* Geometry Factor

$$SNR_{FI,p} = \frac{S}{\sqrt{\chi_{FI}}}$$

$$SNR_{R,p} = \frac{S}{\sqrt{[\chi_R]_{p,p}}}$$

$$\frac{SNR_{R,p}}{SNR_{FI,p}} = \sqrt{\frac{\chi_{FI}}{[\chi_R]_{p,p}}} = \sqrt{\frac{\frac{1}{n_{FI}}\left(C_{FI}^{*}\,\Psi^{-1}C_{FI}\right)^{-1}}{\frac{1}{n_R}\left[\left(C_R^{*}\,\Psi^{-1}C_R\right)^{-1}\right]_{p,p}}}$$

$$= \frac{1}{\sqrt{\frac{n_{FI}}{n_R}}\cdot\underbrace{\sqrt{\left(C_{FI}^{*}\,\Psi^{-1}C_{FI}\right)\left[\left(C_R^{*}\,\Psi^{-1}C_R\right)^{-1}\right]_{p,p}}}}$$

$\sqrt{\frac{n_{FI}}{n_R}} \rightarrow R$; underbrace $\rightarrow g$

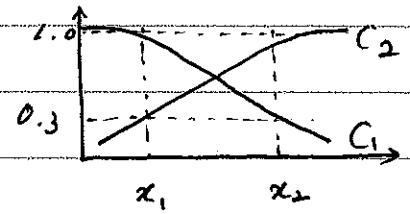
$$\therefore\ g_p = \sqrt{\left(C_{FI}^{*}\,\Psi^{-1}C_{FI}\right)\left[\left(C_R^{*}\,\Psi^{-1}C_R\right)^{-1}\right]_{p,p}}$$

often written as

$$g_p = \sqrt{\left[\left(C^{*}\,\Psi^{-1}C\right)\right]_{p,p}\left[\left(C^{*}\,\Psi^{-1}C\right)^{-1}\right]_{p,p}}$$

where $C = C_R$ & $C_{FI}^{*}\,\Psi^{-1}C_{FI} = \left[C^{*}\,\Psi C\right]_{p,p}$

Ex 1 Good coil sensitivity ($\Psi = I$)



$C_1(x_1) = 1.0$

$C_1(x_2) = 0.3$

$C_2(x_1) = 0.3$

$C_2(x_2) = 1.0$

at x_1

$$C_{FI} = \begin{pmatrix} 1 \\ 0.3 \end{pmatrix}$$

$$C_R = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}$$

$$g = \sqrt{(C_{FI}^* C_{FI})\left[(C_R^* C_R)^{-1}\right]_{1,1}}$$

$$C_{FI}^* C_{FI} = \begin{pmatrix} 1 & 0.3 \end{pmatrix} \begin{pmatrix} 1 \\ 0.3 \end{pmatrix} = 1.09$$

$$C_R^* C_R = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix} = \begin{pmatrix} 1.09 & 0.6 \\ 0.6 & 1.09 \end{pmatrix}$$

$$(C_R^* C_R)^{-1} = \begin{pmatrix} 1.316 & -0.724 \\ -0.724 & 1.316 \end{pmatrix}$$

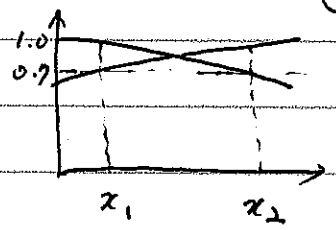
$$g = \sqrt{(1.09)(1.316)} = 1.20$$

20% loss

multi coil loss — condition loss

$$\frac{SNR_R}{SNR_{FI}} = \frac{1}{\sqrt{2} \cdot (1.2)} = 0.59 \qquad 40\% \text{ loss}$$

Ex 2 Good Array Coil



$C_1(x_1) = 1$

$C_1(x_2) = 0.7$

$C_2(x_1) = 0.7$

$C_2(x_2) = 1$

$$C_H = \begin{pmatrix} 1 \\ 0.7 \end{pmatrix} \qquad C_R = \begin{pmatrix} 1 & 0.7 \\ 0.7 & 1 \end{pmatrix}$$

$$C_H^* C_H = 1.49$$

$$C_R^* C_R = \begin{pmatrix} 1.49 & 1.4 \\ 1.4 & 1.49 \end{pmatrix}$$

$$(C_R^* C_R)^{-1} = \begin{pmatrix} 5.72 & -5.38 \\ -5.38 & 5.72 \end{pmatrix}$$

$$g = \sqrt{(1.49)(5.72)} = 2.92$$

$$\frac{SNR_R}{SNR_H} = \frac{1}{\sqrt{2}\ 2.92} = 0.24 \qquad 75\% \text{ loss}$$