Compressed Sensing

M229 Advanced Topics in MRI

Kyung Sung, Ph.D.

5/19/2022

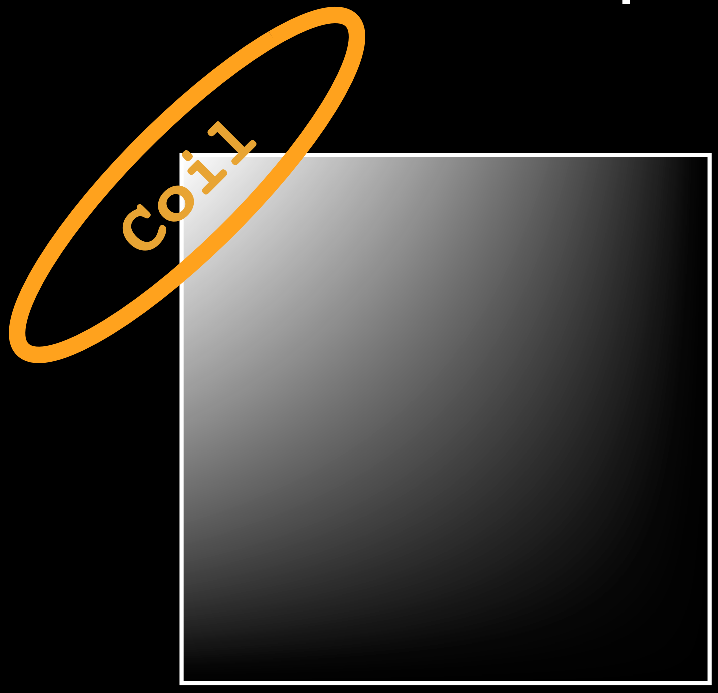
Today's Topics

- Parallel Imaging
  - SMASH review
  - Auto-SMASH
  - GRAPPA
- Compressed sensing
  - Compressibility or sparsity
  - Incoherent measurement
  - Reconstruction

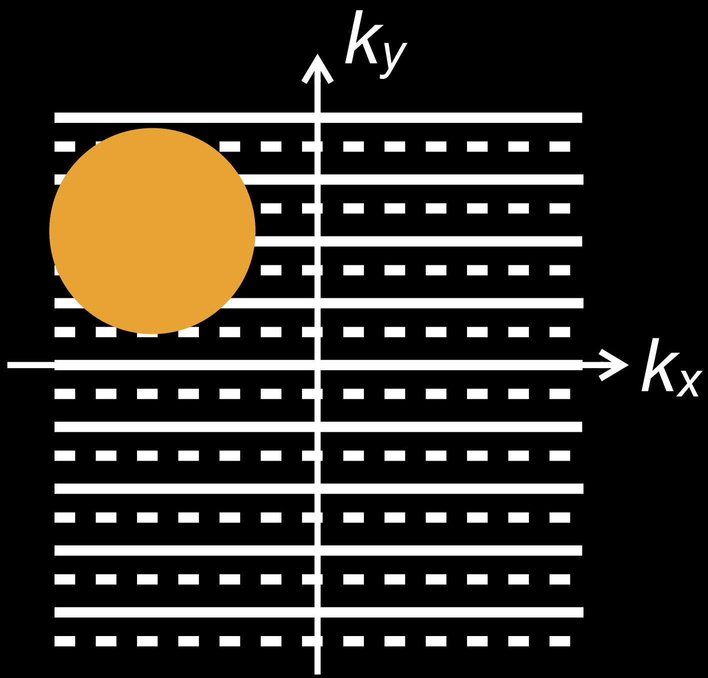
Parallel Imaging
(GRAPPA)

GRAPPA

- Coil sensitivities are
  - Smooth in image space
  - Local in k-space



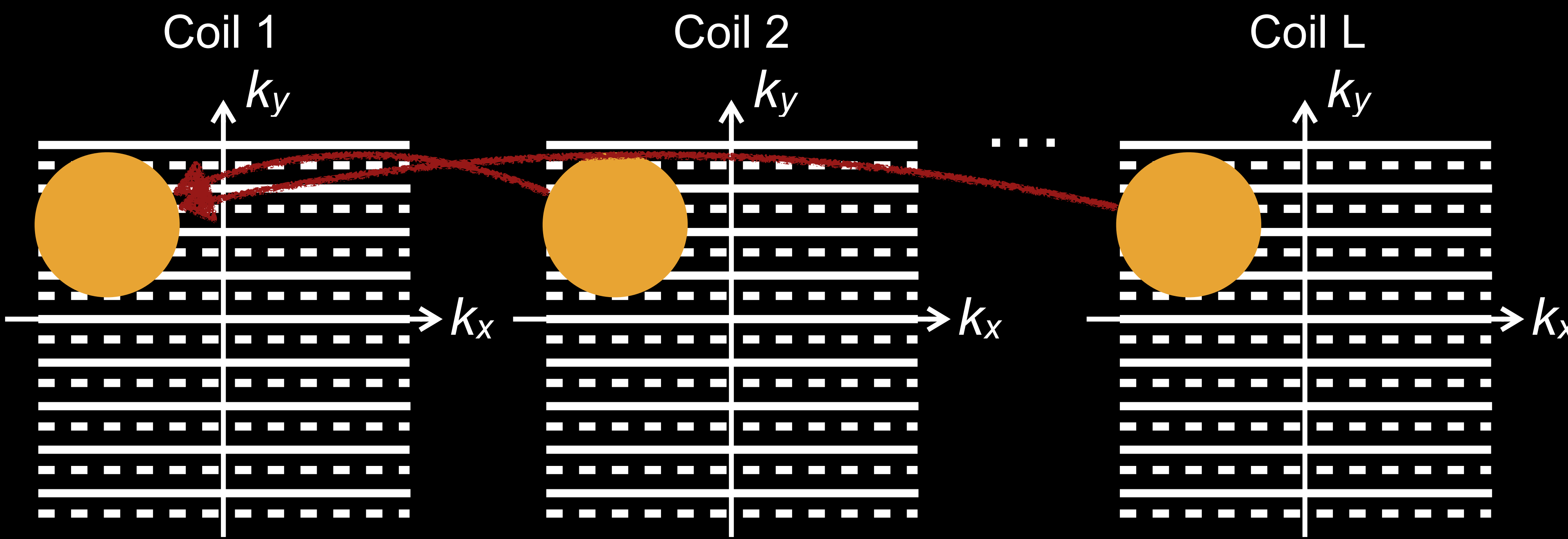
$m(\vec{x})C_j(\vec{x})$



$M(\vec{k}) * C_j(\vec{k})$

GRAPPA

- Missing information is implicitly contained by adjacent data



$\underline{M(\vec{k})} * C_1(\vec{k})$ $\underline{M(\vec{k})} * C_2(\vec{k})$ $\underline{M(\vec{k})} * C_L(\vec{k})$

GRAPPA Reconstruction

- How do we find missing data from these samples?

$$\underbrace{\hat{m}_k(k_x, k_y)}_{\text{missing data for each coil}} = \sum_{i,j,k} \underbrace{a_{i,j,k}}_{\text{weights}} \cdot \underbrace{m_k(k_x + i\Delta k_x, k_y + j\Delta k_y)}_{\text{neighborhood data for each coil}}$$

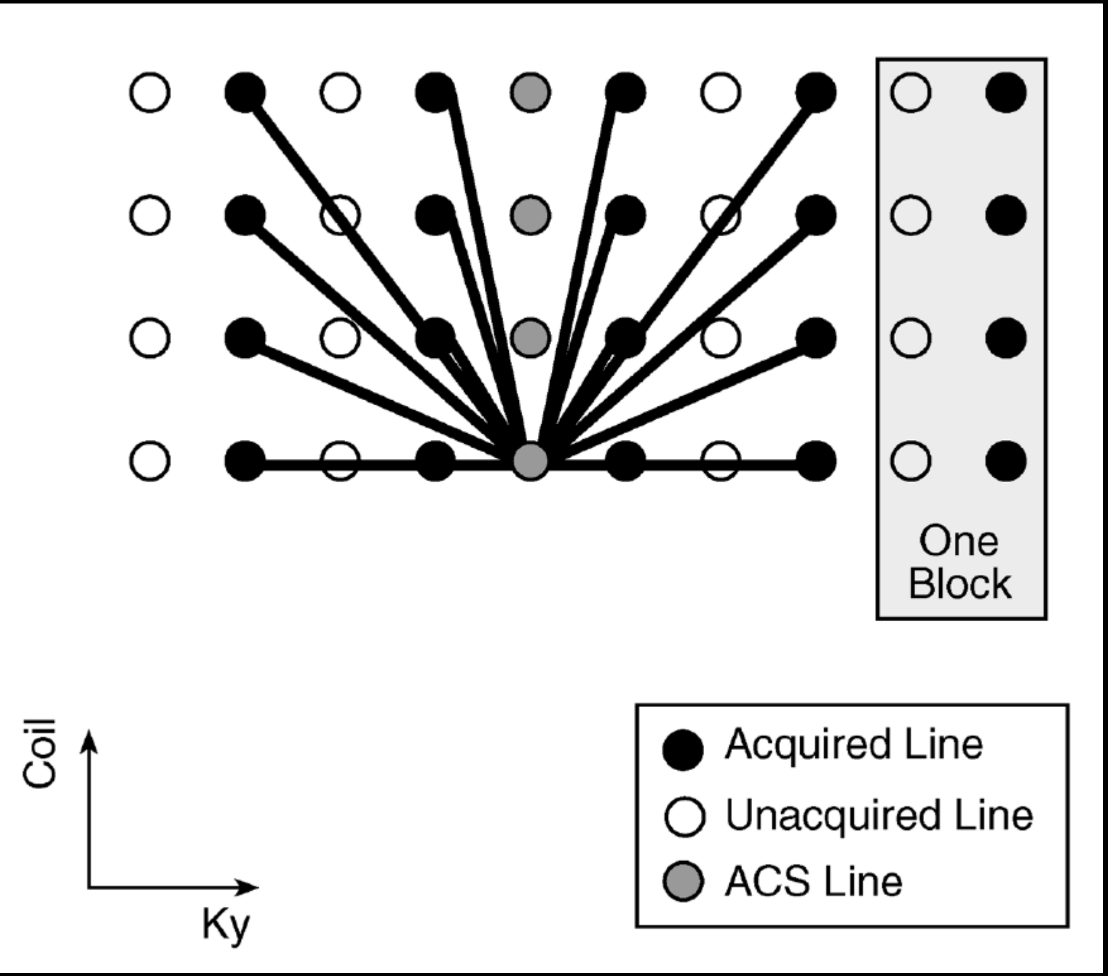
missing data for each coil

weights

neighborhood data for each coil

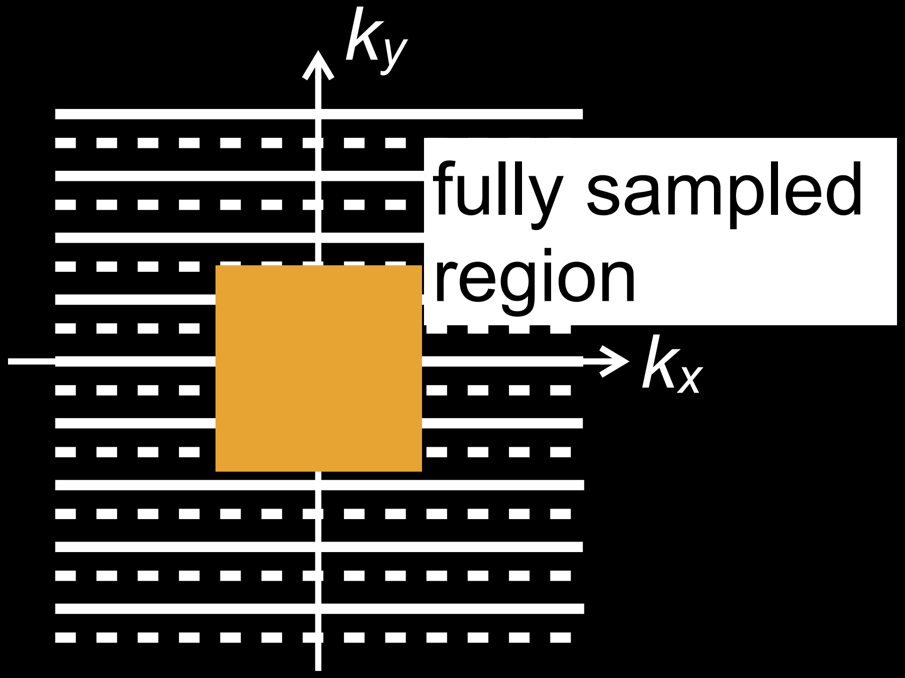
Auto-Calibration

$$\hat{m}_k(k_x, k_y) = \sum_{i,j,k} a_{i,j,k} \cdot m_k(k_x + i\Delta k_x, k_y + j\Delta k_y)$$



Auto-Calibration

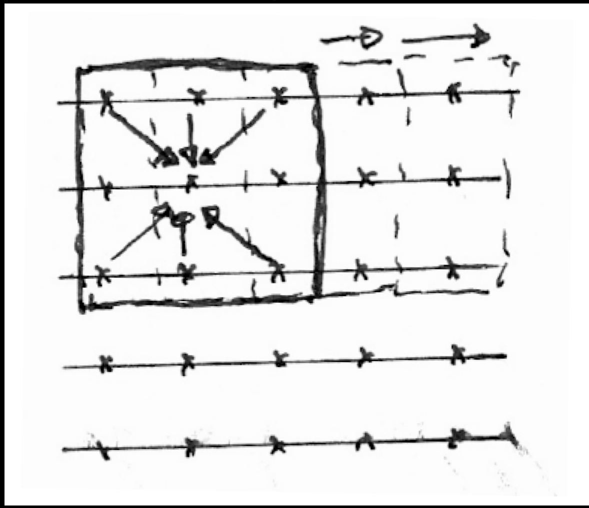
- Assume there is a fully sampled region
- We have samples of what the GRAPPA synthesis equations should produce



- Invert this to solve for GRAPPA weights

Auto-Calibration

- Calibration area has to be larger than the GRAPPA kernel
- Each shift of kernel gives another equation



- Here, 3x3 kernel, 5x5 calibration area gives 9 equations

Auto-Calibration

$$\hat{m}_k(k_x, k_y) = \sum_{i,j,k} a_{i,j,k} \cdot m_k(k_x + i\Delta k_x, k_y + j\Delta k_y)$$

- Write as a matrix equation

GRAPPA Coefficients

$$M_{k,c} = M_A \cdot a_k$$

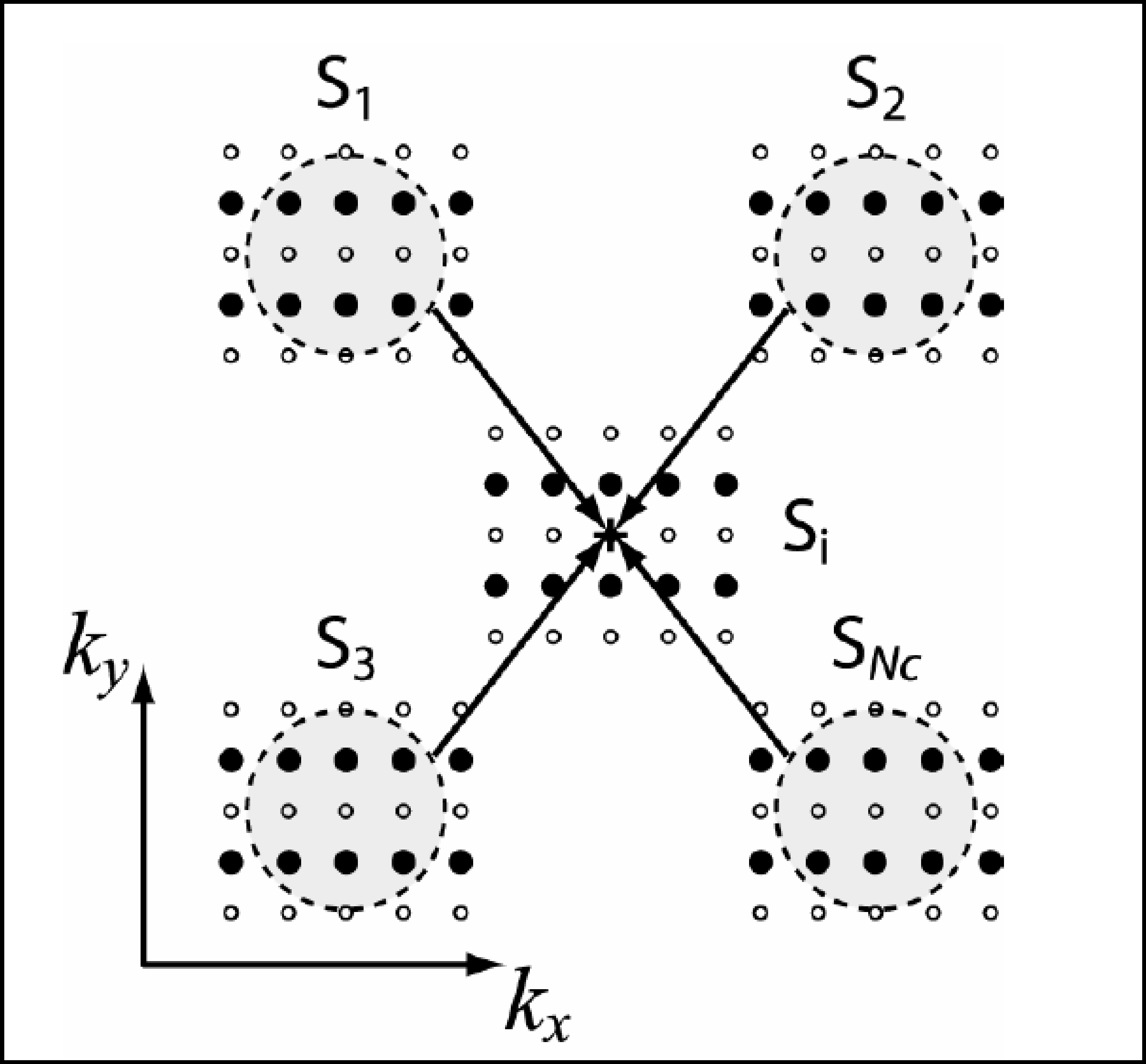
Calibration Data

Neighborhood Data

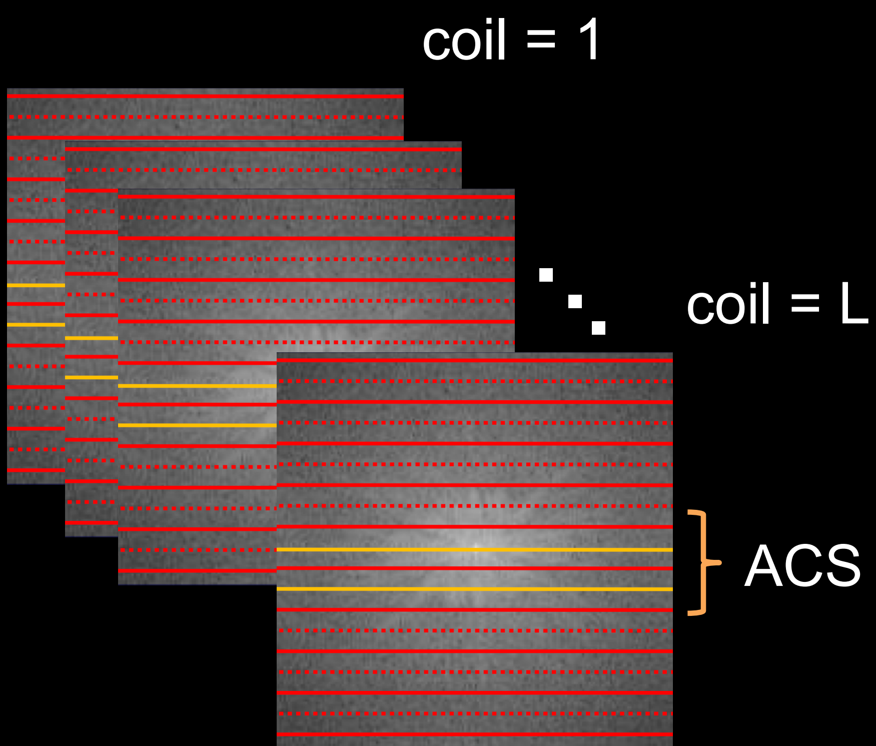
- GRAPPA weights are:

$$a_k = (M_A^* M_A + \lambda I)^{-1} M_A^* M_{k,c}$$

GRAPPA - Synthesis



Auto-Calibration Parallel Imaging



ACS (Auto-Calibration Signal) lines

$$\sum_{l=1}^{L} S_l^{ACS}(k_y - m\Delta k_y) = \sum_{l=1}^{L} n(l, m) S_l(k_y)$$

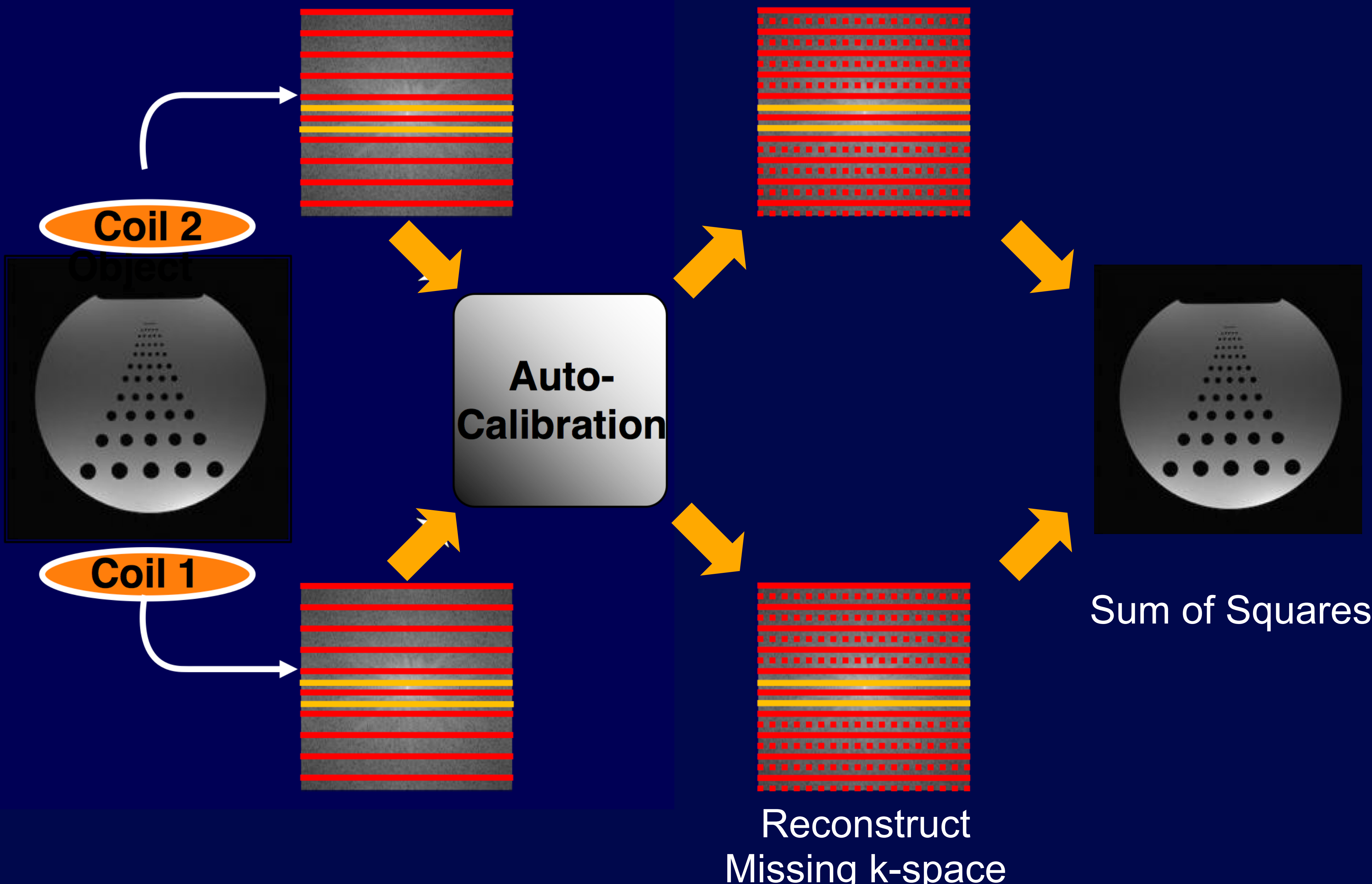
GRAPPA formula to reconstruct signal in one channel

$$S_j(k_y - m\Delta k_y) = \sum_{l=1}^{L} \sum_{b=0}^{N_b-1} n(j, b, l, m) S_l(k_y - bA\Delta k_y)$$

A: Acceleration factor
n(j,b,l,m): GRAPPA weights

Griswold et al. MRM, 47(6):1202-1210 (2002)

GRAPPA Reconstruction

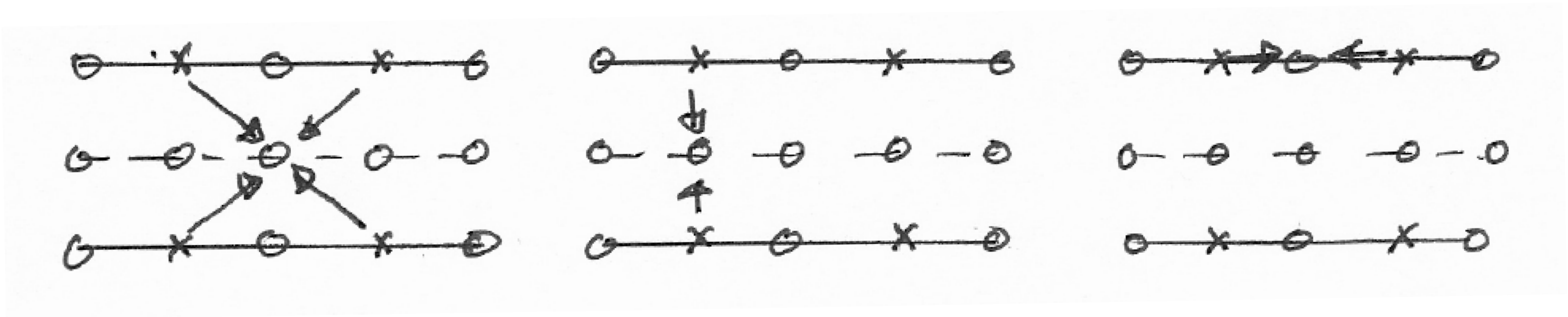


GRAPPA

- Compute GRAPPA weights from calibration region
- Compute missing k-space data using the GRAPPA weights
- Reconstruct individual coil images
- Combine coil images

Considerations of GRAPPA

- Calibration region size
- GRAPPA kernel size
- Sample geometry dependence



GRAPPA

- Compute GRAPPA weights from calibration region
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Summary

- Parallel imaging utilizes coil sensitivities to increase the speed of MRI
- Cases for parallel imaging
  - Higher patient throughput,
  - Real-time imaging/Interventional imaging
  - Motion suppression
- Cases against parallel imaging
  - SNR starving applications

Fast MRI Techniques

- Many reconstruction methods minimize aliasing artifacts by exploiting *information redundancy* (or *prior knowledge*)
  - Parallel imaging
  - Compressed sensing

Donoho, IEEE TIT, 2006
Candes et al., Inverse Problems, 2007

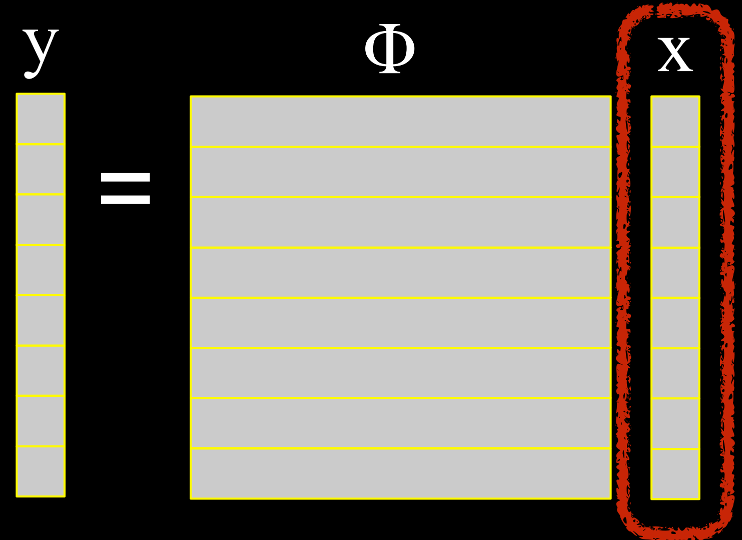
What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

8 Equations
8 Unknowns

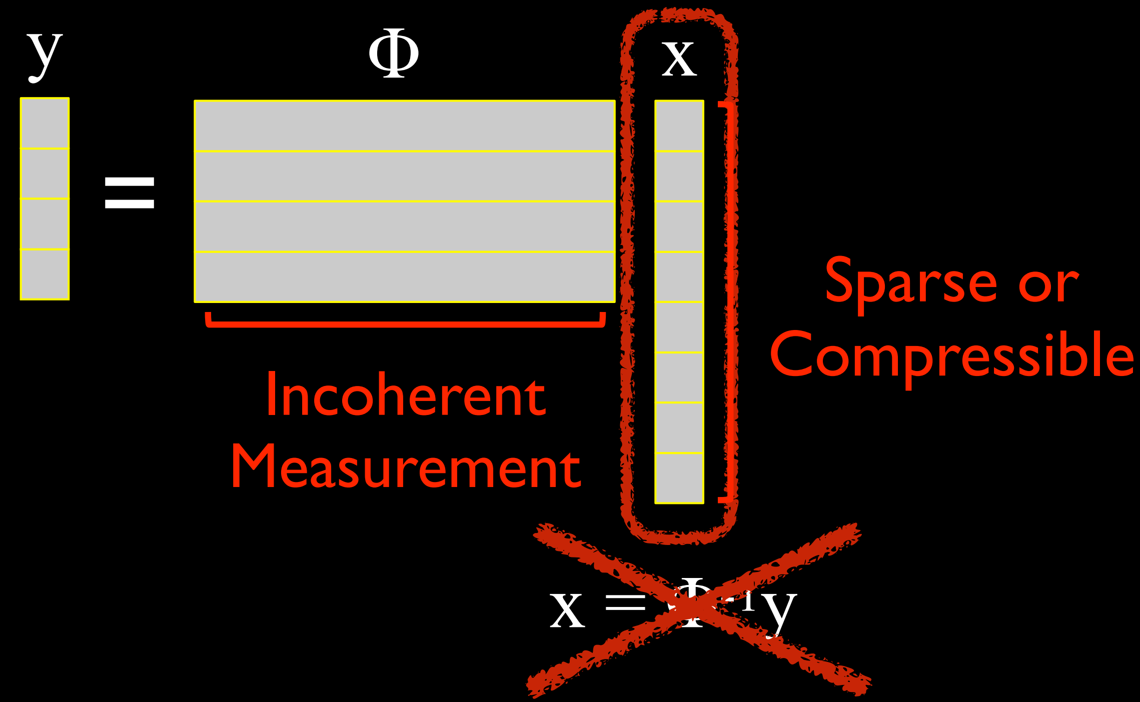


$$x = \Phi^{-1} y$$

What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

4 Equations
8 Unknowns



What is Compressed Sensing?

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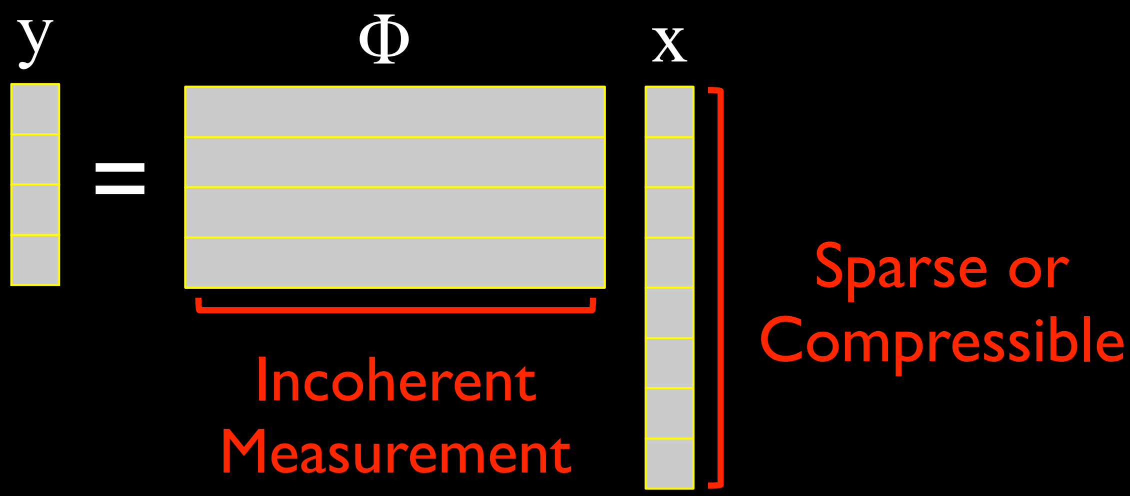
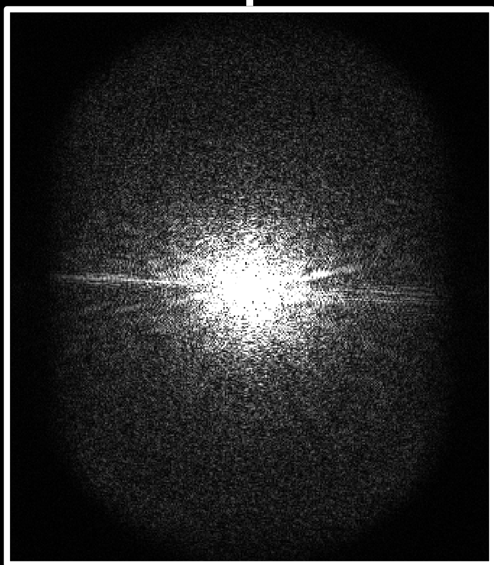
4 Equations
8 Unknowns



We still can find 8 unknowns!

Compressed Sensing MRI

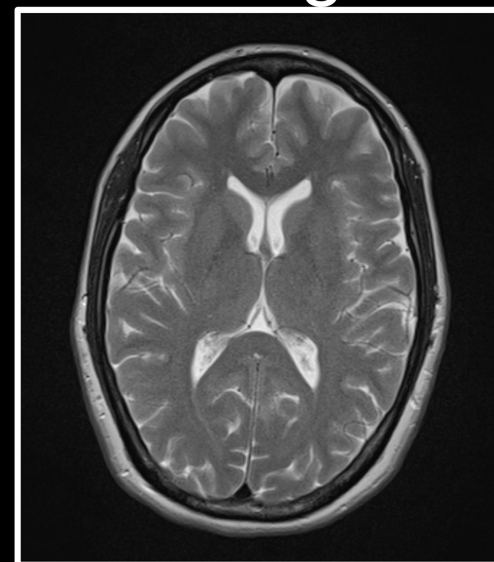
k-space



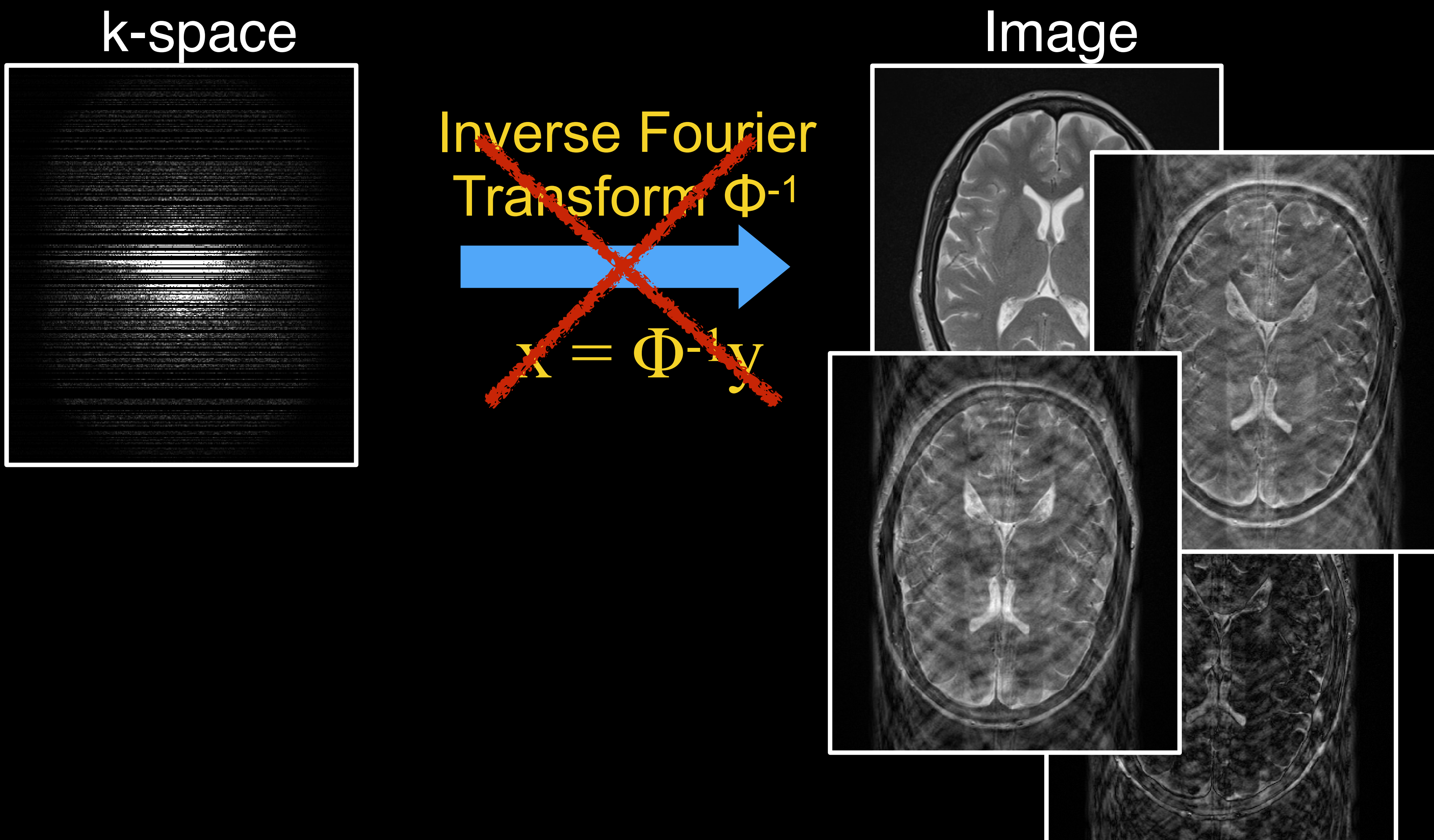
Inverse Fourier Transform Φ^{-1}

$$x = \Phi^{-1} y$$

Image



Compressed Sensing MRI



Compressed Sensing MRI

k-space

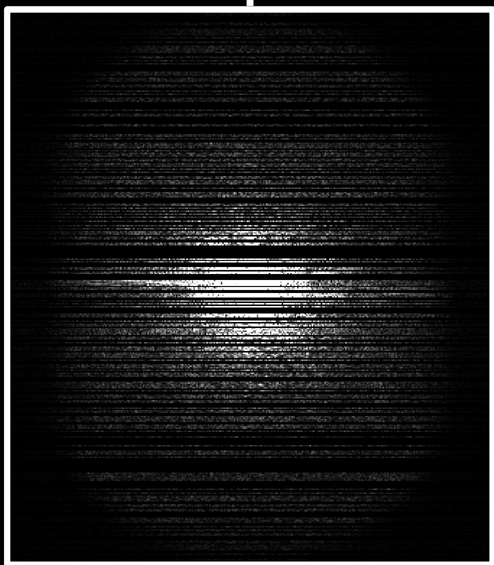


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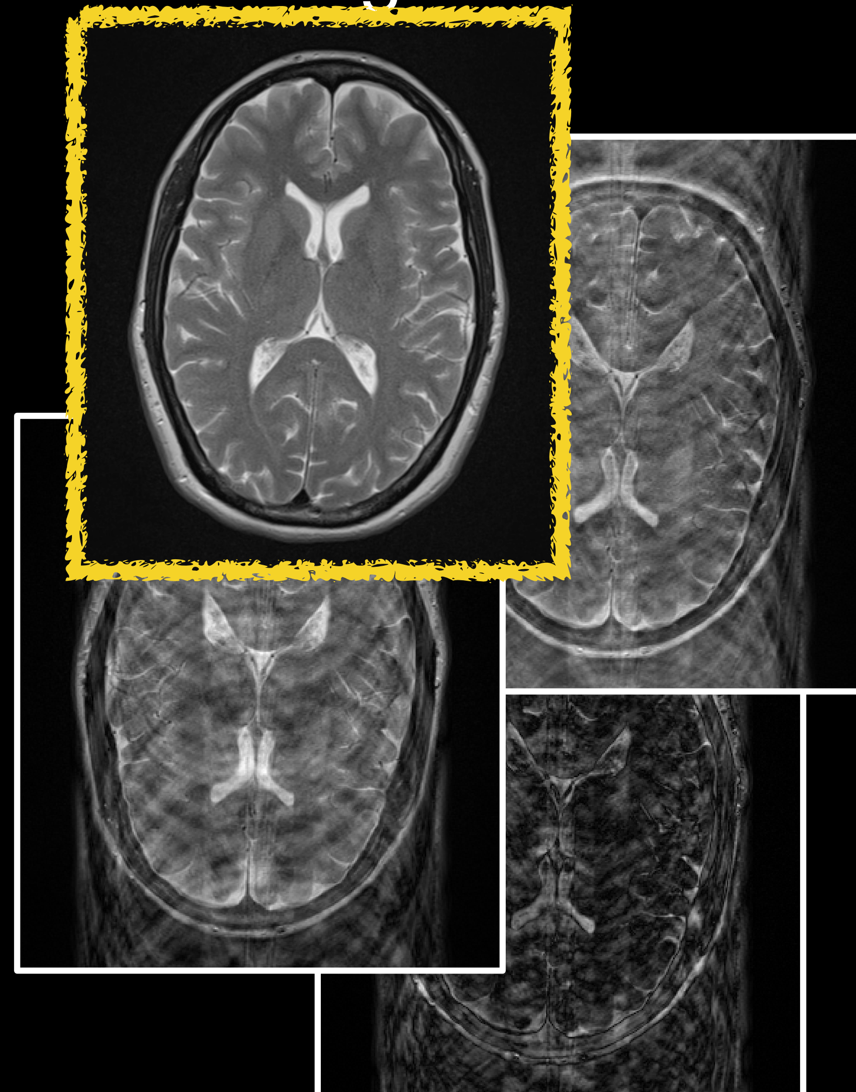
Image



Choose the most compressible image matching data (*systematic optimization*)

Math Background

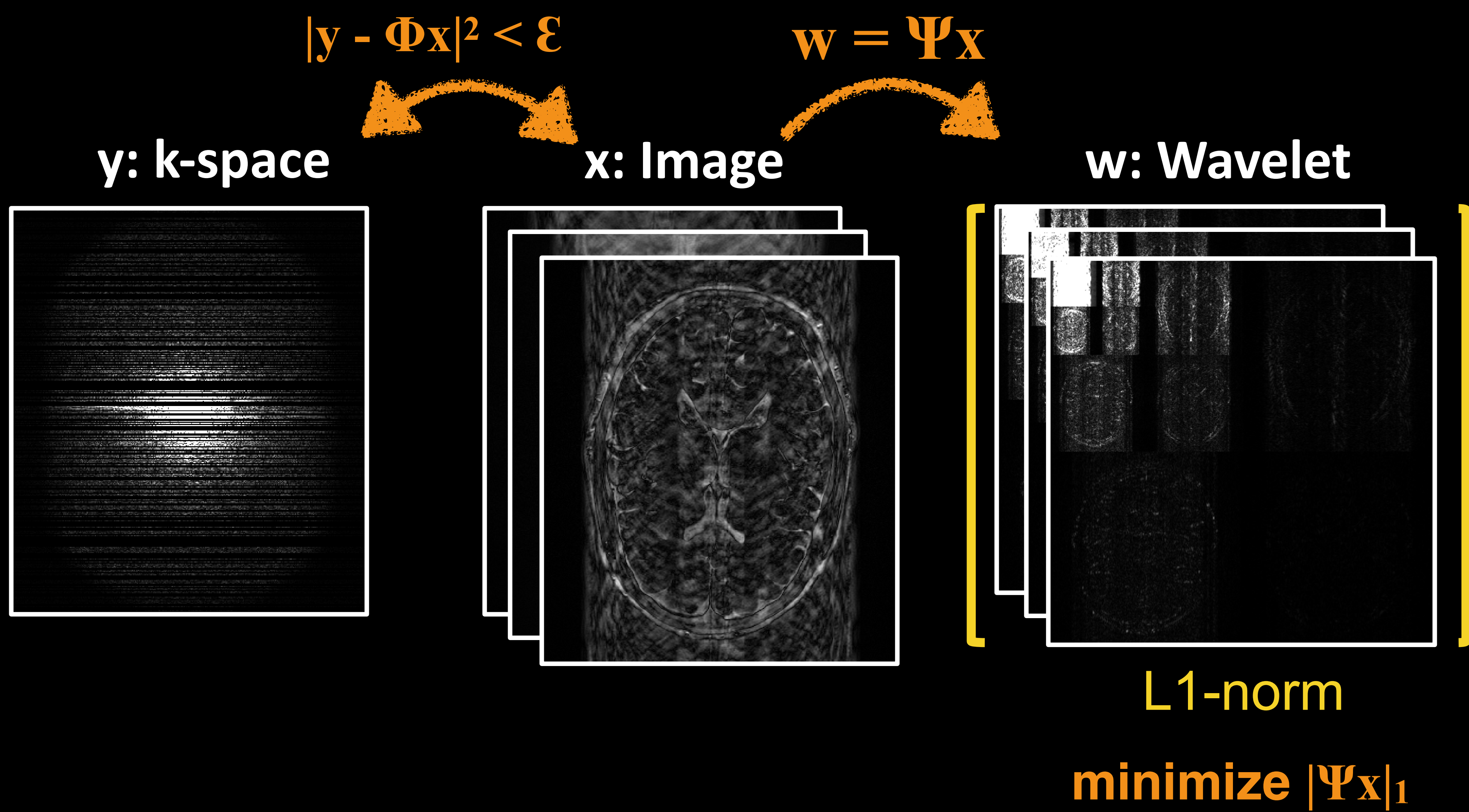
L0-norm ($|x|_0$): a number of non-zero coefficients

L1-norm ($|x|_1$): a sum of absolute values of coefficients

L2-norm ($|x|_2$): a sum of squared values of coefficients

$$x = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}$$

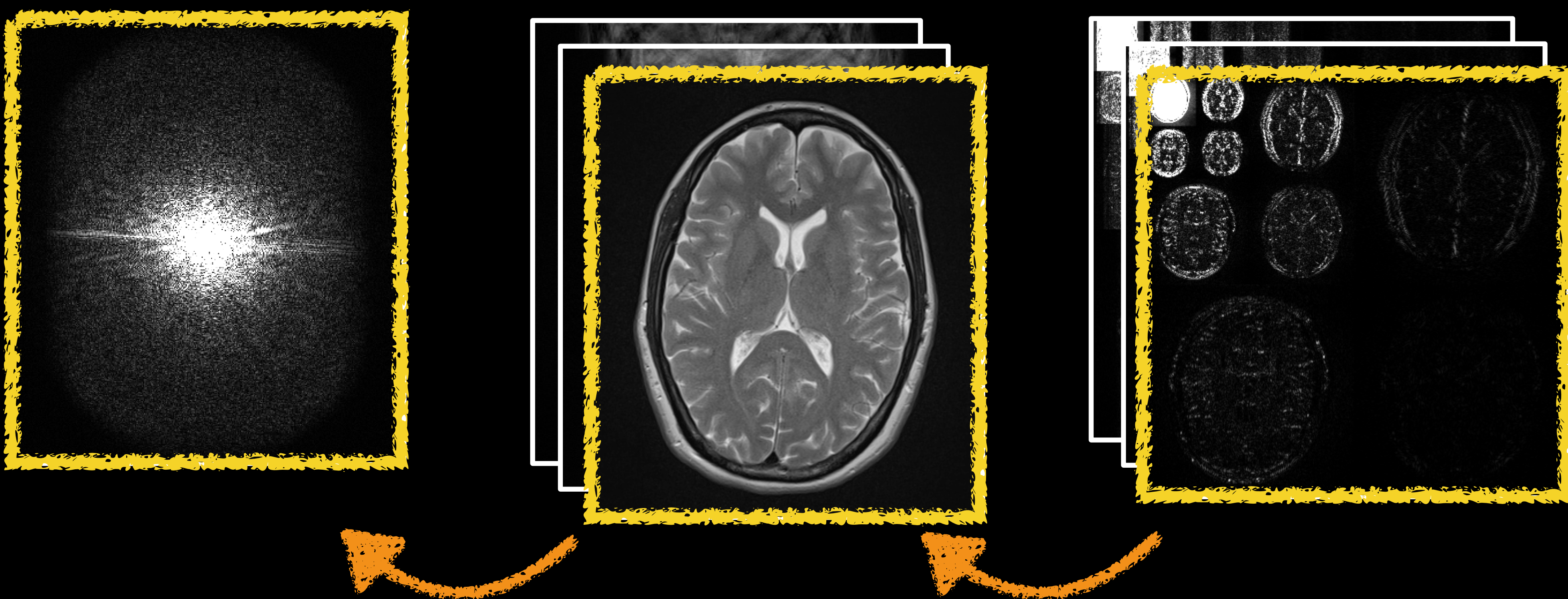
CS-MRI Reconstruction



CS-MRI Reconstruction

minimize $F(x): |y - \Phi x|^2 + R(x)$

y: k-space **x: Image** **w: Wavelet**



$y' = FT(x)$ $x = \Psi^{-1} w$

Three Tenets of CS

minimize $F(x): |y - \Phi x|_2^2 + R(x)$

Data Consistency — Compressibility Constraint

- Three key elements of Compressed Sensing:

Compressibility

Incoherence

Nonlinear Reconstruction

Compressibility Constraint

minimize $F(x): |y - \Phi x|^2_2 + R(x)$

Compressibility Constraint

- $R(x) = \lambda|x|_1$ (*Identity Transform*)
- $R(x) = \lambda|\Psi x|_1$ (*Wavelet Transform*)
- $R(x) = \lambda H(x)$ (*Total Variation*)
- $R(x) = \lambda|x|_*$ (*Rank or Nuclear Norm*)
- Many more…

Wavelet Transform

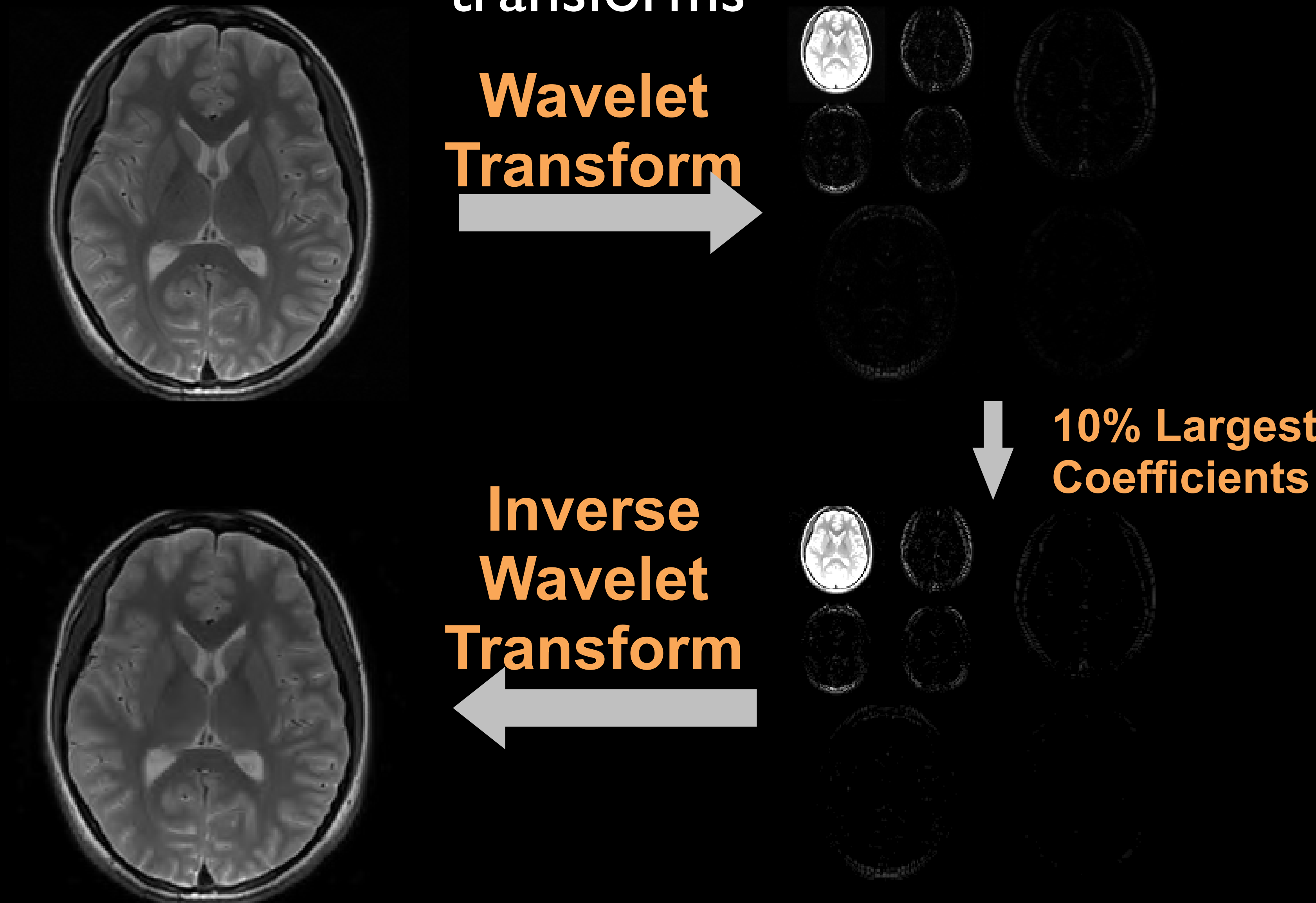
- Natural images are compressible using wavelet transforms

Image Compression Standard: JPEG2000

Images from Wikipedia

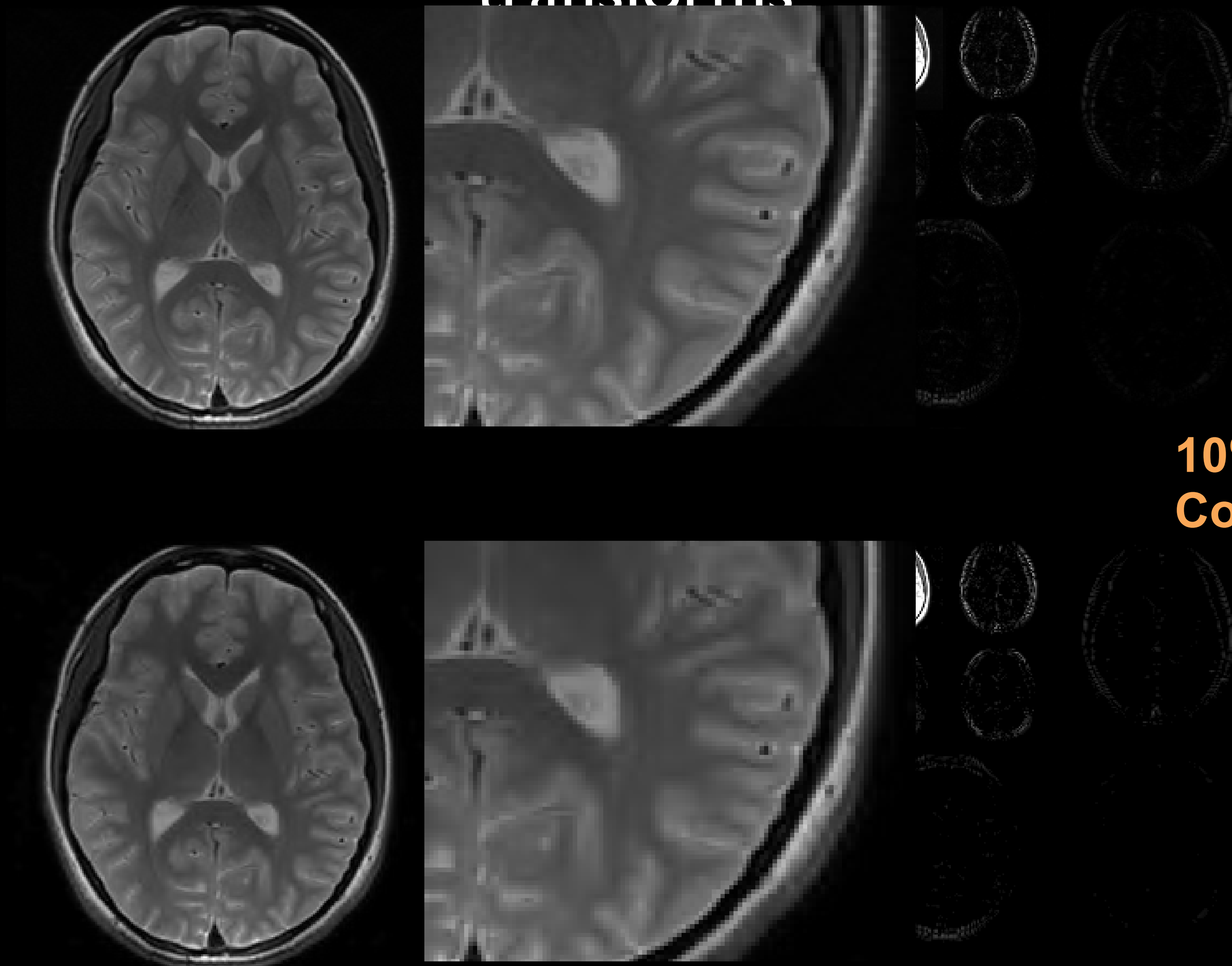
Wavelet Transform

MR images are mostly compressible using wavelet transforms



Wavelet Transform

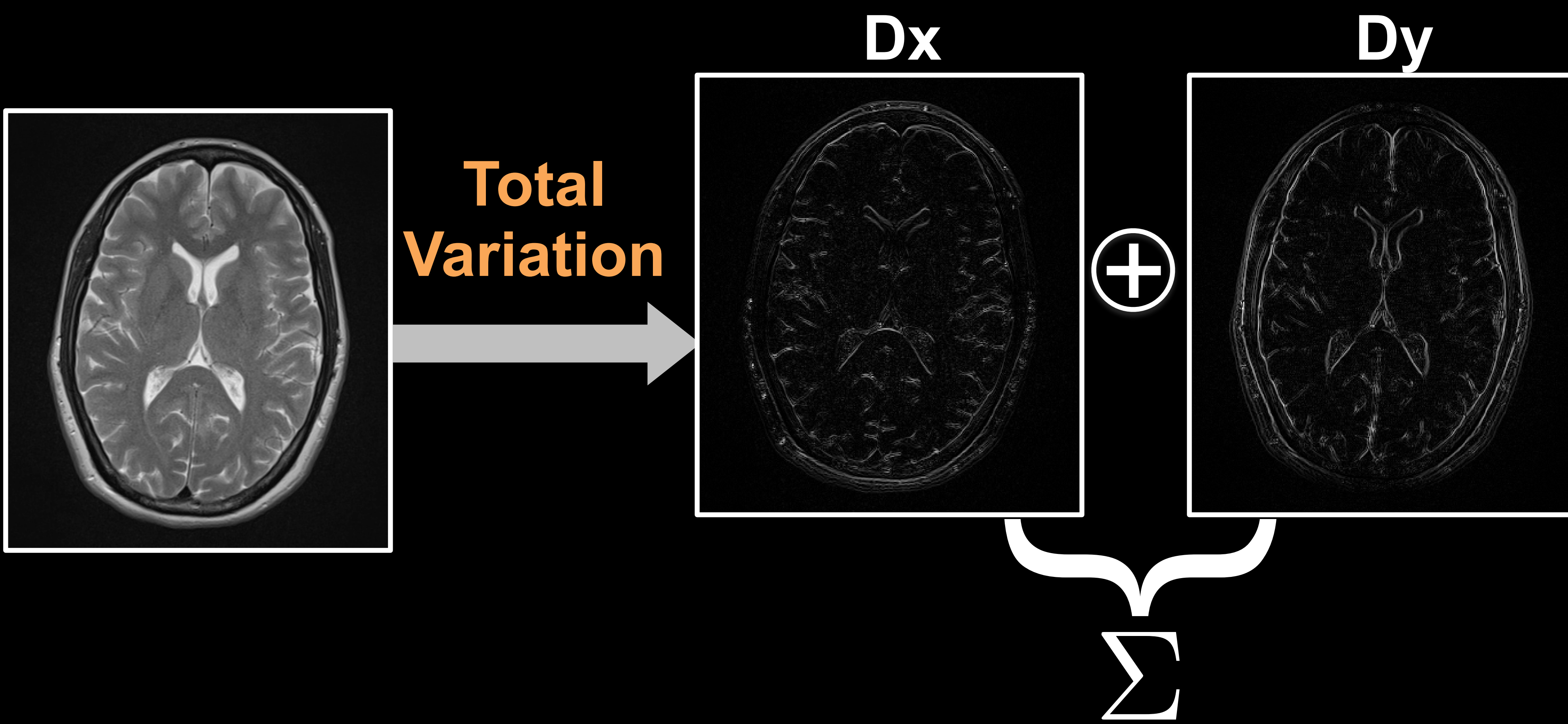
MR images are mostly compressible using wavelet transforms



10% Largest Coefficients

Total Variation

$$H(x) = \sum_{i,j} \sqrt{\underbrace{|x_{i+1,j} - x_{i,j}|^2}_{\text{Dx}} + \underbrace{|x_{i,j+1} - x_{i,j}|^2}_{\text{Dy}}}$$



CS-MRI Reconstruction

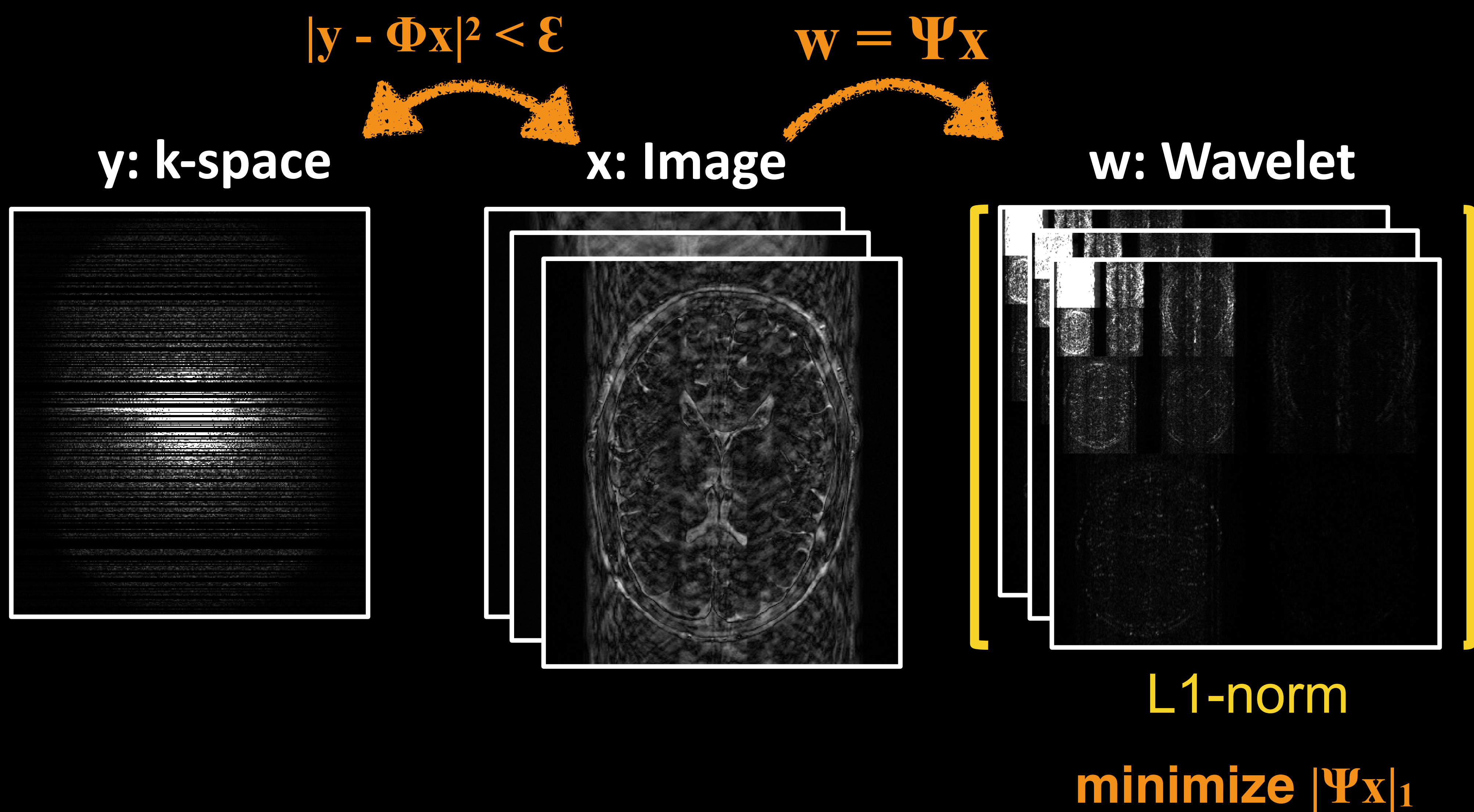
minimize $F(x): |y - \Phi x|_2^2 + R(x)$

- Minimizing F(x) is non-trivial since R(x) is not <u>differentiable</u>
  - Linear programming is challenging due to high computational complexity
- Simple gradient-based algorithms have been developed:
  - Re-weighted L1 / FOCUSS
  - IST / IHT / AMP / FISTA
  - Split Bregman / ADMM

I.F. Gorodnitsky, et al., J. Electroencephalog. Clinical Neurophysiol. 1995 Daubechies I, et al. Commun. Pure Appl. Math. 2004
Elad M, et al. in Proc. SPIE 2007
T. Goldstein, S. Osher, SIAM J. Imaging Sci. 2009

To the board ...

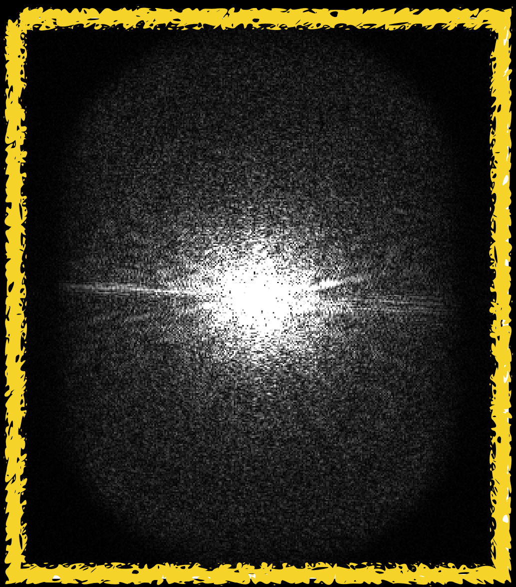
CS-MRI Reconstruction



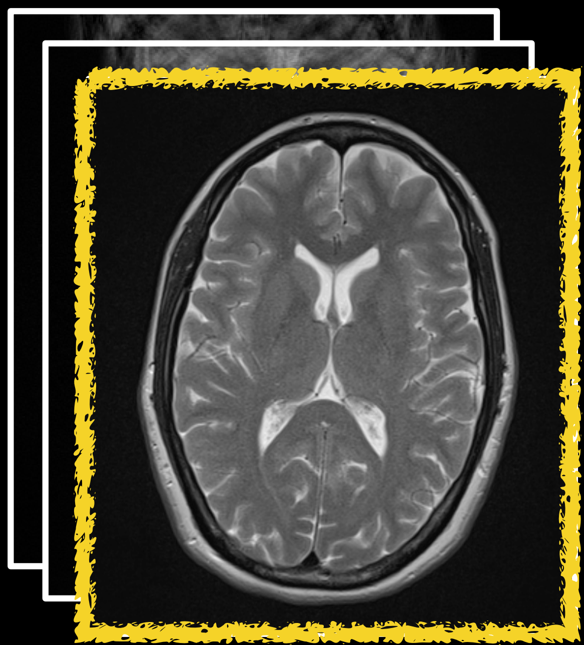
CS-MRI Reconstruction

minimize $F(x): |y - \Phi x|^2 + R(x)$

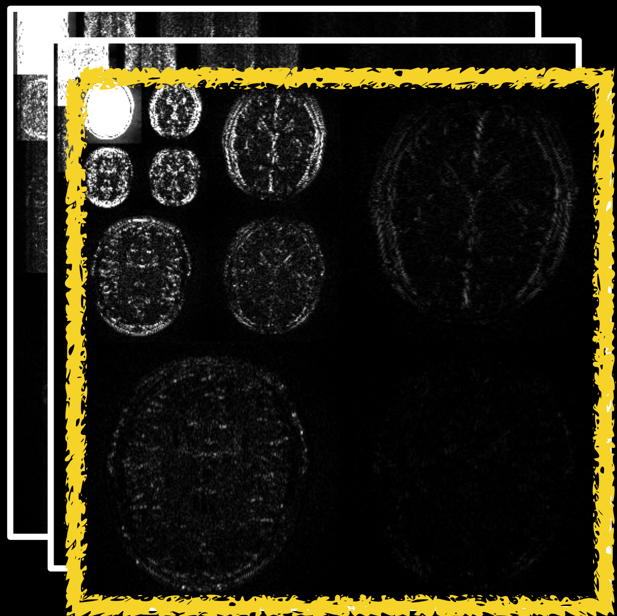
y: k-space



x: Image



w: Wavelet



$y' = FT(x)$

$x = \Psi^{-1} w$

Summary So Far…

minimize $F(x)$: $|y - \Phi x|_2^2 \; + \; R(x)$

Data Consistency — Compressibility Constraint

Compressibility Constraint

Incoherent Measurement

Reconstruction

Cardiac Function

- *Reconstruction Domain*:

  x (dynamic 2D MRI in x-f space)

- *Compressibility Constraint*:

  $|x|_1$: sparsity in x-f

- *Incoherent Measurement*: variable density random undersampling

$$\text{minimize } F(x): |y - \Phi x|_2^2 + \lambda |x|_1$$

- *Reconstruction*: non-linear CG L1 / FOCUSS

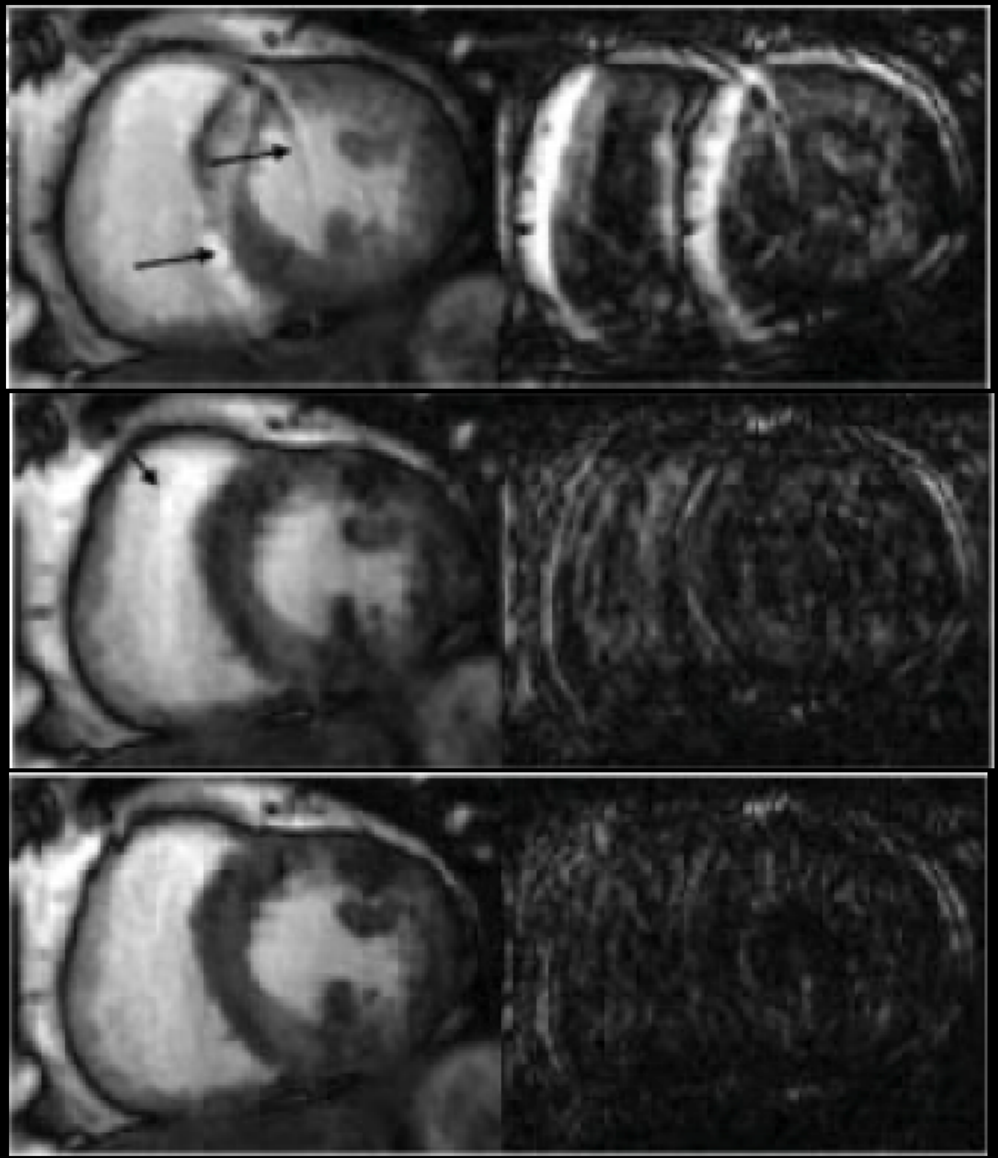
M. Lustig, et al., ISMRM 2006
H. Jung, et al., Physics in Medicine and Biology 2007
H. Jung, et al., MRM 2009

Cardiac Function (k-t FOCUSS)

k-t BLAST

k-t FOCUSS

k-t FOCUSS with ME/MC

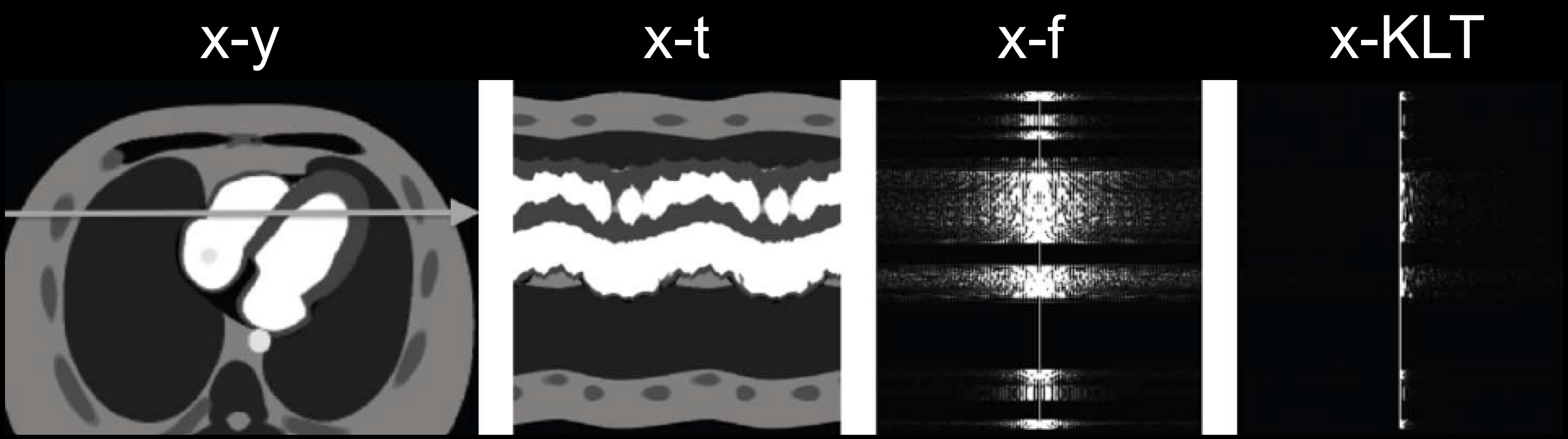


H. Jung, et al., MRM 2009

Cardiac Function (k-t SLR)

- *Compressibility Constraint:*

$$|x|_* = \sum_i (\Sigma_{i,i}) \qquad x = U\Sigma V^*$$



S.G. Lingala, et al., IEEE TMI 2011

Cardiac Function (k-t ISD)

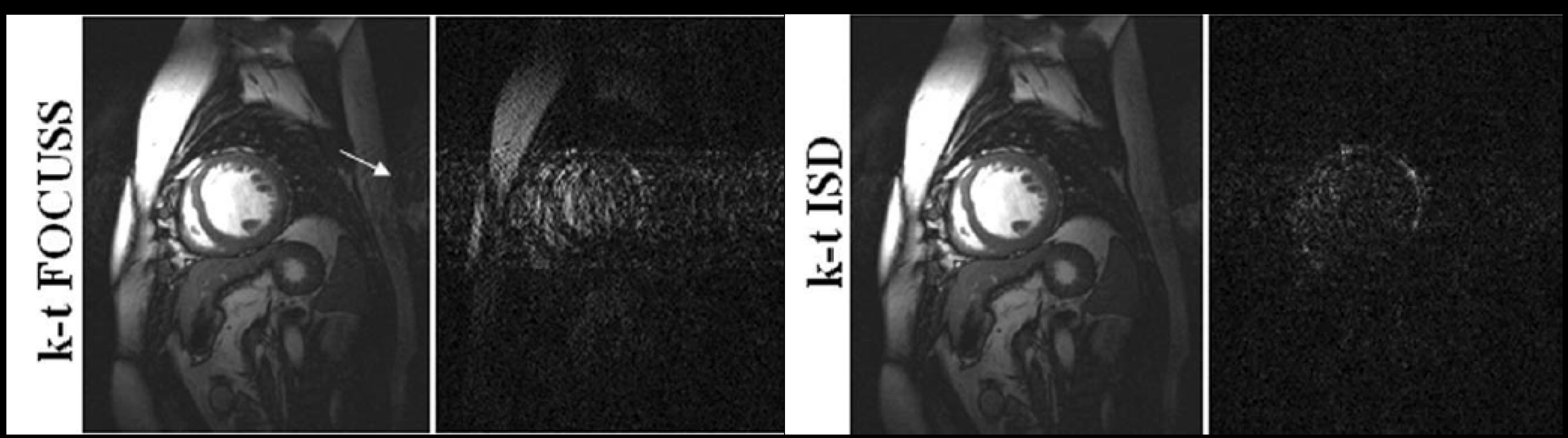
- *Compressibility Constraint*:

  W: Diagonal weighting matrix (known support in x-f)

- *Incoherent Measurement*: variable density random undersampling

$$\text{minimize } F(x): |y - \Phi x|_2^2 + \lambda|Wx|_1$$

- *Reconstruction*: FOCUSS



D. Liang, et al., MRM 2012

Phase Contrast

- <u>*Reconstruction Domain*</u>:

  x_1 (flow-compensated)
  x_2 (flow-encoded)

- <u>*Compressibility Constraint*</u>:

  $H(x_i)$: Total Variation
  $|x_1 - x_2|_1$: Complex Difference

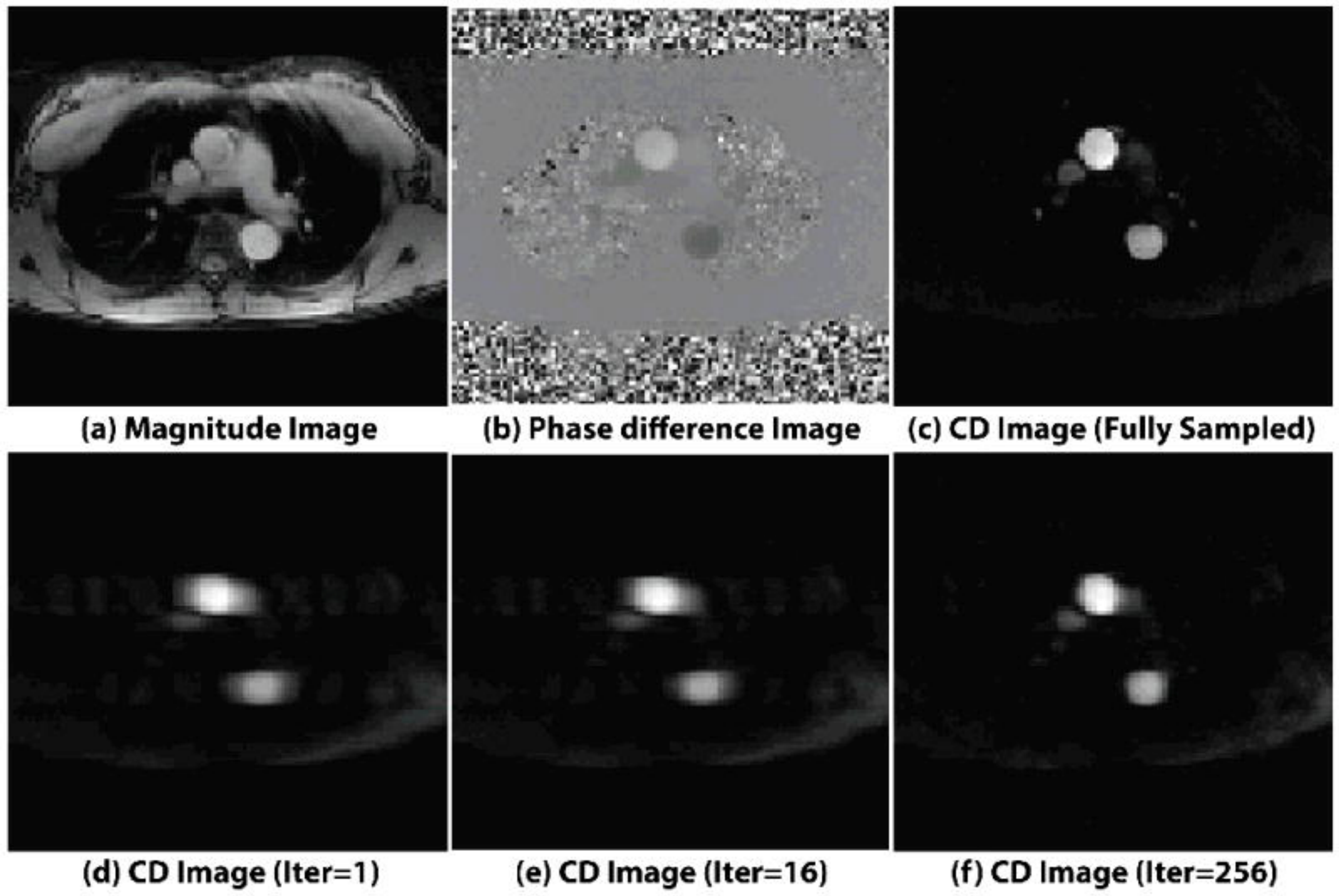
- <u>*Incoherent Measurement*</u>: uniform random undersampling

  minimize $F(x_1)$: $|y - \Phi x_1|_2^2 + \lambda_1 H(x_1) + \lambda_2 |x_1 - x_2|_1$
  minimize $F(x_2)$: $|y - \Phi x_2|_2^2 + \lambda_1 H(x_2) + \lambda_2 |x_1 - x_2|_1$

- <u>*Reconstruction*</u>: Split Bregman

Y Kwak. et al., MRM 2012

Phase Contrast (Complex



(a) Magnitude Image (b) Phase difference Image (c) CD Image (Fully Sampled)

(d) CD Image (Iter=1) (e) CD Image (Iter=16) (f) CD Image (Iter=256)

Y Kwak. et al., MRM 2012

Dynamic CE-MRA

- *Reconstruction Domain*:

  x_i, i = 1,2,3, … (dynamic 3D MRI)

- *Compressibility Constraint:*

  $H(x_i)$: Total Variation

  $\| |x_1| - |x_2| \|_1$: Magnitude Difference

- *Incoherent Measurement*: variable density Poisson disk undersampling

$$\text{minimize } F(x_1): |y - \Phi x_1|^2 + \lambda_1 H(x_1) + \lambda_2 \| |x_1| - |x_2| \|_1$$

$$\text{minimize } F(x_2): |y - \Phi x_2|^2 + \lambda_1 H(x_2) + \lambda_2 \| |x_1| - |x_2| \|_1$$

- *Reconstruction*: Split Bregman

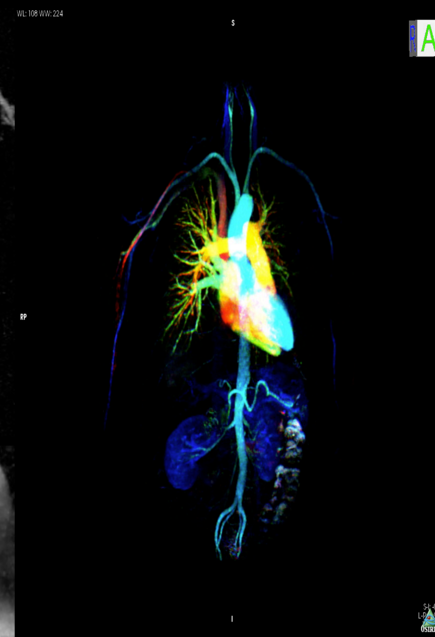
Rapacchi et al. MRM 2014

Dynamic CE-MRA (Mag. Diff.)

- 12X acceleration (1.1 x 1.1 x 2 mm^2)
- 6 volumes (instead of 1) in a single breath-hold

Coronal MIP

Axial MIP



TT=6.9s	12.8s (→Red)	18.7s (→Green)	24.6s (→Blue)	12.8-24.6s(RGB)

Rapacchi et al. MRM 2014

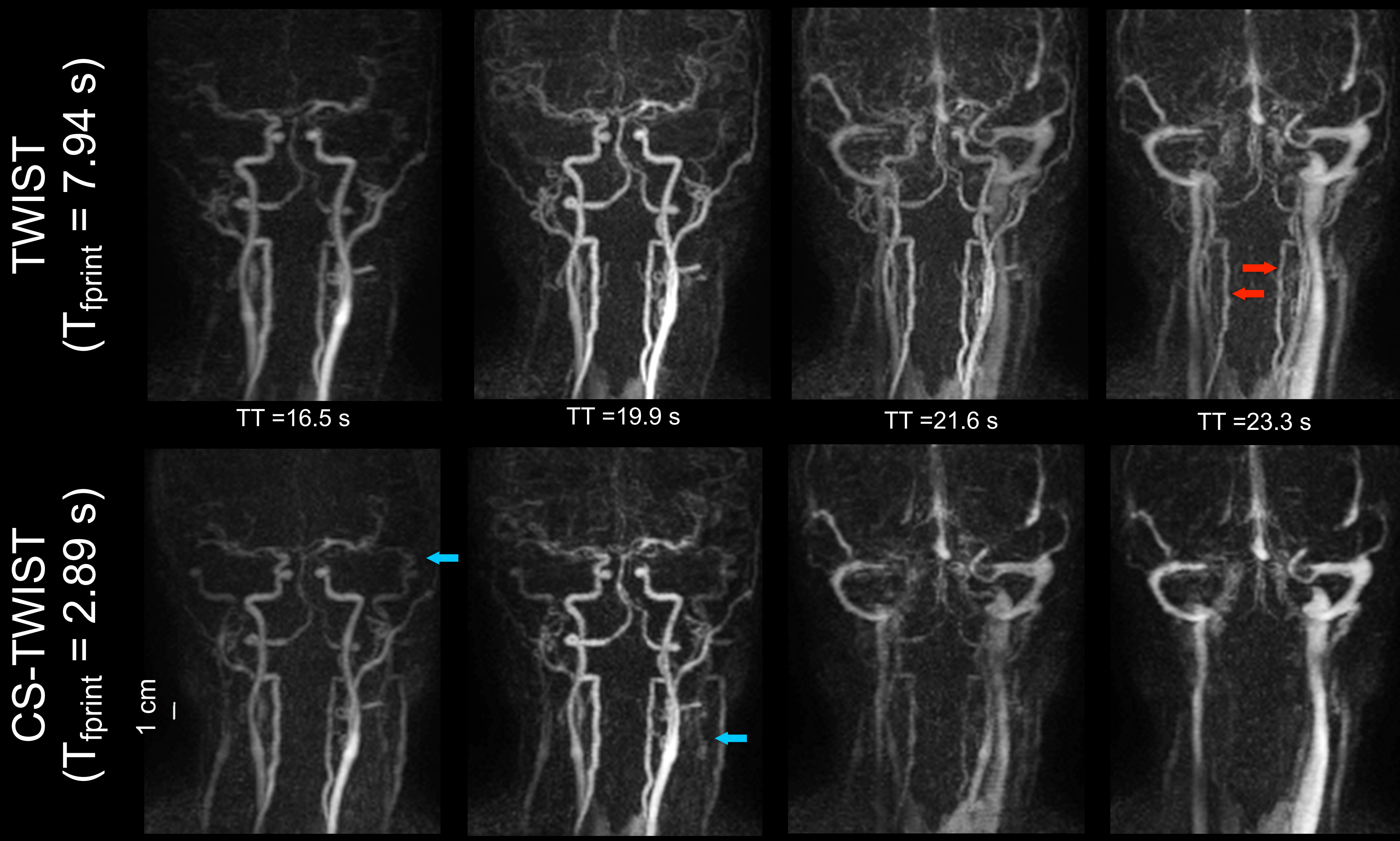
View Sharing vs. CS

TWIST (T_{fprint} = 7.94 s)
view-sharing acceleration

CS-TWIST (T_{fprint} = 2.89 s)
CS acceleration

Rapacchi et al. Int. Soc. Mag. Res. Angio. 2013

View Sharing vs. CS

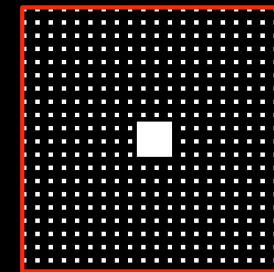


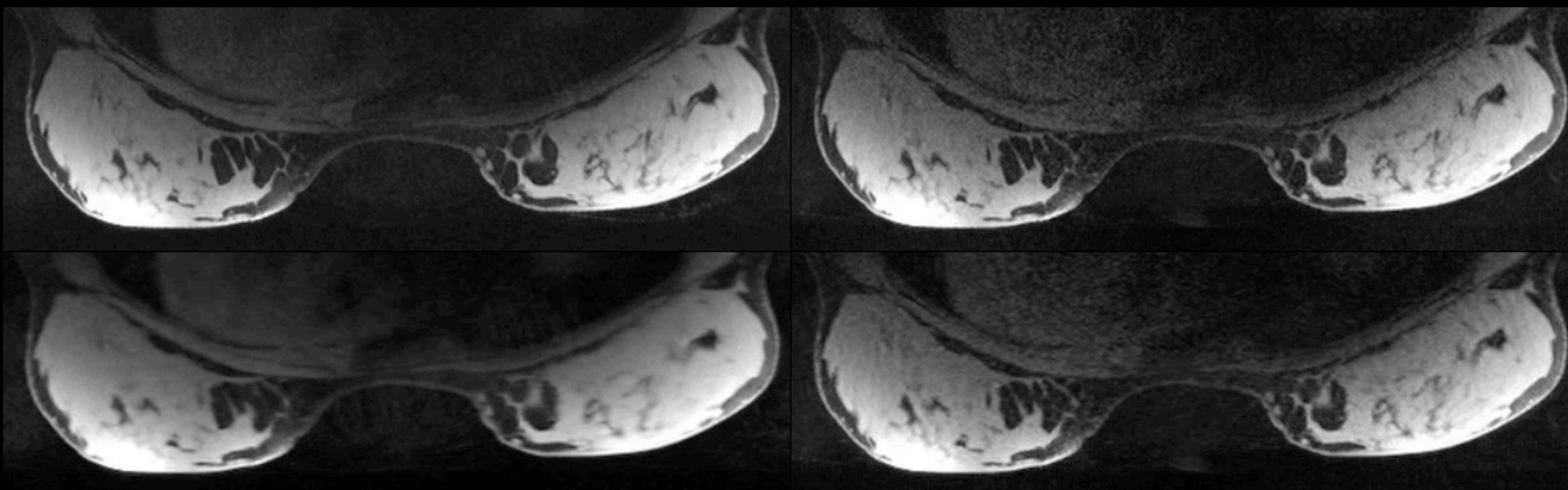
Rapacchi et al. Int. Soc. Mag. Res. Angio. 2013

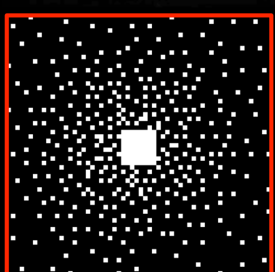
High-Frequency Subband CS

Original

Parallel Imaging (R=5.8)

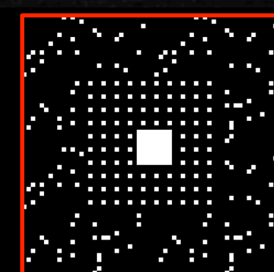






L1 SPIRiT (R=10.7)
Variable Density PD

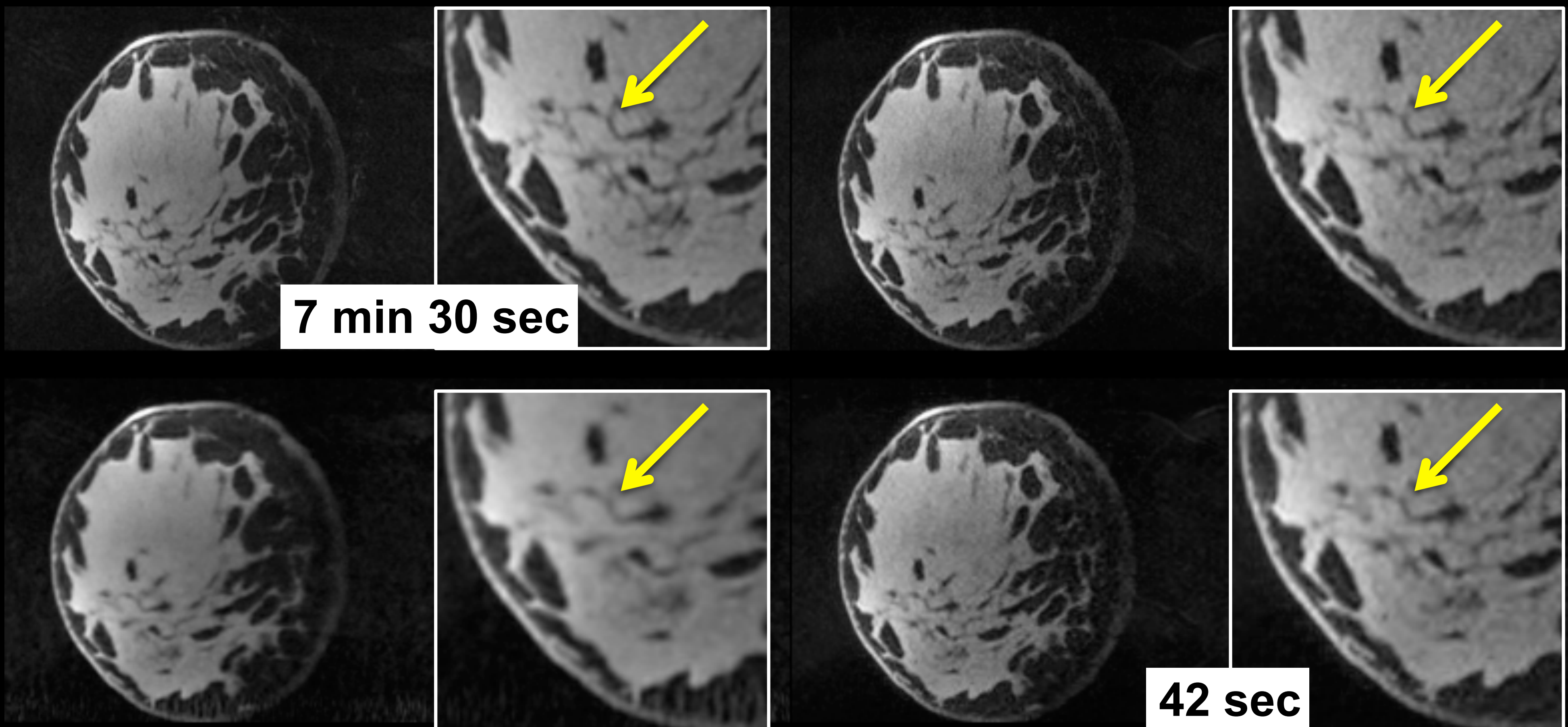
HiSub CS
(R=10.7)



Matrix size = 360 X 360 X 240
Spatial resolution = 0.9 X 0.9 X 0.6 mm

High-Frequency Subband CS

Original | **Parallel Imaging (R=5.8)**

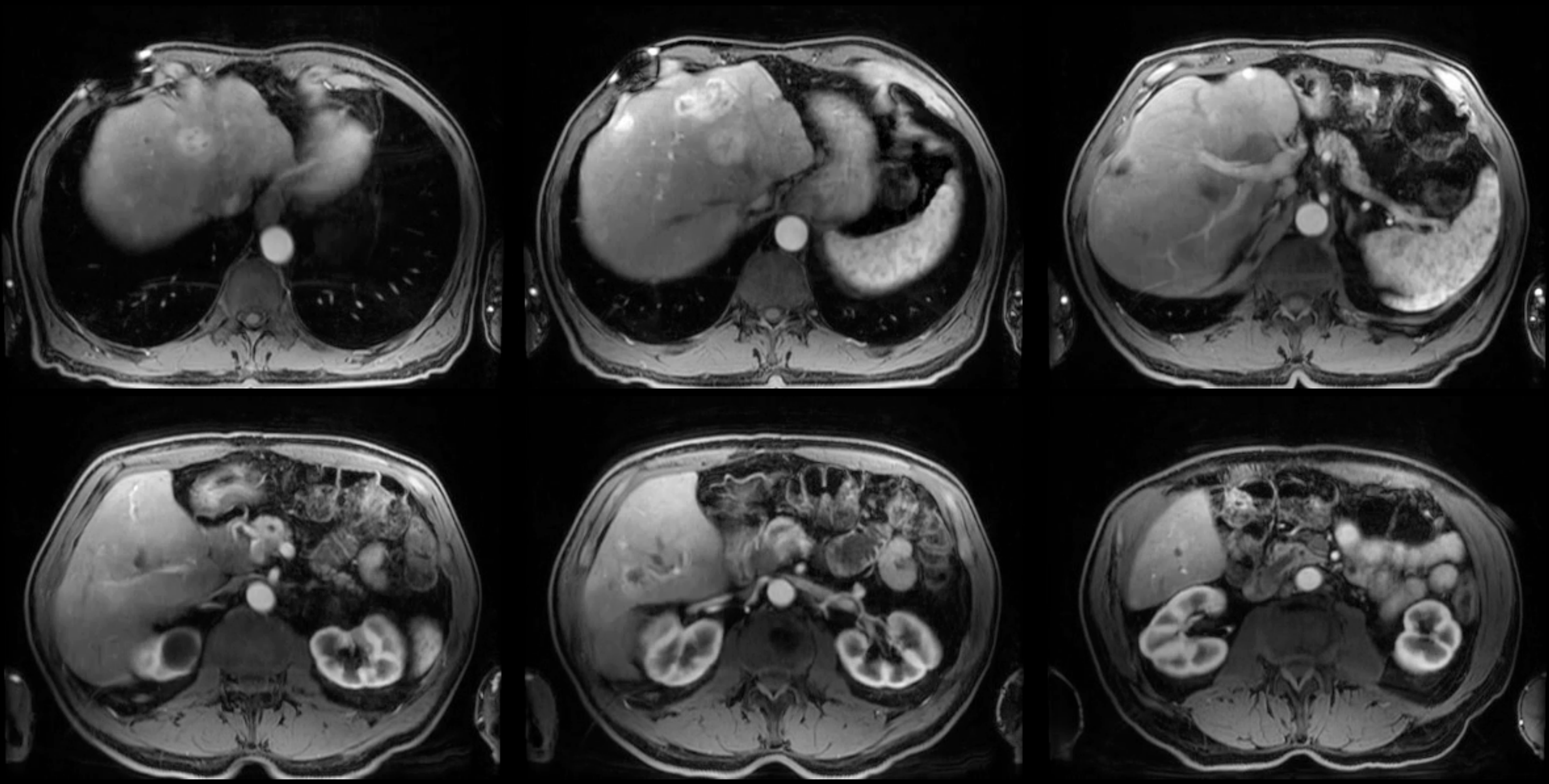


L1 SPIRiT (R=10.7)
Variable Density PD

HiSub CS (R=10.7)

K. Sung, et al. MRM 2013

Liver DCE Imaging (R = 12)



Matrix size = 260 X 202 X 60
Temporal res = 4 sec and # temporal phases = 8
32 channel torso coil

State-of-the-Art CS-MRI

- Reducing possible reconstruction failure
  - Improve sparse transformations
  - Develop k-space undersampling schemes
- Integrating CS with DL/parallel imaging
  - Develop compatible undersampling patterns
  - Develop reconstruction methods

State-of-the-Art CS-MRI

- Methods to evaluate CS reconstructed images
  - RMSE / SSIM / Mutual Information
- Reducing reconstruction time
  - Reduce computational complexity
  - Parallelize reconstruction problems
- Developing stable reconstruction algorithms
  - Minimize / avoid the number of regularization parameters

Further Reading

- Original Compressed Sensing
  - https://ieeexplore.ieee.org/document/1580791
  - https://ieeexplore.ieee.org/document/1614066
- Compressed Sensing MRI
  - https://ieeexplore.ieee.org/abstract/document/4472246

Thanks!

- Next time
  - Artificial Intelligence by Dr. Zabihollahy

Kyung Sung, PhD

ksung@mednet.ucla.edu

https://mrrl.ucla.edu/sunglab/