

# RF Pulse Design

## *RF Pulses / Adiabatic Pulses*

M229 Advanced Topics in MRI

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2019.04.04

4/4/2019

- Ended at 11:45am
- Needed to finish RF pulse review earlier (took an hour to finish)

# Class Business

- Lecture #3 - Friday (4/5)
  - 10-12pm
  - No lecture on 4/9
- Homework 1 will be out on 4/5 (Fri)
  - Due on 4/26
- TA for HW1 is TBD
- Group email list / course website

# Outline

- Review of RF pulses
- Adiabatic passage principle
- Adiabatic inversion
- Applications of adiabatic pulses

# Review of RF Pulses

## Notation and Conventions

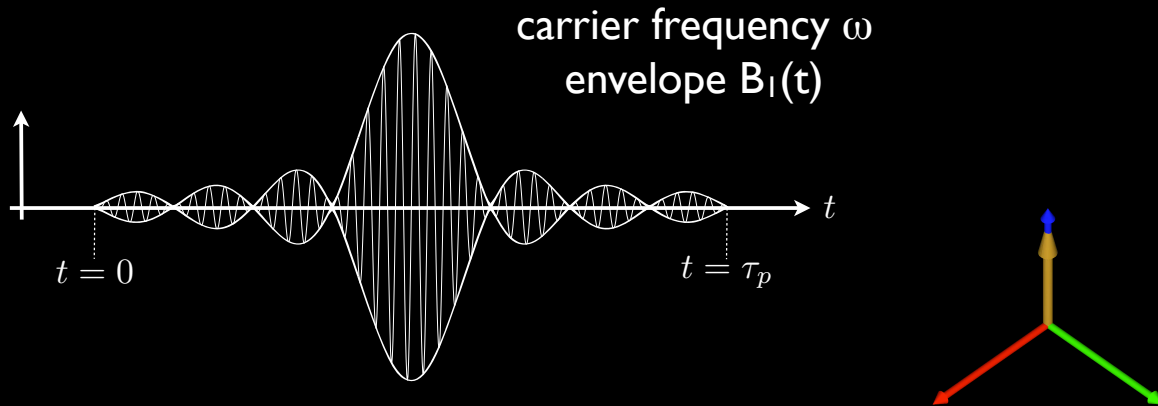
$$\vec{B} = B_0 \hat{k} + B_1(t) [\cos \omega t \hat{i} - \sin \omega t \hat{j}]$$

- $\omega$  = carrier frequency
- $\omega_0$  = resonant frequency
- $B_1(t)$  = complex valued envelop function

# RF Pulse - Excitation

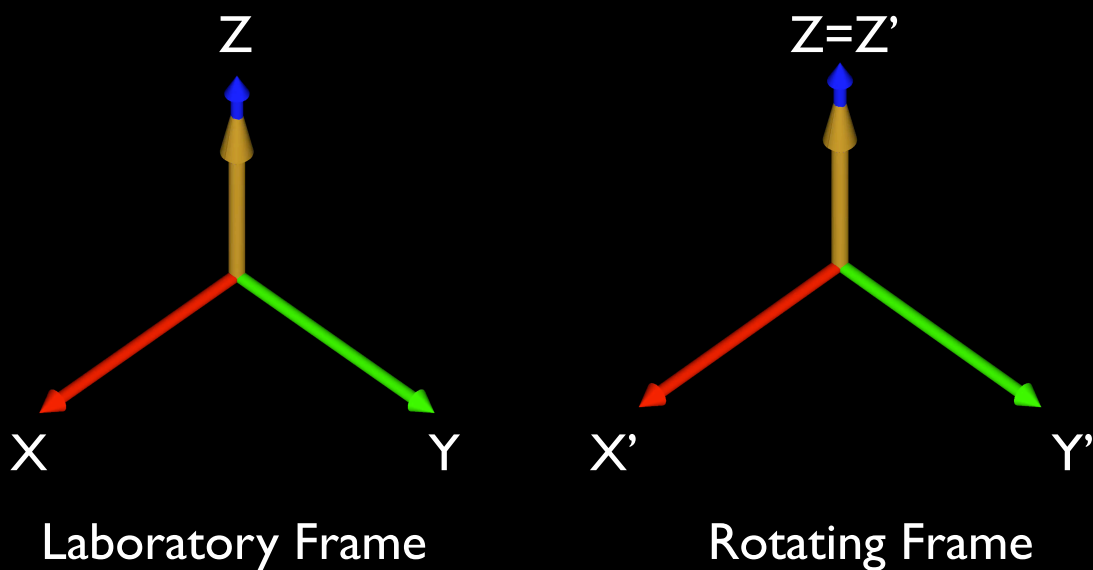
$$\vec{B} = B_0 \hat{k} + B_1(t) [\cos \omega t \hat{i} - \sin \omega t \hat{j}]$$

$$B_1(t) \cdot [\cos(\omega t) \hat{i} - \sin(\omega t) \hat{j}]$$



## Lab vs. Rotating Frame

- The rotating frame simplifies the mathematics and permits more intuitive understanding.



# Rotating Frame

## Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \quad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \quad \begin{array}{l} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{array}$$

$$\vec{M}_{lab}(t) = R_Z(\omega_0 t) \cdot \vec{M}_{rot}(t)$$

$$\vec{B}_{lab}(t) = R_Z(\omega_0 t) \cdot \vec{B}_{rot}(t)$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \longrightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

## Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where  $\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{\omega}_{rot}}{\gamma}$  fictitious field

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix}$$

# Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{\omega}_{rot}}{\gamma}$$

$$\vec{B}_{lab} = \begin{pmatrix} B_1(t) \cos \omega_0 t \\ B_1(t) \sin \omega_0 t \\ B_0 \end{pmatrix} \quad \vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ B_1(t) \\ B_0 \end{pmatrix}$$

Assume real-valued  $B_1(t)$

$$\vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 \end{pmatrix} \quad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix}$$

To the board ...

# Bloch Equation with Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix} \rightarrow \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$$

# Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where  $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \quad \omega_1(t) = \gamma B_1(t)$$

To the board ...

## B1 Variations

- In MRI, B1 field is not always uniform across the imaging volume
- B1 inhomogeneity can cause:
  - Image shading
  - Incomplete saturation (e.g. in fat suppression)
  - Incomplete inversion (e.g. CSF suppression, myocardium suppression in cardiac scar imaging)
  - Inaccurate/imprecise quantification in T1 mapping



## B1 Variations

- It is highly desirable if we can excite tissue homogeneously and produce a uniform flip angle throughout

### → Adiabatic Pulses!

*“Adiabatic pulses are a special class of RF pulses that can excite, refocus or invert magnetization vectors uniformly, even in the presence of a spatially nonuniform B1 field.”*

## Adiabatic Passage Principle

# Adiabatic Pulses

- A special class of RF pulses that can achieve uniform flip angle
- Flip angle is independent of the applied B1 field

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Slice profile of an adiabatic pulse is obtained using Bloch simulations
- Can be used for excitation, inversion and refocusing

# Adiabatic vs. Non-Adiabatic Pulses

## Adiabatic Pulses:

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Amplitude and frequency/phase modulation
- Long duration (8-12 ms)
- Higher B1 amplitude (>12  $\mu\text{T}$ )
- Generally NOT multi-purpose (inversion pulse cannot be used for refocusing, etc.)

## Non-Adiabatic Pulses:

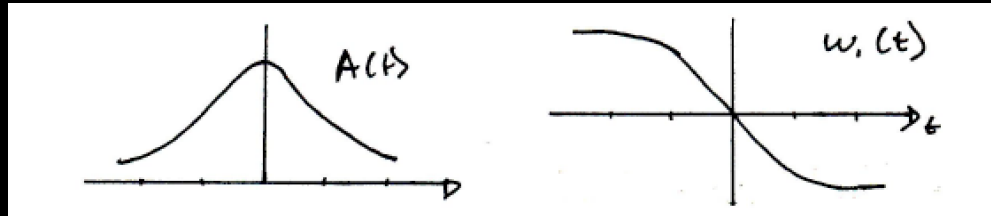
$$\theta = \int_0^T B_1(\tau) d\tau$$

- Amplitude modulation, constant carrier frequency (constant phase)
- Short duration (0.3 ms to 1 ms)
- Lower B1 amplitude
- Generally multi-purpose

# Adiabatic Pulses

- Frequency modulated pulses:

$$B_1(t) = \underbrace{A(t)}_{\text{envelop}} \exp^{-i \int \underbrace{\omega_1(t')}_{\text{frequency sweep}} dt'}$$



- Or phase modulation:

$$B_1(t) = A(t) \exp^{-i\phi(t)}$$

## Bloch Equation (at on-resonance)

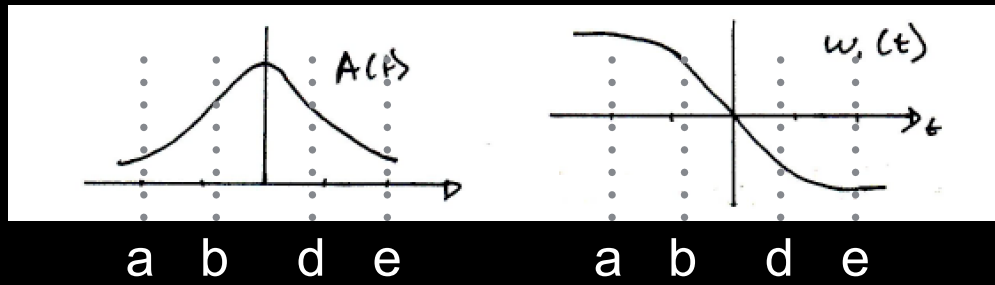
$$B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

where  $\vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \cancel{B_0} \frac{\omega}{\gamma} + \frac{\omega_1(t)}{\gamma} \end{pmatrix}$

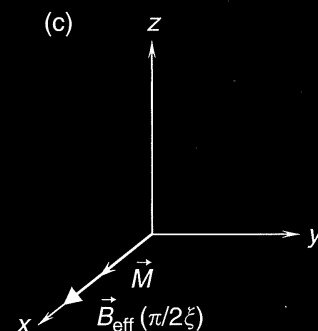
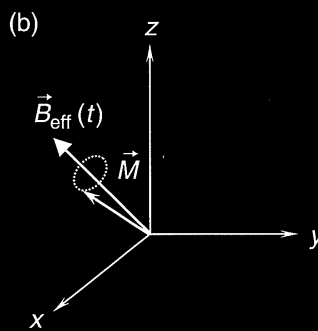
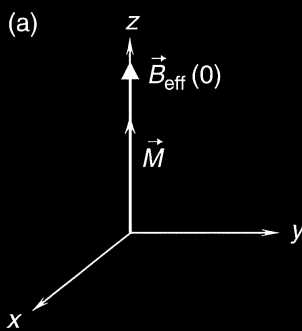
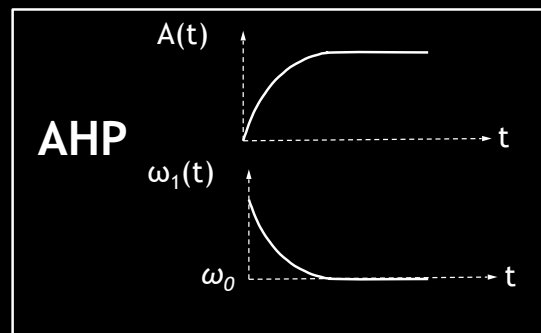
$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega_1(t) & 0 \\ -\omega_1(t) & 0 & \gamma A(t) \\ 0 & -\gamma A(t) & 0 \end{pmatrix} \vec{M}$$

# Magnetization Plot



To the board ...

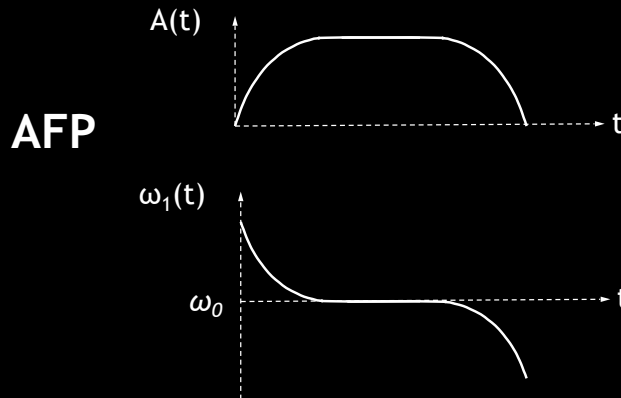
## Adiabatic Excitation



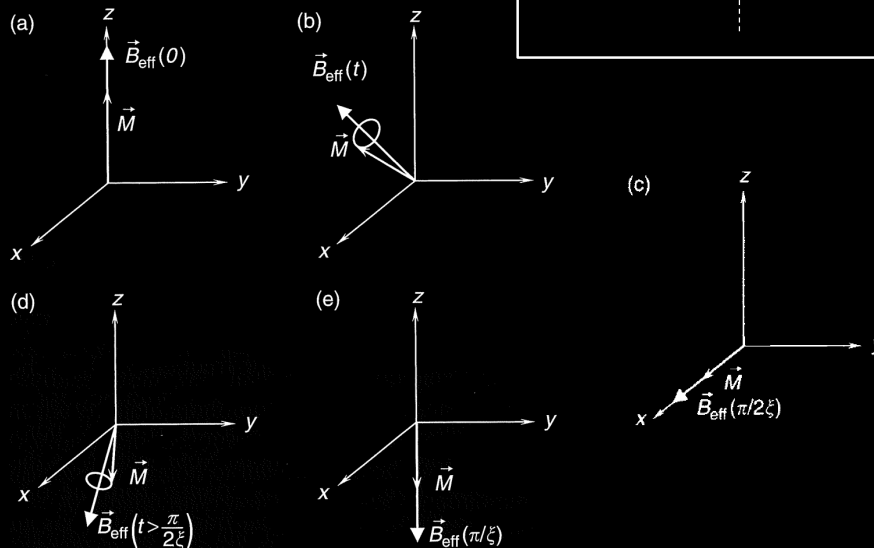
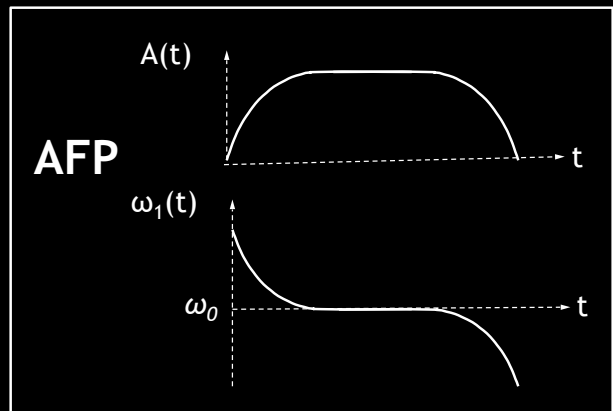
- At the end of the pulse, all the magnetization is in the transverse plane → so we have adiabatic excitation!
- This is also called an **adiabatic half passage (AHP)**

# Adiabatic Inversion

- An adiabatic inversion requires an adiabatic full passage (AFP) pulse:



# Adiabatic Inversion



# Adiabatic Inversion

## Design of Adiabatic Inversion

- General equation for an adiabatic pulse:

$$B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'}$$

- Many different types of adiabatic pulses can be designed by choosing different amplitude and frequency modulation functions
- The most famous one is...

**The Hyperbolic Secant Inversion Pulse!**

# Hyperbolic Secant Pulse Equations

$$B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'}$$

where

$$A(t) = A_0 \operatorname{sech}(\beta t)$$

$$\omega_1(t) = -\mu \beta \tanh(\beta t)$$

$A_0$ : peak amplitude ( $\mu\text{T}$ )

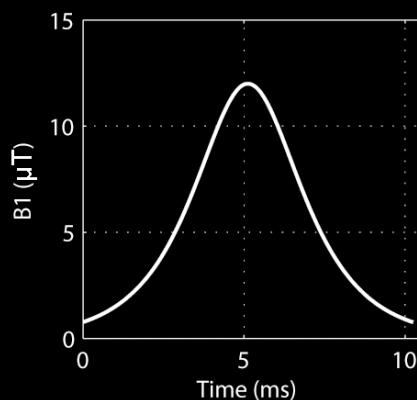
$\beta$ : frequency modulation parameter (rad/s)

$\mu$ : phase modulation parameter (dimensionless)

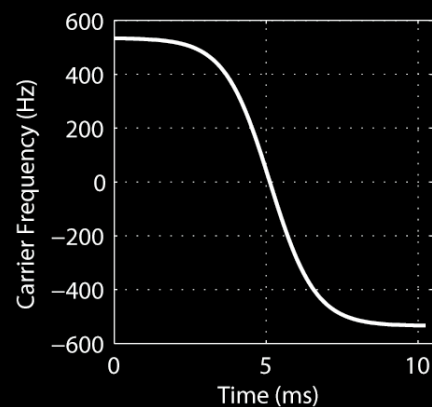
*Handbook of MRI Pulse Sequences, Ch 6.2, pp 194-195*

## Hyperbolic Secant Pulse Example

Amplitude Modulation,  $A(t)$



Frequency Modulation,  $\omega_1(t)$



### Pulse Parameters:

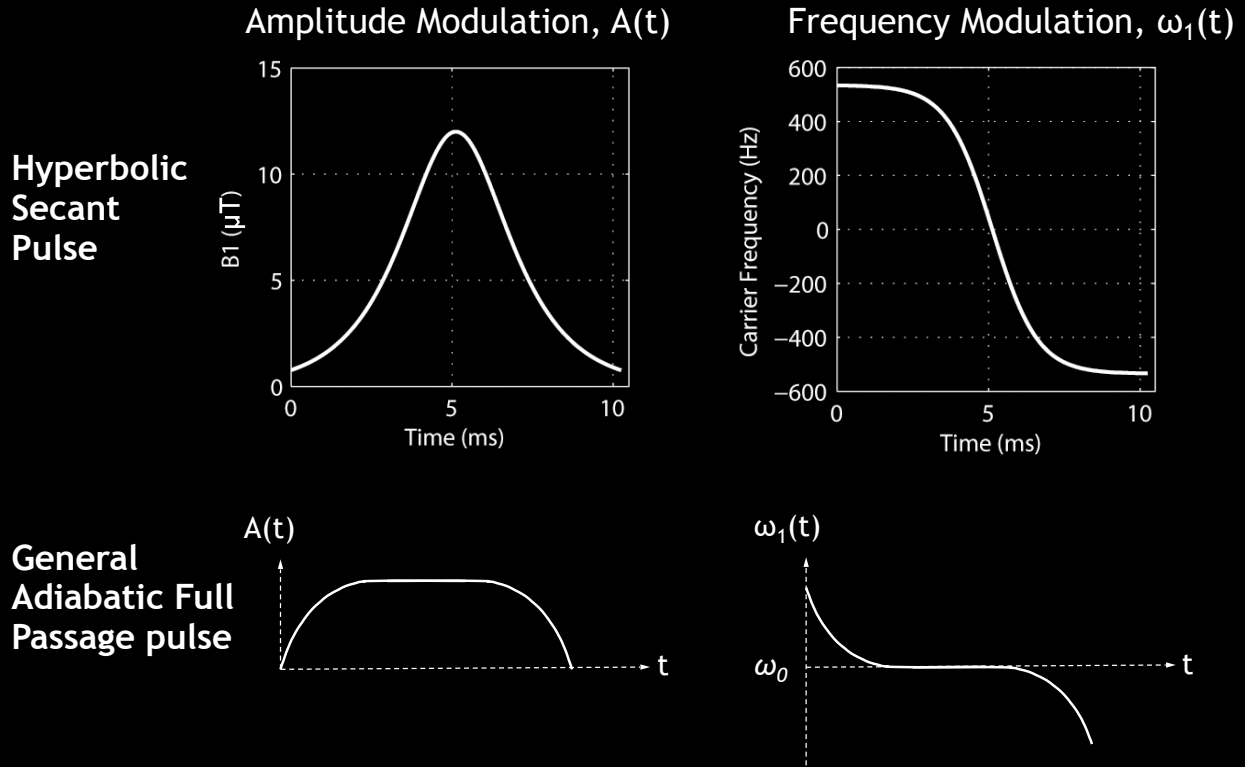
$$A_0 = 12 \mu\text{T}$$

$$\mu = 5$$

$$\beta = 672 \text{ rad/s}$$

$$\text{Duration} = 10.24 \text{ ms}$$

# Comparing Hyperbolic Secant with an AFP Example



# Some Examples of Other Adiabatic Inversion Pulses

Pulse Name	$A(t)$	$\omega_1(t)$
Lorentz	$\frac{1}{1+\beta\tau^2}$	$\frac{\tau}{1+\beta\tau^2} + \frac{1}{\sqrt{\beta}} \tan^{-1}(\sqrt{\beta}\tau)$
HS	$\text{sech}(\beta\tau)$	$\frac{\tanh(\beta\tau)}{\tanh(\beta)}$
Gauss <sup>c</sup>	$\exp\left(-\frac{\beta^2\tau^2}{2}\right)$	$\frac{\text{erf}(\beta\tau)}{\text{erf}(\beta)}$
Hanning	$\frac{1+\cos(\pi\tau)}{2}$	$\tau + \frac{4}{3\pi} \sin(\pi\tau) \left[ 1 + \frac{1}{4} \cos(\pi\tau) \right]$
HSn <sup>c</sup> ( $n=8$ )	$\text{sech}(\beta\tau^n)$	$\int \text{sech}^2(\beta\tau^n) d\tau$
Sin40 <sup>d</sup> ( $n=40$ )	$1 - \left  \sin^n\left(\frac{\pi\tau}{2}\right) \right $	$\tau - \int \sin^n\left(\frac{\pi\tau}{2}\right) \left( 1 + \cos^2\left(\frac{\pi\tau}{2}\right) \right) d\tau$



## Some Examples of Other Adiabatic Inversion Pulses

Pulse Name	A(t)	$\omega_1(t)$
Lorentz	$\frac{1}{1+\beta\tau^2}$	$\frac{\tau}{1+\beta\tau^2} + \frac{1}{\sqrt{\beta}} \tan^{-1}(\sqrt{\beta}\tau)$
HS	$\text{sech}(\beta\tau)$	$\frac{\tanh(\beta\tau)}{\tanh(\beta)}$

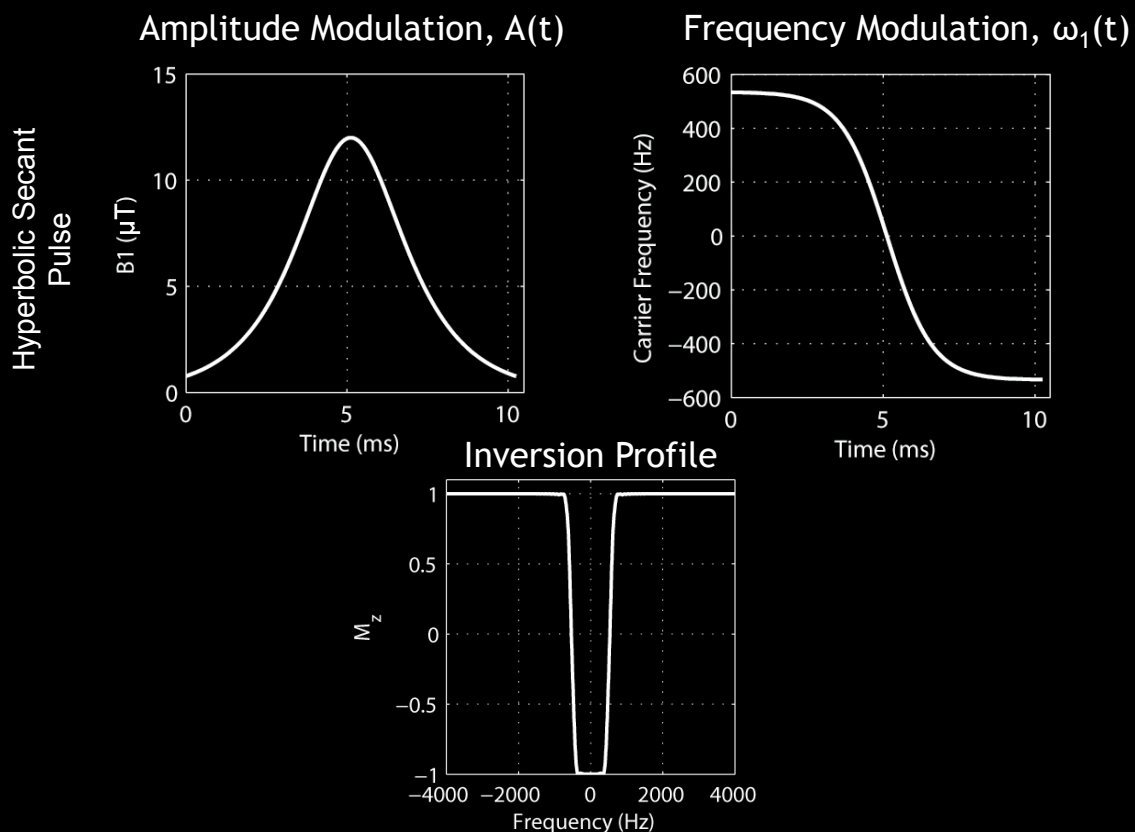
The shape of the inversion profile depends on the choice A(t) and  $\omega_1(t)$ !

$$\text{HSn}^n (n=8) \quad \text{sech}(\beta\tau^n) \quad \int \text{sech}^2(\beta\tau^n) d\tau$$

$$\text{Sin40}^d (n=40) \quad 1 - \left| \sin^n\left(\frac{\pi\tau}{2}\right) \right| \quad \tau - \int \sin^n\left(\frac{\pi\tau}{2}\right) \left(1 + \cos^2\left(\frac{\pi\tau}{2}\right)\right) d\tau$$

Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, vol. 10, p423

## What Will Inversion Profile Look Like?



# Inversion Profiles

- The inversion profile typically calculated using Bloch simulation of the RF pulse (will be covered later) shows us the inversion efficiency and RF bandwidth
- The inversion efficiency depends strongly on the B1 amplitude (as well as pulse duration, T1, T2 and pulse shape)
- For the hyperbolic secant pulse,

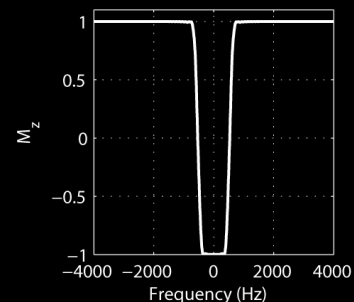
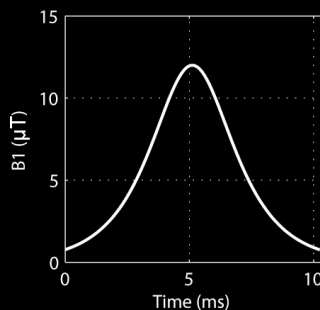
$$\text{RF spectral bandwidth} = \mu\beta$$

$$B_{1\text{max}} \gg (\beta\sqrt{\mu})/\gamma \quad (B_1 \text{ threshold for adiabaticity})$$

## Hyperbolic Secant: Adiabatic Property

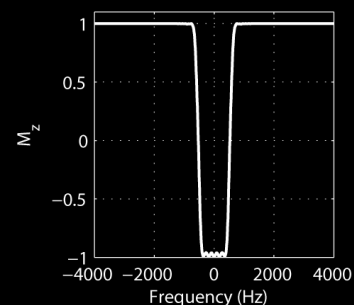
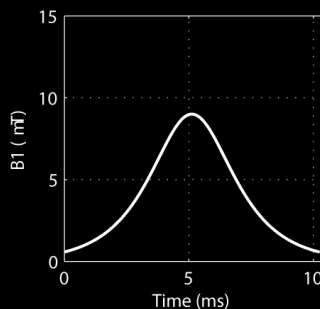
**Original Pulse (100%)**

$$B_{1\text{max}} = 12 \mu\text{T}$$



**75% Attenuated Pulse**

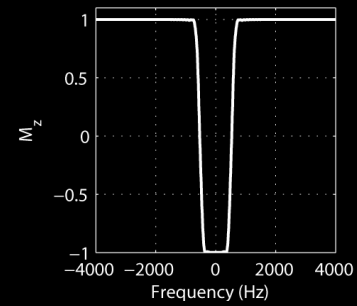
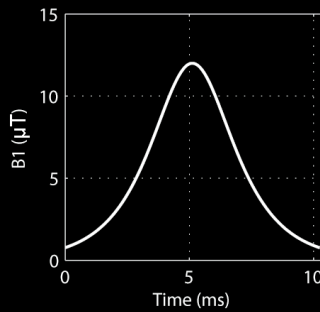
$$B_{1\text{max}} = 9 \mu\text{T}$$



# Hyperbolic Secant: Adiabatic Property

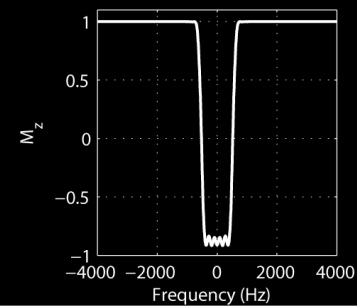
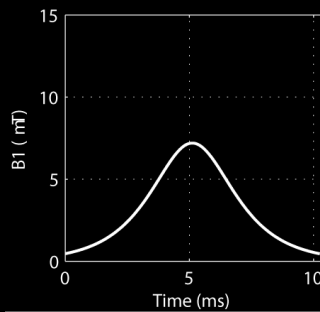
Original Pulse (100%)

$$B1_{\max} = 12 \mu\text{T}$$



60% Attenuated Pulse

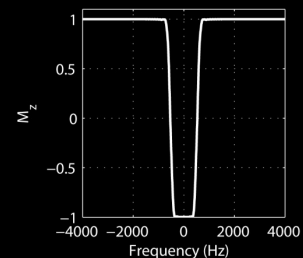
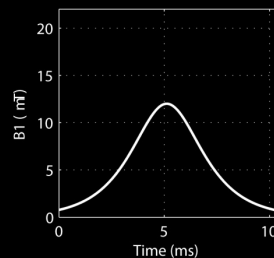
$$B1_{\max} = 7.2 \mu\text{T}$$



B1 Threshold  $\approx 6 \mu\text{T}$

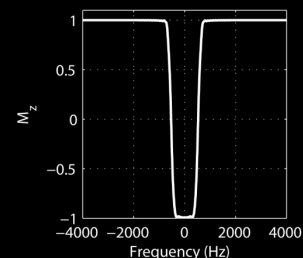
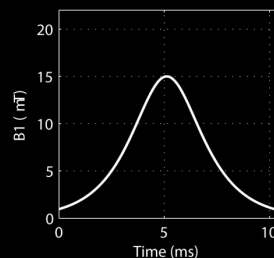
Original Pulse (100%)

$$B1 = 12 \mu\text{T}$$



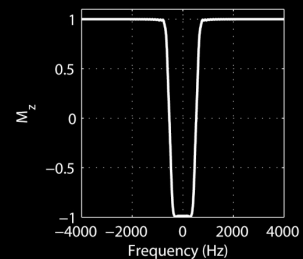
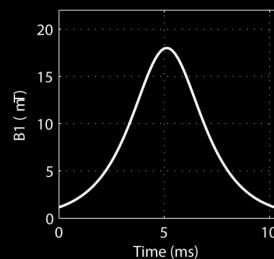
125% Amplified Pulse

$$B1 = 15 \mu\text{T}$$



150% Amplified Pulse

$$B1 = 18 \mu\text{T}$$



## Comments

- Many envelope/modulation functions work
- If a range of adiabaticity is required, optimization can help reduce pulse length
- Hyperbolic Sech needs to be truncated, which can affect the overall performance

## Applications of Adiabatic Pulses

# Adiabatic Pulses

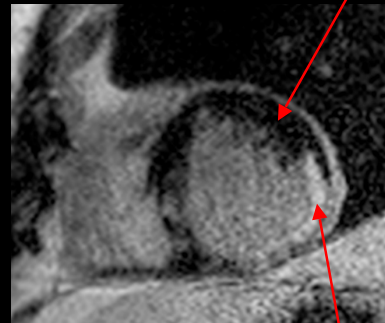
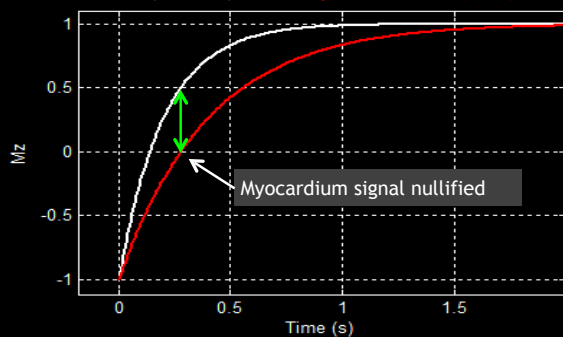
- Fat suppression (STIR)
- CSF suppression (FLAIR)
- Myocardium suppression in cardiac scar imaging (LGE)
- Black blood cardiac imaging (DIR TSE)
- T1 Mapping

# Late Gadolinium Enhancement (LGE)

- Gold standard for detection of scar/myocardial fibrosis
- Spoiled gradient echo (SPGR) sequence with an inversion pulse (inversion recovery SPGR)
  - Inversion pulse is usually hyperbolic secant pulse
  - Healthy myocardium is nulled with the inversion pulse
  - Scar tissue (which has shorter T1 than healthy tissue) appear bright

- The conventional LGE sequence uses an RF-spoiled gradient echo (FLASH) readout with an inversion recovery (IR) pulse as a preparation pulse
- The readout is acquired at a time after inversion at which the healthy myocardium signal reaches zero

Inversion recovery curves of postcontrast scar (white) and myocardium (red)

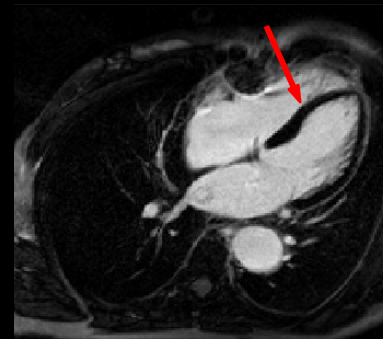
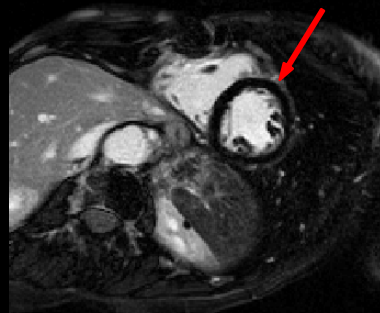


Nullified signal from healthy myocardium

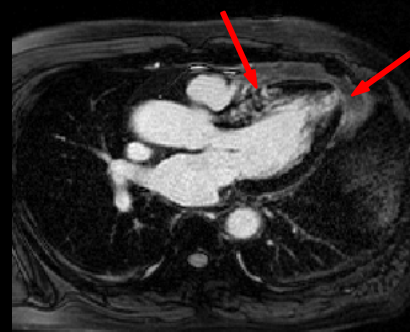
Hyper-enhanced scar region

## Clinical Example

Patient with healthy myocardium



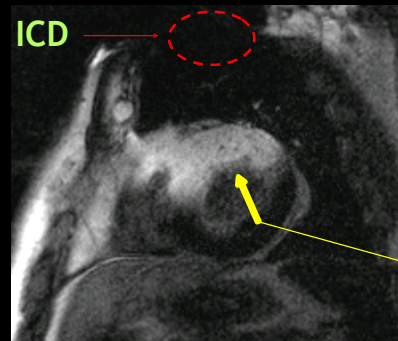
Patient with scar tissue



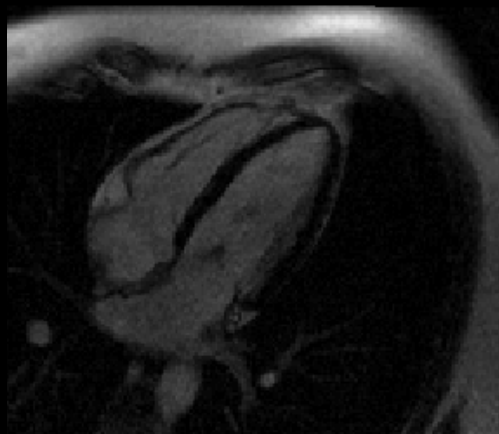
# Clinical Example

Late Gadolinium Enhancement (LGE) in patients with implantable cardiac devices

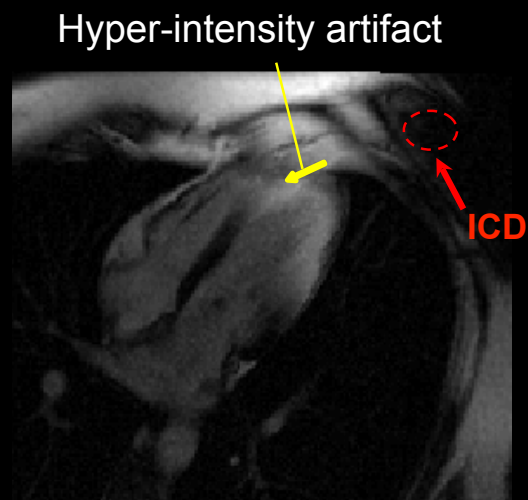
- Presence of an implantable cardiac device in the patients produces an interesting off-resonance artifact



Hyper-intensity Artifacts

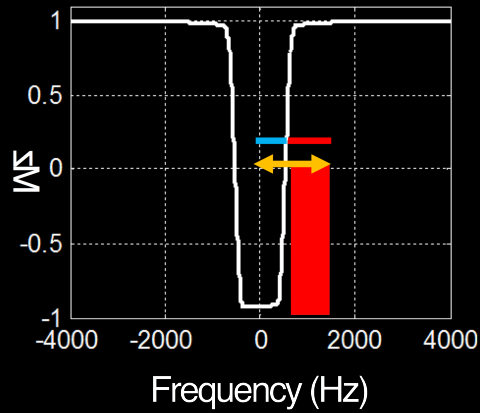


**Conventional IR  
LGE Image**



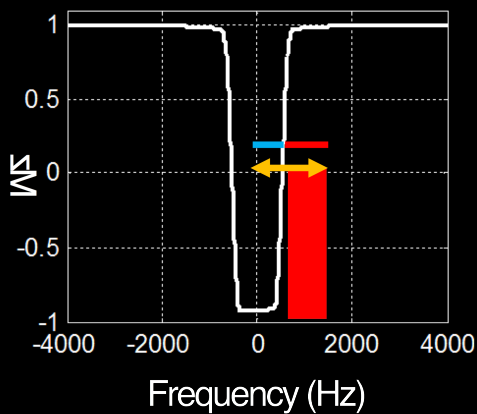
**Conventional IR  
LGE Image**

# Cause of Artifact

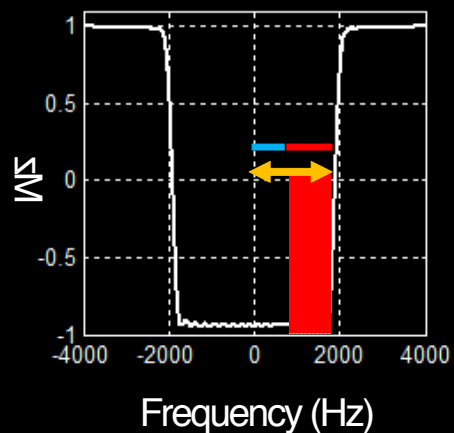


Longitudinal magnetization produced  
by conventional IR pulse  
BW = 1.1 kHz

# Solution: Increase Bandwidth of Inversion Pulse

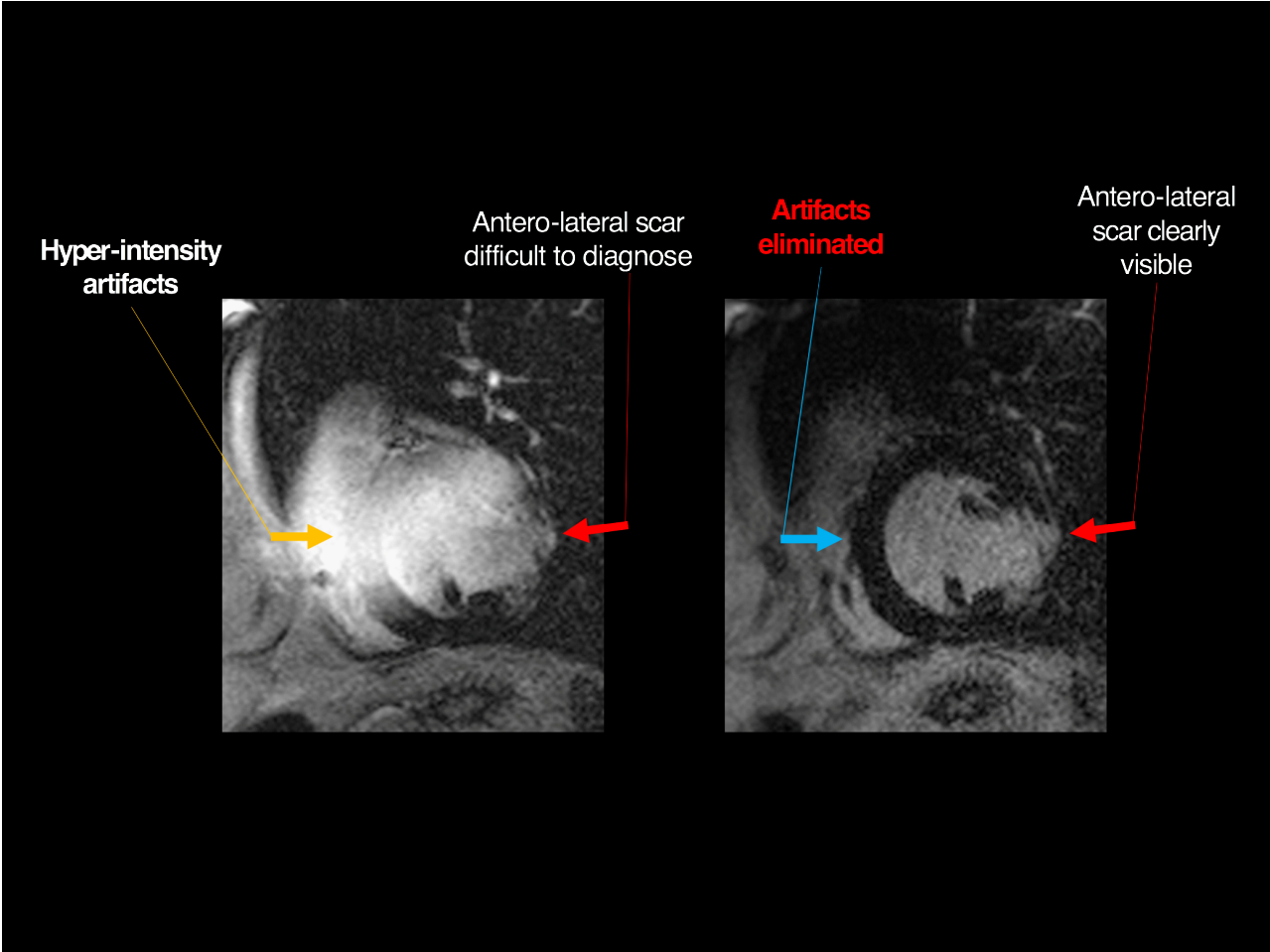
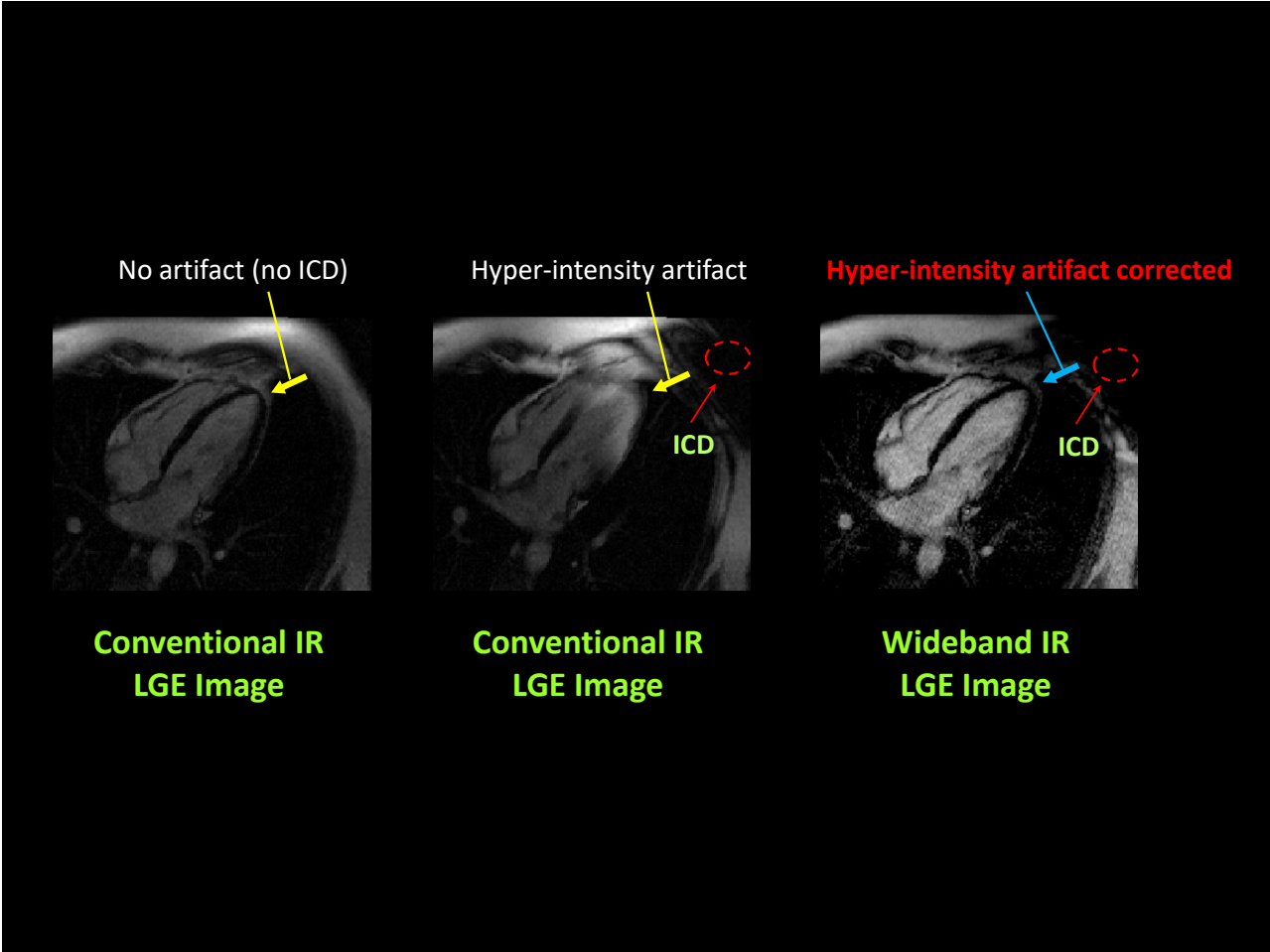


Longitudinal magnetization produced by  
conventional IR pulse  
BW = 1.1 kHz



Longitudinal magnetization produced  
by wideband IR pulse  
BW = 3.8 kHz





# Thank You!

- Further reading
  - Read "Adiabatic Refocusing Pulses" p.200-212
  - Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, Vol. 10, 423-434 (1997)
- Acknowledgments
  - John Pauly's EE469b (RF Pulse Design for MRI)
  - Shams Rashid, Ph.D.

Kyung Sung, PhD  
ksung@mednet.ucla.edu  
<https://mrrl.ucla.edu/sunglab/>