Finding and Verifying Item Clusters in Outcomes Research Using Confirmatory and Exploratory Analyses

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(N208)

Methods to be discussed

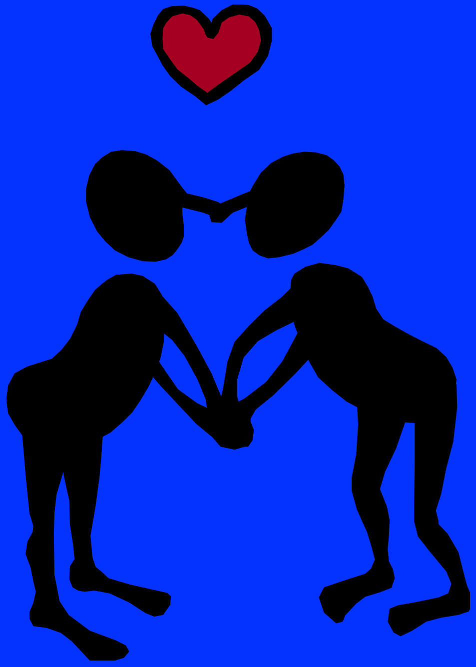
Multitrait Scaling

Confirmatory Factor Analysis

Exploratory Factor Analysis

KISS

"Sometimes, very complex mathematical methods are required for the scientific problem at hand.
However, most situations allow much simpler, direct, and practicable approaches" (Nunnally & Bernstein, 1994, p. 452).



Multitrait Scaling Analysis

- Internal consistency reliability
  - Item convergence
- Item discrimination

http://www.gim.med.ucla.edu/FacultyPages/Hays/

Fake Item-Scale Correlation Matrix

	Scale #1	Scale #2	Scale #3
Item #1	0.80*	0.20	0.20
Item #2	0.80*	0.20	0.20
Item #3	0.80*	0.20	0.20
Item #4	0.20	0.80*	0.20
Item #5	0.20	0.80*	0.20
Item #6	0.20	0.80*	0.20
Item #7	0.20	0.20	0.80*
Item #8	0.20	0.20	0.80*
Item #9	0.20	0.20	0.80*

*Item-scale correlation, corrected for overlap.

Item-Scale Correlation Matrix--Patient Satisfaction Questionnaire

	Technical	Interpersonal	Communication	Financial
Communication				
1	0.58†	0.59†	0.61*	0.26
2	0.47†	0.50†	0.50*	0.25
3	0.58†	0.66†	0.63*	0.23
4	0.66†	0.66†	0.67*	0.25
5	0.66†	0.71†	0.70*	0.25
Financial				
1	0.35	0.35	0.35	0.72*
2	0.17	0.14	0.15	0.65*
3	0.25	0.23	0.23	0.61*
4	0.18	0.15	0.16	0.67*
5	0.31	0.27	0.29	0.70*
6	0.24	0.23	0.22	0.73*
7	0.25	0.23	0.25	0.55*
8	0.34	0.31	0.31	
0.64*				
Cronbach's alpha	**0.80**	**0.82**	**0.82**	**0.88**

Note – Standard error of correlation is 0.03. Technical = satisfaction with technical quality. Interpersonal = satisfaction with the interpersonal aspects. Communication = satisfaction with communication. Financial = satisfaction with financial arrangements.

***Item-scale correlations for hypothesized scales (corrected for item overlap). †Correlation within two standard errors of the correlation of the item with its hypothesized scale.**

>> **Rosenberg's Self-Esteem Scale**

STATEMENT	Strongly Agree	Agree	Disagree	Strongly Disagree
1. I feel that I am a person of worth, at least on an equal plane with others.	○	○	○	○
2. I feel that I have a number of good qualities..	○	○	○	○
3. All in all, I am inclined to feel that I am a failure.	○	○	○	○
4. I am able to do things as well as most other people.	○	○	○	○
5. I feel I do not have much to be proud of.	○	○	○	○
6. I take a positive attitude toward myself.	○	○	○	○
7. On the whole, I am satisfied with myself.	○	○	○	○
8. I wish I could have more respect for myself.	○	○	○	○
9. I certainly feel useless at times.	○	○	○	○
10. At times I think I am no good at all.	○	○	○	○

Score Results Reset

Your score on the Rosenberg self-esteem scale is: ____ .

Scores are calculated as follows:

Correlations for 10 Self-Esteem Items

	1	2	3	4	5	6	7	8	9
SELF1	1.00								
SELF2	0.18	1.00							
SELF3	0.45	0.05	1.00						
SELF4	0.40	0.21	0.35	1.00					
SELF5	0.41	0.25	0.40	0.37	1.00				
SELF6	0.26	0.25	0.21	0.42	0.34	1.00			
SELF7	0.39	0.23	0.38	0.47	**0.45**	0.47	1.00		
SELF8	0.35	0.05	0.43	0.28	0.46	0.21	0.32	1.00	
SELF9	0.36	0.28	0.28	0.36	0.32	0.50	0.58	0.30	1.00
SELF10	0.20	0.27	0.33	0.22	0.42	0.19	0.31	0.37	0.23

Factor Loadings for 10 Self-Esteem Items

	FACTOR 1	FACTOR 2
SELF7	0.76	0.21
SELF5	0.71	-.21
SELF9	0.68	0.37
SELF4	0.66	0.14
SELF1	0.65	-.16
SELF3	0.63	-.45
SELF6	0.62	0.47
SELF8	0.60	-.49
SELF10	0.55	-.26
SELF2	0.39	0.46

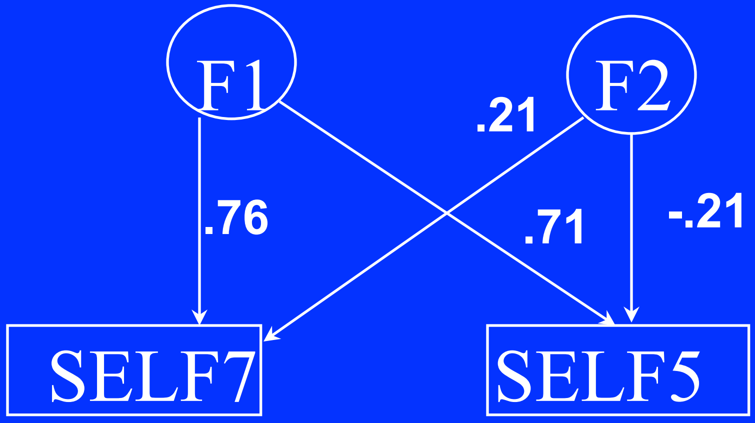
Reproducing self7-self5 correlation: EFA

observed r = 0.446

reproduced r = 0.75570 (0.71255) + 0.21195(-.2077)

= 0.538474 - 0.0440241 = **0.494**

residual = 0.446 - 0.494 = **-.048**



Confirmatory Factor Analysis

- Compares observed covariances with covariances generated by hypothesized model
- Statistical and practical tests of fit
- Factor loadings
- Correlations between factors
- Regression coefficients

Fit Indices

- Normed fit index: $$\frac{\chi^2_{null} - \chi^2_{model}}{\chi^2_{null}}$$
- Non-normed fit index: $$\frac{\dfrac{\chi^2_{null}}{df_{null}} - \dfrac{\chi^2_{model}}{df_{model}}}{\left(\dfrac{\chi^2_{null}}{df_{null}} - 1\right)}$$
- Comparative fit index: $$1 - \left(\frac{\chi^2_{model} - df_{model}}{\chi^2_{null} - df_{null}}\right)$$

Hypothesized Model

- Getting Needed Care
- Promptness of Care
- Provider Communication
- Office Staff Helpfulness
- Plan Customer Service

Marshall et al. (2001, Psychological Assessment)

Multiple Groups

- Latino, Medicaid (n = 609)
- Whites, Medicaid (n = 5,480)
- Latino, Commercial (n = 1,020)
- Whites, Commercial (n = 7,983)

Factor Loadings for Five-Factor Model in Four Groups

	Sample			
Item and Dimension	**L-Med**	**W-Med**	**L-Com**	**W-Com**
Access to Care				
Q5	.43	.51	.51	.46
Q8	-.12	-.15	.24	.17
Q11	.36	.39	.32	.32
Q13	.49	.52	.43	.45
Q36	.61	.66	.55	.58
Timeliness of Care				
Q16	.51	.58	.54	.54
Q17	.42	.52	.42	.51
Q19	.54	.58	.49	.52
Q21	.57	.56	.48	.51
Q25	-.41	-.46	-.34	-.27
Provider Communication				
Q28	.75	.81	.68	.73
Q29	.65	.63	.60	.66
Q30	.75	.81	.68	.72
Q31	.73	.77	.65	.69
Q32	.64	.66	.61	.65
Q34	.50	.53	.43	.53
Health Plan Consumer Service				
Q38	-.08 (n.s)	-.20	-.15	-.13
Q40	.28	.29	.22	.35
Q42	.46	.52	.51	.54
Q43	.70	.67	.69	.68
Q44	.66	.68	.68	.71
Office Staff Helpfulness				
Q26	.71	.72	.61	.65
Q27	.79	.81	.76	.76

Note. All items significant at $p < .001$. L = Latino; W = white; Med = Medicaid; Com=Commercial.
χ^2 = 306.71, 972.91, 368.44 and 2769.46 in the L-Med, W-Med, L-Com and W-Com subgroups, respectively.

Correlations Among Factors in Four Subgroups

Factor and Sample	Factor I	II	III	IV	V
I. Access to Care					
H-Med	- - -	.72	.77	.64	.62
C-Med	- - -	.84	.83	.69	.70
H-Com	- - -	.85	.89	.71	.69
C-Com	- - -	.78	.80	.63	.63
II. Timeliness of Care					
H-Med		- - -	.80	.62	.82
C-Med		- - -	.78	.63	.77
H-Com		- - -	.80	.62	.70
C-Com		- - -	.73	.57	.71
III. Provider Communication					
H-Med			- - -	.60	.83
C-Med			- - -	.52	.81
H-Com			- - -	.56	.79
C-Com			- - -	.52	.76
IV. Health Plan Consumer Service					
H-Med				- - -	.48
C-Med				- - -	.47
H-Com				- - -	.54
C-Com				- - -	.45
V. Office Staff Helpfulness					
H-Med					- - -
C-Med					- - -
H-Com					- - -
C-Com					- - -

Note. All correlations significant at $p < .001$. H = Hispanic; C = white; Med = Medicaid; Com = Commercial

Configurally invariant model

- no cross-group equality constraints

Weak factorial invariance

- cross-group constraints on loadings

Partial weak factorial invariance

- released some constrains

Factor correlation invariance

- constraints on 10 factor correlations

Partial factor correlation invariance

Multi-Group Models: Commercial Sample

	Model	χ^2	df	NFI	NNFI	CFI
1.	Configurally-invariant model	3137.89	440	0.93	0.93	0.94
2.	Weak factorial invariance	3177.96	463	0.92	0.93	0.94
3.	Partial weak factorial invariance	3149.19	458	0.93	0.93	0.94
4.	Factor correlation invariance	3171.72	468	0.93	0.93	0.94
5.	Partial factor corr. invariance	3159.93	465	0.93	0.93	0.94

Multi-Group Models: Medicaid Sample

	Model	χ^2	df	NFI	NNFI	CFI
1.	Configurally-invariant model	1279.60	440	0.97	0.98	0.98
2.	Weak factorial invariance	1319.79	463	0.97	0.98	0.98
3.	Partial weak factorial invariance	1296.24	460	0.97	0.98	0.98
4.	Factor correlation invariance	1319.32	470	0.97	0.98	0.98
5.	Partial factor correlation invariance	1304.61	468	0.97	0.98	0.98

Two Steps in Factor Analysis

Identify number of dimensions or factors

http://www.gim.med.ucla.edu/FacultyPages/Hays/

Rotate to simple structure

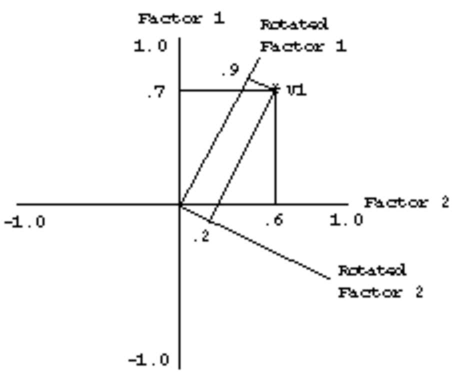
Factor Rotation

Unrotated factors are complex and hard to interpret

Rotation improves “simple” structure (more high and low loadings) and interpretability lyst wants to confirm the hypothesis or replicate the previous study, then a factor analysis with the prespecified number of factors can be run. The NFACTOR=*n* N=*n*) option in PROC FACTOR extracts the user-supplied number of factors. Ultimately, the criterion for determining the number of factors should be the licability of the solution. It is important to extract only factors that can be expected to replicate themselves when a new sample of subjects is employed.

The Rotation of Factors

ce you decide on the number of factors to extract, the next logical step is to determine the method of rotation. The fundamental theorem of factor analysis is ariant within rotations. That is, the initial factor pattern matrix is not unique. We can get an infinite number of solutions, which produce the same correlation matrix rotating the reference axes of the factor solution to simplify the factor structure and to achieve a more meaningful and interpretable solution. The idea of simple ucture has provided the most common basis for rotation, the goal being to rotate the factors simultaneously so as to have as many zero loadings on each factor as ssible. The following figure is a simplified example of rotation, showing only one variable from a set of several variables.



e variable V1 initially has factor loadings (correlations) of .7 and .6 on factor 1 and factor 2 respectively. However, after rotation the factor loadings have nged to .9 and .2 on the rotated factor 1 and factor 2 respectively, which is closer to a simple structure and easier to interpret.

e simplest case of rotation is an *orthogonal rotation* in which the angle between the reference axes of factors are maintained at 90 degrees. More complicated ms of rotation allow the angle between the reference axes to be other than a right angle, i.e., factors are allowed to be correlated with each other. These types of ational procedures are referred to as *oblique rotations*. Orthogonal rotation procedures are more commonly used than oblique rotation procedures. In some ations, theory may mandate that underlying latent constructs be uncorrelated with each other, and therefore oblique rotation procedures will not be appropriate. other situations where the correlations between the underlying constructs are not assumed to be zero, oblique rotation procedures may yield simpler and more rpretable factor patterns.

Rotation

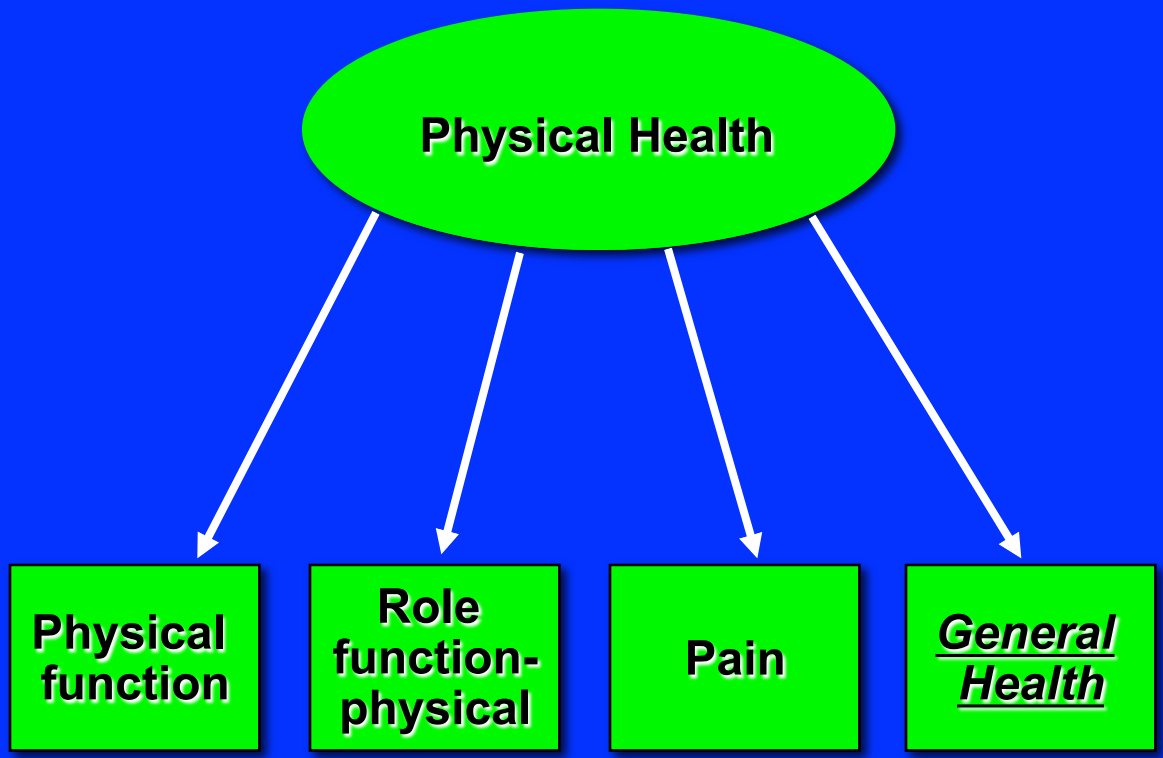
Communalities unchanged by rotation

Cumulative % of variance explained by common factors unchanged

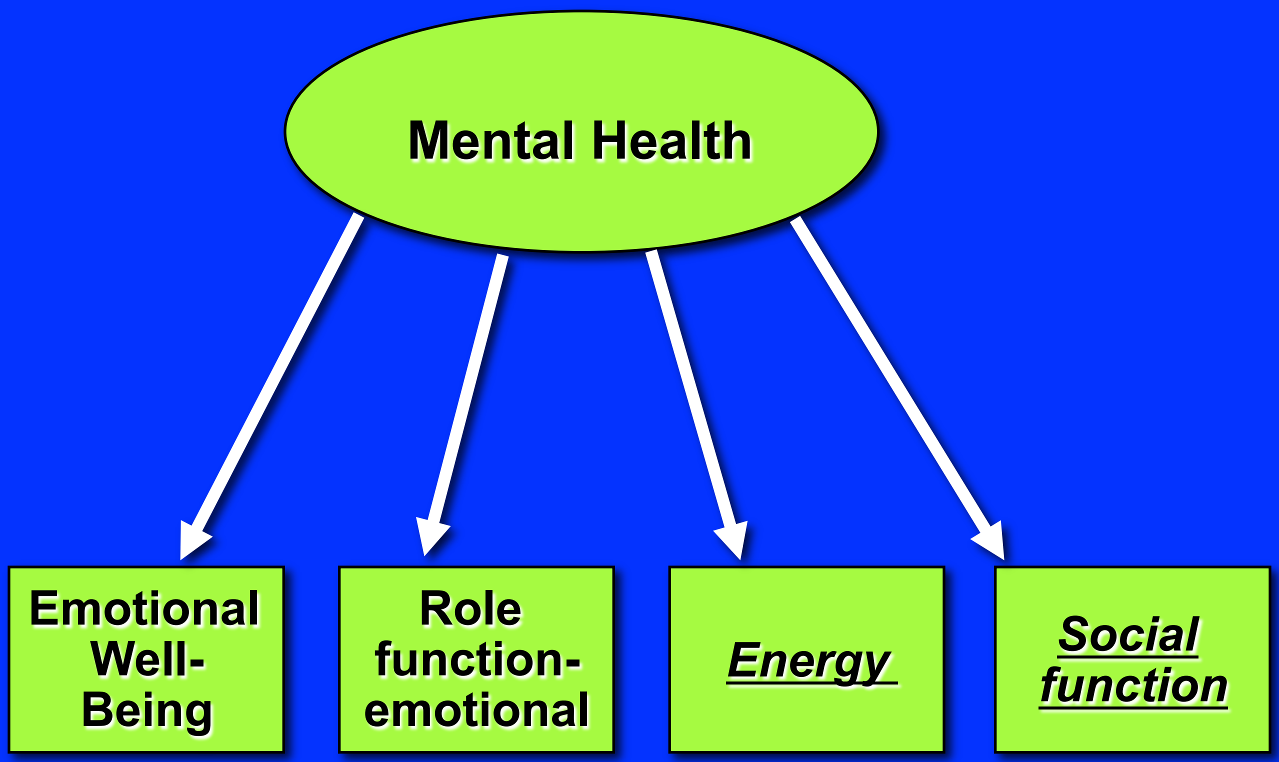
Varimax (orthogonal rotation) maximizes sum of squared factor loadings (after dividing each loading by the item's communality)

Promax allows factors to be correlated

SF-36 Physical Health



SF-36 Mental Health



SF-36 PCS and MCS

PCS = (PF_Z * .42402) + (RP_Z * .35119) + (BP_Z * .31754) +
(GH_Z * .24954) + (EF_Z * .02877) + (SF_Z * -.00753) + (RE_
* -.19206) + (EW_Z * -.22069)

MCS = (PF_Z * -.22999) + (RP_Z * -.12329) + (BP_Z * -.09731) +
(GH_Z * -.01571) + (EF_Z * .23534) + (SF_Z * .26876) +
(RE_Z * .43407) + (EW_Z * .48581)

T-score Transformation

PCS = (PCS_z*10) + 50

MCS = (MCS_z*10) + 50

SF-36 Factor Analysis in Singapore

	English		Chinese		United States	
	Physical	Mental	Physical	Mental	Physical	Mental
PF	0.60	0.14	0.75	0.03	0.85	0.12
RP	0.85	0.12	0.78	0.25	0.81	0.27
BP	0.46	0.53	0.53	0.51	0.76	0.28
GH	0.14	0.74	0.32	0.66	0.69	0.37
VT	0.15	0.84	0.16	0.83	0.47	0.64
SF	0.49	0.56	0.48	0.56	0.42	0.67
RE	0.77	0.18	0.62	0.36	0.17	0.78
MH	0.12	0.83	0.10	0.86	0.17	0.87

Cluster Analysis of Factor Loadings

Cluster 1

- Denmark
- France
- Germany
- Italy
- Netherlands
- Norway
- Spain
- Sweden
- United Kingdom
- United States

- **Cluster 2**
  - Japan
  - Singapore English
  - Singapore Chinese

Factor loadings taken from J Clin Epidemiol 1998: 51: 1159-65

Slide from Dr. Julian Thumboo

What Factor Analysis of SF-36 Tells Us

Patterns of subscale loadings vary among subgroups

Distinct scoring protocols may be needed for east versus western countries

Principal Components Analysis

- Component is linear combination of items
- First component accounts for as much of the total item variance as possible
- Second component accounts for as much variance as possible, but uncorrelated with the first component
- $C_1 = a_1 * x_1 + b_1 * x_2$
- $C_2 = a_2 * x_1 + b_2 * x_2$
- Mean of C_1 & C_2 = 0

Eigenvalues

$$S_{c_1}^2 = a_1^2 s_{x_1}^2 + b_1^2 s_{x_2}^2 + 2(a_1)(b_1)\, r_{x_1, x_2} s_{x_1} s_{x_2}$$

$$S_{c_2}^2 = a_2^2 s_{x_1}^2 + b_2^2 s_{x_2}^2 + 2(a_2)(b_2)\, r_{x_1, x_2} s_{x_1} s_{x_2}$$

$S_{c_1}^2$ is maximized

C_1 & C_2 are uncorrelated

$$a_1^2 + b_1^2 = a_2^2 + b_2^2$$

$$S_{c_1} + S_{c_2} = S_{x_1} + S_{x_2}$$

Correlations Between Component and Item

$$r_{1,c_1} = \frac{a_1 (s_{c_1})}{S_{x_1}}$$

$$r_{2,c_1} = \frac{b_1 (s_{c_1})}{S_{x_2}}$$

Common Factor Analysis

- Each item represented as a linear combination of unobserved common and unique factors

$$X_1 = a_1 F_1 + b_1 F_2 + e_1$$

$$X_2 = a_2 F_1 + b_2 F_2 + e_2$$

- F_1 and F_2 are standardized common factors
- a's and b's are factor loadings; e's are unique factors
- Factors are independent variables (components are dependent variables)

Hypothetical Factor Loadings, Communalities, and Specificities

	Factor Loadings		Communality	Specificity
Variable	F_1	F_2	h^2	u^2
X_1	0.511	0.782	0.873	0.127
X_2	0.553	0.754	0.875	0.125
X_3	0.631	-0.433	0.586	0.414
X_4	0.861	-0.386	0.898	0.102
X_5	0.929	-0.225	0.913	0.087
Variance explained	2.578	1.567	4.145	0.855
Percentage	51.6%	31.3%	82.9%	17.1%

From Afifi and Clark, Computer-Aided Multivariate Analysis, 1984, p. 338

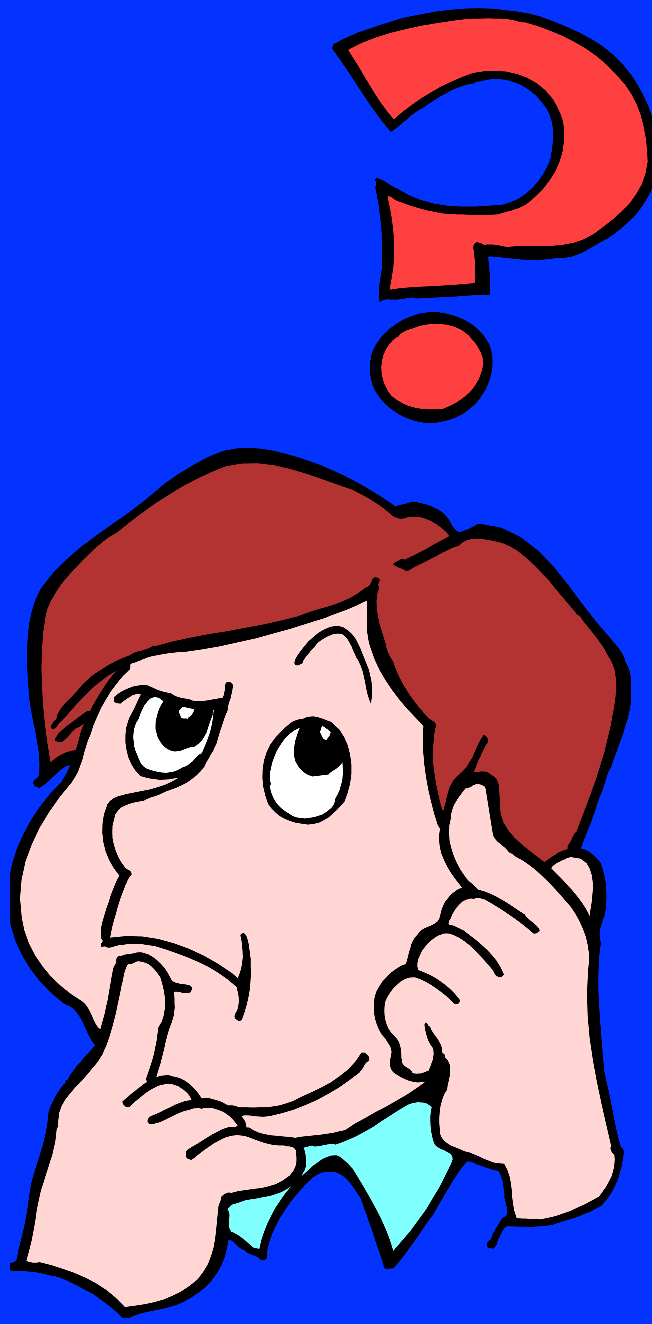
Ratio of Items/Factors & Cases/Items

At least 5
- items per factor
- 5 cases per item
- 5 cases per parameter estimate

Recommended Readings

Floyd, F. J., & Widaman, K. F. (1995). Factor analysis in the development and refinement of clinical assessment instruments. Psychological Assessment, 7, 286-299.

Fayers, P. M., & Machin, D. (1998). Factor analysis. In M. Staquet, R. **Hays**, & P. Fayers (eds.), Quality of Life Assessment in Clinical Trials: Methods and Practice (pp. 191-223). Oxford: Oxford University Press.



Finding and Verifying Item Clusters in Outcomes Research

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