RF Pulse Design:
RF Pulses, Adiabatic Pulses

M229 Advanced Topics in MRI

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Class Business

- Office hours
  - Holden: by appointment
  - Wenqi (HW1): 10-12 on 4/18 Fri
  - Timo (HW2): 4/18, 4/24, 4/25
  - Email beforehand
- Homework 1 due on 4/21 Mon
- Homework 2 due on 4/28 Mon
- Final project
  - Start thinking

Outline

- Review of RF pulses
- Adiabatic passage principle
- Adiabatic inversion
- Applications of adiabatic pulses
- MATLAB demo

Review of RF Pulses

RF Pulses

- What do RF pulses do?
- Challenges at higher B_0 fields?

Notation and Conventions

$$\vec{B} = B_0\hat{k} + B_1(t)[\cos\omega t\hat{i} - \sin\omega t\hat{j}]$$

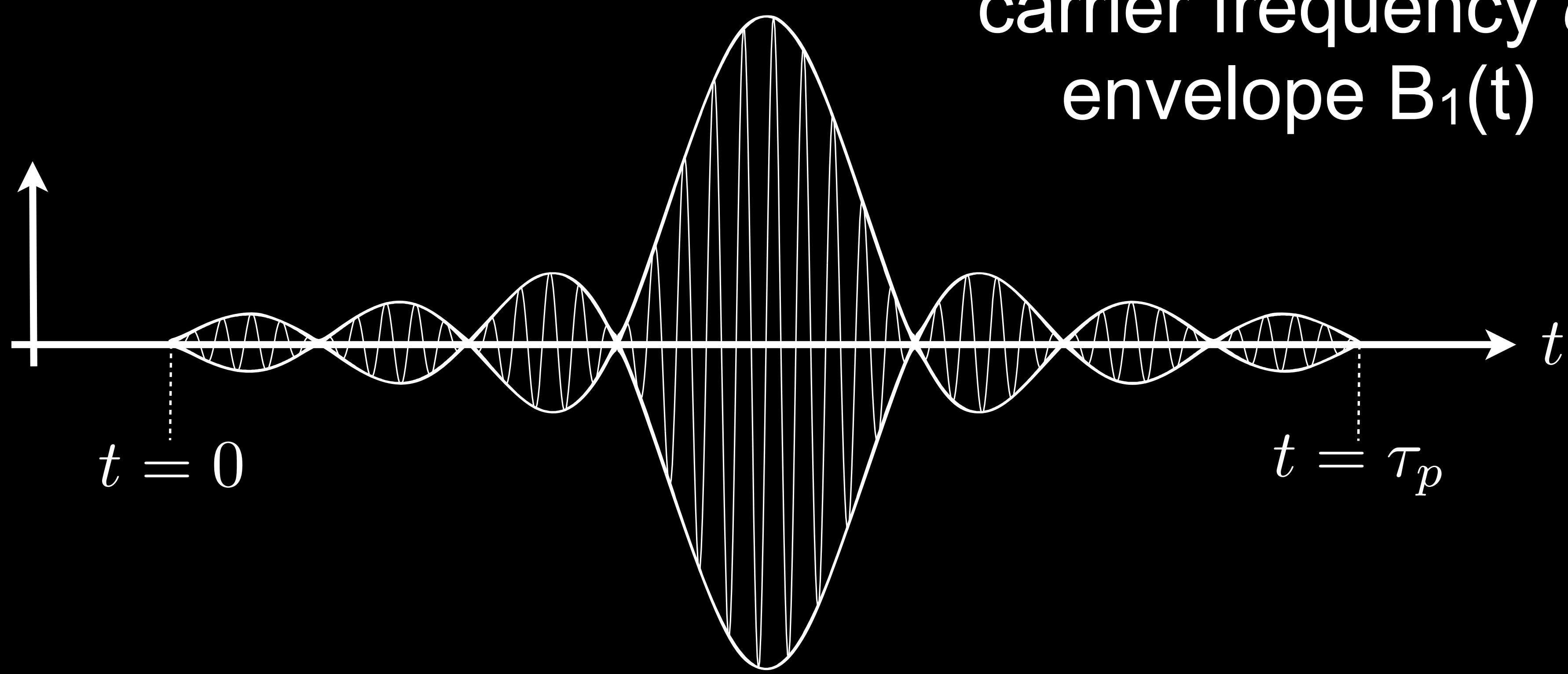
- ω = carrier frequency
- ω_0 = resonant frequency
- $B_1(t)$ = complex valued envelope function

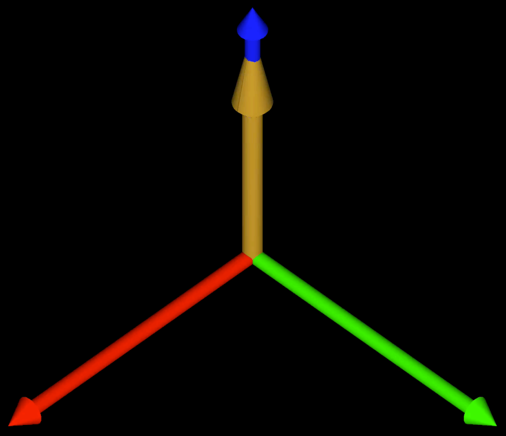
RF Pulse - Excitation

$$\vec{B} = B_0\hat{k} + B_1(t)[\cos\omega t\hat{i} - \sin\omega t\hat{j}]$$

$$B_1(t) \cdot [\cos(\omega t)\hat{i} - \sin(\omega t)\hat{j}]$$

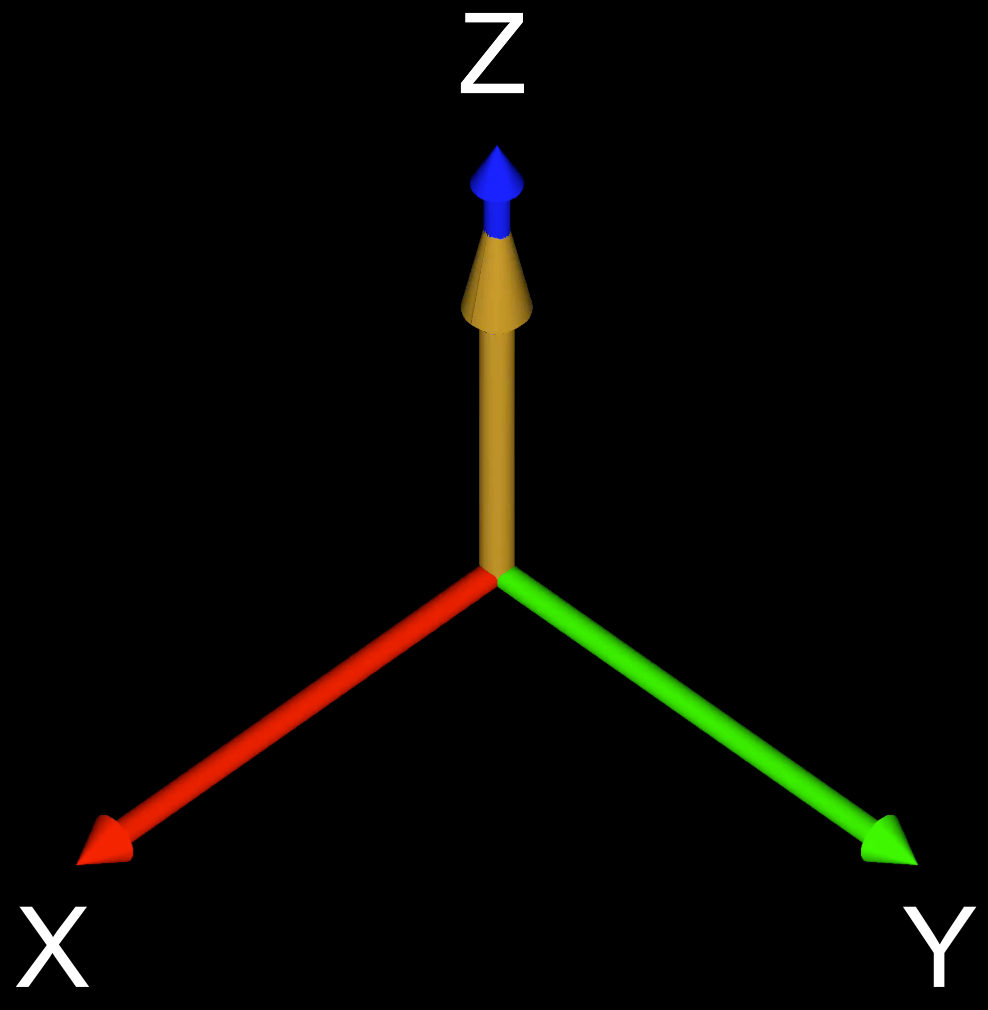
carrier frequency ω
envelope $B_1(t)$



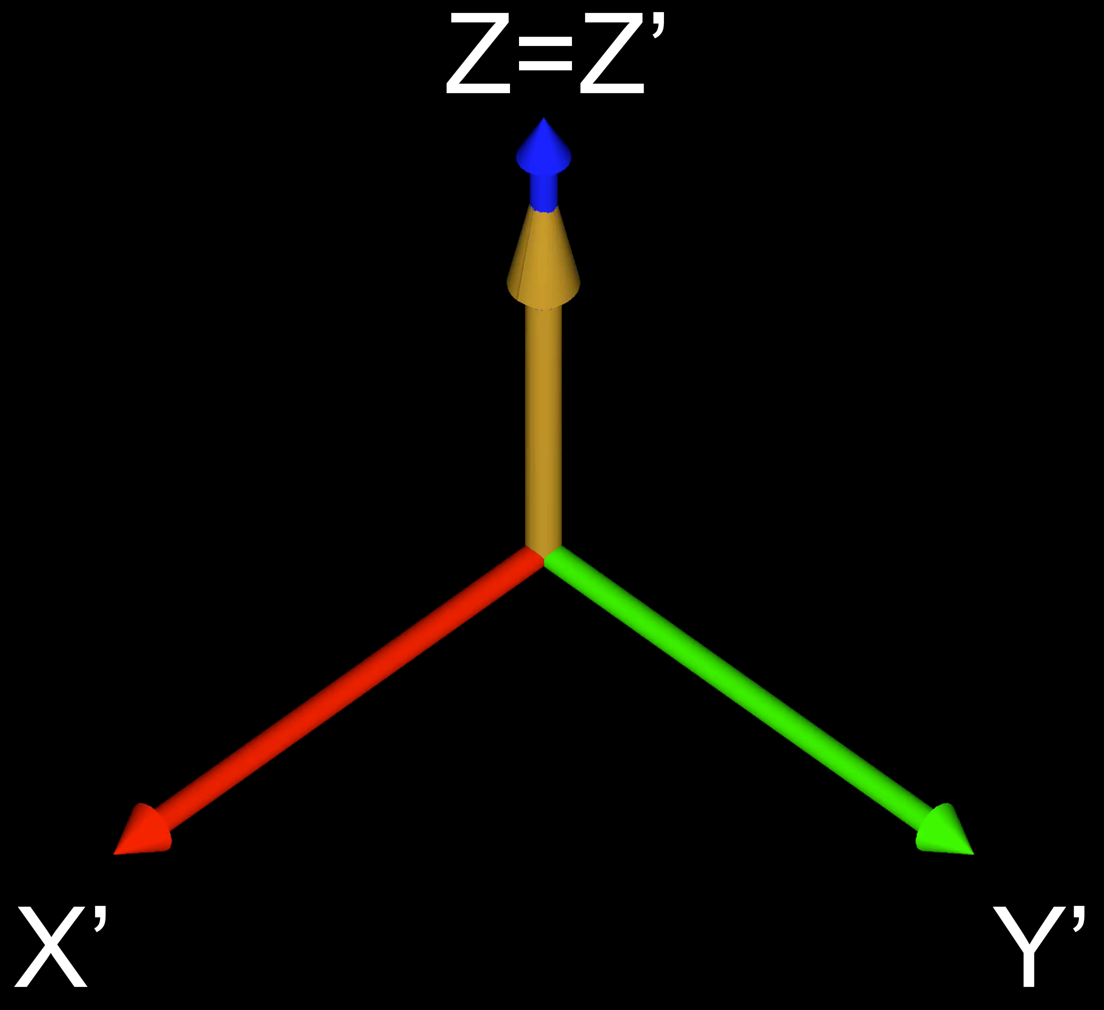


Lab vs. Rotating Frame

- The rotating frame simplifies the mathematics and permits more intuitive understanding.



Laboratory Frame



Rotating Frame

Rotating Frame

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{matrix} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{matrix}$$

$$\vec{M}_{lab}(t) = R_Z(w_0 t) \cdot \vec{M}_{rot}(t)$$

$$\vec{B}_{lab}(t) = R_Z(w_0 t) \cdot \vec{B}_{rot}(t)$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \Rightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{w}_{rot}}{\gamma}$ fictitious field

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix}$$

Bloch Equation (Rotating Frame)

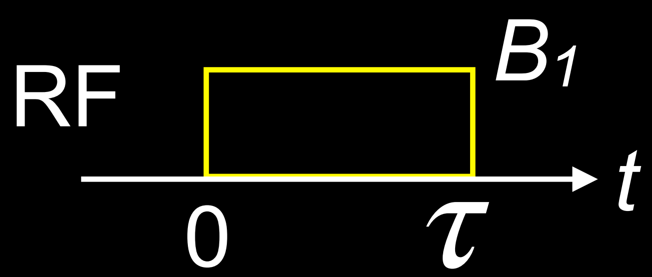
$$\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{w}_{rot}}{\gamma}$$

$$\vec{B}_{lab} = \begin{pmatrix} B_1(t)\cos\omega_0 t \\ B_1(t)\sin\omega_0 t \\ B_0 \end{pmatrix} \qquad \vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ B_1(t) \\ B_0 \end{pmatrix}$$

Assume real-valued $B_1(t)$

$$\vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix}$$

Bloch Equation (Rotating Frame)



$$B_1(t) = B_1; 0 \le t \le \tau$$

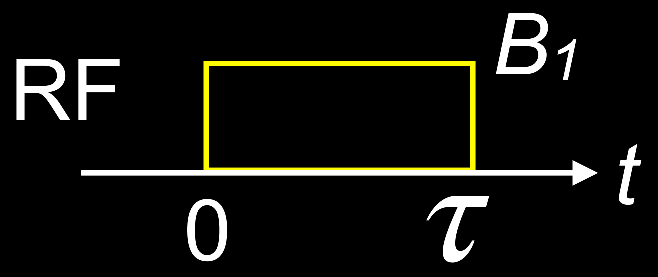
$$\vec{B}_{eff} = \begin{pmatrix} B_1 \\ 0 \\ 0 \end{pmatrix}$$

On resonance: $B_0 - \frac{\omega_0}{\gamma} = 0$

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff} \qquad \theta = \int_0^{\tau} \gamma B_1(t')dt' = \gamma B_1 \tau$$

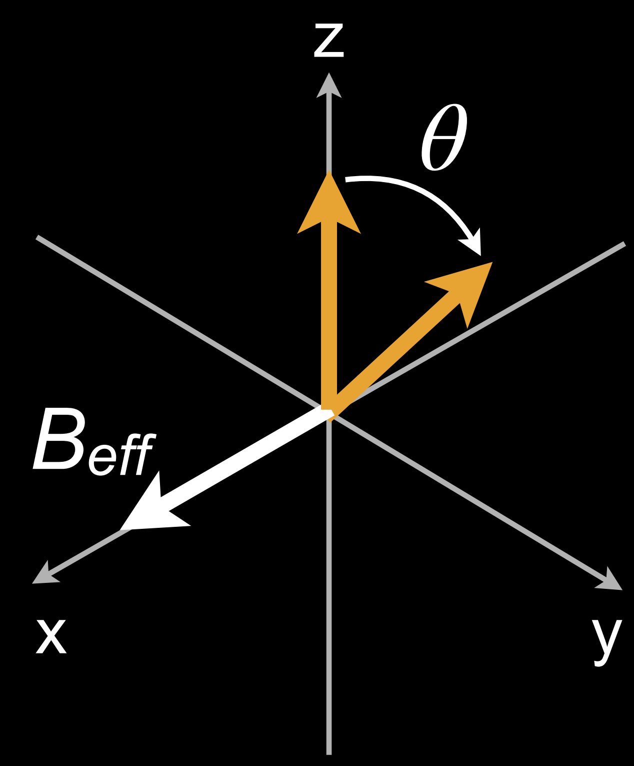
$$\Rightarrow \vec{M}_{rot}(t) = R_x(\theta = \gamma B_1 t) \begin{bmatrix} 0 \\ 0 \\ M_0 \end{bmatrix} = \begin{bmatrix} 0 \\ M_0 sin(\theta) \\ M_0 cos(\theta) \end{bmatrix}$$

Bloch Equation (Rotating Frame)



$$B_1(t) = B_1; 0 \leq t \leq \tau$$

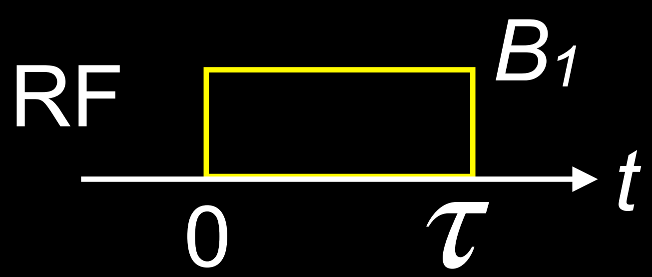
$$\vec{M}_{rot}(t) = \begin{bmatrix} 0 \\ M_0 sin(\theta) \\ M_0 cos(\theta) \end{bmatrix}$$



$$\theta = 90^\circ = \frac{\pi}{2}, \tau = 1ms$$

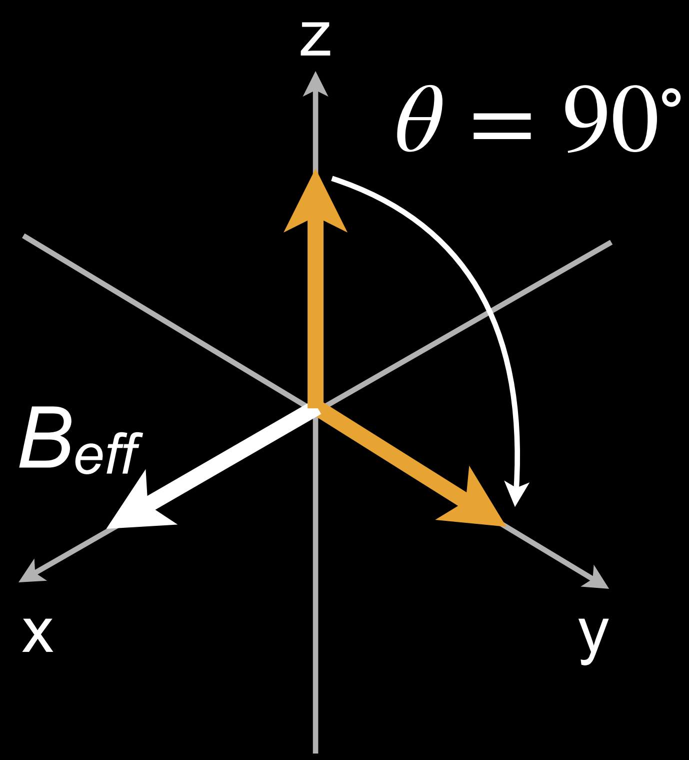
$$\frac{\pi}{2} = \gamma \cdot 1ms \cdot B_1 \quad \Rightarrow B_1 \approx 0.66G = 6\mu T$$

Bloch Equation (Rotating Frame)

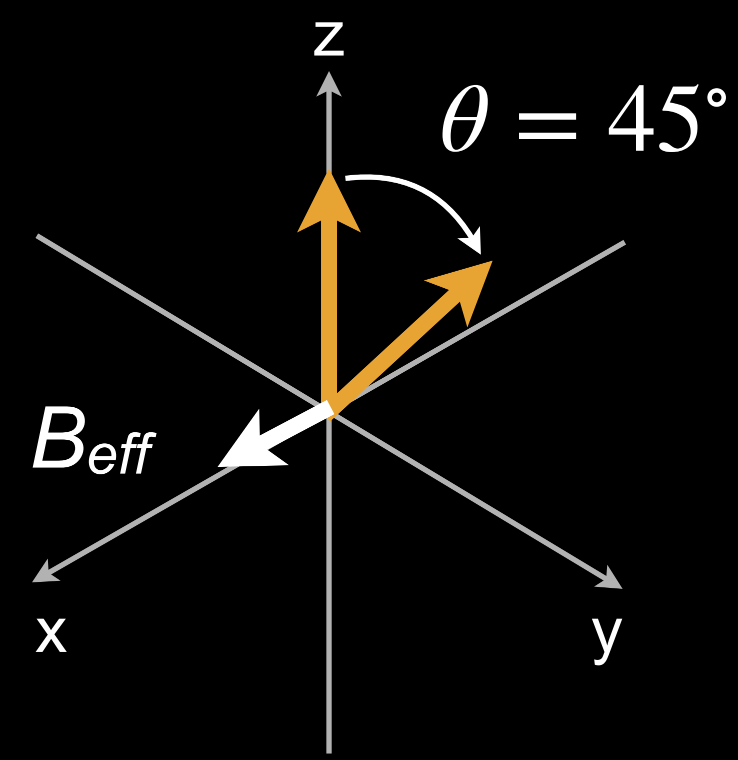


$$B_1(t) = B_1; 0 \leq t \leq \tau$$

B_1 inhomogeneity:



B_1 reduced by 50%



Bloch Equation with Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

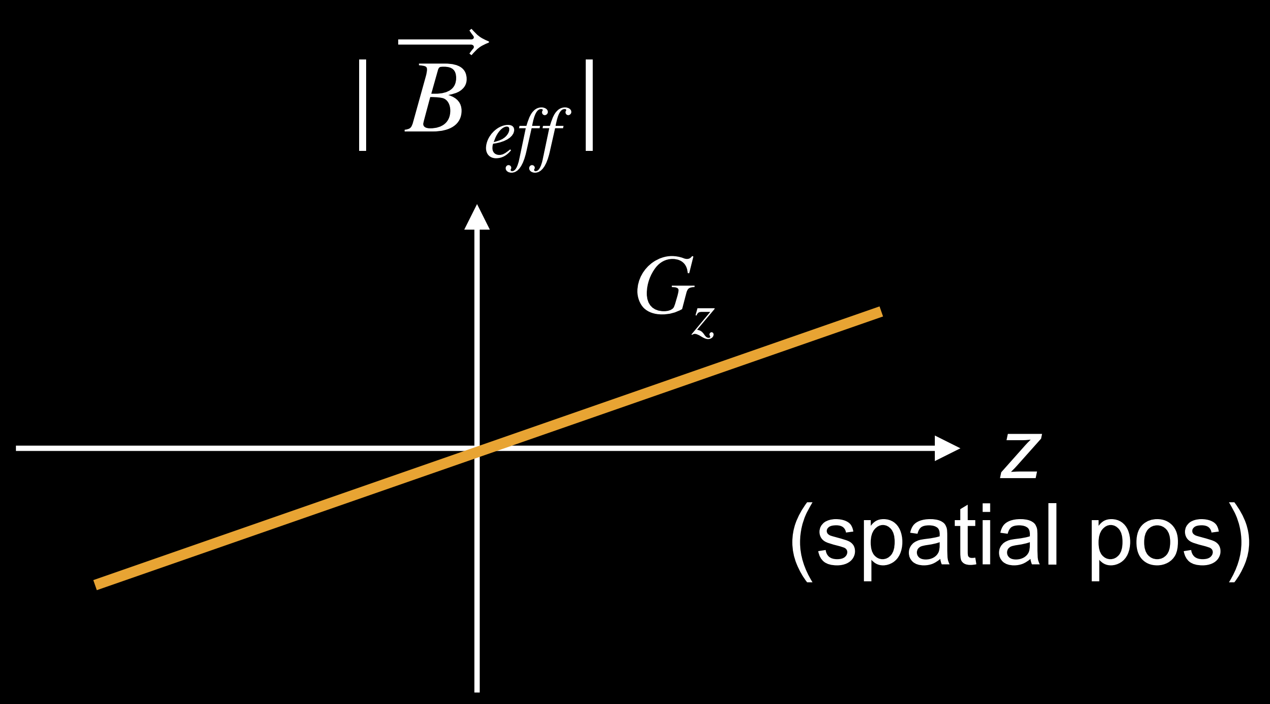
$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix} \Rightarrow \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$$

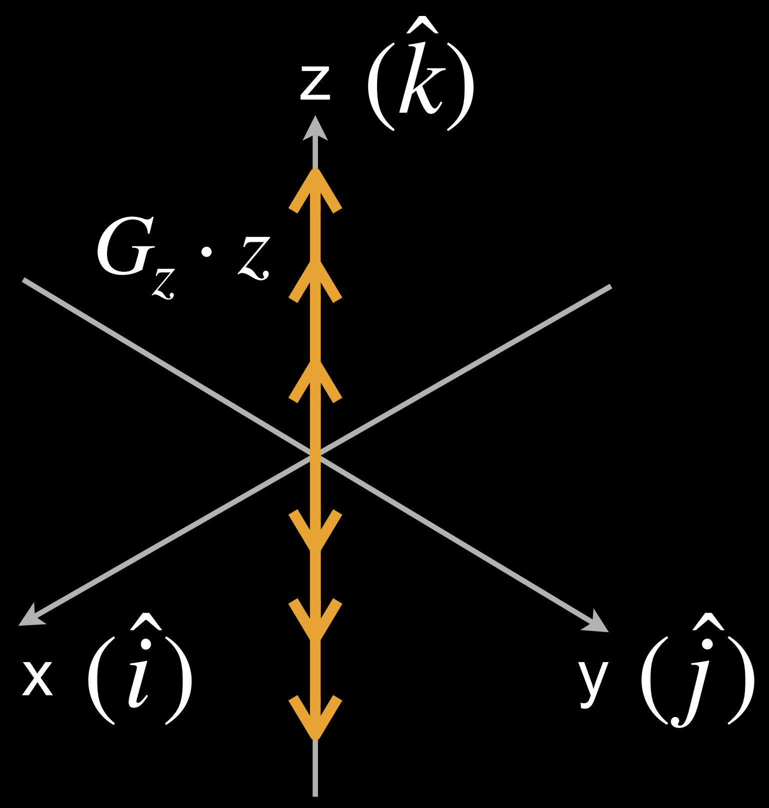
Bloch Equation with Gradient

No RF Pulse (B_1=0), with G_z on

$$\vec{B} = (B_0 + G_z \cdot z)\hat{k} \qquad \vec{B}_{eff} = (B_0 + G_z \cdot z - \frac{\omega_{RF}}{\gamma})\hat{k}$$

On resonance: $\omega_{RF} = \omega_0$, $\vec{B}_{eff} = (G_z \cdot z)\hat{k}$





Bloch Equation (at on-resonance)

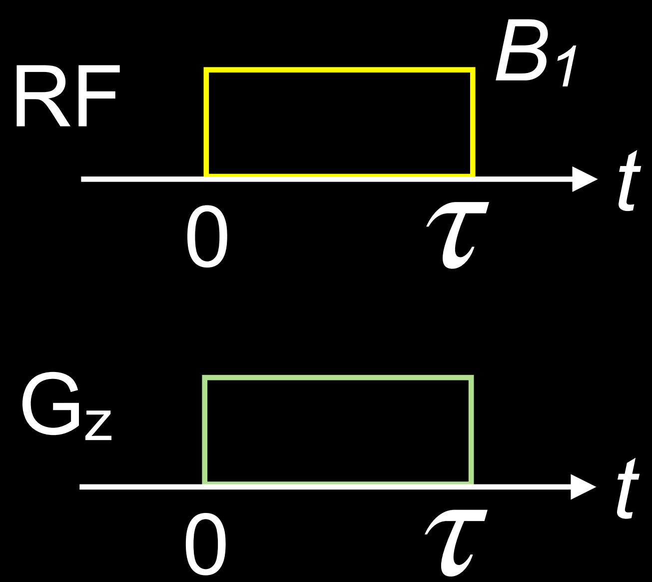
$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

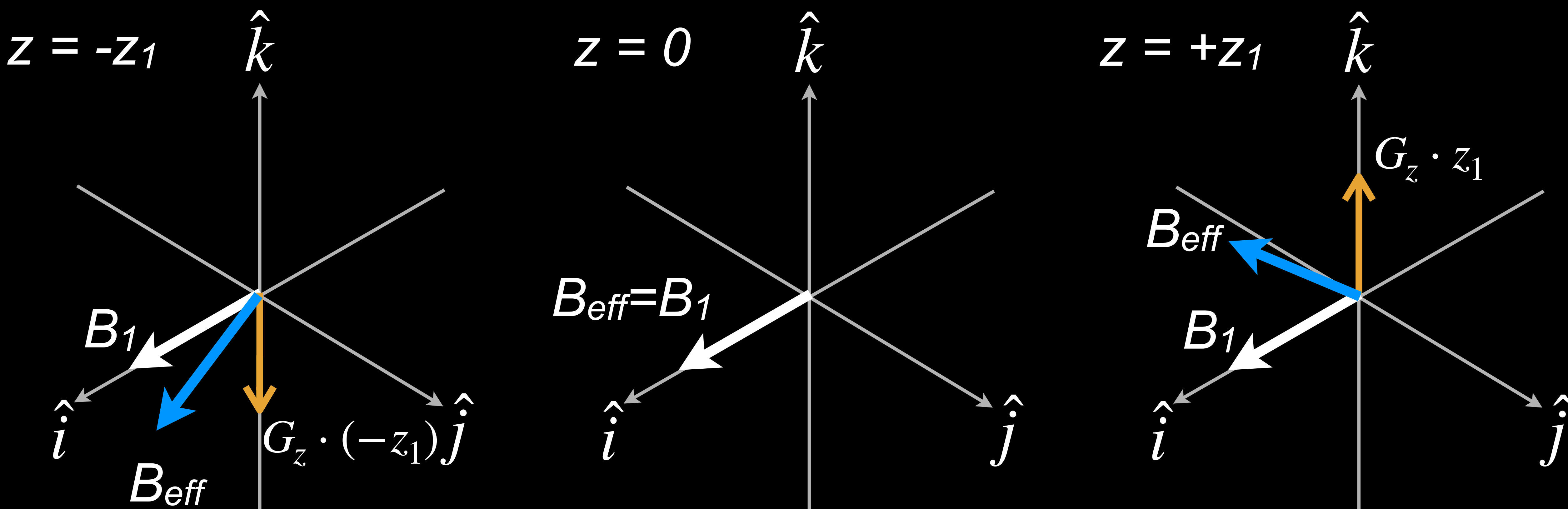
Bloch Equation (at on-resonance)



$$\overrightarrow{B}_{eff} = (B_0 + G_z \cdot z - \frac{\omega_{RF}}{\gamma})\hat{k} + B_1\hat{i}$$

On resonance: $\omega_{RF} = \omega_0$

$$\overrightarrow{B}_{eff} = (G_z \cdot z)\hat{k} + B_1\hat{i}$$



What happens when z is very far from z=0?

B_1 Variations

- In MRI, the B_1 field is not always uniform across the imaging volume
- B_1 inhomogeneity can cause:
  - Image shading
  - Incomplete saturation (e.g. in fat suppression)
  - Incomplete inversion (e.g. CSF suppression, myocardium suppression in cardiac scar imaging)
  - Inaccurate/imprecise quantification in T_1 mapping

B_1 Variations

- It is highly desirable if we can excite tissue homogeneously and produce a uniform flip angle throughout

→ **Adiabatic Pulses!**

"Adiabatic pulses are a special class of RF pulses that can excite, refocus or invert magnetization vectors uniformly, even in the presence of a spatially nonuniform B_1 field."

Adiabatic Passage Principle

Adiabatic Pulses

- A special class of RF pulses that can achieve uniform flip angle
- Flip angle is independent of the applied B_1 field

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Slice profile of an adiabatic pulse is obtained using Bloch simulations
- Can be used for excitation, inversion and refocusing

Adiabatic vs. Non-Adiabatic Pulses

Adiabatic Pulses:

$$\theta \neq \int_0^T B_1(\tau)d\tau$$

- Amplitude and frequency/phase modulation
- Long duration (8-12 ms)
- Higher B_1 amplitude (>12 μT)
- Generally NOT multi-purpose (inversion pulse cannot be used for refocusing, etc.)

Non-Adiabatic Pulses:

$$\theta = \int_0^T B_1(\tau)d\tau$$

- Amplitude modulation, constant carrier frequency (constant phase)
- Short duration (0.3 ms to 1 ms)
- Lower B_1 amplitude
- Generally multi-purpose

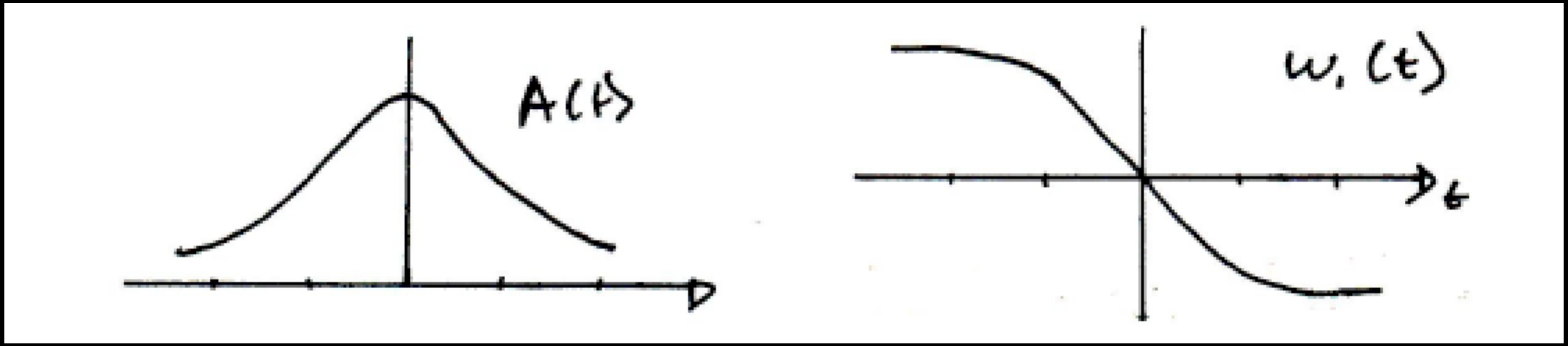
Adiabatic Pulses

- Frequency modulated pulses:

$$B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'}$$

envelope

frequency sweep



- Or phase modulation:

$$B_1(t) = A(t) \exp^{-i\phi(t)}$$

Bloch Equation (at on-resonance)

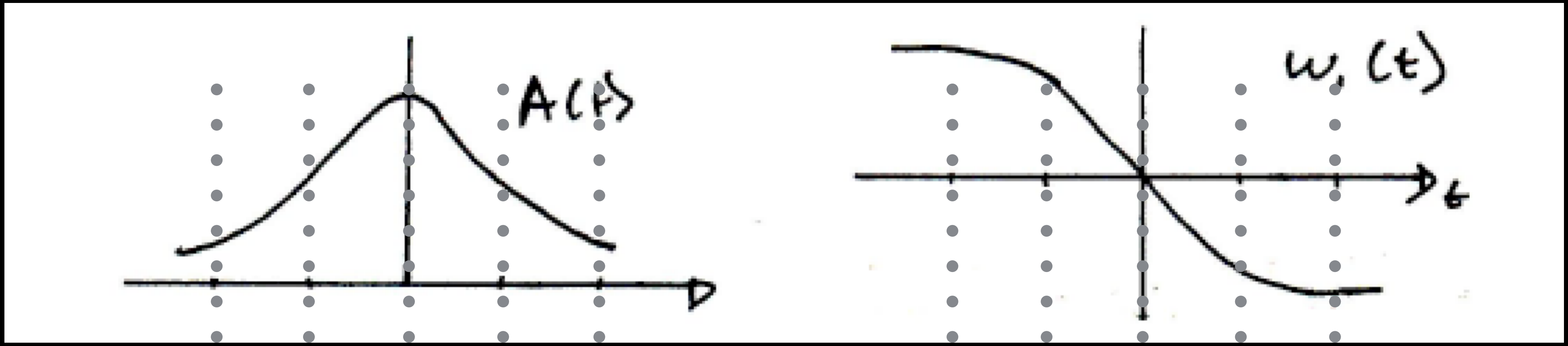
$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + \frac{\omega_1(t)}{\gamma} \end{pmatrix}$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega_1(t) & 0 \\ -\omega_1(t) & 0 & \gamma A(t) \\ 0 & -\gamma A(t) & 0 \end{pmatrix} \vec{M}$$

Magnetization Plot

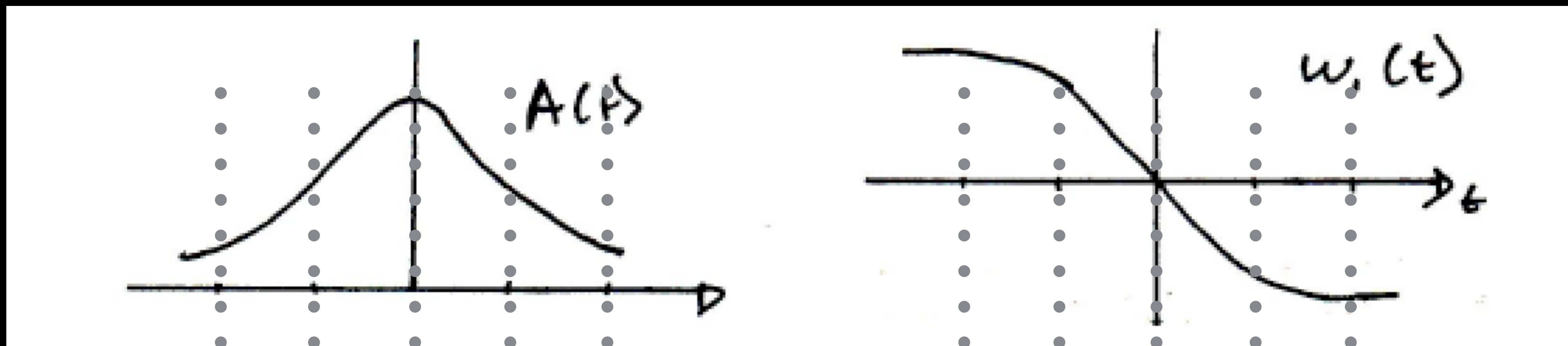


a b c d e a b c d e

$$B_1(t) = A(t)e^{-i\omega_1(t)\cdot t}$$

$$\overrightarrow{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \frac{\omega_1(t)}{\gamma} \end{pmatrix}$$

Magnetization Plot

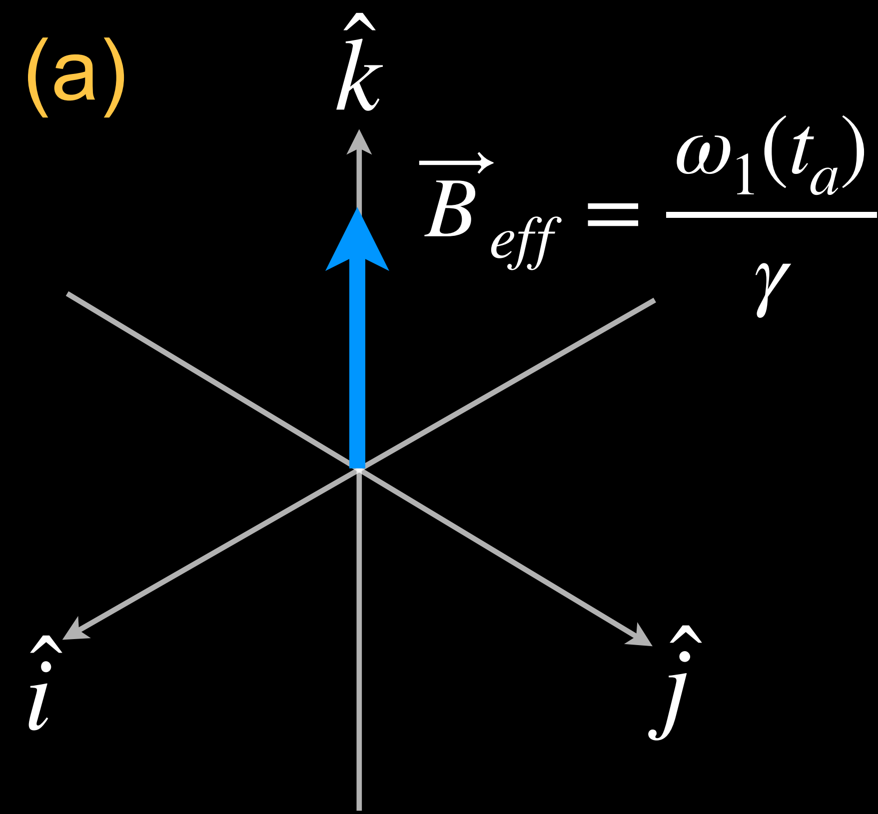


a b c d e   a b c d e

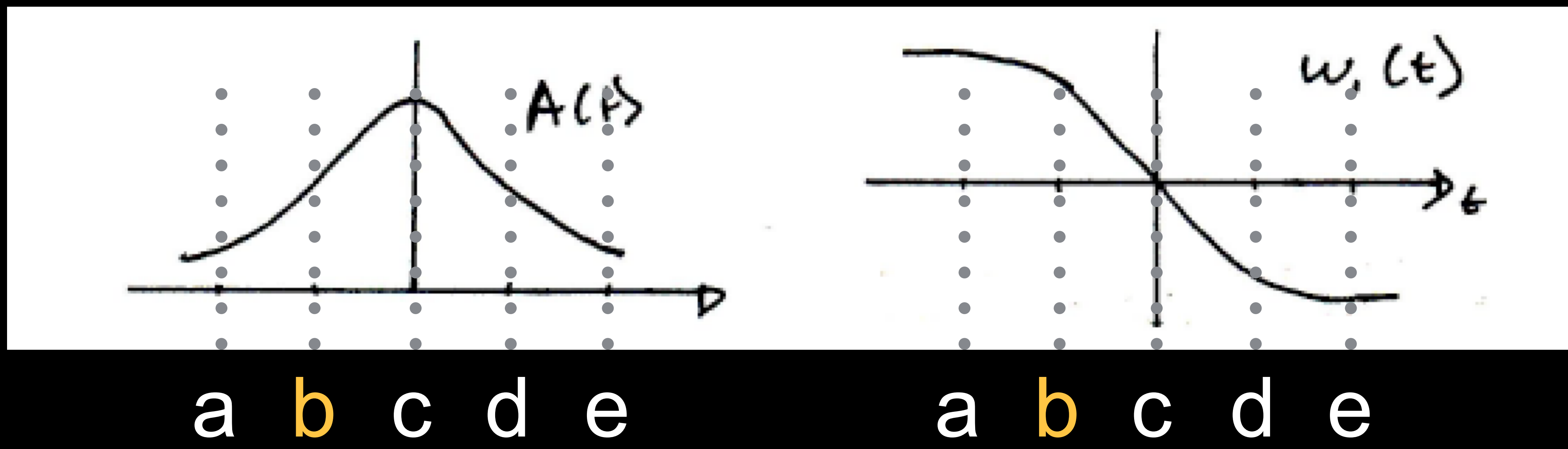
$$B_1(t) = A(t)e^{-i\omega_1(t)\cdot t}$$

$$\overrightarrow{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \frac{\omega_1(t)}{\gamma} \end{pmatrix}$$

(a)

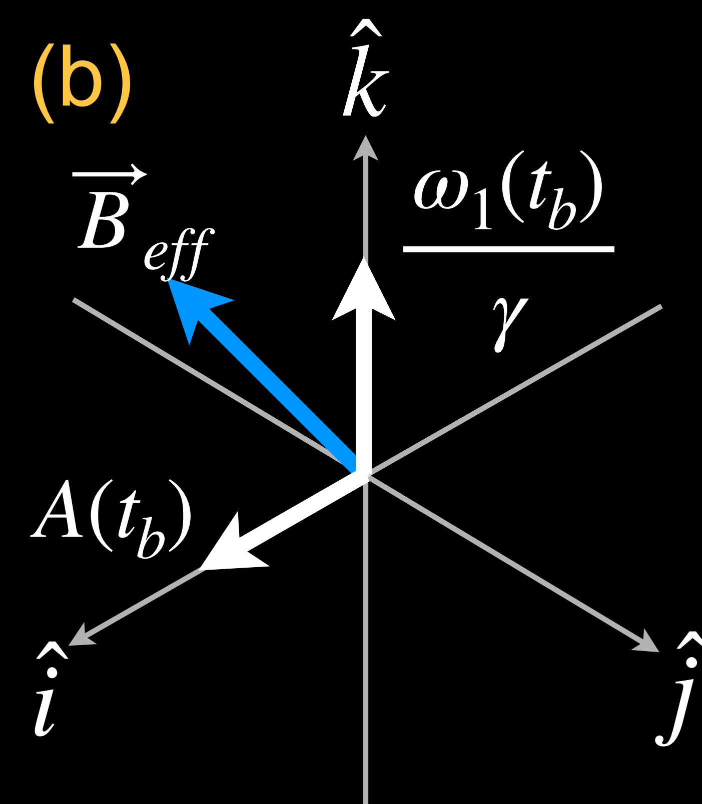


Magnetization Plot

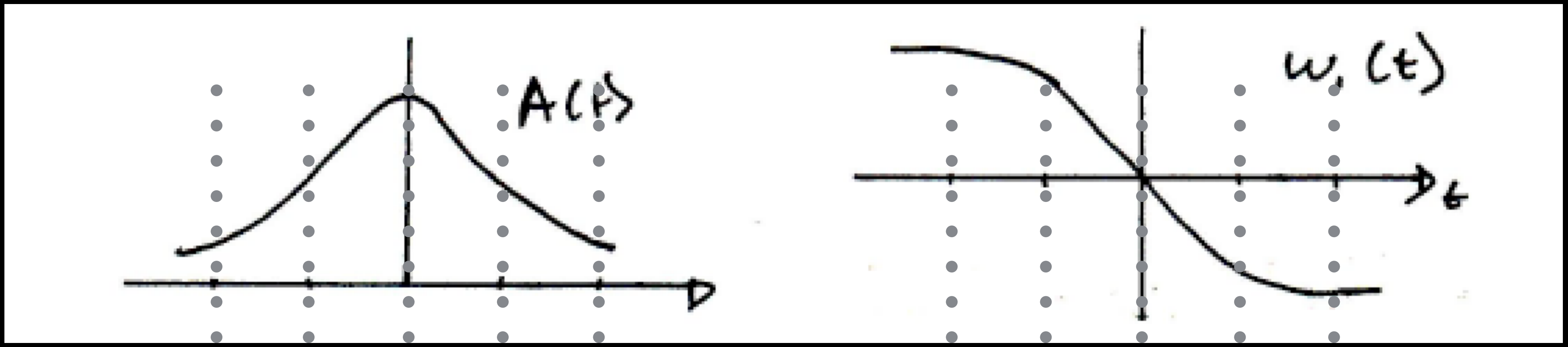


$$B_1(t) = A(t)e^{-i\omega_1(t)\cdot t}$$

$$\overrightarrow{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \dfrac{\omega_1(t)}{\gamma} \end{pmatrix}$$



Magnetization Plot



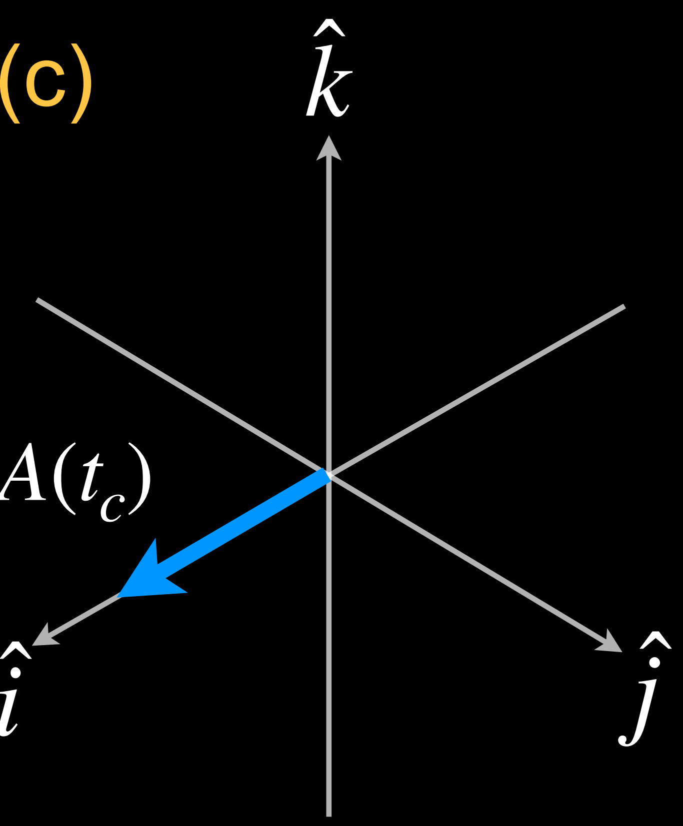
a b c d e   a b c d e

$$B_1(t) = A(t)e^{-i\omega_1(t)\cdot t}$$

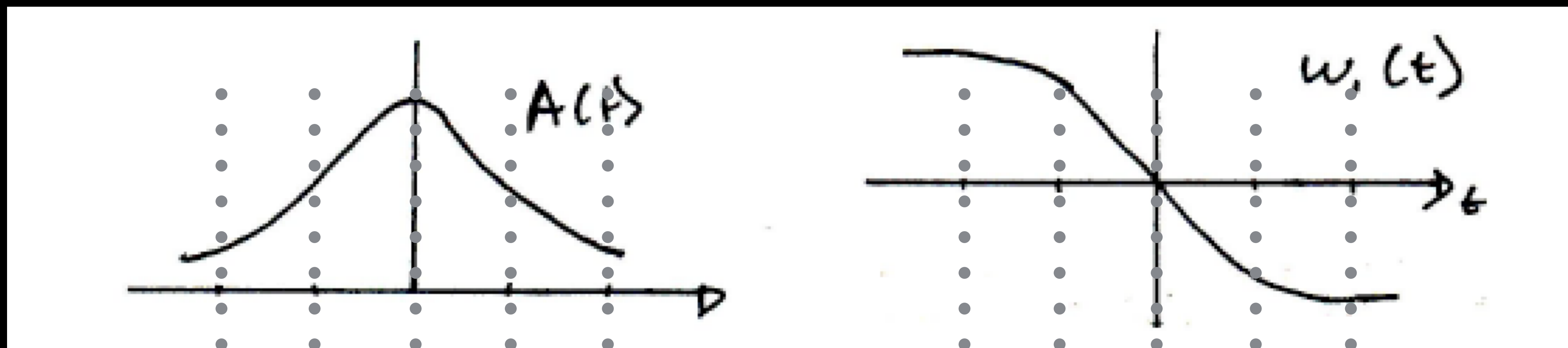
$$\overrightarrow{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \dfrac{\omega_1(t)}{\gamma} \end{pmatrix}$$

(c)

$$\overrightarrow{B}_{eff} = A(t_c)$$



Magnetization Plot



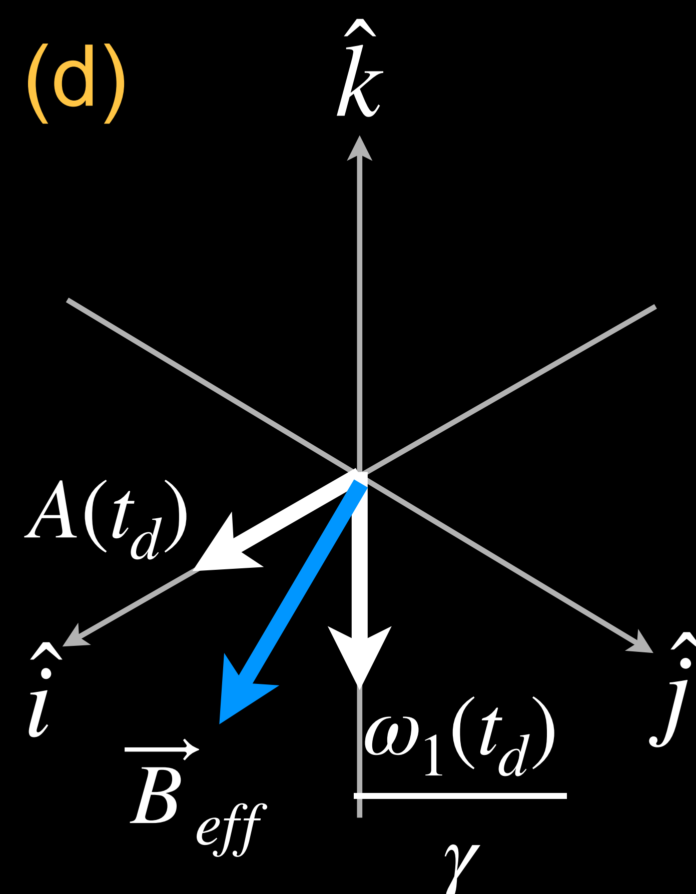
a b c d e

a b c d e

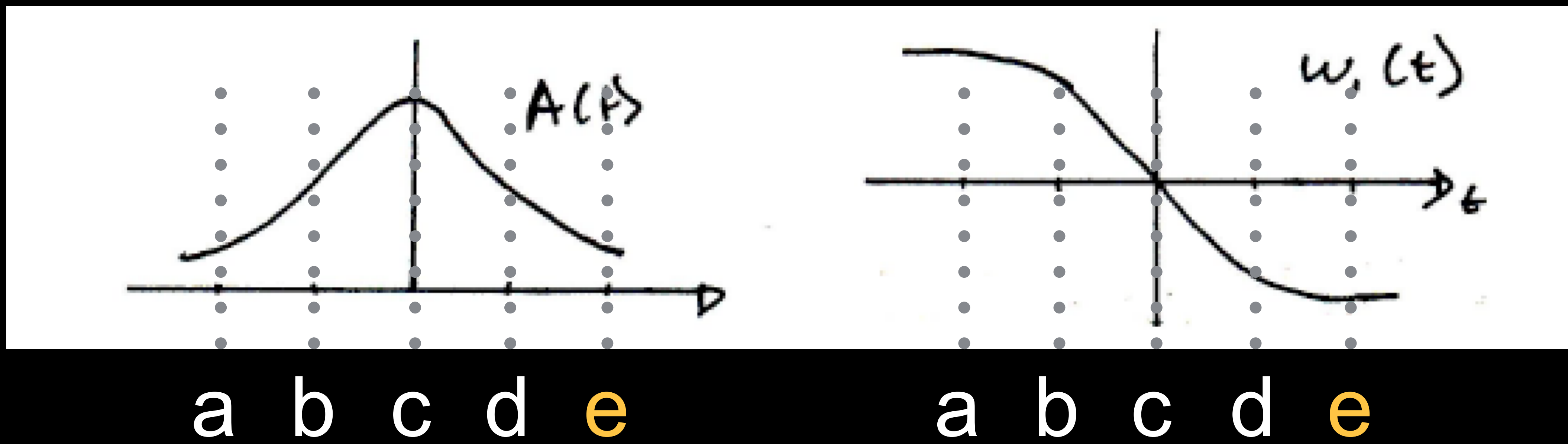
$$B_1(t) = A(t)e^{-i\omega_1(t)\cdot t}$$

$$\overrightarrow{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \dfrac{\omega_1(t)}{\gamma} \end{pmatrix}$$

(d)

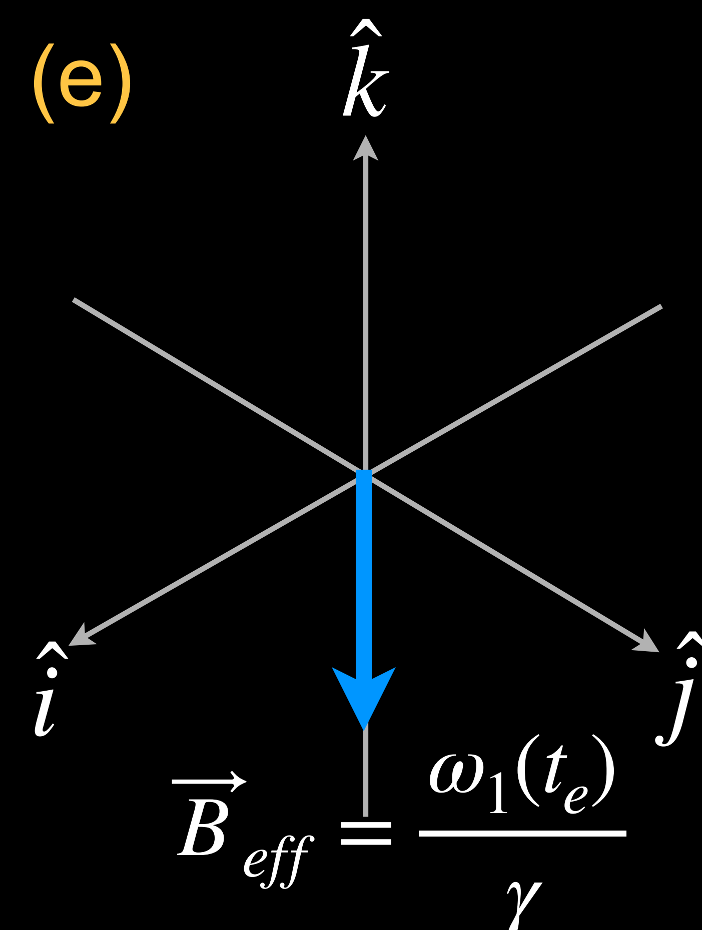


Magnetization Plot

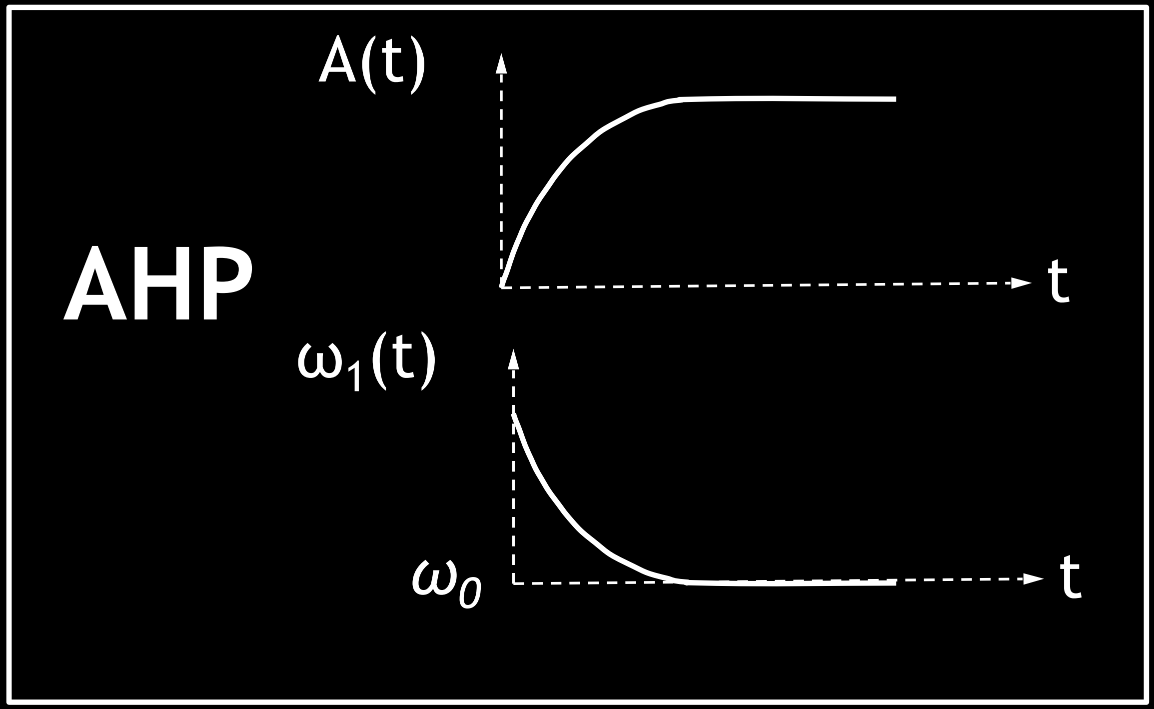


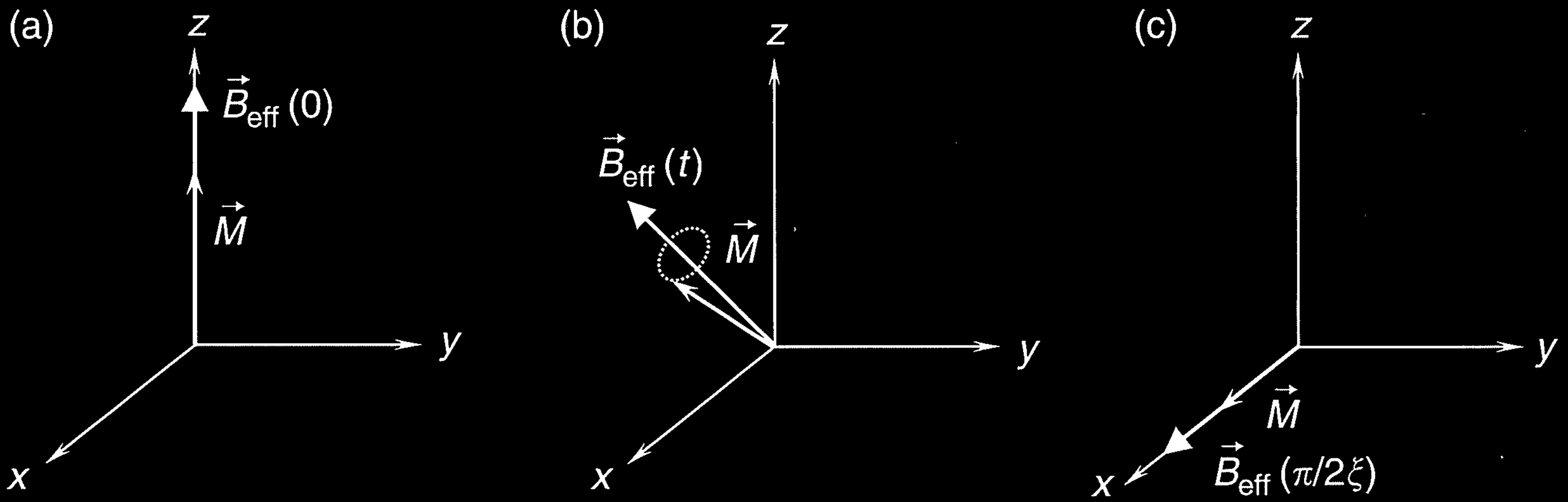
$$B_1(t) = A(t)e^{-i\omega_1(t)\cdot t}$$

$$\overrightarrow{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \frac{\omega_1(t)}{\gamma} \end{pmatrix}$$



Adiabatic Excitation



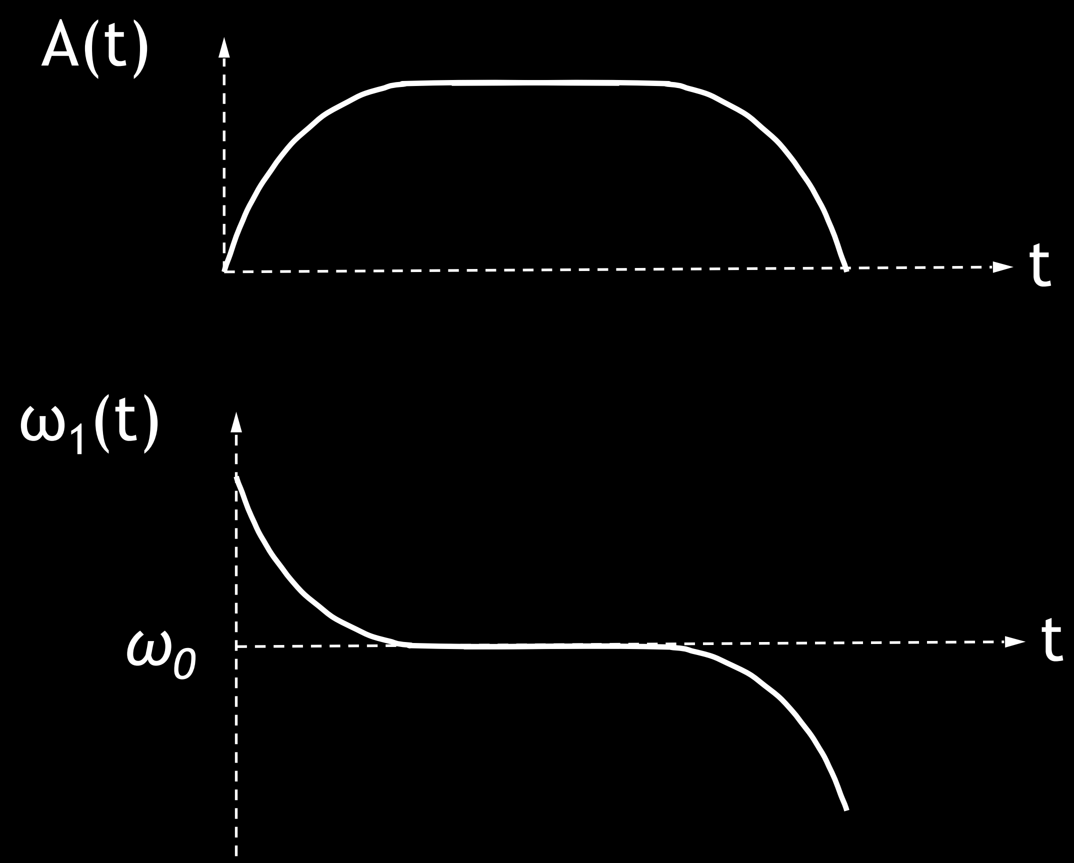


- At the end of the pulse, all the magnetization is in the transverse plane, so we have adiabatic excitation!
- This is also called an **adiabatic half passage (AHP)**

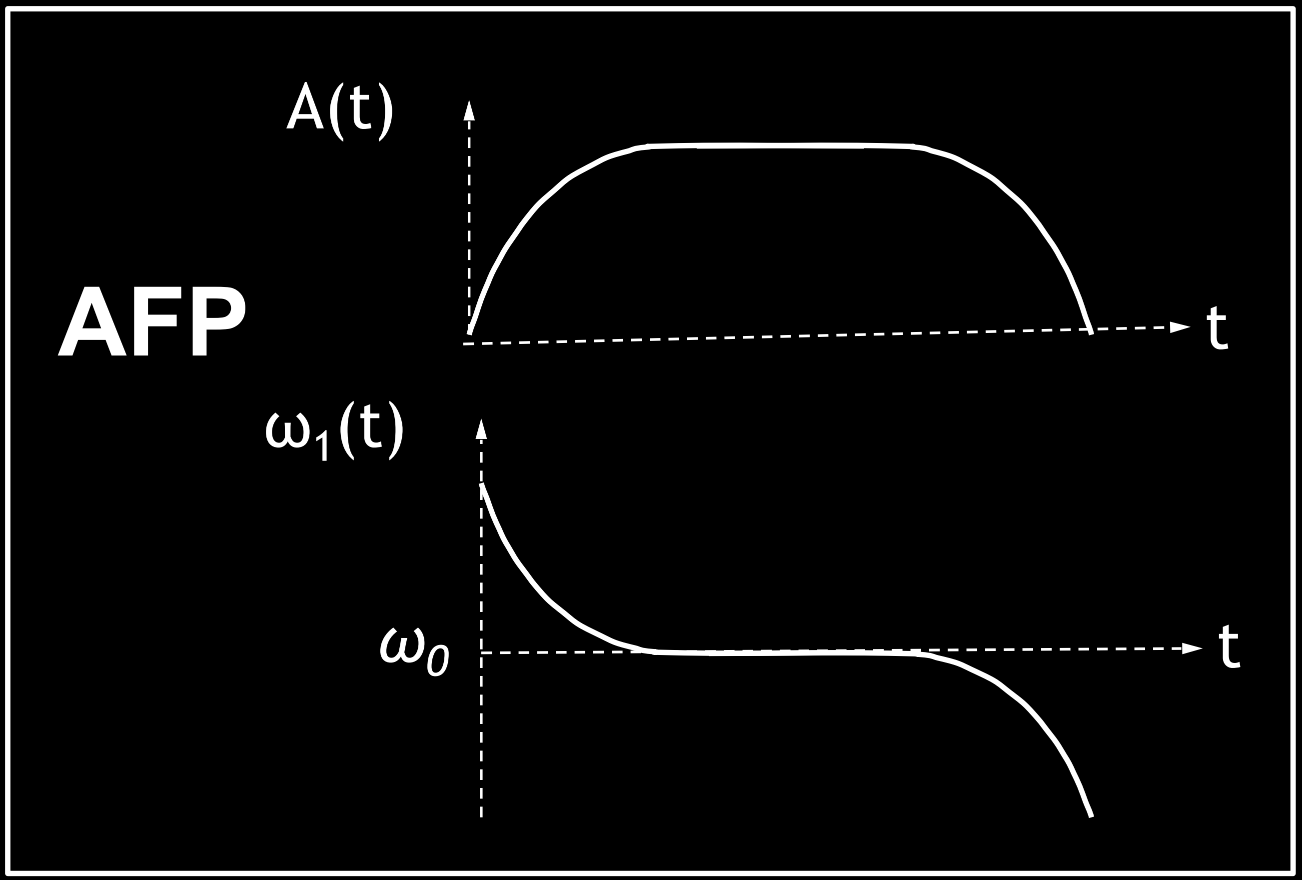
Adiabatic Inversion

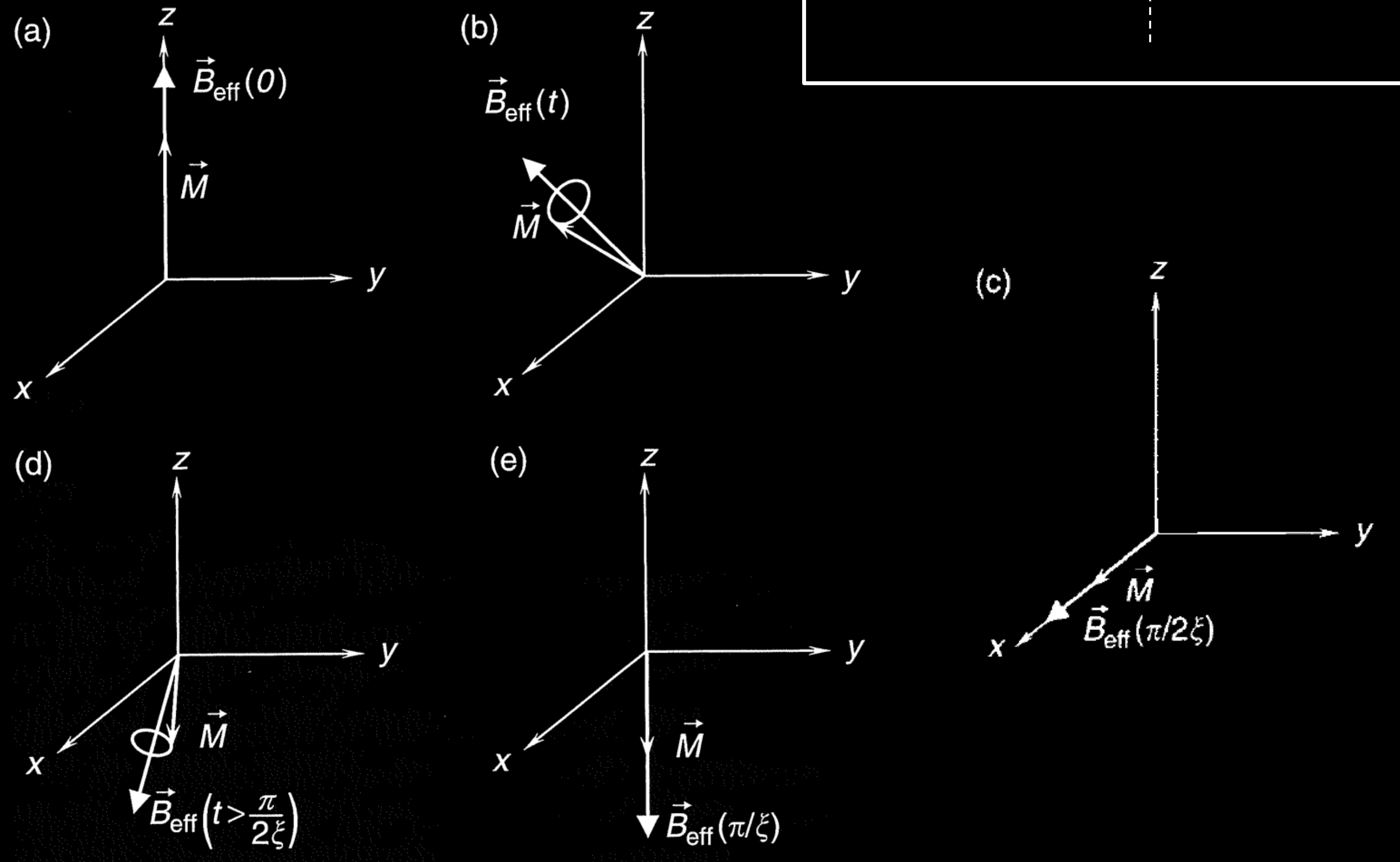
- An adiabatic inversion requires an adiabatic full passage (AFP) pulse:

AFP



Adiabatic Inversion





Adiabatic Inversion

Design of Adiabatic Inversion

- General equation for an adiabatic pulse:

$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

- Many different types of adiabatic pulses can be designed by choosing different amplitude and frequency modulation functions
- The most famous one is…

The Hyperbolic Secant Inversion Pulse!

Hyperbolic Secant Pulse Equations

$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

where

$$A(t) = A_0 sech(\beta t)$$

$$\omega_1(t) = -\mu\beta tanh(\beta t)$$

A_0: peak amplitude (µT)

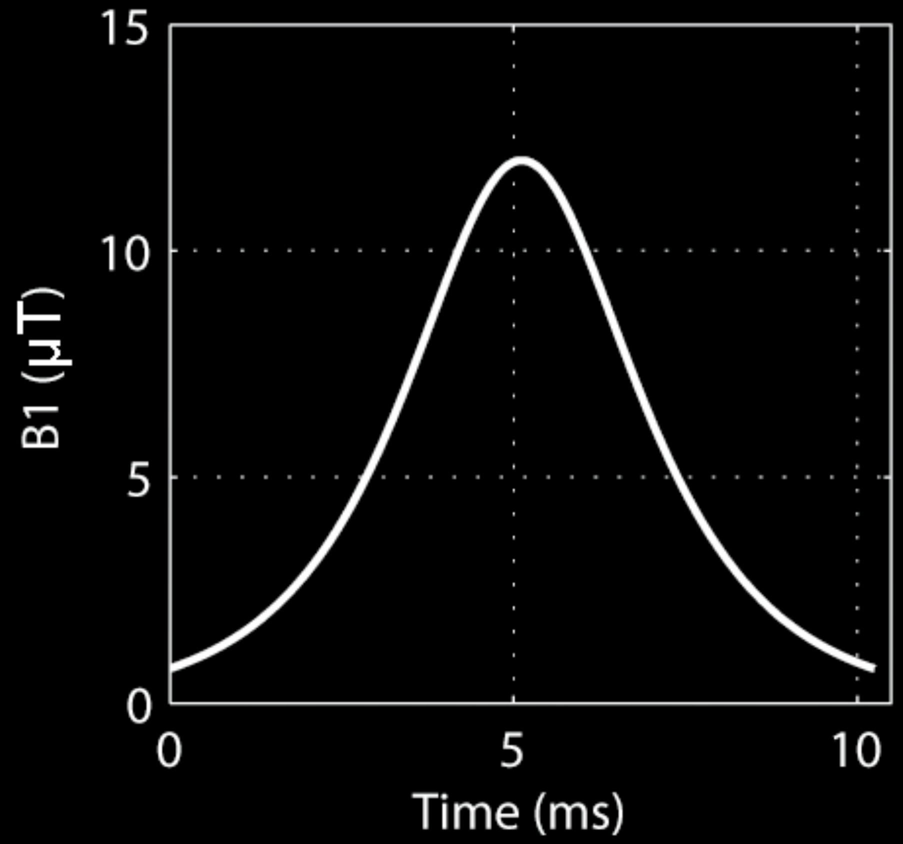
β: frequency modulation parameter (rad/s)

μ: phase modulation parameter (dimensionless)

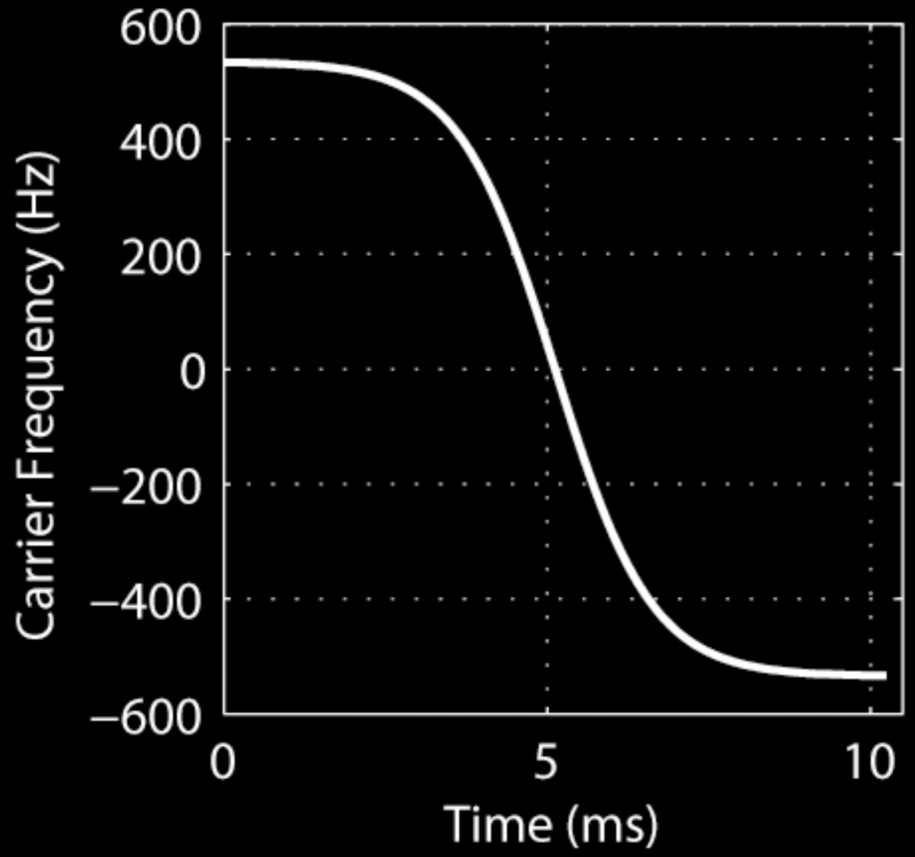
Handbook of MRI Pulse Sequences, Ch 6.2, pp 194-195

Hyperbolic Secant Pulse Example

Amplitude Modulation, A(t)



Frequency Modulation, $\omega_1(t)$



Pulse Parameters:

A_0 = 12 μT

μ = 5

β = 672 rad/s

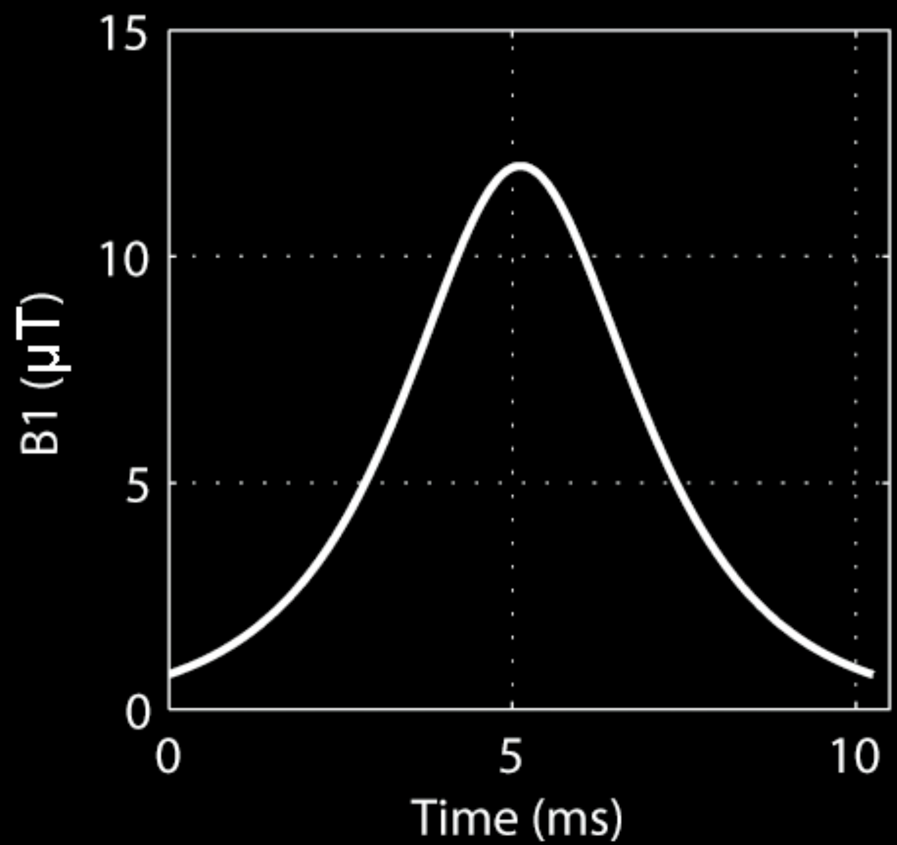
Duration = 10.24 ms

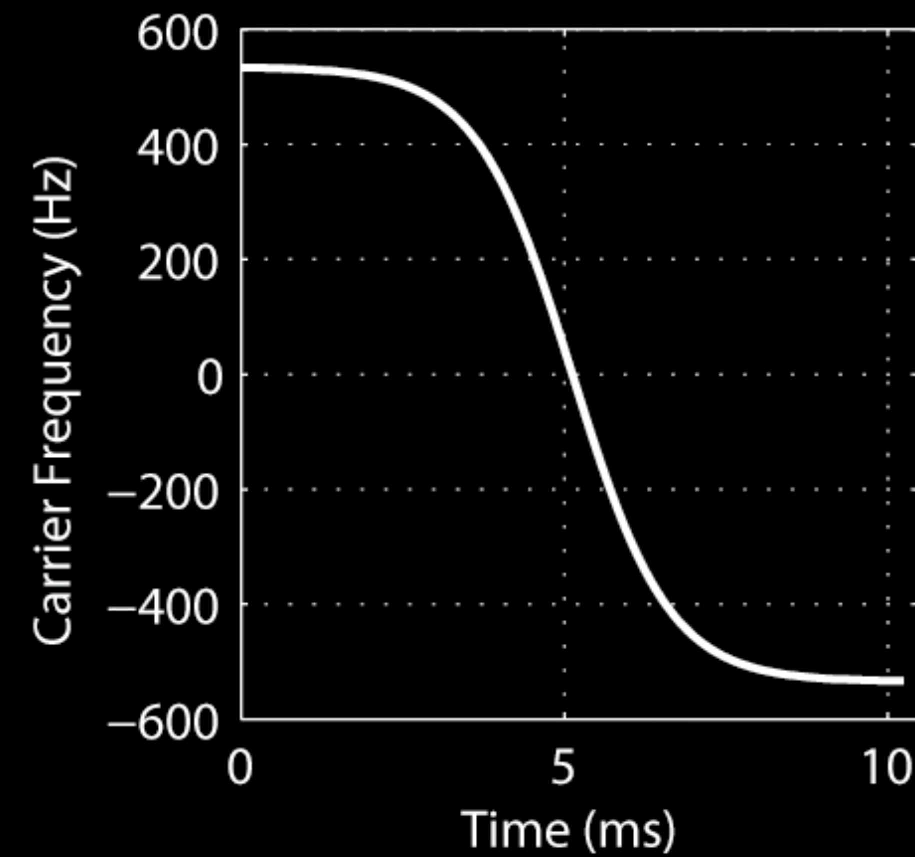
Comparing Hyperbolic Secant with an AFP Example

Amplitude Modulation, A(t)

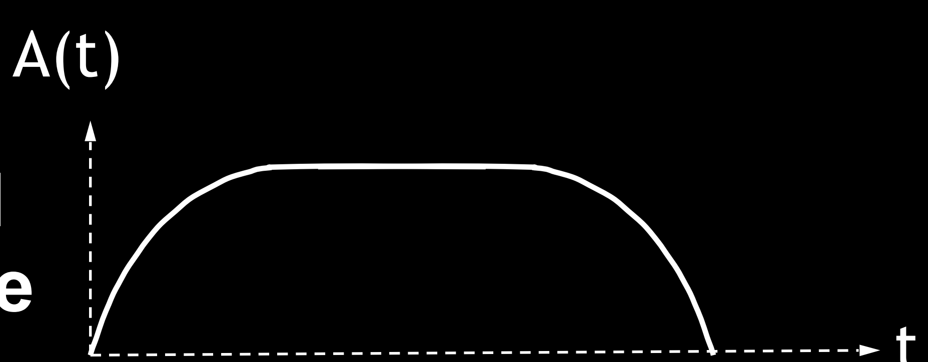
Frequency Modulation, $\omega_1(t)$

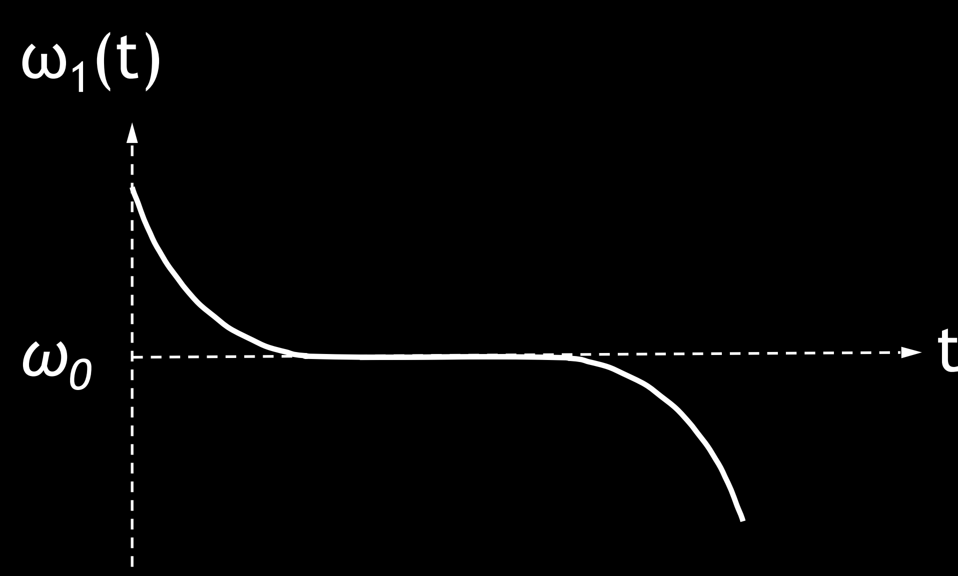
Hyperbolic Secant Pulse





General Adiabatic Full Passage pulse





Some Examples of Other Adiabatic Inversion Pulses

Pulse Name	A(t)	$\omega_1(t)$
Lorentz	$\frac{1}{1+\beta\tau^2}$	$\frac{\tau}{1+\beta\tau^2}+\frac{1}{\sqrt{\beta}}\tan^{-1}(\sqrt{\beta}\tau)$
HS	$\mathrm{sech}(\beta\tau)$	$\frac{\tanh(\beta\tau)}{\tanh(\beta)}$
Gauss[c]	$\exp\left(-\frac{\beta^2\tau^2}{2}\right)$	$\frac{\mathrm{erf}(\beta\tau)}{\mathrm{erf}(\beta)}$
Hanning	$\frac{1+\cos(\pi\tau)}{2}$	$\tau+\frac{4}{3\pi}\sin(\pi\tau)\left[1+\frac{1}{4}\cos(\pi\tau)\right]$
HSn[c] ($n=8$)	$\mathrm{sech}(\beta\tau^n)$	$\int \mathrm{sech}^2(\beta\tau^n)\,d\tau$
Sin40[d] ($n=40$)	$1-\left\lvert \sin^n\left(\frac{\pi\tau}{2}\right)\right\rvert$	$\tau-\int \sin^n\left(\frac{\pi\tau}{2}\right)\left(1+\cos^2\left(\frac{\pi\tau}{2}\right)\right)d\tau$

Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, vol. 10, p423

Some Examples of Other Adiabatic Inversion Pulses

Pulse Name	A(t)	ω_1(t)
Lorentz	$\frac{1}{1+\beta\tau^2}$	$\frac{\tau}{1+\beta\tau^2}+\frac{1}{\sqrt{\beta}}\tan^{-1}(\sqrt{\beta}\tau)$
HS	$\mathrm{sech}(\beta\tau)$	$\frac{\tanh(\beta\tau)}{\tanh(\beta)}$
HSn[c] ($n=8$)	$\mathrm{sech}(\beta\tau^n)$	$\int \mathrm{sech}^2(\beta\tau^n)\,d\tau$
Sin40[d] ($n=40$)	$1-\left\lvert \sin^n\left(\frac{\pi\tau}{2}\right)\right\rvert$	$\tau-\int \sin^n\left(\frac{\pi\tau}{2}\right)\left(1+\cos^2\left(\frac{\pi\tau}{2}\right)\right)d\tau$

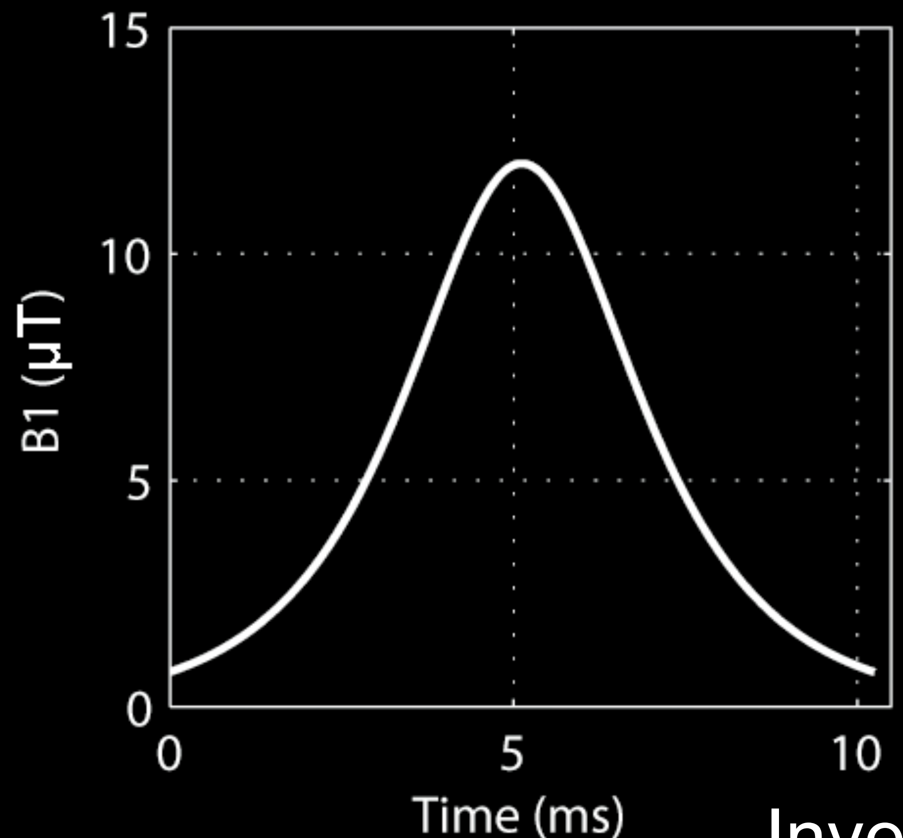
The shape of the inversion profile depends on the choice A(t) and ω_1(t)!

Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, vol. 10, p423

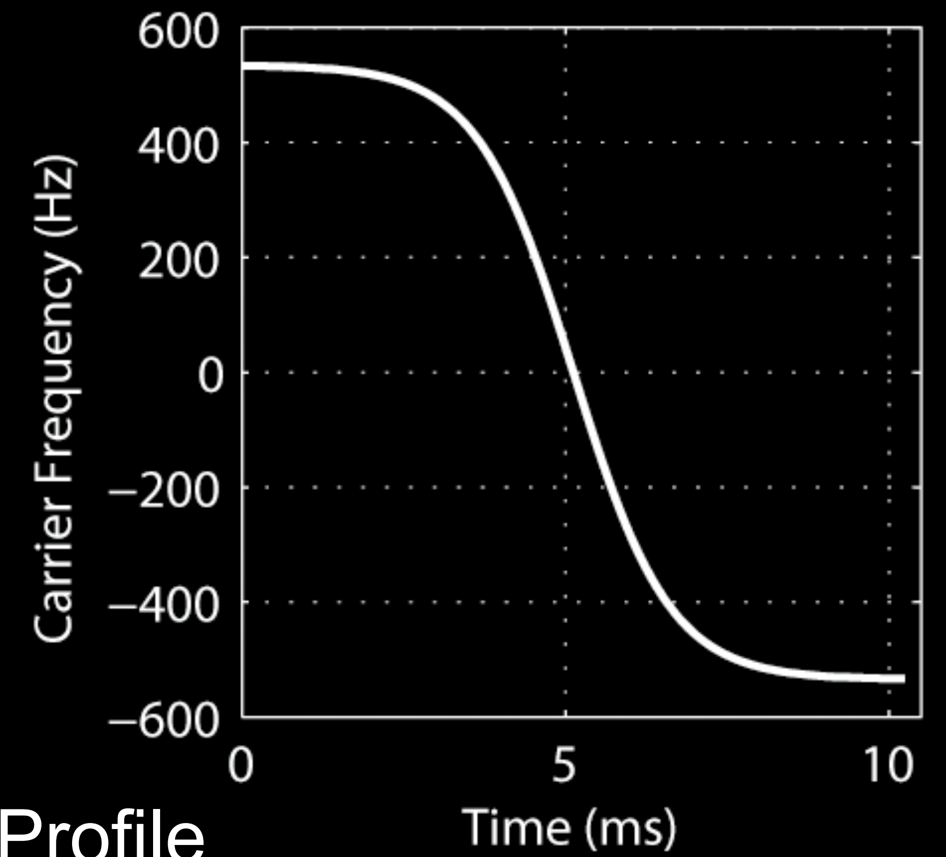
What Will Inversion Profile Look Like?

Hyperbolic Secant Pulse

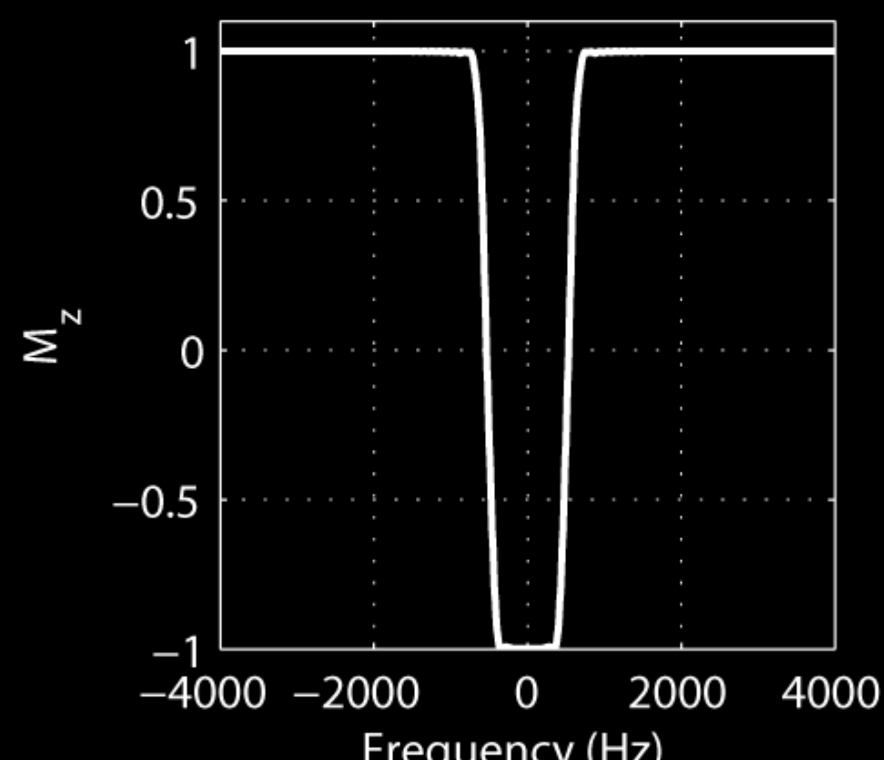
Amplitude Modulation, A(t)



Frequency Modulation, $\omega_1(t)$



Inversion Profile



Inversion Profiles

- The inversion profile typically calculated using Bloch simulation of the RF pulse (will be covered later) shows us the inversion efficiency and RF bandwidth
- The inversion efficiency depends strongly on the B_1 amplitude (as well as pulse duration, T_1, T_2 and pulse shape)
- For the hyperbolic secant pulse,

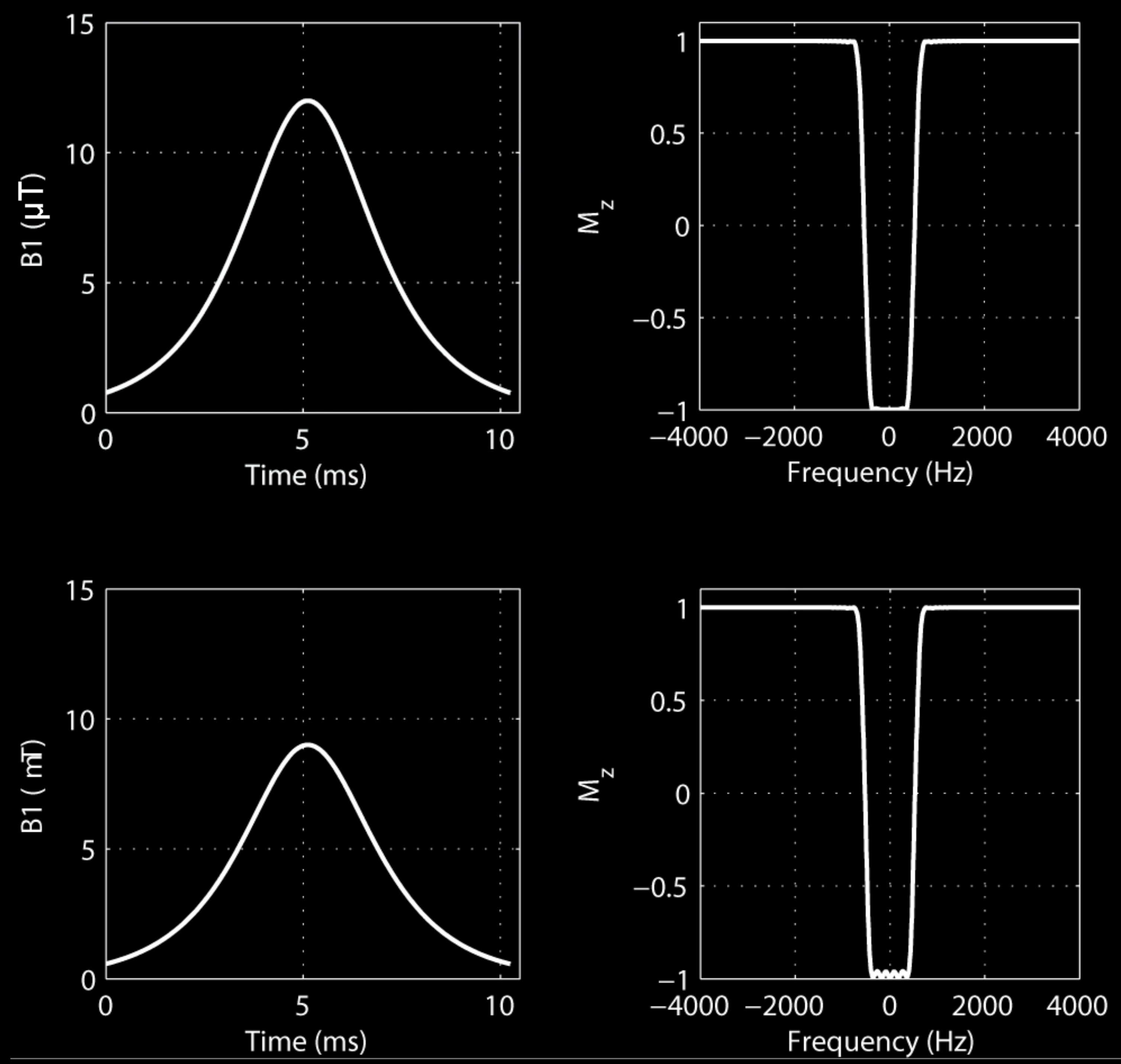
$$\text{RF spectral bandwidth} = \mu\beta$$

$$B_{1max} \gg (\beta\sqrt{\mu})/\gamma \quad (B_1 \text{ threshold for adiabaticity})$$

Hyperbolic Secant: Adiabatic Property

Original Pulse (100%)
B_{1max} = 12 μT

75% Attenuated Pulse
B_{1max} = 9 μT

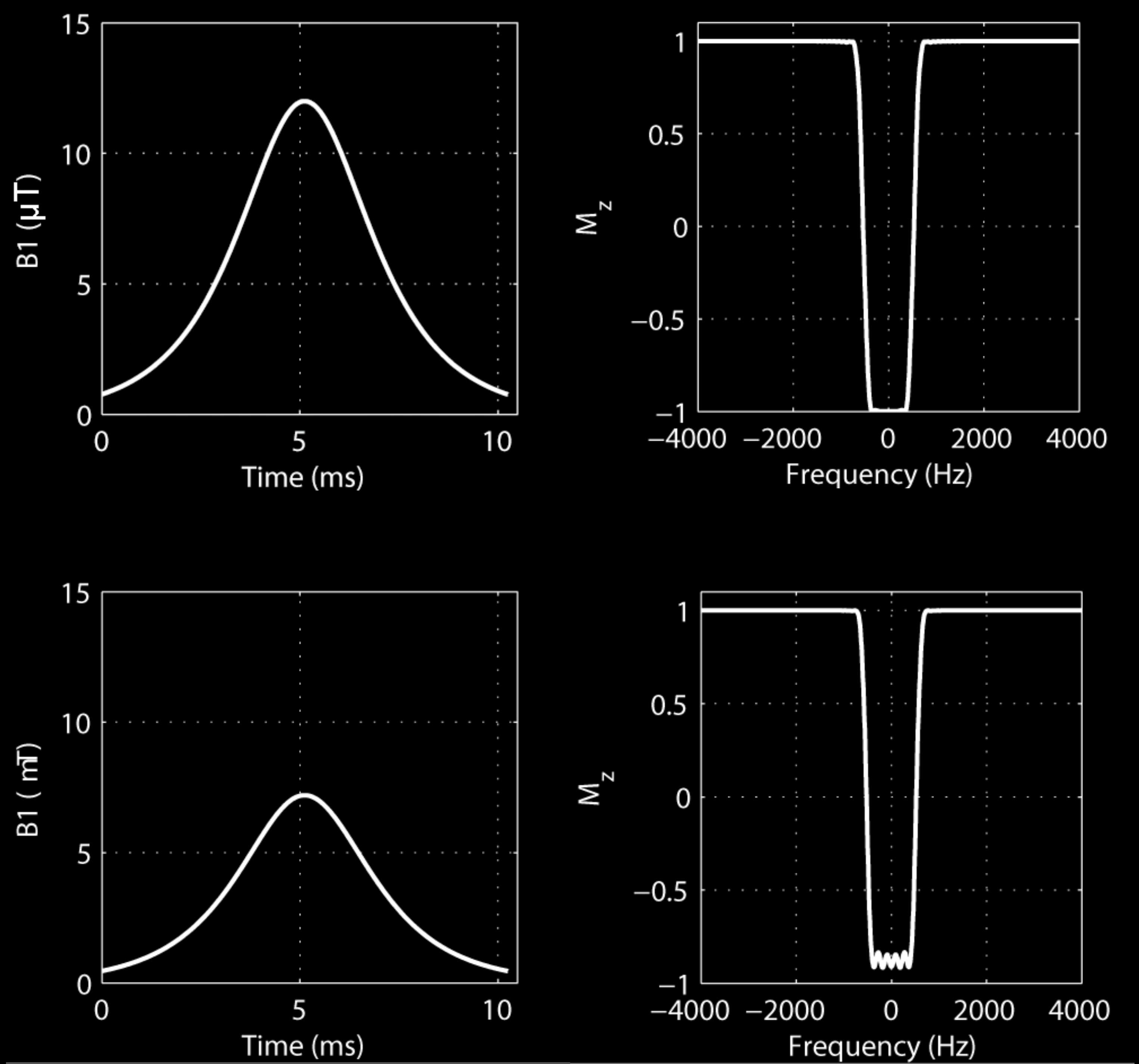


Hyperbolic Secant: Adiabatic Property

Original Pulse (100%)
B_{1max} = 12 μT

60% Attenuated Pulse
B_{1max} = 7.2 μT

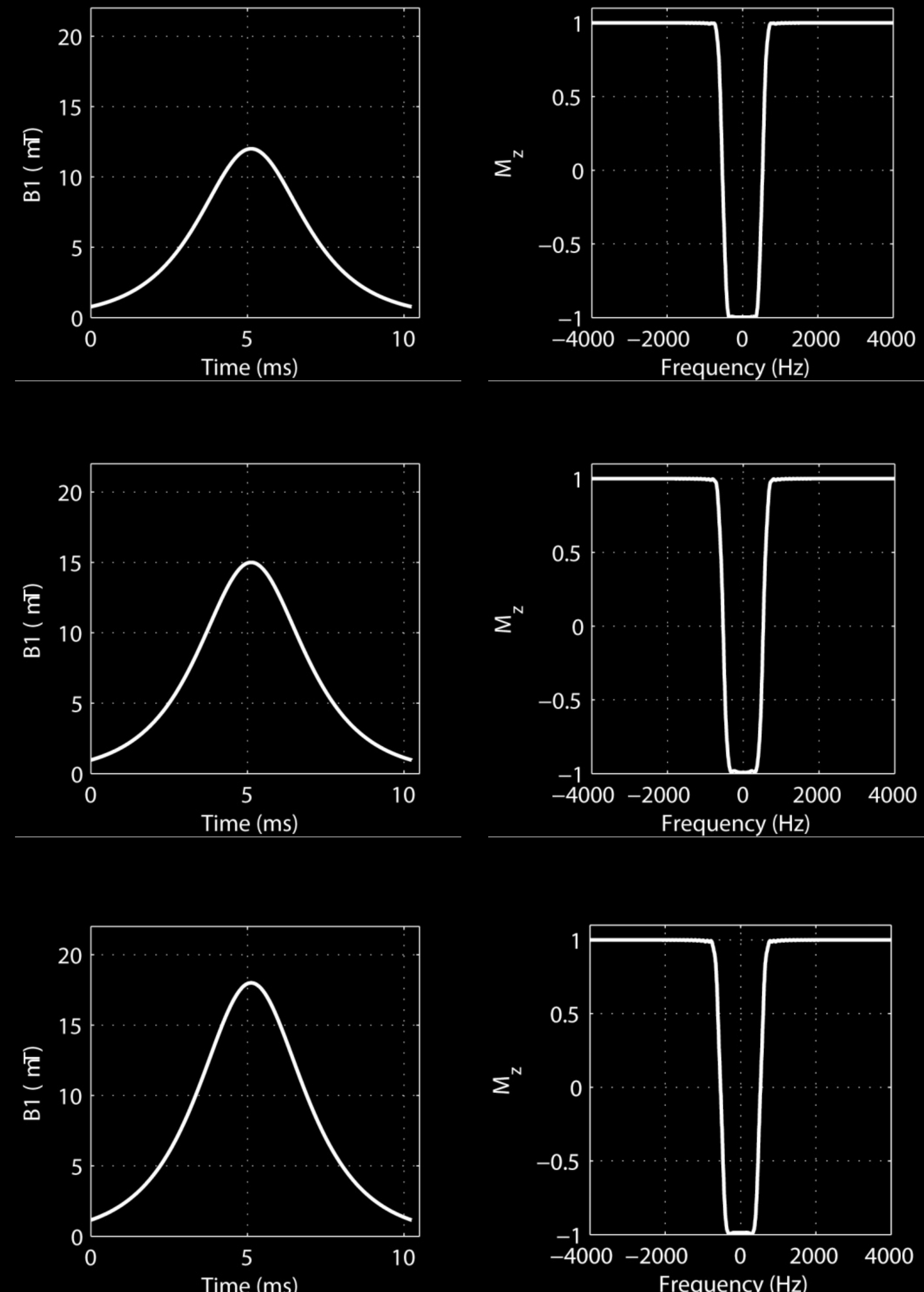
B_1 Threshold ≈ 6 μT



Original Pulse (100%)
B_1 = 12 µT

125% Amplified Pulse
B_1 = 15 µT

150% Amplified Pulse
B_1 = 18 µT



Comments

- Many envelope/modulation functions work
- If a range of adiabaticity is required, optimization can help reduce pulse length
- Hyperbolic secant needs to be truncated, which can affect the overall performance

Applications of Adiabatic Pulses

Adiabatic Pulses

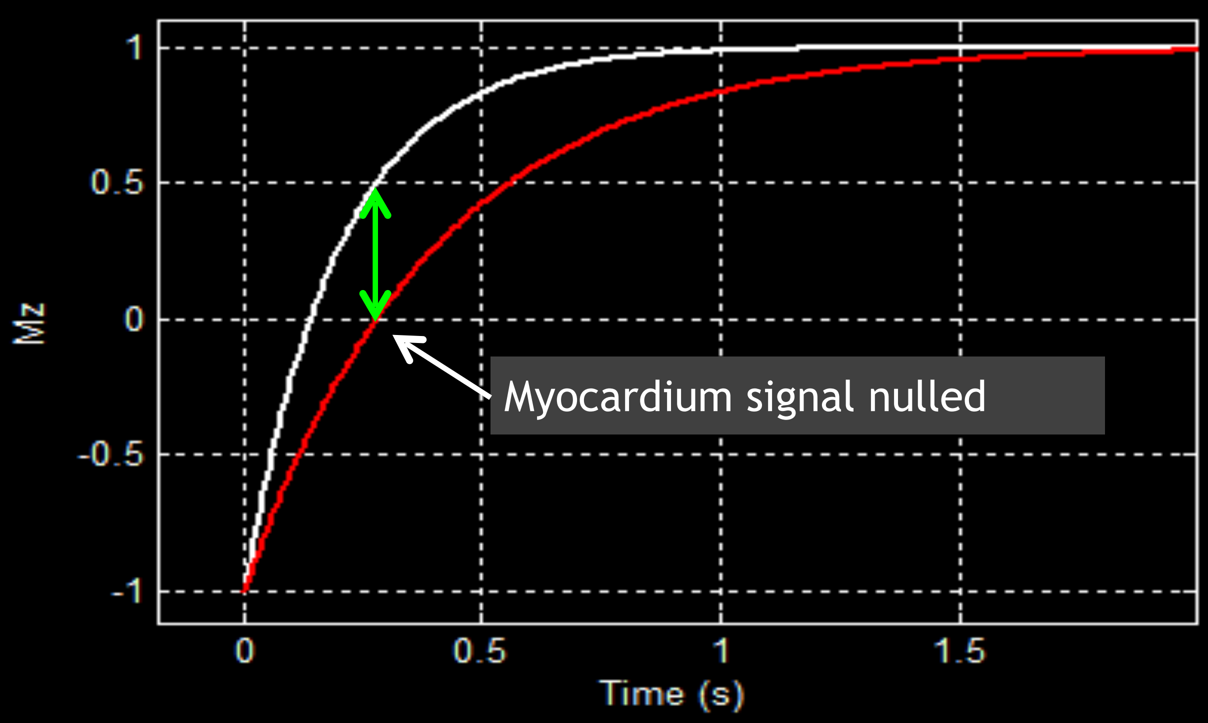
- Fat suppression (STIR)
- CSF suppression (FLAIR)
- Myocardium suppression in cardiac scar imaging (LGE)
- Black blood cardiac imaging (DIR TSE)
- T_1 Mapping

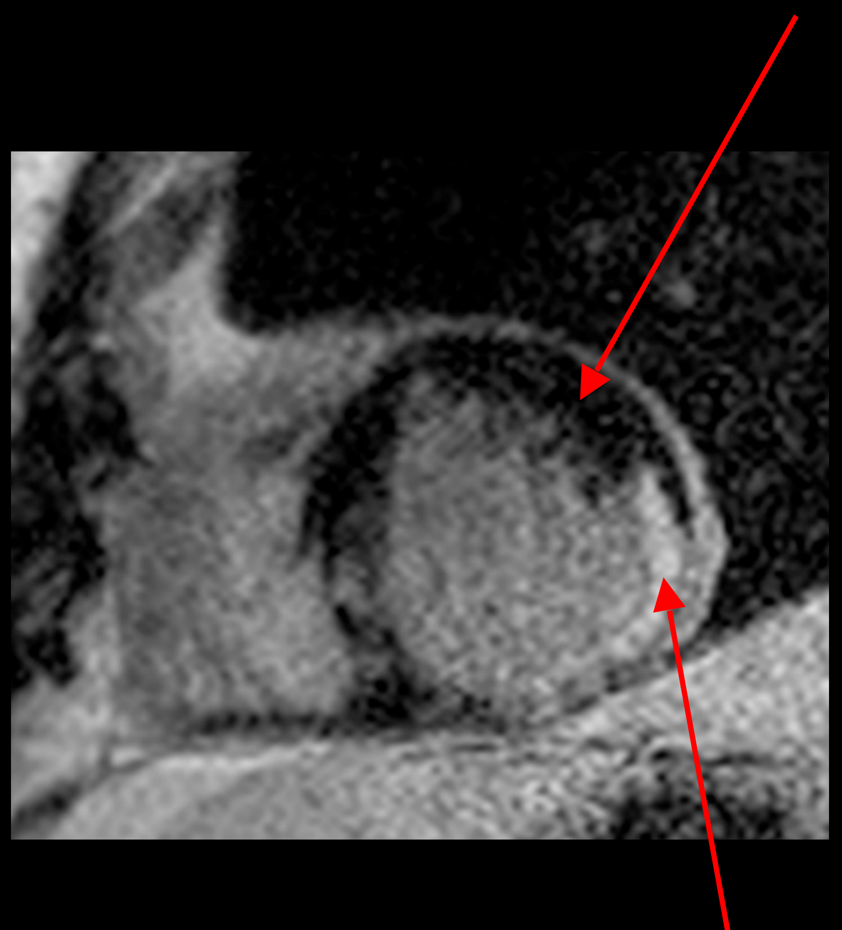
Late Gadolinium Enhancement (LGE)

- Gold standard for detection of scar/myocardial fibrosis
- Spoiled gradient echo (SPGR) sequence with an inversion pulse (inversion recovery SPGR)
  - Inversion pulse is usually hyperbolic secant pulse
  - Healthy myocardium is nulled with the inversion pulse
  - Scar tissue (which has shorter T_1 than healthy tissue) appear bright

- The conventional LGE sequence uses an RF-spoiled gradient echo (FLASH) readout with an inversion recovery (IR) pulse as a preparation pulse
- The readout is acquired at a time after inversion at which the healthy myocardium signal reaches zero

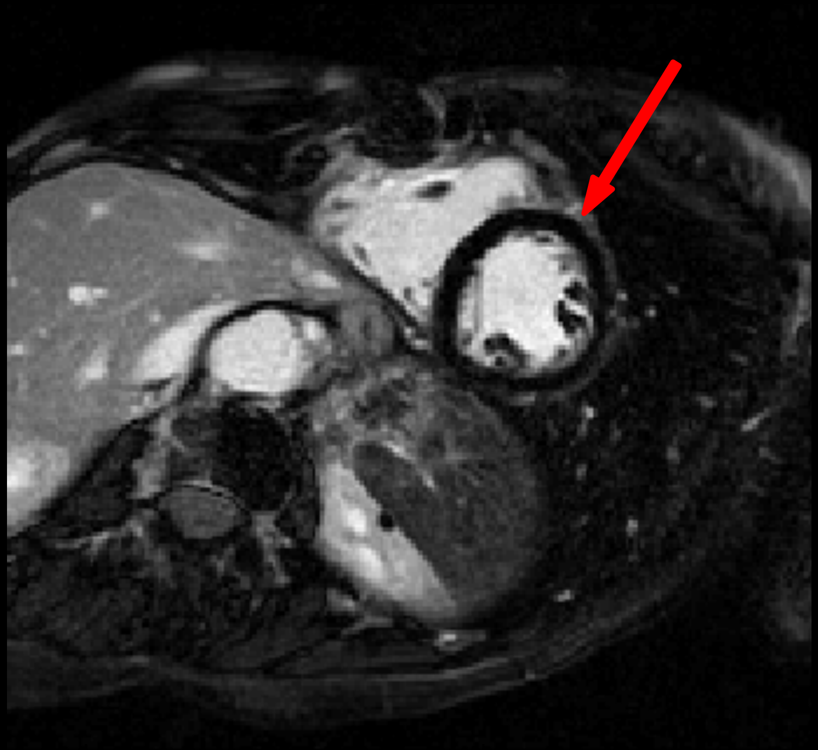
Inversion recovery curves of postcontrast scar (white) and myocardium (red)

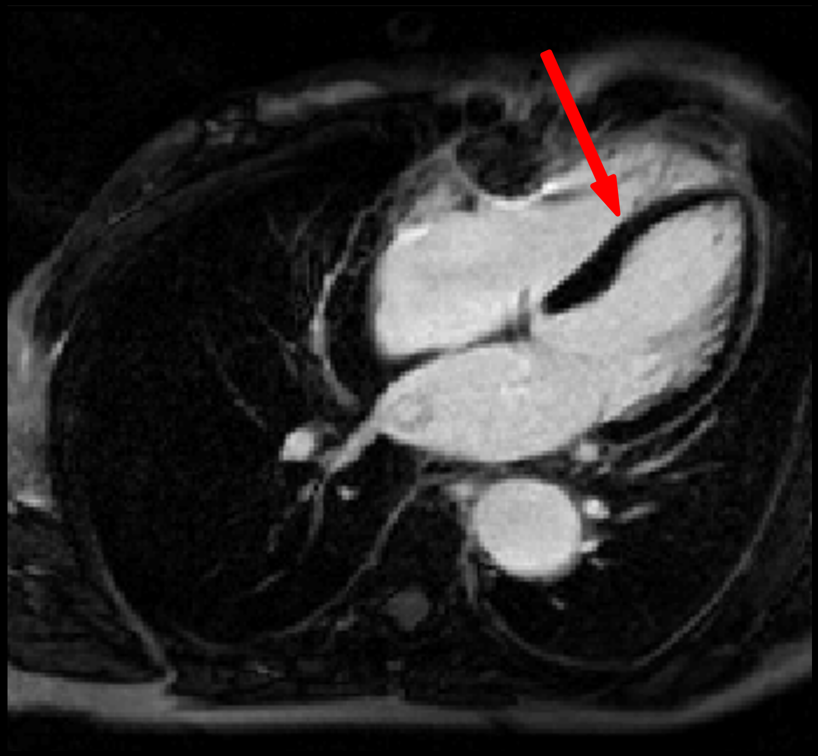




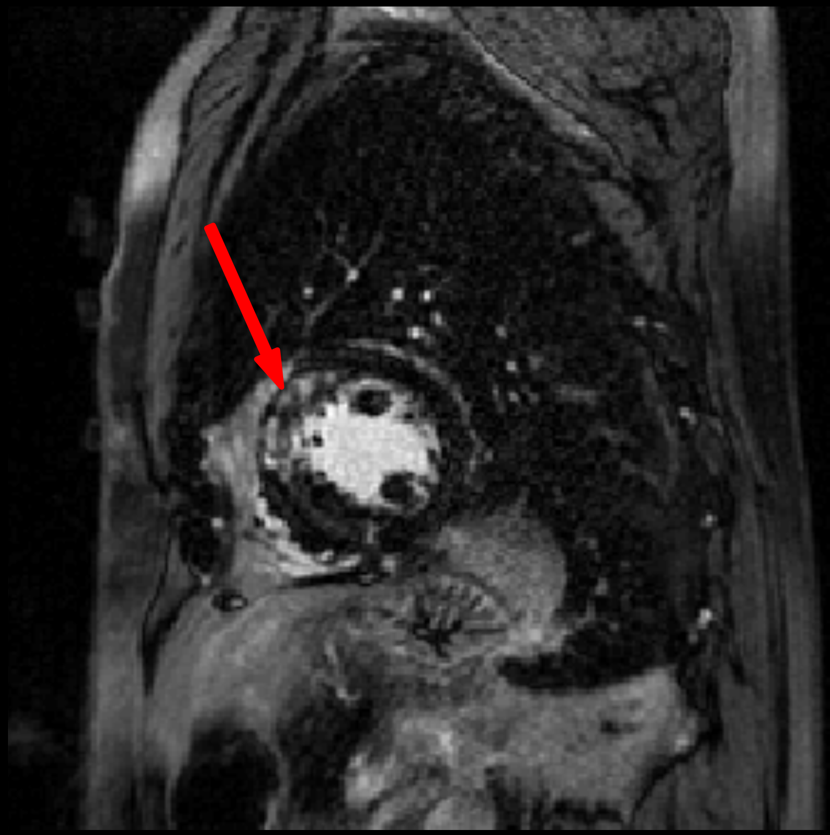
Clinical Example

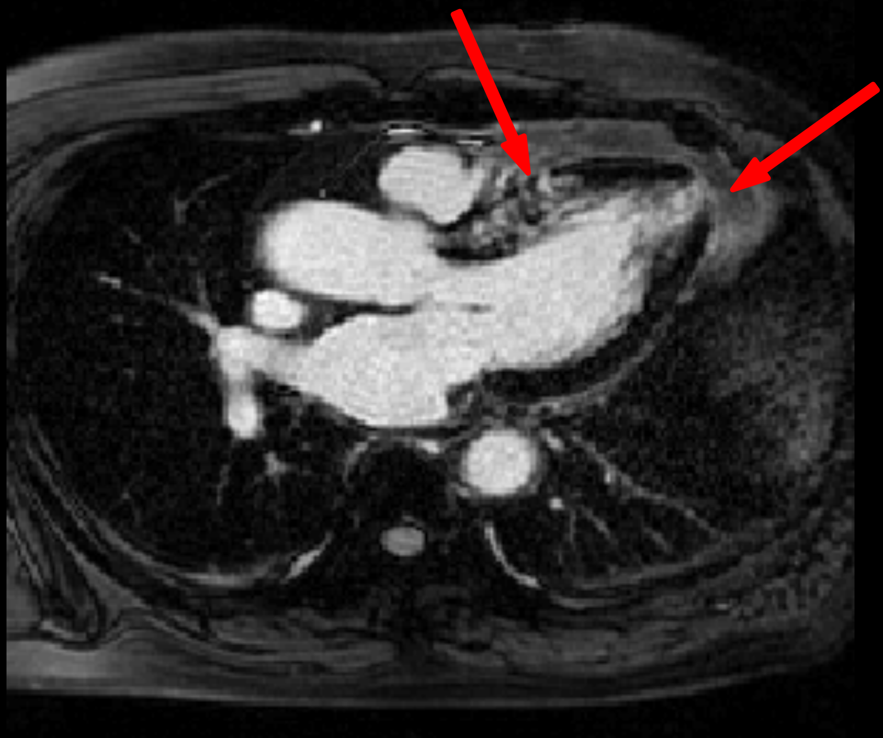
Patient with healthy myocardium





Patient with scar tissue

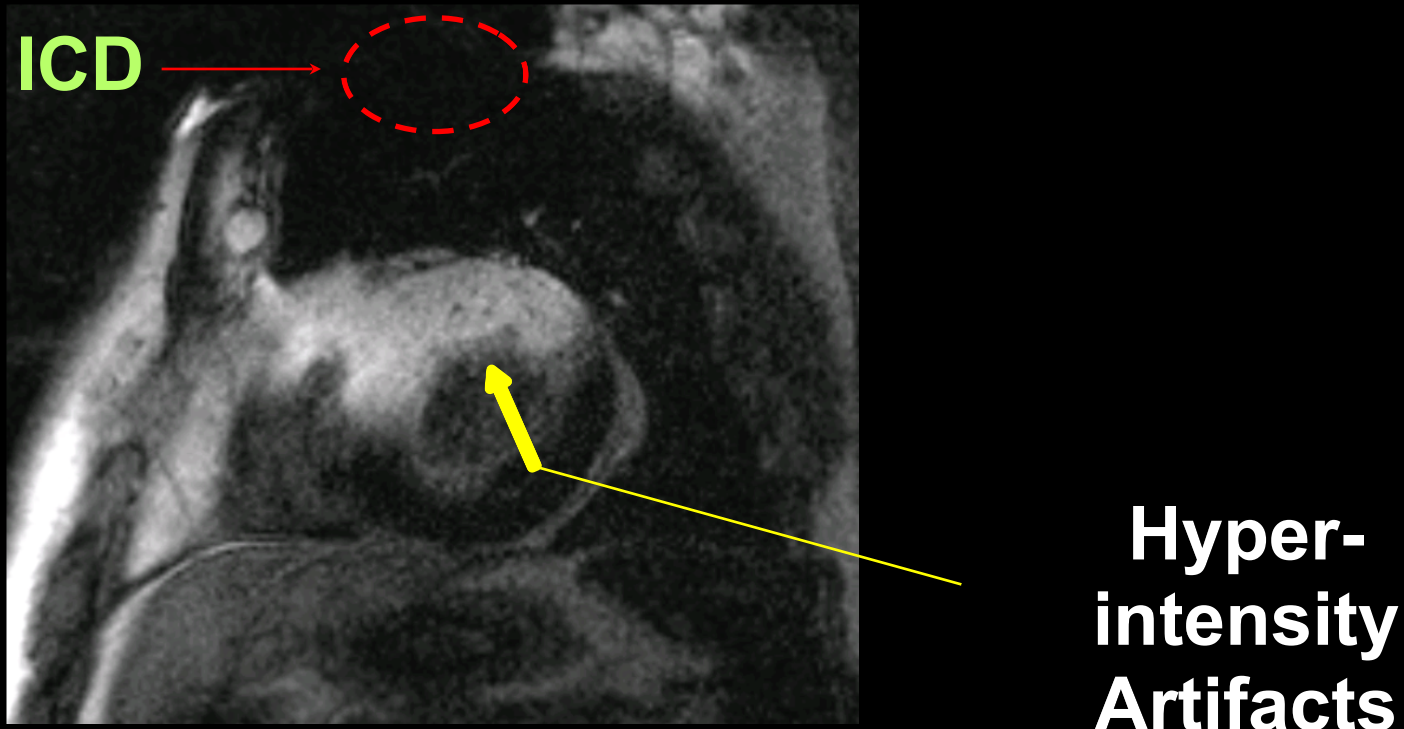


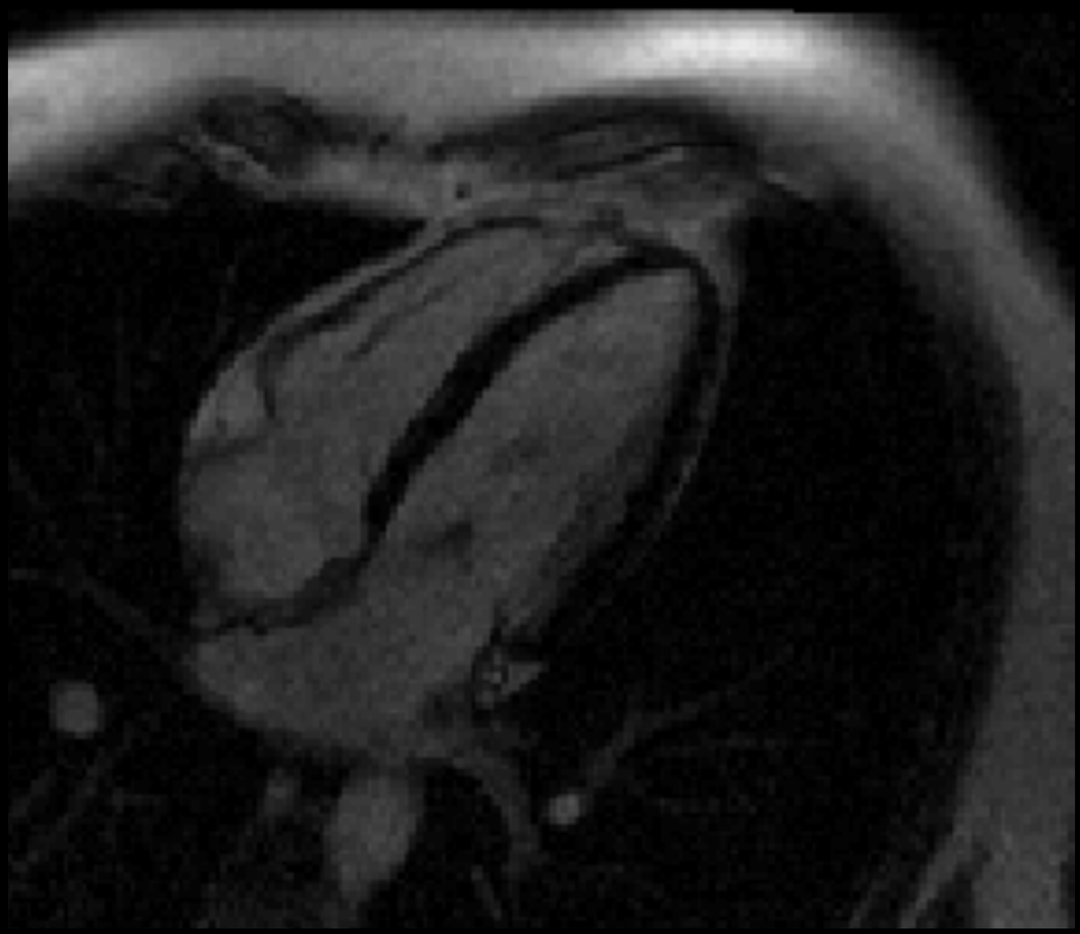


Clinical Example

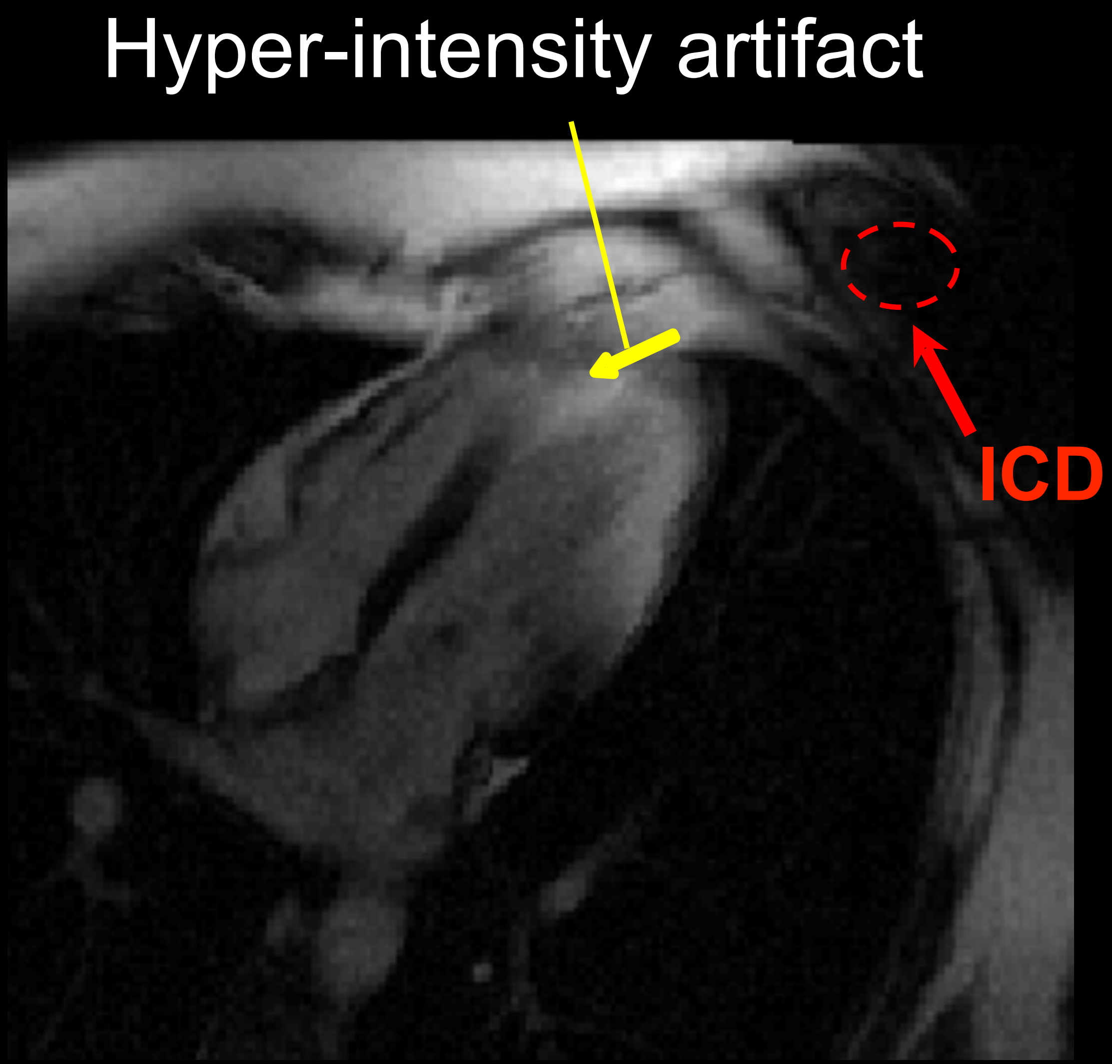
Late Gadolinium Enhancement (LGE) in patients with implantable cardiac devices

- Presence of an implantable cardiac device in the patients produces an interesting off-resonance artifact



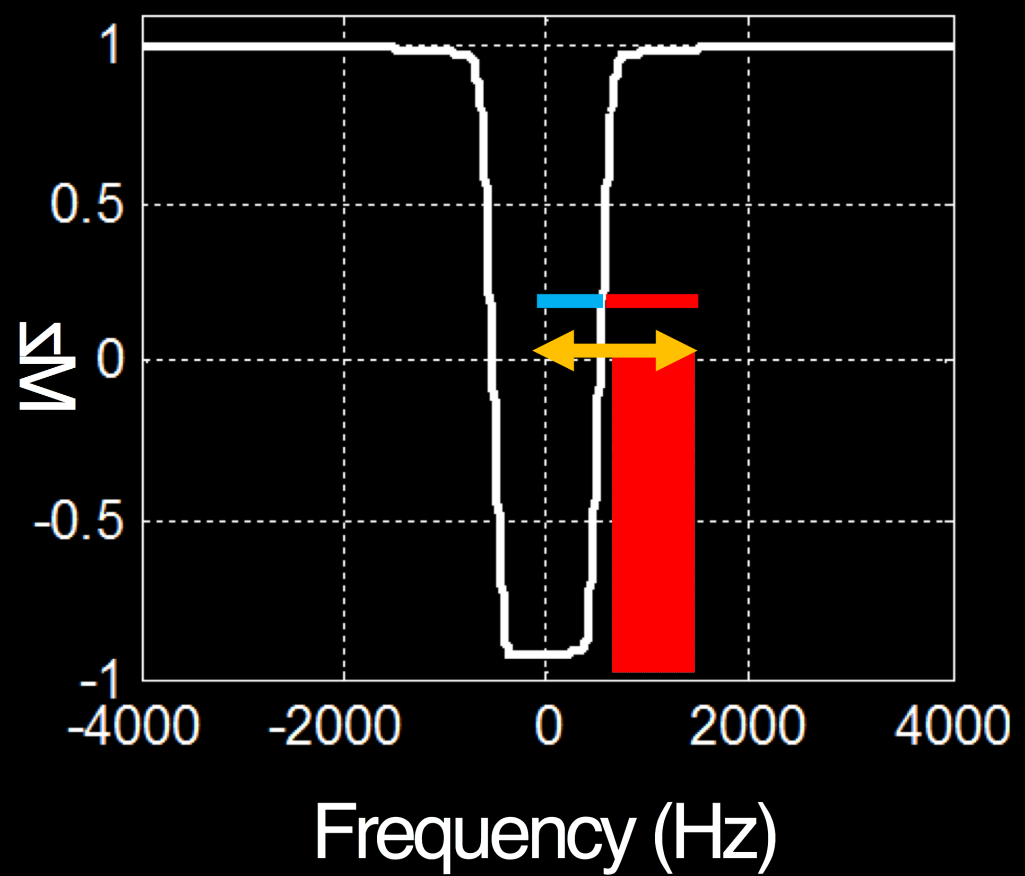


Conventional IR LGE Image



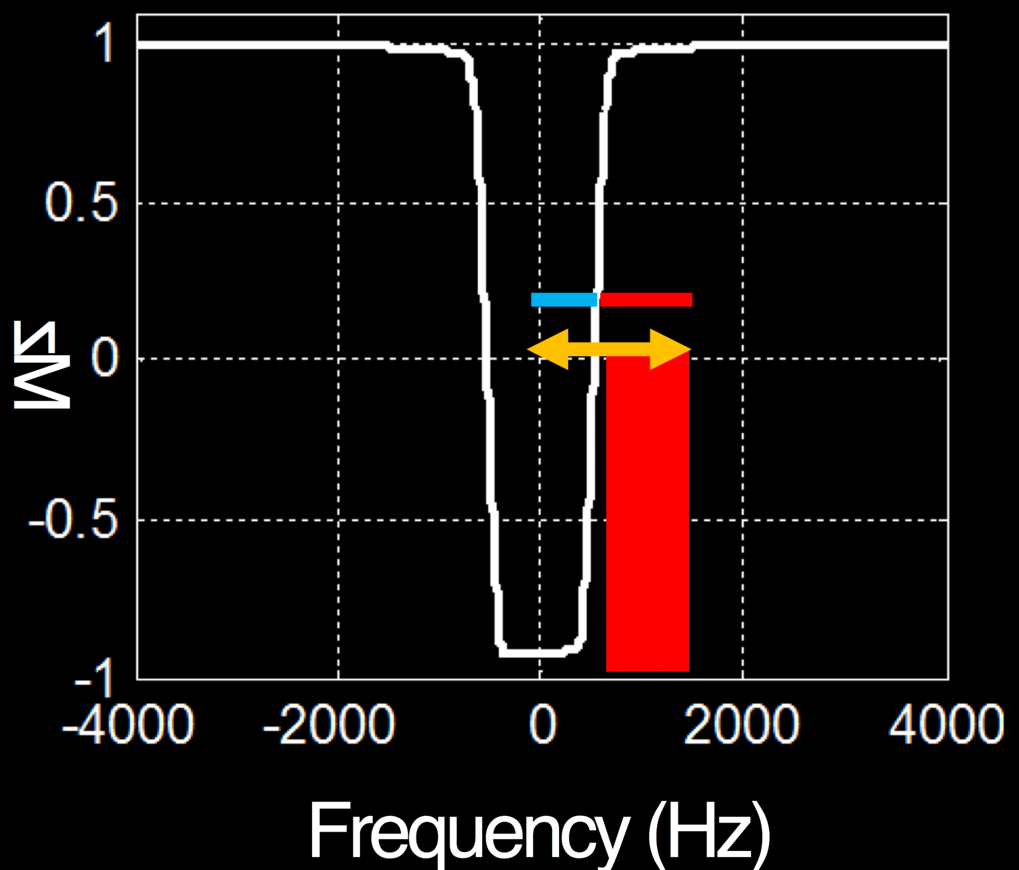
Conventional IR LGE Image

Cause of Artifact

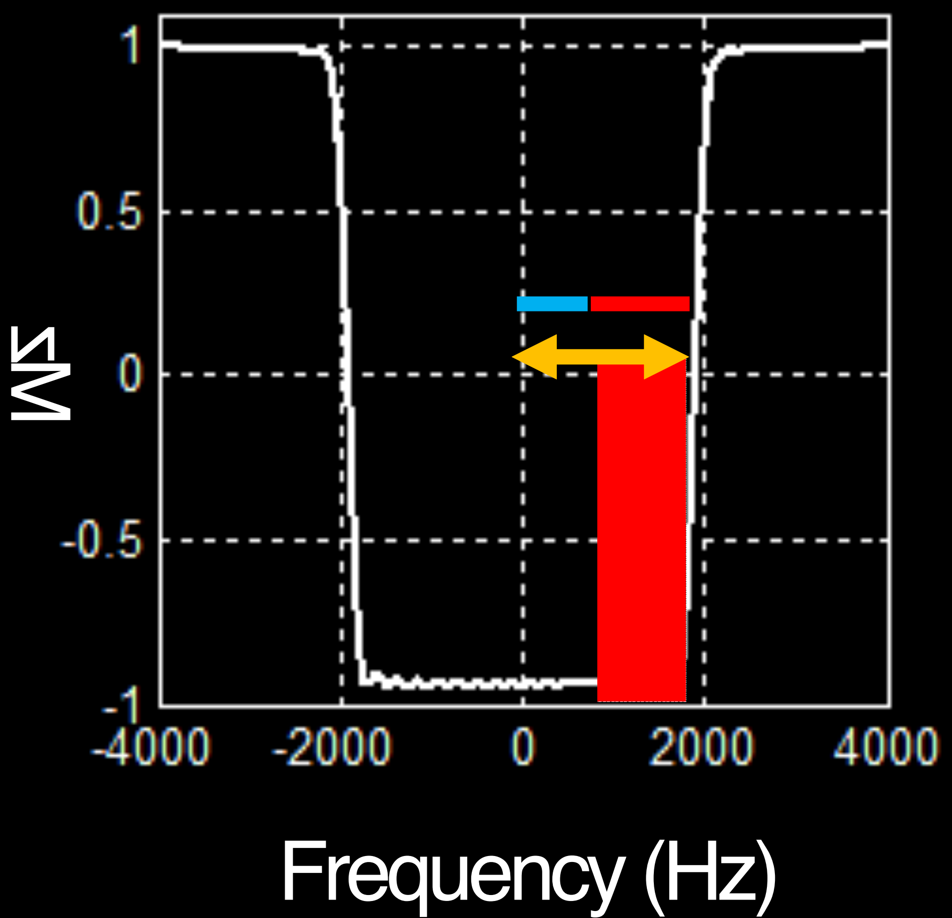


Longitudinal magnetization produced by conventional IR pulse
BW = 1.1 kHz

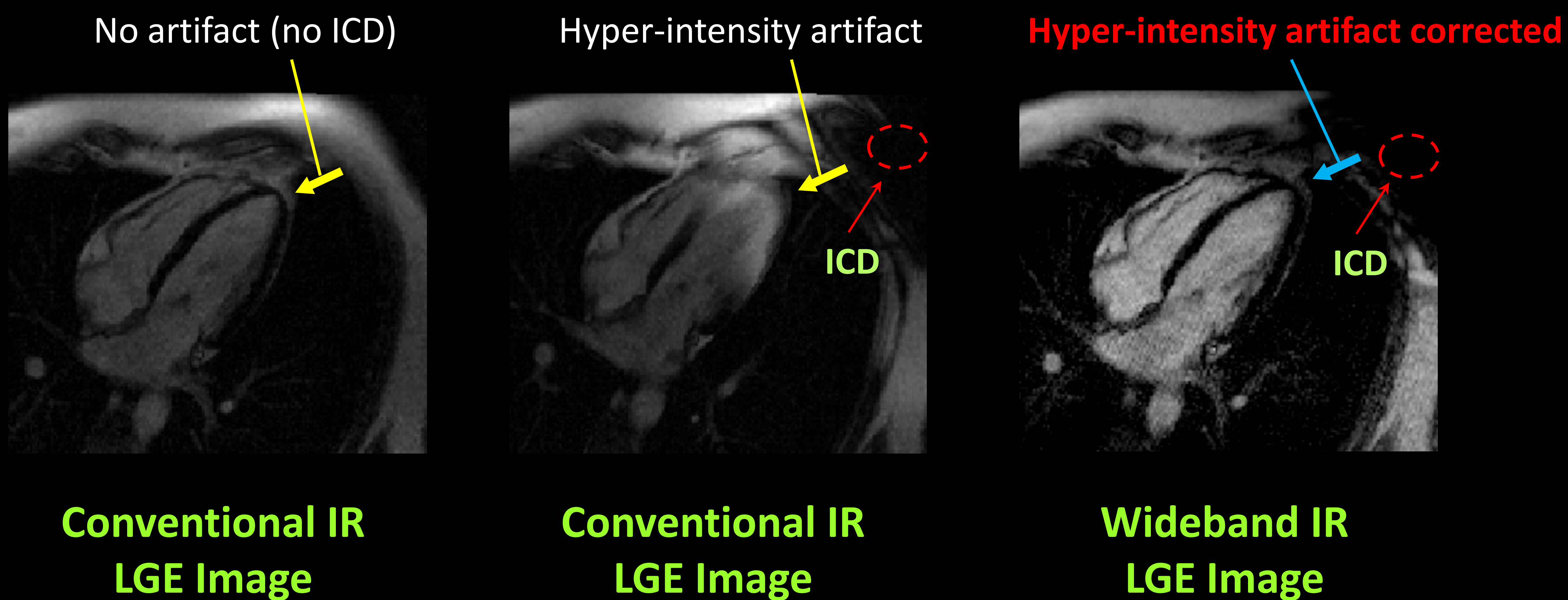
Solution: Increase Bandwidth of Inversion Pulse

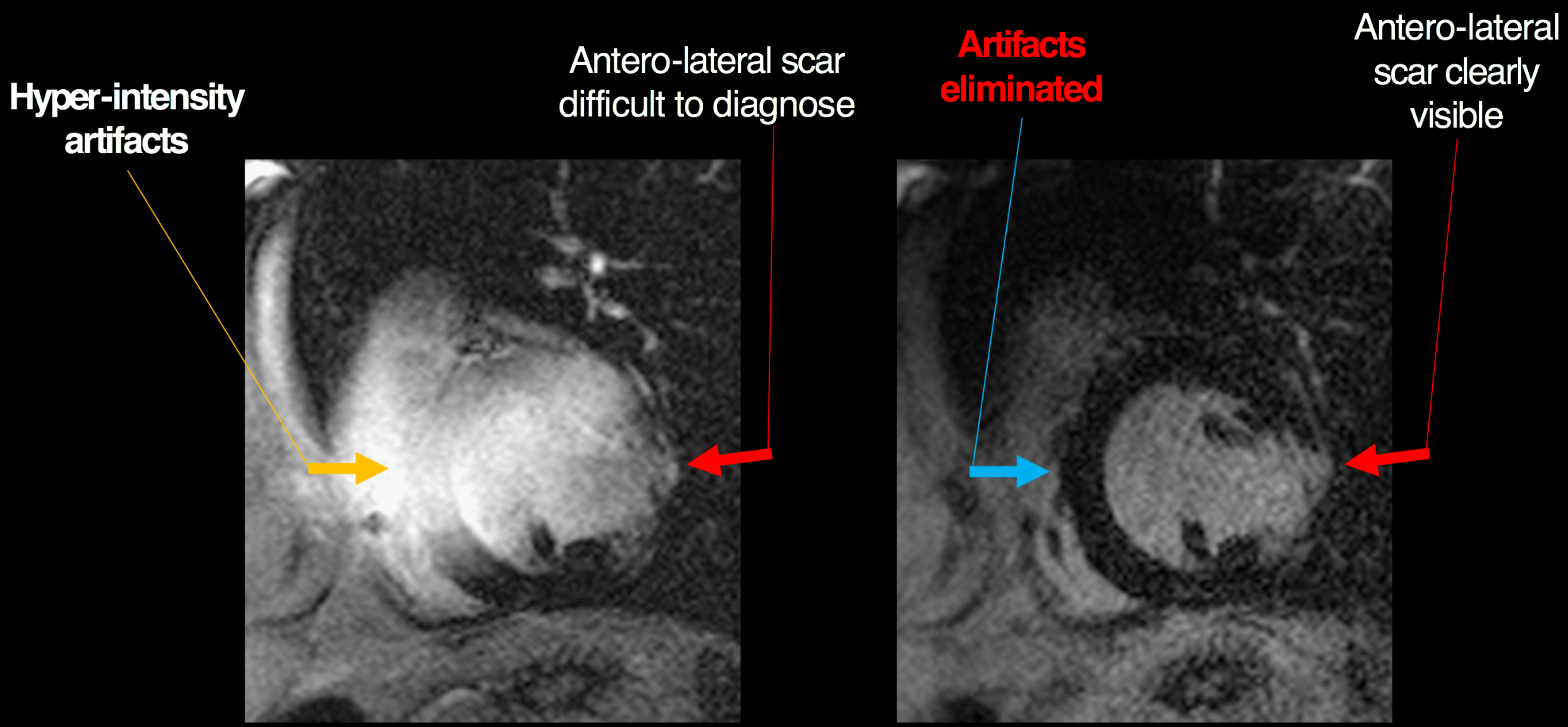


Longitudinal magnetization produced by conventional IR pulse
BW = 1.1 kHz



Longitudinal magnetization produced by wideband IR pulse
BW = 3.8 kHz


No artifact (no ICD)
Hyper-intensity artifact
Hyper-intensity artifact corrected
ICD
ICD
Conventional IR
LGE Image
Conventional IR
LGE Image
Wideband IR
LGE Image


Hyper-intensity artifacts
Antero-lateral scar difficult to diagnose
Artifacts eliminated
Antero-lateral scar clearly visible

MATLAB Demo

```
%%% User inputs:
mu = 5;   % Phase modulation parameter [dimensionless]
beta1 = 672;   % Frequency modulation parameter [rad/s]
pulseWidth = 10.24;   % RF pulse duration [ms]
A0 = 0.12;   % Peak B1 amplitude [Gauss].

%%%%%%

nSamples = 512;          % number of samples in the RF pulse
dt = pulseWidth/nSamples/1000;  % time step, [seconds]
tim_sech = linspace(-pulseWidth/2,pulseWidth/2,nSamples)./1000';
% time scale to calculate the RF waveforms in seconds.

% Amplitude modulation function B1(t):
B1 = A0.* sech(beta1.*tim_sech);

% Carrier frequency modulation function w(t):
w = -mu.*beta1.*tanh(beta1.*tim_sech)./(2*pi);
% The 2*PI scaling factor at the end converts the unit from rad/s to Hz

% Phase modulation function phi(t):
phi = mu .* log(sech(beta1.*tim_sech));

% Put together complex RF pulse waveform:
rf_pulse = B1 .* exp(1i.*phi);

% Generate a time scale for the Bloch simulation:
tim_bloch = [0:(nSamples-1)]*dt;
```

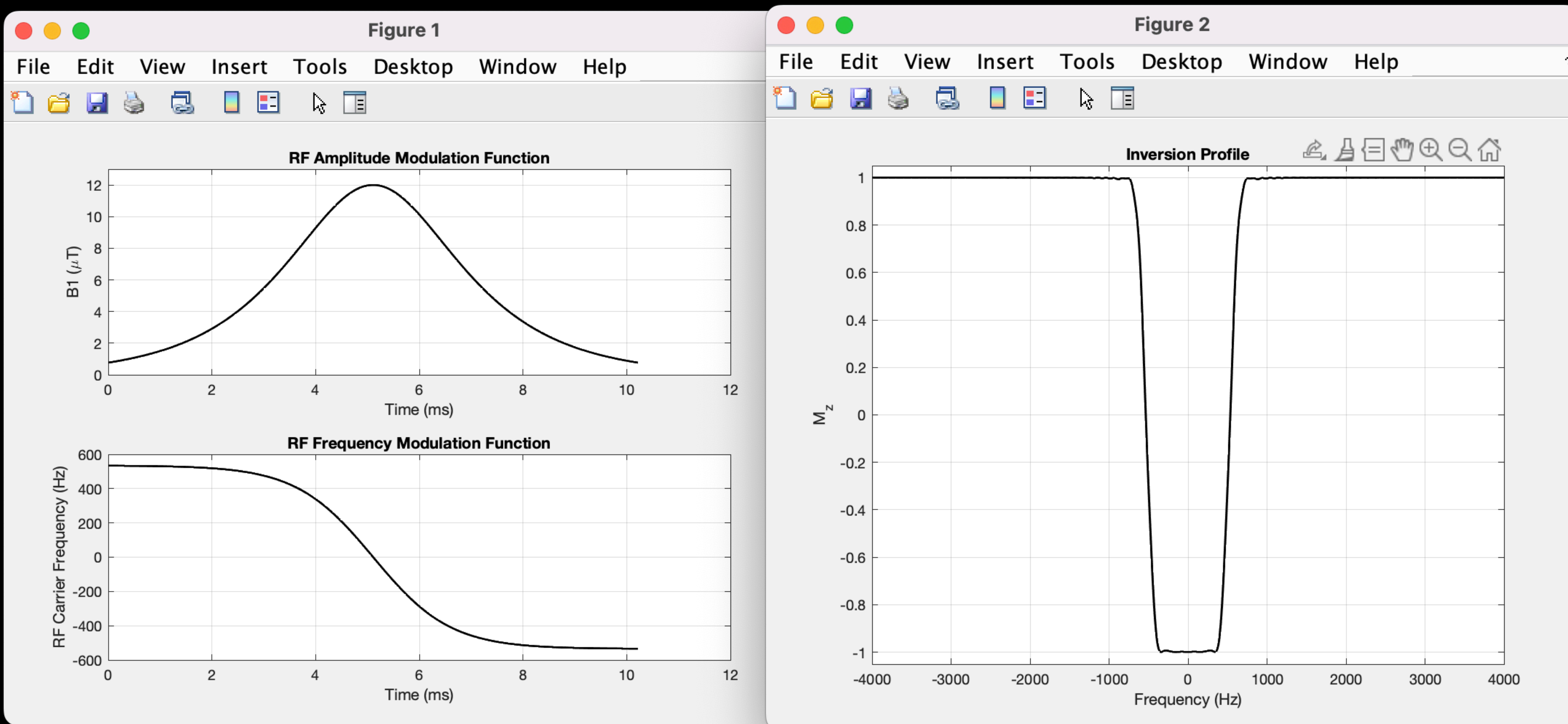


Figure 1
File Edit View Insert Tools Desktop Window Help
RF Amplitude Modulation Function
B1 (μT)
Time (ms)
RF Frequency Modulation Function
RF Carrier Frequency (Hz)
Time (ms)
Figure 2
File Edit View Insert Tools Desktop Window Help
Inversion Profile
Mz
Frequency (Hz)

Thank You!

- Further reading
  - Read "Adiabatic Refocusing Pulses" p.200-212
  - Tannus et al., “Adiabatic Pulses”, NMR in Biomedicine, Vol. 10, 423-434 (1997)

- Acknowledgments
  - John Pauly’s EE469B (RF Pulse Design for MRI)
  - Shams Rashid
  - Kyung Sung

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