Factor Analysis

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N208, Factor 5-255 (3-6pm)

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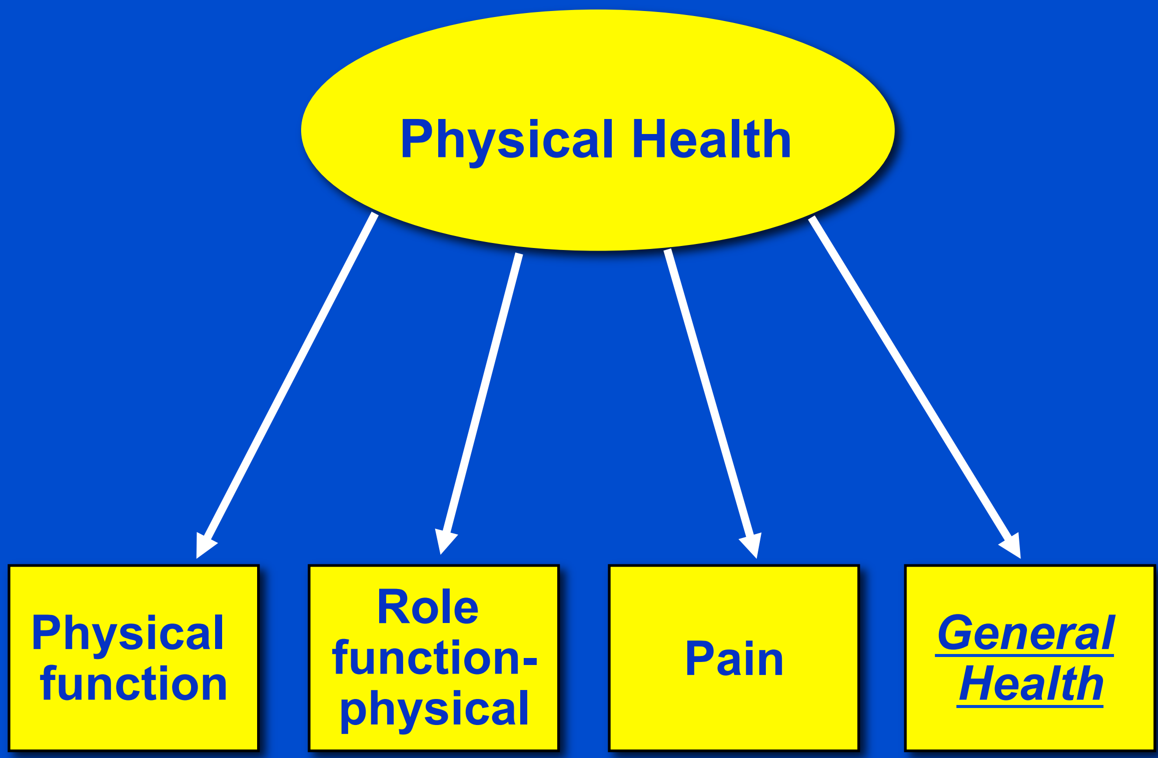
Issues to be discussed

- SF-36 Factor Analysis
  - Equivalence by subgroup
  - Orthogonal or Oblique model
- Exploratory Factor Analysis
- Confirmatory Analysis
  - Factor
  - Multitrait Scaling

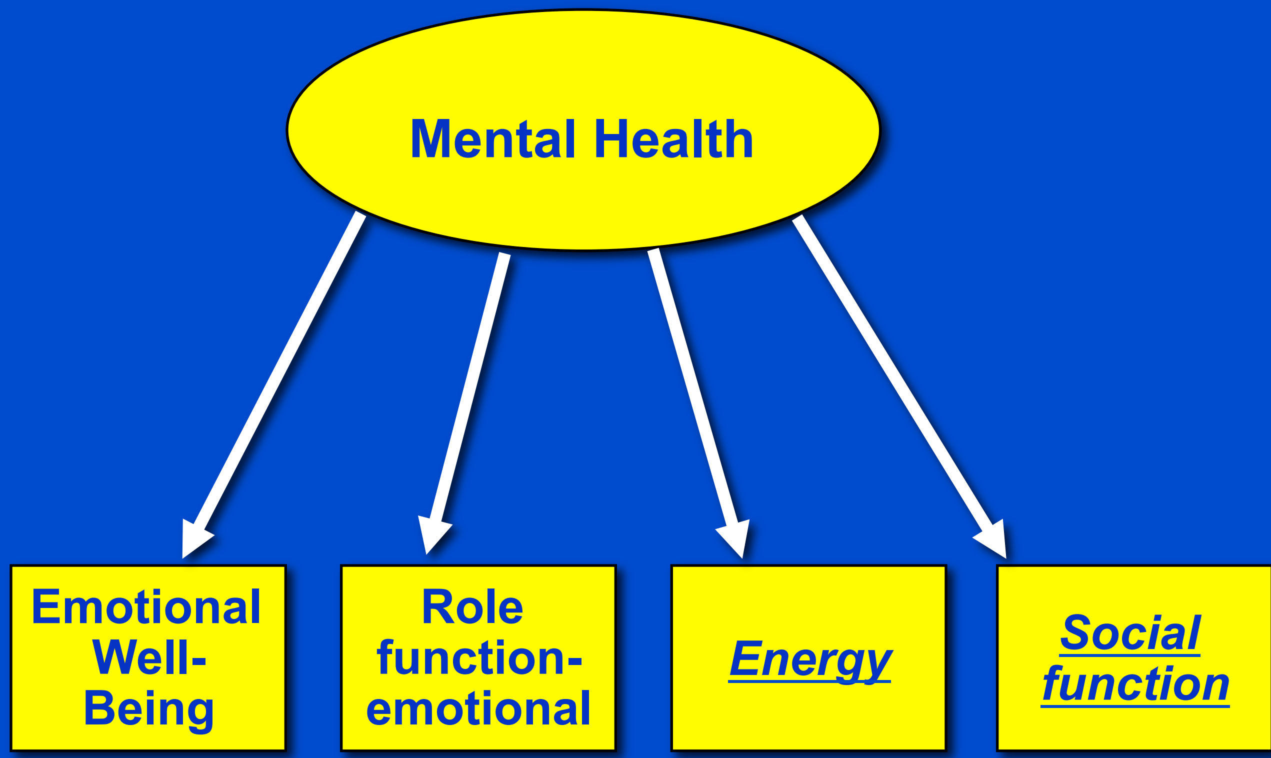
Generic HRQOL: 8 SF-36 Scales

- Physical functioning
- Role limitations/physical
- Pain
- General health perceptions
- Social functioning
- Energy/fatigue
- Role limitations/emotional
- Emotional well-being

Physical Health



Mental Health



Correlations among SF-36 Scales

	PF	RP	P	GH	EW	RE	E
PF	1.00						
RP	*0.54*	1.00					
P	*0.47*	*0.60*	1.00				
GH	*0.49*	*0.53*	*0.53*	1.00			
EW	0.20	0.28	0.35	0.44	1.00		
RE	0.26	0.41	0.32	0.35	*0.53*	1.00	
E	0.38	*0.50*	*0.52*	*0.61*	*0.61*	0.44	1.00
SF	0.37	*0.49*	*0.46*	0.42	0.45	0.44	*0.48*

Larger Correlations

0.61 Energy and General Health/Emotional well-being

0.60 Role-Physical and Pain

0.54 Physical Function and Role-Physical

0.53 General Health and Role-Physical/Pain

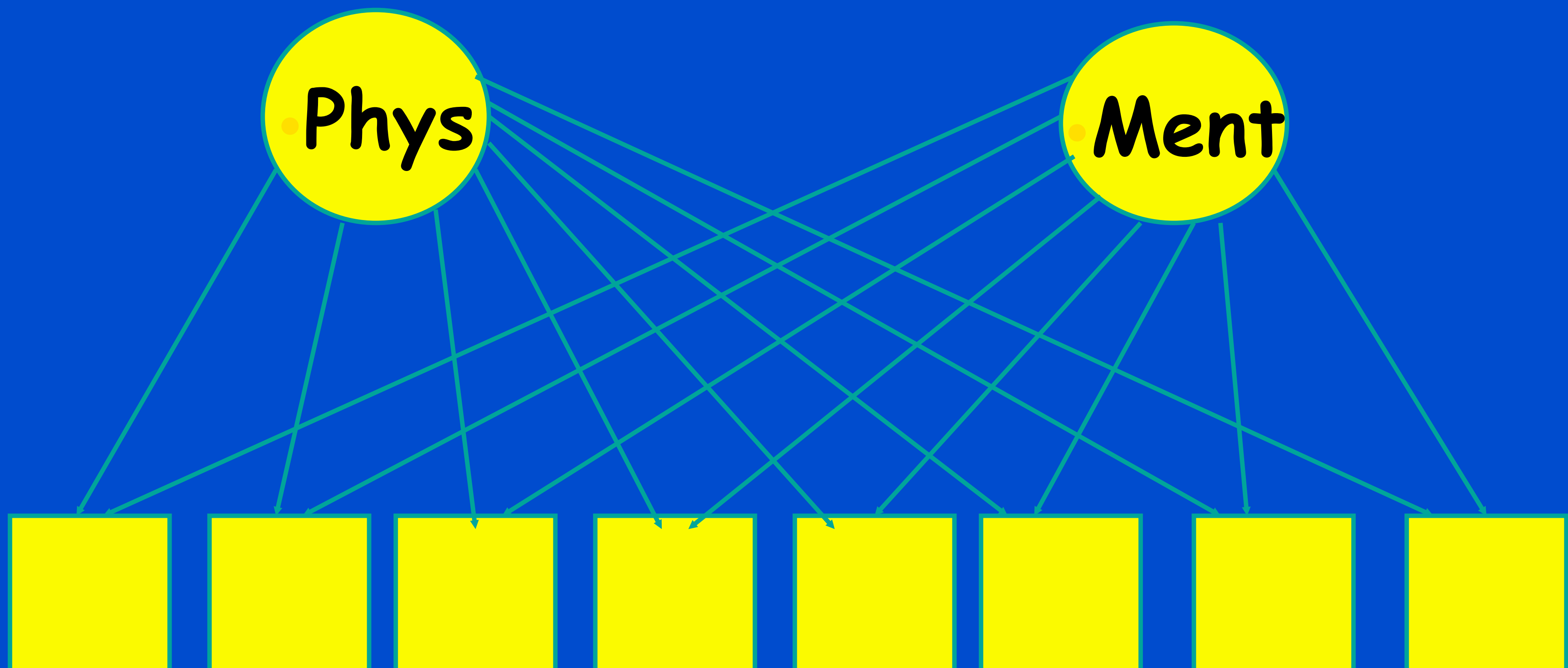
0.52 Energy and Pain

0.50 Energy and Role—Physical

0.49 General Health and Physical Function; Role-Physical and Social Functioning

SF-36 Factor Analysis in United States

	United States	
	Physical	Mental
PF	0.85	0.12
RP	0.81	0.27
BP	0.76	0.28
GH	0.69	0.37
VT	0.47	0.64
SF	0.42	0.67
RE	0.17	0.78
MH	0.17	0.87

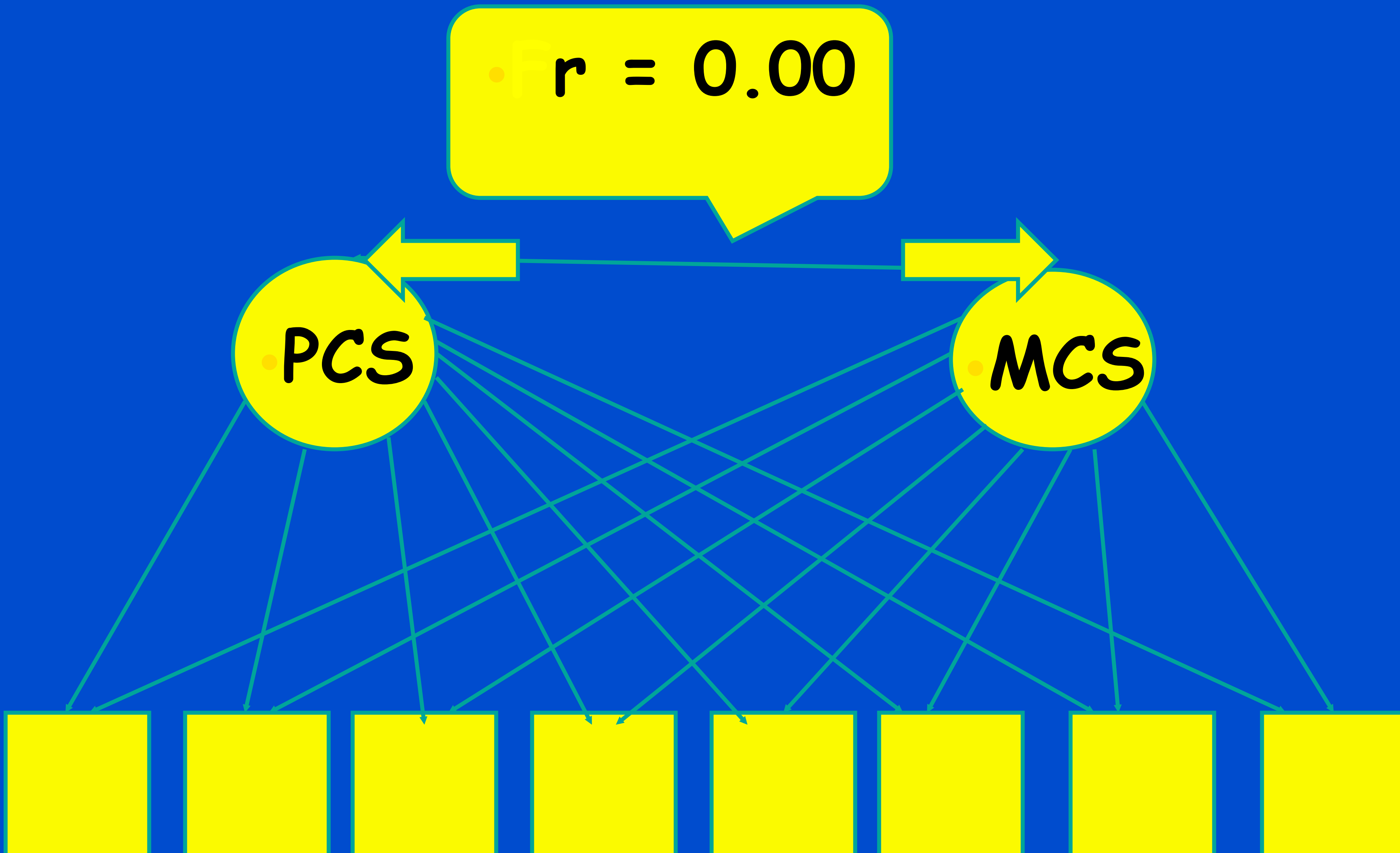

Phys
Ment

SF-36 Factor Analysis in US

	English		Spanish		United States	
	Physical	Mental	Physical	Mental	Physical	Mental
PF	0.69	---	**0.25**	---	0.85	**0.12**
RP	0.81	---	0.53	---	0.81	**0.27**
BP	0.69	**0.12**	0.88	**0.02 &**	0.76	**0.28**
GH	0.56	**0.28**	**0.39**	**0.34**	0.69	**0.37**
VT	**0.38**	0.56	**0.22**	0.69	0.47	0.64
SF	0.41	**0.35**	0.45	**0.36**	0.42	0.67
RE	---	0.62	---	0.44	**0.17**	0.78
MH	---	0.86	---	0.93	**0.17**	0.87

SF-36 Factor Analysis in Singapore vs. US

	English		Chinese		United States	
	Physical	Mental	Physical	Mental	Physical	Mental
PF	0.60	**0.14**	0.75	**0.03**	0.85	**0.12**
RP	0.85	**0.12**	0.78	**0.25**	0.81	**0.27**
BP	0.46	0.53	0.53	0.51	0.76	**0.28**
GH	**0.14**	0.74	**0.32**	0.66	0.69	**0.37**
VT	**0.15**	0.84	**0.16**	0.83	0.47	0.64
SF	0.49	0.56	0.48	0.56	0.42	0.67
RE	0.77	**0.18**	0.62	**0.36**	**0.17**	0.78
MH	**0.12**	0.83	**0.10**	0.86	**0.17**	0.87



r = 0.00
PCS
MCS

SF-36 PCS and MCS

PCS_z = (PF_z * .42402) + (RP_z * .35119) + (BP_z * .31754) + (GH_z * .24954) + (EF_z * .02877) + (SF_z * -.00753) + (RE_z * -.19206) + (EW_z * -.22069)

MCS_z = (PF_z * -.22999) + (RP_z * -.12329) + (BP_z * -.09731) + (GH_z * -.01571) + (EF_z * .23534) + (SF_z * .26876) + (RE_z * .43407) + (EW_z * .48581)

T-score Transformation

$Z_X = (X - \text{x-bar}) / SD_x$

PCS = (PCS_z*10) + 50

MCS = (MCS_z*10) + 50

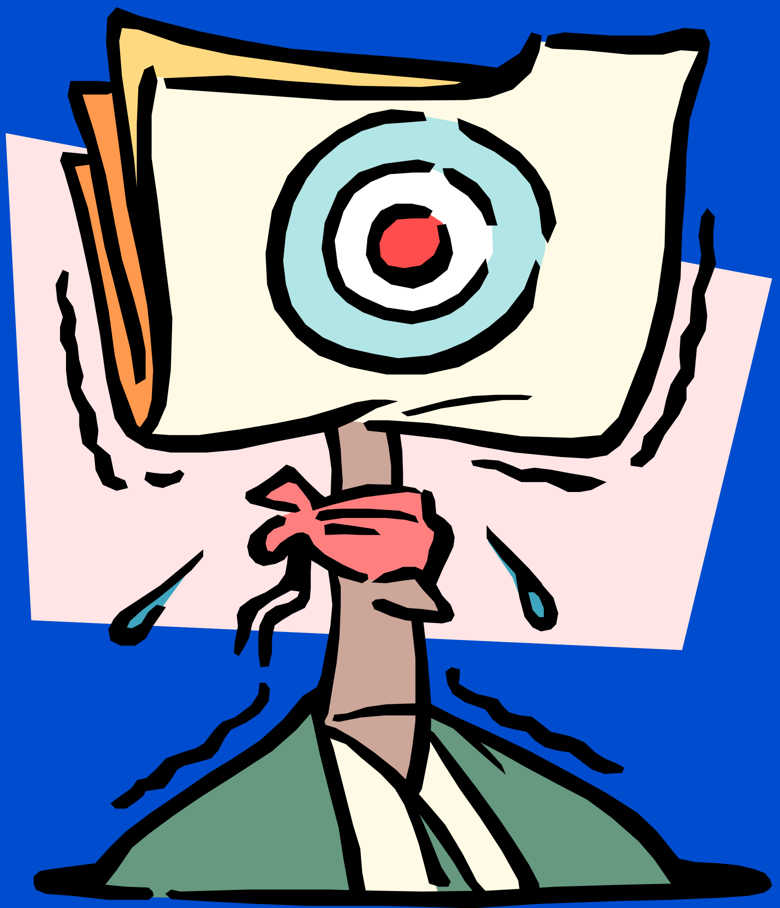
Debate About Summary Scores

- Taft, C., Karlsson, J., & Sullivan, M. (2001). Do SF-36 component score accurately summarize subscale scores? Quality of Life Research, 10, 395-404.
- Ware, J. E., & Kosinski, M. (2001). Interpreting SF-36 summary health measures: A response. Quality of Life Research, 10, 405-413.
- Taft, C., Karlsson, J., & Sullivan, M. (2001). Reply to Drs Ware and Kosinski. Quality of Life Research, 10, 415-420.

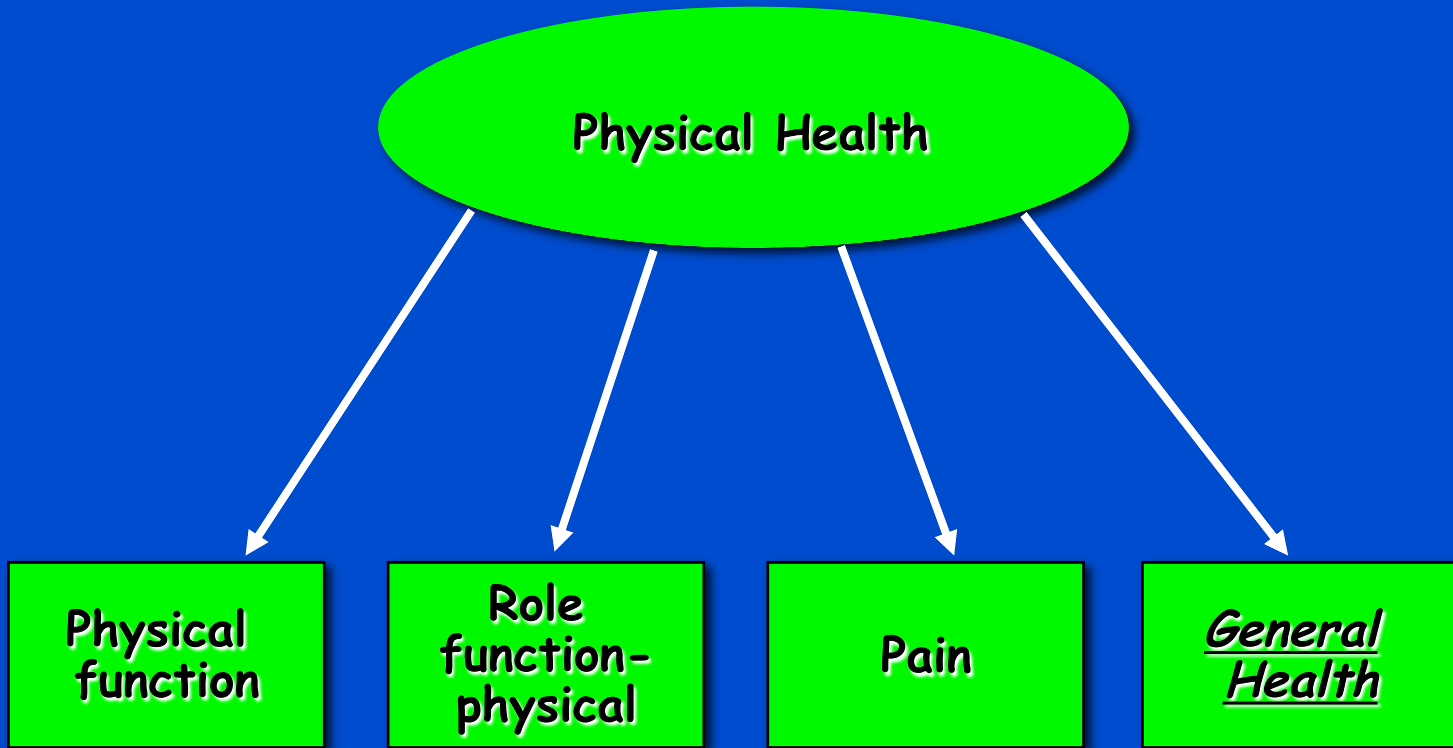
536 Primary Care Patients Initiating Antidepressant Tx

- 3-month improvements in physical functioning, role—physical, pain, and general health perceptions ranging from 0.28 to 0.49 SDs.
- Yet SF-36 PCS did not improve.

- *Simon et al. (Med Care, 1998)*



Four scales improve 0.28-0.49 SD, but physical health summary score doesn't change

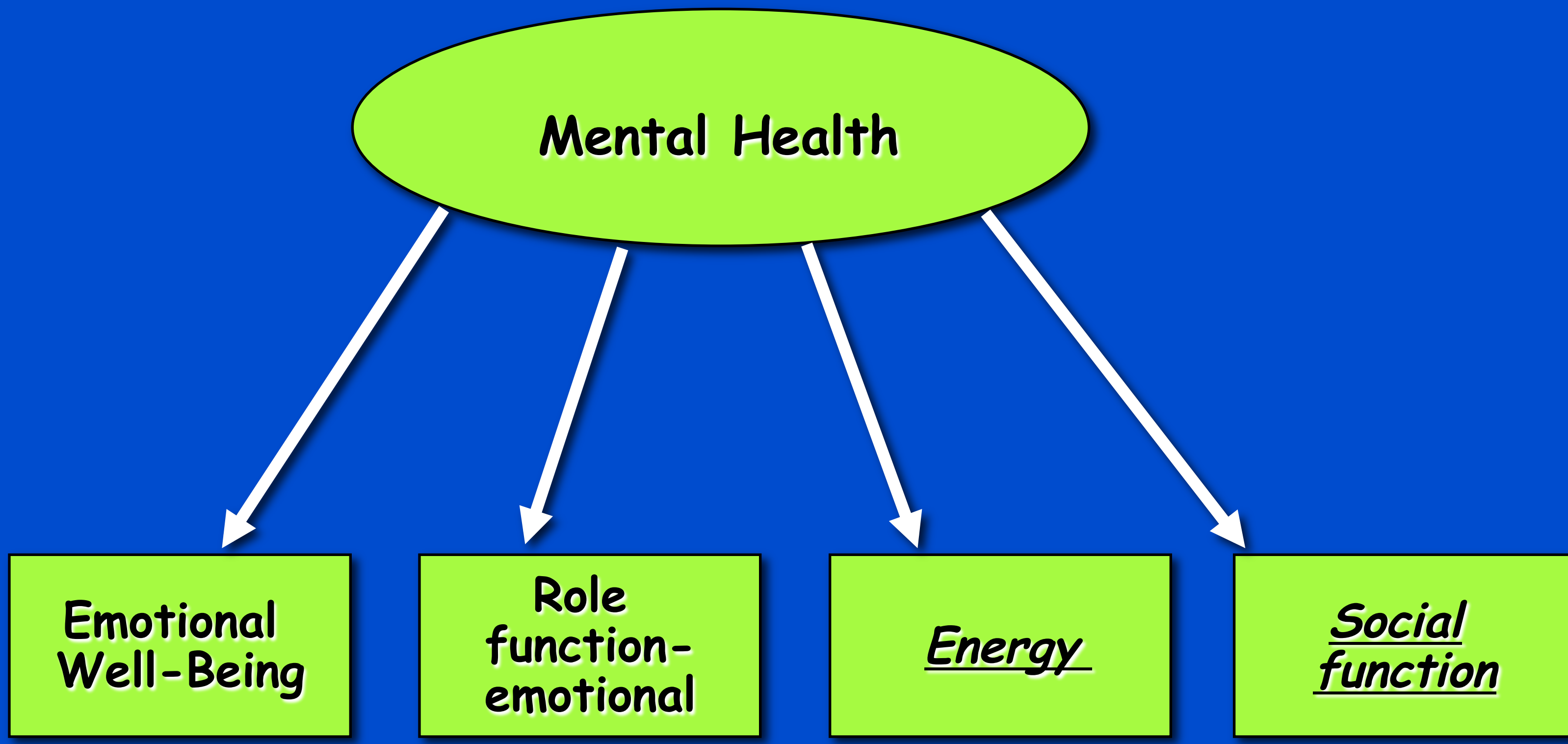


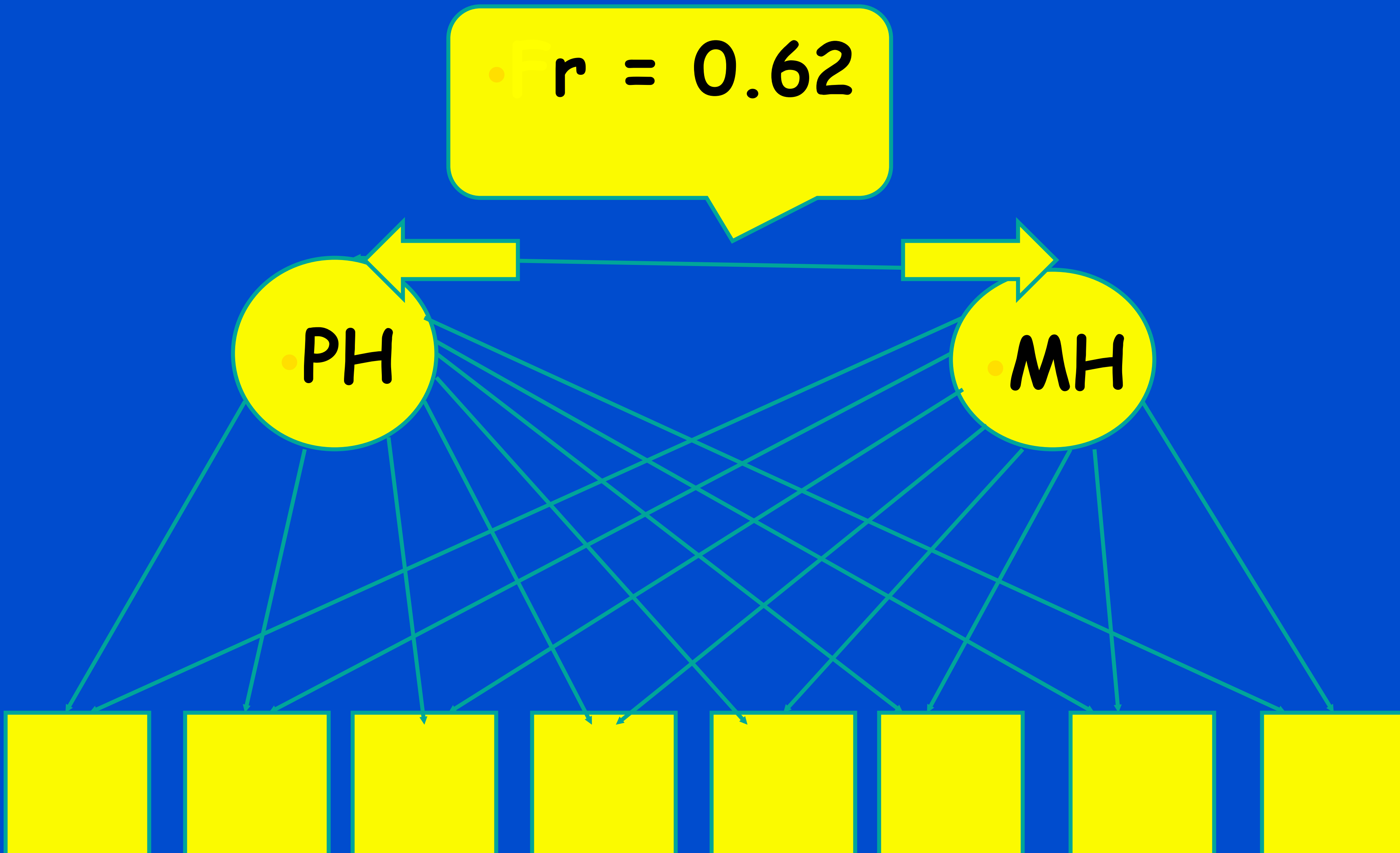
n = 194 with Multiple Sclerosis

- Lower scores than general population on
  - Emotional well-being (↓ 0.3 SD)
  - Role—emotional (↓ 0.7 SD)
  - Energy (↓1.0 SD)
  - Social functioning (↓1.0 SD)
- Yet SF-36 MCS was only 0.2 SD lower.
- RAND-36 mental health was 0.9 SD lower.

Nortvedt et al. (Med Care, 2000)

Four scales 0.3-1.0 SD lower, but mental health summary score only 0.2 SD lower




r = 0.62
PH
MH

Alternative Weights for SF-36 PCS and MCS

PCS_z = (PF_z * .20) + (RP_z * .31) + (BP_z * .23) + (GH_z * .20) + (EF_z * .13) + (SF_z * .11) + (RE_z * .03) + (EW_z * -.03)

MCS_z = (PF_z * -.02) + (RP_z * .03) + (BP_z * .04) + (GH_z * .10) + (EF_z * .29) + (SF_z * .14) + (RE_z * .20) + (EW_z * .35)

Farivar, S. S., & Hays, R.D. (2004, November). Constructing correlated physical and mental health summary scores for the SF-36 health survey. Paper presented at the annual meeting of the International Society for Quality of Life Research, Hong Kong. (<u>Quality of Life Research</u>, 13 (9), 1550).

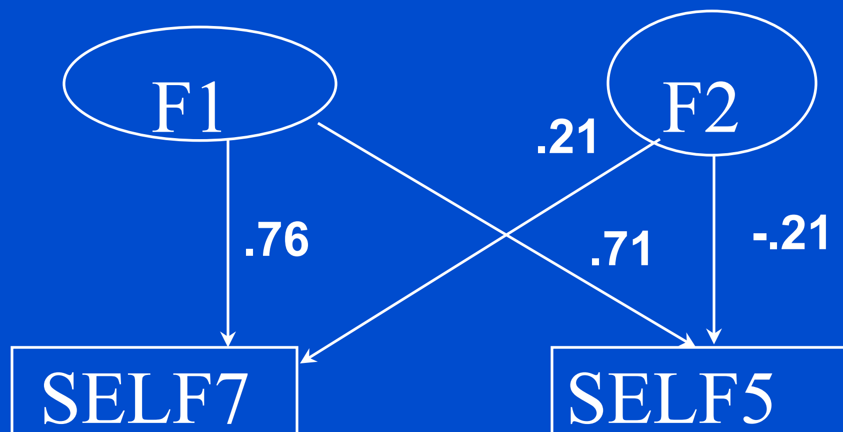
What is Factor Analysis Doing?

observed r = 0.446

reproduced r = 0.75570 (0.71255) + 0.21195(-.2077)

= 0.538474 - 0.0440241 = 0.494

residual = 0.446 - 0.494 = -.048



Rosenberg's Self-Esteem Scale - Microsoft Internet Explorer

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Address http://www.wwnorton.com/psychsci/media/rosenberg.htm

>> Rosenberg's Self-Esteem Scale

STATEMENT	Strongly Agree	Agree	Disagree	Strongly Disagree
1. I feel that I am a person of worth, at least on an equal plane with others.				
2. I feel that I have a number of good qualities..				
3. All in all, I am inclined to feel that I am a failure.				
4. I am able to do things as well as most other people.				
5. I feel I do not have much to be proud of.				
6. I take a positive attitude toward myself.				
7. On the whole, I am satisfied with myself.				
8. I wish I could have more respect for myself.				
9. I certainly feel useless at times.				
10. At times I think I am no good at all.				

Score Results Reset

Your score on the Rosenberg self-esteem scale is: .

Correlations for 10 Self-Esteem Items

	1	2	3	4	5	6	7	8	9
SELF1	1.00								
SELF2	0.18	1.00							
SELF3	0.45	0.05	1.00						
SELF4	0.40	0.21	0.35	1.00					
SELF5	0.41	0.25	0.40	0.37	1.00				
SELF6	0.26	0.25	0.21	0.42	0.34	1.00			
SELF7	0.39	0.23	0.38	0.47	0.45	0.47	1.00		
SELF8	0.35	0.05	0.43	0.28	0.46	0.21	0.32	1.00	
SELF9	0.36	0.28	0.28	0.36	0.32	0.50	0.58	0.30	1.00
SELF10	0.20	0.27	0.33	0.22	0.42	0.19	0.31	0.37	0.23

Factor Loadings for 10 Self-Esteem Items

	FACTOR 1	FACTOR 2
SELF7	0.76	0.21
SELF5	0.71	-.21
SELF9	0.68	0.37
SELF4	0.66	0.14
SELF1	0.65	-.16
SELF3	0.63	-.45
SELF6	0.62	0.47
SELF8	0.60	-.49
SELF10	0.55	-.26
SELF2	0.39	0.46

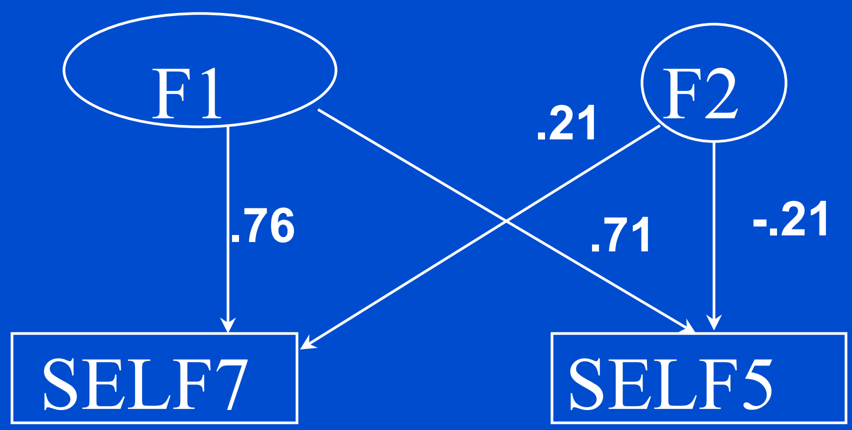
Reproducing self7-self5 correlation: EFA

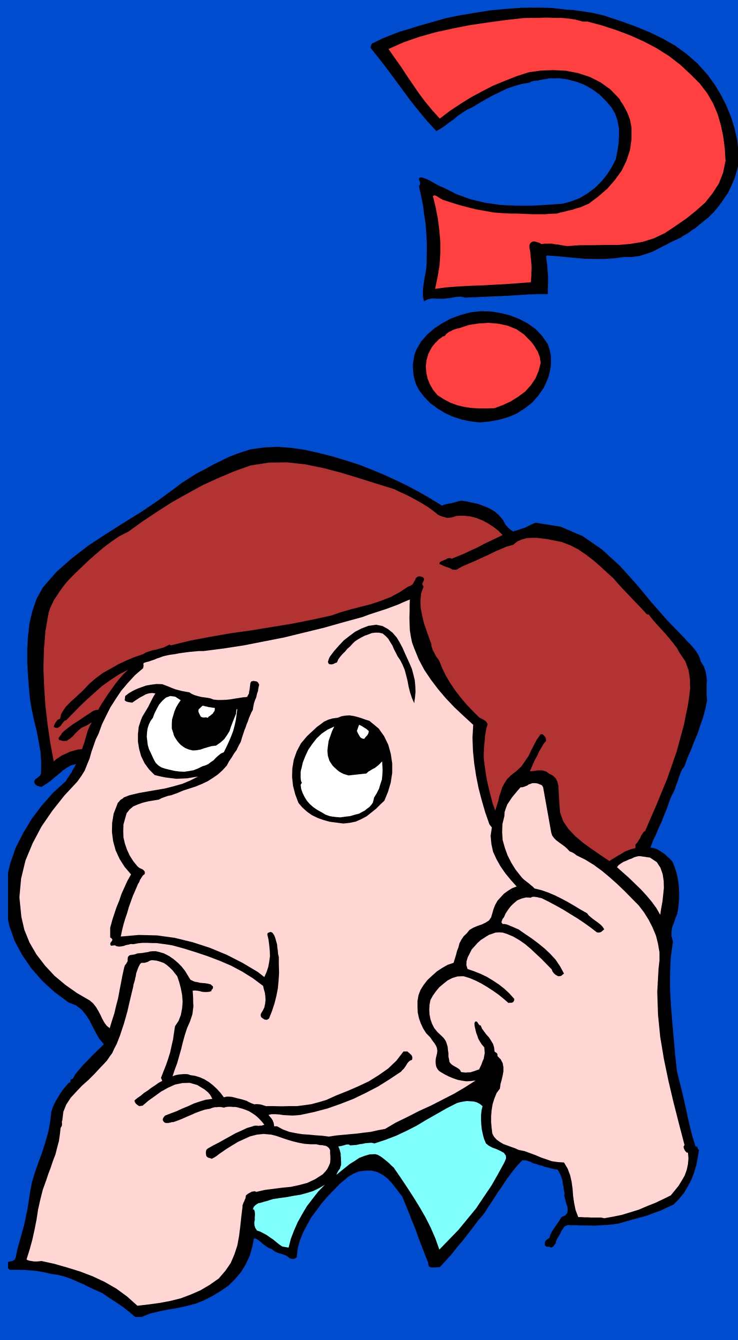
observed r = 0.446

reproduced r = 0.75570 (0.71255) + 0.21195(-.2077)

= 0.538474 - 0.0440241 = 0.494

residual = 0.446 - 0.494 = -.048





Three Steps in Exploratory Factor Analysis

Check correlation matrix for problems

Identify number of dimensions or factors

Rotate to simple structure

http://www.gim.med.ucla.edu/FacultyPages/Hays/

Checking Correlation Matrix

Determinant of correlation matrix ranges between 0-1

Determinant = 0 if there is linear dependency in the matrix (singular, not positive definite, matrix has no inverse)

Determinant = 1 if all off diagonal elements in matrix are zero (identity matrix)

Measurement Error

observed = true score + systematic error + random error

(bias)

Partitioning of Variance Among Items

observed = Common + Specific + Error

Standardize items: $Z_X = (X - \text{x-bar}) / SD_x$

Principal Components Analysis

Try to explain ALL variance in items, summarizing interrelations among items by smaller set of orthogonal principal components that are linear combinations of the items.

* First component is linear combination that explains maximum amount of variance in correlation matrix.

* Second component explains maximum amount of variance in residual correlation matrix.

Factor loadings represent correlation of each item with the component.

Eigenvalue (max = number of items) is sum of squared factor loadings for the component (column) and represents amount of variance in items explained by it.

Principal Components Analysis

- Use 1.0 as initial estimate of communality (variance in item explained by the factors) for each item
- Component is linear combination of items
- First component accounts for as much of the total item variance as possible
- Second component accounts for as much variance as possible, but uncorrelated with the first component
- $C_1 = a_1*x_1 + b_1*x_2$
- $C_2 = a_2*x_1 + b_2*x_2$
- Mean of C_1 & C_2 = 0

Common Factor Analysis

Factors are not linear combinations of items but are hypothetical constructs estimated from the items.

These factors are estimated from the common variance (not total) of the items; diagonal elements (communality estimates) of the correlation matrix estimated as less than 1.0.

Common Factor Analysis

- Each item represented as a linear combination of unobserved common and unique factors

$$X_1 = a_1 F_1 + b_1 F_2 + e_1$$

$$X_2 = a_2 F_1 + b_2 F_2 + e_2$$

- F_1 and F_2 are standardized common factors
- a's and b's are factor loadings; e's are unique factors
- Factors are independent variables (components are dependent variables)

Hypothetical Factor Loadings, Communalities, and Specificities

Variable	Factor Loadings F_1	F_2	Communality h^2	Specificity u^2
X_1	0.511	0.782	0.873	0.127
X_2	0.553	0.754	0.875	0.125
X_3	0.631	-0.433	0.586	0.414
X_4	0.861	-0.386	0.898	0.102
X_5	0.929	-0.225	0.913	0.087
Variance explained	2.578	1.567	4.145	0.855
Percentage	51.6%	31.3%	82.9%	17.1%

From Afifi and Clark, Computer-Aided Multivariate Analysis, 1984, p. 338 (Principal components example)

Number of factors decision

Guttman's weakest lower bound

PCA eigenvalues > 1.0

Parallel analysis

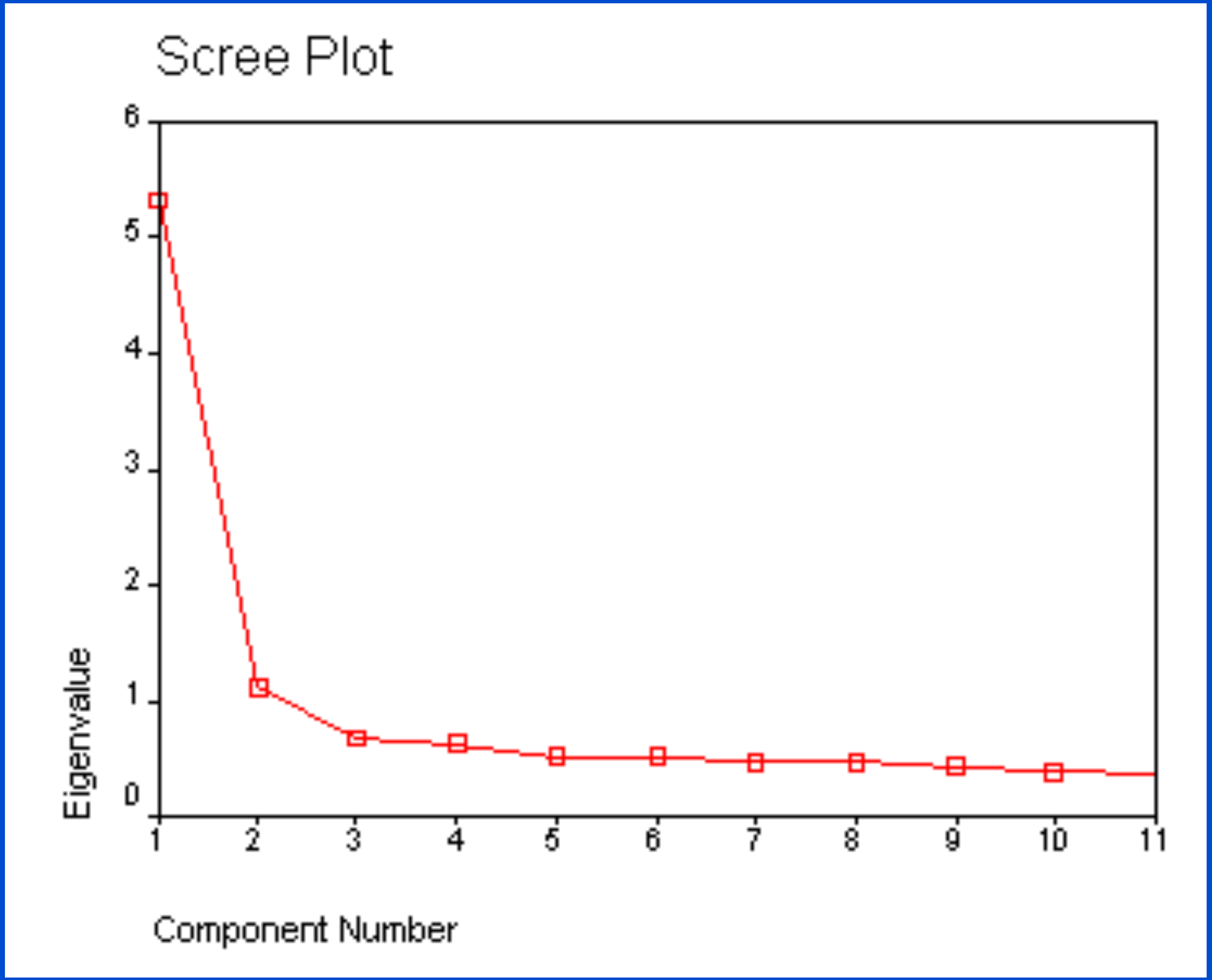
Scree test

ML and Tucker's rho

Parallel Analysis

```
PARALLEL.EXE: LATENT ROOTS OF RANDOM DATA CORRELATION MATRICES PROGRAM
PROGRAMMER: RON HAYS, RAND CORPORATION
FOR 3000 SUBJECTS AND 15 VARIABLES
*****************************************************************
Hays, R. D.  (1987).  PARALLEL:  A program for performing parallel
  analysis.  Applied Psychological Measurement, 11, 58.
*****************************************************************

EIGENVALUES FOR FACTOR ANALYSIS SMC ESTIMATES FOLLOW:

                OBSERVED         RANDOM          SLOPE
               =========       =========       =========
LAMBDA    1= 7.790000       0.111727       ---------
                                           -6.880000
LAMBDA    2= 0.910000       0.084649       ---------
                                           -0.490000 ***
LAMBDA    3= 0.420000       0.068458       ---------
                                           -0.160000 ***
LAMBDA    4= 0.260000       0.057218       ---------
                                           -0.130000 ***
LAMBDA    5= 0.130000       0.043949       ---------
                                           -0.030000
LAMBDA    6= 0.100000       0.033773       ---------
                                           -0.095000 ***
LAMBDA    7= 0.005000       0.021966       ---------


(CAN'T COMPUTE LAMBDA 8 :LOG OF ZERO OR NEGATIVE IS UNDEFINED)

Results of Parallel Analysis Indicate Maximum of    6 Factors.
Slopes followed by asterisks indicate discontinuity points
that may be suggestive of the number of factors to retain.
```

Scree Test


Scree Plot
6
5
4
3
2
1
0
Eigenvalue
1
2
3
4
5
6
7
8
9
10
11
Component Number

ML and Tucker's rho

Significance Tests Based on 3000 Observations

Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors HA: At least one common factor	105	30632.0250	<.0001
H0: 4 Factors are sufficient HA: More factors are needed	51	937.9183	<.0001

Chi-Square without Bartlett's Correction	940.58422
Tucker and Lewis's Reliability Coefficient	0.94018

Factor Rotation

Unrotated factors are complex and hard to interpret

Rotation improves "simple" structure (more high and low loadings) and interpretability

Rotation

Communalities unchanged by rotation

Cumulative % of variance explained by common factors unchanged

Varimax (orthogonal rotation) maximizes sum of squared factor loadings (after dividing each loading by the item's communality)

Promax allows factors to be correlated

* Structure, pattern, and factor correlation matrix

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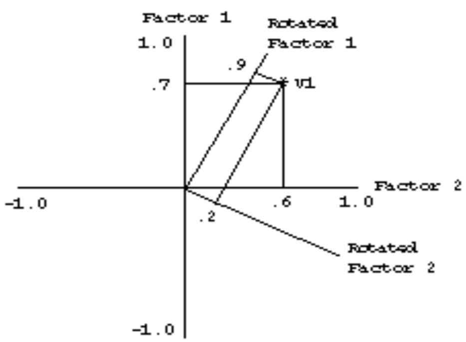
analyst wants to confirm the hypothesis or replicate the previous study, then a factor analysis with the prespecified number of factors can be run. The NFACTOR=*n* (or N=*n*) option in PROC FACTOR extracts the user-supplied number of factors. Ultimately, the criterion for determining the number of factors should be the replicability of the solution. It is important to extract only factors that can be expected to replicate themselves when a new sample of subjects is employed.

5. The Rotation of Factors

Once you decide on the number of factors to extract, the next logical step is to determine the method of rotation. The fundamental theorem of factor analysis is invariant within rotations. That is, the initial factor pattern matrix is not unique. We can get an infinite number of solutions, which produce the same correlation matrix, by rotating the reference axes of the factor solution to simplify the factor structure and to achieve a more meaningful and interpretable solution. The idea of simple structure has provided the most common basis for rotation, the goal being to rotate the factors simultaneously so as to have as many zero loadings on each factor as possible. The following figure is a simplified example of rotation, showing only one variable from a set of several variables.



The variable V1 initially has factor loadings (correlations) of .7 and .6 on factor 1 and factor 2 respectively. However, after rotation the factor loadings have changed to .9 and .2 on the rotated factor 1 and factor 2 respectively, which is closer to a simple structure and easier to interpret.

The simplest case of rotation is an *orthogonal rotation* in which the angle between the reference axes of factors are maintained at 90 degrees. More complicated forms of rotation allow the angle between the reference axes to be other than a right angle, i.e., factors are allowed to be correlated with each other. These types of rotational procedures are referred to as *oblique rotations*. Orthogonal rotation procedures are more commonly used than oblique rotation procedures. In some situations, theory may mandate that underlying latent constructs be uncorrelated with each other, and therefore oblique rotation procedures will not be appropriate. In other situations where the correlations between the underlying constructs are not assumed to be zero, oblique rotation procedures may yield simpler and more interpretable factor patterns.

Items/Factors and Cases/Items

At least 5

- items per factor
- cases per item
- cases per parameter estimate

Confirmatory Factor Analysis

- Compares observed covariances with covariances generated by hypothesized model
- Statistical and practical tests of fit
- Factor loadings
- Correlations between factors
- Regression coefficients

Fit Indices

- Normed fit index: $$\frac{\chi^2_{null} - \chi^2_{model}}{\chi^2_{null}}$$
- Non-normed fit index: $$\frac{\dfrac{\chi^2_{null}}{df_{null}} - \dfrac{\chi^2_{model}}{df_{model}}}{\left(\dfrac{\chi^2_{null}}{df_{null}} - 1\right)}$$
- Comparative fit index: $$1 - \left(\frac{\chi^2_{model} - df_{model}}{\chi^2_{null} - df_{null}}\right)$$

Software

SAS PROC CALIS

EQS

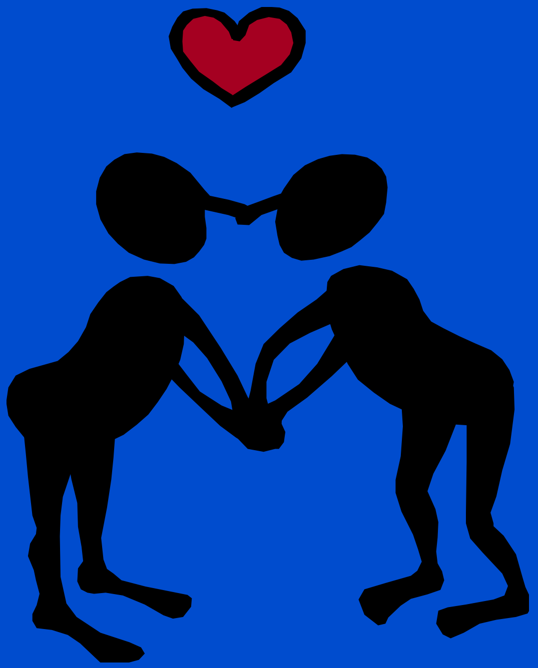
LISREL

MPLUS

KISS

"Sometimes, very complex mathematical methods are required for the scientific problem at hand.
However, most situations allow much simpler, direct, and practicable approaches"

(Nunnally & Bernstein, 1994, p. 452).



Multitrait Scaling Analysis

- Internal consistency reliability
  - Item convergence
- Item discrimination

Hypothetical Multitrait/Multi-Item Correlation Matrix

	Trait #1	Trait #2	Trait #3
Item #1	0.80*	0.20	0.20
Item #2	0.80*	0.20	0.20
Item #3	0.80*	0.20	0.20
Item #4	0.20	0.80*	0.20
Item #5	0.20	0.80*	0.20
Item #6	0.20	0.80*	0.20
Item #7	0.20	0.20	0.80*
Item #8	0.20	0.20	0.80*
Item #9	0.20	0.20	0.80*

*Item-scale correlation, corrected for overlap.

Multitrait/Multi-Item Correlation Matrix for Patient Satisfaction Ratings

	Technical	Interpersonal	Communication	Financial
Technical				
1	0.66*	0.63†	0.67†	0.28
2	0.55*	0.54†	0.50†	0.25
3	0.48*	0.41	0.44†	0.26
4	0.59*	0.53	0.56†	0.26
5	0.55*	0.60†	0.56†	0.16
6	0.59*	0.58†	0.57†	0.23
Interpersonal				
1	0.58	0.68*	0.63†	0.24
2	0.59†	0.58*	0.61†	0.18
3	0.62†	0.65*	0.67†	0.19
4	0.53†	0.57*	0.60†	0.32
5	0.54	0.62*	0.58†	0.18
6	0.48†	0.48*	0.46†	0.24

Note – Standard error of correlation is 0.03. Technical = satisfaction with technical quality. Interpersonal = satisfaction with the interpersonal aspects. Communication = satisfaction with communication. Financial = satisfaction with financial arrangements. *Item-scale correlations for hypothesized scales (corrected for item overlap). †Correlation within two standard errors of the correlation of the item with its hypothesized scale.

Recommended Readings

Pett, M. A., Lackey, N. R., & Sullivan, J. J. (2003). Making sense of factor analysis: The use of factor analysis for instrument development in health care research. Thousand Oaks: Sage.

Floyd, F. J., & Widaman, K. F. (1995). Factor analysis in the development and refinement of clinical assessment instruments. Psychological Assessment, 7, 286-299

Hays, R. D., Revicki, D., & Coyne, K. (in press). Application of structural equation modeling to health outcomes research. Evaluation and the Health Professions.

Appendix: Eigenvalues

$$S_{c_1}^2 = a_1^2 s_{x_1}^2 + b_1^2 s_{x_2}^2 + 2(a_1)(b_1)\, r_{x_1, x_2} s_{x_1} s_{x_2}$$

$$S_{c_2}^2 = a_2^2 s_{x_1}^2 + b_2^2 s_{x_2}^2 + 2(a_2)(b_2)\, r_{x_1, x_2} s_{x_1} s_{x_2}$$

$S_{c_1}^2$ is maximized

C_1 & C_2 are uncorrelated

$$a_1^2 + b_1^2 = a_2^2 + b_2^2$$

$$S_{c_1} + S_{c_2} = S_{x_1} + S_{x_2}$$

Appendix: Correlations Between Component and Item

$$r_{1,c_1} = \frac{a_1 (s_{c_1})}{s_{x_1}}$$

$$r_{2,c_1} = \frac{b_1 (s_{c_1})}{s_{x_2}}$$