

**Homework 2: RF Pulse Design**  
M229 Advanced Topics in MRI (Spring 2022)  
Assigned: 4/19/2022, Due: 5/6/2022 at 5 pm by email

1. Consider a nonselective excitation in which a constant pulse of amplitude  $B_1$  and duration  $\tau$  is applied in the presence of  $B_0$ . If the carrier frequency  $\omega$  is not exactly tuned to  $\omega_0$  (i.e.,  $\omega \neq \omega_0$ ), determine the resultant transverse magnetization output at time  $\tau$ . Use the small tip angle solution based in the rotating frame at the carrier frequency  $\omega$ .

## 2. Adiabatic Full Passage Pulse Design using Hyperbolic Secant Pulse Equations

Design an adiabatic RF pulse using the method outlined in class:

$$\begin{aligned} B_1(t) &= A(t)e^{-i\int \omega(t')t'} \\ A(t) &= A_0 \operatorname{sech}(\beta t) \\ \omega_1(t) &= -\mu\beta \tanh(\beta t) \end{aligned}$$

### a) amplitude modulation function

Design an amplitude modulation function,  $A\_t$ , with *sech*. Use  $\beta = 672$  [rad/s] and  $A_0 = 0.12$  [G]. Use 512 samples for the pulse duration of 10.24 ms.

```
>> beta = 672;
>> pulseWidth = 10.24; % in [ms]
>> A0 = 0.12 % in [G]
>> nSamples = 512; % the number of samples
>> dt = pulseWidth/nSamples/1000; % time step in [sec]
```

Plot the amplitude modulation function in *Gauss*.

```
>> plot(time, A_t); grid on;
>> title('Amplitude Modulation Function'); xlabel('Time (ms)'); ylabel('A(t) (G)');
```

### b) frequency modulation function

Design a frequency modulation function,  $w1\_t$ , with *tanh*. Use  $\mu = 5$  [dimensionless].

```
>> mu = 5; % [dimensionless]
```

Plot the frequency modulation function in *Hz*.

```
>> plot(time,w1_t); grid on;
>> title('Frequency Modulation Function'); xlabel('Time (ms)'); ylabel('\omega_1(t) (Hz)');
```

### c) inversion profile using Bloch simulation

Combine amplitude and frequency modulation functions:

```
>> rf_pulse = A_t .* exp(1i .* cumsum(w1_t)*dt);
```

Or, you can use phase modulation function instead:

$$\phi(t) = \mu \ln(\operatorname{sech} \beta t)$$

```
>> rf_pulse = A_t .* exp(1i .* phi);
```

Simulate the inversion profile over a sufficient range of frequency (e.g. -4,000 Hz to 4,000 Hz) using Bloch simulation. Plot the inversion profile.

```
>> plot(freq_range, mz);  
>> title('Inversion Profile'); xlabel('Frequency (Hz)'); ylabel('M_z'); grid on;
```

#### **d) inversion profiles with different B1+ variation**

Simulate the inversion profiles with different B1+ variations (60% attenuated, 30% attenuated, and 150% amplified pulses). Plot the inversion profiles.

### **3. 2D EPI Pulse Design**

Design a 2D separable EPI RF pulse using the method outlined in class.

#### **a) gradient design**

Design a blipped EPI trajectory with  $k_{x,max} = k_{y,max} = 0.5$  cycles/cm, and nine lines ( $L=9$ ). Assume the trapezoid ramps ( $\tau_R$ ) are 1/8 ms long, and the trapezoids themselves ( $\tau$ ) are 1 ms long. The blips are 1/4 ms long. What is the maximum amplitude of  $G_x$  and  $G_y$ ?

Sample the RF and gradient waveforms at 5 us (200 samples per trapezoid and 25 samples per trapezoid ramp). Include a refocusing lobe at the end to bring the trajectory back to  $k_x = k_y = 0$ , using 1 ms trapezoids on x and y. Plot the gradient waveforms,  $G_x$  and  $G_y$ , with the axes labeled.

#### **b) RF pulse design**

Design the RF waveform with TBW = 4 for the subpulses, and the envelope. This will produce a 4 cm by 4 cm excited volume.

```
>> tbw = 4;  
>> rf_fast = wsinc(tbw,samples);  
>> rf_slow = wsinc(tbw,L);
```

Apply the “flat-top only design” (RF only played flat part), and use the RF waveform to be zero during the refocusing gradient. Scale the RF to a flip angle of 1 radian (i.e.  $\sum(\text{rf}) = 1$ ). Plot the RF waveform in Gauss.

#### **c) 2D Bloch simulation**

Simulate the pulse over a sufficient range (e.g., -12cm to 12cm in x and y) at on-resonance. Plot the profile as an image using

```
>> imshow(abs(mxy),[]);
```

and cross-section plots along x ( $M_{xy}$  vs.  $x$ ) and y ( $M_{xy}$  vs.  $y$ )

```
>> subplot(211); plot(x,abs(mxy(:,round(length(y)/2))));
```

```
>> subplot(212); plot(y,abs(mxy(round(length(x)/2),:)));
```