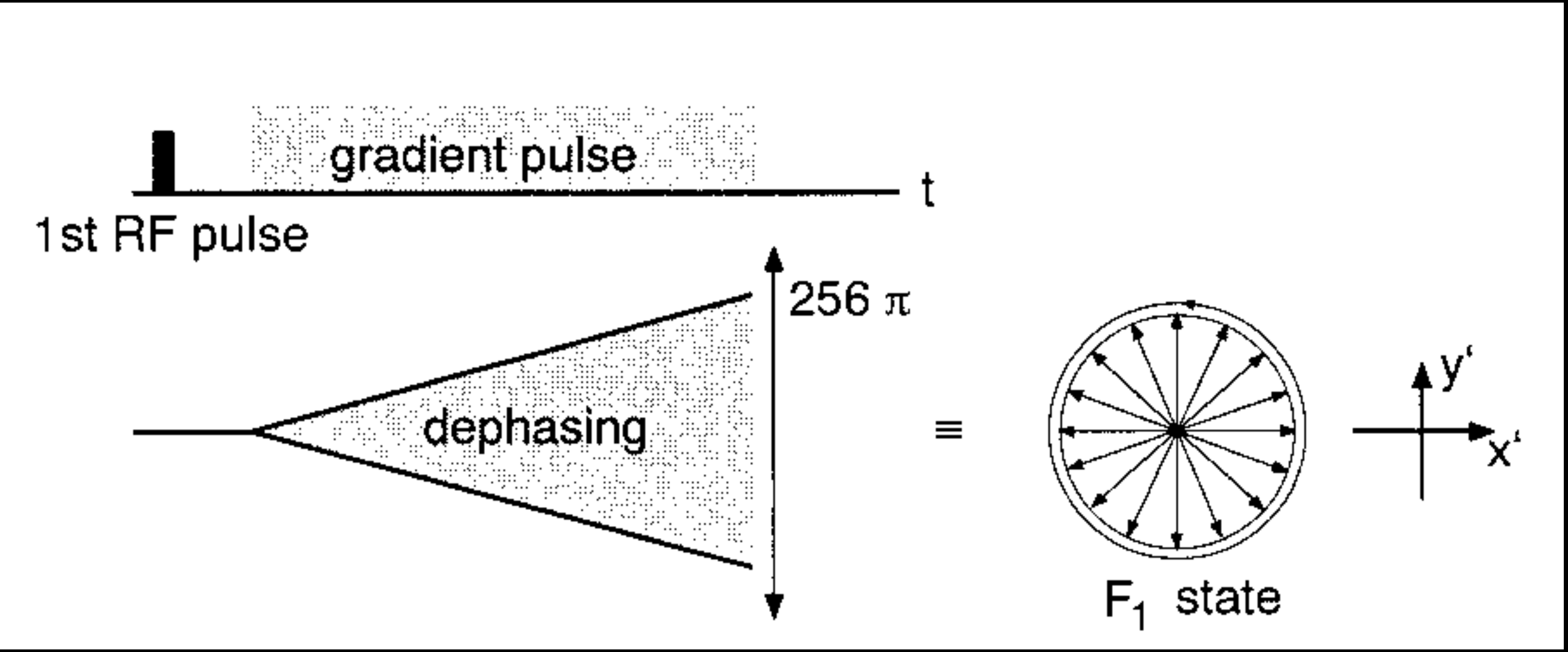
Spin Echo (2 pulses)



Stimulated Echo (3 pulses)

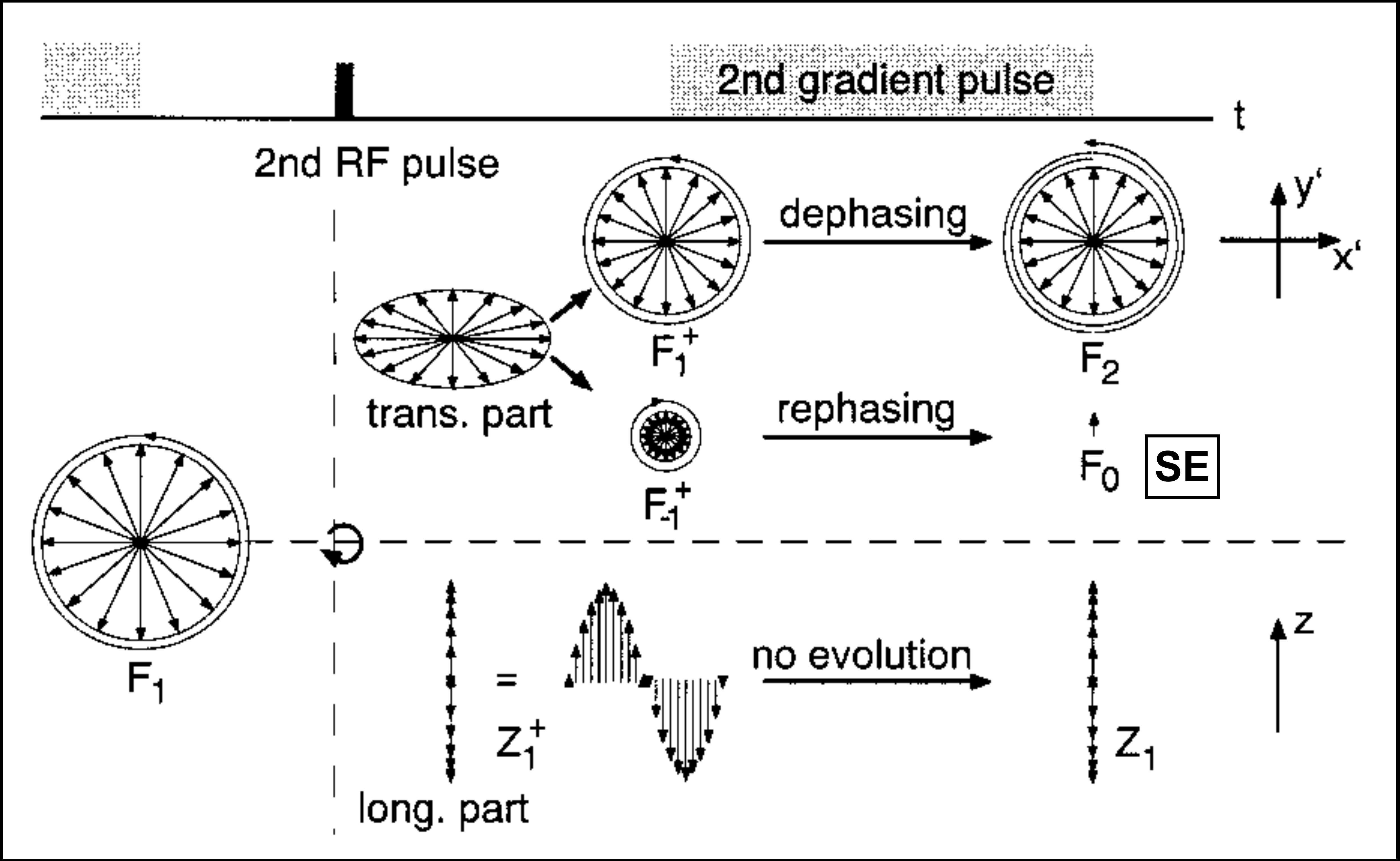


Multiple Pulse Experiments



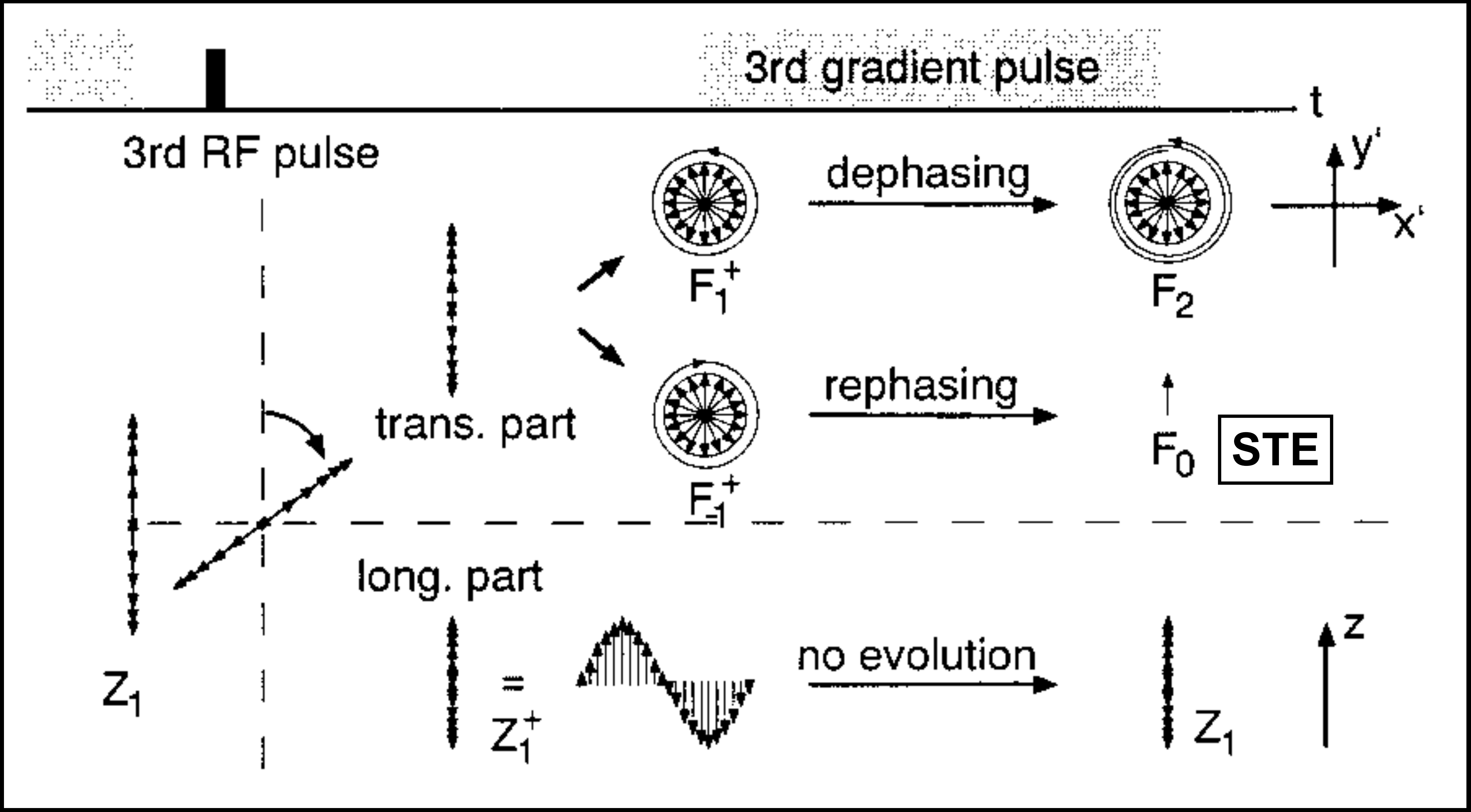
Scheffler, Concepts in MR 1999

Multiple Pulse Experiments



Scheffler, Concepts in MR 1999

Multiple Pulse Experiments



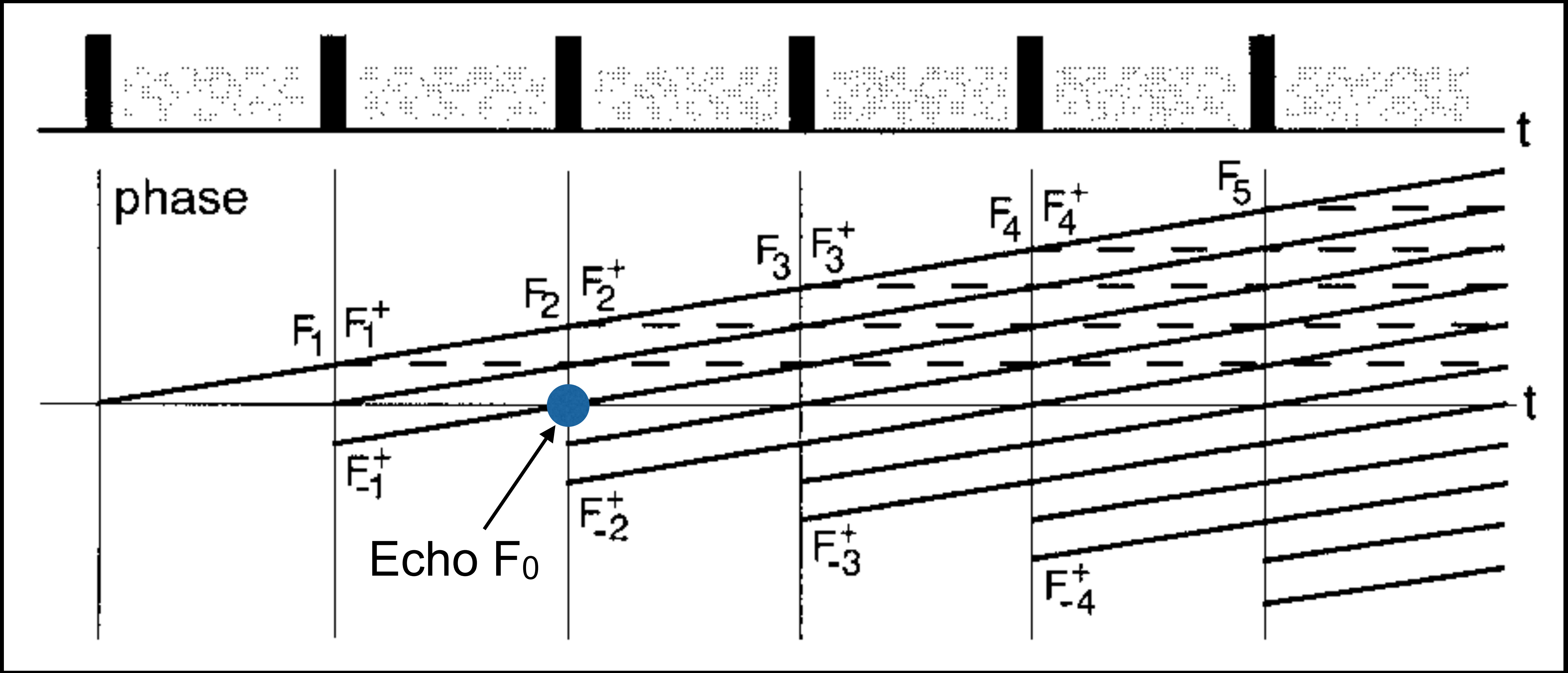
Scheffler, Concepts in MR 1999

Multiple Pulse Experiments

- RF pulses act on an ensemble of spins
  - M_z to M_{xy}
  - M_{xy} to M_z, M_{xy} and M_{xy}^*
- Transverse F states
  - $F = M_x + iM_y = F_{pos}$; $F^* = M_x - iM_y = F_{neg}$
- Longitudinal Z states

Multiple Pulse Experiments

Signal Pathways on a Phase Diagram (i.e. **EPG**)



Z states appear as broken lines; *F_0* states are echoes

Scheffler, Concepts in MR 1999

Extended Phase Graphs

- MR signal is a sum of all dephased spins
- Bloch equation
  - tracks evolution of magnetization for each spin
  - exact, but hard to visualize intuitively
- EPG
  - considers groups of spins under constant gradients
  - decomposes the spin system into several dephased states: F_k and F_{-k}; Z_k

Hennig, JMR 1988; 78:397-407

Extended Phase Graphs

- Based on Fourier space coordinate *k*

$$k_n(t) = \gamma \int_{t'=0}^{t} G_n(t')dt' = \int_{t'=0}^{t} g_n(t')dt',$$

- Magnetization represented by Fourier transforms

$$F_+(\mathbf{k}) = \int_V \{M_x(\mathbf{r}) + iM_y(\mathbf{r})\} \exp(-i\mathbf{kr})d^3r,$$
$$F_-(\mathbf{k}) = \int_V \{M_x(\mathbf{r}) - iM_y(\mathbf{r})\} \exp(-i\mathbf{kr})d^3r,$$
$$Z(\mathbf{k}) = \int_V M_z(\mathbf{r}) \exp(-i\mathbf{kr})d^3r,$$

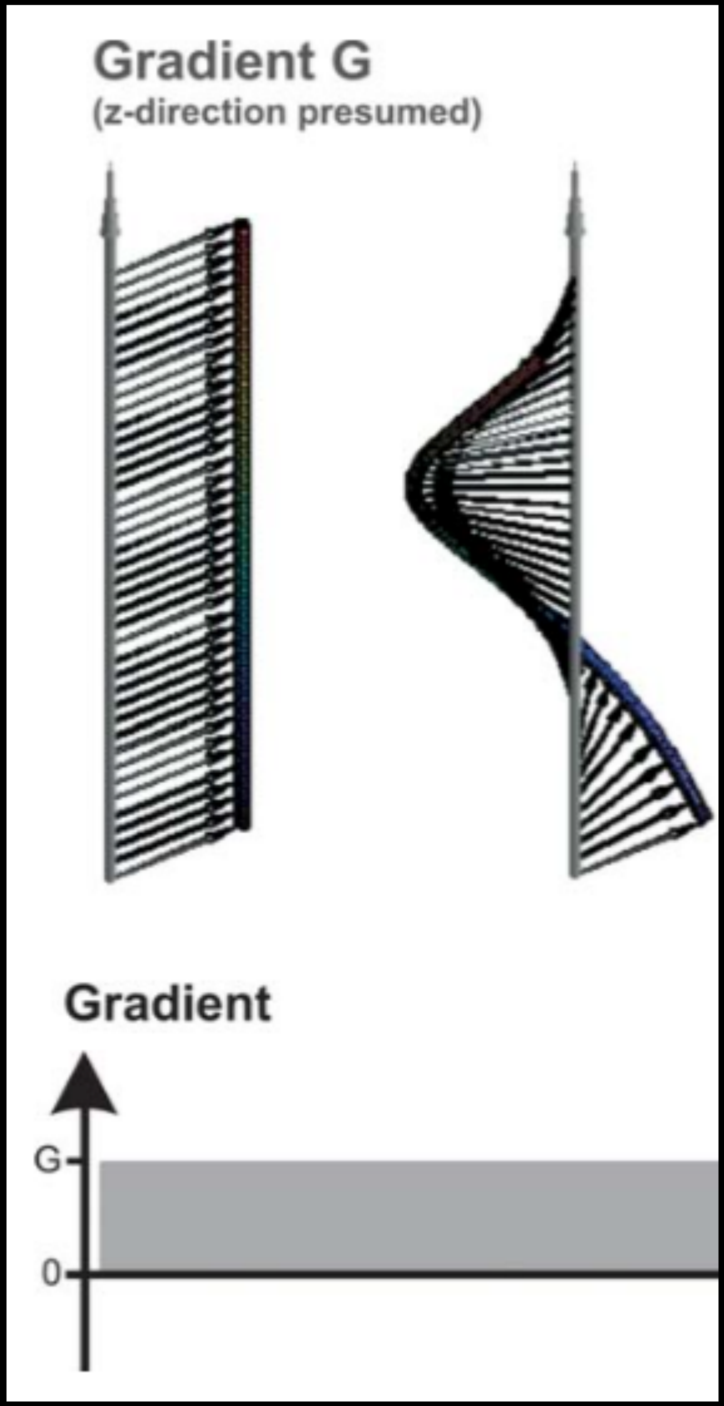
- Complete magnetization is described by vector F of various EPG partitions states with different *k*

$$\mathbf{F} = (F_0 Z_0 F_1 F_{-1} Z_1 F_2 F_{-2} Z_2 \cdots F_{+\mathrm{k}} F_{-\mathrm{k}} Z_{\mathrm{k}})^{\mathrm{T}}.$$

Gradient Dephasing

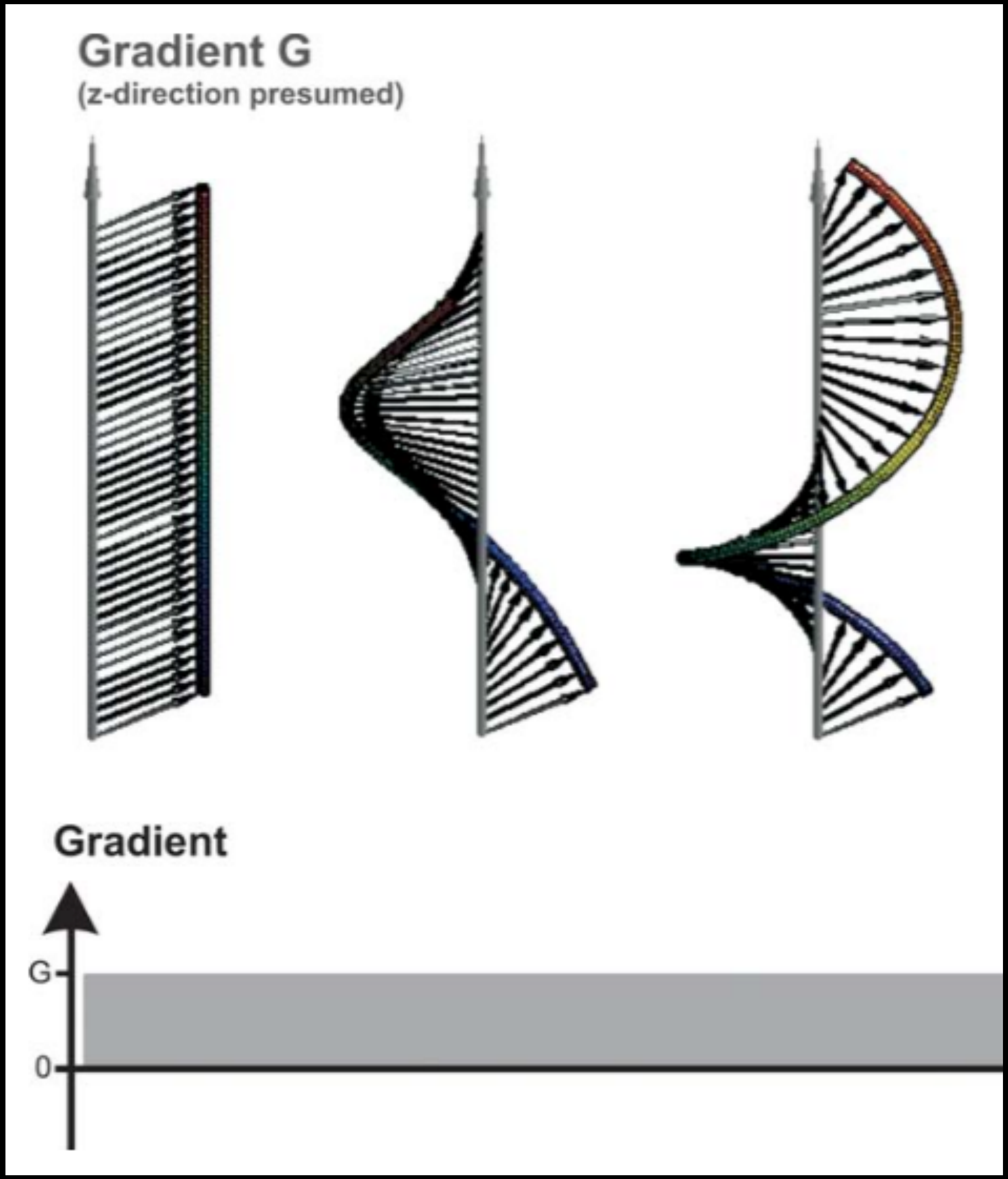
Weigel, JMRI 2015;41:266–295.

Gradient Dephasing



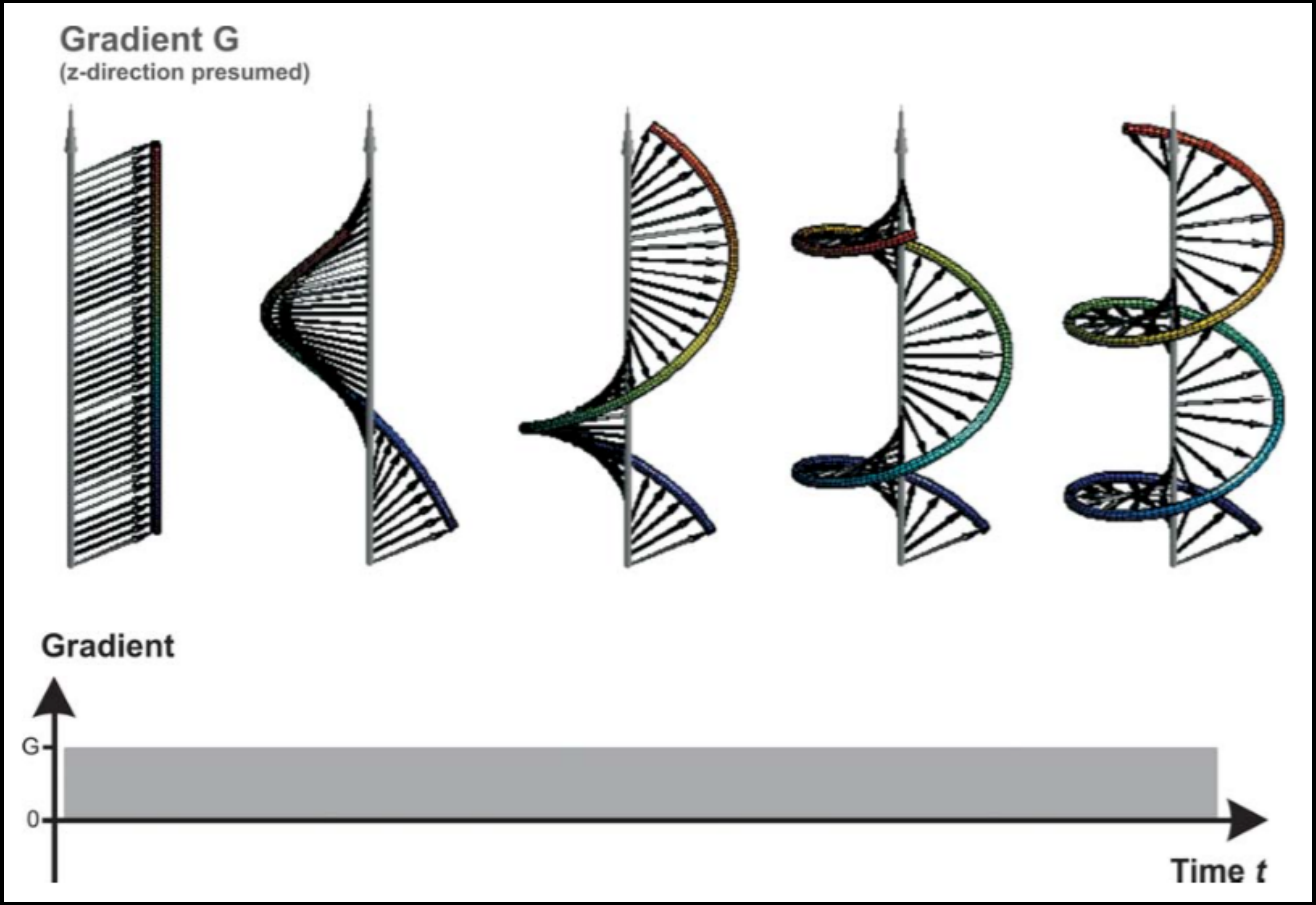
Weigel, JMRI 2015;41:266–295.

Gradient Dephasing

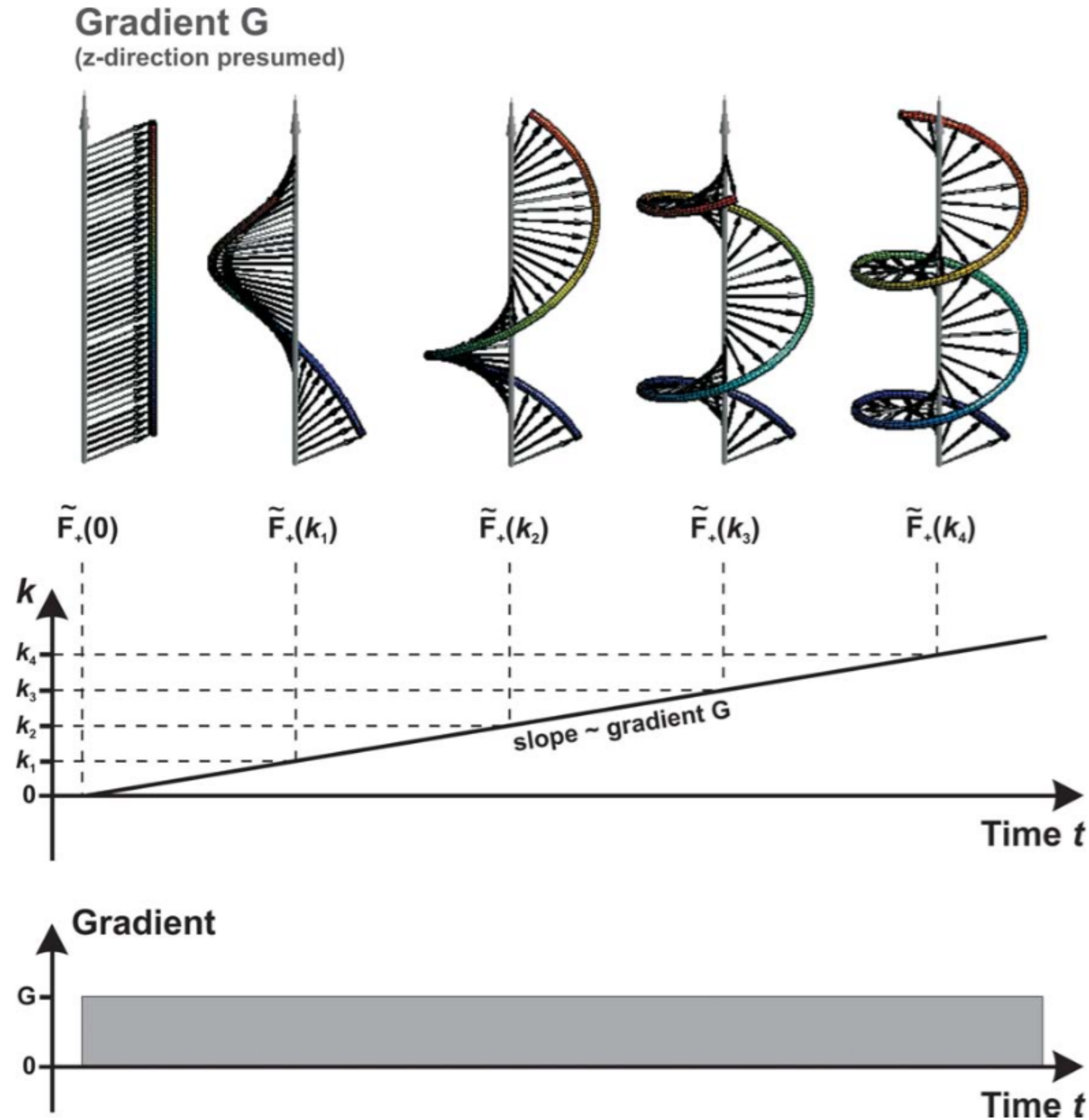


Weigel, JMRI 2015;41:266–295.

Gradient Dephasing

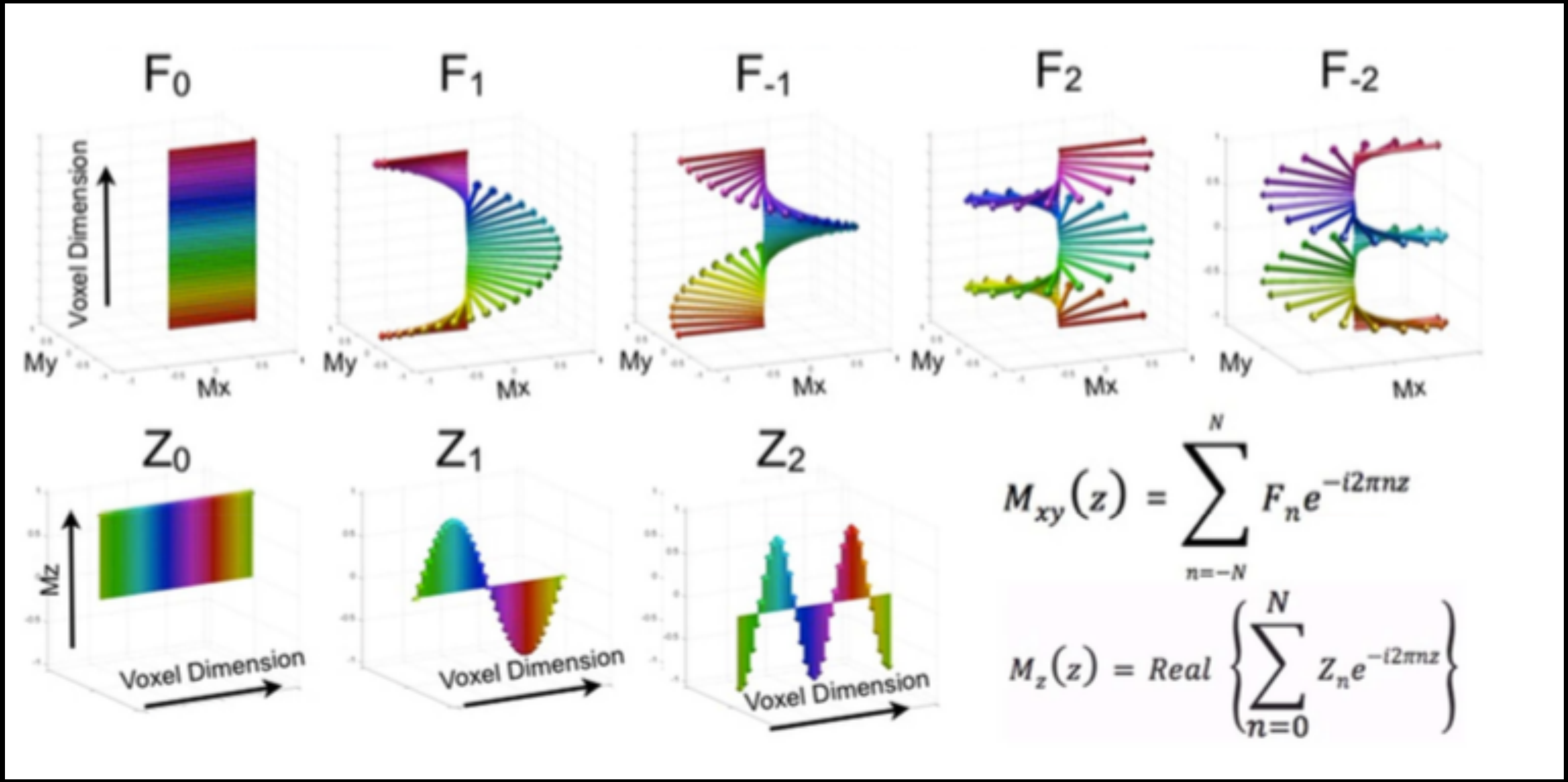


Weigel, JMRI 2015;41:266–295.



Gradient G
(z-direction presumed)
$\tilde{F}_+(0)$
$\tilde{F}_+(k_1)$
$\tilde{F}_+(k_2)$
$\tilde{F}_+(k_3)$
$\tilde{F}_+(k_4)$
k
k_4
k_3
k_2
k_1
0
slope ~ gradient G
Time t
Gradient
G
0
Time t

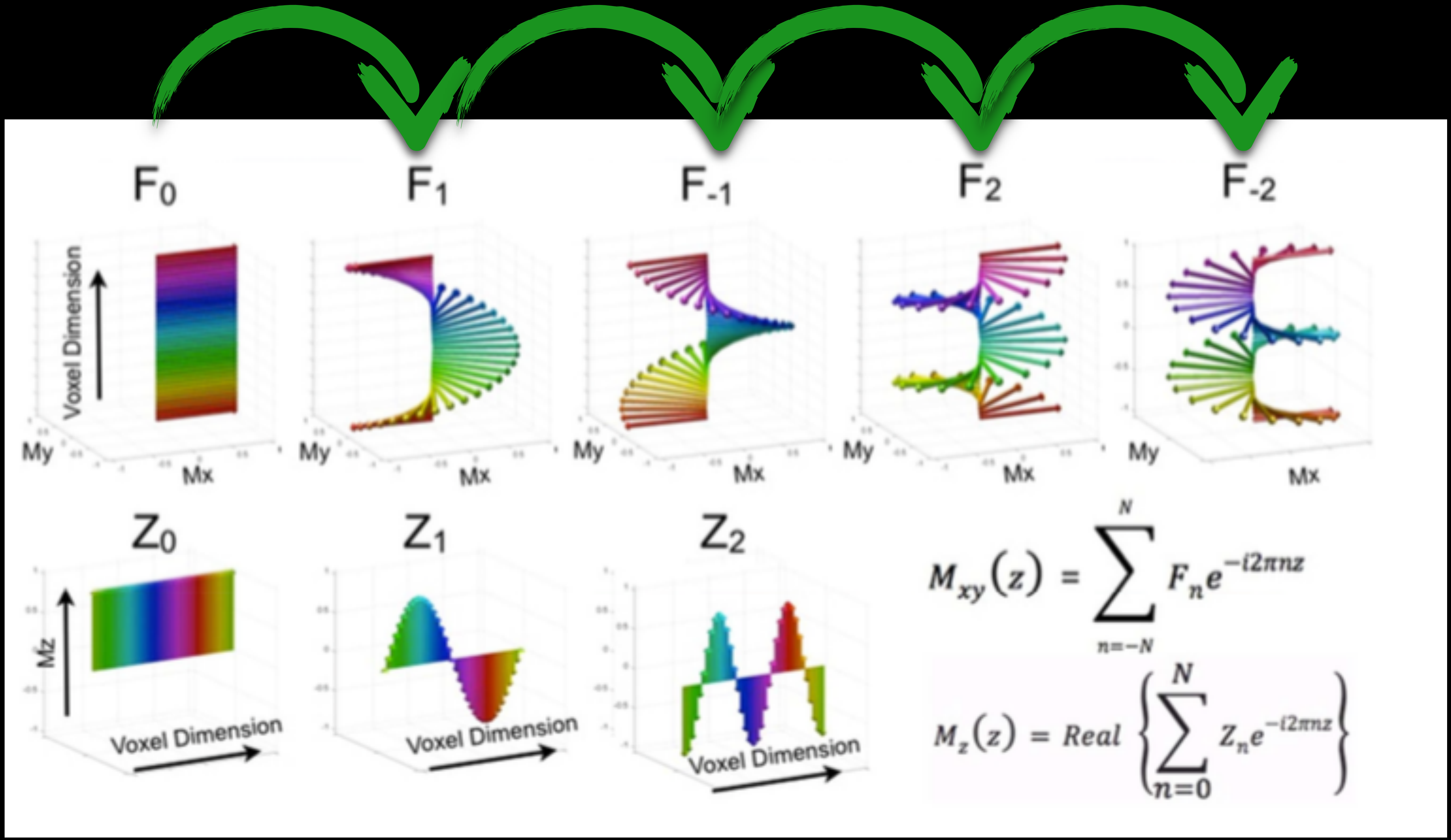
Gradient Dephasing



Brian Hargreaves and Karla Miller ISMRM 2013: Educational E-Poster #3718

“Discrete” Gradient Dephasing

transition between states

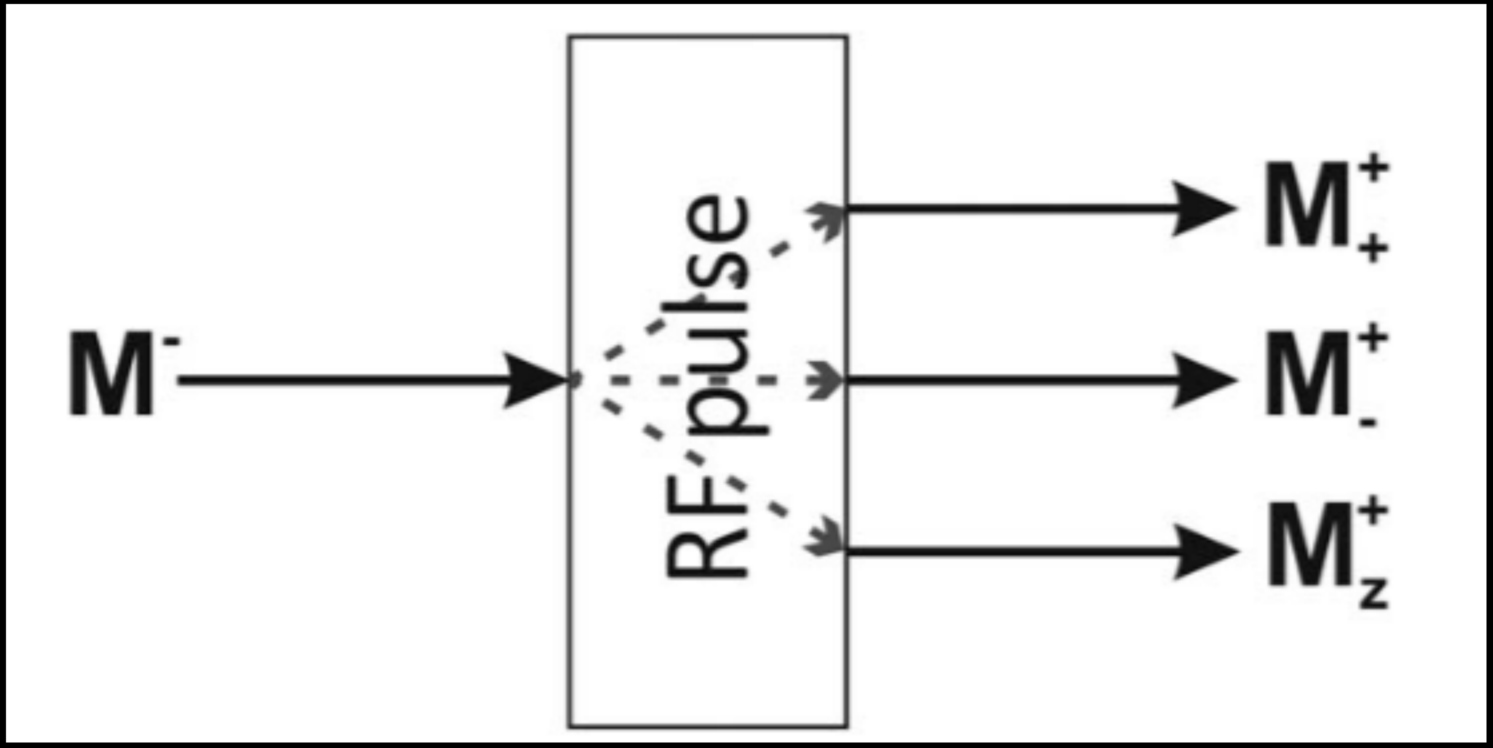


k is the number of twists/cycles across a voxel

Brian Hargreaves and Karla Miller ISMRM 2013: Educational E-Poster #3718

RF Pulse

- Woessner Decomposition
  magnetization after an RF pulse can be regarded as a composition of 3 components:
  - transversal component that is unaffected (0^o-pulse)
  - transversal component that is refocused (180^o-pulse)
  - a longitudinal component



rephasing

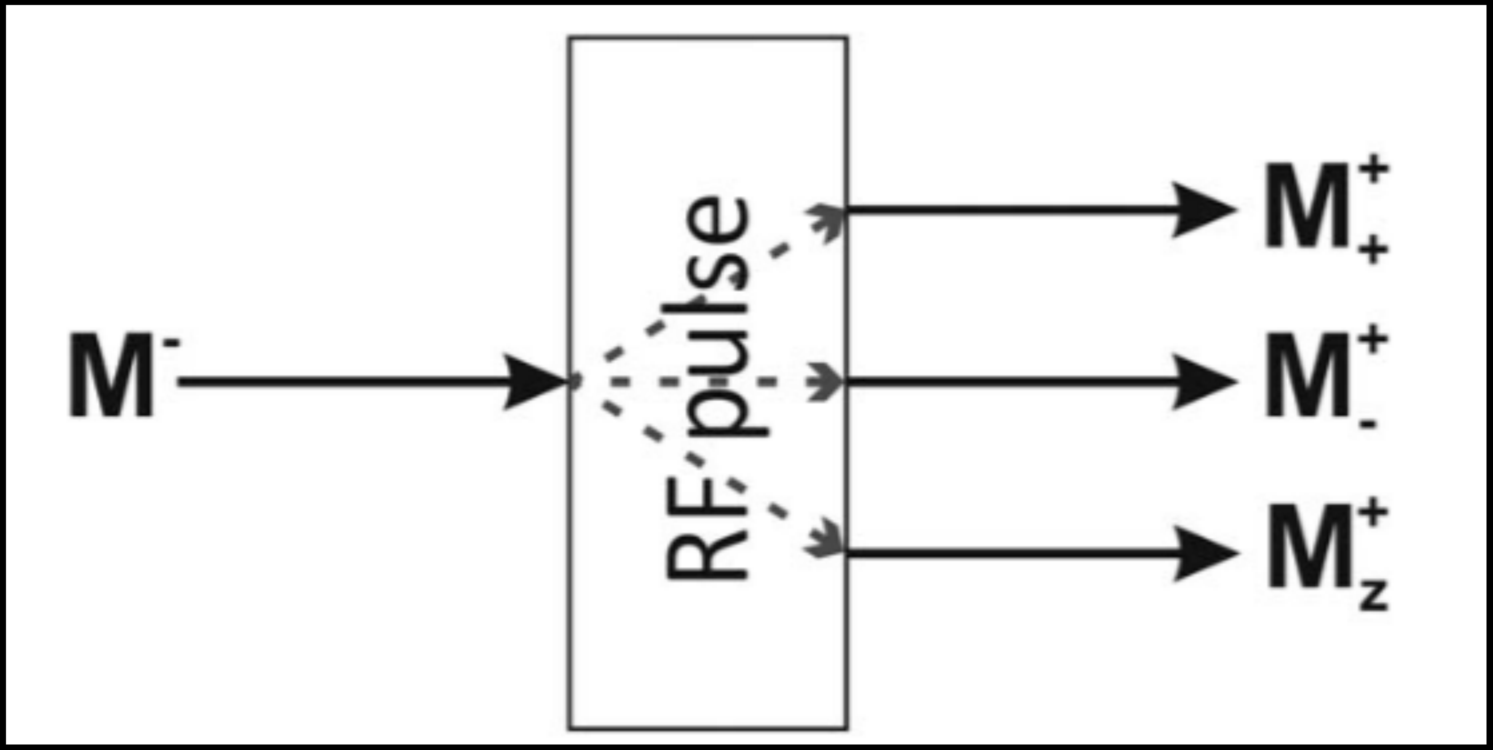
dephasing

longitudinal

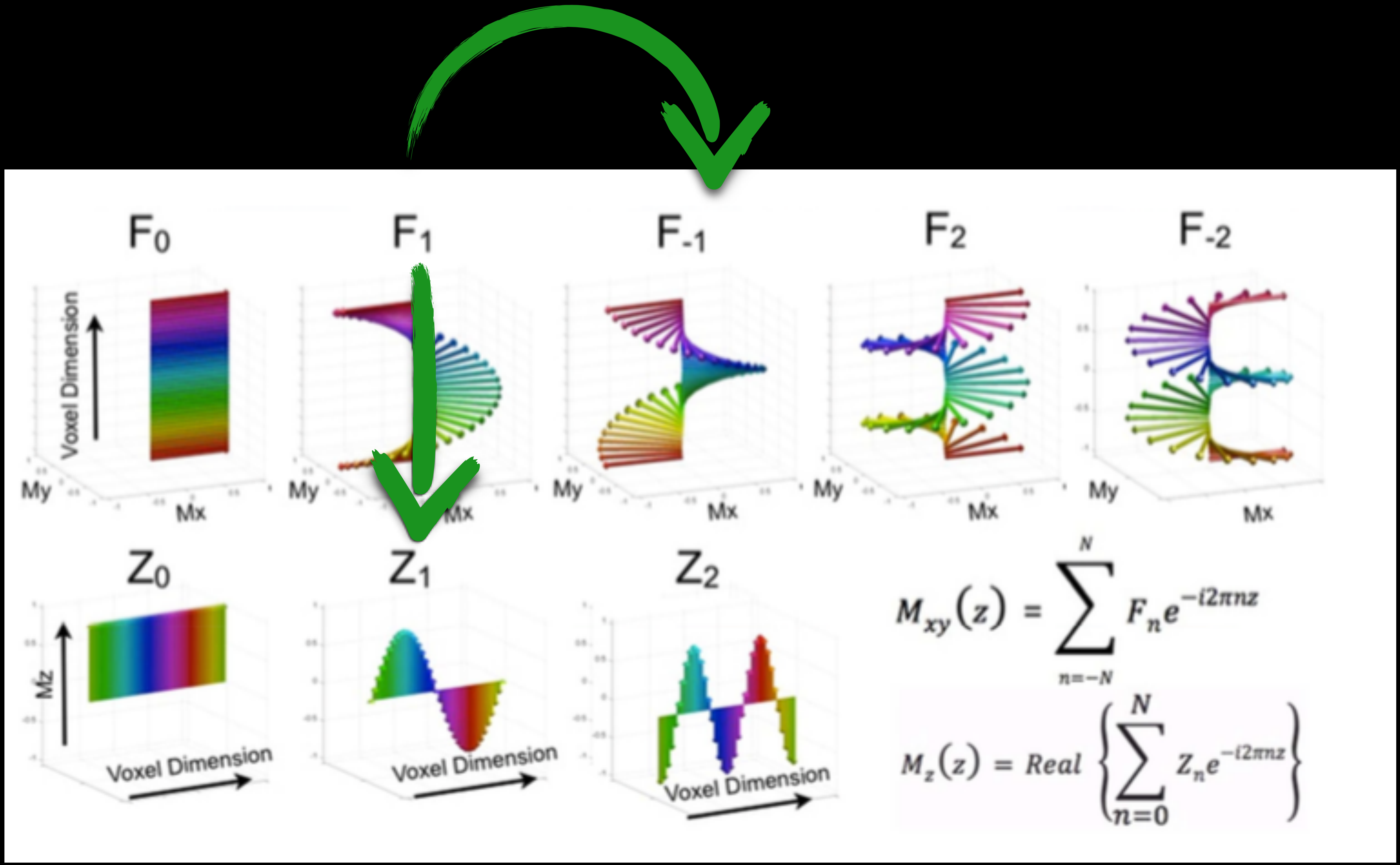
Woessner DE. J Chem Phys 1961; 34: 2057–2061.

RF Pulse

- The RF pulse operator splits any given EPG state with dephasing order *k* into 3 different new states:
  - a transversal state with identical *k*
  - a transversal state with inverted *k*
  - a longitudinal state with identical *k*



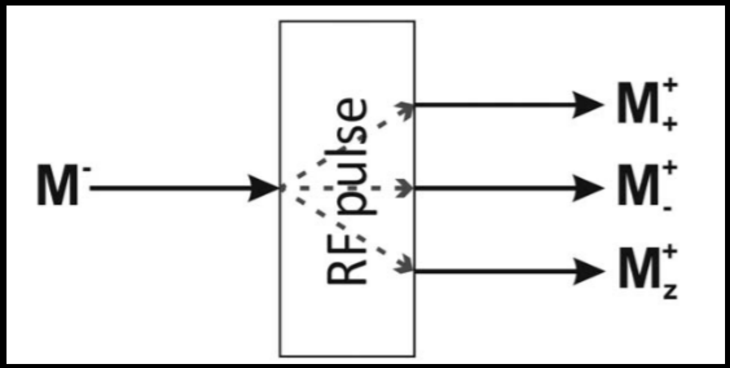
RF Pulse

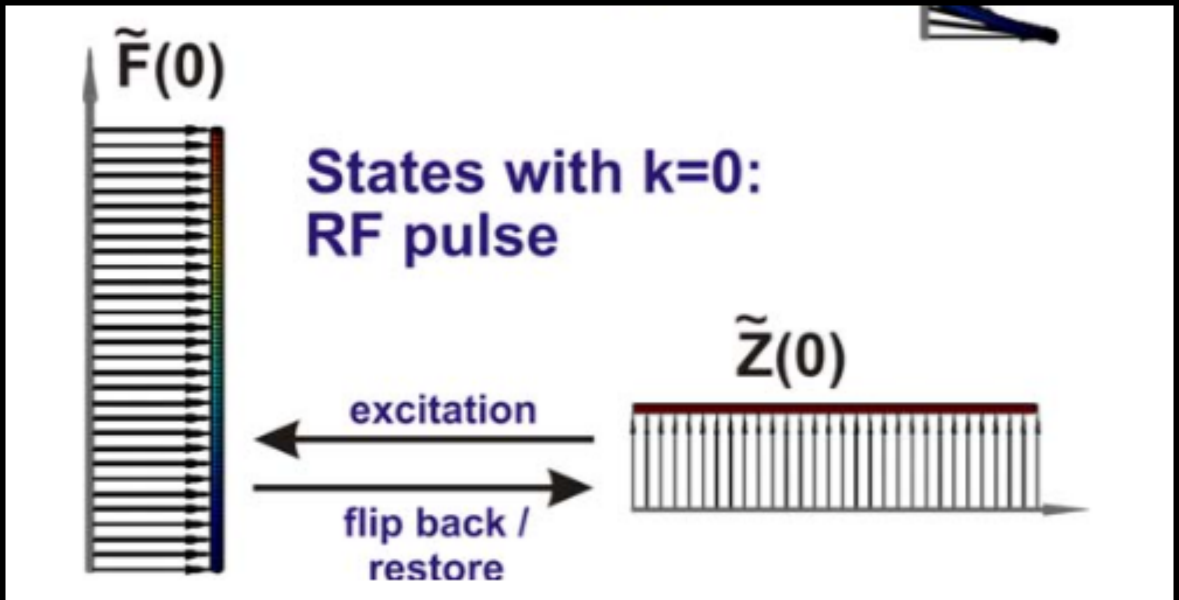


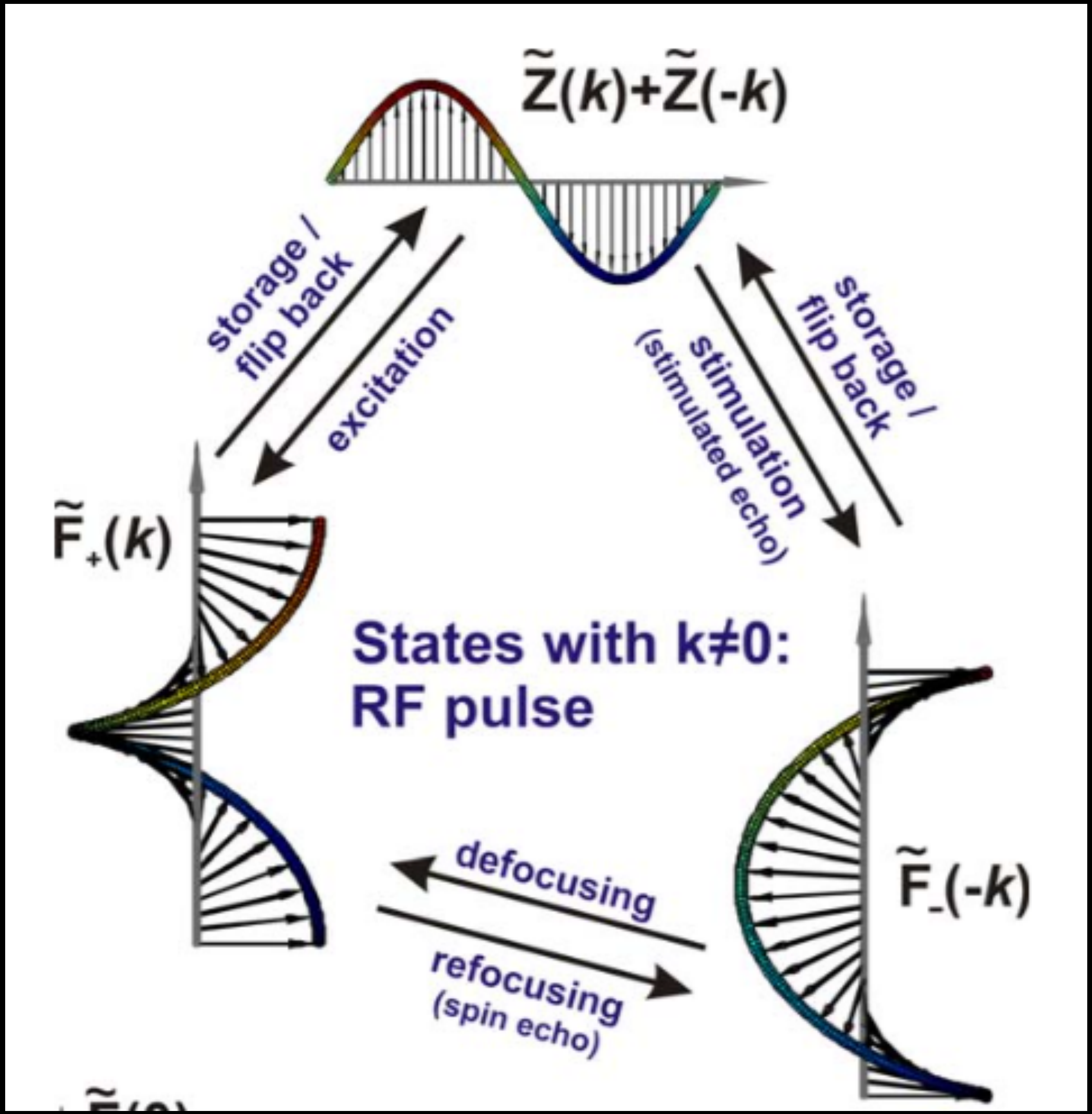
mixes *F* and *Z* states!

Brian Hargreaves and Karla Miller ISMRM 2013: Educational E-Poster #3718

RF Pulse





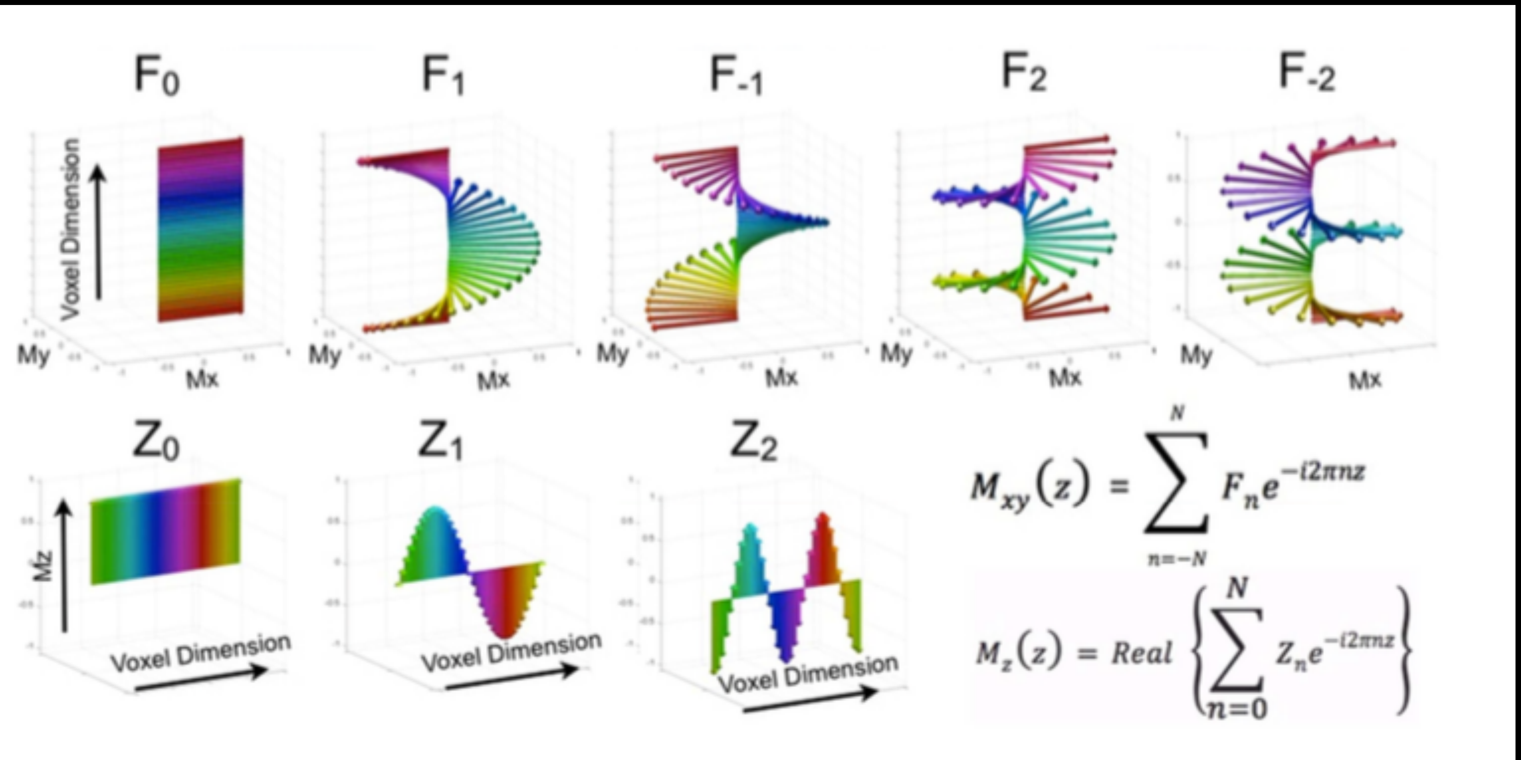


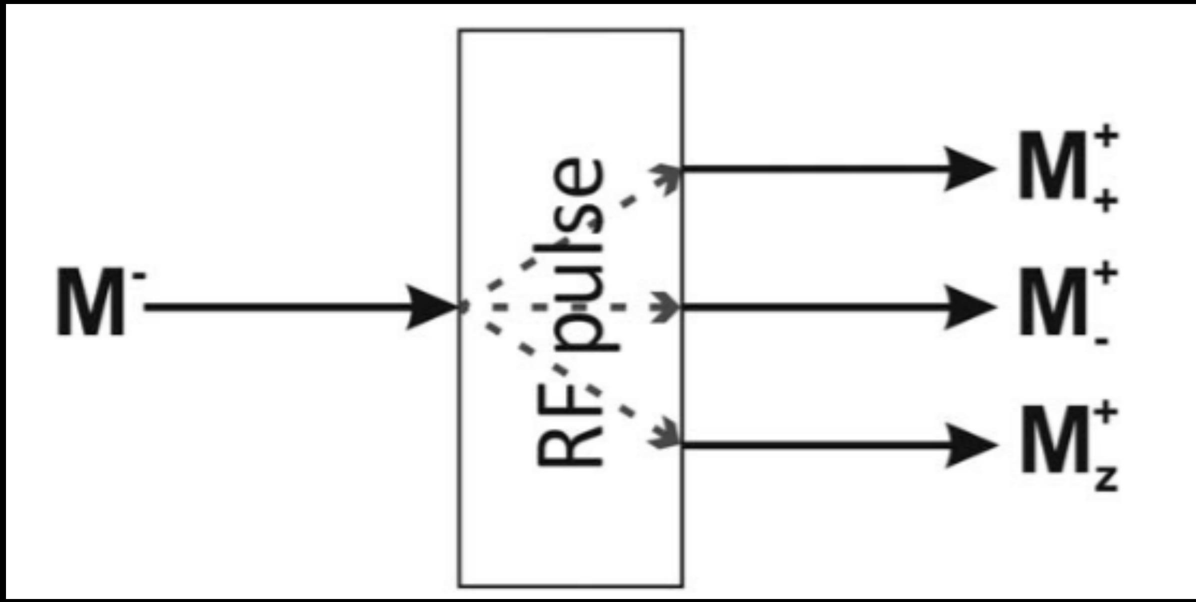
Weigel, JMRI 2015;41:266–295.

EPG Concept Summary

Fourier based configuration states

RF pulse partitioning

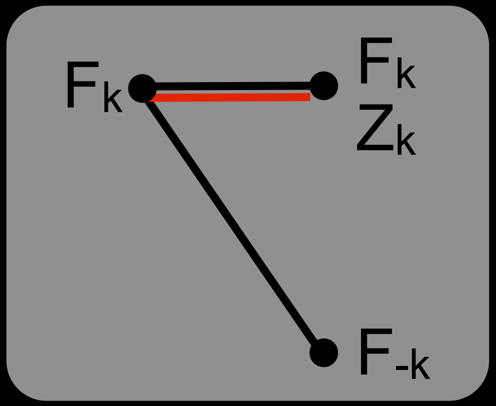




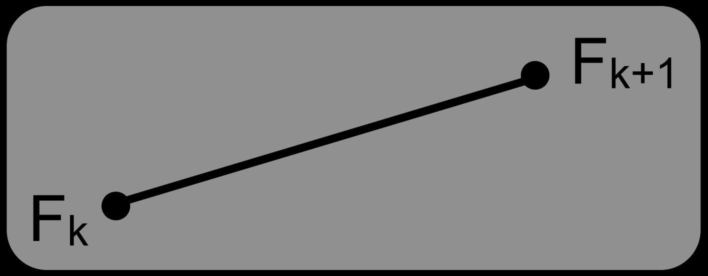
Phase graph approach that depicts the evolution of a complete isochromat ensemble.

EPG "Calculus"

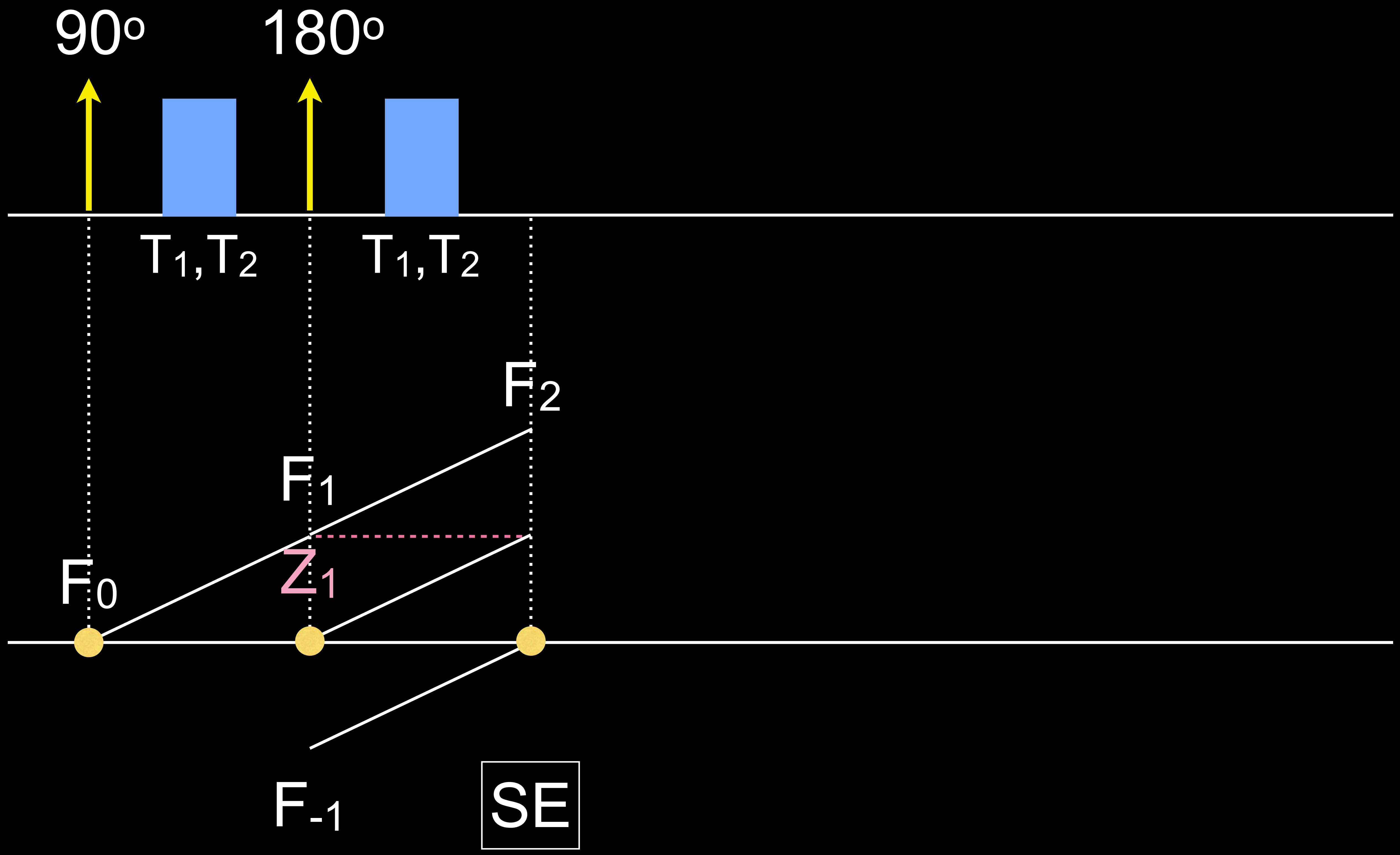
- RF pulse for state *k*:
  - Produces signal in longitudinal state *k* and transverse states *k* and -*k*



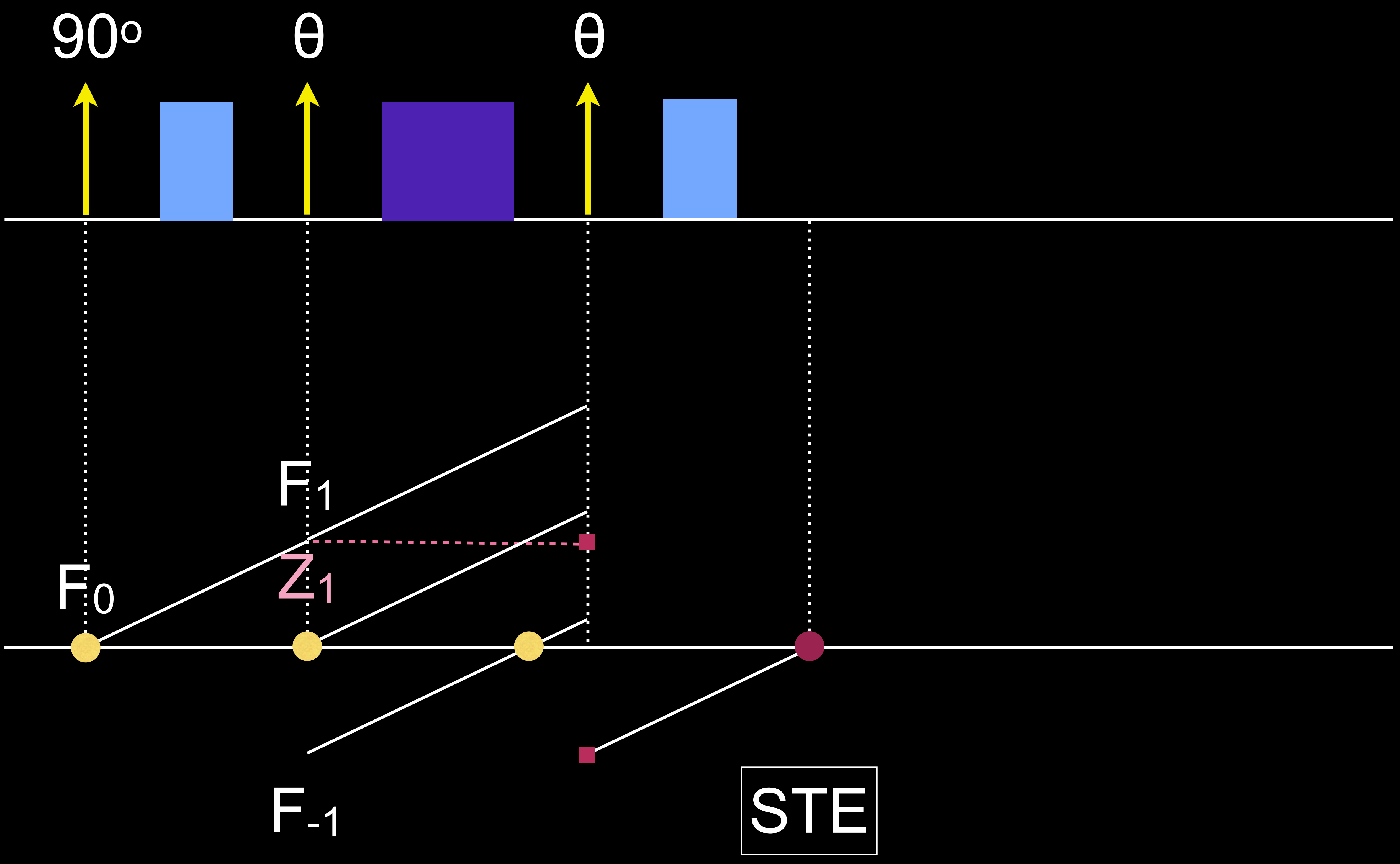
- Gradient dephaser for state *k*:
  - Moves transverse magnetization to *k*+1
  - Does not affect longitudinal magnetization



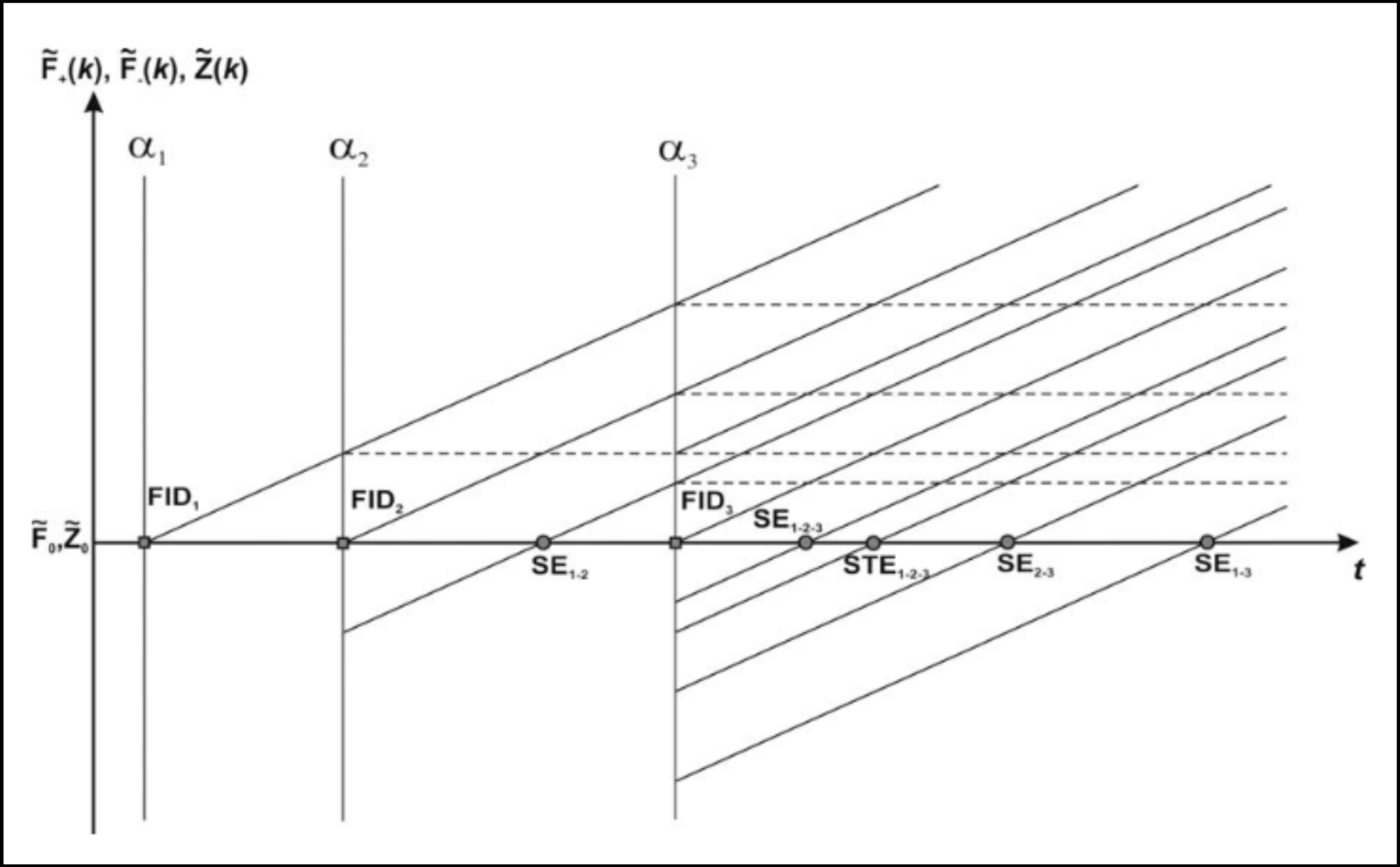
EPG: Spin Echo



EPG: Stimulated Echo

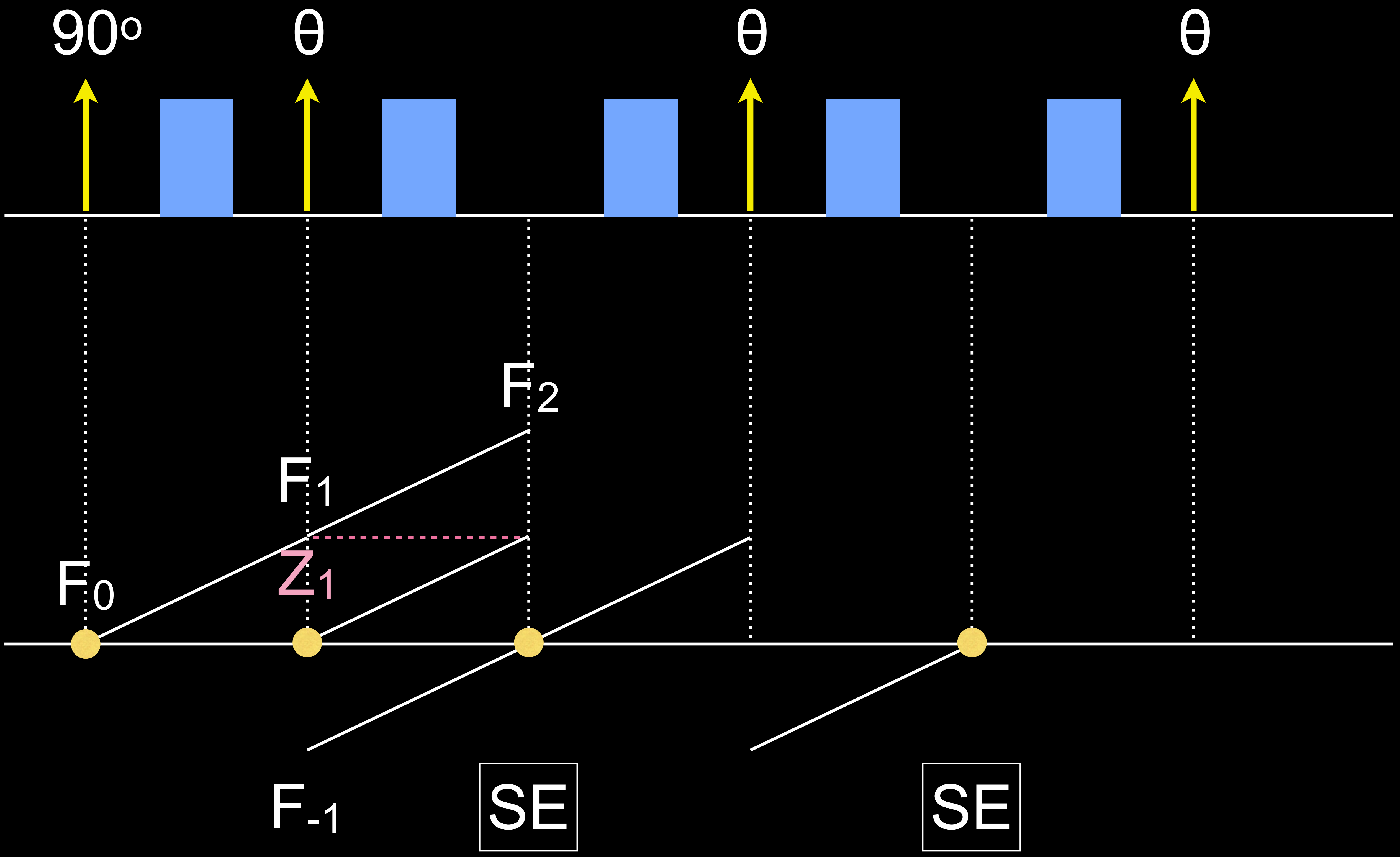


EPG: 3-Pulse Experiment

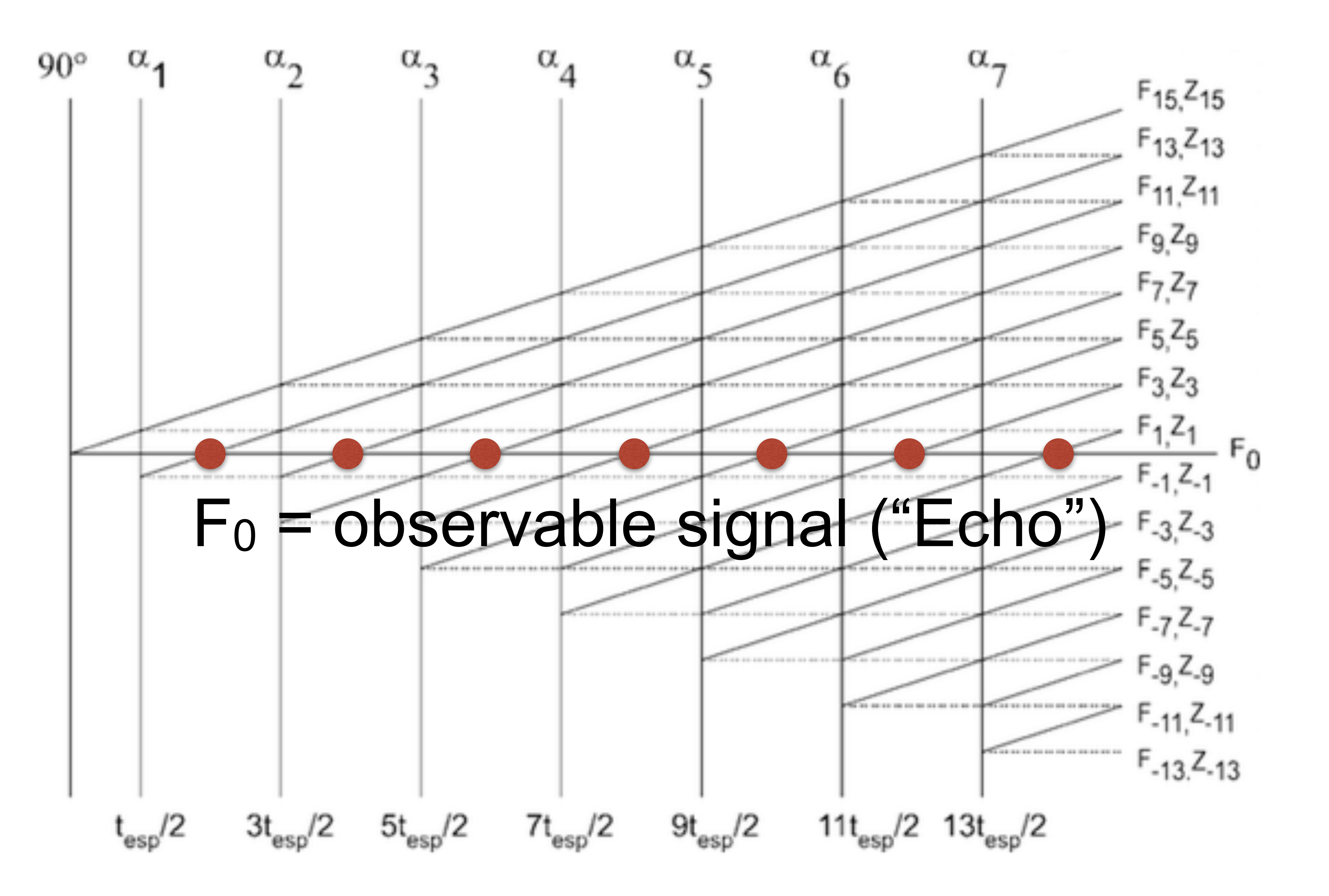


Weigel, JMRI 2015;41:266–295.

EPG: Train of Spin Echo



EPG: CPMG



F_0 = observable signal ("Echo")

EPG: Matrix formulation

- Phase states
  - Can represent as a matrix:

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & \\ F_0^* & F_{-1} & F_{-2} & \cdots \\ Z_0 & Z_1 & Z_2 & \end{bmatrix}$$

EPG: Matrix formulation

- RF pulses
  - invert state (e.g., F_3 to F_{-3}) or can transfer between F and Z states
  - Simple pre-multiplication P' = RP, where R is

$$\begin{pmatrix} \cos^2\frac{\alpha}{2} & e^{2i\phi}\sin^2\frac{\alpha}{2} & -ie^{i\phi}\sin\alpha \\ e^{-2i\phi}\sin^2\frac{\alpha}{2} & \cos^2\frac{\alpha}{2} & ie^{-i\phi}\sin\alpha \\ -\frac{i}{2}e^{-i\phi}\sin\alpha & \frac{i}{2}e^{i\phi}\sin\alpha & \cos\alpha \end{pmatrix}$$

for an RF pulse with flip angle α and phase ϕ

Scheffler, Concepts in MR 1999

EPG: Matrix formulation

- Gradients (in discretized units)
  - Increase number of states by 1
  - Replace all F_k states with F_{k-1} (e.g., F_0 becomes F_1)
  - Replace F_0 using F_0^*
  - Do not change Z states

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & & 0 \\ F_0^* & F_{-1} & F_{-2} & \cdots & 0 \\ Z_0 & Z_1 & Z_2 & & 0 \end{bmatrix}$$

phase states grow linearly w.r.t. TSE ETL

EPG: Matrix formulation

- Relaxation
  - Transverse:
    All *F* states attenuated by $E_2 = \exp(-T/T_2)$
  - Longitudinal:
    All *Z* states attenuated by $E_1 = \exp(-T/T_1)$
    Z_0 state only has recovery of $M_0(1-E_1)$

EPG: Extensions

- Non-ideal slice profiles
- Variable RF flip angle and phase
- Motion / flow effects
- Diffusion effects
  - Weigel M, et al., JMR 2010; 205: 276-285

EPG Simulation

- Phase state propagation
  - RF pulse
  - T_1, T_2 decay
  - free precession
  - gradient pulse

EPG Simulation

Phase states:

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & \cdots \\ F_0* & F_{-1} & F_{-2} & \cdots \\ Z_0 & Z_1 & Z_2 & \cdots \end{bmatrix}$$

RF pulse (θ, ϕ), P^+ = RP:

$$R_{\{\theta,\phi\}} = \begin{bmatrix} \cos^2\frac{\theta}{2} & e^{2i\phi}\sin^2\frac{\theta}{2} & -ie^{i\phi}\sin\theta \\ e^{-2i\phi}\sin^2\frac{\theta}{2} & \cos^2\frac{\theta}{2} & ie^{-i\phi}\sin\theta \\ -\frac{i}{2}e^{-i\phi}\sin\theta & \frac{i}{2}e^{i\phi}\sin\theta & \cos\theta \end{bmatrix}$$

EPG Simulation

Gradients:

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & \cdots \\ F_0* & F_{-1} & F_{-2} & \cdots \\ Z_0 & Z_1 & Z_2 & \cdots \end{bmatrix}$$

Relaxation:

$$F_k \longrightarrow E_2 \, F_k$$

$$Z_k \longrightarrow E_1 \, Z_k \qquad (k>0)$$

$$Z_0 \longrightarrow E_1 \, Z_0 + \mathrm{M}_0(1 - E_1)$$

EPG Simulation

- Transient state; steady state
- Different seq/tissue params

- Brian's MATLAB EPG sim code
  - will be emailed to class email list

EPG Simulation

- *<u>Example</u>*: Turbo Spin Echo
  - epg_rf.m
  - epg_grelax.m, epg_grad.m, epg_mgrad.m
  - epg_cpmg_hhw.m
  - EPGSim_CPMG_hhw.m

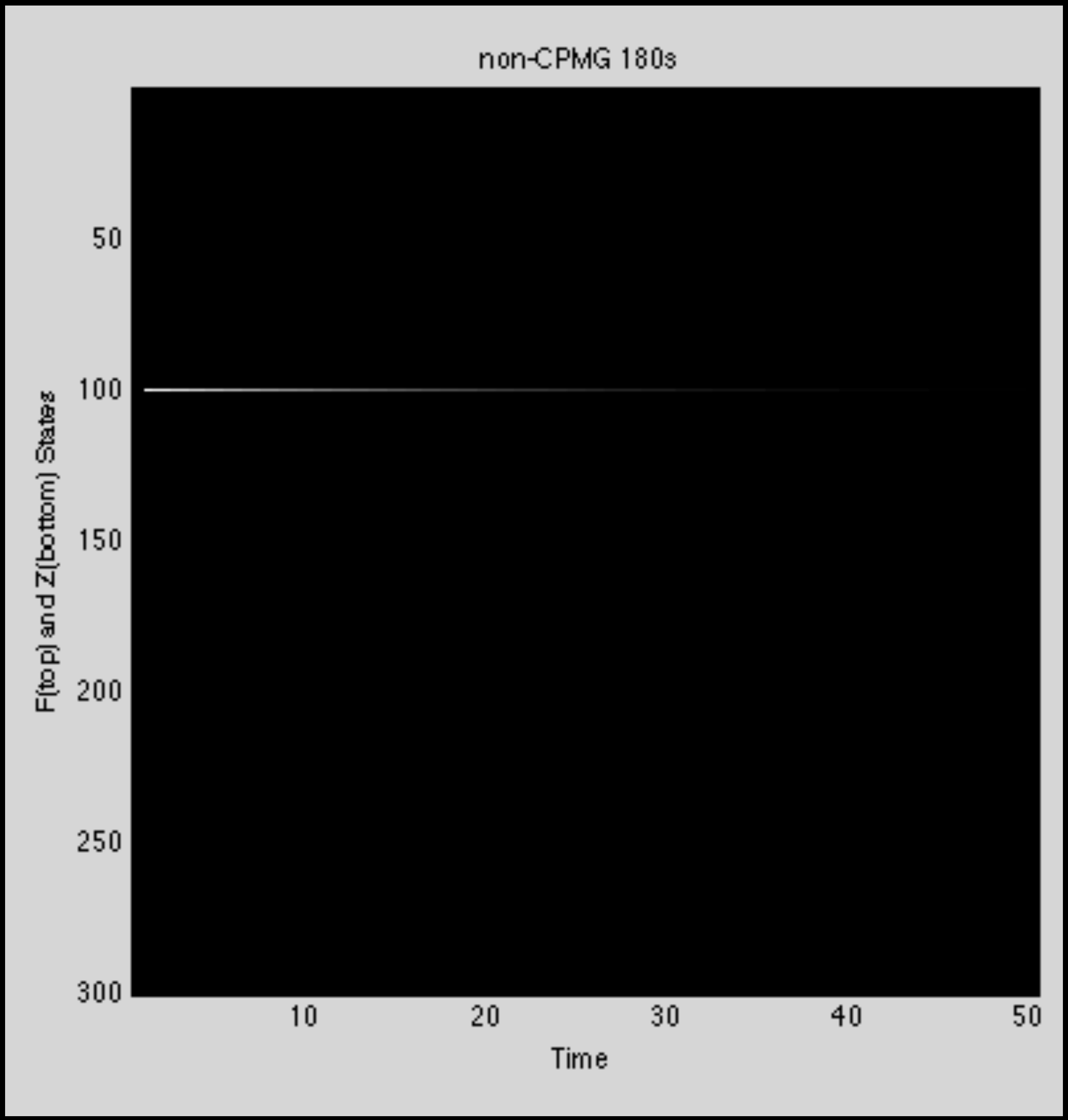
  - can look at different refocusing RF trains

EPG Simulations: TSE

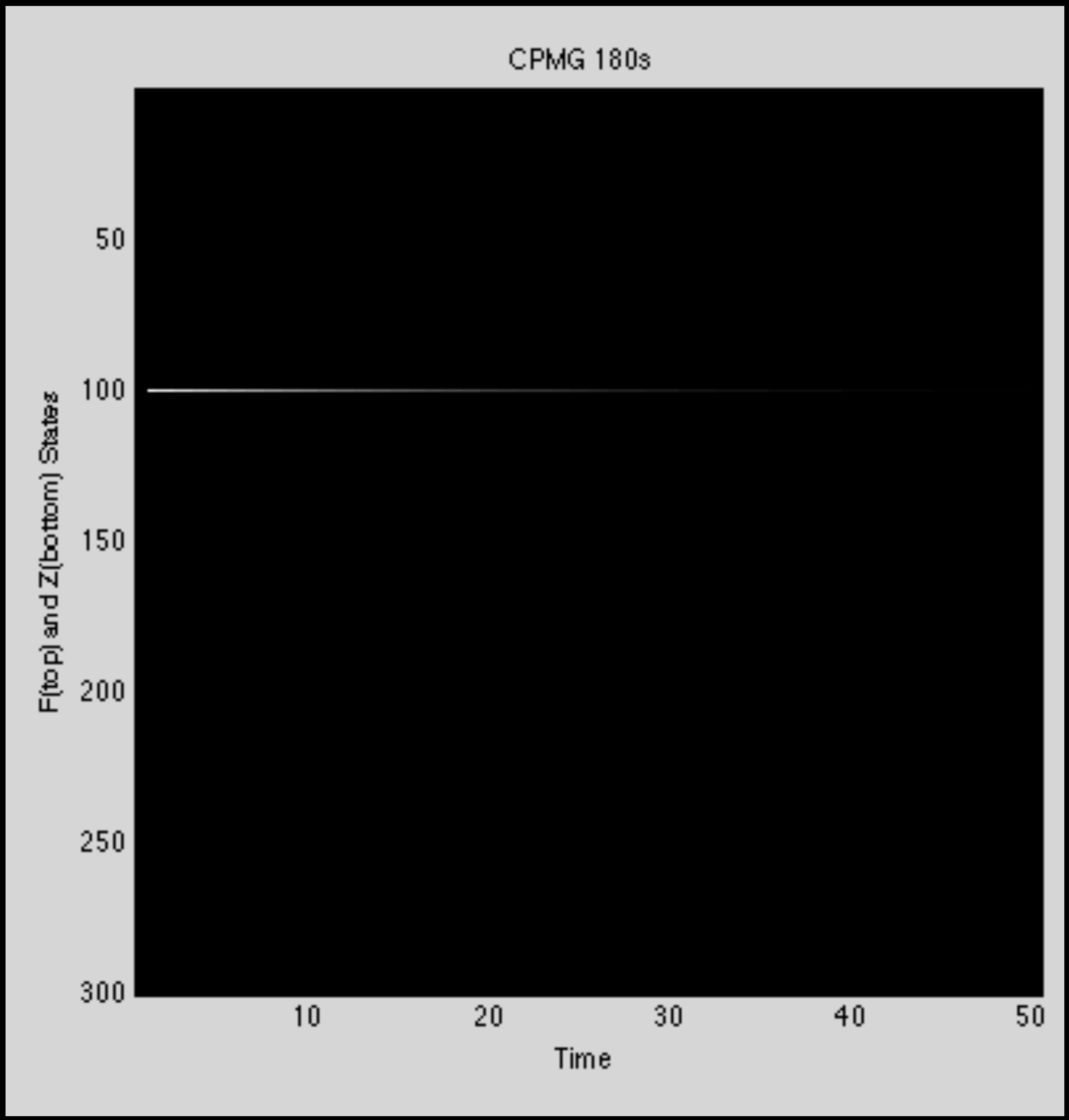
- non-CPMG 180s: 90x-180x-180x-…
- CPMG 180s: 90x-180y-180y-…
- non-CPMG 120s: 90x-120x-120x-…
- CPMG 120s: 90x-120y-120y-…
- CPMG 120s +prep: 90x-150y-120y-…

EPG Simulations: TSE

non-CPMG 180s



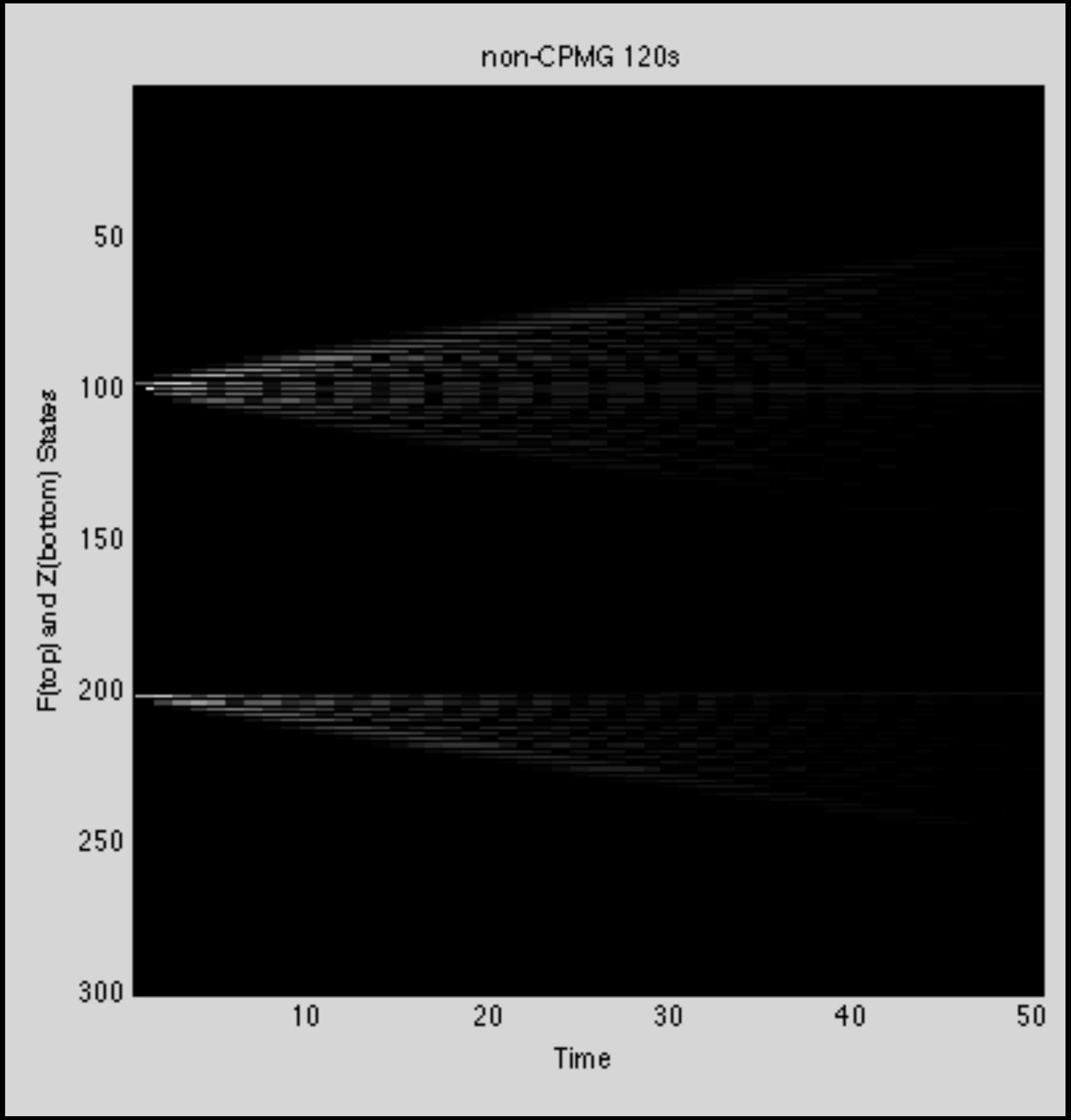
CPMG 180s



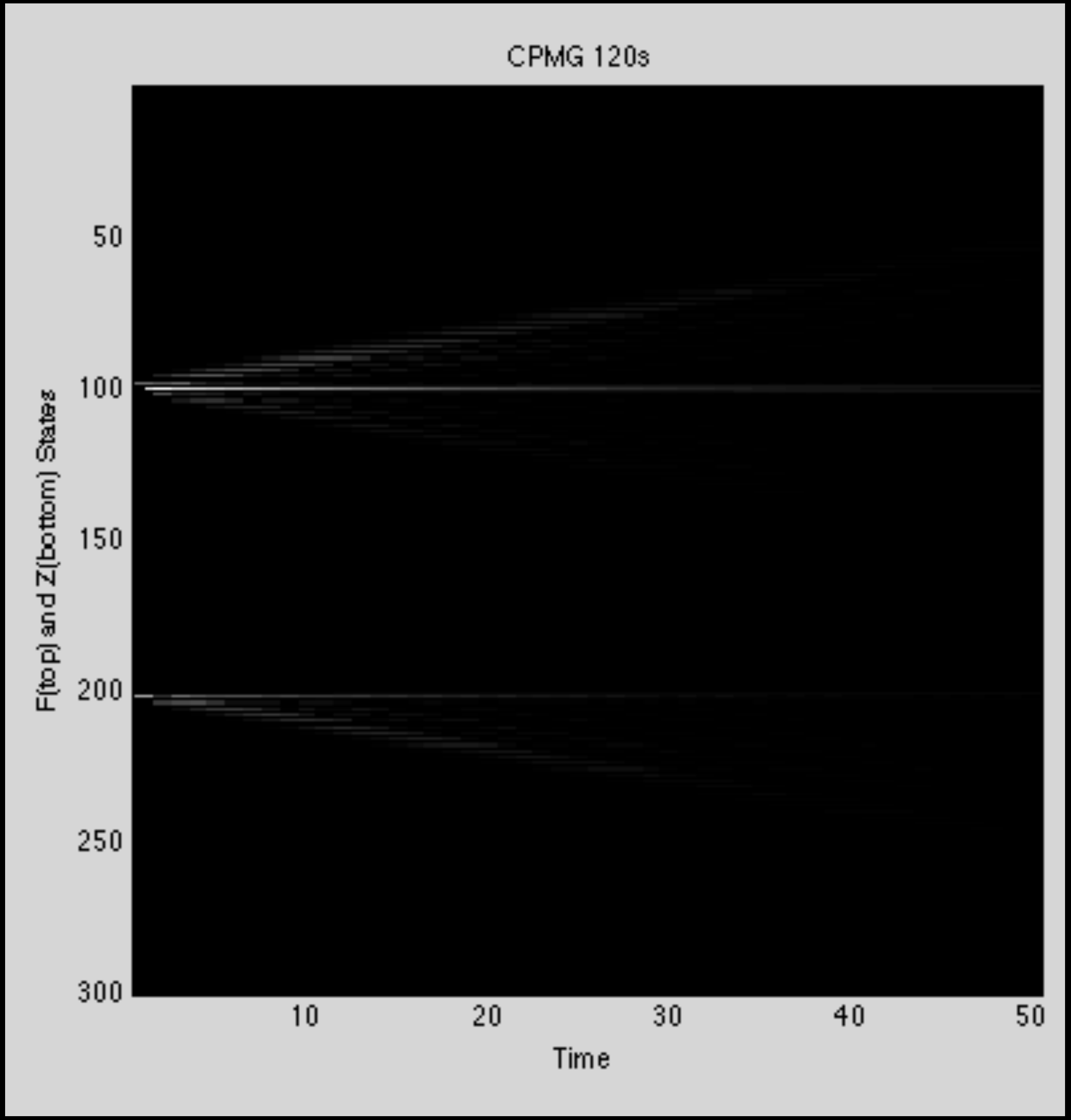
T_1 = 1000 ms, T_2 = 100 ms, ETL = 50, ESP = 10 ms

EPG Simulations: TSE

non-CPMG 120s



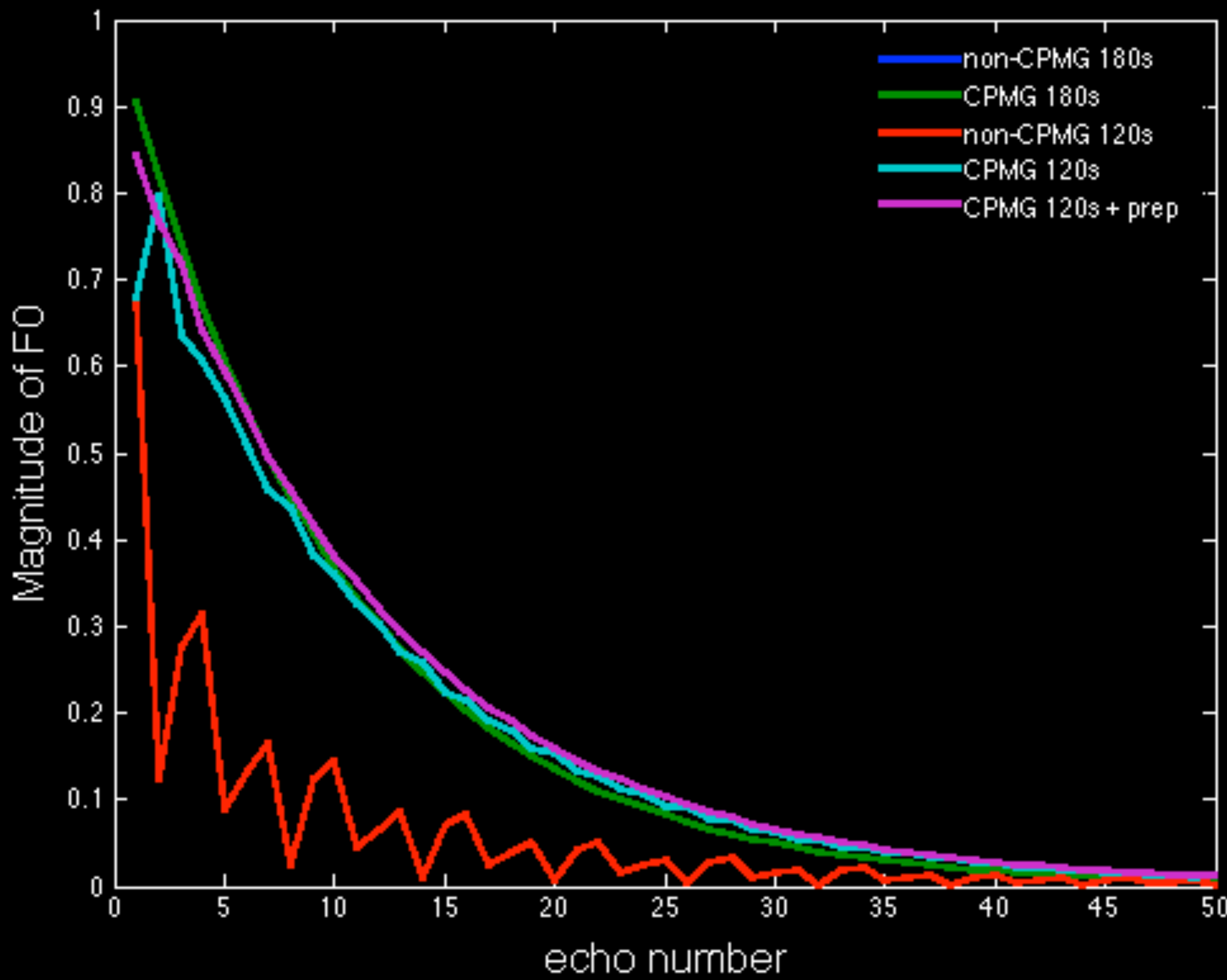
CPMG 120s



T_1 = 1000 ms, T_2 = 100 ms, ETL = 50, ESP = 10 ms

EPG Simulations: TSE

F_0 vs. echo number



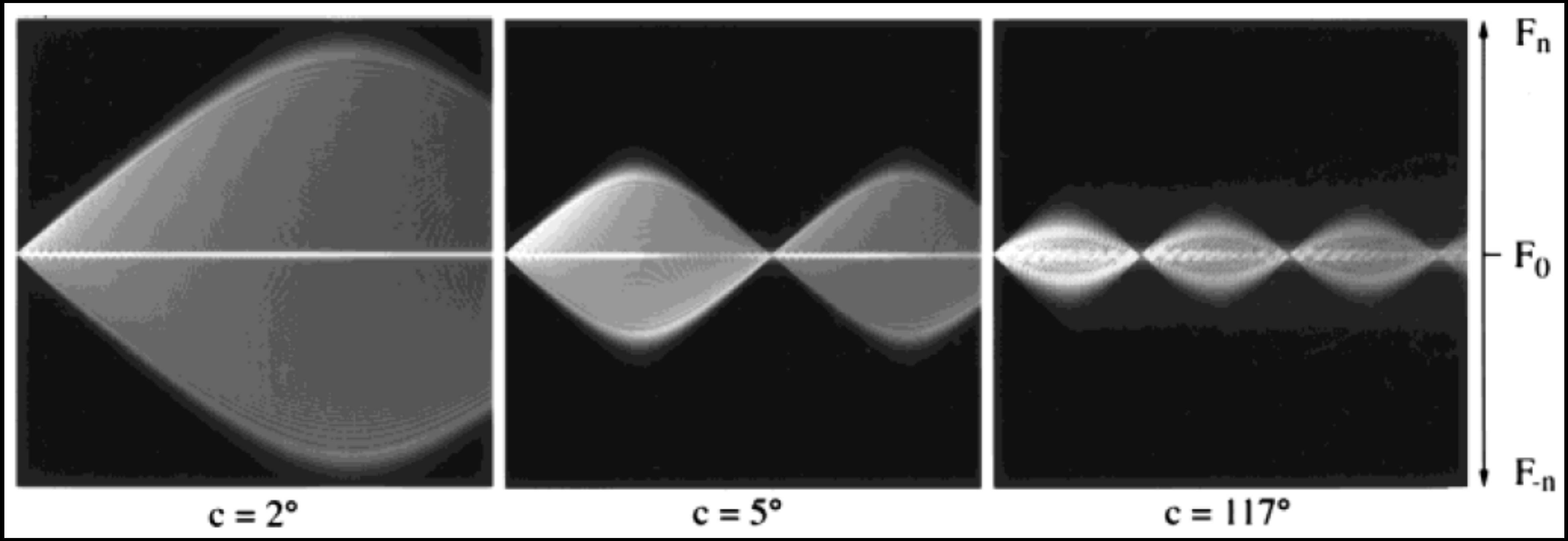
T_1 = 1000 ms, T_2 = 100 ms, ETL = 50, ESP = 10 ms

EPG Simulation

- Homework 1, part 2A
  - Gradient-spoiled GRE (SSFP-FID)

EPG Simulation

- Homework 1, part 2B
  - RF-spoiled GRE



Scheffler, Concepts in MR 1999, Fig. 11

Homework 1

- Pulse Sequence Simulations
  - 1. Bloch: Steady state comparison, bSSFP transient state and catalyzation
  - 2. EPG: SSFP-FID, RF-spoiled GRE
- Due 5 pm, Mon, 4/21 by email
  - PDF and MATLAB code

Summary

- Multiple RF pulses -> multiple echoes
- EPG analysis
  - consider groups of spins
  - explicit treatment of pathways and echoes
  - flexible and powerful
  - you can do it!

Thanks!

- Web resources
  - ISMRM 2010 Edu: Miller, Weigel
  - ISMRM 2011 Edu: Miller, Weigel
- Further reading
  - Bernstein et al., Handbook of MRI Sequences
  - Haacke et al., Magnetic Resonance Imaging
  - Scheffler, Concepts in MR 1999; 11:291-304
  - Hennig, JMR 1988; 78:397-407
  - Weigel, JMRI 2015; 41:266-295

Thanks!

- Acknowledgments
  - Brian Hargreaves's EPG slides and code
  - Kyung Sung's EPG slides
  - Isabel Dregely's EPG slides

Holden H. Wu, Ph.D.

HoldenWu@mednet.ucla.edu

http://mrrl.ucla.edu/wulab