Class Business

- Homework 1 due 4/26
- Homework 2 due 5/3
- Final project
  - start thinking
  - come to office hours
  - discussion on 4/23 Tue
  - proposal due 5/10 (not graded)
Outline

• Multi-Pulse Experiments
• Extended Phase Graphs (EPG)

• EPG Simulations
  - Homework 2

• Spin Bench Demo
Multi-Pulse Experiments

• Multiple RF pulses
  - always have echoes (many types)
  - do not need perfect $90^\circ + 180^\circ$ to form SE, etc.

• Analysis
  - Bloch Equations
  - Extended Phase Graphs (EPG)
Spin Echo (2 pulses)
Stimulated Echo (3 pulses)

- $90^\circ_x$
- FP, $T_2$
- FP, $T_2$
- $90^\circ_y$
- FP, $T_2$
- $T_1$
Multiple Pulse Experiments

Scheffler, Concepts in MR 1999
Multiple Pulse Experiments

Scheffler, Concepts in MR 1999
Multiple Pulse Experiments

Scheffler, Concepts in MR 1999
Multiple Pulse Experiments

- RF pulses act on an ensemble of spins
  - $M_z$ to $M_{xy}$
  - $M_{xy}$ to $M_z$, $M_{xy}$ and $M_{xy}^*$

- Transverse $F$ states
  - $F = M_x + iM_y = F_{pos}$; $F^* = M_x - iM_y = F_{neg}$

- Longitudinal $Z$ states
Multiple Pulse Experiments

Signal Pathways on a Phase Diagram (i.e. EPG)

Z states appear as broken lines; $F_0$ states are echoes

Scheffler, Concepts in MR 1999
Extended Phase Graphs

- MR signal is a sum of all dephased spins
- Bloch equation
  - tracks evolution of magnetization for each spin
  - exact, but hard to visualize intuitively
- EPG
  - considers groups of spins under constant gradients
  - decomposes the spin system into several dephased states: $F_k$ and $F_{-k}; Z_k$

Hennig, JMR 1988; 78:397-407
Extended Phase Graphs

- Based on Fourier space coordinate $k$

$$k_n(t) = \gamma \int_{t'=0}^{t} G_n(t')dt' = \int_{t'=0}^{t} g_n(t')dt',$$

- Magnetization represented by Fourier transforms

$$F_+(k) = \int_{V} \{M_x(r) + iM_y(r)\} \exp(-ikr)d^3r,$$

$$F_-(k) = \int_{V} \{M_x(r) - iM_y(r)\} \exp(-ikr)d^3r,$$

$$Z(k) = \int_{V} M_z(r) \exp(-ikr)d^3r,$$

- Complete magnetization is described by vector $F$ of various EPG partitions states with different $k$

$$F = (F_0Z_0F_1F_{-1}Z_1F_2F_{-2}Z_2 \cdots F_{+k}F_{-k}Z_k)^T.$$
Gradient Dephasing

Gradient Dephasing

Gradient Dephasing

Gradient $G$
(z-direction presumed)

\[ \tilde{F}_+(k_1), \tilde{F}_+(k_2), \tilde{F}_+(k_3), \tilde{F}_+(k_4) \]

$slope \sim gradient G$

\[ G \]

Time $t$
Gradient Dephasing

Brian Hargreaves and Karla Miller ISMRM 2013: Educational E-Poster #3718
“Discrete” Gradient Dephasing

$k$ is the number of twists/cycles across a voxel

Brian Hargreaves and Karla Miller ISMRM 2013: Educational E-Poster #3718
RF Pulse

- Woessner Decomposition magnetization after an RF pulse can be regarded as a composition of 3 components:
  - transversal component that is unaffected (0°-pulse)
  - transversal component that is refocused (180°-pulse)
  - a longitudinal component

RF Pulse

The RF pulse operator splits any given EPG state with dephasing order $k$ into 3 different new states:
- a transversal state with identical $k$
- a transversal state with inverted $k$
- a longitudinal state with identical $k$
mixes $F$ and $Z$ states!

Brian Hargreaves and Karla Miller ISMRM 2013: Educational E-Poster #3718
RF Pulse

EPG Concept Summary

Fourier based configuration states

Phase graph approach that depicts the evolution of a complete isochromat ensemble.

RF pulse partitioning

\[
M_{xy}(z) = \sum_{n=-N}^{N} F_n e^{-i2\pi nz}
\]

\[
M_z(z) = \text{Real} \left\{ \sum_{n=0}^{N} Z_n e^{-i2\pi mz} \right\}
\]
EPG "Calculus"

• RF pulse for state $k$:
  - Produces signal in longitudinal state $k$ and transverse states $k$ and $-k$

• Gradient dephaser for state $k$:
  - Moves transverse magnetization to $k+1$
  - Does not affect longitudinal magnetization
EPG: Spin Echo

- Rotation angles: 90°, 180°
- Time points: T₁, T₂
- Frequency points: F₀, F₁, F₂, F₋₁
- Echo points: Z₁
- Sequence: SE
EPG: Stimulated Echo
EPG: 3-Pulse Experiment

EPG: Train of Spin Echo
EPG: CPMG

\( F_0 = \) observable signal ("Echo")
EPG: Matrix formulation

- Phase states
  - Can represent as a matrix:

\[
P = \begin{bmatrix}
F_0 & F_1 & F_2 \\
F_0^* & F_{-1} & F_{-2} \\
Z_0 & Z_1 & Z_2 \\
\end{bmatrix}
\]
EPG: Matrix formulation

• RF pulses
  - invert state (e.g., $F_3$ to $F_{-3}$) or can transfer between $F$ and $Z$ states
  - Simple pre-multiplication $P' = RP$, where $R$ is

$$
\begin{pmatrix}
\cos^2 \frac{\alpha}{2} & e^{2i\Phi} \sin^2 \frac{\alpha}{2} & -ie^{i\Phi} \sin \alpha \\
\frac{e^{-2i\Phi} \sin^2 \frac{\alpha}{2}}{2} & \frac{\cos^2 \frac{\alpha}{2}}{2} & ie^{-i\Phi} \sin \alpha \\
\frac{-ie^{-i\Phi} \sin \alpha}{2} & \frac{i e^{i\Phi} \sin \alpha}{2} & \cos \alpha
\end{pmatrix}
$$

for an RF pulse with flip angle $\alpha$ and phase $\phi$

Scheffler, Concepts in MR 1999
EPG: Matrix formulation

- Gradients (in discretized units)
  - Increase number of states by 1
  - Replace all $F_k$ states with $F_{k-1}$
    (e.g., $F_0$ becomes $F_1$)
  - Replace $F_0$ using $F_0^*$
  - Do not change $Z$ states

# phase states grow linearly w.r.t. TSE ETL
EPG: Matrix formulation

- Relaxation
  - Transverse:
    All \( F \) states attenuated by \( E_2 = \exp(-T/T_2) \)
  - Longitudinal:
    All \( Z \) states attenuated by \( E_1 = \exp(-T/T_1) \)
    \( Z_0 \) state only has recovery of \( M_0(1-E_1) \)
EPG: Extensions

- Non-ideal slice profiles
- Variable RF flip angle and phase
- Motion / flow effects
- Diffusion effects
  - Weigel M, et al., JMR 2010; 205: 276-285
EPG Simulation

- Phase state propagation
  - RF pulse
  - $T_1$, $T_2$ decay
  - free precession
  - gradient pulse
Phase states:

\[ P = \begin{bmatrix}
F_0 & F_1 & F_2 & \cdots \\
F_0^* & F_{-1} & F_{-2} & \cdots \\
Z_0 & Z_1 & Z_2 & \cdots 
\end{bmatrix} \]

RF pulse \((\theta, \phi)\), \(P^+ = RP\):

\[
R\{\theta,\phi\} = \begin{bmatrix}
\cos^2 \frac{\theta}{2} & e^{2i\phi} \sin^2 \frac{\theta}{2} & -ie^{i\phi} \sin \theta \\
e^{-2i\phi} \sin^2 \frac{\theta}{2} & \cos^2 \frac{\theta}{2} & ie^{-i\phi} \sin \theta \\
-i \frac{1}{2} e^{-i\phi} \sin \theta & i \frac{1}{2} e^{i\phi} \sin \theta & \cos \theta 
\end{bmatrix}
\]
EPG Simulation

Gradients:

\[ P = \begin{bmatrix} F_0 & F_1 & F_2 & \cdots \\ F_0 \times & F_{-1} & F_{-2} & \cdots \\ Z_0 & Z_1 & Z_2 & \cdots \end{bmatrix} \]

Relaxation:

\[ F_k \rightarrow E_2 \, F_k \]

\[ Z_k \rightarrow E_1 \, Z_k \quad (k>0) \]

\[ Z_0 \rightarrow E_1 \, Z_0 + M_0(1 - E_1) \]
EPG Simulation

- Transient state; steady state
- Different seq/tissue params

- Brian’s MATLAB EPG sim code
  - will be emailed to class mailing list
EPG Simulation

- **Example**: Turbo Spin Echo
  - epg_rf.m
  - epg_grelax.m, epg_grad.m, epg_mgrad.m
  - epg_cpmg_hhw.m
  - EPGSim_CPMG_hhw.m

- can look at different refocusing RF trains
EPG Simulations: FSE

- non-CPMG 180s: 90x-180x-180x-…
- CPMG 180s: 90x-180y-180y-…
- non-CPMG 120s: 90x-120x-120x-…
- CPMG 120s: 90x-120y-120y-…
- CPMG 120s +prep: 90x-150y-120y-…
$T_1 = 1000 \text{ ms}, \ T_2 = 100 \text{ ms}, \ \text{ETL} = 50, \ \text{ESP} = 10 \text{ ms}$
$T_1 = 1000 \text{ ms}, \; T_2 = 100 \text{ ms}, \; \text{ETL} = 50, \; \text{ESP} = 10 \text{ ms}$
$F_0$ vs. echo number

$T_1 = 1000 \text{ ms, } T_2 = 100 \text{ ms, ETL = 50, ESP = 10 ms}$
EPG Simulation

- Homework 2, part 2A
  - Gradient-spoiled GRE (SSFP-FID)
EPG Simulation

- Homework 2, part 2B
  - RF-spoiled GRE

*Scheffler, Concepts in MR 1999, Fig. 11*
Homework 2

- Pulse Sequence Simulations
  - 1. Bloch: Steady state comparison, bSSFP transient state and catalyzation
  - 2. EPG: SSFP-FID, RF-spoiled GRE

- Due 5 pm, Fri, 5/3 by email
  - PDF and MATLAB code
Summary

- Multiple RF pulses -> multiple echoes
- EPG analysis
  - consider groups of spins
  - explicit treatment of pathways and echoes
  - flexible and powerful
  - you can do it!
Spin Bench Demo

- bSSFP and other examples
  - phase cycling, …
Thanks!

- **Web resources**
  - ISMRM 2010 Edu: Miller, Weigel
  - ISMRM 2011 Edu: Miller, Weigel

- **Further reading**
  - Bernstein et al., Handbook of MRI Sequences
  - Haacke et al., Magnetic Resonance Imaging
  - Hennig, JMR 1988; 78:397-407
  - Weigel, JMRI 2015; 41:266-295
Thanks!

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Holden H. Wu, Ph.D.

HoldenWu@mednet.ucla.edu
http://mrrl.ucla.edu/wulab