Final Session of Summer: Psychometric Evaluation

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Four Types of Data Collection Errors

- Coverage Error
  Does each person in population have an equal chance of selection?

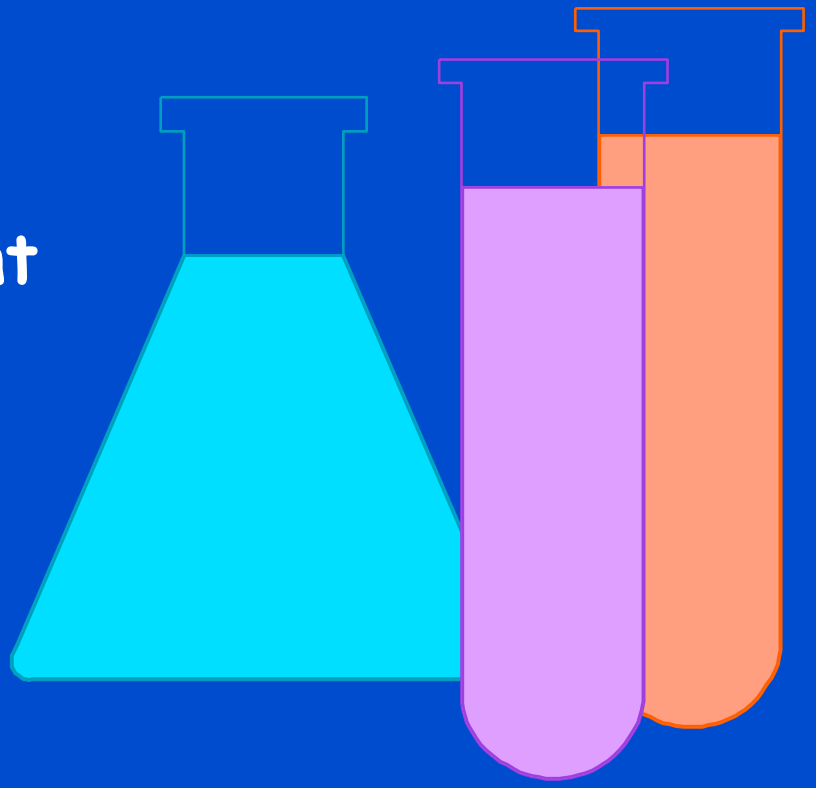
- Sampling Error
  Are only some members of the population sampled?

- Nonresponse Error
  Do people in the sample who respond differ from those who do not?

- Measurement Error
  Are inaccurate answers given to survey questions?

What's a Good Measure?

- Same person gets same score (reliability)
- Different people get different scores (validity)
- People get scores you expect (validity)
- It is practical to use (feasibility)



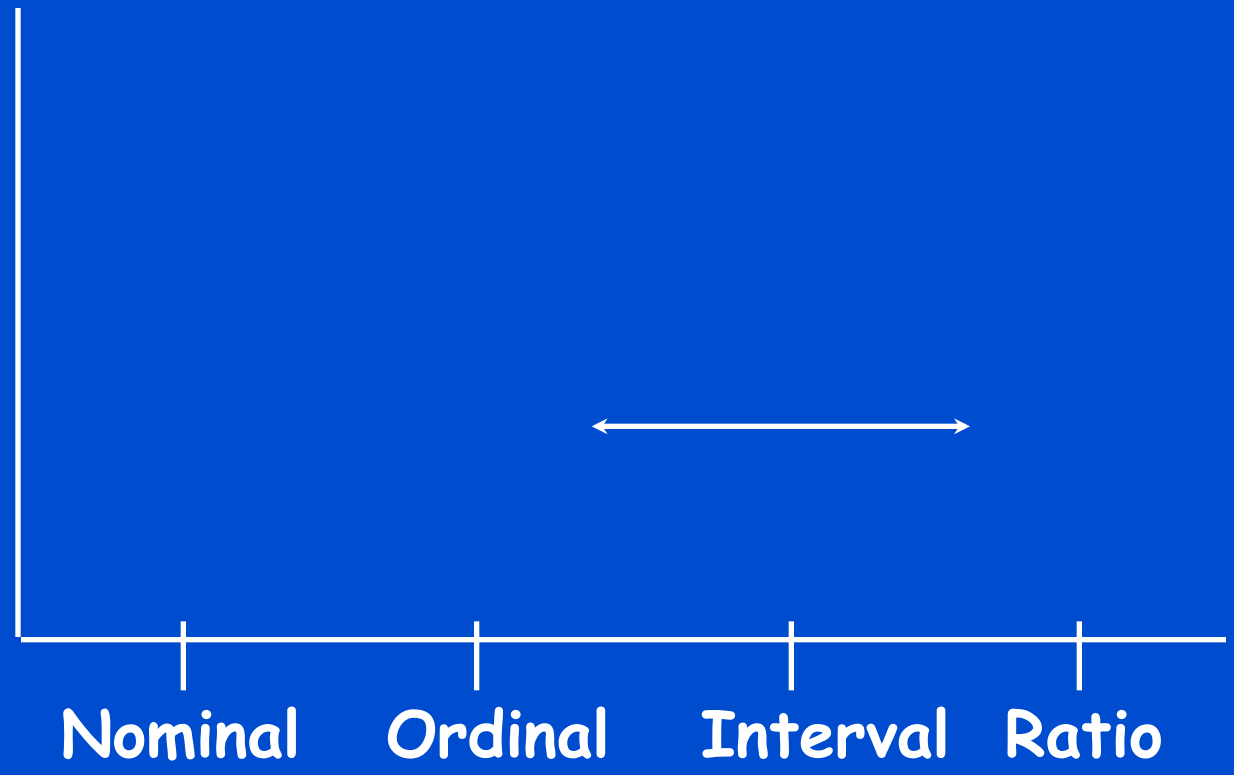
How Are Good Measures Developed?

- Review literature
- Expert input (patients and clinicians)
- Define constructs you are interested in
- Draft items (item generation)
- Pretest
  - Cognitive interviews
  - Field and pilot testing
- Revise and test again
- Translate/harmonize across languages

Scales of Measurement and Their Properties

Type of Scale	Property of Numbers: Rank Order	Equal Interval	Absolute 0
Nominal	No	No	No
Ordinal	Yes	No	No
Interval	Yes	Yes	No
Ratio	Yes	Yes	Yes

Measurement Range for Health Outcome Measures



Indicators of Acceptability

- Response rate
- Administration time
- Missing data (item, scale)

Variability

- All scale levels are represented
- Distribution approximates bell-shaped "normal"

Measurement Error

observed = true score + systematic error + random error

(bias)

Flavors of Reliability

- Inter-rater (rater)
- Equivalent forms (forms)
- Internal consistency (items)
- Test-retest (administrations)

Test-retest Reliability of MMPI 317-362
r = 0.75

MMPI 362 \ MMPI 317	True	False	
True	169	15	184
False	21	95	116
	190	110	

I am more sensitive than most other people.

Kappa Coefficient of Agreement (Corrects for Chance)

$$\text{kappa} = \frac{(\text{observed} - \text{chance})}{(1 - \text{chance})}$$

- Kappa can only reach 1.0 if the marginal distributions are equal
- Adjusted kappa divides kappa by the maximum possible given the marginals

Example of Computing KAPPA

		Rater A					
		1	2	3	4	5	Row Sum
Rater B	1	1	1				2
	2		2				2
	3			2			2
	4				2		2
	5					2	2
Column Sum		1	3	2	2	2	10

Example of Computing KAPPA (Continued)

$$P_c = \frac{(1 \times 2) + (3 \times 2) + (2 \times 2) + (2 \times 2) + (2 \times 2)}{(10 \times 10)} = \boxed{0.20}$$

$$P_{obs.} = \frac{9}{10} = \boxed{0.90}$$

$$\text{Kappa} = \frac{0.90 - 0.20}{1 - 0.20} = \boxed{0.87}$$

Guidelines for Interpreting Kappa

Conclusion	Kappa
Poor	< .40
Fair	.40 - .59
Good	.60 - .74
Excellent	> .74

Fleiss (1981)

Conclusion	Kappa
Poor	< 0.0
Slight	.00 - .20
Fair	.21 - .40
Moderate	.41 - .60
Substantial	.61 - .80
Almost perfect	.81 - 1.00

Landis and Koch (1977)

Ratings of Height of Houseplants

Plant		Baseline Height	Follow-up Height	Experimental Condition
A1				
	R1	120	121	1
	R2	118	120	
A2				
	R1	084	085	2
	R2	096	088	
B1				
	R1	107	108	2
	R2	105	104	
B2				
	R1	094	100	1
	R2	097	104	
C1				
	R1	085	088	2
	R2	091	096	

Ratings of Height of Houseplants (Cont.)

Plant		Baseline Height	Follow-up Height	Experimental Condition
C2				
	R1	079	086	1
	R2	078	092	
D1				
	R1	070	076	1
	R2	072	080	
D2				
	R1	054	056	2
	R2	056	060	
E1				
	R1	085	101	1
	R2	097	108	
E2				
	R1	090	084	2
	R2	092	096	

Reliability of Baseline Houseplant Ratings

Ratings of Height of Plants: 10 plants, 2 raters

Baseline Results

Source	DF	SS	MS	F
Plants	9	5658	628.667	35.52
Within	10	177	17.700	
Raters	1	57.8	57.800	
Raters x Plants	9	119.2	13.244	
Total	19	5835		

Sources of Variance in Baseline Houseplant Height

Source	dfs	MS	
Plants (N)	9	628.67	(BMS)
Within	10	17.70	(WMS)
Raters (K)	1	57.80	(JMS)
Raters x Plants	9	13.24	(EMS)
Total	19		

Intraclass Correlation and Reliability

Model	Reliability	Intraclass Correlation
One-Way	$\frac{MS_{BMS} - MS_{WMS}}{MS_{BMS}}$	$\frac{MS_{BMS} - MS_{WMS}}{MS_{BMS} + (K-1)MS_{WMS}}$
Two-Way Fixed	$\frac{MS_{BMS} - MS_{EMS}}{MS_{BMS}}$	$\frac{MS_{BMS} - MS_{EMS}}{MS_{EMS} + (K-1)MS_{EMS}}$
Two-Way Random	$\frac{N(MS_{BMS} - MS_{EMS})}{NMS_{BMS} + MS_{JMS} - MS_{EMS}}$	$\frac{MS_{BMS} - MS_{EMS}}{MS_{BMS} + (K-1)MS_{EMS} + K(MS_{JMS} - MS_{EMS})/N}$

Summary of Reliability of Plant Ratings

	Baseline		Follow-up	
	R_{TT}	R_{II}	R_{TT}	R_{II}
One-Way Anova	0.97	0.95	0.97	0.94
Two-Way Random Effects	0.97	0.95	0.97	0.94
Two-Way Fixed Effects	0.98	0.96	0.98	0.97

Source	Label	Baseline MS
Plants	BMS	628.667
Within	WMS	17.700
Raters	JMS	57.800
Raters X Plants	EMS	13.244

$$ICC(1,1) = \frac{BMS - WMS}{BMS + (K - 1) * WMS}$$

$$ICC(2,1) = \frac{BMS - EMS}{BMS + (K - 1) * EMS + K(JMS - EMS)/n}$$

$$ICC(3,1) = \frac{BMS - EMS}{BMS + (K - 1) * EMS}$$

Cronbach's Alpha

Source	df	SS	MS
Respondents (BMS)	4	11.6	2.9
Items (JMS)	1	0.1	0.1
Resp. x Items (EMS)	4	4.4	1.1
Total	9	16.1	

$$\text{Alpha} = \frac{2.9 - 1.1}{2.9} = \frac{1.8}{2.9} = \boxed{0.62}$$

Alpha by Number of Items and Inter-item Correlations

$$alpha_{st} = \frac{K\,\bar{r}}{1 + (K - 1)\,\bar{r}}$$

K = number of items in scale

Alpha for Different Numbers of Items and Homogeneity

Number of Items	Average Inter-item Correlation ($\bar{r}$)					
	.0	.2	.4	.6	.8	1.0
2	.000	.333	.572	.750	.889	1.000
4	.000	.500	.727	.857	.941	1.000
6	.000	.600	.800	.900	.960	1.000
8	.000	.666	.842	.924	.970	1.000

Number of Items and Reliability for Three Versions of the Mental Health Inventory (MHI)

Measure	Number of Items	Completion time (min.)	Reliability
MHI-32	32	5-8	.98
MHI-18	18	3-5	.96
MHI-5	5	1 or less	.90

Data from McHorney et al. 1992

Spearman-Brown Prophecy Formula

$$\text{alpha}_{y} = \left(\frac{N \cdot \text{alpha}_{x}}{1 + (N - 1) * \text{alpha}_{x}} \right)$$

N = how much longer scale y is than scale x

Reliability Minimum Standards

For Group Comparisons

-0.70 or above

For Individual Assessment

-0.90 or higher

Reliability of a Composite Score

$$\text{Mosier} = 1 - \frac{\Sigma(w_j^2)(S_j^2) - \Sigma(w_j^2)(S_j^2)(\alpha_j)}{\Sigma(w_j^2)(S_j^2) + 2\Sigma(w_j)(w_K)(S_j)(S_K)(r_{jK})}$$

w_j = weight given to component J

w_K = weight given to component K

S_j = standard deviation of J

α_j = reliability of J

r_{jK} = correlation between J and K

Hypothetical Multitrait/Multi-Item Correlation Matrix

	Trait #1	Trait #2	Trait #3
Item #1	0.80*	0.20	0.20
Item #2	0.80*	0.20	0.20
Item #3	0.80*	0.20	0.20
Item #4	0.20	0.80*	0.20
Item #5	0.20	0.80*	0.20
Item #6	0.20	0.80*	0.20
Item #7	0.20	0.20	0.80*
Item #8	0.20	0.20	0.80*
Item #9	0.20	0.20	0.80*

*Item-scale correlation, corrected for overlap.

Multitrait/Multi-Item Correlation Matrix for Patient Satisfaction Ratings

	Technical	Interpersonal	Communication	Financial
Technical				
1	0.66*	0.63†	0.67†	0.28
2	0.55*	0.54†	0.50†	0.25
3	0.48*	0.41	0.44†	0.26
4	0.59*	0.53	0.56†	0.26
5	0.55*	0.60†	0.56†	0.16
6	0.59*	0.58†	0.57†	0.23
Interpersonal				
1	0.58	0.68*	0.63†	0.24
2	0.59†	0.58*	0.61†	0.18
3	0.62†	0.65*	0.67†	0.19
4	0.53†	0.57*	0.60†	0.32
5	0.54	0.62*	0.58†	0.18
6	0.48†	0.48*	0.46†	0.24

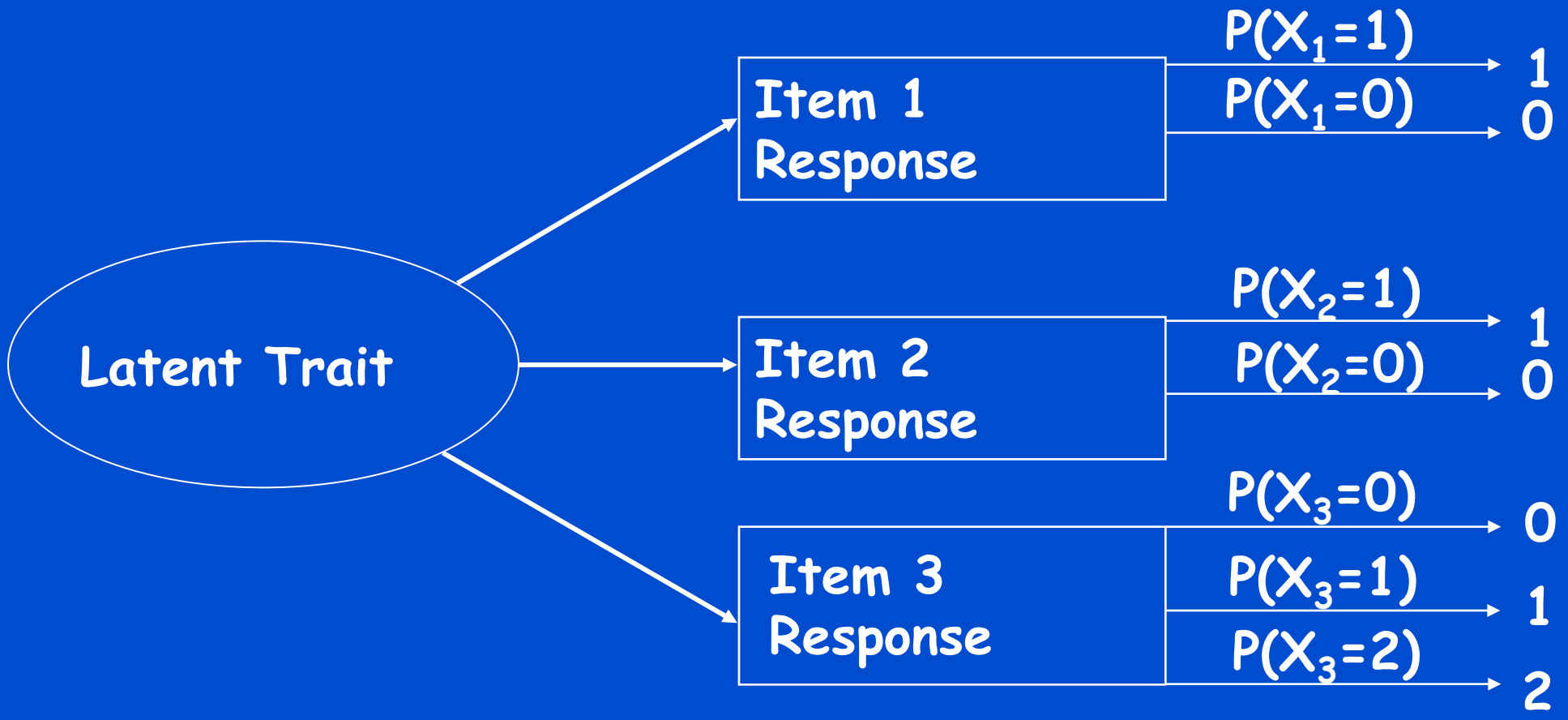
Note – Standard error of correlation is 0.03. Technical = satisfaction with technical quality. Interpersonal = satisfaction with the interpersonal aspects. Communication = satisfaction with communication. Financial = satisfaction with financial arrangements. *Item-scale correlations for hypothesized scales (corrected for item overlap). †Correlation within two standard errors of the correlation of the item with its hypothesized scale.

IRT

What are IRT Models?

Mathematical equations that relate observed survey responses to a persons location on an unobservable latent trait (i.e., intelligence, patient satisfaction).

Latent Trait and Item Responses



Types of IRT Models

- Unidimensional and multidimensional
- Dichotomous and polytomous
- Parameterization
  - One parameter: difficulty (location)
  - Two Parameter: difficulty and slope (discrimination)
  - Three Parameters: difficulty, slope, and guessing

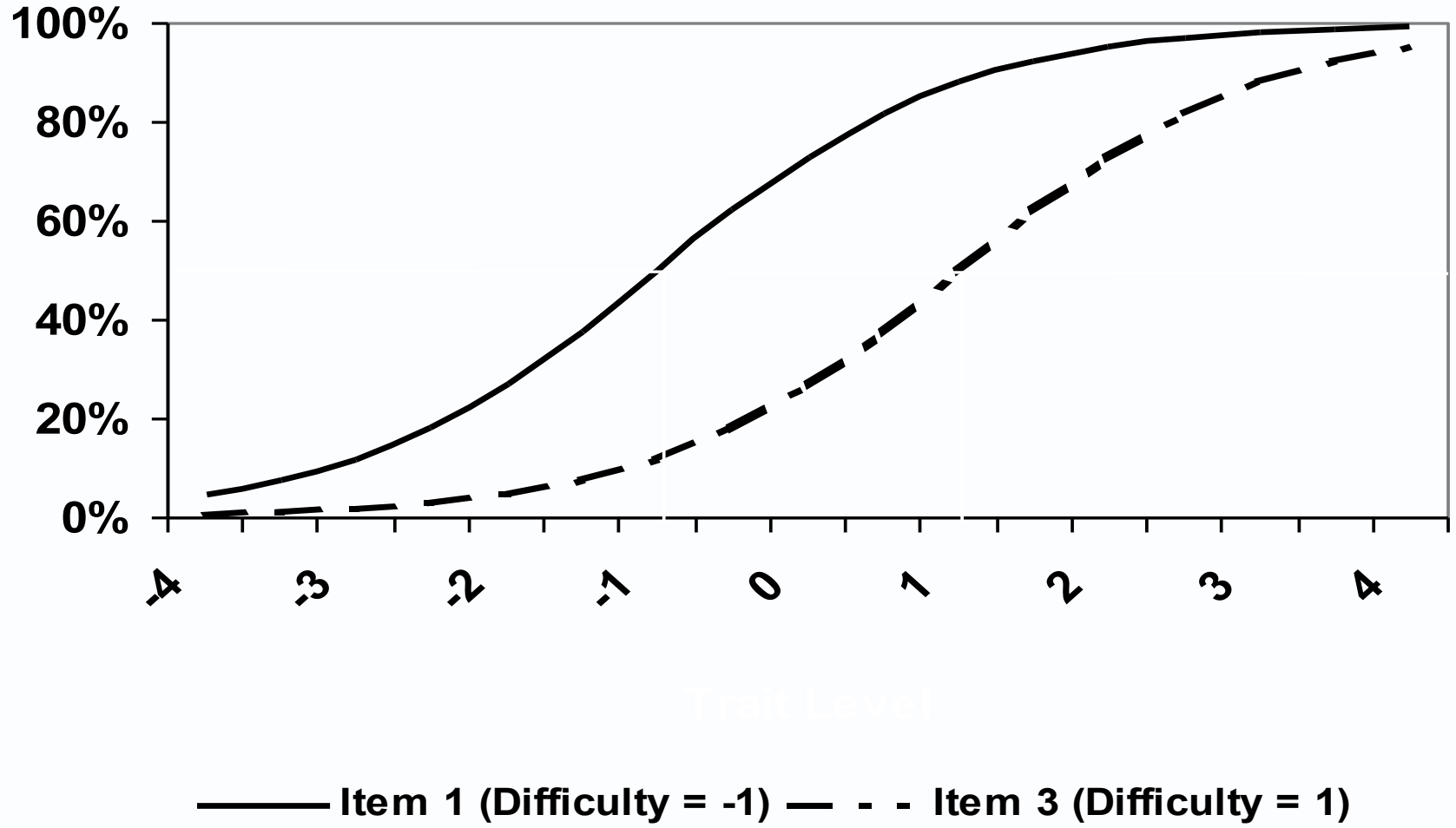
1-Parameter Logistic Model for (Dichotomous Outcomes)

$$P_i(\Theta) = \frac{e^{(\Theta - b_i)}}{1 + e^{(\Theta - b_i)}}$$

$P_i(\Theta)$ Probability that a randomly selected respondent with ability Θ (trait level) answers item i correctly.

b_i Item i difficulty.

Item Characteristic Curves (1-Parameter Model)



2-Parameter Logistic Model
(Dichotomous Outcomes)

$$P_i(\Theta) = \frac{e^{Da_i(\Theta - b_i)}}{1 + e^{Da_i(\Theta - b_i)}}$$

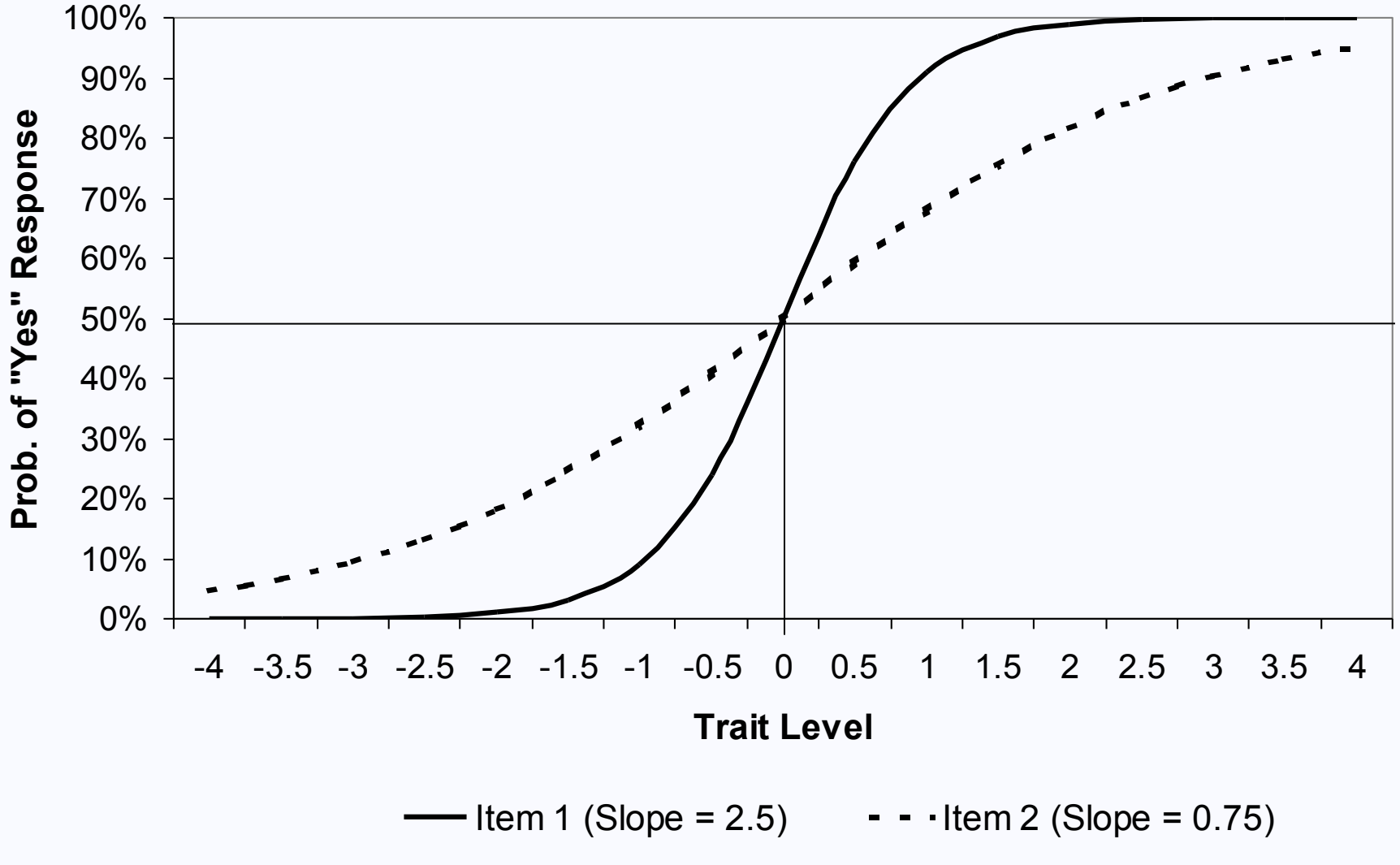
$P_i(\Theta)$ Probability that a randomly selected respondent with ability Θ (trait level) answers item i correctly.

b_i Item i difficulty.

a_i Item i slope.

D Scaling constant.

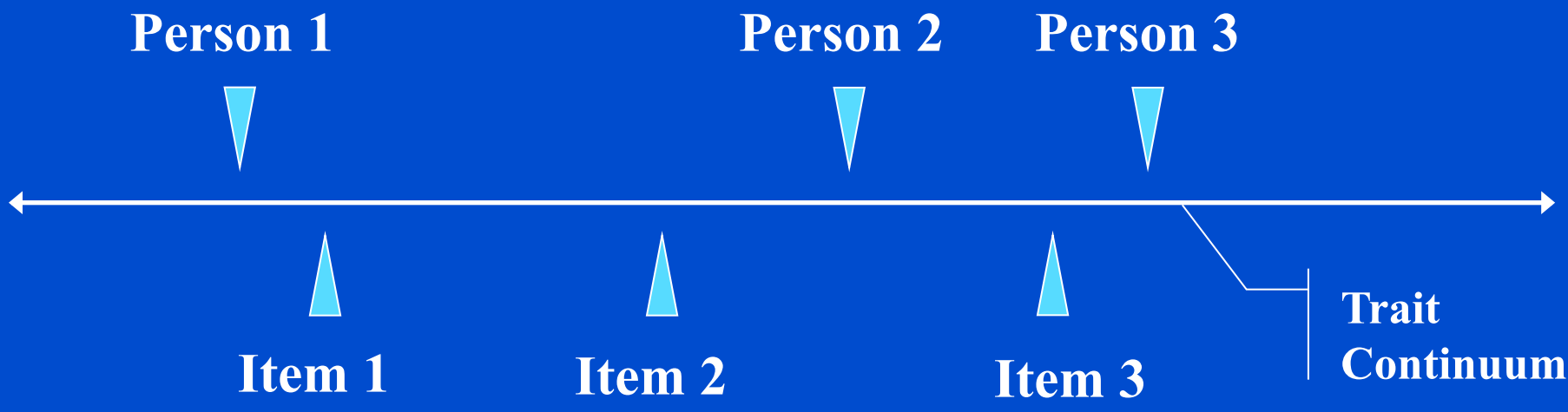
Item Characteristic Curves (2-Parameter Model)



IRT Model Assumptions

- Unidimensionality
  - One construct measured by items in scale.
- Local Independence
  - Items uncorrelated when latent trait(s) have been controlled for.

Item Responses and Trait Levels



Information Conditional on Trait Level

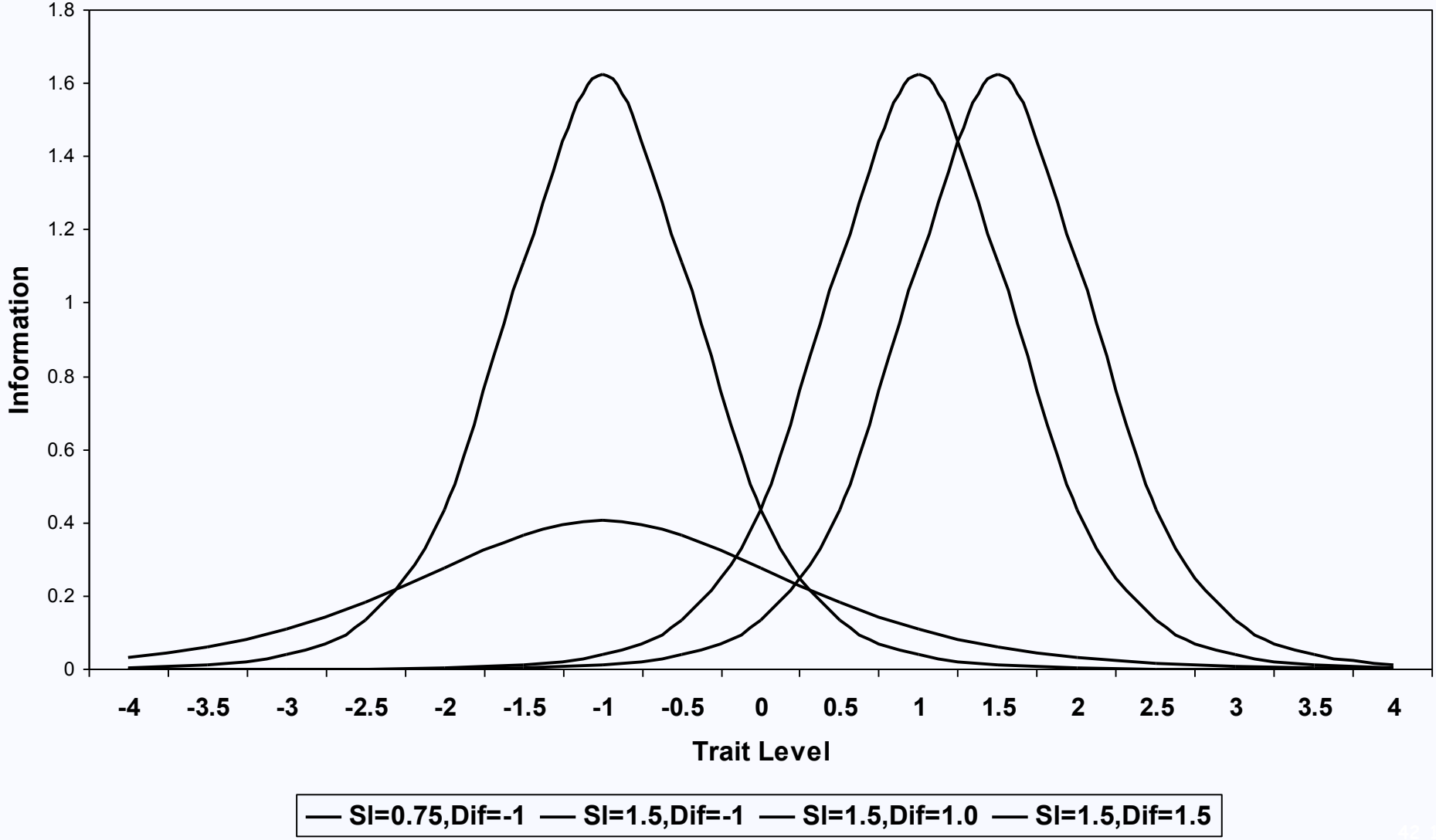
- Item information proportional to inverse of standard error:

$$SE(\Theta) = \frac{1}{\sqrt{I(\Theta)}}$$

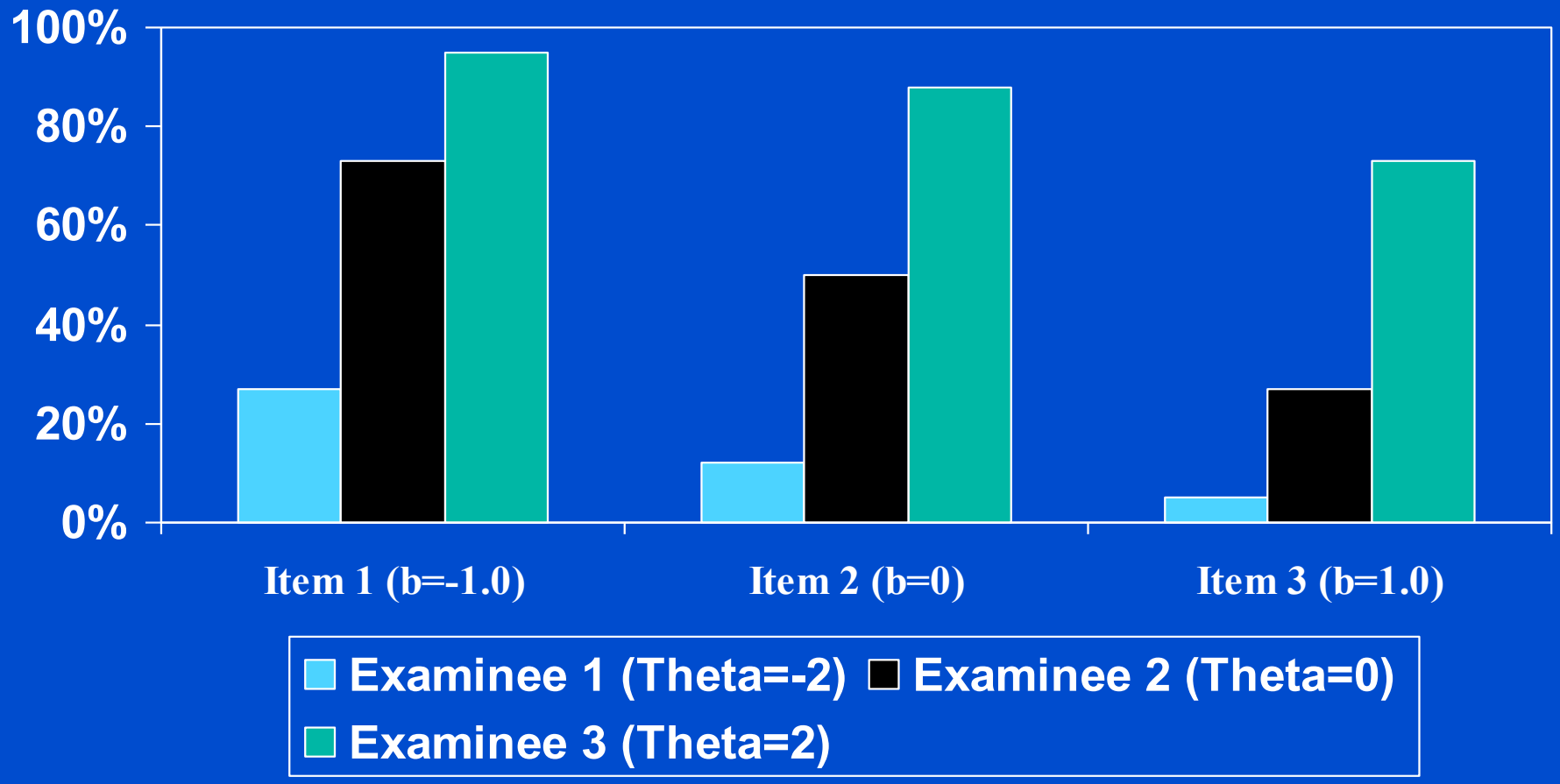
- Scale/Test information is the sum over item information:

$$I(\Theta) = \sum_{i=1}^{n} I_i(\Theta)$$

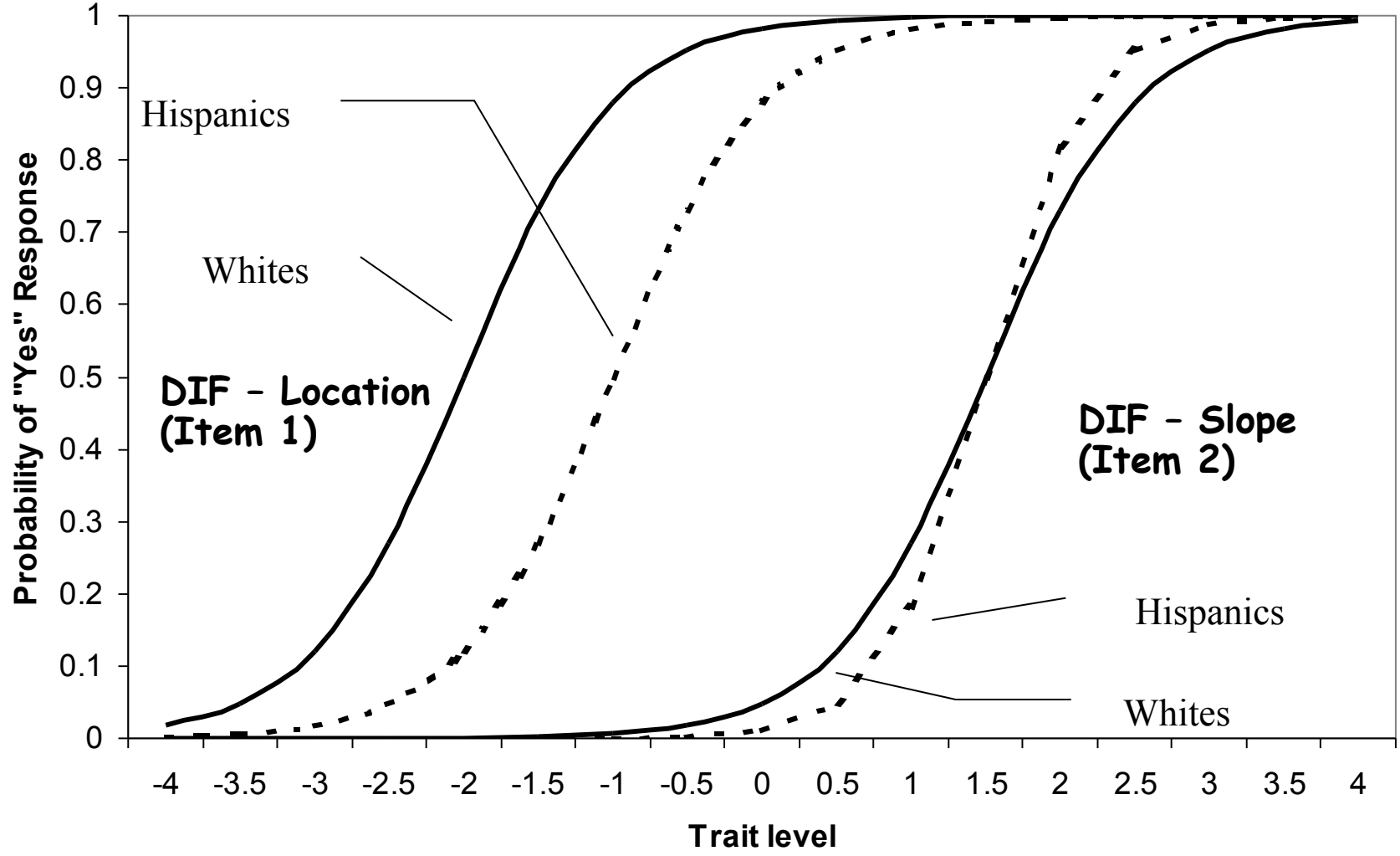
Item Information (2-parameter model)



Linking Item Content to Trait Estimates



Dichotomous Items Showing DIF
(2-Parameter Model)



Forms of Validity

- Content
- Criterion
- Construct Validity

Construct Validity

- Does measure relate to other measures in ways consistent with hypotheses?
- Responsiveness to change

Relative Validity Analyses

- Form of "known groups" validity
- Relative sensitivity of measure to important clinical difference
- One-way between group ANOVA

Relative Validity Example

	Severity of Heart Disease				
	None	Mild	Severe	F-ratio	Relative Validity
Scale #1	87	90	91	2	---
Scale #2	74	78	88	10	5
Scale #3	77	87	95	20	10

Responsiveness to Change and Minimally Important Difference

- HRQOL measures should be responsive to interventions that changes HRQOL

- Evaluating responsiveness requires assessment of HRQOL
  - pre-post intervention of known efficacy
  - at two times in tandem with gold standard

Two Essential Elements

- External indicator of change (Anchors)
  - mean change in HRQOL scores among people who have a "minimal" change in HRQOL.
- Amount of HRQOL change

External Indicator of Change (A)

Overall has there been any change in your asthma since the beginning of the study?

Much improved; Moderately improved; Minimally improved

No change

Much worse; Moderately worse; Minimally worse

External Indicator of Change (B)

Rate your overall condition. This rating should encompass factors such as social activities, performance at work or school, seizures, alertness, and functional capacity; that is, your overall quality of life.

7 response categories; ranging from no impairment to extremely severe impairment

External Indicator of Change (C)

- "changed" group = seizure free (100% reduction in seizure frequency)
- "unchanged" group = < 50% change in seizure frequency

Responsiveness Indices

(1) Effect size (ES) = D/SD

(2) Standardized Response Mean (SRM) = $D/SD^{\dagger}$

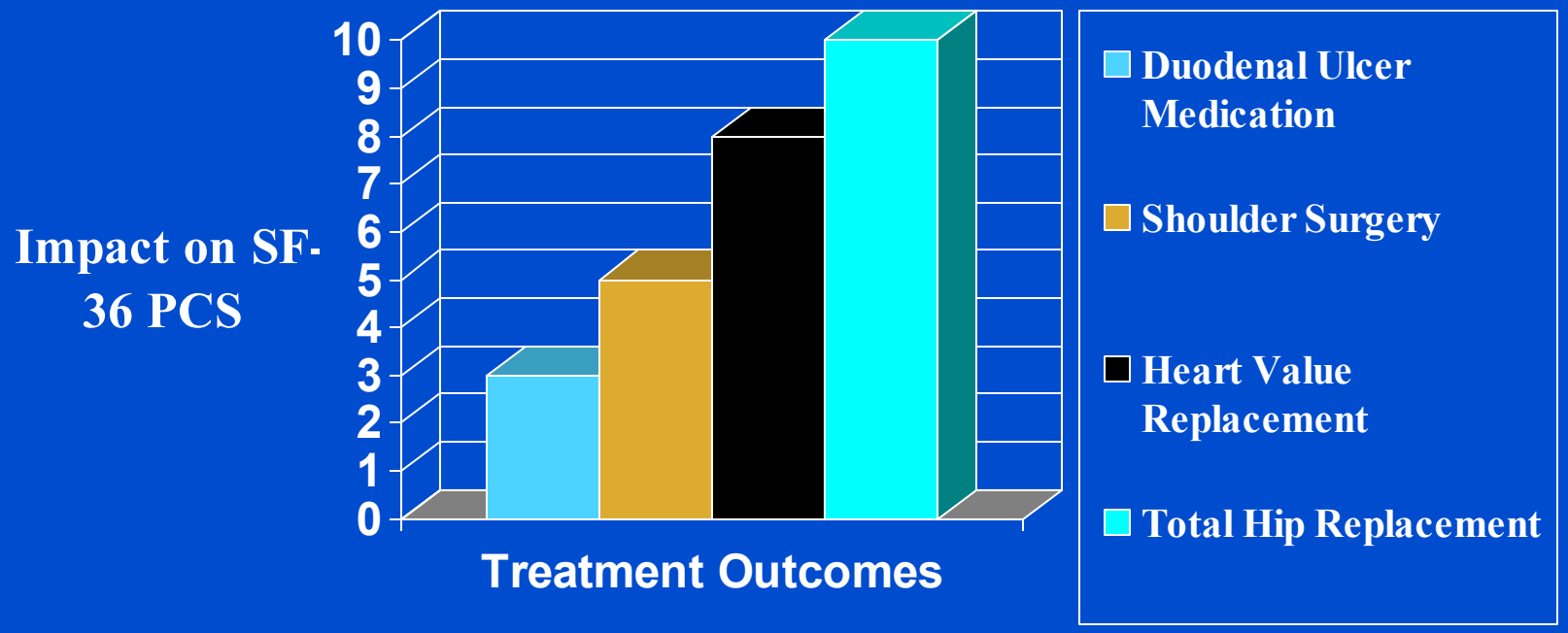
(3) Guyatt responsiveness statistic (RS) = $D/SD^{\ddagger}$

D = raw score change in "changed" group;
SD = baseline SD;
$SD^{\dagger}$ = SD of D;
$SD^{\ddagger}$ = SD of D among "unchanged"

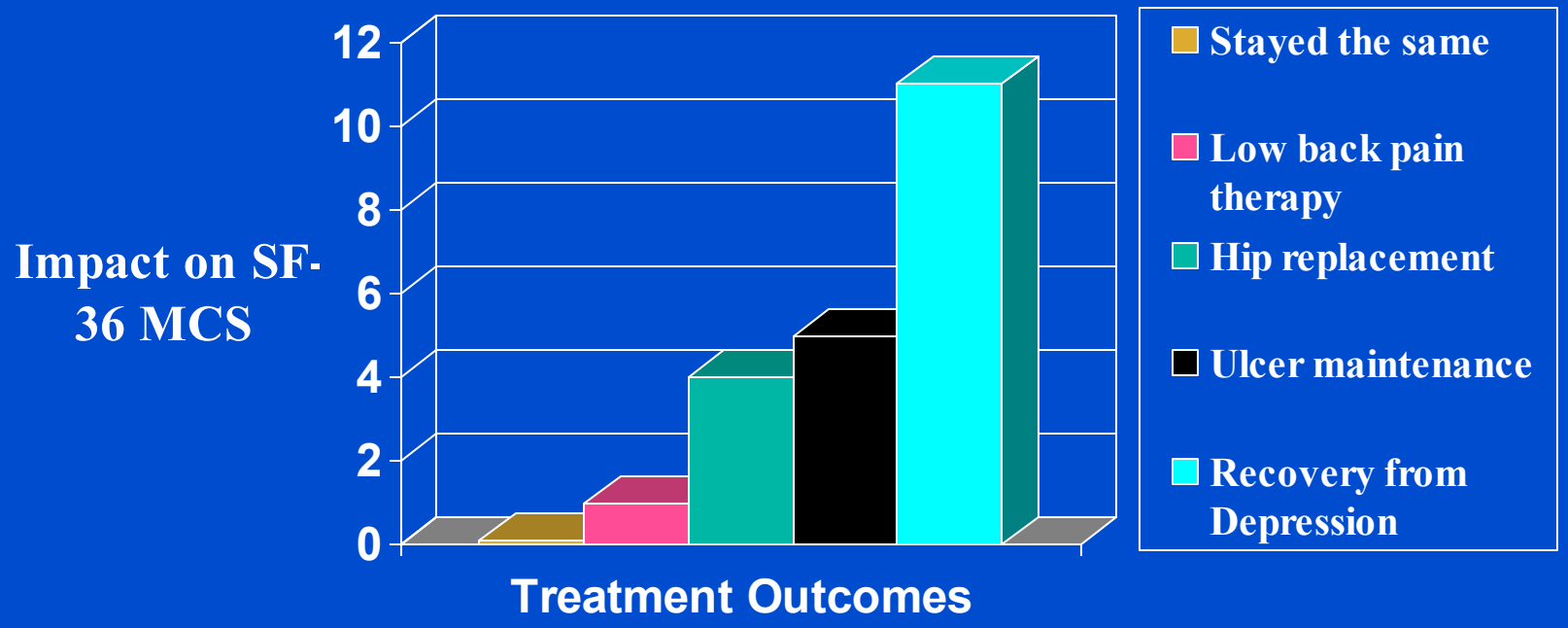
Effect Size Benchmarks

- Small: 0.20->0.49
- Moderate: 0.50->0.79
- Large: 0.80 or above

Treatment Impact on PCS



Treatment Impact on MCS



Treatment Impact on SF-12 MCS

- Prozac for depressed (3 points)
- Erythropoetin for dialysis patients (5 points)