

# Evaluating Multi-Item Scales Using Item-Scale Correlations and Confirmatory Factor Analysis

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Drew Cobb Room 131/Broxton 2<sup>nd</sup> Floor Conference Room

<http://gim.med.ucla.edu/FacultyPages/Hays/>

# Development and Evaluation of PROMIS GI Distress Scale (n = 574)

- Dinesh Khanna and Brennan Spiegel, PIs
- Irritable Bowel Syndrome, Irritable Bowel Disease, Scleroderma
- 61% female; median age = 48 (18-85 yr. range)
- Some college = median educational level
- Race/ethnicity
  - 13% Hispanic
  - 17% non-Hispanic black
  - 12% non-Hispanic Asian
  - 5% non-Hispanic other

# 8 Hypothesized Scales (101 items)

1. Gastroesophageal reflux (29 items)
2. Disrupted swallowing (7 items)
3. Diarrhea (7 items)
4. Bowel incontinence/soilage (4 items)
5. Nausea and vomiting (13 items)
6. Constipation (15 items)
7. Belly pain (8 items)
8. Gas/bloat/flatulence (18 items)

# Gastroesophageal Reflux (29 items)

In the past 7 days ...

- How often did you have regurgitation—that is, food or liquid coming back up into your throat or mouth without vomiting?
  - Never
  - One day
  - 2-6 days
  - Once a day
  - More than once a day

# Disrupted swallowing (7 items)

In the past 7 days ...

- How often did food get stuck in your throat when you were eating?
  - Never
  - Rarely
  - Sometimes
  - Often
  - Always

# Diarrhea (7 items)

In the past 7 days ...

- How many days did you have loose or watery stools?
  - No days
  - 1 day
  - 2 days
  - 3-5 days
  - 6-7 days

# Bowel incontinence (4 items)

In the past 7 days ...

- How often did you have bowel incontinence—that is, have an accident because you could not make it to the bathroom in time?
  - No days
  - 1 day
  - 2-3 days
  - 4-5 days
  - 6-7 days

# Nausea and vomiting (13 items)

In the past 7 days ...

- How often did you have nausea—that is, a feeling like you could vomit?
  - Never
  - Rarely
  - Sometimes
  - Often
  - Always

# Constipation (14 items)

In the past 7 days ...

- How often did you strain while trying to have bowel movements?
  - Never
  - Rarely
  - Sometimes
  - Often
  - Always

# Belly pain (8 items)

In the past 7 days ...

- How often did you have belly pain?
  - Never
  - Rarely
  - Sometimes
  - Often
  - Always

# Gas/bloat/flatulence (18 items)

In the past 7 days ...

- How often did you feel bloated?
  - Never
  - Rarely
  - Sometimes
  - Often
  - Always

# Problematic Item\*

*Constipation (Gisx62)*

In the last 7 days ...

**How often did you have a bowel movement?**

- Never {3}
- One day {2}
- 2-6 days {1}
- Once a day {1}
- More than once a day {1}

# Problematic Item #2

## *Gastroesophageal Reflux (Gisx22)*

In the last 7 days ...

**How often did you burp?**

- Never {1}
- One day {2}
- 2-6 days {3}
- Once a day {3}
- More than once a day {4}

# Problematic Item #3

*Gas/Bloat/Flatulence (Gisx105)*

In the last 7 days ...

**How often did you pass gas?**

- Never {1}
- Rarely (only once or twice a day) {2}
- About every 3-4 hours {3}
- About every 2 hours {4}
- About every hour {4}

# Internal Consistency Reliability for 8 Scales (101 items)

| Scale                   | Number of items | Alpha |
|-------------------------|-----------------|-------|
| Gastroesophageal Reflux | 29              | 0.94  |
| Disrupted swallowing    | 7               | 0.90  |
| Diarrhea                | 7               | 0.89  |
| Bowel incontinence      | 4               | 0.87  |
| Nausea/vomiting         | 13              | 0.93  |
| Constipation            | 14              | 0.93  |
| Belly pain              | 8               | 0.93  |
| Gas/bloat               | 18              | 0.96  |

Reliability Standards:

0.70 or above for group comparisons; 0.90 for individual assessment

*How often did you have a bowel movement?* did not go with Constipation scale.

# Item-scale correlation matrix

|         | <u>Depress</u> | <u>Anxiety</u> | <u>Anger</u> |
|---------|----------------|----------------|--------------|
| Item #1 | 0.80*          | 0.20           | 0.20         |
| Item #2 | 0.80*          | 0.20           | 0.20         |
| Item #3 | 0.80*          | 0.20           | 0.20         |
| Item #4 | 0.20           | 0.80*          | 0.20         |
| Item #5 | 0.20           | 0.80*          | 0.20         |
| Item #6 | 0.20           | 0.80*          | 0.20         |
| Item #7 | 0.20           | 0.20           | 0.80*        |
| Item #8 | 0.20           | 0.20           | 0.80*        |
| Item #9 | 0.20           | 0.20           | 0.80*        |



\*Item-scale correlation, corrected for overlap.

# Item-scale correlation matrix

|         | <u>Depress</u> | <u>Anxiety</u> | <u>Anger</u> |
|---------|----------------|----------------|--------------|
| Item #1 | 0.50*          | 0.50           | 0.50         |
| Item #2 | 0.50*          | 0.50           | 0.50         |
| Item #3 | 0.50*          | 0.50           | 0.50         |
| Item #4 | 0.50           | 0.50*          | 0.50         |
| Item #5 | 0.50           | 0.50*          | 0.50         |
| Item #6 | 0.50           | 0.50*          | 0.50         |
| Item #7 | 0.50           | 0.50           | 0.50*        |
| Item #8 | 0.50           | 0.50           | 0.50*        |
| Item #9 | 0.50           | 0.50           | 0.50*        |



\*Item-scale correlation, corrected for overlap.

MULTI -- MULTITRAIT SCALING PROGRAM: gisx105rec item-scale correlations based on separate MULTI.

SAMPsize = 573 SEOFCORR = 0.04

| item    | Gastro       | Disrup      | Diarr | Bowel | Nause | Consti | Belly | Gas  |
|---------|--------------|-------------|-------|-------|-------|--------|-------|------|
| gisx2   | 0.70*        | 0.50        | 0.18  | 0.15  | 0.52  | 0.23   | 0.30  | 0.29 |
| gisx3   | 0.59*        | 0.40        | 0.18  | 0.17  | 0.55  | 0.19   | 0.31  | 0.29 |
| gisx4   | 0.63*        | 0.49        | 0.14  | 0.15  | 0.50  | 0.17   | 0.22  | 0.22 |
| gisx5   | 0.65*        | 0.52        | 0.11  | 0.18  | 0.39  | 0.20   | 0.22  | 0.22 |
| gisx6   | 0.71*        | 0.52        | 0.20  | 0.17  | 0.55  | 0.19   | 0.27  | 0.31 |
| gisx7   | 0.66*        | 0.48        | 0.12  | 0.19  | 0.42  | 0.17   | 0.18  | 0.20 |
| gisx8   | 0.74*        | 0.54        | 0.19  | 0.18  | 0.54  | 0.24   | 0.28  | 0.31 |
| gisx9   | 0.75*        | 0.59        | 0.21  | 0.17  | 0.54  | 0.30   | 0.31  | 0.30 |
| gisx10  | 0.51*        | 0.42        | 0.16  | 0.13  | 0.33  | 0.19   | 0.20  | 0.14 |
| gisx11  | 0.68*        | 0.51        | 0.11  | 0.11  | 0.41  | 0.22   | 0.23  | 0.26 |
| gisx12  | 0.42*        | 0.31        | 0.09  | 0.11  | 0.35  | 0.25   | 0.21  | 0.18 |
| gisx13  | 0.71*        | 0.55        | 0.15  | 0.12  | 0.40  | 0.27   | 0.28  | 0.31 |
| gisx14  | 0.65*        | 0.48        | 0.13  | 0.12  | 0.38  | 0.26   | 0.30  | 0.31 |
| gisx15  | 0.72*        | 0.52        | 0.15  | 0.15  | 0.45  | 0.30   | 0.34  | 0.39 |
| gisx16  | 0.74*        | 0.51        | 0.14  | 0.15  | 0.44  | 0.28   | 0.35  | 0.36 |
| gisx17  | 0.56*        | 0.43        | 0.07  | 0.05  | 0.35  | 0.26   | 0.21  | 0.31 |
| gisx18  | 0.70*        | 0.50        | 0.11  | 0.11  | 0.40  | 0.28   | 0.25  | 0.35 |
| gisx19  | 0.71*        | 0.50        | 0.14  | 0.16  | 0.40  | 0.28   | 0.31  | 0.37 |
| gisx20  | 0.44*        | 0.25        | 0.12  | 0.07  | 0.25  | 0.11   | 0.21  | 0.24 |
| gisx21  | 0.74*        | 0.60        | 0.15  | 0.16  | 0.40  | 0.21   | 0.30  | 0.32 |
| gi22rec | <u>0.30*</u> | 0.22        | 0.09  | 0.05  | 0.19  | 0.11   | 0.12  | 0.19 |
| gisx23  | 0.57*        | 0.39        | 0.14  | 0.13  | 0.39  | 0.23   | 0.20  | 0.31 |
| gisx24  | 0.59*        | 0.43        | 0.14  | 0.07  | 0.44  | 0.31   | 0.23  | 0.34 |
| gisx25  | 0.41*        | 0.29        | 0.12  | 0.19  | 0.26  | 0.19   | 0.15  | 0.19 |
| gisx26  | 0.42*        | 0.29        | 0.17  | 0.15  | 0.36  | 0.20   | 0.15  | 0.19 |
| gisx27  | 0.46*        | 0.28        | 0.11  | 0.11  | 0.31  | 0.19   | 0.16  | 0.20 |
| gisx28  | 0.63*        | <u>0.64</u> | 0.11  | 0.13  | 0.36  | 0.28   | 0.25  | 0.28 |
| gisx29  | 0.62*        | <u>0.62</u> | 0.18  | 0.20  | 0.42  | 0.27   | 0.23  | 0.27 |
| gisx30  | 0.63*        | <u>0.63</u> | 0.14  | 0.13  | 0.39  | 0.27   | 0.24  | 0.27 |
| gisx31  | 0.59         | 0.71*       | 0.13  | 0.18  | 0.37  | 0.26   | 0.18  | 0.19 |
| gisx32  | 0.60         | 0.80*       | 0.14  | 0.24  | 0.38  | 0.27   | 0.18  | 0.25 |
| gisx33  | 0.68         | 0.68*       | 0.14  | 0.18  | 0.41  | 0.32   | 0.29  | 0.29 |
| gisx34  | 0.60         | 0.81*       | 0.15  | 0.26  | 0.37  | 0.25   | 0.18  | 0.23 |
| gisx35  | 0.56         | 0.74*       | 0.13  | 0.26  | 0.35  | 0.22   | 0.17  | 0.25 |
| gisx36  | 0.56         | 0.70*       | 0.09  | 0.19  | 0.31  | 0.24   | 0.18  | 0.21 |
| gisx37  | 0.48         | 0.62*       | 0.09  | 0.19  | 0.32  | 0.24   | 0.16  | 0.23 |
| gisx38  | 0.06         | 0.02        | 0.69* | 0.34  | 0.20  | 0.04   | 0.26  | 0.18 |
| gisx39  | 0.06         | 0.05        | 0.63* | 0.39  | 0.20  | 0.11   | 0.24  | 0.17 |
| gisx40  | 0.23         | 0.14        | 0.84* | 0.46  | 0.37  | 0.22   | 0.40  | 0.33 |
| gisx41  | 0.20         | 0.13        | 0.83* | 0.40  | 0.34  | 0.21   | 0.37  | 0.31 |
| gisx42  | 0.20         | 0.17        | 0.71* | 0.51  | 0.26  | 0.20   | 0.32  | 0.27 |
| gisx43  | 0.26         | 0.20        | 0.82* | 0.47  | 0.33  | 0.28   | 0.38  | 0.36 |
| gisx44  | 0.23         | 0.19        | 0.80* | 0.44  | 0.32  | 0.29   | 0.37  | 0.34 |
| gisx45  | 0.13         | 0.19        | 0.42  | 0.76* | 0.22  | 0.10   | 0.08  | 0.12 |
| gisx46  | 0.15         | 0.23        | 0.46  | 0.84* | 0.20  | 0.10   | 0.04  | 0.11 |
| gisx47  | 0.13         | 0.21        | 0.41  | 0.80* | 0.17  | 0.07   | 0.03  | 0.09 |
| gisx48  | 0.28         | 0.28        | 0.49  | 0.60* | 0.28  | 0.19   | 0.19  | 0.27 |

| item            | Gastro      | Disrup      | Diarr       | Bowel       | Nause       | Consti      | Belly       | Gas          |
|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|
| gissx49         | 0.52        | 0.39        | 0.30        | 0.19        | 0.79*       | 0.31        | 0.46        | 0.37         |
| gissx50         | 0.51        | 0.35        | 0.30        | 0.16        | 0.84*       | 0.32        | 0.48        | 0.38         |
| gissx51         | 0.53        | 0.35        | 0.31        | 0.16        | 0.82*       | 0.32        | 0.49        | 0.36         |
| gissx52         | 0.45        | 0.37        | 0.23        | 0.17        | 0.73*       | 0.27        | 0.33        | 0.31         |
| gissx53         | 0.52        | 0.36        | 0.35        | 0.16        | 0.84*       | 0.35        | 0.46        | 0.41         |
| gissx54         | 0.52        | 0.35        | 0.32        | 0.15        | 0.84*       | 0.34        | 0.49        | 0.40         |
| gissx55         | 0.39        | 0.34        | 0.31        | 0.25        | 0.52*       | 0.28        | 0.35        | 0.30         |
| gissx56         | 0.40        | 0.32        | 0.19        | 0.21        | 0.59*       | 0.21        | 0.25        | 0.18         |
| gissx57         | 0.43        | 0.34        | 0.21        | 0.22        | 0.63*       | 0.26        | 0.26        | 0.23         |
| gissx58         | 0.46        | 0.37        | 0.20        | 0.21        | 0.63*       | 0.26        | 0.29        | 0.25         |
| gissx59         | 0.47        | 0.29        | 0.16        | 0.18        | 0.62*       | 0.15        | 0.26        | 0.21         |
| gissx60         | 0.49        | 0.31        | 0.22        | 0.21        | 0.67*       | 0.18        | 0.28        | 0.25         |
| gissx61         | 0.41        | 0.26        | 0.21        | 0.21        | 0.61*       | 0.11        | 0.24        | 0.23         |
| gissx63         | 0.22        | 0.19        | -.04        | -.00        | 0.13        | 0.44*       | 0.17        | 0.21         |
| gissx64         | 0.31        | 0.29        | 0.02        | 0.01        | 0.22        | 0.66*       | 0.32        | 0.38         |
| gissx65         | 0.24        | 0.24        | 0.09        | 0.05        | 0.25        | 0.74*       | 0.32        | 0.35         |
| gissx66         | 0.25        | 0.25        | 0.08        | 0.04        | 0.25        | 0.78*       | 0.33        | 0.35         |
| gissx67         | 0.23        | 0.22        | 0.14        | 0.05        | 0.25        | 0.82*       | 0.37        | 0.40         |
| gissx68         | 0.28        | 0.26        | 0.21        | 0.11        | 0.26        | 0.79*       | 0.34        | 0.37         |
| gissx69         | 0.29        | 0.27        | 0.23        | 0.09        | 0.27        | 0.78*       | 0.38        | 0.37         |
| gissx70         | 0.31        | 0.25        | 0.24        | 0.12        | 0.30        | 0.79*       | 0.39        | 0.39         |
| gissx71         | 0.28        | 0.26        | 0.24        | 0.08        | 0.27        | 0.82*       | 0.37        | 0.37         |
| gissx72         | 0.24        | 0.22        | 0.30        | 0.16        | 0.27        | 0.63*       | 0.42        | 0.44         |
| gissx73         | 0.24        | 0.17        | 0.36        | 0.15        | 0.31        | 0.68*       | 0.45        | 0.50         |
| gissx74         | 0.27        | 0.30        | 0.17        | 0.25        | 0.25        | 0.43*       | 0.18        | 0.23         |
| gissx75         | 0.24        | 0.17        | 0.20        | 0.21        | 0.29        | 0.48*       | 0.25        | 0.24         |
| gissx76         | 0.25        | 0.18        | 0.21        | 0.16        | 0.30        | 0.53*       | 0.26        | 0.31         |
| gissx77         | 0.34        | 0.23        | 0.35        | 0.09        | 0.42        | 0.41        | 0.89*       | 0.55         |
| gissx78         | 0.25        | 0.15        | 0.31        | 0.09        | 0.39        | 0.33        | 0.81*       | 0.49         |
| gissx79         | 0.36        | 0.21        | 0.37        | 0.08        | 0.45        | 0.39        | 0.87*       | 0.53         |
| gissx80         | 0.37        | 0.23        | 0.35        | 0.06        | 0.46        | 0.43        | 0.89*       | 0.52         |
| gi8189ct        | 0.31        | 0.21        | 0.29        | 0.04        | 0.33        | 0.31        | 0.72*       | 0.49         |
| gissx90         | 0.35        | 0.21        | 0.44        | 0.14        | 0.48        | 0.43        | 0.84*       | 0.57         |
| gissx91         | 0.35        | 0.22        | 0.39        | 0.10        | 0.46        | 0.47        | 0.89*       | 0.58         |
| gissx92         | 0.37        | 0.24        | 0.36        | 0.11        | 0.46        | 0.41        | 0.83*       | 0.61         |
| gissx93         | 0.39        | 0.30        | 0.24        | 0.17        | 0.32        | 0.39        | 0.53        | 0.82*        |
| gissx94         | 0.38        | 0.28        | 0.24        | 0.17        | 0.34        | 0.38        | 0.52        | 0.84*        |
| gissx95         | 0.37        | 0.26        | 0.24        | 0.15        | 0.34        | 0.36        | 0.51        | 0.84*        |
| gissx96         | 0.41        | 0.28        | 0.30        | 0.17        | 0.41        | 0.41        | 0.57        | 0.81*        |
| gissx97         | 0.38        | 0.28        | 0.24        | 0.11        | 0.37        | 0.42        | 0.54        | 0.85*        |
| gissx98         | 0.31        | 0.23        | 0.27        | 0.14        | 0.31        | 0.41        | 0.53        | 0.82*        |
| gissx99         | 0.37        | 0.26        | 0.26        | 0.15        | 0.35        | 0.41        | 0.52        | 0.88*        |
| gissx100        | 0.34        | 0.23        | 0.23        | 0.10        | 0.32        | 0.41        | 0.53        | 0.86*        |
| gissx101        | 0.38        | 0.27        | 0.24        | 0.10        | 0.36        | 0.43        | 0.55        | 0.87*        |
| gissx102        | 0.25        | 0.23        | 0.14        | 0.05        | 0.26        | 0.34        | 0.39        | 0.65*        |
| gissx103        | 0.38        | 0.26        | 0.26        | 0.14        | 0.35        | 0.43        | 0.53        | 0.85*        |
| gissx104        | 0.38        | 0.26        | 0.22        | 0.08        | 0.33        | 0.43        | 0.51        | 0.87*        |
| gissx106        | 0.30        | 0.18        | 0.34        | 0.22        | 0.25        | 0.36        | 0.32        | 0.56*        |
| gissx107        | 0.30        | 0.17        | 0.35        | 0.21        | 0.23        | 0.40        | 0.34        | 0.55*        |
| gissx108        | 0.21        | 0.09        | 0.35        | 0.11        | 0.27        | 0.24        | 0.42        | 0.48*        |
| gissx109        | 0.21        | 0.11        | 0.34        | 0.09        | 0.28        | 0.29        | 0.43        | 0.49*        |
| gissx110        | 0.29        | 0.17        | 0.33        | 0.14        | 0.31        | 0.37        | 0.44        | 0.61*        |
| <b>gi105rec</b> | <b>0.18</b> | <b>0.07</b> | <b>0.19</b> | <b>0.05</b> | <b>0.10</b> | <b>0.25</b> | <b>0.24</b> | <b>0.39*</b> |

# Summary of Item-Scale Correlations

- Gisx22 (burp) correlates only 0.30 with *Gastroesophageal Reflux* scale
- Gisx33 (pain in chest when swallowing food) correlates as highly with *Gastroesophageal Reflux* as with *Disrupted swallowing* scale
- Three items correlate as highly with *Disrupted Swallowing* as with *Gastroesophageal Reflux* scale:
  - Gisx28 (lump in throat frequency)
  - Gisx29 (lump in throat interfere)
  - Gisx30 (lump in throat bother)

# Confirmatory Factor Analysis

|                | <u>Depress</u> | <u>Anxiety</u> | <u>Anger</u> |
|----------------|----------------|----------------|--------------|
| <b>Item #1</b> | <b>0.80*</b>   | 0.00           | 0.00         |
| <b>Item #2</b> | <b>0.80*</b>   | 0.00           | 0.00         |
| <b>Item #3</b> | <b>0.80*</b>   | 0.00           | 0.00         |
| <b>Item #4</b> | 0.00           | <b>0.80*</b>   | 0.00         |
| <b>Item #5</b> | 0.00           | <b>0.80*</b>   | 0.00         |
| <b>Item #6</b> | 0.00           | <b>0.80*</b>   | 0.00         |
| <b>Item #7</b> | 0.00           | 0.00           | <b>0.80*</b> |
| <b>Item #8</b> | 0.00           | 0.00           | <b>0.80*</b> |
| <b>Item #9</b> | 0.00           | 0.00           | <b>0.80*</b> |

\*Factor loading.

# Confirmatory Factor Analysis

- Eight-factor categorical model fit the data well (CFI=0.952 and RMSEA=0.049).
- Lower loadings on *Gastroesophageal Reflux* for:
  - Gisx22 (burp frequency): 0.377
  - Gisx20 (pain behind breastbone): 0.391
- Secondary loadings on *Gas* for:
  - Glsx48 (thought gas but liquid; BI): 0.222
  - Glsx72 (unfinished frequency; Con.): 0.224
  - Glsx73 (feeling unfinished bother; Con.): 0.265

# Correlated Residuals

- **r = 0.58 (Gas/bloat/flatulence)**
  - GI108 (freq. gurgling in belly) with GI109 (freq. gurgling when not hungry)
- **r = 0.53 (Constipation)**
  - GI75 (freq. pain in anus) with GI76 (bother pain in anus)
- **r = 0.50 (Gas/bloat/flatulence)**
  - GI106 (gas interference) with GI107 (gas bother)

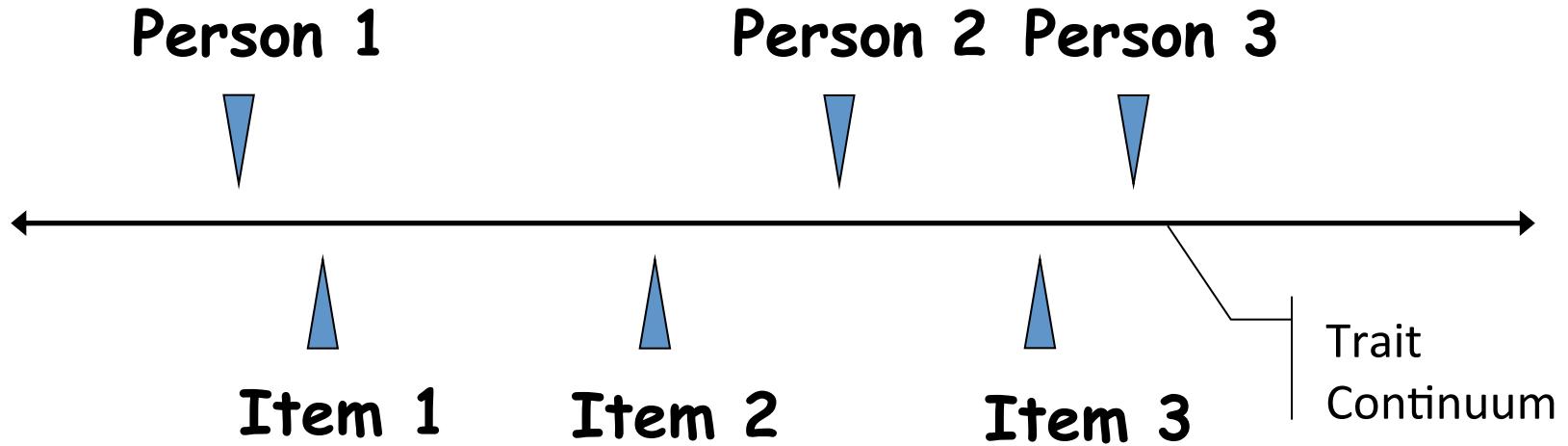
# Item Response Theory (IRT)

IRT models the relationship between a person's response  $Y_i$  to the question (i) and his or her level of the latent construct  $\theta$  being measured by positing

$$\Pr(Y_i \geq k) = \frac{1}{1 + \exp(-a_i\theta + b_{ik})}$$

$b_{ik}$  estimates how difficult it is for the item (i) to have a score of k or more and the discrimination parameter  $a_i$  estimates the discriminatory power of the item.

# Item Responses and Trait Levels



# Bowel Incontinence Items

- How often did you soil or dirty your underwear before getting to a bathroom?
- How often did you have bowel incontinence—that is, have an accident because you could not make it to the bathroom in time?
- How often did you leak stool or soil your underwear?
- How often did you think you were going to pass gas, but stool or liquid came out instead?

# Graded Response Model Parameters (Incontinence)

| Item   | Slope | Threshold 1 | Threshold 2 | Threshold 3 | Threshold 4 |
|--------|-------|-------------|-------------|-------------|-------------|
| Gisx46 | 11.4  | 0.7         | 1.3         | 1.7         | 2.1         |
| Gisx45 | 5.0   | 0.9         | 1.5         | 1.9         | 2.4         |
| Gisx47 | 4.8   | 0.7         | 1.2         | 1.7         | 2.1         |
| Gisx48 | 2.0   | 0.1         | 1.0         | 2.0         | 3.1         |
|        |       |             |             |             |             |

1. How often did you soil or dirty your underwear before getting to a bathroom?
2. How often did you have bowel incontinence—that is, have an accident because you could not make it to the bathroom in time?
3. How often did you leak stool or soil your underwear?

**[No days; 1 day; 2-3 days; 4-5 days; 6-7 days]**

4. How often did you think you were going to pass gas, but stool or liquid came out instead?

**[Never; Rarely; Sometimes; Often; Always]**

# Questions?

**NO FARTING**



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# Coefficient Alpha Formula is Two-Way Fixed Model Below

| Model          | Reliability  | Intraclass Correlation  |
|----------------|--|---|
| One-way        | $\frac{MS_{BMS} - MS_{WMS}}{MS_{BMS}}$                           | $\frac{MS_{BMS} - MS_{WMS}}{MS_{BMS} + (k - 1)MS_{WMS}}$                              |
| Two-way fixed  | $\frac{MS_{BMS} - MS_{EMS}}{MS_{BMS}}$                           | $\frac{MS_{BMS} - MS_{EMS}}{MS_{BMS} + (k - 1)MS_{EMS}}$                              |
| Two-way random | $\frac{N(MS_{BMS} - MS_{EMS})}{NMS_{BMS} + MS_{JMS} - MS_{EMS}}$ | $\frac{MS_{BMS} - MS_{EMS}}{MS_{BMS} + (k - 1)MS_{EMS} + k(MS_{JMS} - MS_{EMS}) / N}$ |

BMS = Between Ratee Mean Square

WMS = Within Mean Square

JMS = Item or Rater Mean Square

EMS = Ratee x Item (Rater) Mean Square

N = n of ratees

k = n of replicates

# *Confirmatory Factor Analysis*

## *Fit Indices*

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- Normed fit index:

$$\frac{\chi_{\text{null}}^2 - \chi_{\text{model}}^2}{\chi_{\text{null}}^2}$$

$$\frac{\chi_{\text{null}}^2 - \chi_{\text{model}}^2}{\text{df}_{\text{null}} - \text{df}_{\text{model}}}$$

- Non-normed fit index:

$$\left[ \frac{\chi_{\text{null}}^2}{\text{df}_{\text{null}}} - 1 \right]$$

- Comparative fit index:

$$1 - \left[ \frac{\chi_{\text{model}}^2 - \text{df}_{\text{model}}}{\chi_{\text{null}}^2 - \text{df}_{\text{null}}} \right]$$

$$\text{RMSEA} = \text{SQRT} (\lambda^2 - \text{df}) / \text{SQRT} (\text{df} (N - 1))$$