

Fast Imaging Trajectories: Non-Cartesian Sampling (2)

M229 Advanced Topics in MRI

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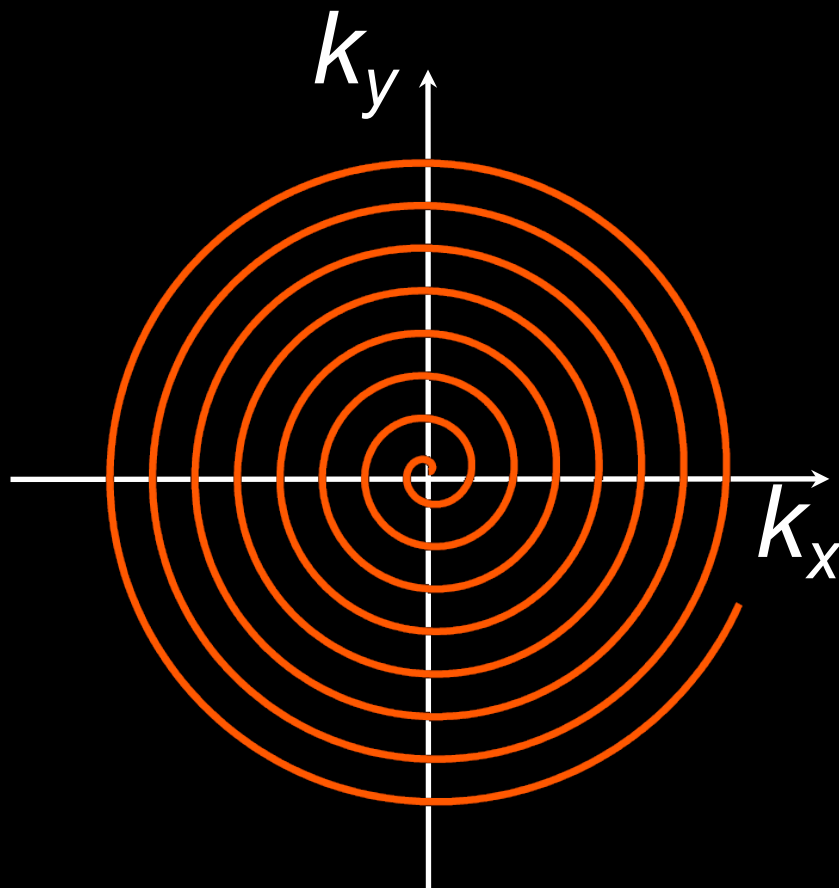
Class Business

- Homework 2 - being graded
- Final project
 - Proposal due 5/9 Fri by 5 pm
 - Abstract due 6/6 Fri by 5 pm
 - Presentations and Q&A on 6/10, 6/12

Outline

- Spiral Trajectory
- Non-Cartesian 3D Trajectories
 - 3D stack of radial
 - 3D radial
 - 3D cones
- Non-Cartesian Image Reconstruction
 - Gridding reconstruction
 - Gradient measurement
 - Off-resonance correction

Spirals



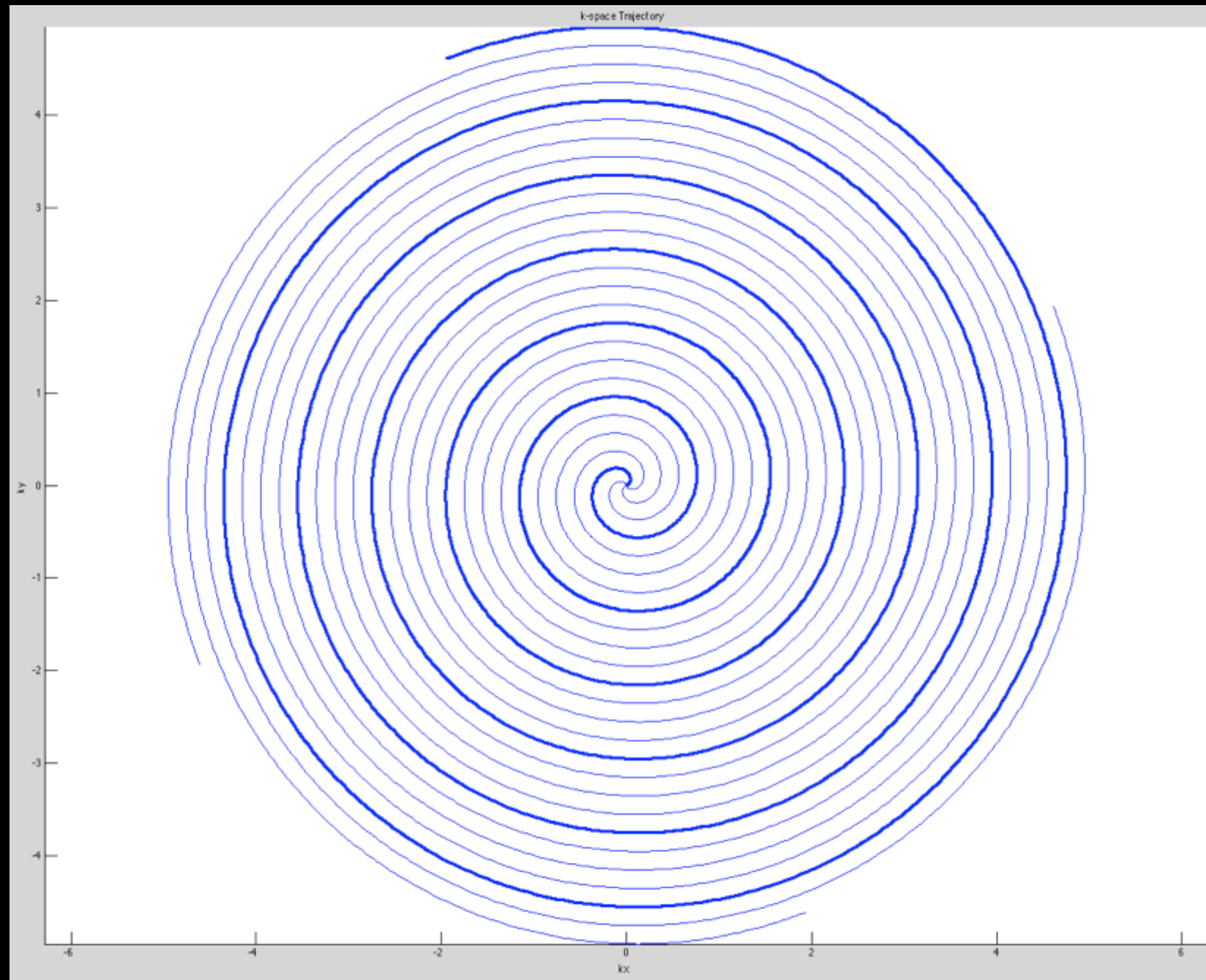
“THE” non-Cartesian trajectory

Highly robust to motion/flow effects

Very fast!

- optimal use of gradients in 2D
- can acquire one image in ~ 100 ms

Spirals: Sampling Requirements



N interleaves

$$2 k_{r,max} = 1 / dx$$

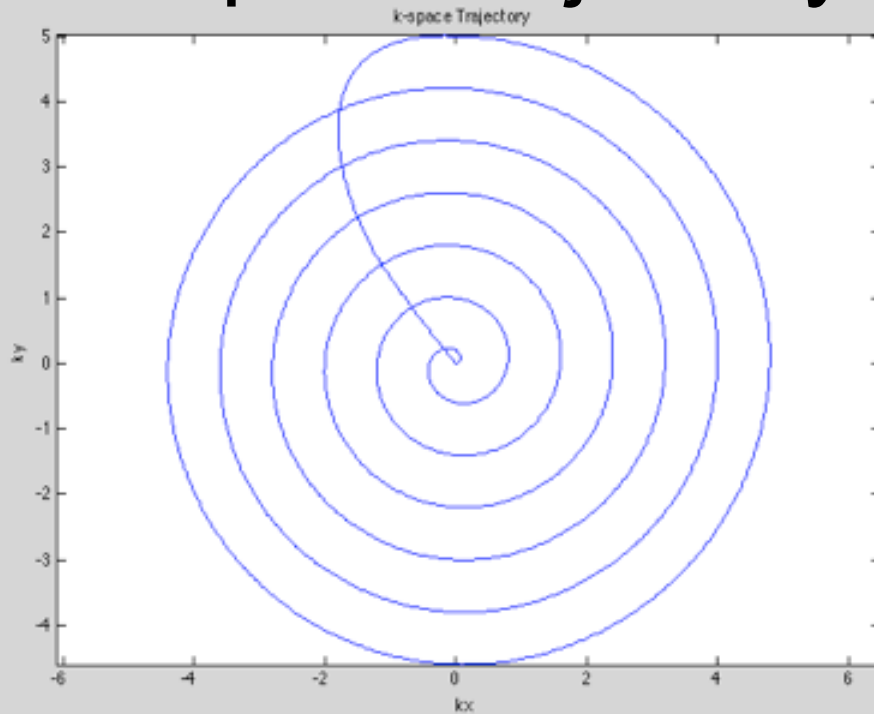
$$dk = 1 / FOV$$

Design 1 interleaf
and rotate

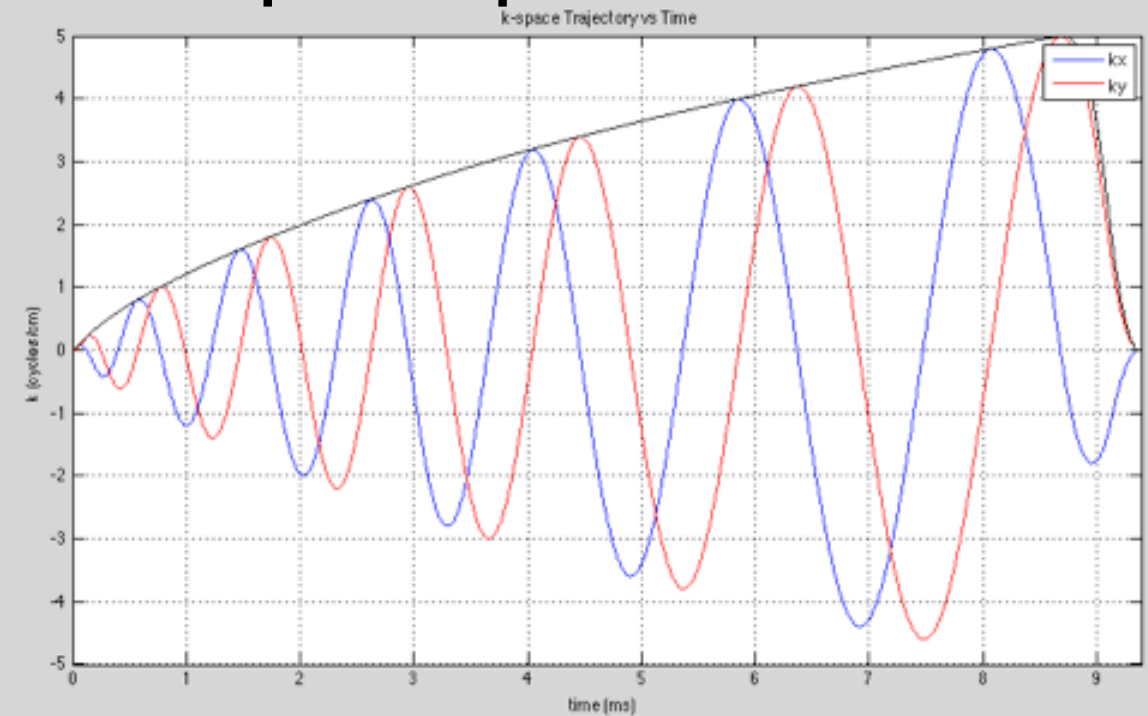
Subject to HW limits

Spirals: Gradient Design

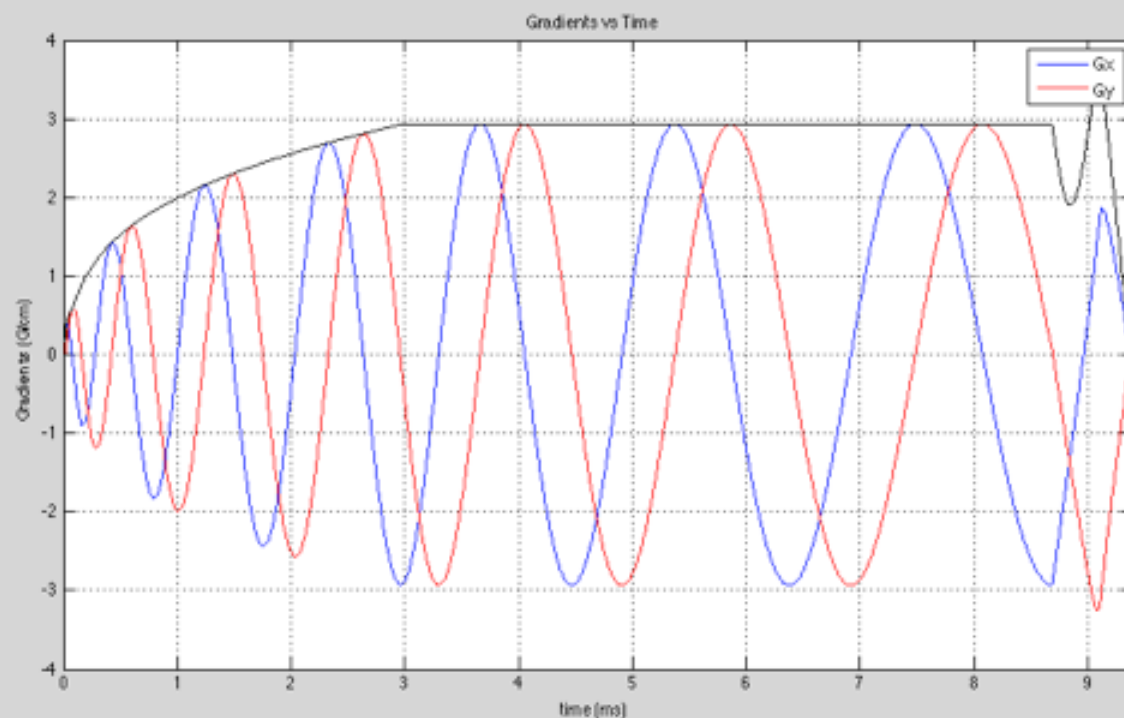
k-space trajectory



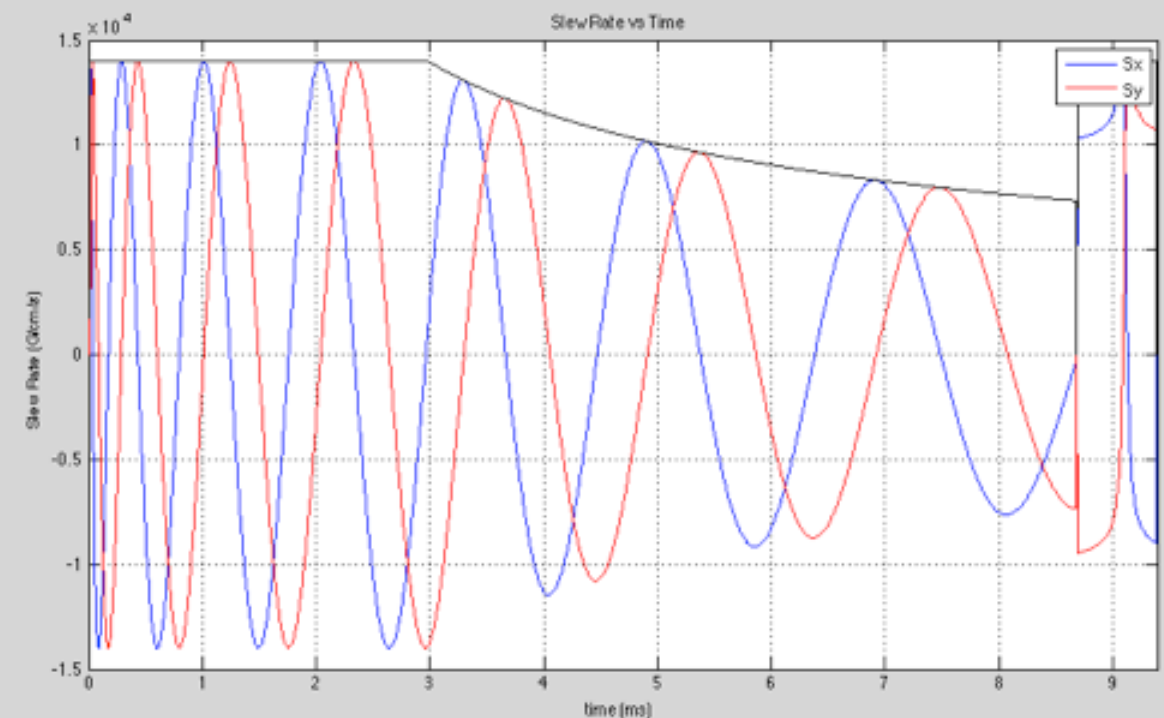
k-space pos vs. time



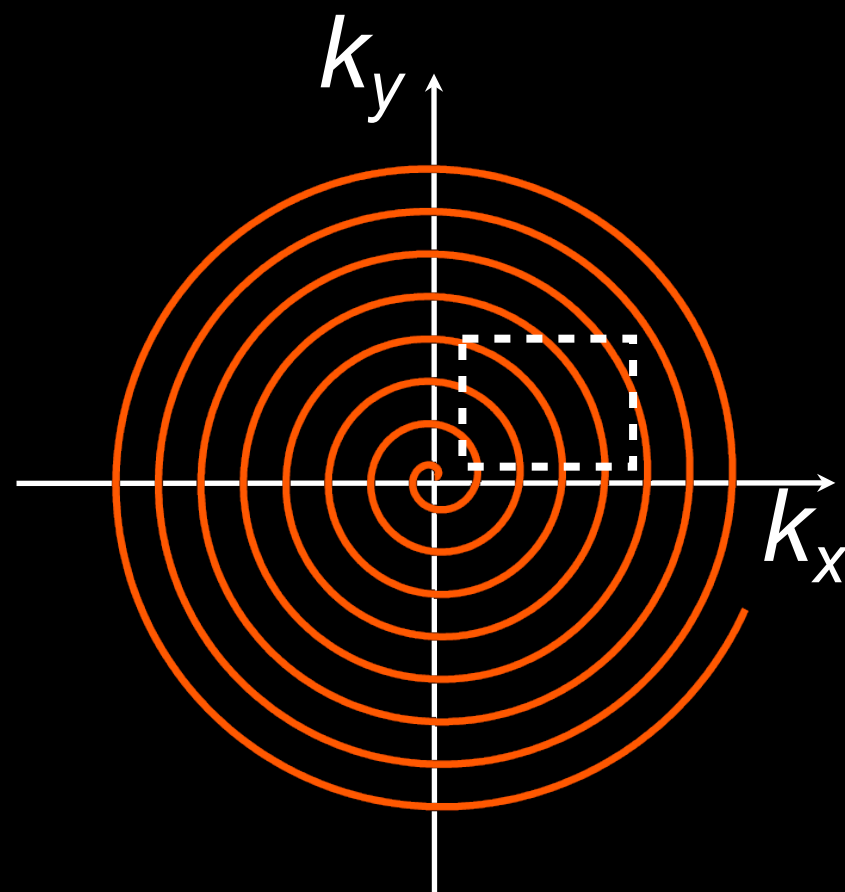
Gradients vs. time



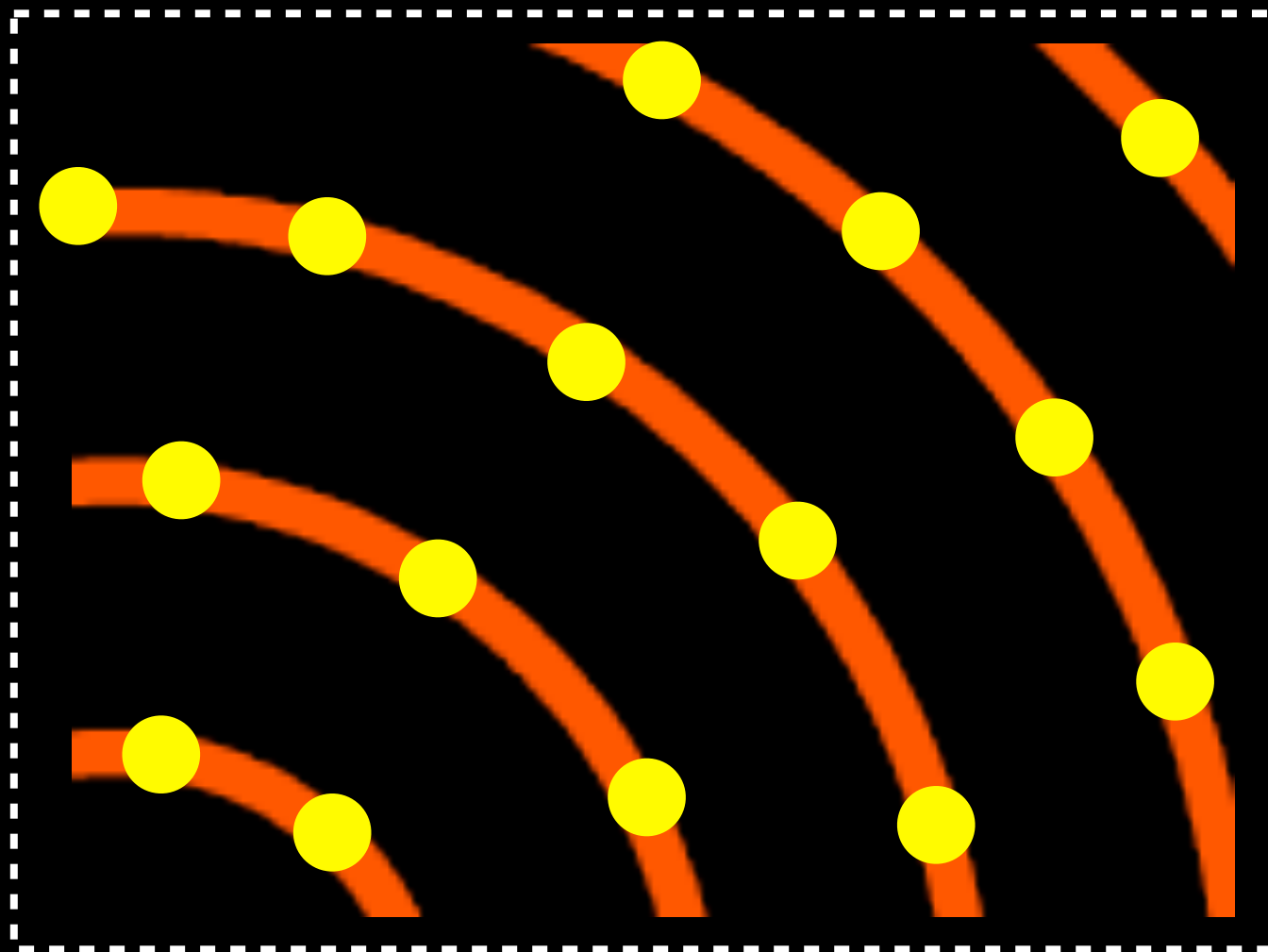
Slew rate vs. time



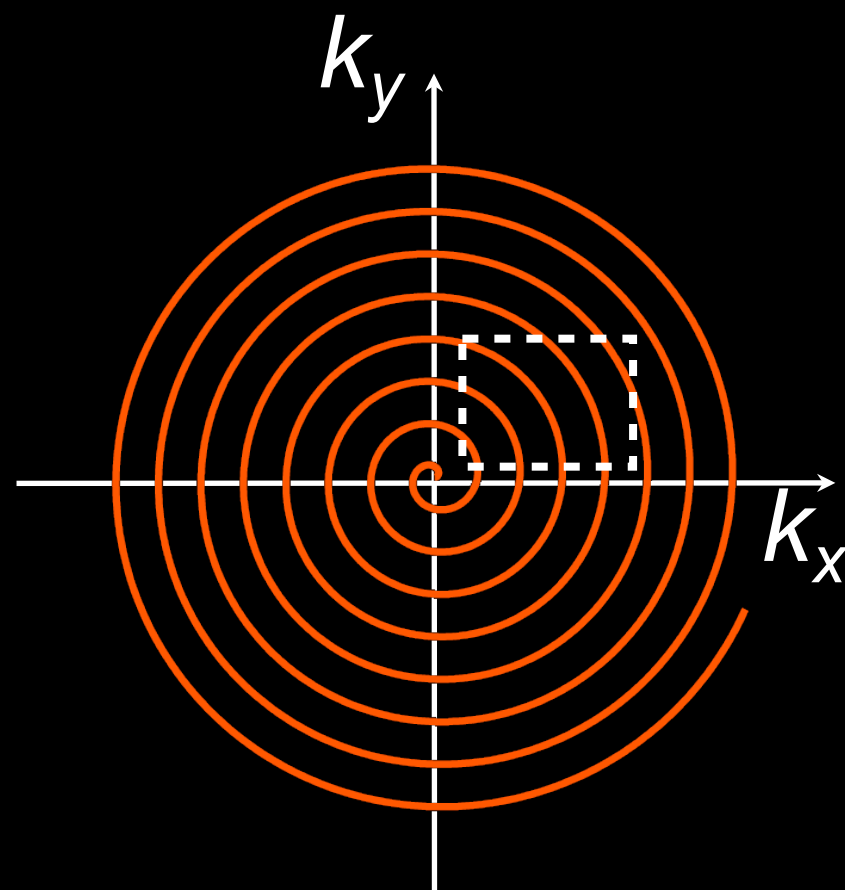
Spirals: Image Reconstruction



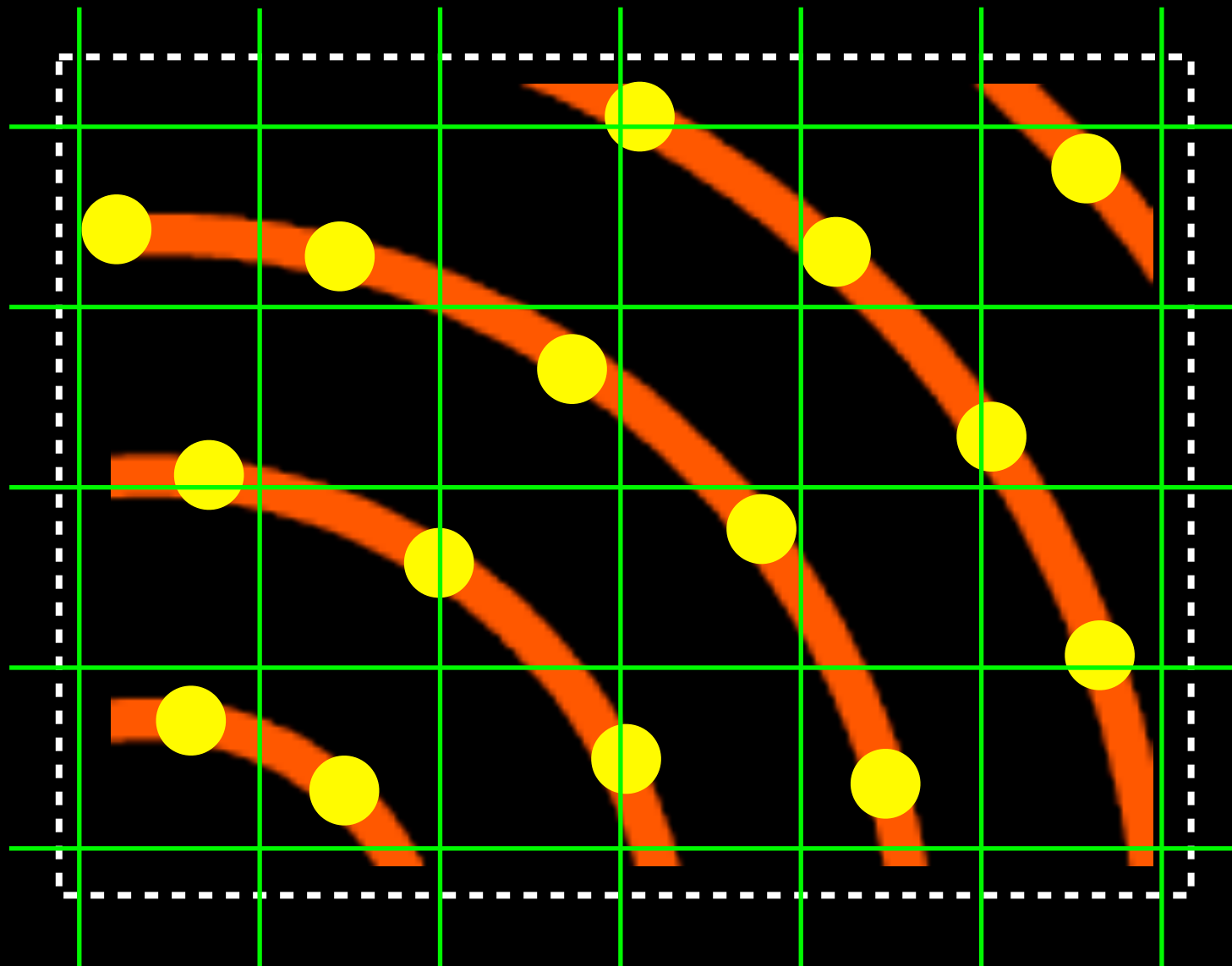
Gridding Algorithm



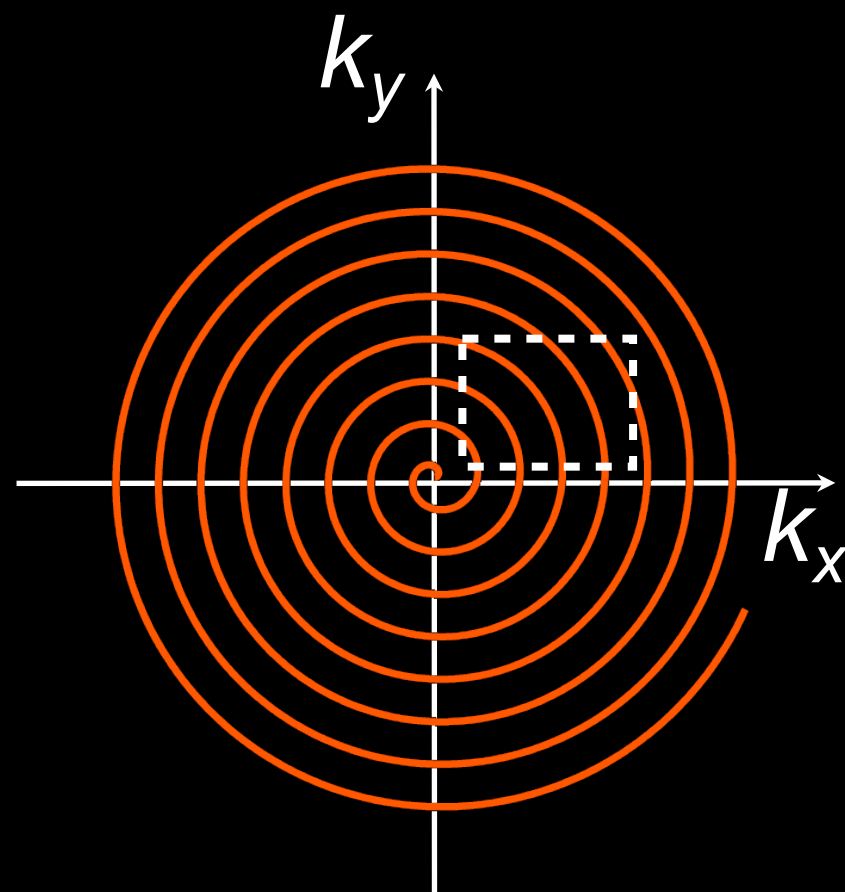
Spirals: Image Reconstruction



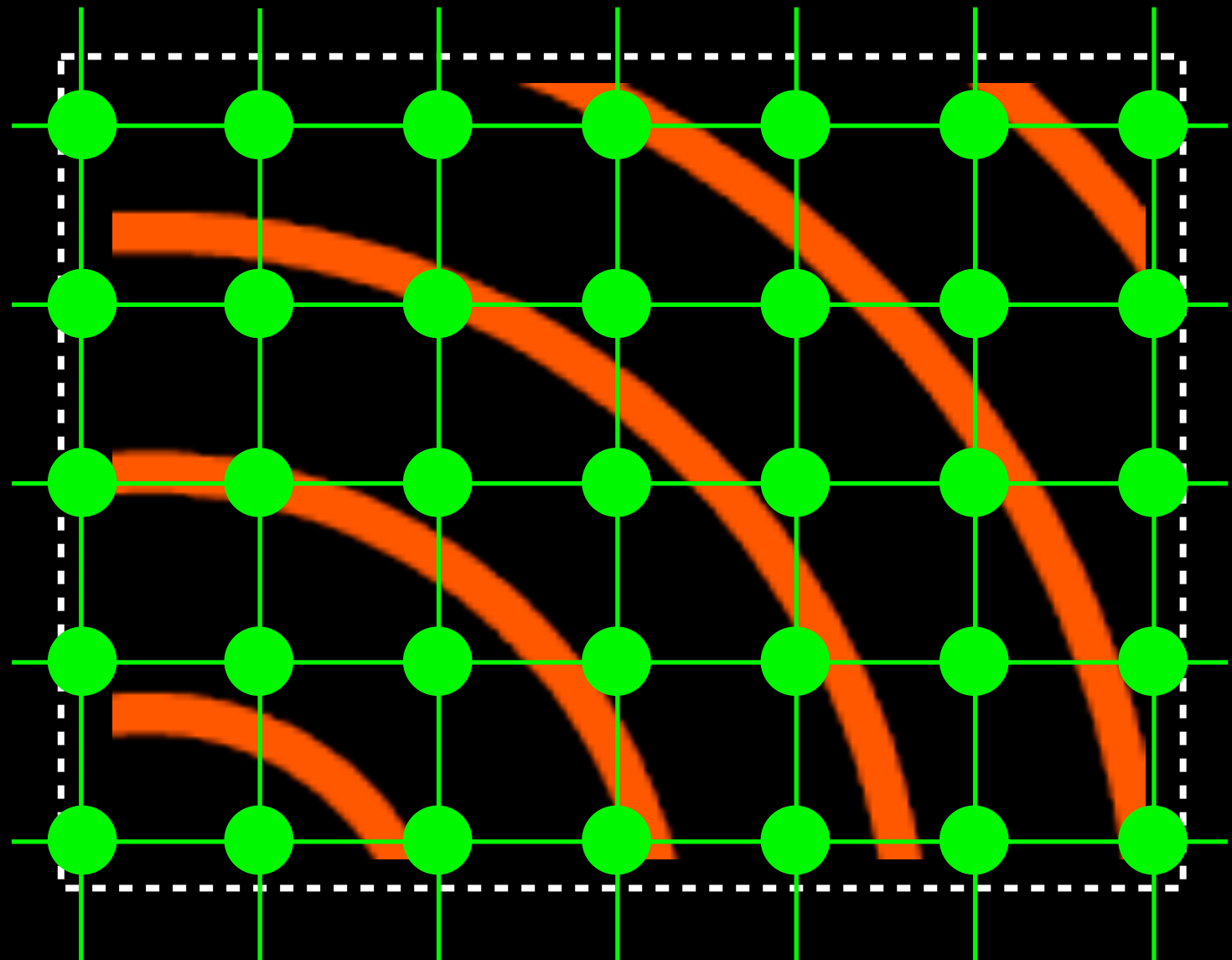
Gridding Algorithm



Spirals: Image Reconstruction



Gridding Algorithm



Follow with 2D Fourier Transform ...

Spirals: Gradient Delays



2 sample delay

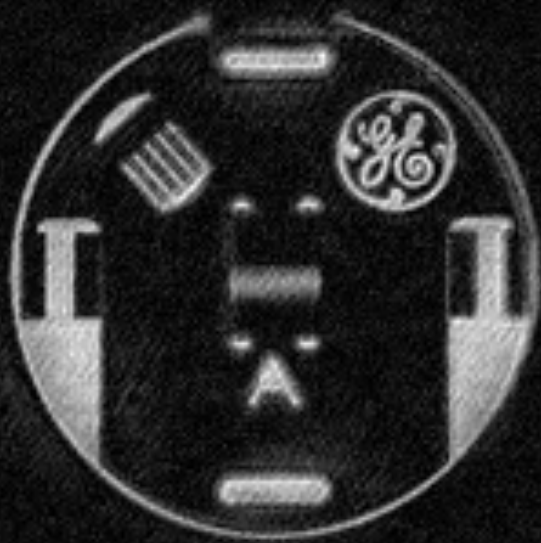


1 sample delay



calibrated

Spirals: Off-Resonance Effects



$N_{\text{intlv}} = 8$

$T_{\text{rd}} = 26.67 \text{ ms}$



$N_{\text{intlv}} = 16$

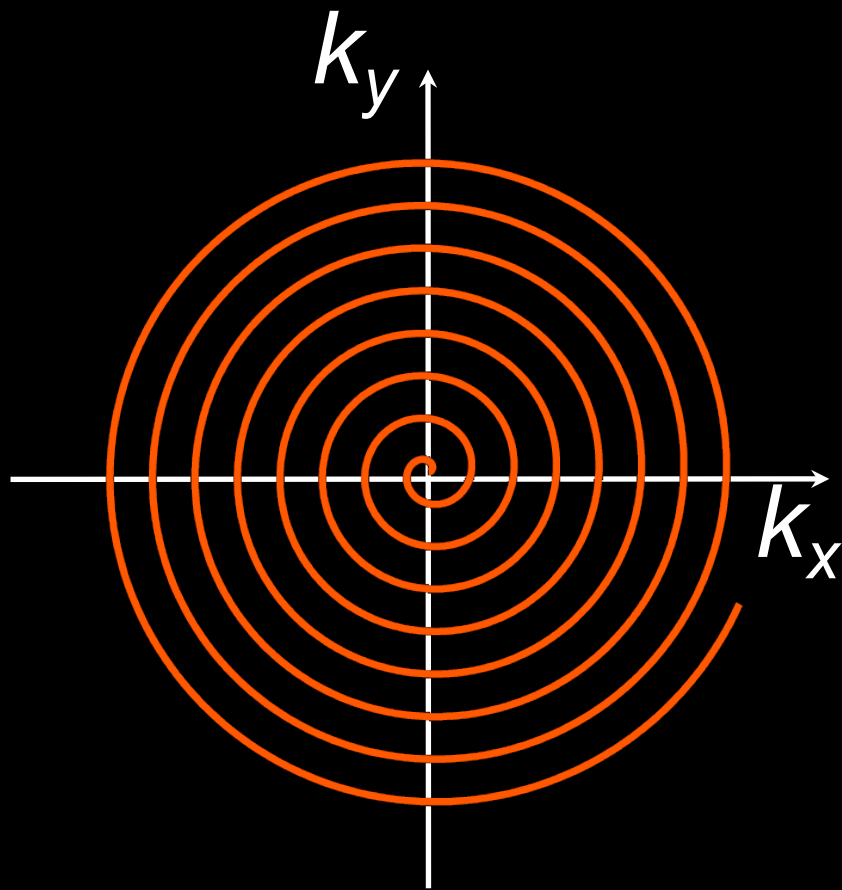
$T_{\text{rd}} = 13.41 \text{ ms}$



$N_{\text{intlv}} = 48$

$T_{\text{rd}} = 4.61 \text{ ms}$

Spirals: Practical Considerations



Trajectory design

Gradient waveform calibration

k-Space density compensation

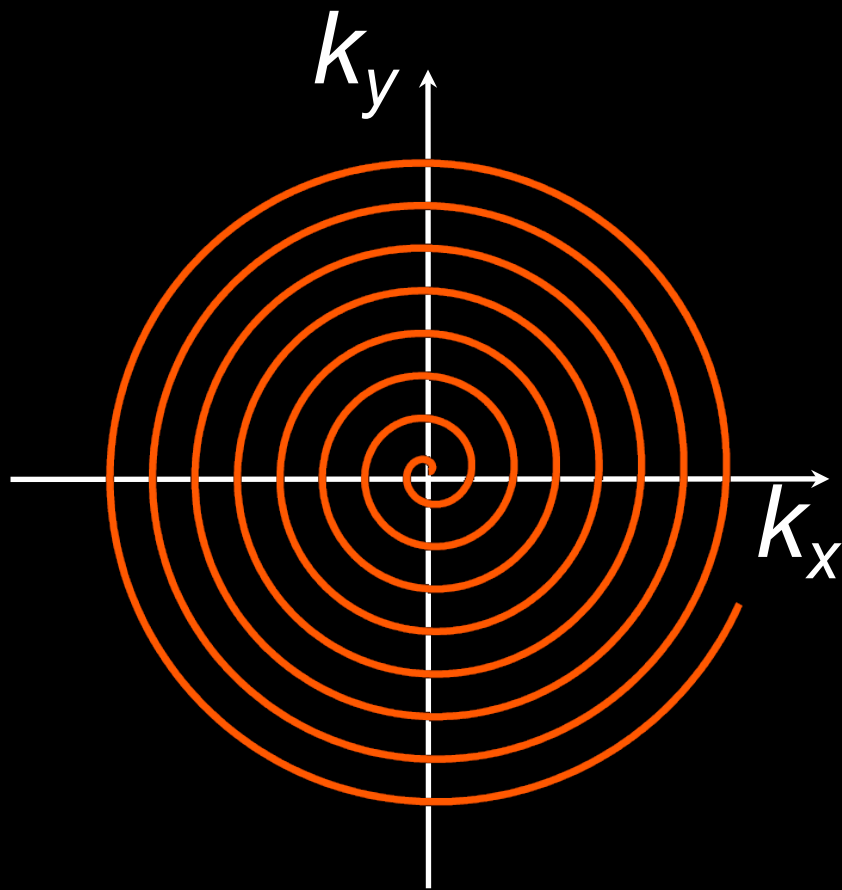
Off-resonance correction

Fat suppression

Gridding reconstruction

applies to non-Cartesian MRI in general

Spirals: Pros and Cons



Pros

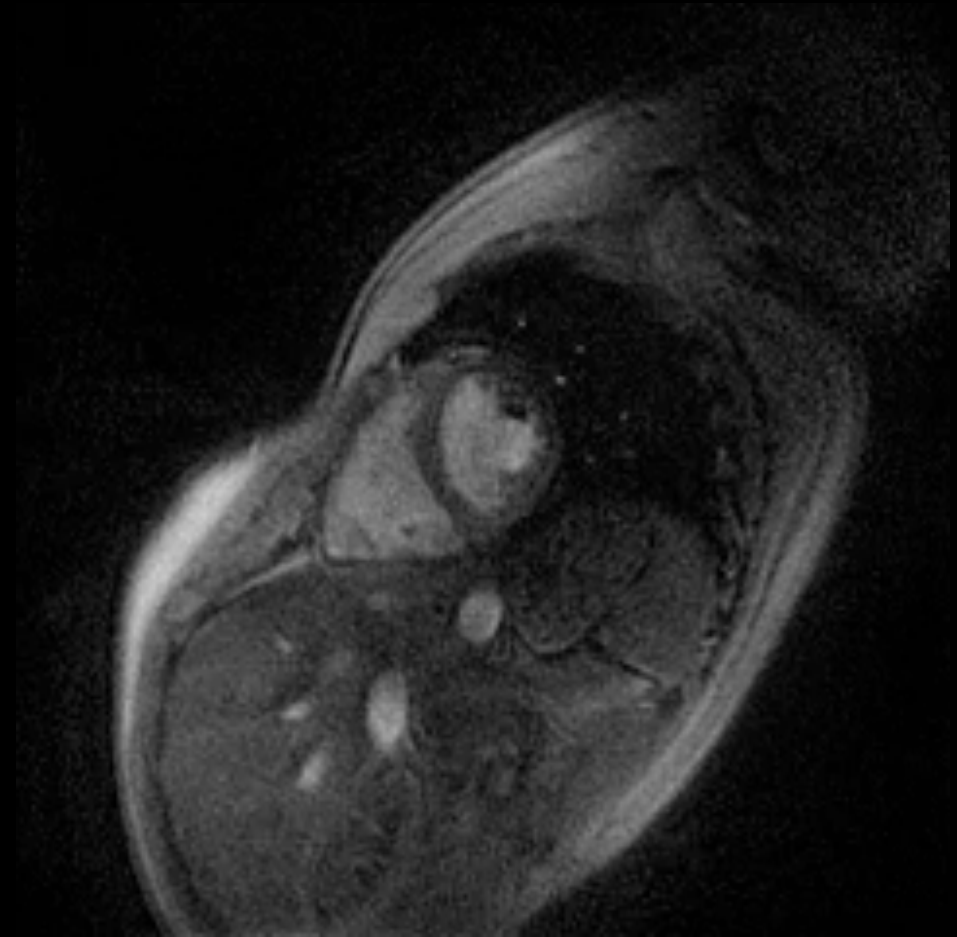
- Very fast (up to single shot)
- Very short TE
- Robust to motion/flow effects

Cons

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

Spirals: Real-Time Cardiac MRI

- Healthy subject; 1.5 T; 8-ch array
- Golden-angle ordering
- Spiral 2D GRE; 8-mm slice
- Spatial resolution = 1.6 mm
- SPIRiT recon with $R = 2$
- 40 cm, 1.6 mm
- 250x250 matrix @ 6 fps
- 12-fold reduction in #TRs (vs. 2DFT)
- 8-TR sliding window display (16 fps)



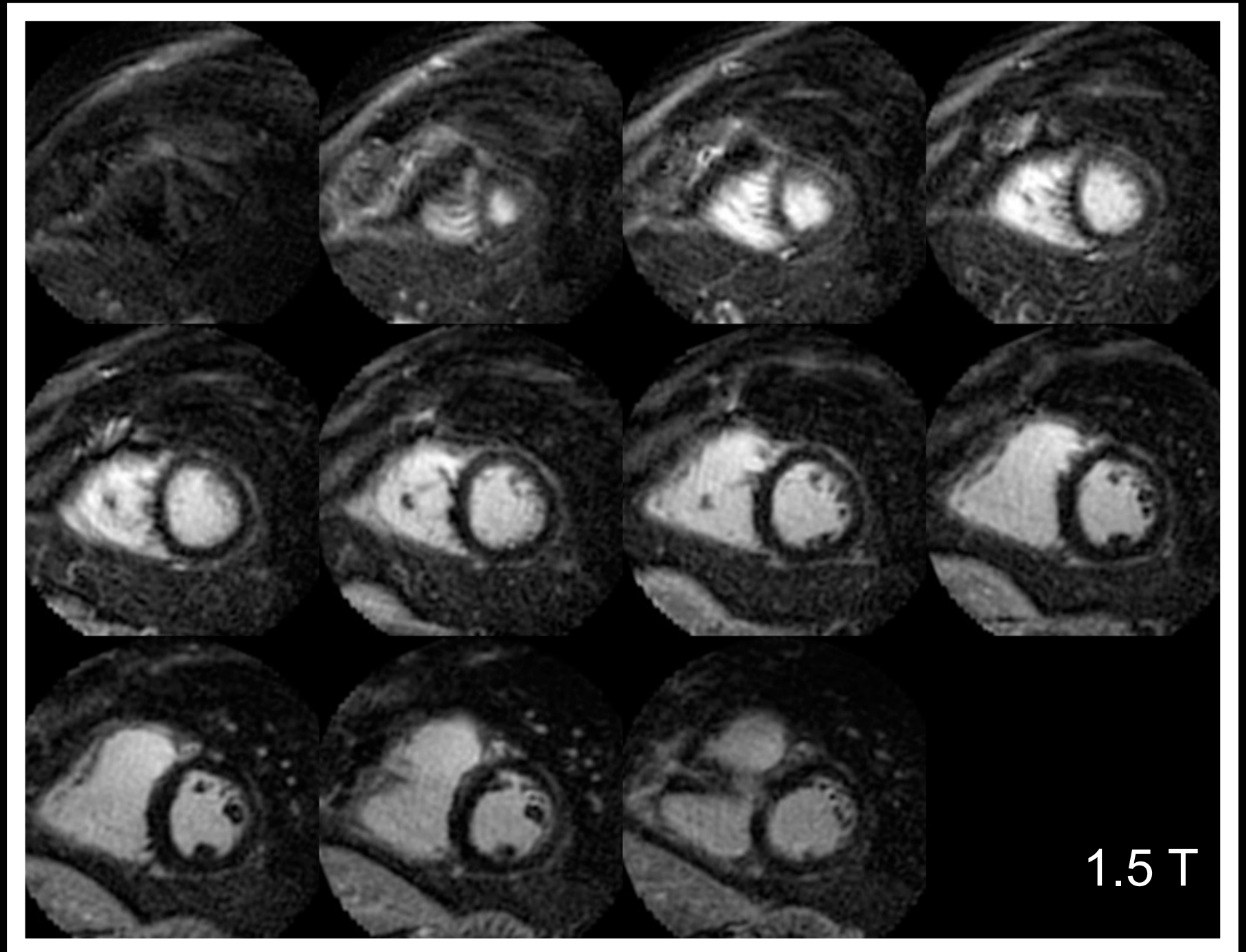
Spirals: 3D LGE MRI

3D Spiral IR-GRE

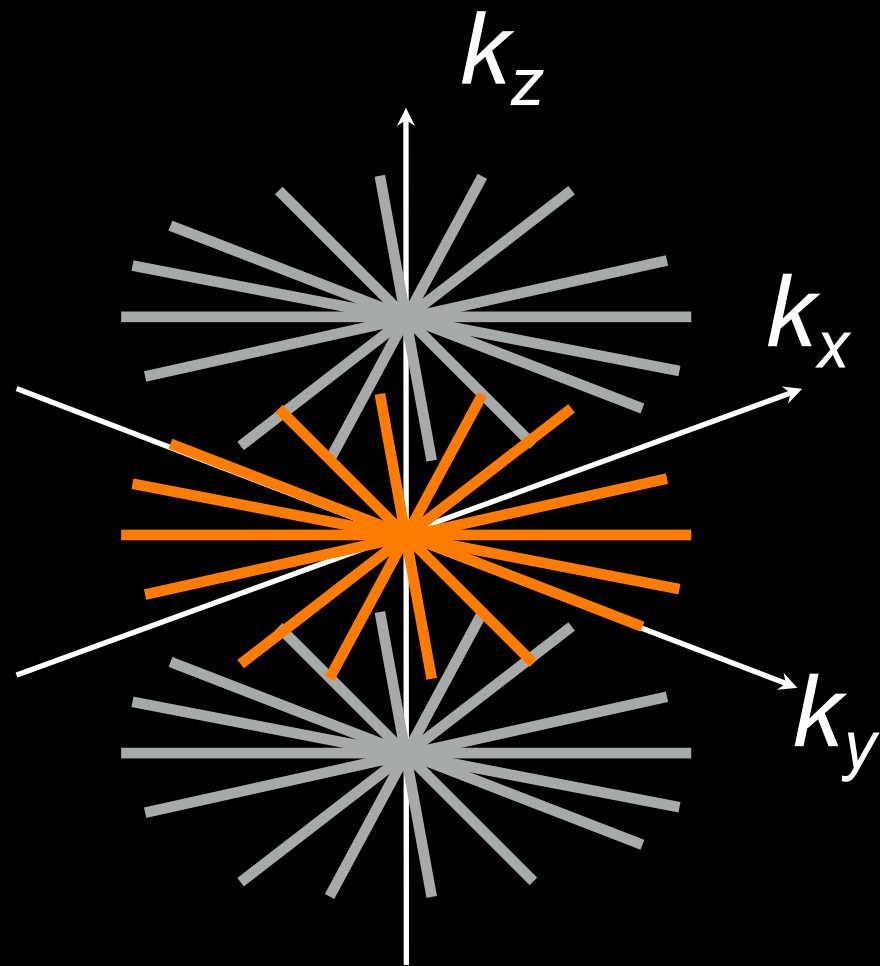
- 6-interleaf VD spiral
- 7.5-ms readout
- 90 x 90 x 11 matrix
- outer volume suppr
- water-only RF exc
- TR = 15.48 ms
- 8-HB BH scan

Reconstruction

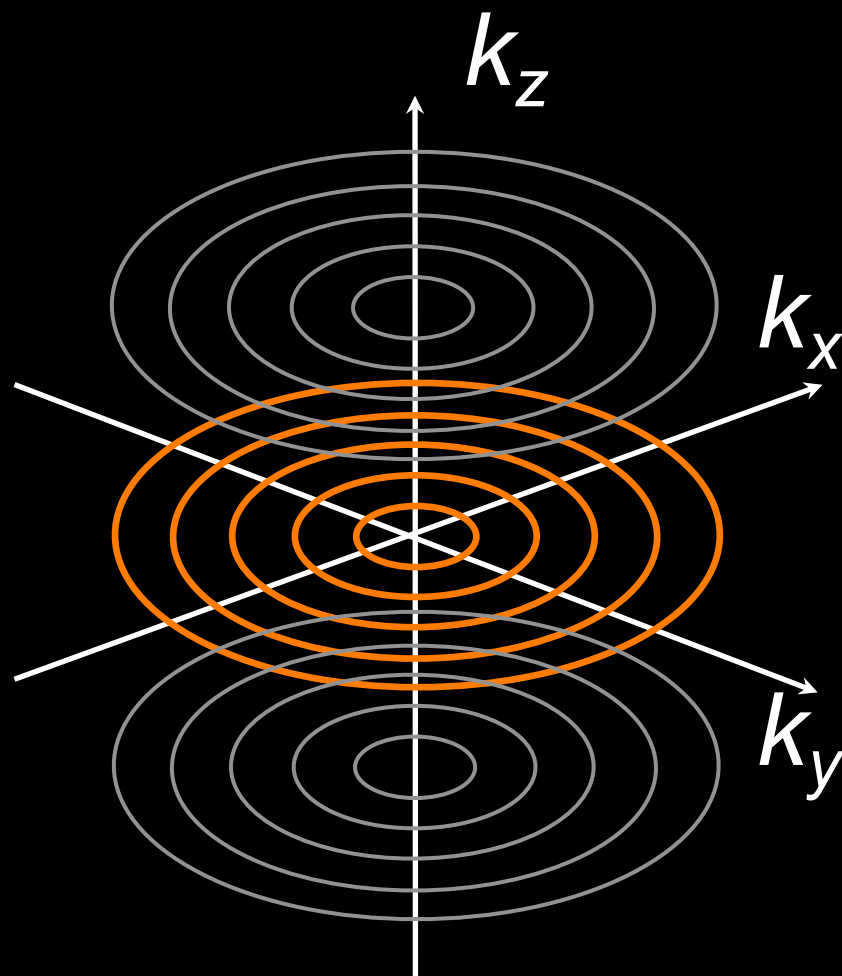
- SPIRiT ($R = 2$)
- ~5-sec recon



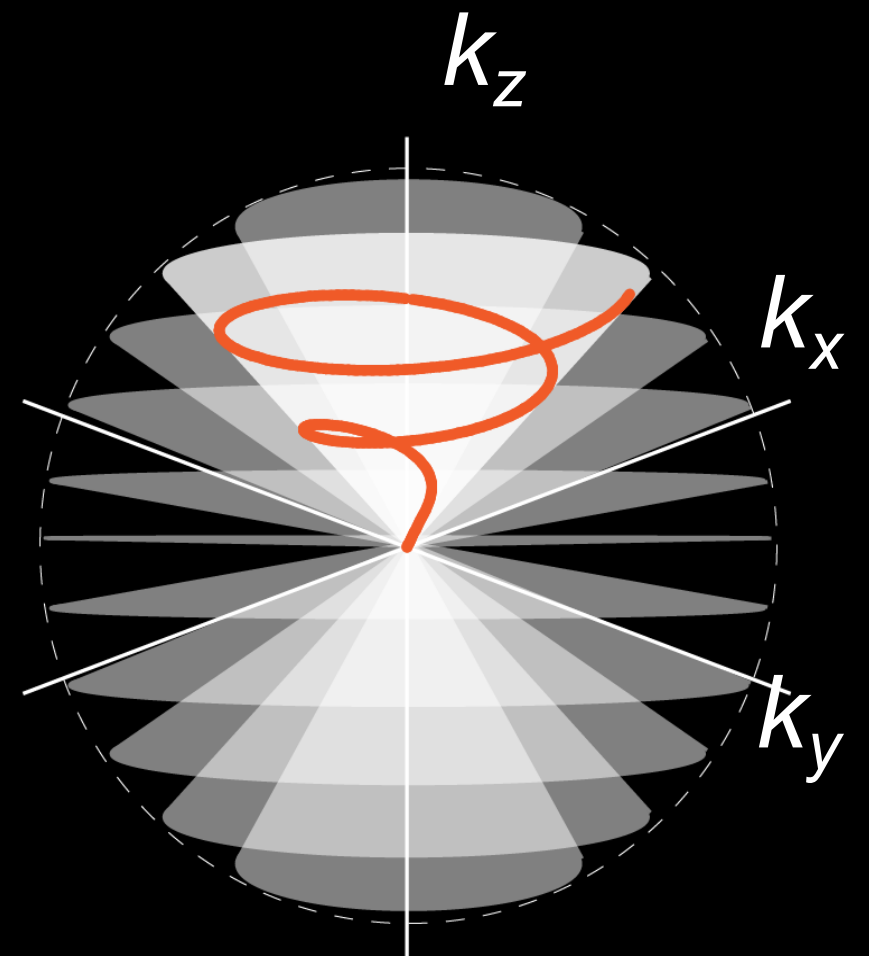
3D Non-Cartesian Sampling



3D Stack of Stars



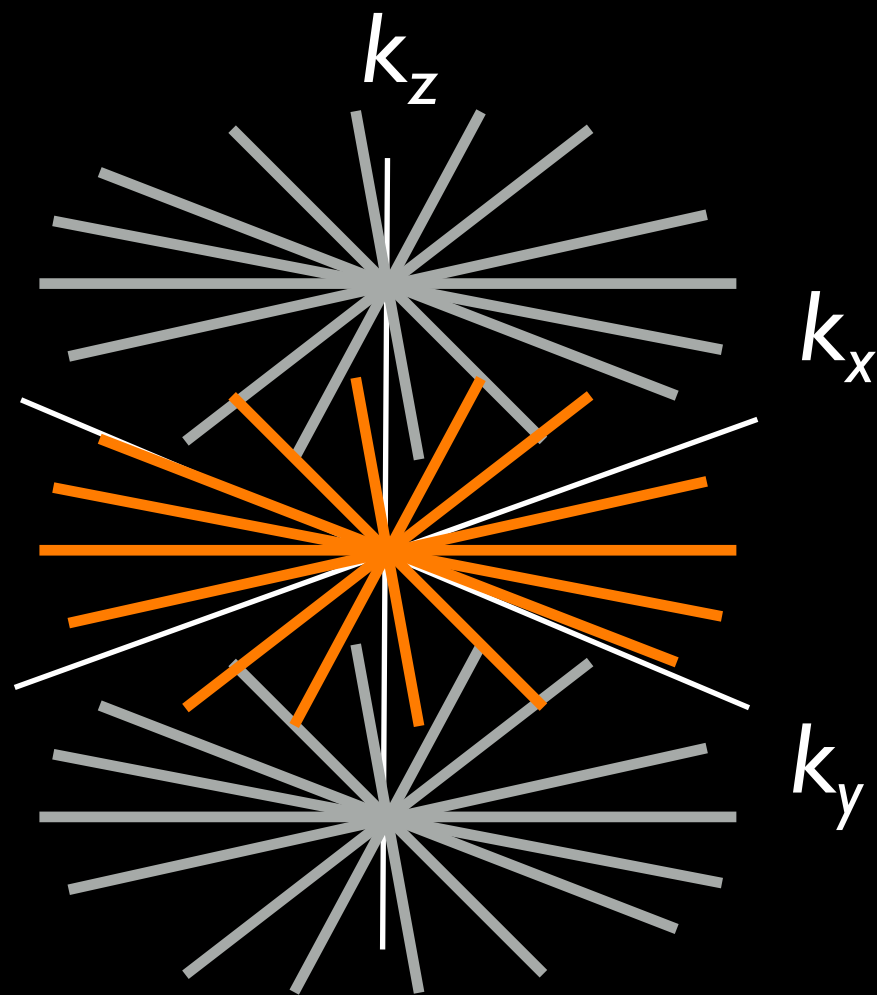
3D Stack of Rings



3D Cones

and much more ...

3D Stack-of-Radial



aka Stack-of-Stars

Pros

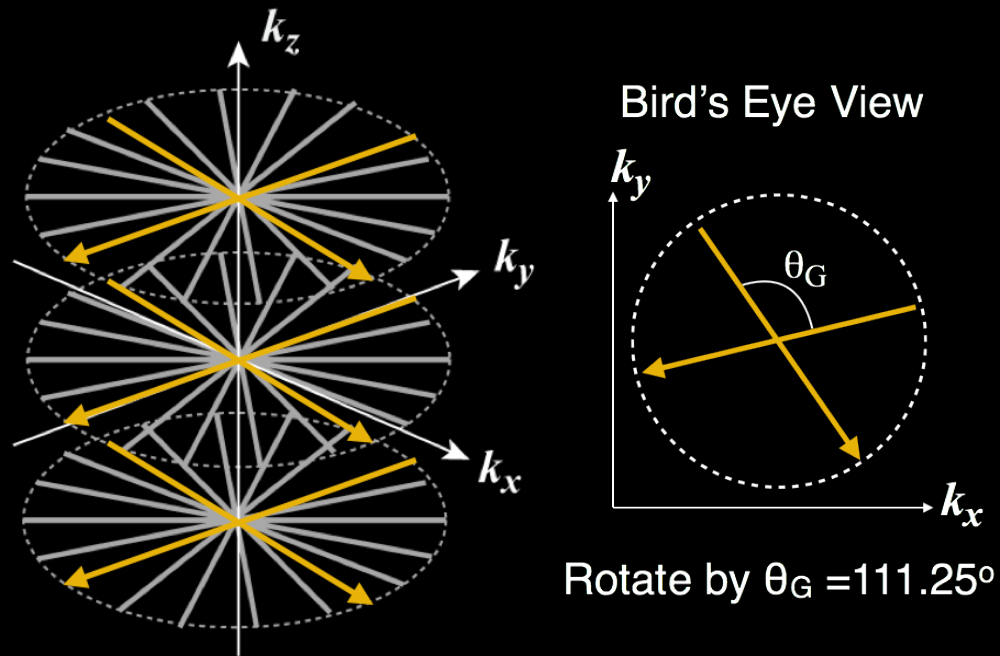
- Straightforward extension of radial
- Robust to motion
- Supports self-navigation
- Can tolerate a lot of undersampling

Cons

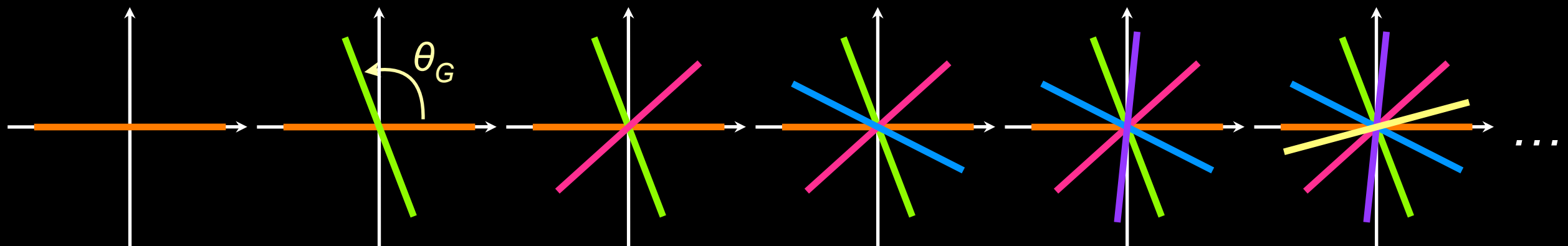
- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

3D Stack-of-Radial

Can Combine with Golden-Angle Ordering

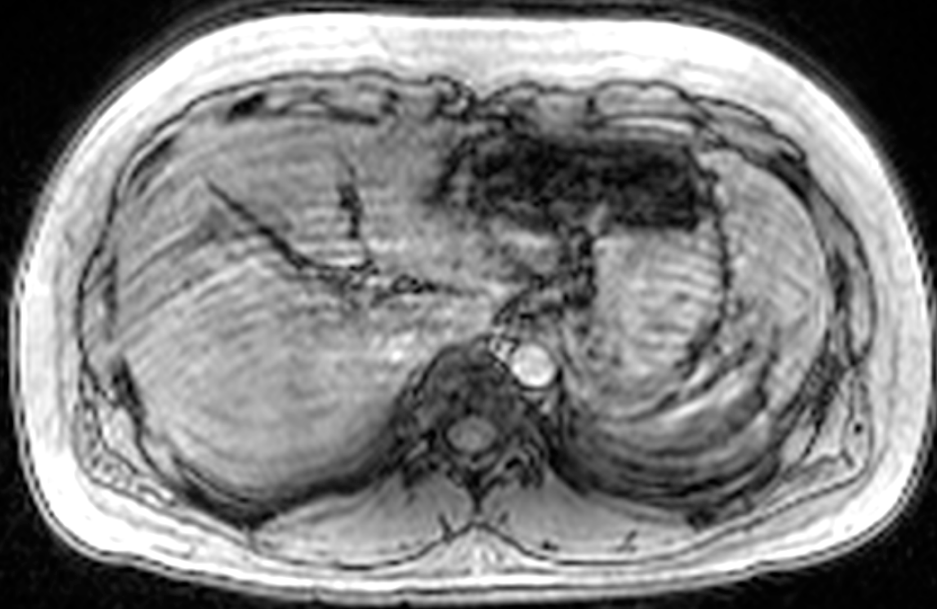


Golden Angle Ordering



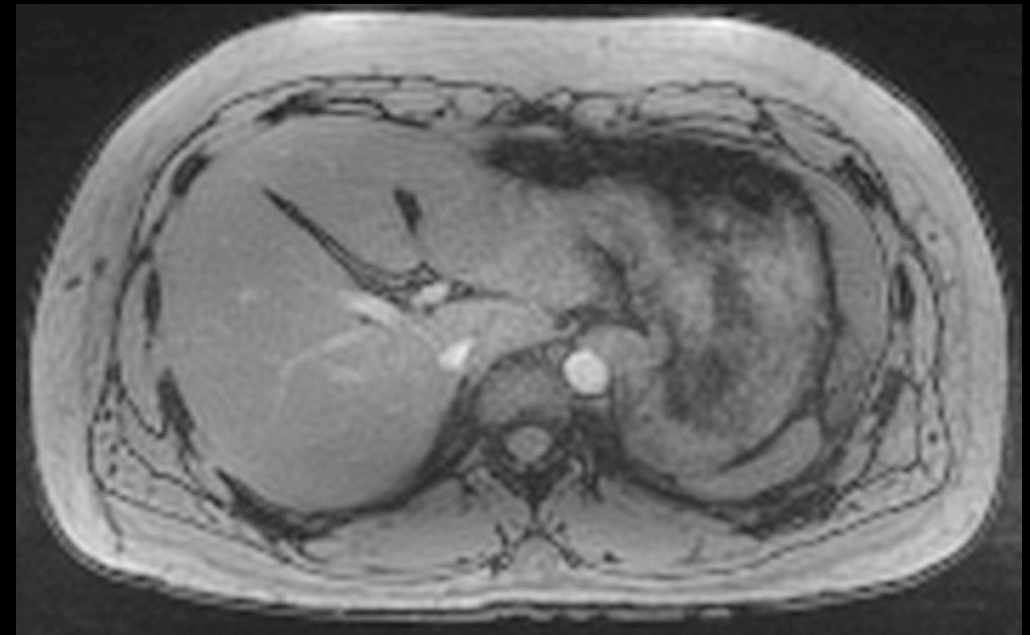
3D Stack-of-Radial: Liver MRI

3D Cartesian MRI

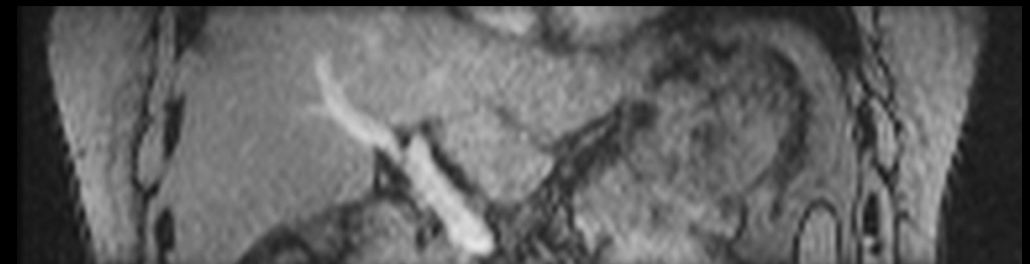


Insufficient breath-holding

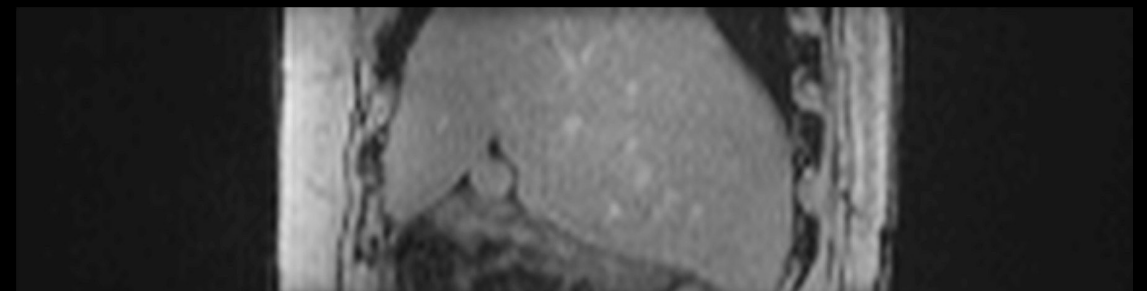
Free-breathing 3D Stack-of-Radial MRI



Axial



Coronal

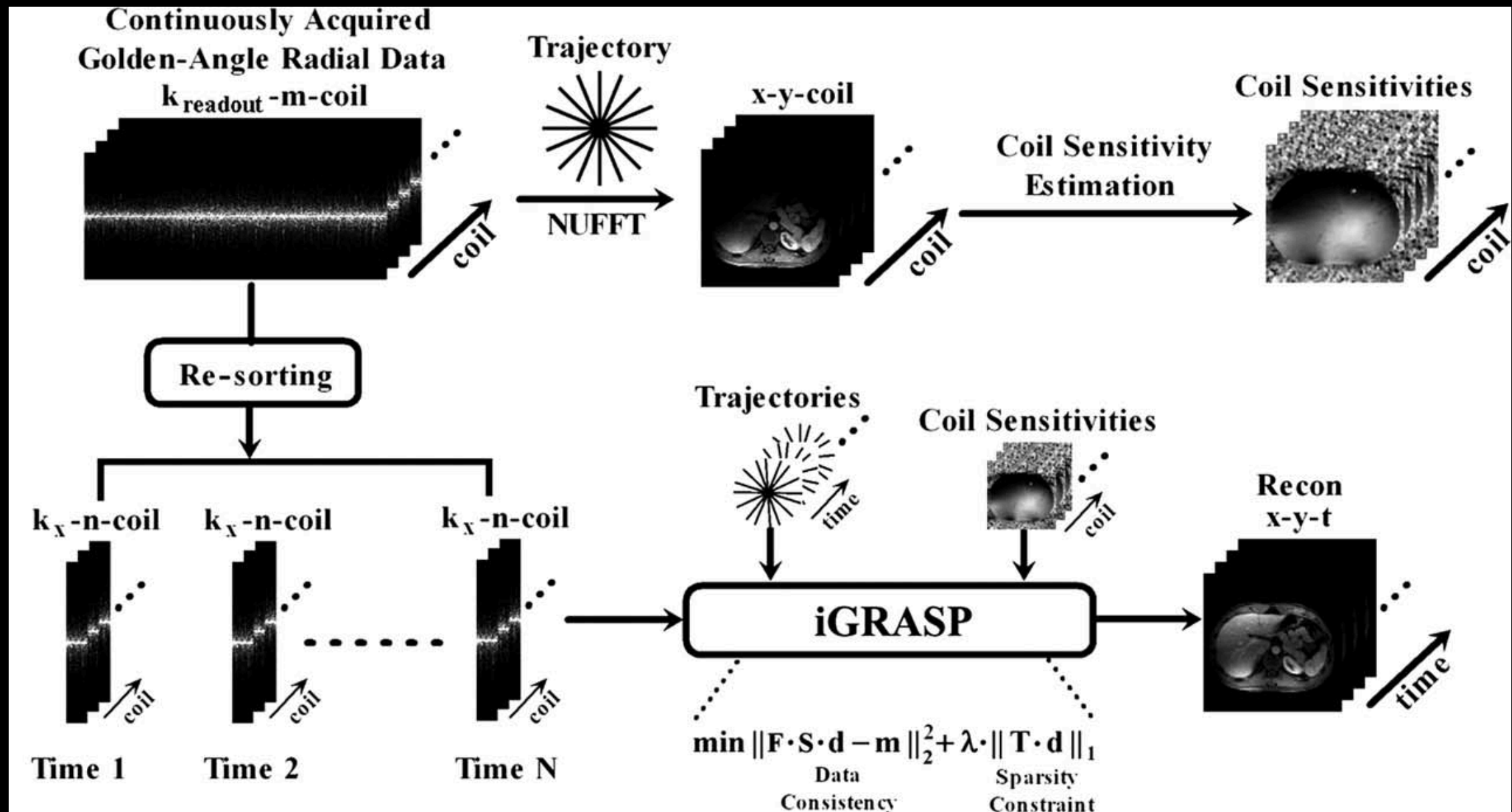


Sagittal

courtesy of Tess Armstrong

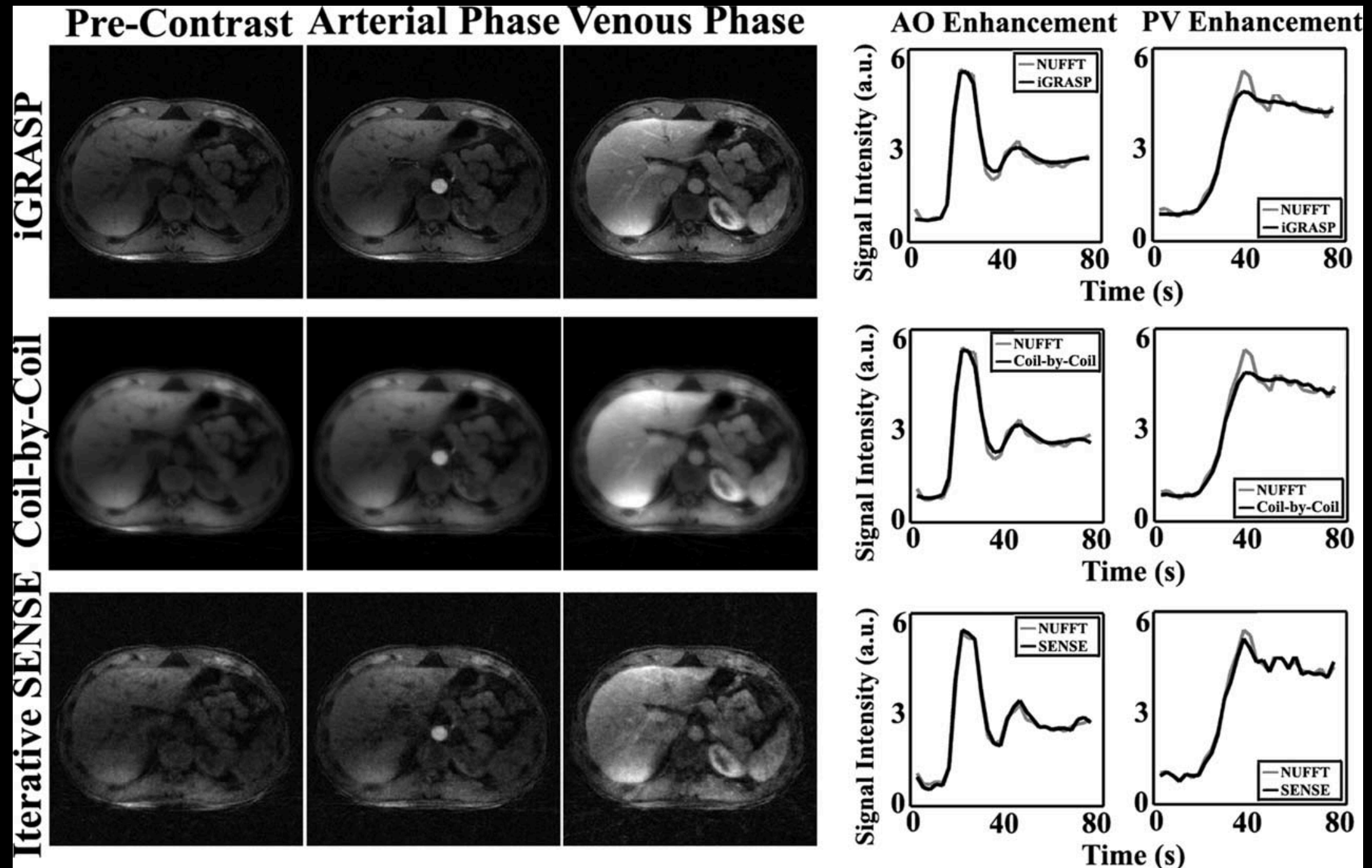
3D Stack-of-Radial: Liver MRI

GRASP: Golden-Angle Radial Sparse Parallel MRI

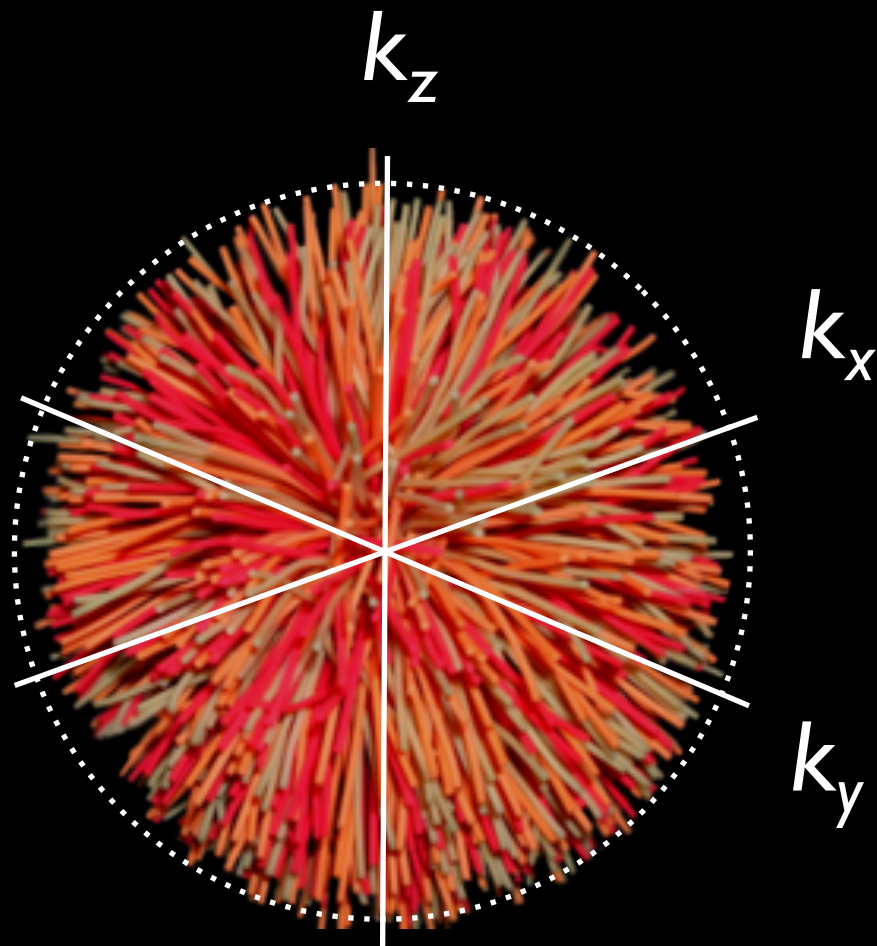


3D Stack-of-Radial: Liver MRI

GRASP: Golden-Angle Radial Sparse Parallel MRI



3D Radial



Pros

- Robust to motion (get DC every TR)
- Supports self-navigation
- Can tolerate a lot of undersampling
- Half-spoke PR has very short TE

Cons

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

3D Radial: Coronary MRA

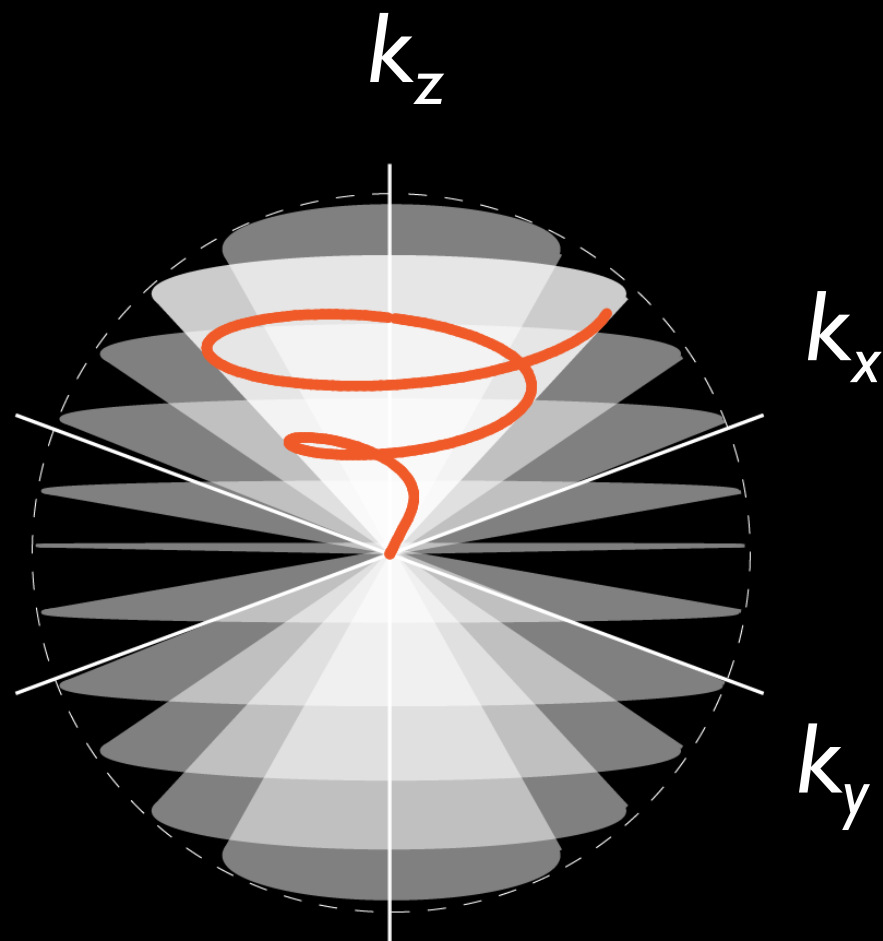
Contrast-Enhanced MRA at 3.0T



ECG-gated, fat-saturated, inversion-recovery prepared spoiled gradient echo sequence
(1.0 mm)³ spatial resolution, 1D self navigation, CG-SENSE recon, 5.4 min scan time

courtesy of Debiao Li and J Pang (Cedars-Sinai)

3D Cones



Pros

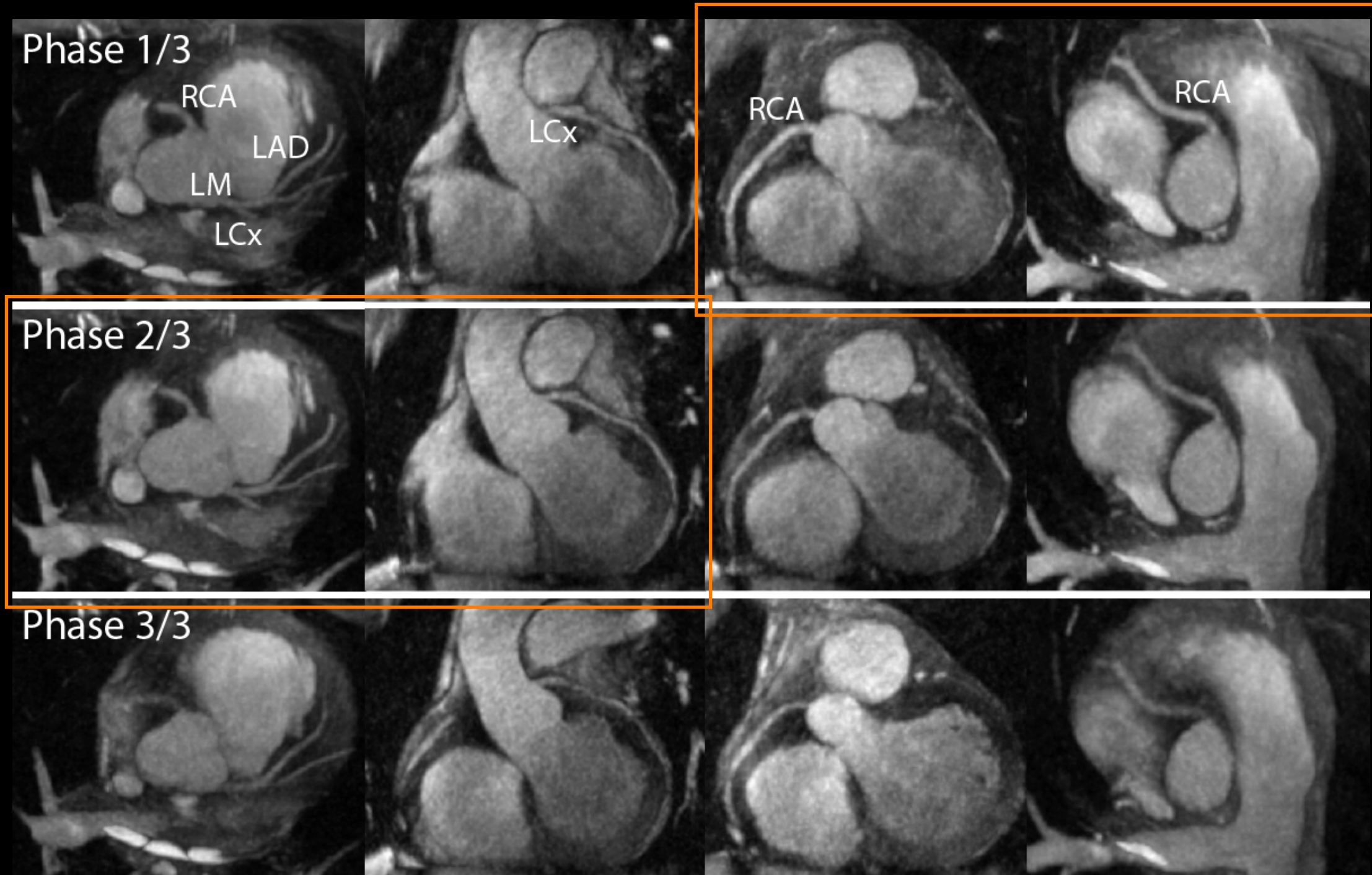
- Very fast (3-8x vs. Cartesian)
- Very short TE
- Flexible readout length
- Robust to motion/flow effects

Cons

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

3D Cones: Coronary MRA

Multi-Phase Thin-Slab MIP Reformats

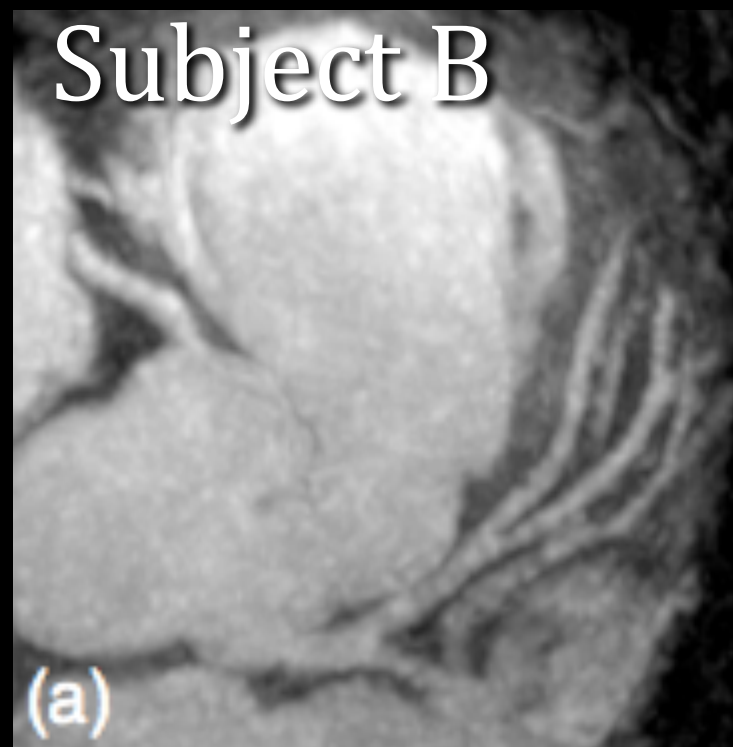


3D Cones: Hi-res CMRA

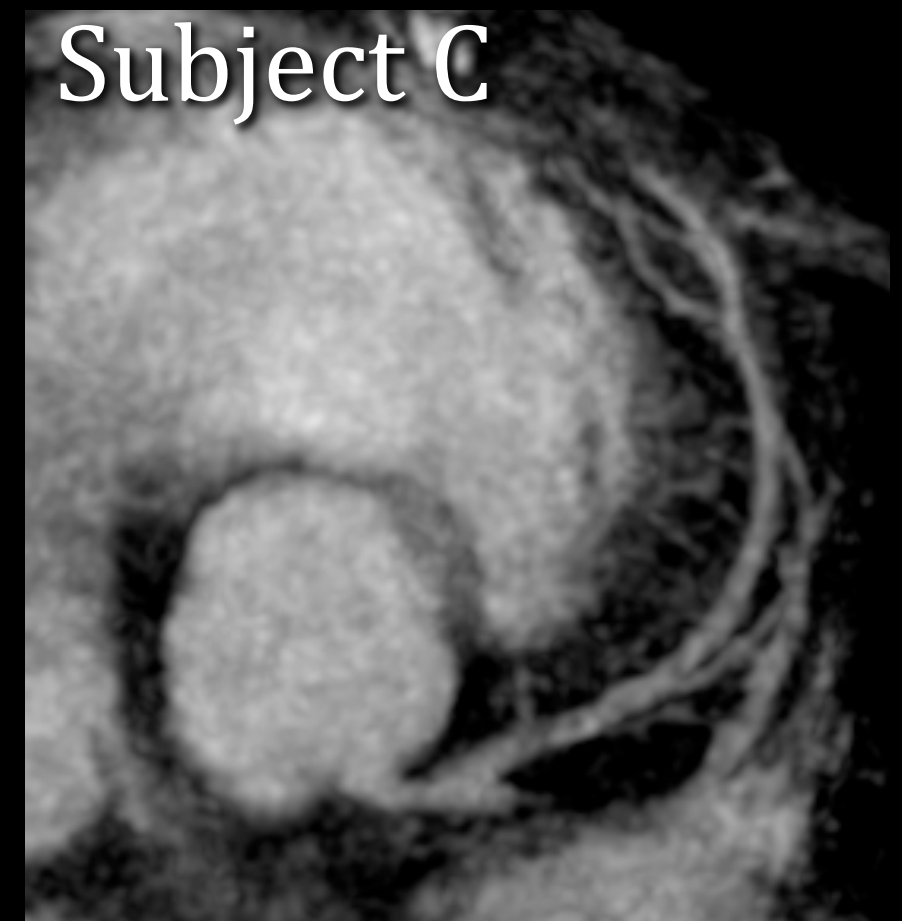
Thin-Slab MIP Reformats: 0.8 mm isotropic



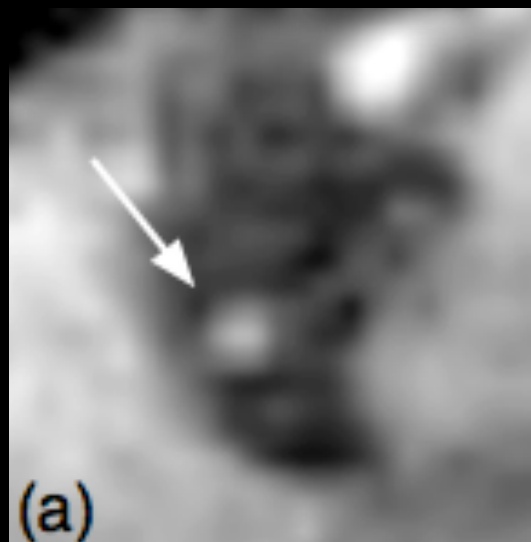
1.2 mm



0.8 mm



Right coronary
artery cross
section



1.5 T; 8-channel cardiac coil

Non-Cartesian Image Reconstruction

- Gridding reconstruction
- Gradient measurement
- Off-resonance correction

MRI Signal Equation

$$\begin{aligned} s(t) &= \iint_{X,Y} m(x,y) \cdot \exp(-i2\pi \cdot [k_x(t)x + k_y(t)y]) \, dx \, dy \\ &= \mathcal{FT}(m(x,y)) = M(k_x(t), k_y(t)) \end{aligned}$$

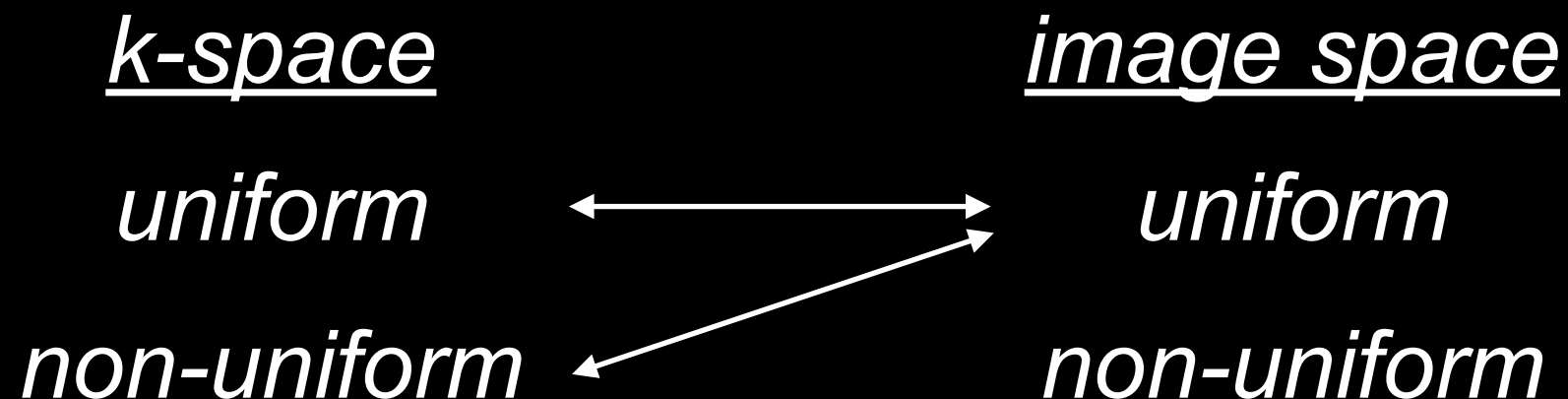
General definition of k -space:

$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) \, d\tau, \quad k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) \, d\tau$$

MRI Reconstruction

$$m(x, y) = \mathcal{FT}^{-1}(M(k_x, k_y))$$

$$m(x, y) = \iint_{k_x, k_y} M(k_x, k_y) \cdot \exp(i2\pi \cdot [k_x x + k_y y]) \, dk_x \, dk_y$$



simple for Cartesian (k_x, k_y) to Cartesian (x, y) : 2D FFT

time consuming for non-Cartesian (k_x, k_y) to Cartesian (x, y)

Non-Cartesian Reconstruction

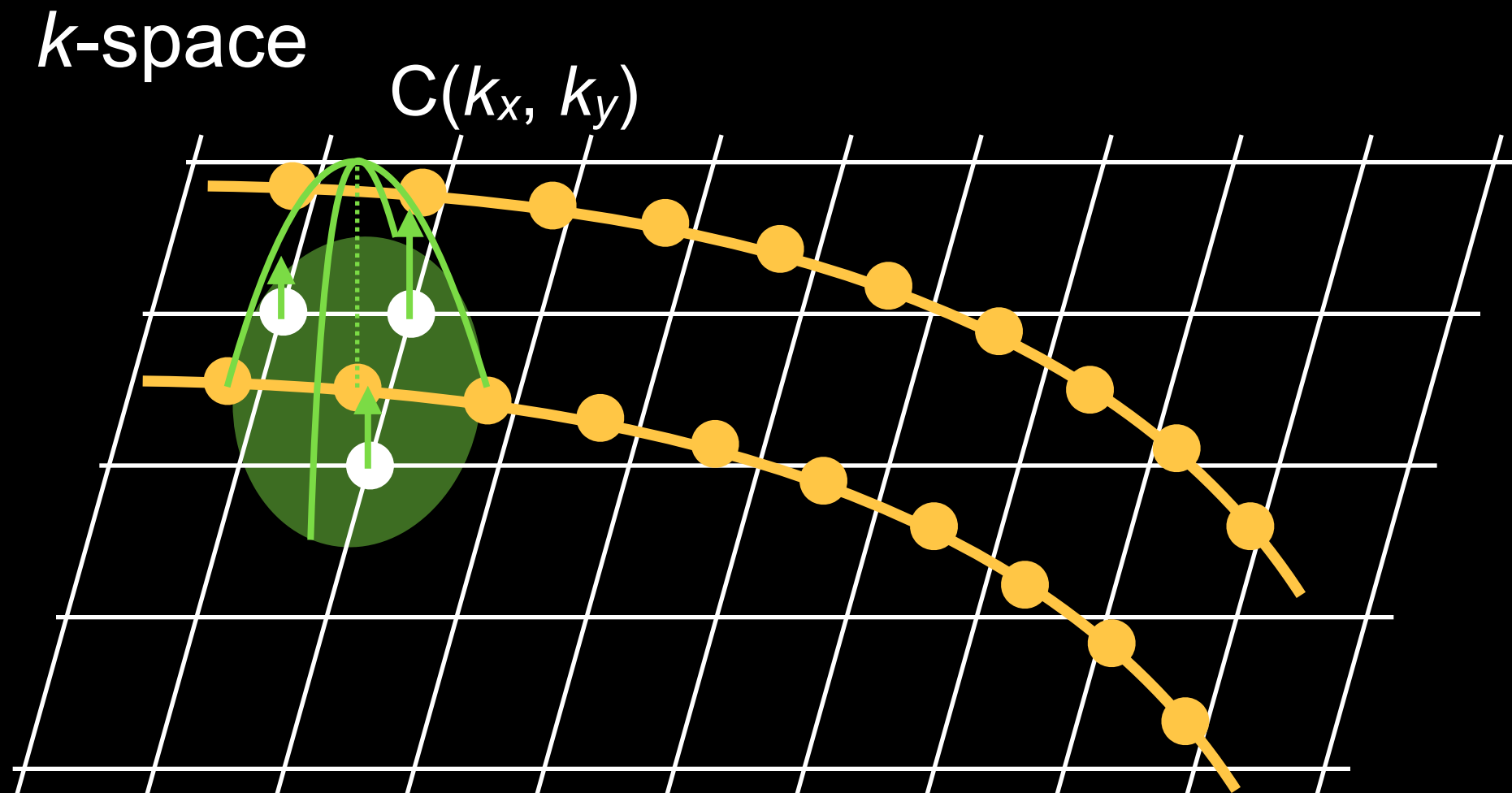
- Inverse Fourier transform
 - aka conjugate phase reconstruction
- Gridding (+FFT)¹
 - grid driven interpolation
 - data driven interpolation (more popular)
 - forward and reverse (inverse)
- Non-uniform FFT (NUFFT)²
- Block Uniform ReSampling (BURS)³

¹ O'Sullivan JD, *IEEE TMI* 1985; 4: 200-207

² Fessler JA et al., *IEEE TSP* 2003; 51: 560-574

³ Rosenfeld D, *MRM* 2002; 48: 193-202

Gridding: Basic Idea



convolve each acquired data point with kernel $C(k_x, k_y)$
resample the convolution onto Cartesian grid points
2D inverse FFT; de-apodization and FOV cropping

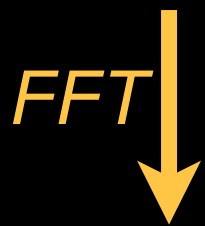
Gridding: Basic Math

Sampling pattern: $S(k_x, k_y) = \sum_j^2 \delta(k_x - k_{x,j}, k_y - k_{y,j})$

Convolution kernel: $C(k_x, k_y)$ Grid: $\text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)$

Gridding recon:

$$\hat{M}(k_x, k_y) = \underbrace{[M(k_x, k_y) \cdot S(k_x, k_y)]}_{\text{non-Cartesian dataset}} \underbrace{* C(k_x, k_y)}_{\text{interpolation}} \cdot \underbrace{\text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)}_{\text{resample to grid}}$$



$$\hat{m}(x, y) = \underbrace{[m(x, y) * s(x, y)]}_{\text{remove by deap}} \cdot \underbrace{c(x, y)}_{\text{remove by cropping}} * \text{III}\left(\frac{x}{\text{FOV}_x}, \frac{y}{\text{FOV}_y}\right)$$

→ $m(x, y)$

Gridding: Design Issues

- Convolution kernel
 - apodization; aliasing
- Sampling grid density (Cartesian)
 - aliasing
- Sampling pattern (non-Cartesian)
 - impulse response and side lobes
 - density characterization / compensation

Gridding: Design - Kernel

- Ideal convolution kernel: SINC
 - don't need de-apodization
 - infinite extent impractical to implement
 - windowed version has limited performance
- Desired kernel characteristics
 - compact support (finite width) in k-space
 - minimal aliasing effects in image (sharp transition)

Gridding: Design - Kernel

Combine with grid oversampling

$$\Delta k_x = \frac{1}{\text{FOV}_x}, \Delta k_y = \frac{1}{\text{FOV}_y}$$

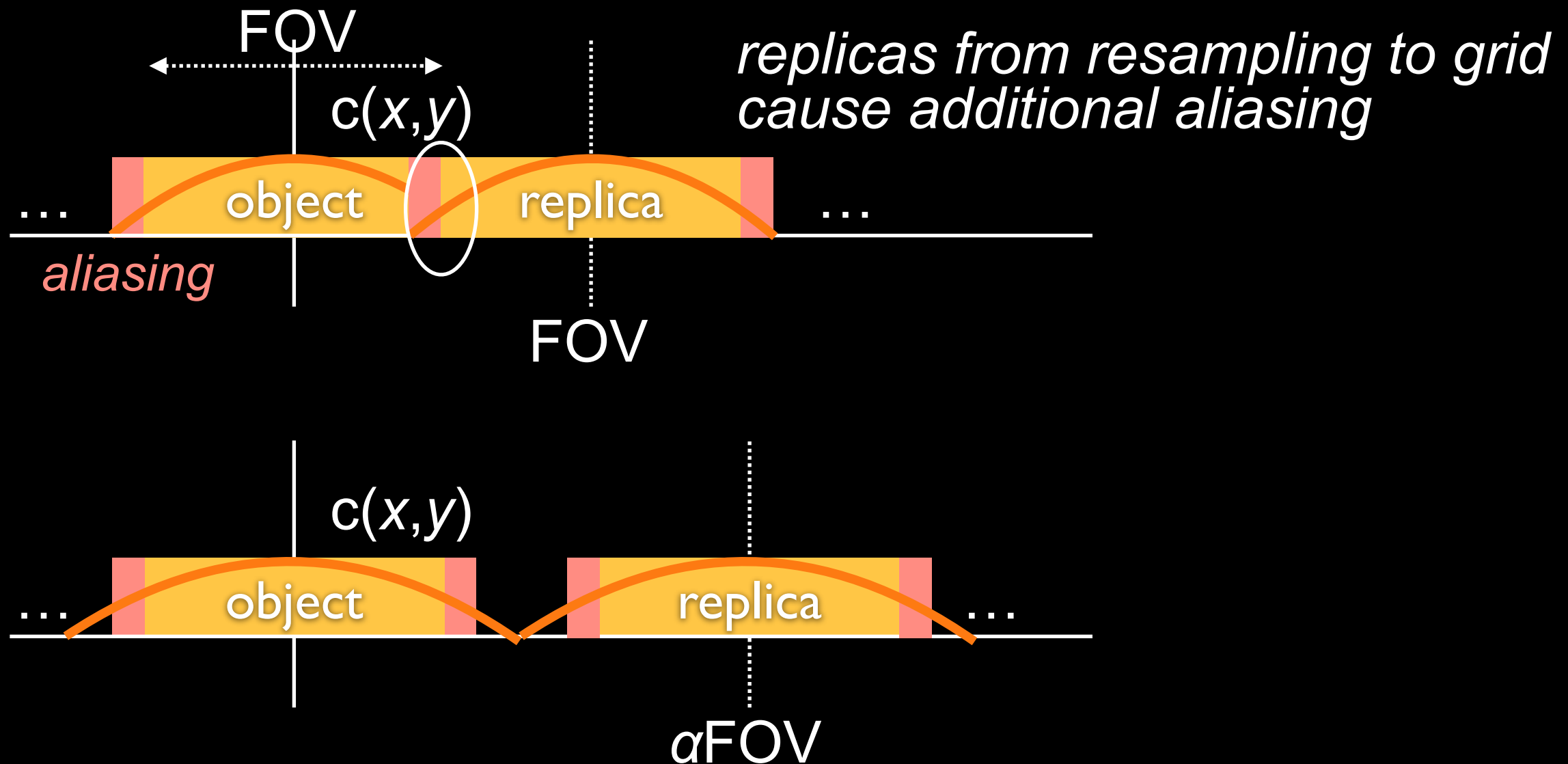
$$\frac{\Delta k_x}{\alpha} = \frac{1}{\alpha \text{FOV}_x}, \frac{\Delta k_y}{\alpha} = \frac{1}{\alpha \text{FOV}_y} \quad \alpha > 1$$

$$\hat{M}(k_x, k_y) = [(M(k_x, k_y) \cdot S(k_x, k_y)) * C(k_x, k_y)] \cdot \text{III}\left(\frac{k_x}{\Delta k_x / \alpha}, \frac{k_y}{\Delta k_y / \alpha}\right)$$

$$\hat{m}(x, y) = [(m(x, y) * s(x, y)) \cdot c(x, y)] * \text{III}\left(\frac{x}{\alpha \text{FOV}_x}, \frac{y}{\alpha \text{FOV}_y}\right)$$

Gridding: Design - Kernel

Combine with grid oversampling



$\alpha = 2$ very forgiving; many kernels work well; apodization minimal expensive ... especially for 3D gridding

Gridding: Design - Kernel

- Jointly consider α and kernel
 - minimize aliasing energy
 - characterize trade-offs
 - numerical designs possible
 - Kaiser-Bessel window works very well, with proper choice of β and $kw^{1,2}$; precompute a lookup table to speedup calculations²

$$C_{KB}(k_x) = I_0 \left(\beta \sqrt{1 - \left(\frac{k_x}{kw/2} \right)^2} \right)$$

¹Jackson et al., *IEEE TMI* 1991; 10: 473-478

²Beatty et al., *IEEE TMI* 2005; 24: 799-808

Gridding: Design - Density

Sampling density of $S(k_x, k_y)$ not uniform: $\rho(k_x, k_y)$

Pre-compensation of sampling density:

$$\hat{M}(k_x, k_y) = \left[\left(M(k_x, k_y) \cdot \frac{S(k_x, k_y)}{\rho(k_x, k_y)} \right) * C(k_x, k_y) \right] \cdot \text{III}$$

density corrected on a data point basis before convolution
need to know $\rho(k_x, k_y)$

from geometrical analysis, numerical analysis (Voronoi), etc.

inverse of ρ known as the density compensation function (DCF)

Gridding: Design - Density

Post-compensation of sampling density:

$$\hat{M}(k_x, k_y) = \frac{[(M(k_x, k_y) \cdot S(k_x, k_y)) * C(k_x, k_y)] \cdot \text{III}}{\rho(k_x, k_y)}$$

density corrected on a grid point basis after convolution

can estimate ρ along with gridding; grid all 1s:

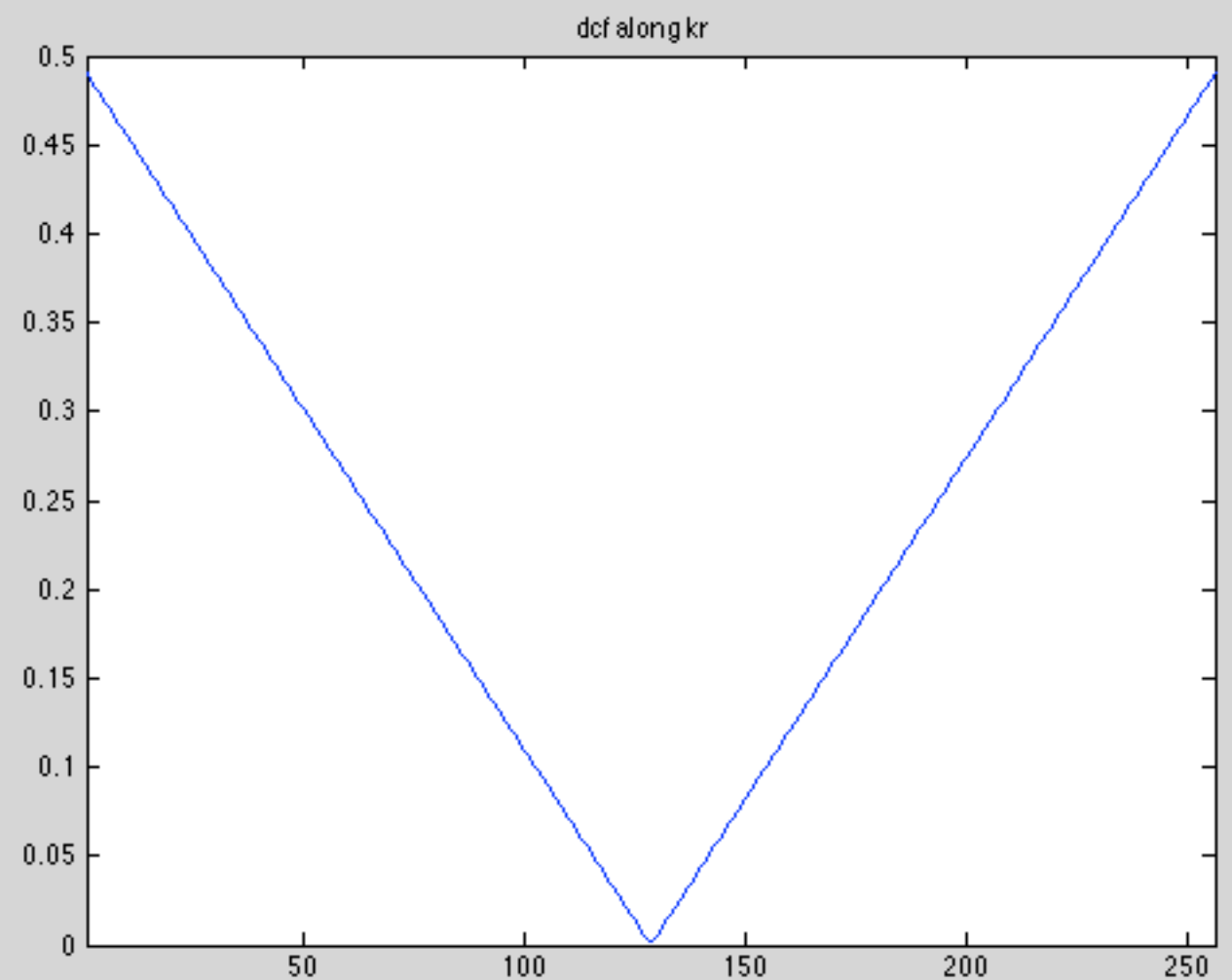
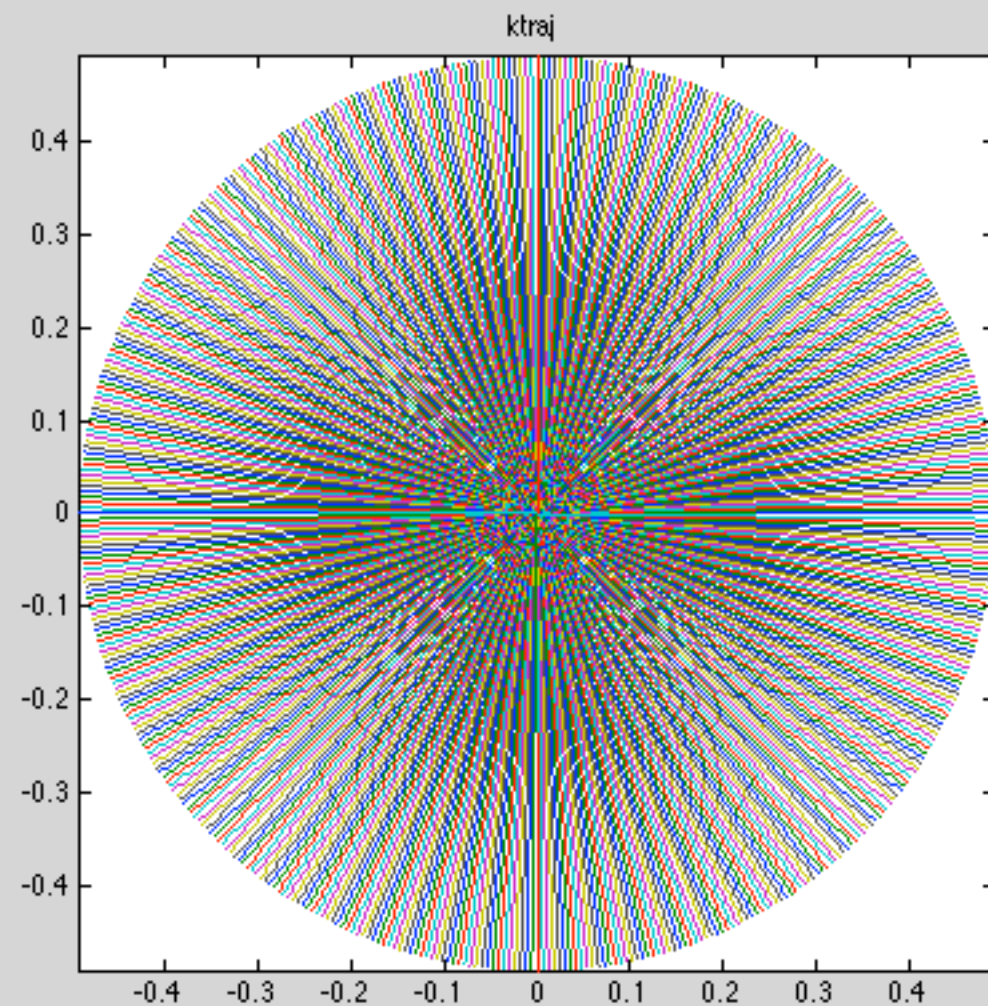
$$\hat{\rho}(k_x, k_y) = [S(k_x, k_y) * C(k_x, k_y)] \cdot \text{III}$$

may be okay if S changes slowly

... but only an approximation and fails when S changes rapidly

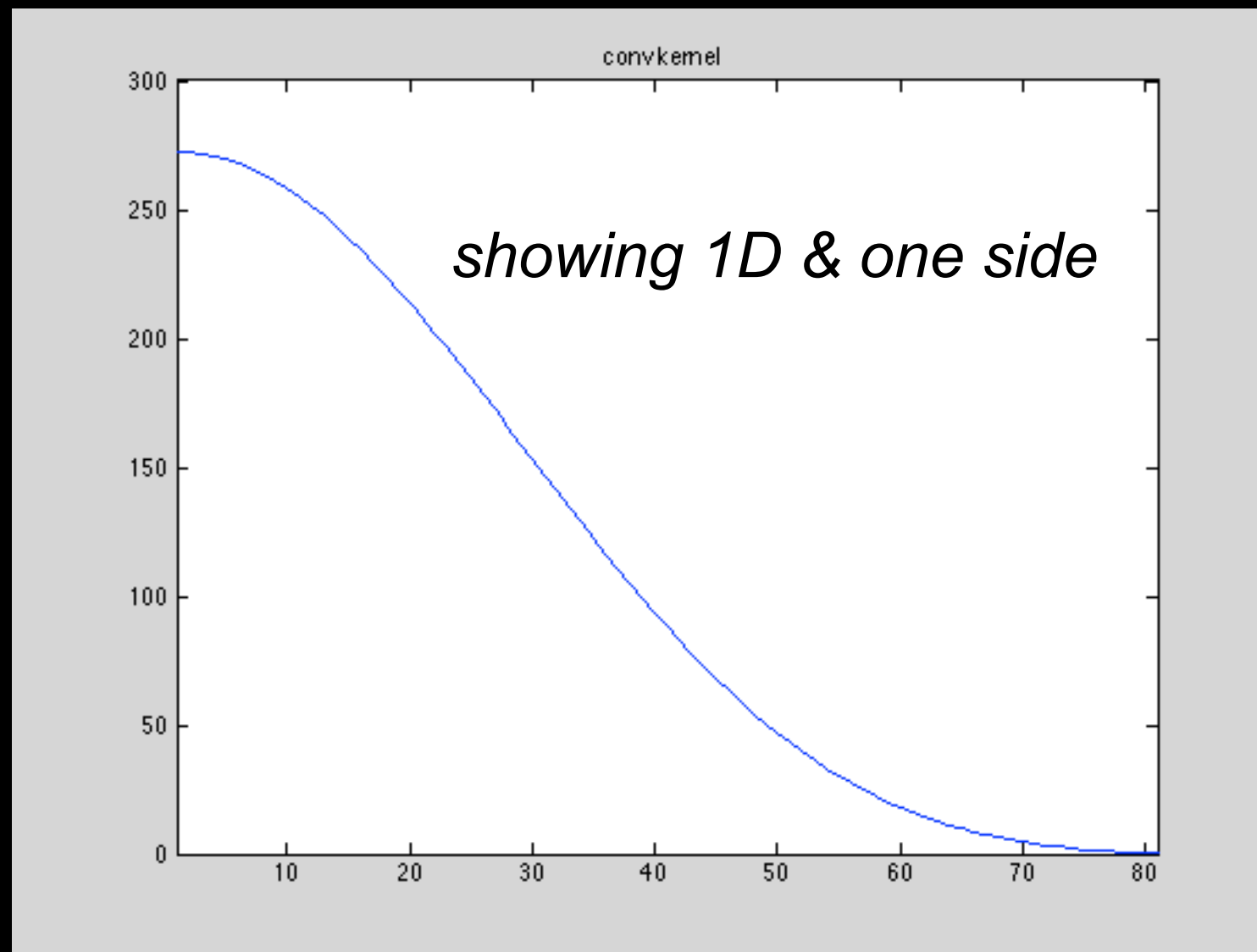
Gridding: 2D Radial Example

Radial trajectory [256x256] with ramp DCF



Gridding: 2D Radial Example

Kaiser-Bessel convolution kernel with linear lookup table¹

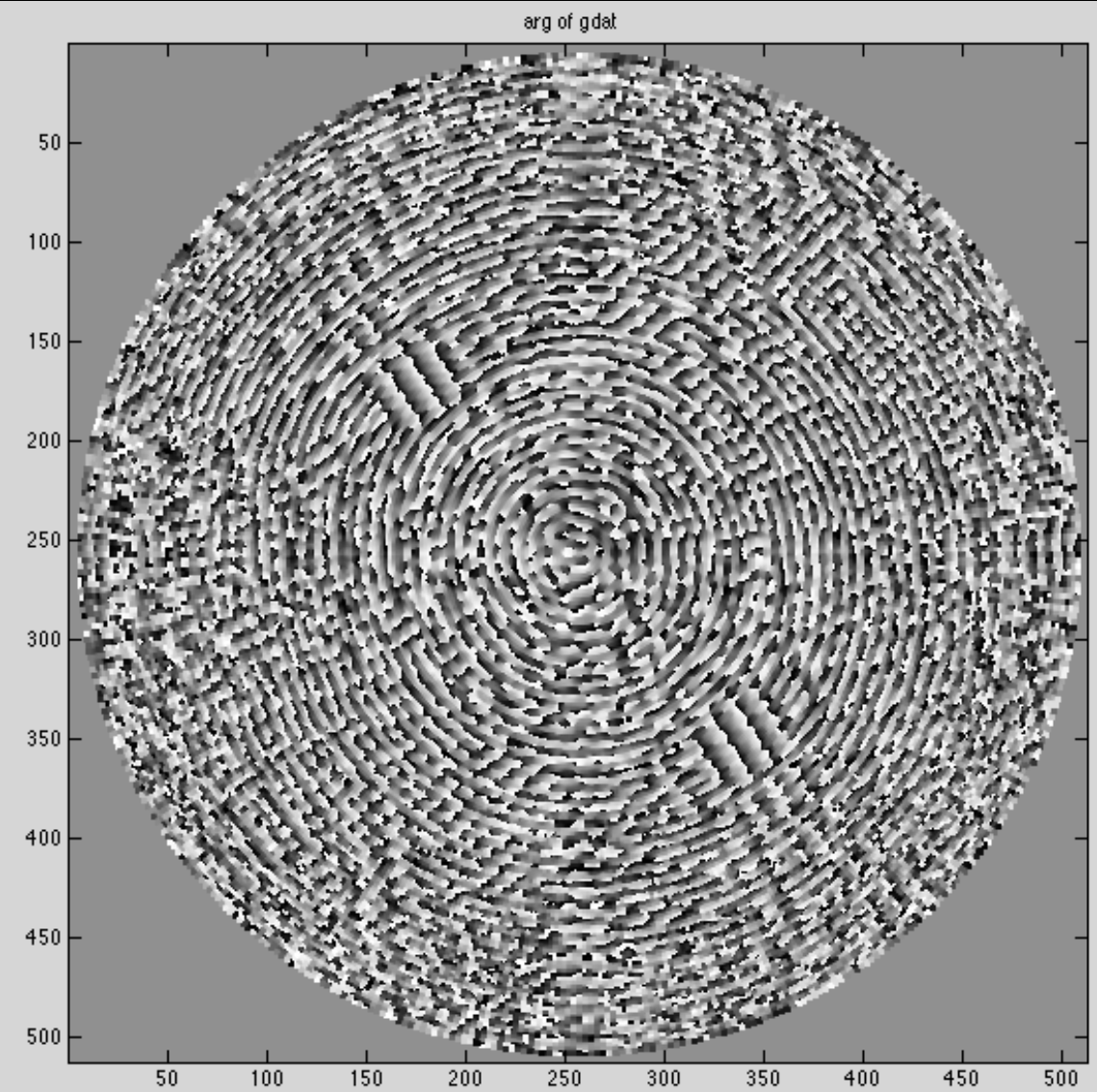
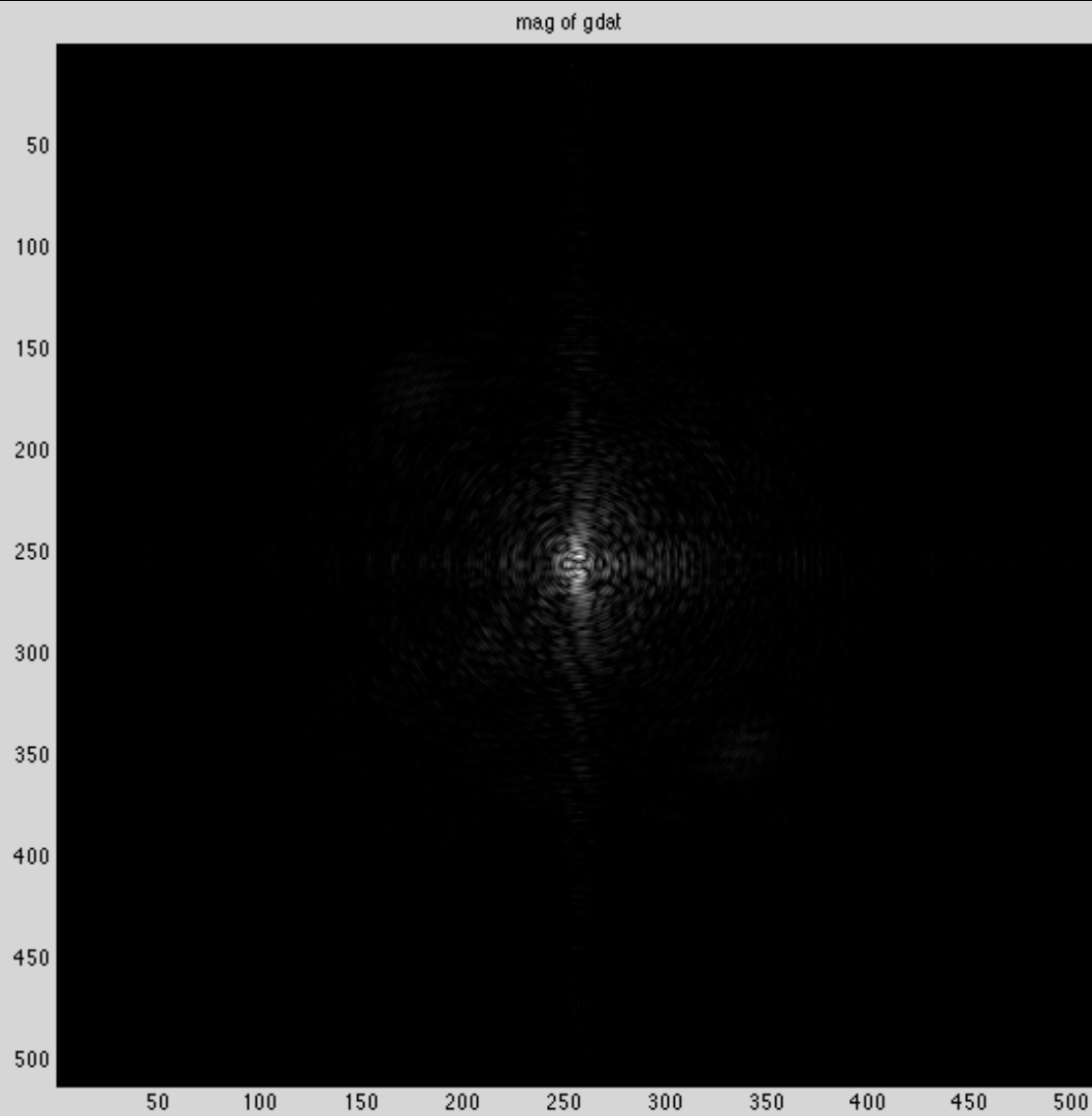


$\alpha = 2$; grid size = 2x[256 256]; kw = 4;

¹Beatty et al., *IEEE TMI* 2005; 24: 799-808

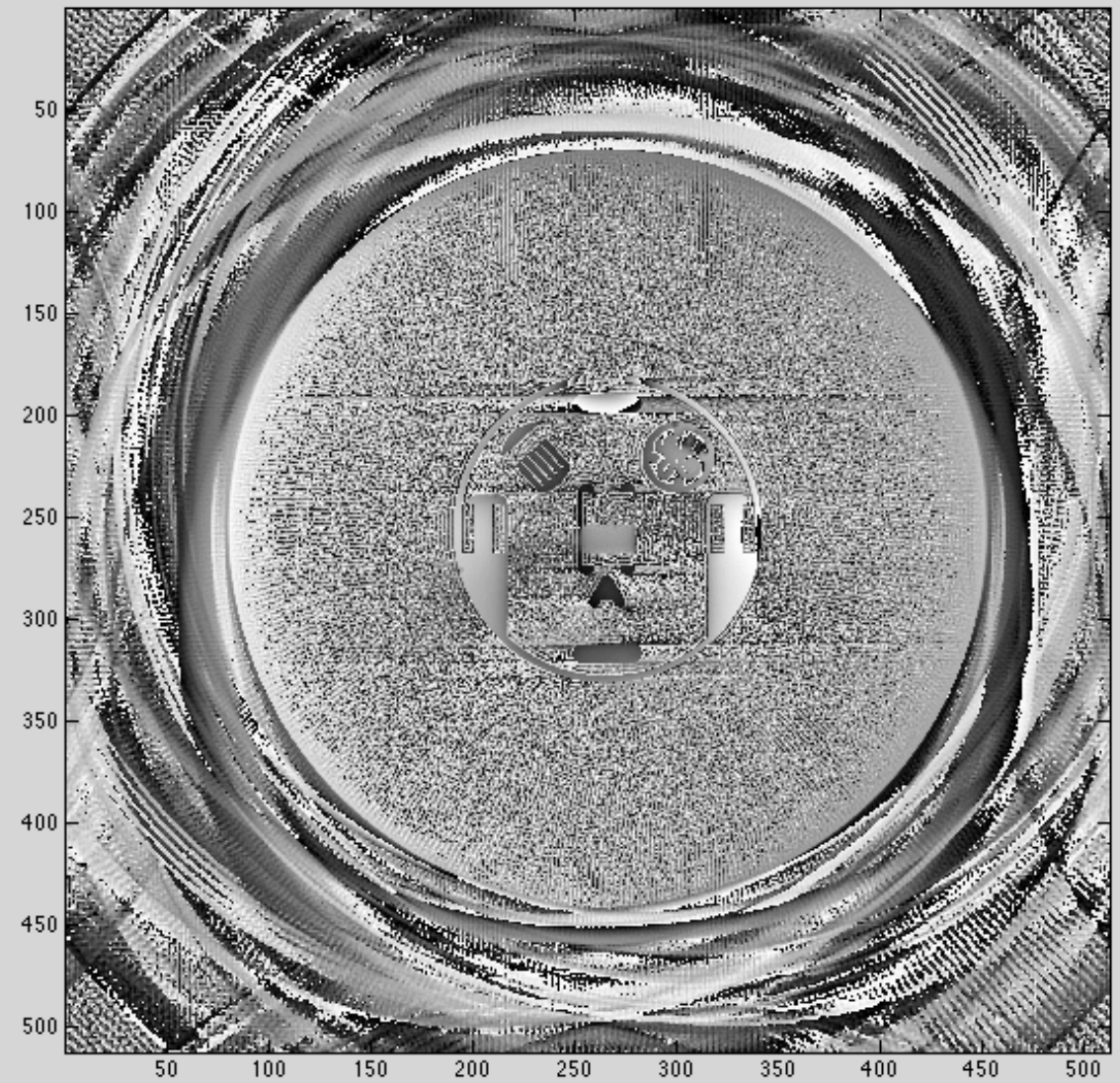
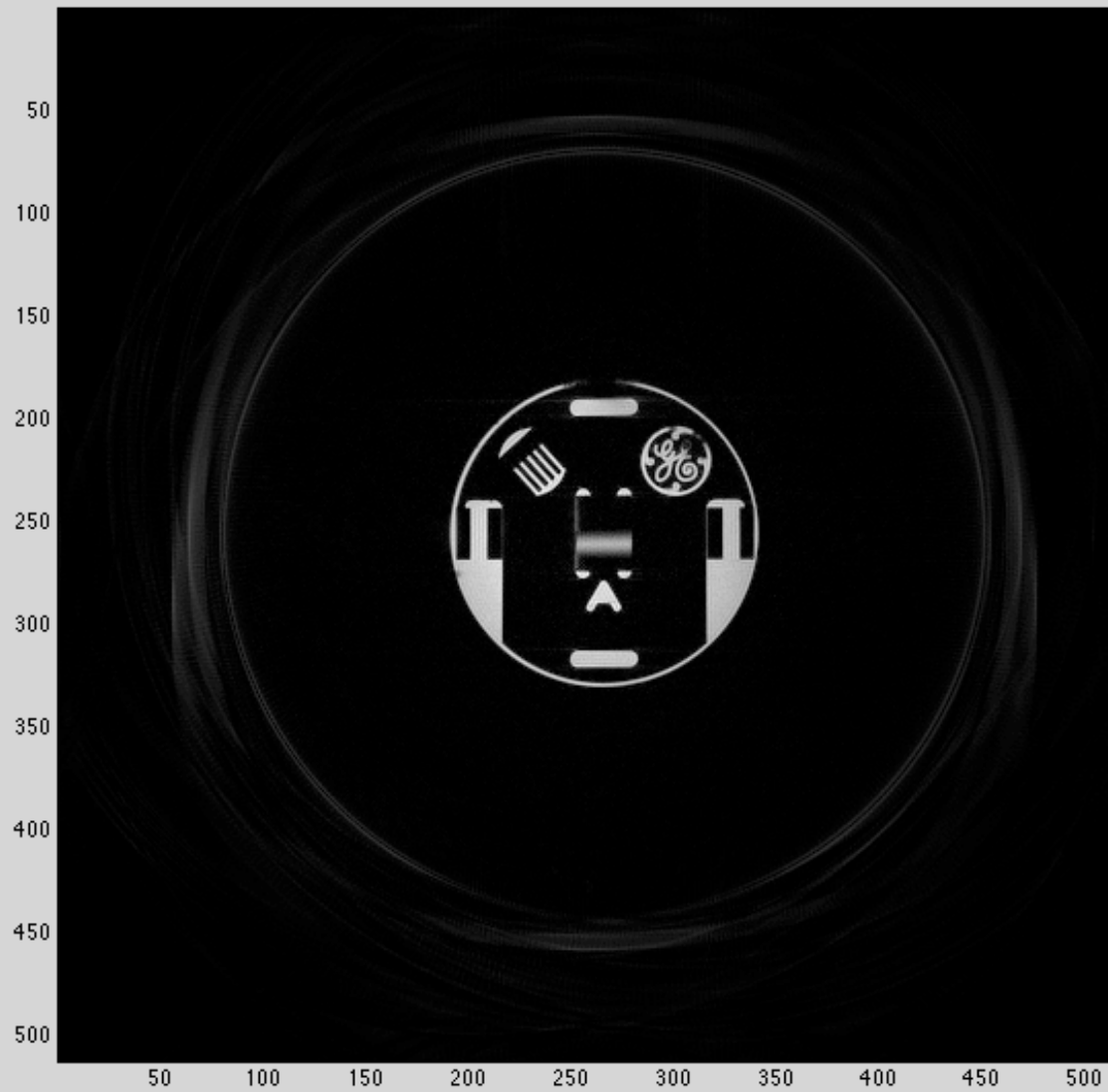
Gridding: 2D Radial Example

Gridded data on [512x512] grid



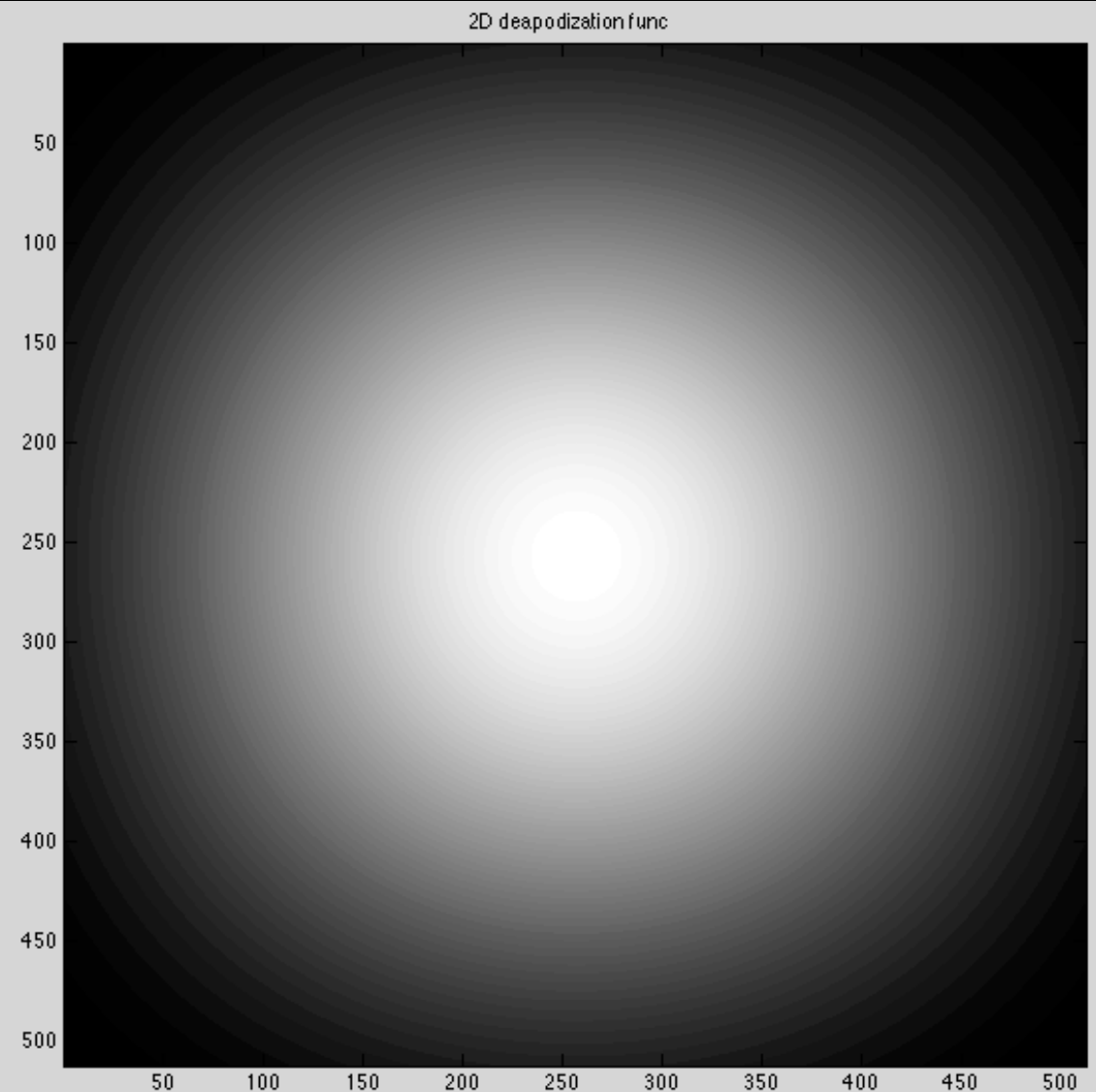
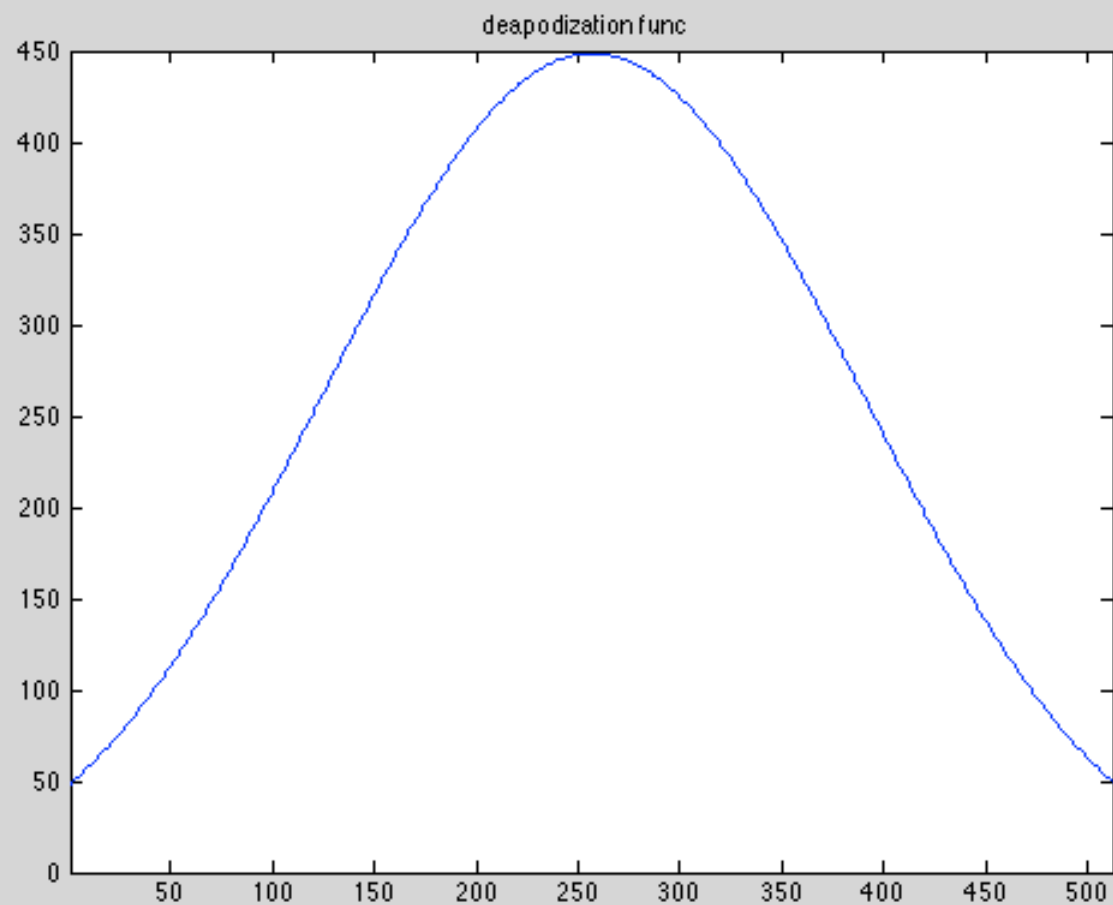
Gridding: 2D Radial Example

Inverse 2D FFT produces image with 2x FOV



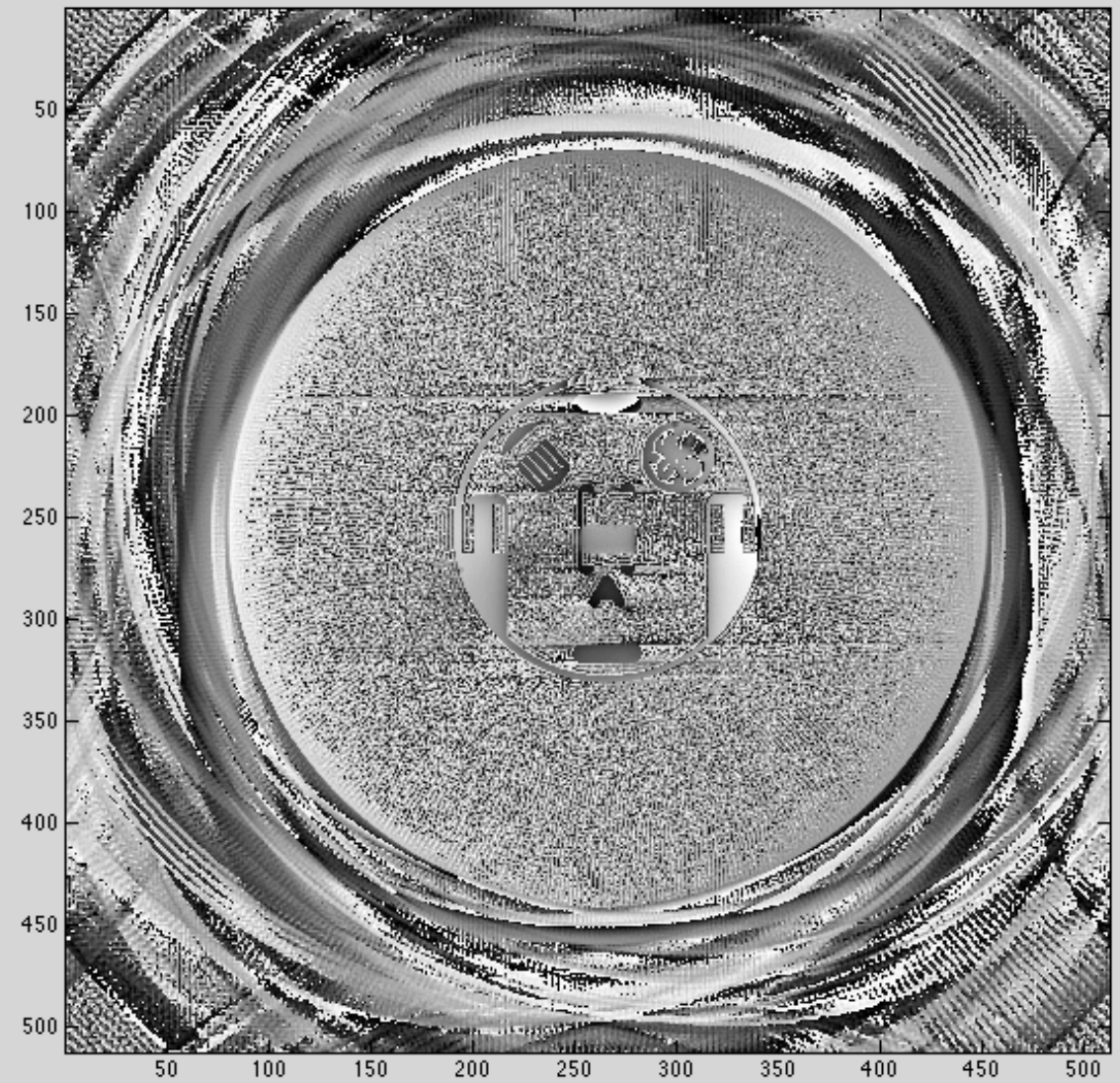
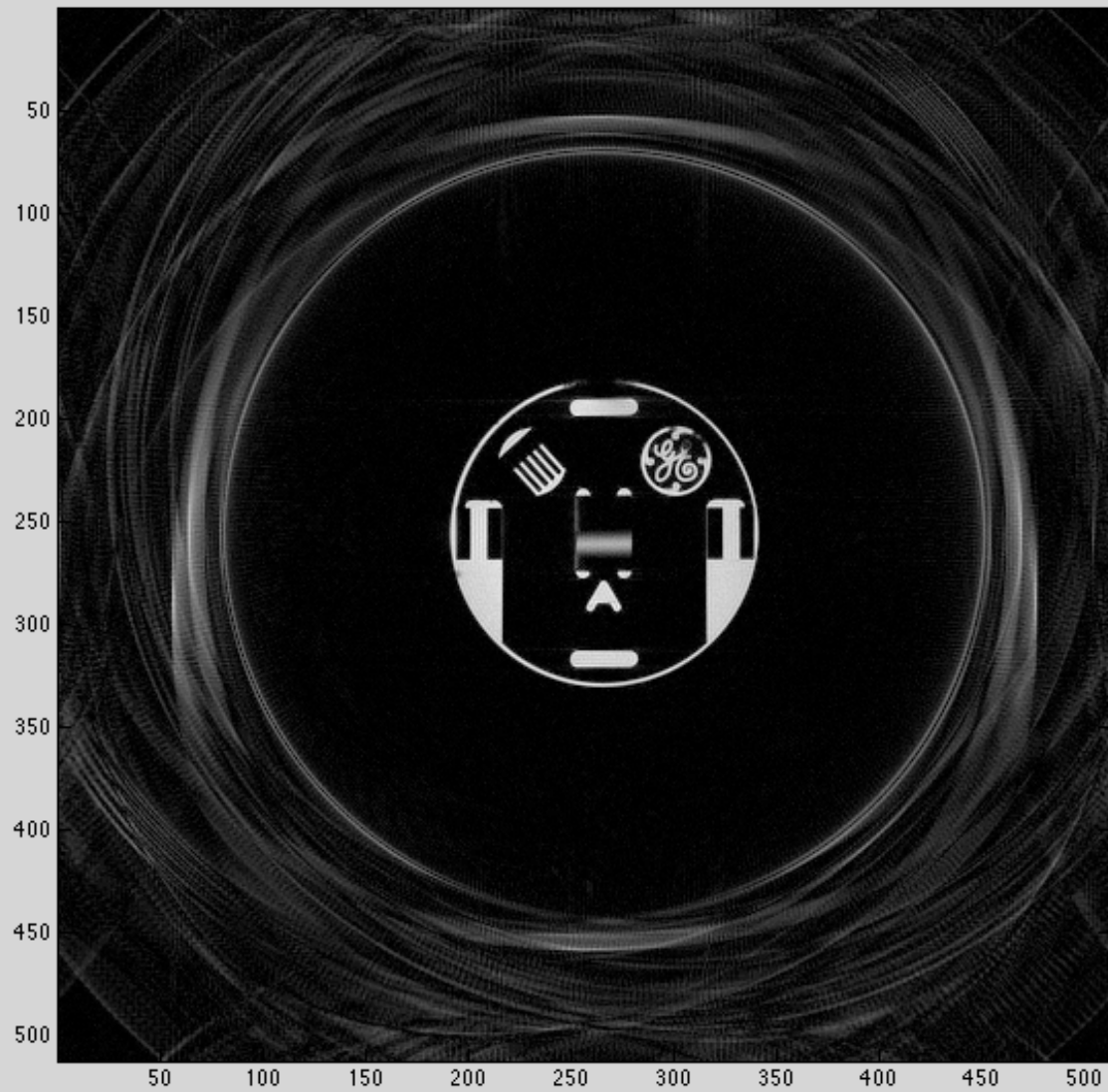
Gridding: 2D Radial Example

Deapodization function is FT of KB convolution kernel



Gridding: 2D Radial Example

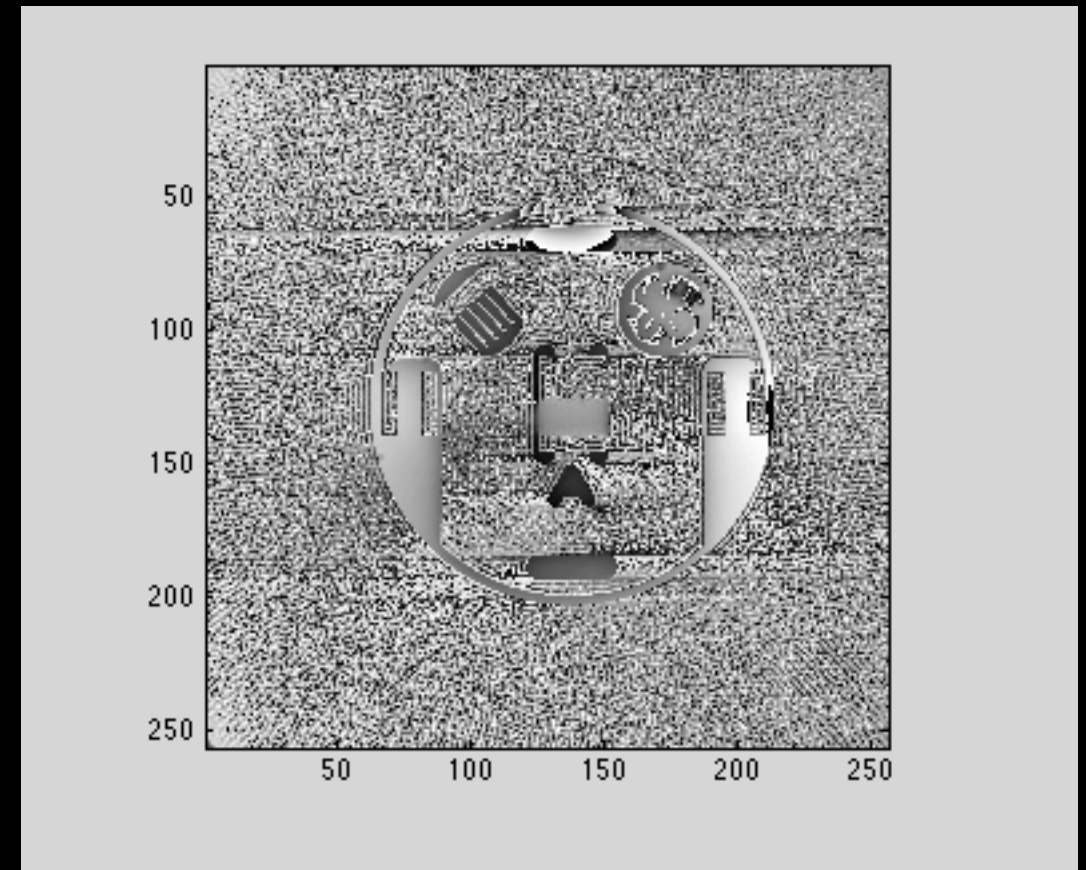
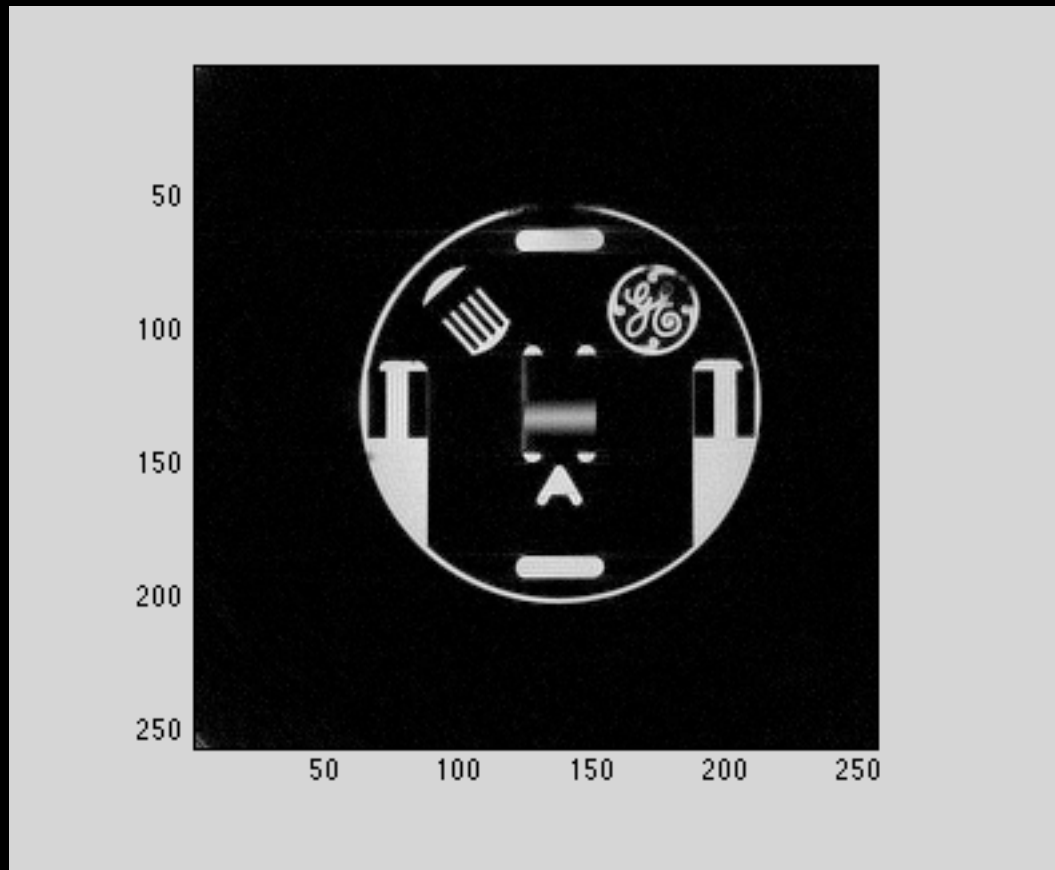
Deapodized image



Gridding: 2D Radial Example

FOV cropped to extract desired [256x256] image

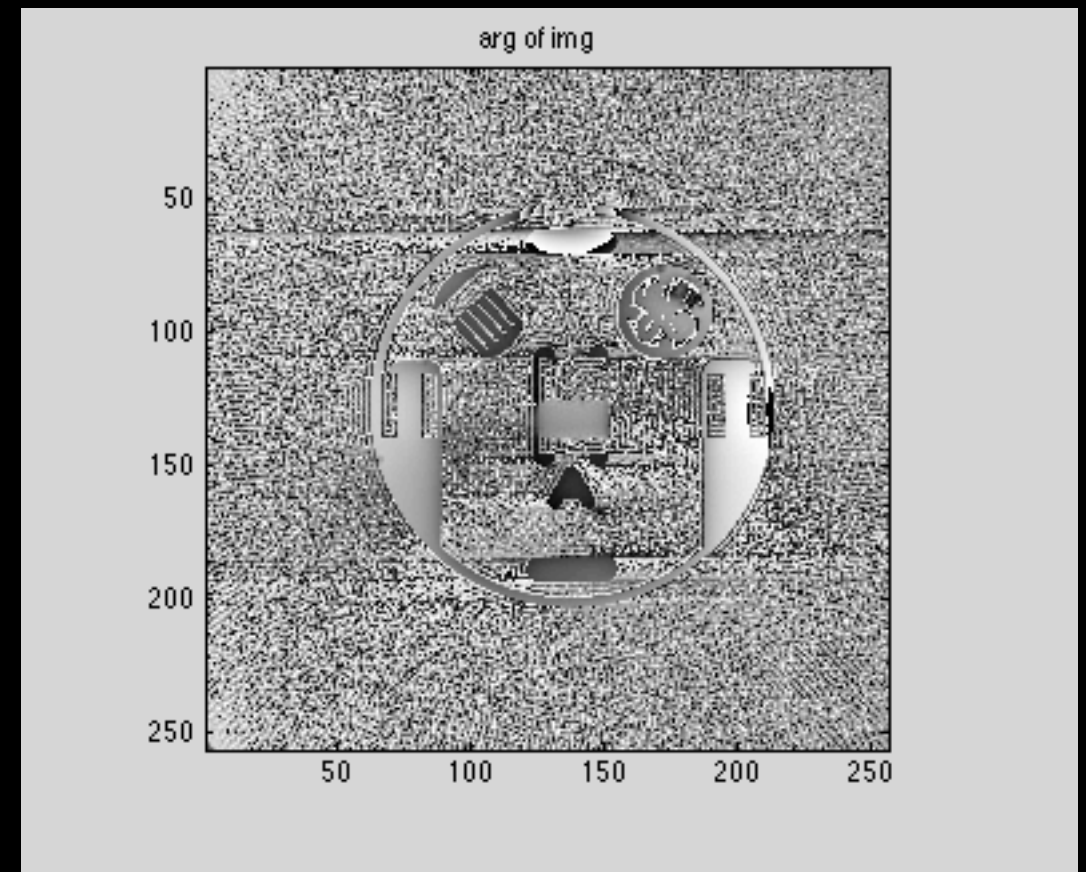
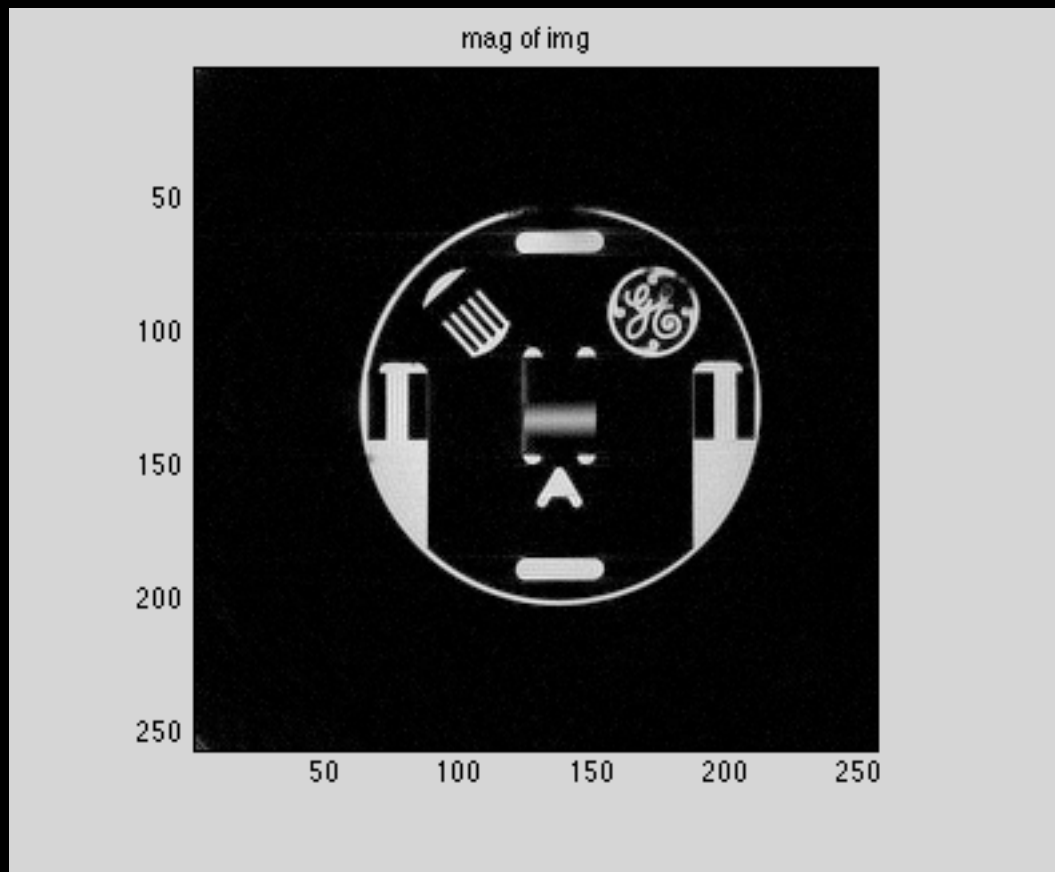
$$\alpha = 2, \text{ kw} = 4$$



Gridding: 2D Radial Example

FOV cropped to extract desired [256x256] image

$$\alpha = 1.375, kw = 5^1$$



Gridding: Summary

- Data input
 - k-space data
 - k-space traj (usually normalized), DCF
- Gridding params
 - target image dimensions [MxN]
 - grid oversampling factor α
 - kernel type and width
- Data output
 - gridded Cartesian k-space
 - reconstructed image

Gradient Measurement

- Non-Cartesian recon requires
 - k-space trajectory
 - density compensation function
- Both depend on actual gradient waveforms on scanner
 - can deviate from desired
- Knowledge of k-space trajectory also important for RF design

Gradient Measurement

- Gradient imperfections cause artifacts
 - FOV scaling, shifting
 - signal loss, shading
 - image blurring, geometric distortion
- Sources of gradient errors
 - eddy currents (B_0 , linear)
 - group delays (RF filters, A/D)
 - amplifier limitations (BW, freq response)
 - gradient warping
 - other ...

Gradient Measurement

- General techniques
 - off-iso slice technique^{1,2}, and more
- Trajectory-specific techniques
 - radial³, spiral⁴, and more
- Characterize gradient system
 - assume linear time-invariant model⁵

1 Duyn JH et al., JMR 1998; 132: 150-153

4 Robison RK et al., MRM 2010; 63: 1683-90

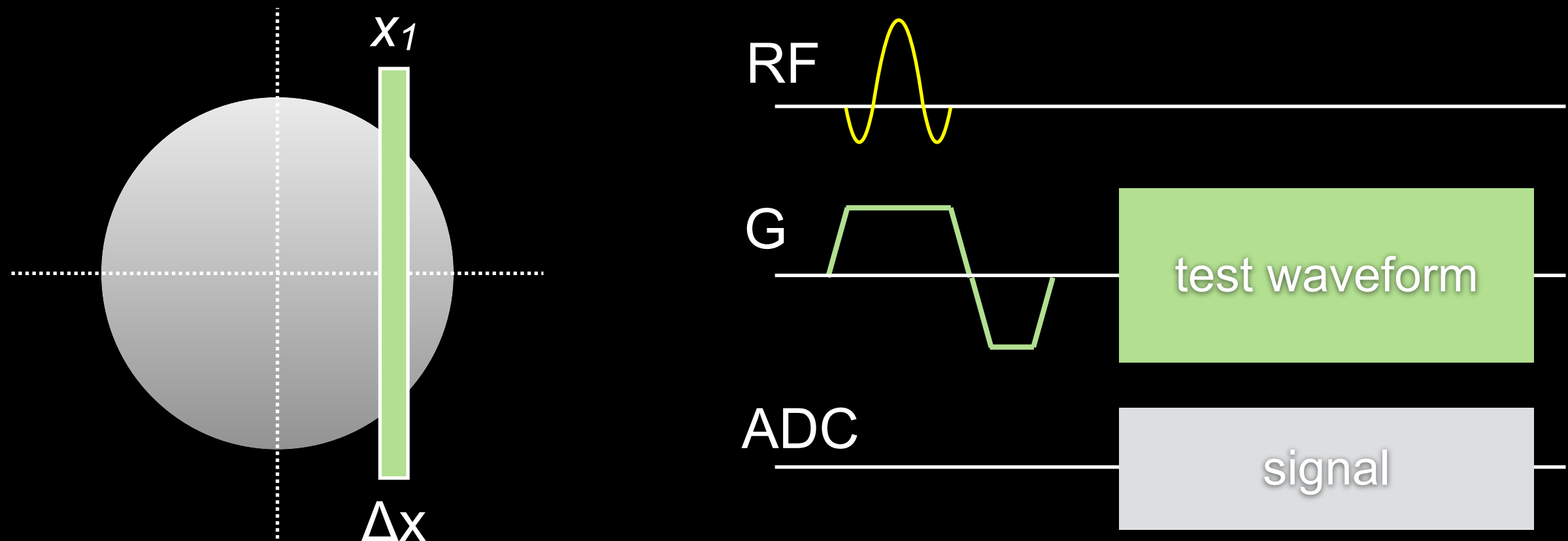
2 Beaumont M et al., MRM 2007; 58: 200-205

5 Addy NO et al., MRM 2012; 68: 120-129

3 Peters DC et al., MRM 2003; 50: 1-6

Gradient Measurement

Off-isocenter slice measurement technique



Can repeat on all three axes G_x , G_y , G_z

Gradient Measurement

Off-isocenter slice measurement technique

Waveform ON:

$$s_{x1,Gon}(t) = \iint_{Y,Z} m(x_1, y, z) e^{-i\phi_0(x_1, y, z, t)} \cdot e^{-i2\pi \cdot [\frac{\gamma}{2\pi} \int_0^t G(\tau) d\tau] \cdot x_1} dy dz$$

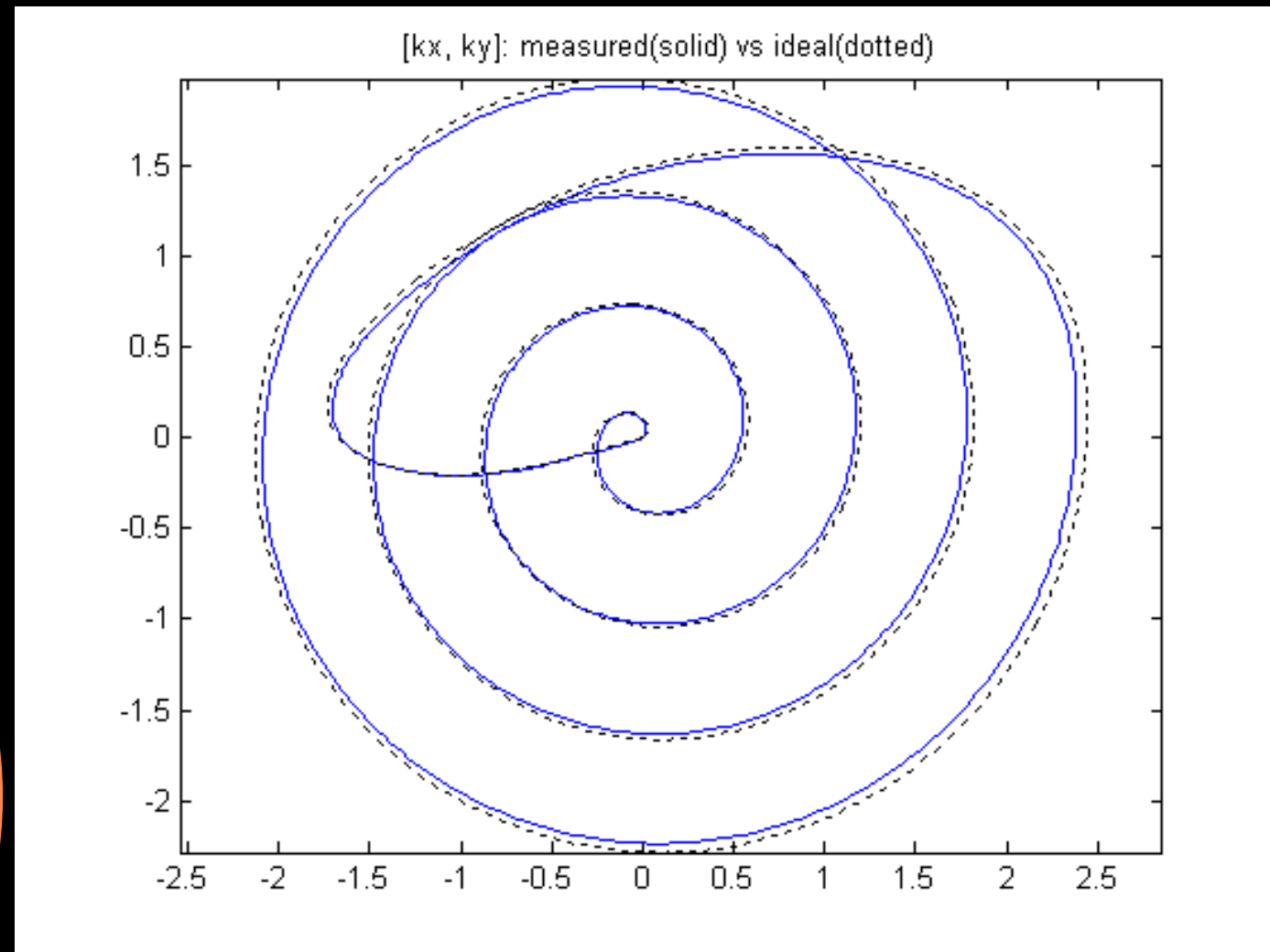
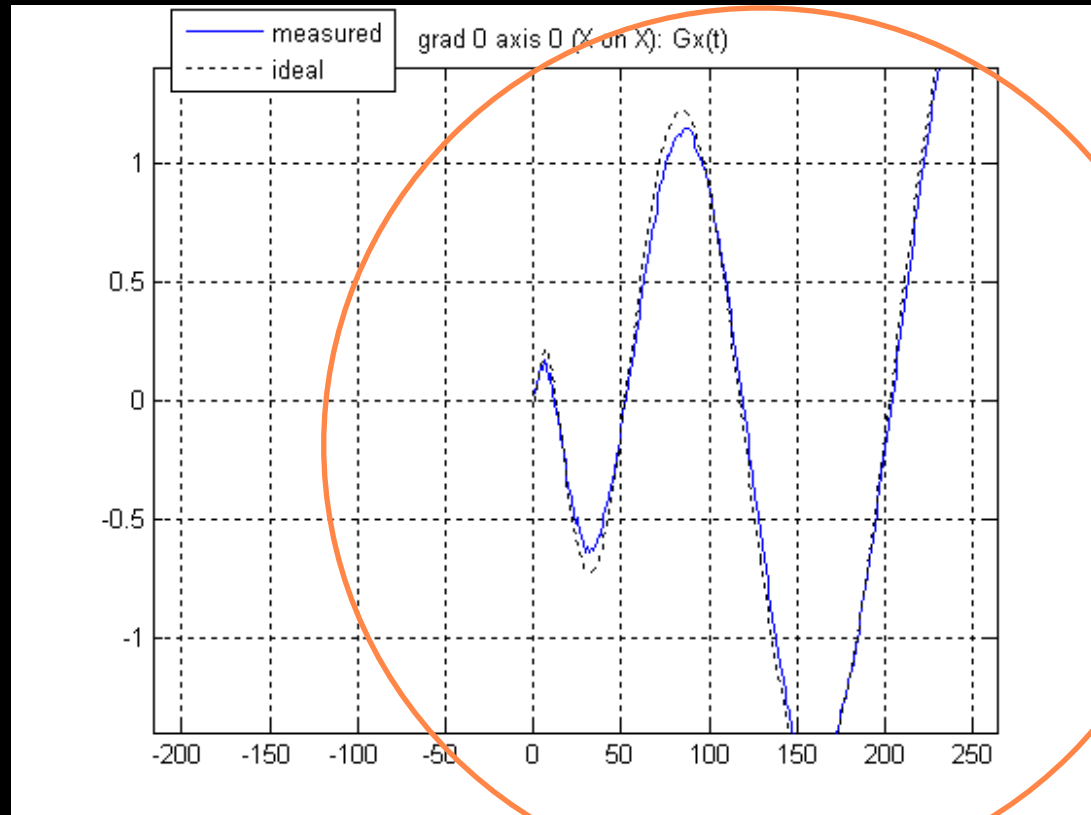
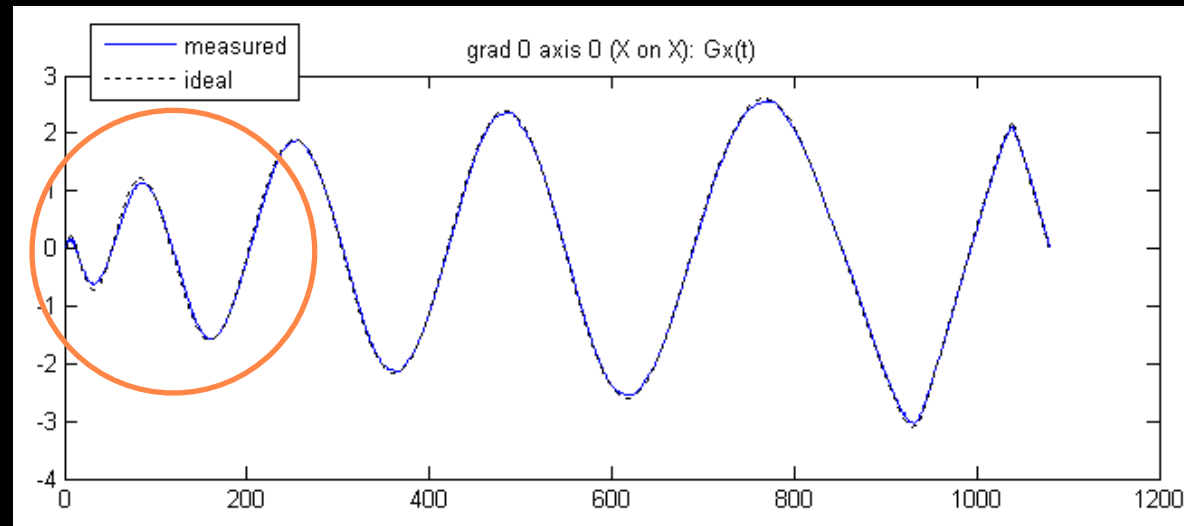
Waveform OFF:

$$s_{x1,Goff}(t) = \iint_{Y,Z} m(x_1, y, z) e^{-i\phi_0(x_1, y, z, t)} dy dz$$

Phase difference:

$$\Delta\phi_{x1}(t) = \gamma \int_0^t G(\tau) \cdot x_1 d\tau = x_1 \cdot k(t)$$

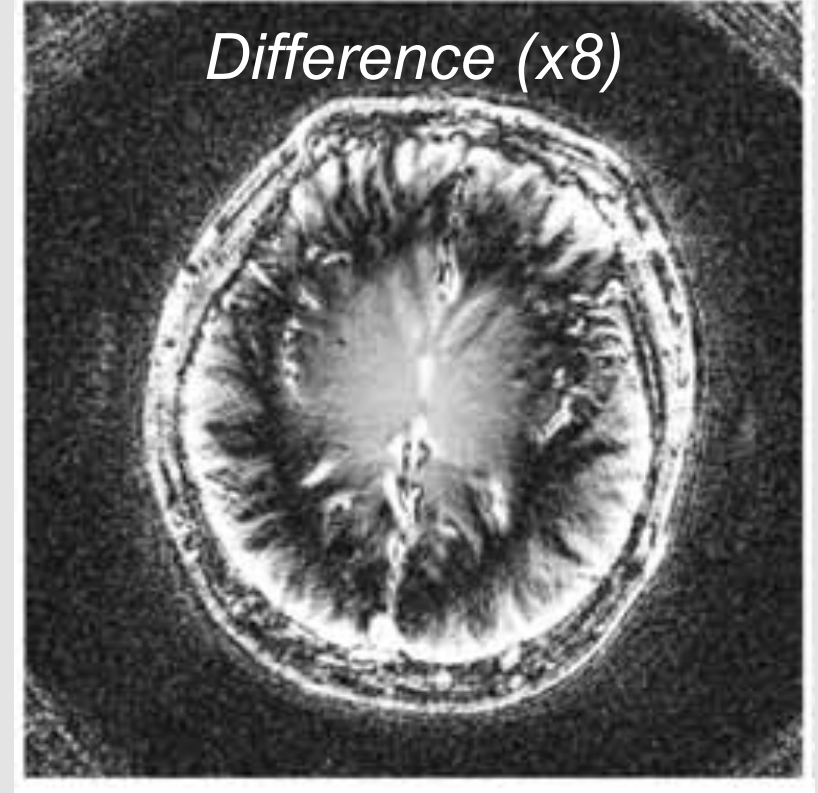
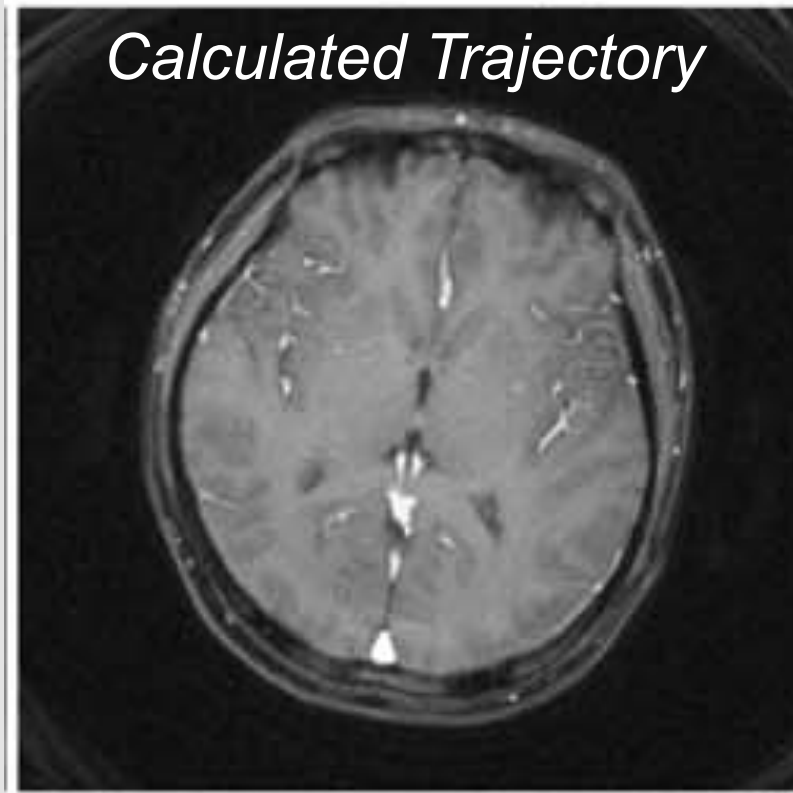
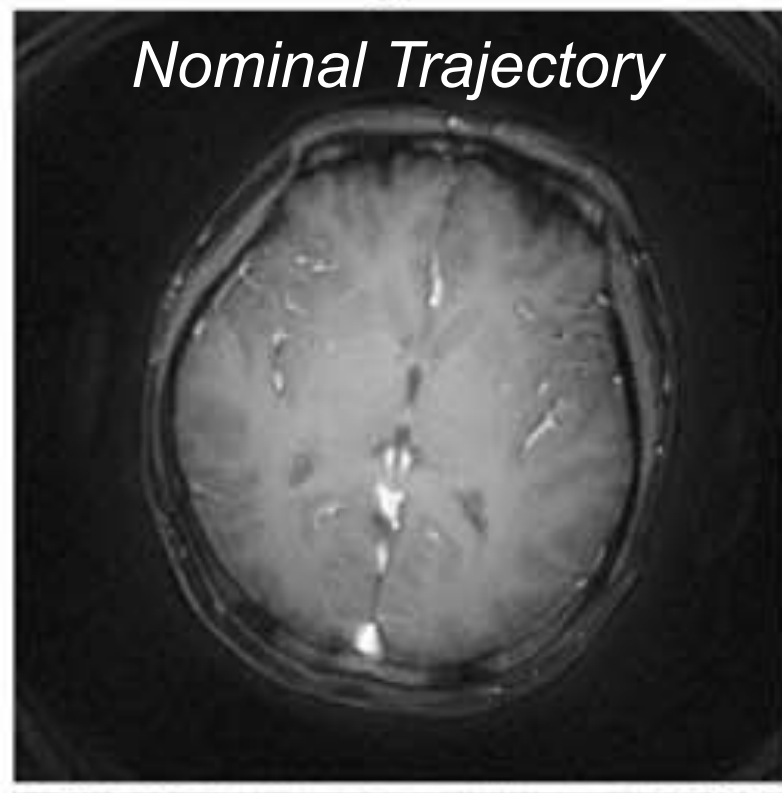
Gradient Measurement



Gradient Measurement

- Gradient (trajectory) correction
 - use actual trajectory for recon
 - pre-tune bulk gradient delay

Example: Axial Spiral at 1.5 T



Gradient Measurement

- Off-iso slice measurement technique
 - two measurements per axis
 - can measure X on X, Y on Y, Z on Z, and also cross terms; linearly combine
 - Δx should be small (may need avging)
 - need to account for phase wrapping
 - use spin echo for long waveforms
 - can acquire multiple slice offsets and gradient polarities to model individual gradient error terms

Gradient Measurement

- Delay calibration
 - gradient errors (e.g., linear eddy currents) mainly cause an apparent bulk delay
 - adjust ADC window w.r.t. gradients
 - delays may be different for each axis

Off-resonance Correction

- Off resonance effects (ΔB_0 , fat, etc.)

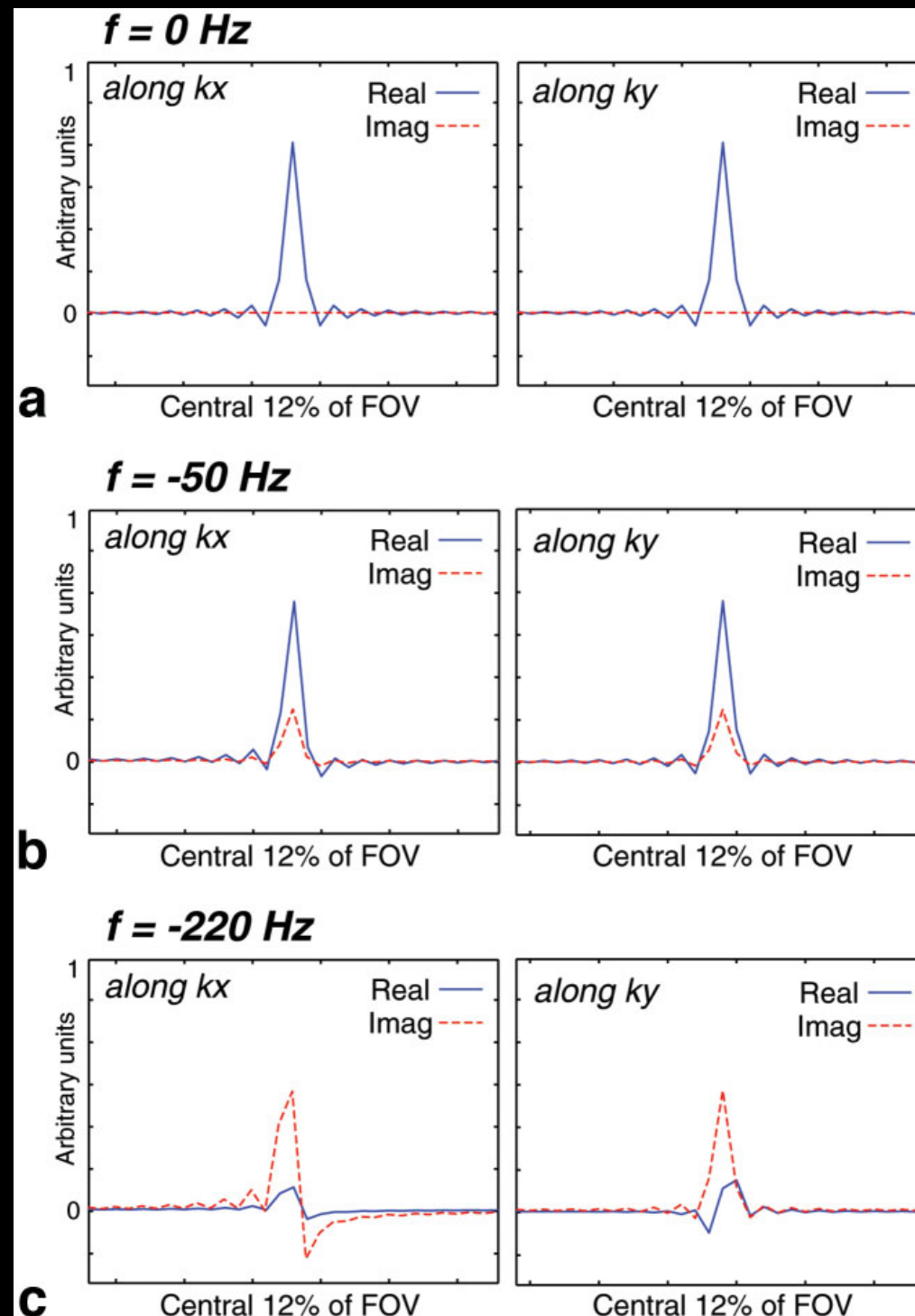
$$s(t) = \iint_{X,Y} m(x,y) \cdot e^{-i\phi(x,y,t)} \cdot e^{-i2\pi \cdot [k_x(t)x + k_y(t)y]} dx dy$$

$$\phi(x,y,t) = 2\pi\psi(x,y)t$$

- patient (scan) dependent
- pre-scan shim calibration helps
- usually negligible for Cartesian MRI
- non-Cartesian MRI: signal loss, spatial blurring, geometric distortion

Off-resonance Correction

Effects of off-res for concentric rings: PSF blurring



Off-resonance Correction

- Account for field inhomogeneity
 - use shorter readouts
 - measure/estimate field map

$$s(\text{TE}_1) \longrightarrow I_1 = M'(x, y) \cdot e^{-i2\pi\psi(x, y)\text{TE}_1}$$

$$s(\text{TE}_2) \longrightarrow I_2 = M'(x, y) \cdot e^{-i2\pi\psi(x, y)\text{TE}_2}$$

$$\hat{\psi}(x, y) = \arg(I_1 \cdot I_2^*) / 2\pi(\Delta\text{TE}) \quad [\pm 1/2\pi\Delta\text{TE}]$$

and then correct (during recon)^{1,2,3}

time-segmented, freq-segmented, etc.

¹ Noll DC et al., *IEEE TMI* 1991; 10: 629-637

² Noll DC et al., *MRM* 1992; 25: 319-333

³ Chen JY et al., *MRM* 2011; 66: 390-401

Off-resonance Correction

Linear Correction

$$\psi(x, y) = f_0 + f_x x + f_y y \quad (\text{can fit to this model})$$

$$\phi(x, y) = 2\pi f_0 t + 2\pi \Delta k_x(t) x + 2\pi \Delta k_y(t) y$$

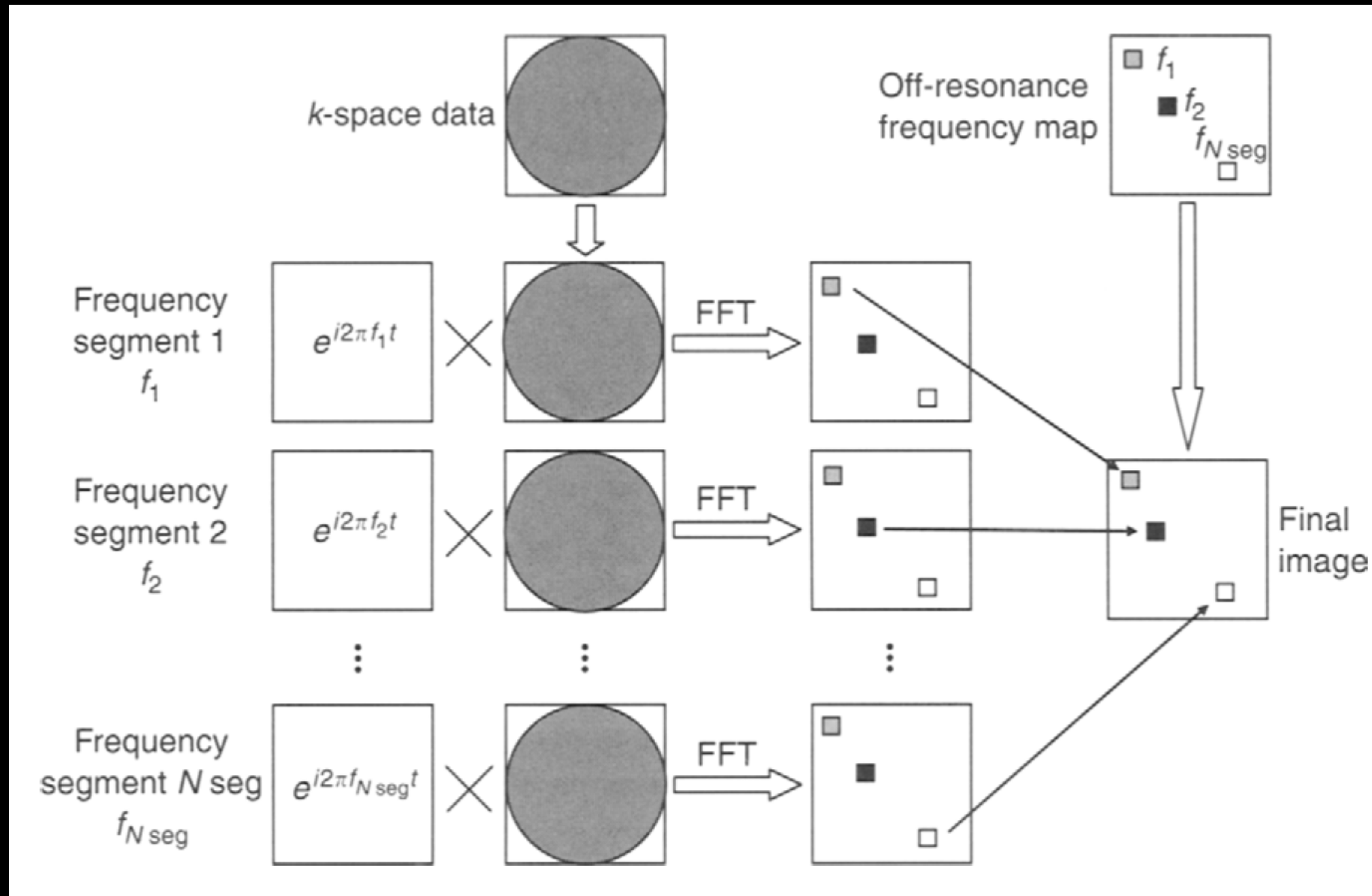
$$\Delta k_x(t) = f_x t, \quad \Delta k_y(t) = f_y t$$

$$s(t) = \underbrace{e^{-i2\pi f_0 t}}_{\text{demod}} \iint_{X,Y} m(x, y) \cdot e^{-i2\pi \cdot \underbrace{[(k_x(t) + \Delta k_x(t)) x + (k_y(t) + \Delta k_y(t)) y]}_{\text{shift } k\text{-space trajectory}}} dx dy$$

Can follow with frequency-segmented off-res correction

Off-resonance Correction

Frequency-segmented correction



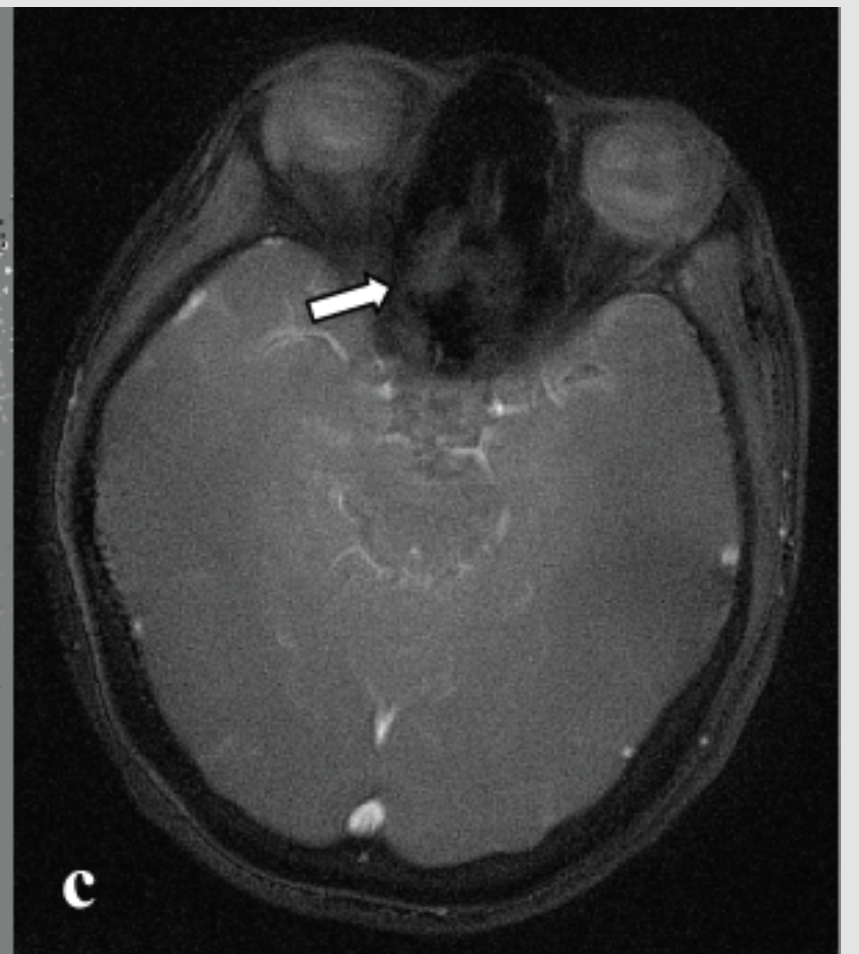
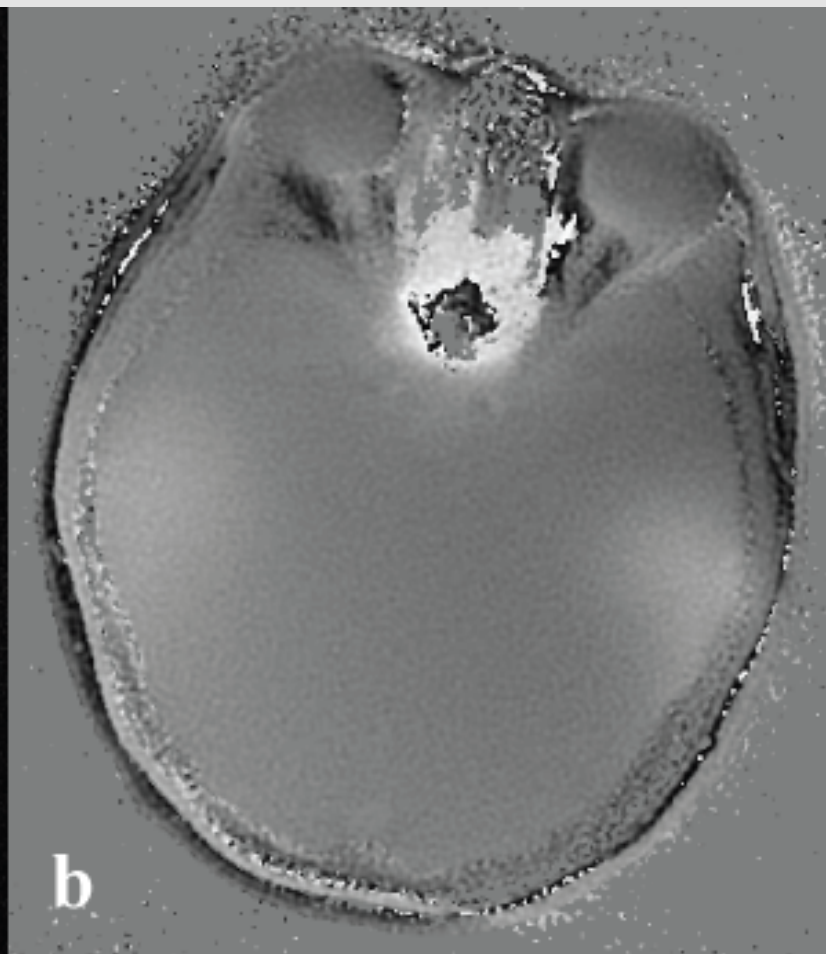
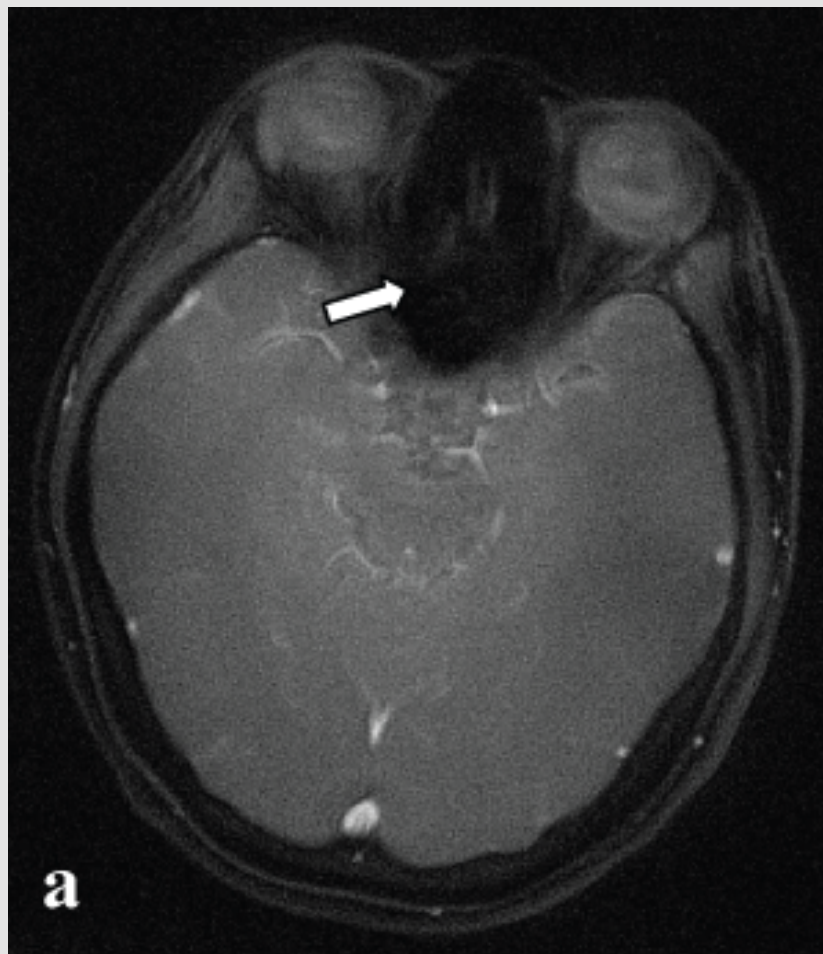
Off-resonance Correction

Example: Axial Concentric Rings at 1.5 T

Regular Recon

Field Map

ORC Image



Off-resonance Correction

- Field map measurement
- Segmented correction methods
 - Need to recon multiple images,
 $N_{\text{bins}} \sim 4(f_{\text{max}} - f_{\text{min}})T_{\text{acq}}$
- Other sources of off resonance
 - concomitant gradients
 - chemical shift (e.g., fat)
- Other ORC algorithms
 - autofocusing (field map optional)
 - combine with image reconstruction

Thanks!

- Further reading
 - references on each slide
 - further reading section on website
- Acknowledgments
 - John Pauly's EE369C class notes (Stanford)

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