Multitrait Scaling and IRT: Part I

Ron D. Hays, Ph.D.
(drhays@ucla.edu)

http://www.gim.med.ucla.edu/FacultyPages/Hays/

Questionnaire Design and Testing Workshop

2nd Floor Conference Room

December 4, 2007, 10 am-12 noon

Class

Kristina Cordasco

Jason Fish

Ying-Ying Goh

Corita Grudzen <-

Anish Mahajan

Veronica Meneses

Ana-Claire Meyer

Anisha Patel <-

Rashmi Shetgiri

<u>Benjamin Springgate (Ann Yochim)</u>

Multitrait Scaling Analysis

- Internal consistency reliability
  - Item convergence
- Item discrimination

Measurement Error

observed	=	true score	+	systematic error	+	random error
				(bias)		

Cronbach's Alpha

01 55
02 45
03 42
04 35
05 22

Source	df	SS	MS
Respondents (BMS)	4	11.6	2.9
Items (JMS)	1	0.1	0.1
Resp. x Items (EMS)	4	4.4	1.1
Total	9	16.1	

$$\text{Alpha} = \frac{2.9 - 1.1}{2.9} = \frac{1.8}{2.9} = \boxed{0.62}$$

Intraclass Correlation and Reliability

Model	Reliability	Intraclass Correlation
One-Way	$\frac{MS_{BMS} - MS_{WMS}}{MS_{BMS}}$	$\frac{MS_{BMS} - MS_{WMS}}{MS_{BMS} + (K-1)MS_{WMS}}$
Two-Way Fixed	$\frac{MS_{BMS} - MS_{EMS}}{MS_{BMS}}$	$\frac{MS_{BMS} - MS_{EMS}}{MS_{EMS} + (K-1)MS_{EMS}}$
Two-Way Random	$\frac{N(MS_{BMS} - MS_{EMS})}{NMS_{BMS} + MS_{JMS} - MS_{EMS}}$	$\frac{MS_{BMS} - MS_{EMS}}{MS_{BMS} + (K-1)MS_{EMS} + K(MS_{JMS} - MS_{EMS})/N}$

Alpha for Different Numbers of Items and Homogeneity

Number of Items (k)	Average Inter-item Correlation ($\bar{r}$) .0	.2	.4	.6	.8	1.0
2	.000	.333	.572	.750	.889	1.000
4	.000	.500	.727	.857	.941	1.000
6	.000	.600	.800	.900	.960	1.000
8	.000	.666	.842	.924	.970	1.000

$$\text{Alpha}_{st} = \frac{k * \bar{r}}{1 + (k - 1) * \bar{r}}$$

Spearman-Brown Prophecy Formula

$$\text{alpha}_y = \left(\frac{N \cdot \text{alpha}_x}{1 + (N - 1) * \text{alpha}_x} \right)$$

N = how much longer scale y is than scale x

Example Spearman-Brown Calculations

MHI-18

$$\frac{18/32\ (0.98)}{(1+(18/32\ -1)*0.98}$$

$= 0.55125/0.57125 = 0.96$

Number of Items and Reliability for Three Versions of the Mental Health Inventory (MHI)

Measure	Number of Items	Completion time (min.)	Reliability
MHI-32	32	5-8	.98
MHI-18	18	3-5	.96
MHI-5	5	1 or less	.90

From McHorney et al. 1992

Reliability Minimum Standards

0.70 or above (for group comparisons)

0.90 or higher (for individual assessment)

- SEM = SD $(1 - \text{reliability})^{1/2}$

Hypothetical Multitrait/Multi-Item Correlation Matrix

	Trait #1	Trait #2	Trait #3
Item #1	0.80*	0.20	0.20
Item #2	0.80*	0.20	0.20
Item #3	0.80*	0.20	0.20
Item #4	0.20	0.80*	0.20
Item #5	0.20	0.80*	0.20
Item #6	0.20	0.80*	0.20
Item #7	0.20	0.20	0.80*
Item #8	0.20	0.20	0.80*
Item #9	0.20	0.20	0.80*

*Item-scale correlation, corrected for overlap.

Multitrait/Multi-Item Correlation Matrix for Patient Satisfaction Ratings

	Technical	Interpersonal	Communication	Financial
Technical				
1	$\boxed{0.66*}$	0.63†	0.67†	0.28
2	$\boxed{0.55*}$	0.54†	0.50†	0.25
3	$\boxed{0.48*}$	0.41	0.44†	0.26
4	$\boxed{0.59*}$	0.53	0.56†	0.26
5	$\boxed{0.55*}$	0.60†	0.56†	0.16
6	$\boxed{0.59*}$	0.58†	0.57†	0.23
Interpersonal				
1	0.58	$\boxed{0.68*}$	0.63†	0.24
2	0.59†	$\boxed{0.58*}$	0.61†	0.18
3	0.62†	$\boxed{0.65*}$	0.67†	0.19
4	0.53†	$\boxed{0.57*}$	0.60†	0.32
5	0.54	$\boxed{0.62*}$	0.58†	0.18
6	0.48†	$\boxed{0.48*}$	0.46†	0.24

Note – Standard error of correlation is 0.03. Technical = satisfaction with technical quality. Interpersonal = satisfaction with the interpersonal aspects. Communication = satisfaction with communication. Financial = satisfaction with financial arrangements. *Item-scale correlations for hypothesized scales (corrected for item overlap). †Correlation within two standard errors of the correlation of the item with its hypothesized scale.

Confirmatory Factor Analysis

- Compares observed covariances with covariances generated by hypothesized model
- Statistical and practical tests of fit
- Factor loadings
- Correlations between factors
- Regression coefficients

Fit Indices

- Normed fit index: $$\frac{\chi^2_{null} - \chi^2_{model}}{\chi^2_{null}}$$
- Non-normed fit index: $$\frac{\dfrac{\chi^2_{null}}{df_{null}} - \dfrac{\chi^2_{model}}{df_{model}}}{\left(\dfrac{\chi^2_{null}}{df_{null}} - 1\right)}$$
- Comparative fit index: $$1 - \left(\frac{\chi^2_{model} - df_{model}}{\chi^2_{null} - df_{null}}\right)$$

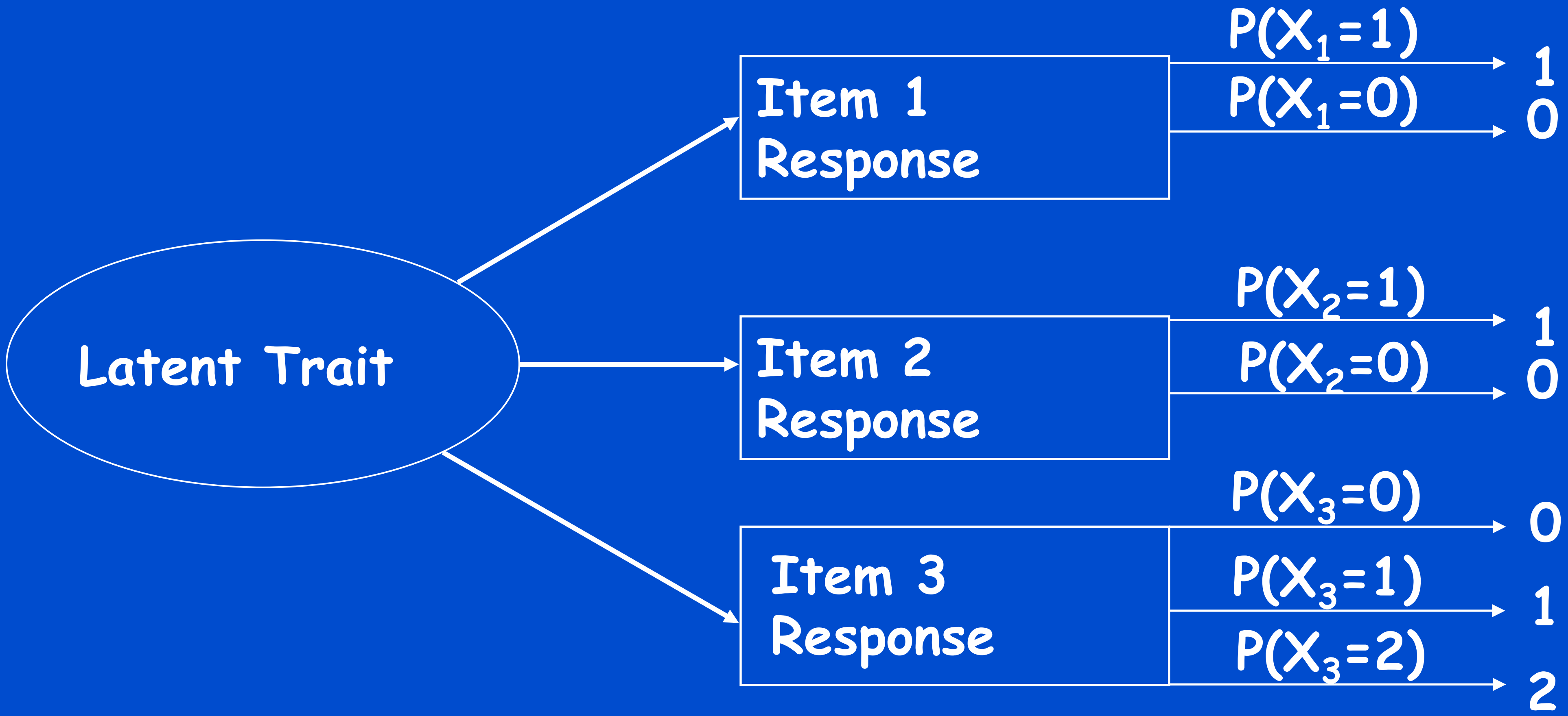
Three Steps in Exploratory Factor Analysis

Check correlation matrix for problems

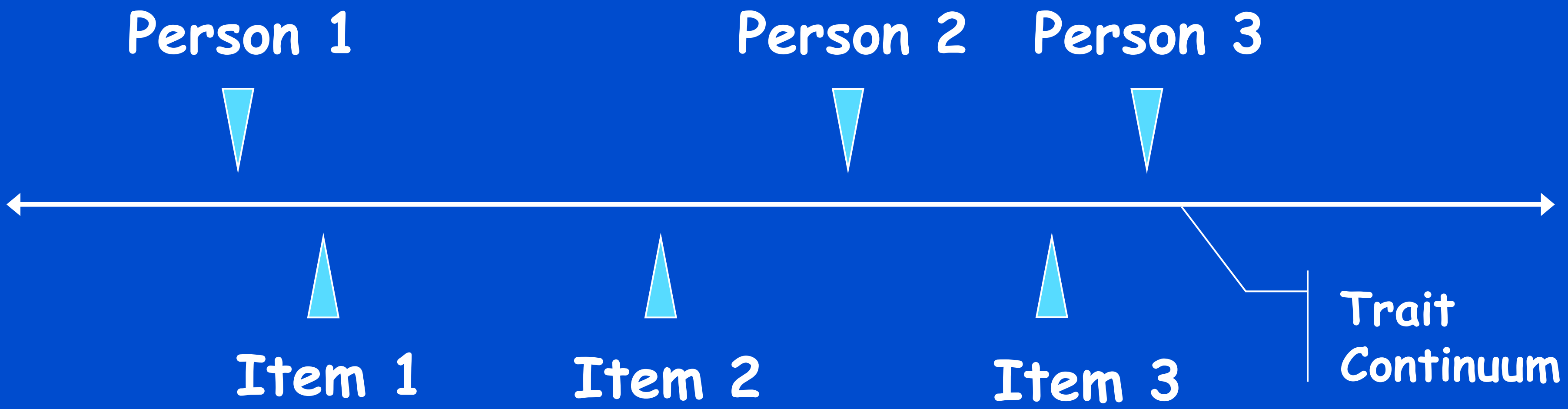
Identify number of dimensions or factors

Rotate to simple structure

Latent Trait and Item Responses



Item Responses and Trait Levels



Item Response Theory (IRT)

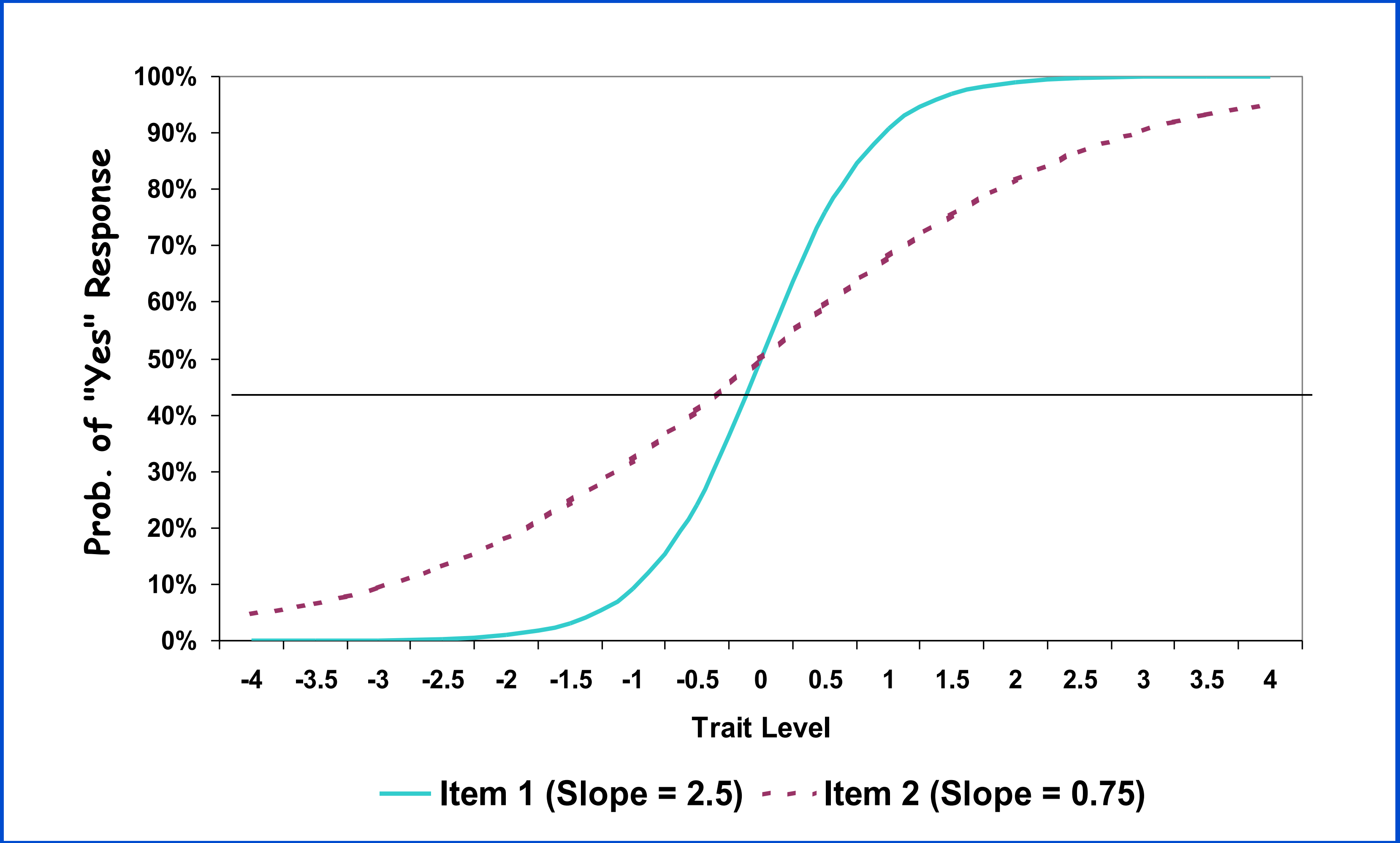
IRT models the relationship between a person's response Y_i to the question (i) and his or her level of the latent construct θ being measured by positing

$$\Pr(Y_i \geq k) = \frac{1}{1 + \exp(-a_i\theta + b_{ik})}$$

b_{ik} estimates how difficult it is for the item (i) to have a score of k or more and the discrimination parameter a_i estimates the discriminatory power of the item.

If for one group versus another at the same level θ we observe systematically different probabilities of scoring k or above then we will say that the item i displays DIF

Item Characteristic Curves (2-Parameter Model)



PROMIS Assessment Center

http://www.nihpromis.org/

http://www.assessmentcenter.net/ac1/

Appendix: Exploratory Factor Analysis

Check correlation matrix for problems

Identify number of dimensions or factors

Rotate to simple structure

http://www.gim.med.ucla.edu/FacultyPages/Hays/

Checking Correlation Matrix

Determinant of correlation matrix ranges between 0-1

Determinant = 0 if there is linear dependency in the matrix (singular, not positive definite, matrix has no inverse)

Determinant = 1 if all off diagonal elements in matrix are zero (identity matrix)

Partitioning of Variance Among Items

observed = Common + Specific + Error

Principal Components Analysis

Try to explain ALL variance in items, summarizing interrelations among items by smaller set of orthogonal principal components that are linear combinations of the items.

* First component is linear combination that explains maximum amount of variance in correlation matrix.

* Second component explains maximum amount of variance in residual correlation matrix.

Factor loadings represent correlation of each item with the component.

Eigenvalue (max = number of items) is sum of squared factor loadings for the component (column) and represents amount of variance in items explained by it.

Principal Components Analysis

- Standardize items: $Z_X = (X - \text{x-bar}) / SD_x$
- Use 1.0 as initial estimate of communality (variance in item explained by the factors) for each item
- Component is linear combination of items
- First component accounts for as much of the total item variance as possible
- Second component accounts for as much variance as possible, but uncorrelated with the first component
- $C_1 = a_1 * x_1 + b_1 * x_2$
- $C_2 = a_2 * x_1 + b_2 * x_2$
- Mean of C_1 & C_2 = 0

Common Factor Analysis

Factors are not linear combinations of items but are hypothetical constructs estimated from the items.

These factors are estimated from the common variance (not total) of the items and thus the diagonal elements (communality estimates) of the correlation is set to less than 1.0.

Common Factor Analysis

- Each item represented as a linear combination of unobserved common and unique factors

$$X_1 = a_1 F_1 + b_1 F_2 + e_1$$

$$X_2 = a_2 F_1 + b_2 F_2 + e_2$$

- F_1 and F_2 are standardized common factors
- a's and b's are factor loadings; e's are unique factors
- Factors are independent variables (components are dependent variables)

Hypothetical Factor Loadings, Communalities, and Specificities

	Factor Loadings		Communality	Specificity
Variable	F_1	F_2	h^2	u^2
X_1	0.511	0.782	0.873	0.127
X_2	0.553	0.754	0.875	0.125
X_3	0.631	-0.433	0.586	0.414
X_4	0.861	-0.386	0.898	0.102
X_5	0.929	-0.225	0.913	0.087
Variance explained	2.578	1.567	4.145	0.855
Percentage	51.6%	31.3%	82.9%	17.1%

From Afifi and Clark, Computer-Aided Multivariate Analysis, 1984, p. 338

Number of factors decision

Guttman's weakest lower bound

PCA eigenvalues > 1.0

Parallel analysis

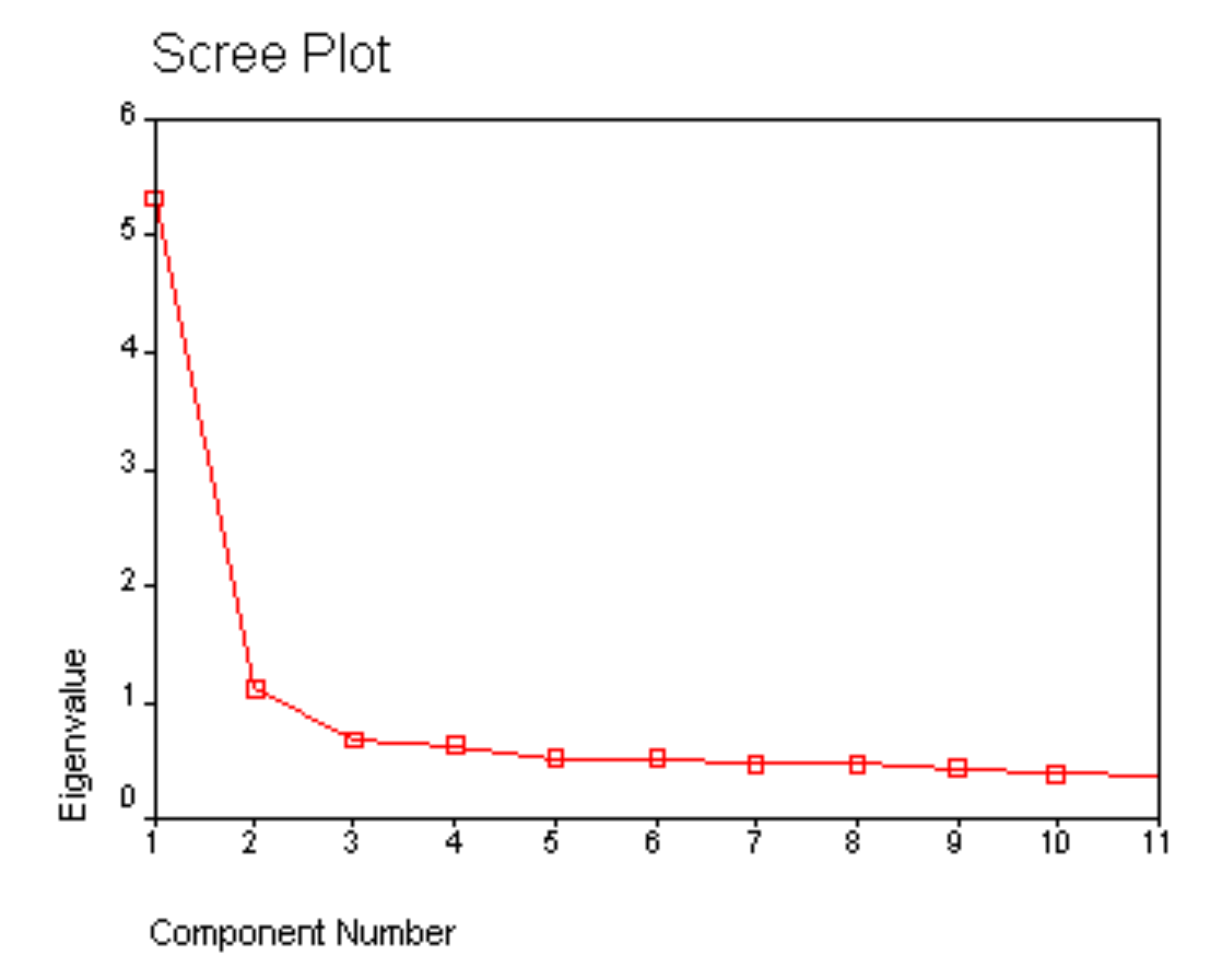
Scree test

ML and Tucker's rho

Parallel Analysis

```
PARALLEL.EXE: LATENT ROOTS OF RANDOM DATA CORRELATION MATRICES PROGRAM
PROGRAMMER: RON HAYS, RAND CORPORATION
FOR 3000 SUBJECTS AND 15 VARIABLES
*****************************************************************
Hays, R. D.  (1987).  PARALLEL:  A program for performing parallel
  analysis.  Applied Psychological Measurement, 11, 58.
*****************************************************************

EIGENVALUES FOR FACTOR ANALYSIS SMC ESTIMATES FOLLOW:

                 OBSERVED       RANDOM        SLOPE
                =========     =========     =========
LAMBDA    1= 7.790000     0.111727      ---------
                                        -6.880000
LAMBDA    2= 0.910000     0.084649      ---------
                                        -0.490000 ***
LAMBDA    3= 0.420000     0.068458      ---------
                                        -0.160000 ***
LAMBDA    4= 0.260000     0.057218      ---------
                                        -0.130000 ***
LAMBDA    5= 0.130000     0.043949      ---------
                                        -0.030000
LAMBDA    6= 0.100000     0.033773      ---------
                                        -0.095000 ***
LAMBDA    7= 0.005000     0.021966      ---------


(CAN'T COMPUTE LAMBDA 8 :LOG OF ZERO OR NEGATIVE IS UNDEFINED)

Results of Parallel Analysis Indicate Maximum of    6 Factors.
Slopes followed by asterisks indicate discontinuity points
that may be suggestive of the number of factors to retain.
```

Scree Test



Scree Plot
6
5
4
3
2
1
0
Eigenvalue
1
2
3
4
5
6
7
8
9
10
11
Component Number

ML and Tucker's rho

Significance Tests Based on 3000 Observations

Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors HA: At least one common factor	105	30632.0250	<.0001
H0: 4 Factors are sufficient HA: More factors are needed	51	937.9183	<.0001

Chi-Square without Bartlett's Correction	940.58422
Tucker and Lewis's Reliability Coefficient	0.94018

Factor Rotation

Unrotated factors are complex and hard to interpret

Rotation improves “simple” structure (more high and low loadings) and interpretability

Rotation

Communalities unchanged by rotation

Cumulative % of variance explained by common factors unchanged

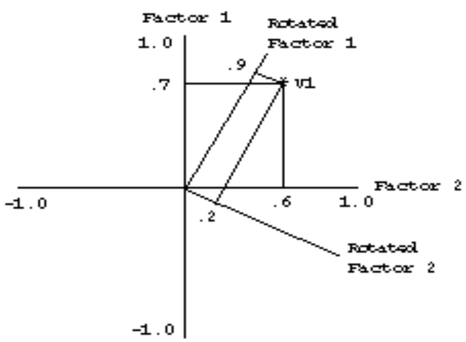
Varimax (orthogonal rotation) maximizes sum of squared factor loadings (after dividing each loading by the item's communality)

Promax allows factors to be correlated

Structure, pattern, and factor correlation matrix analyst wants to confirm the hypothesis or replicate the previous study, then a factor analysis with the prespecified number of factors can be run. The NFACTOR=*n* (or N=*n*) option in PROC FACTOR extracts the user-supplied number of factors. Ultimately, the criterion for determining the number of factors should be the replicability of the solution. It is important to extract only factors that can be expected to replicate themselves when a new sample of subjects is employed.

5. The Rotation of Factors

Once you decide on the number of factors to extract, the next logical step is to determine the method of rotation. The fundamental theorem of factor analysis is invariant within rotations. That is, the initial factor pattern matrix is not unique. We can get an infinite number of solutions, which produce the same correlation matrix, by rotating the reference axes of the factor solution to simplify the factor structure and to achieve a more meaningful and interpretable solution. The idea of simple structure has provided the most common basis for rotation, the goal being to rotate the factors simultaneously so as to have as many zero loadings on each factor as possible. The following figure is a simplified example of rotation, showing only one variable from a set of several variables.



The variable V1 initially has factor loadings (correlations) of .7 and .6 on factor 1 and factor 2 respectively. However, after rotation the factor loadings have changed to .9 and .2 on the rotated factor 1 and factor 2 respectively, which is closer to a simple structure and easier to interpret.

The simplest case of rotation is an *orthogonal rotation* in which the angle between the reference axes of factors are maintained at 90 degrees. More complicated forms of rotation allow the angle between the reference axes to be other than a right angle, i.e., factors are allowed to be correlated with each other. These types of rotational procedures are referred to as *oblique rotations*. Orthogonal rotation procedures are more commonly used than oblique rotation procedures. In some situations, theory may mandate that underlying latent constructs be uncorrelated with each other, and therefore oblique rotation procedures will not be appropriate. In other situations where the correlations between the underlying constructs are not assumed to be zero, oblique rotation procedures may yield simpler and more interpretable factor patterns.

Items/Factors and Cases/Items

At least 5

- items per factor
- cases per item
- cases per parameter estimate