Pulse Sequences:
EPG and Simulations

M229 Advanced Topics in MRI

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Class Business

- Office hours
  - Instructor: Fri 10-11 am (Zoom)

- Homework 1 due on 4/22 Fri

- Final project
  - Start thinking
  - Discuss over email or during office hours
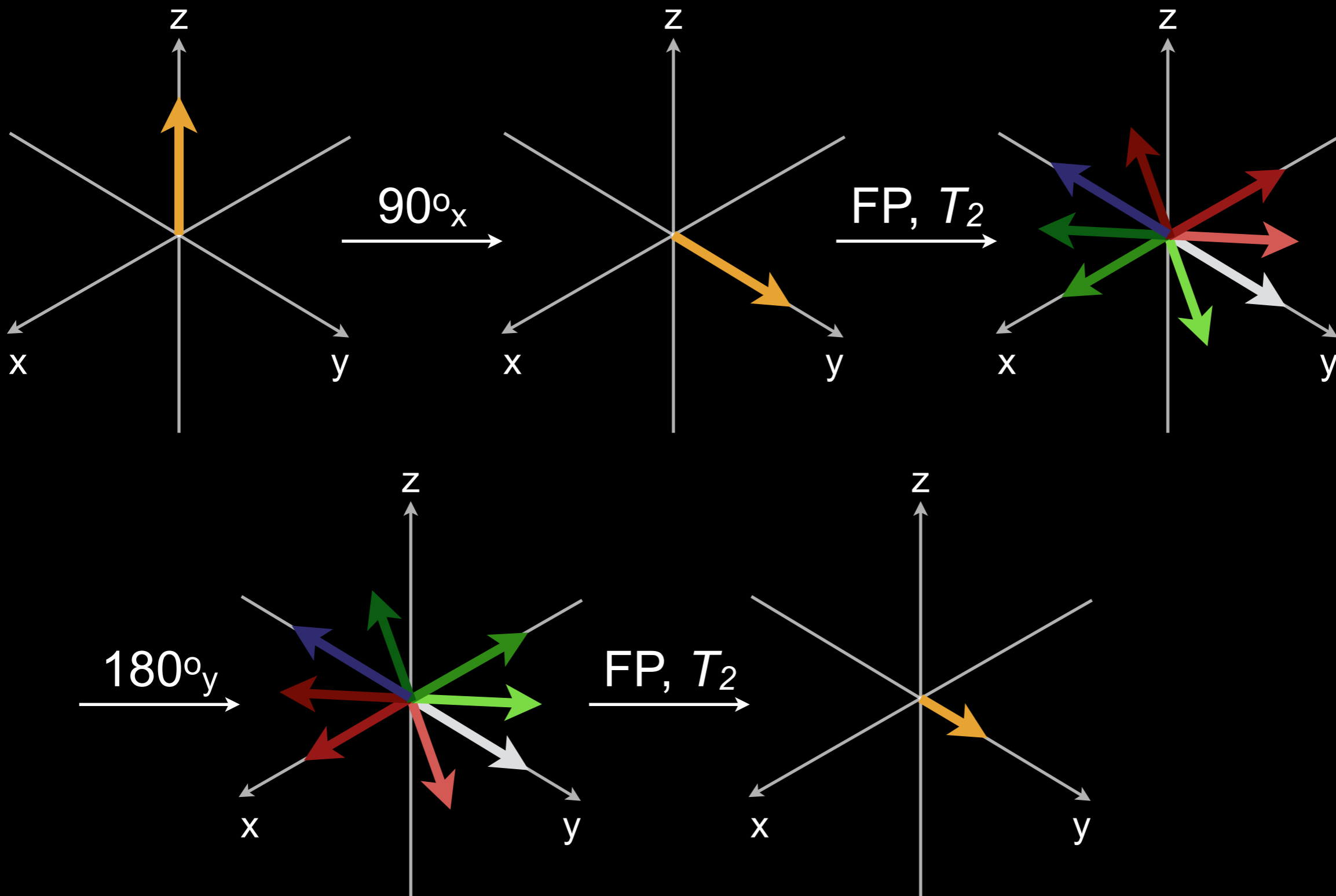 - Discussion in class on 4/21 Thu

Outline

- Multi-Pulse Experiments

- Extended Phase Graphs (EPG)

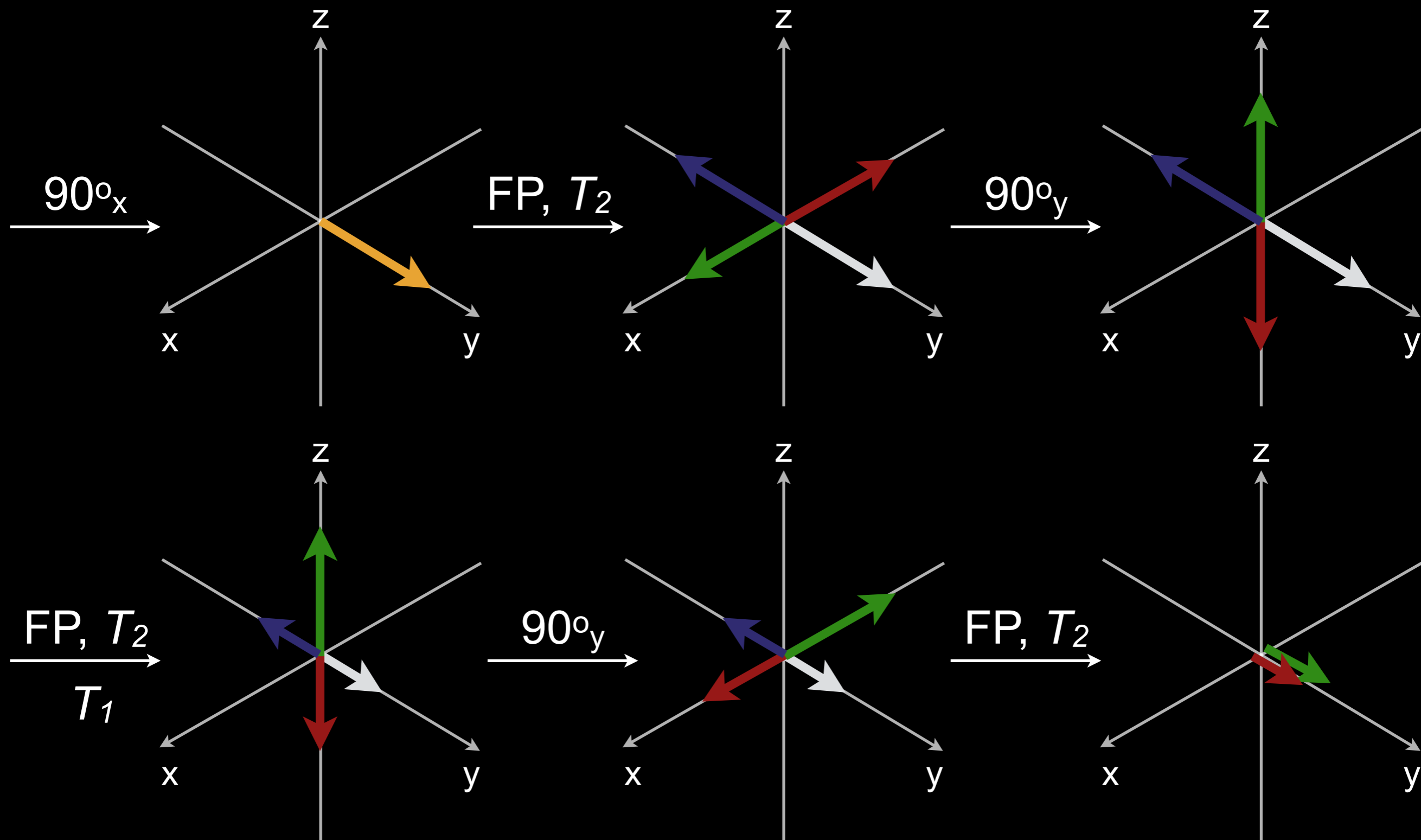- EPG Simulations
  - Homework 1

- Spin Bench

Multi-Pulse Experiments

- Multiple RF pulses
  - always have echoes (many types)
  - do not need perfect $90^o + 180^o$ to form SE, etc.

- Analysis
  - Bloch Equations
  - Extended Phase Graphs (EPG)
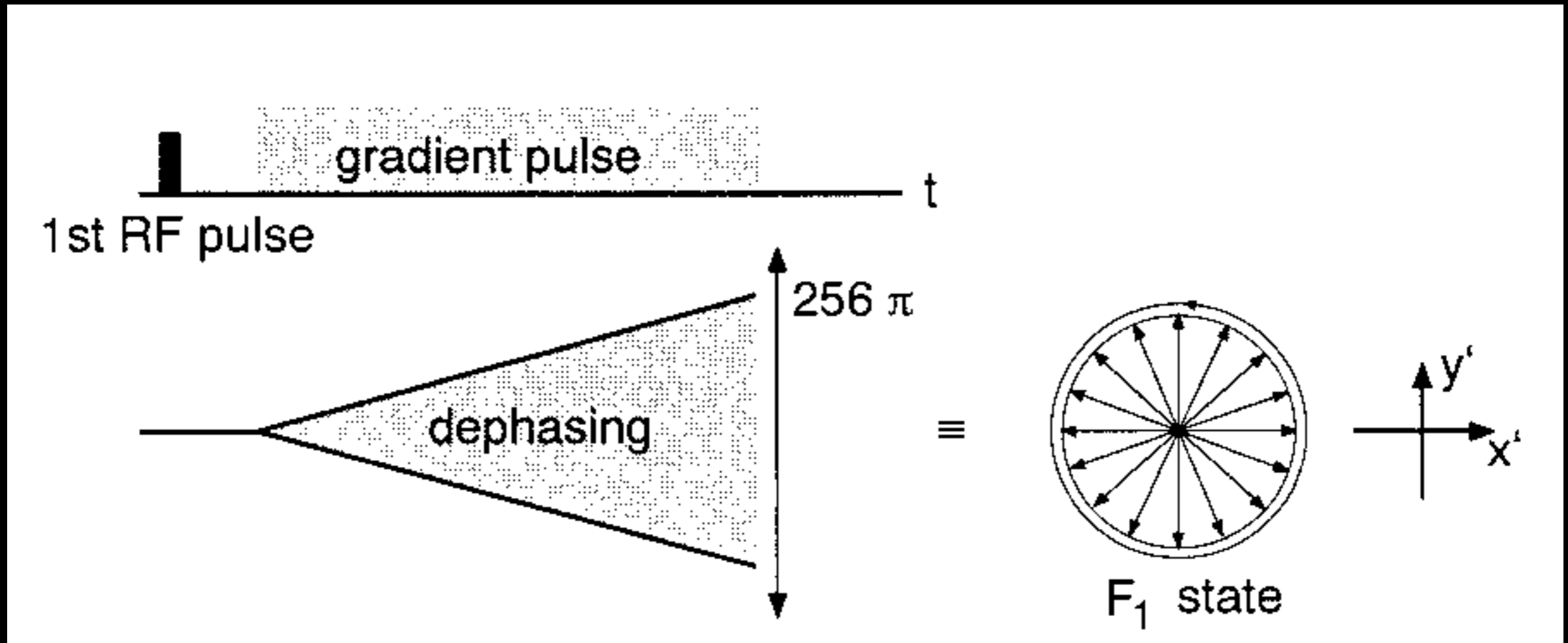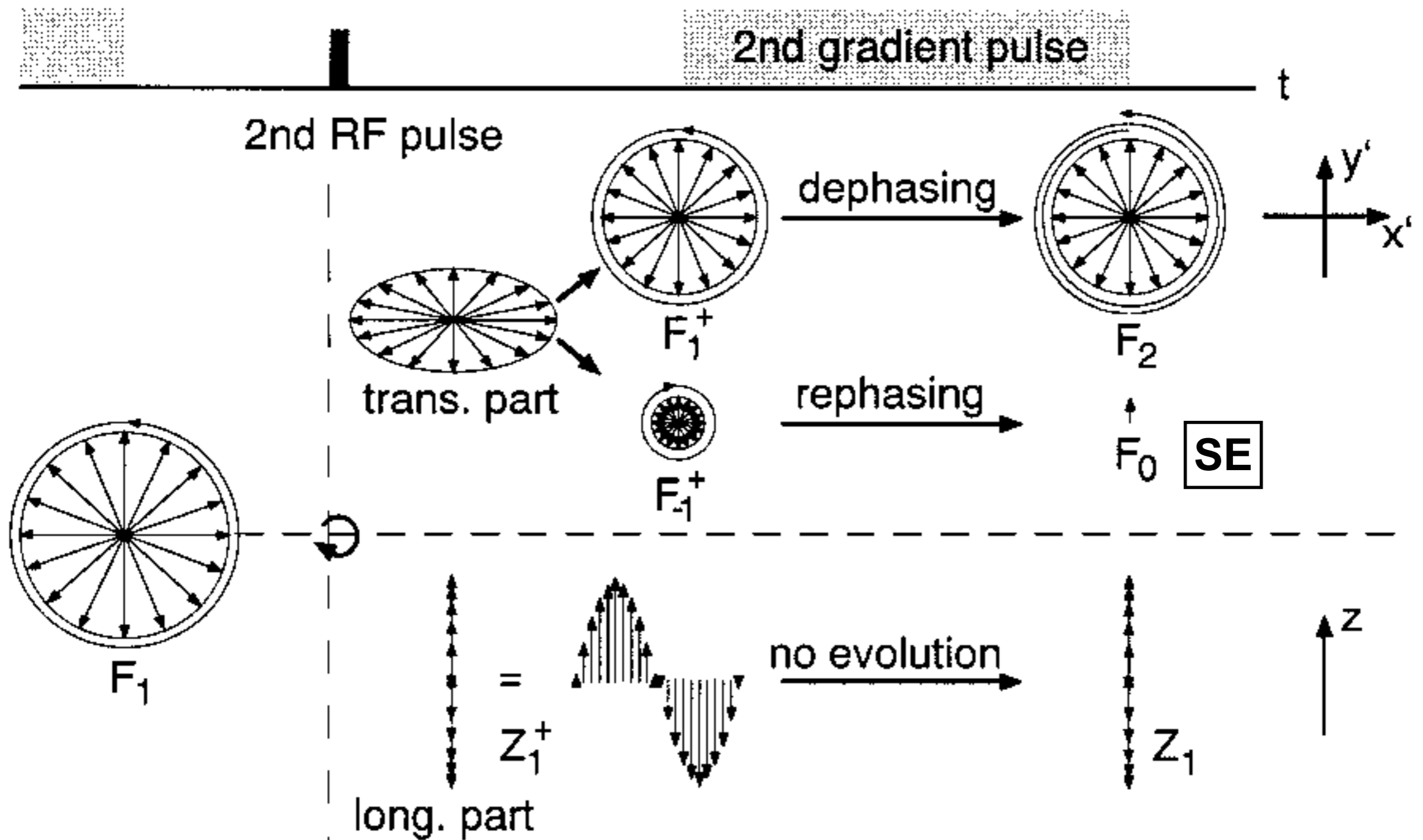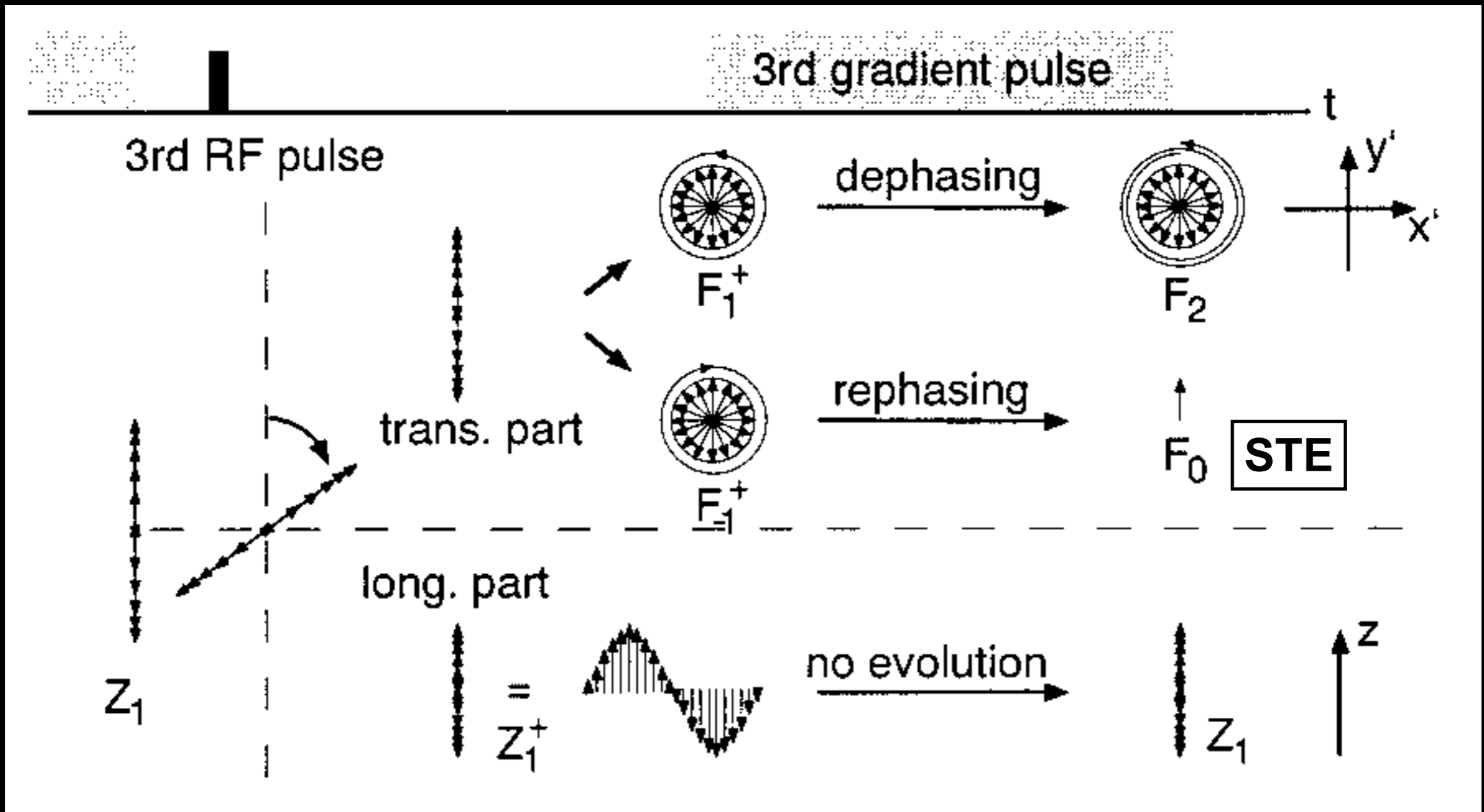
Spin Echo (2 pulses)

Stimulated Echo (3 pulses)

Multiple Pulse Experiments

Scheffler, Concepts in MR 1999

Multiple Pulse Experiments

Multiple Pulse Experiments
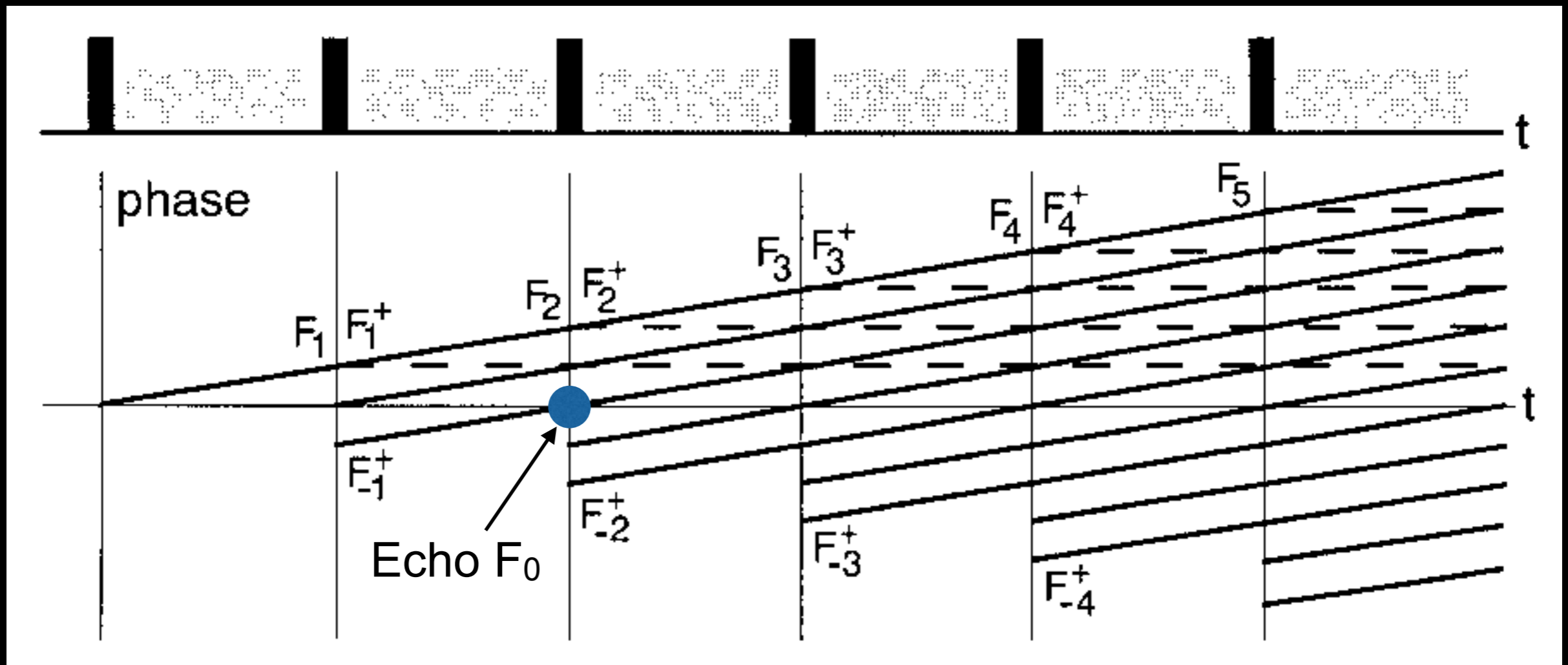

Scheffler, Concepts in MR 1999

Multiple Pulse Experiments

- RF pulses act on an ensemble of spins
  - M_z to M_{xy}
  - M_{xy} to M_z, M_{xy} and M_{xy}^*

- Transverse F states
  - $F = M_x + iM_y = F_{pos}$; $F^* = M_x - iM_y = F_{neg}$

- Longitudinal Z states

Multiple Pulse Experiments

Signal Pathways on a Phase Diagram (i.e. **EPG**)

Z states appear as broken lines; F_0 states are echoes

Extended Phase Graphs

- MR signal is a sum of all dephased spins

- Bloch equation
  - tracks evolution of magnetization for each spin
  - exact, but hard to visualize intuitively

- EPG
  - considers groups of spins under constant gradients
  - decomposes the spin system into several dephased states: F_k and F_{-k}; Z_k

Extended Phase Graphs

- Based on Fourier space coordinate *k*

$$k_n(t) = \gamma \int_{t'=0}^{t} G_n(t')dt' = \int_{t'=0}^{t} g_n(t')dt',$$

- Magnetization represented by Fourier transforms

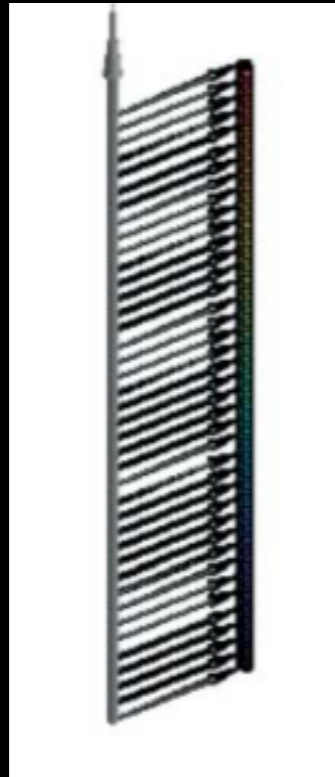$$F_+(\mathbf{k}) = \int_V \{M_x(\mathbf{r}) + iM_y(\mathbf{r})\} \exp(-i\mathbf{k}\mathbf{r})d^3r,$$

$$F_-(\mathbf{k}) = \int_V \{M_x(\mathbf{r}) - iM_y(\mathbf{r})\} \exp(-i\mathbf{k}\mathbf{r})d^3r,$$

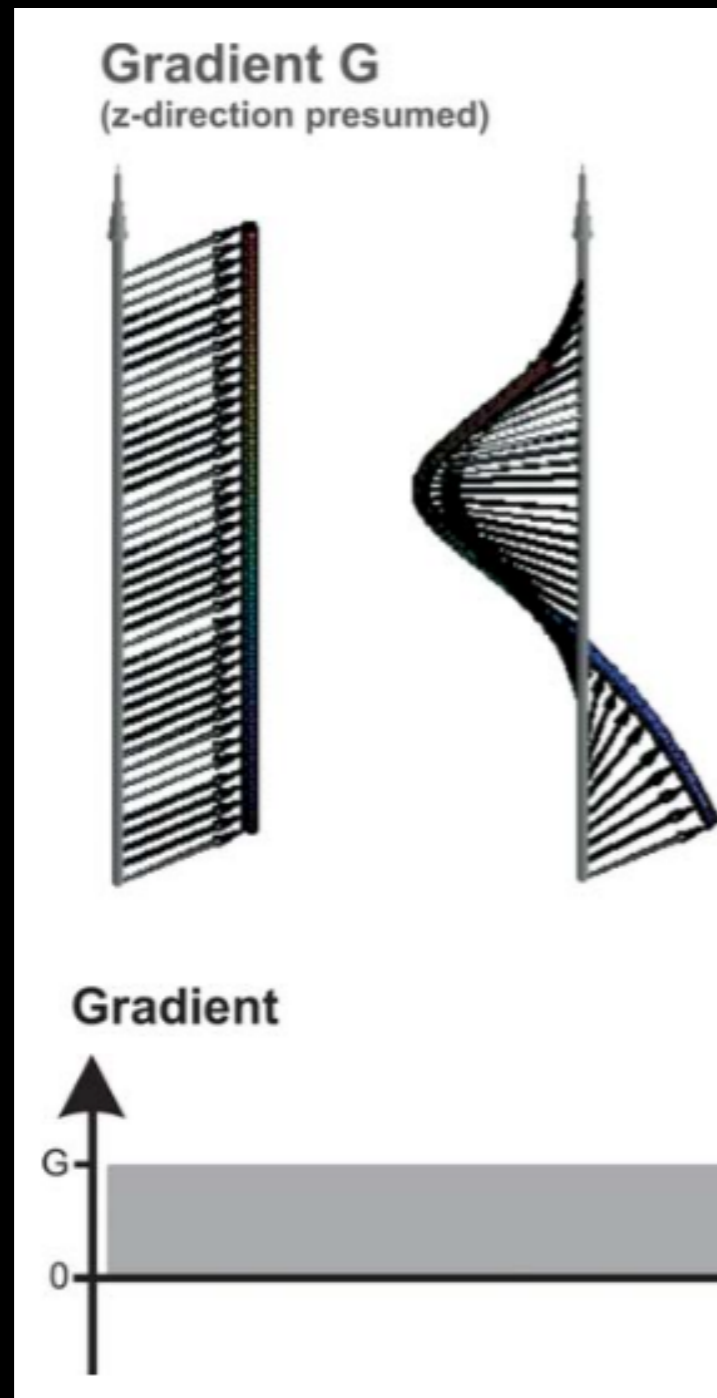$$Z(\mathbf{k}) = \int_V M_z(\mathbf{r}) \exp(-i\mathbf{k}\mathbf{r})d^3r,$$

- Complete magnetization is described by vector F of various EPG partitions states with different *k*

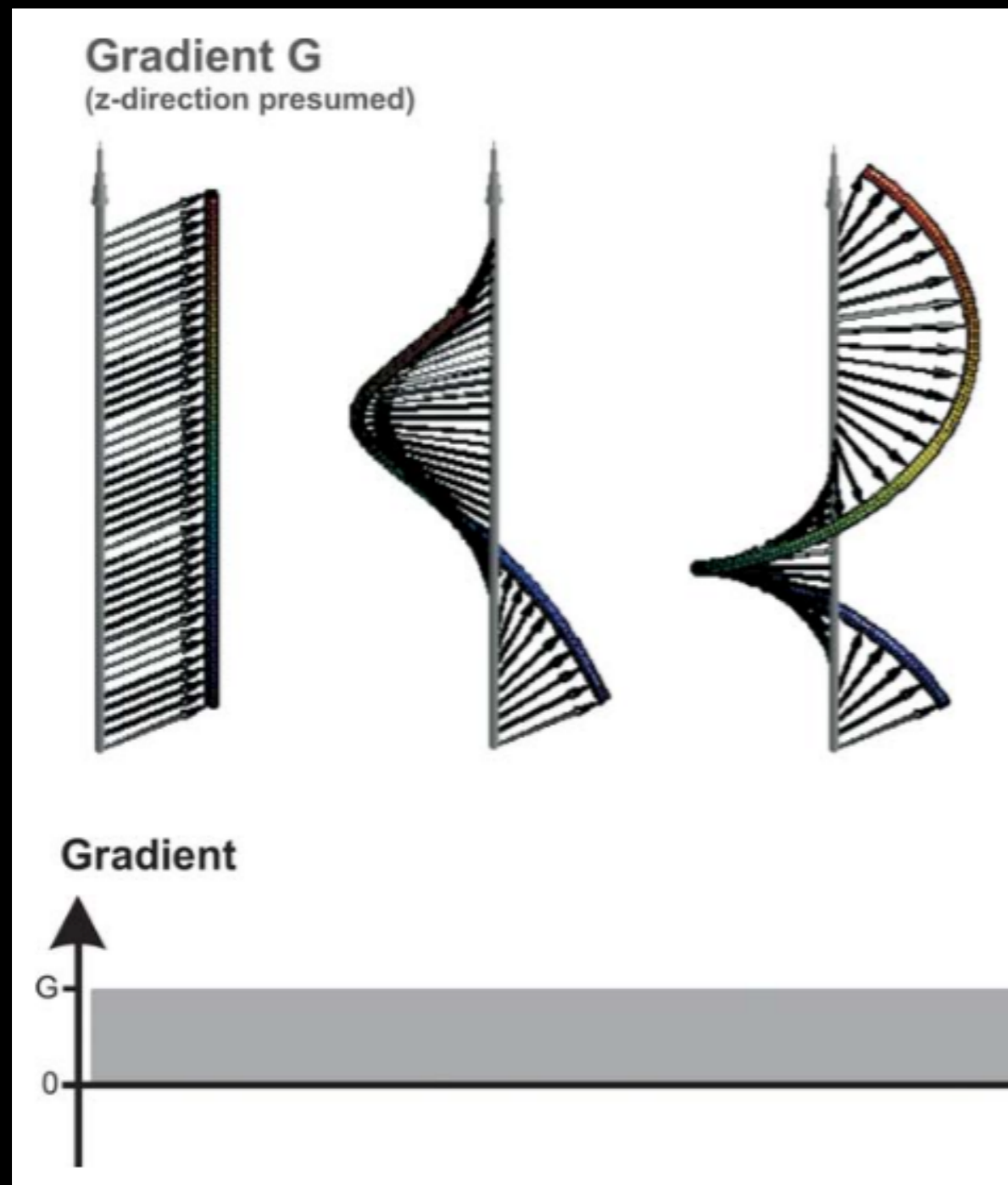$$\mathbf{F} = (F_0 Z_0 F_1 F_{-1} Z_1 F_2 F_{-2} Z_2 \cdots F_{+k} F_{-k} Z_k)^{\mathrm{T}}.$$

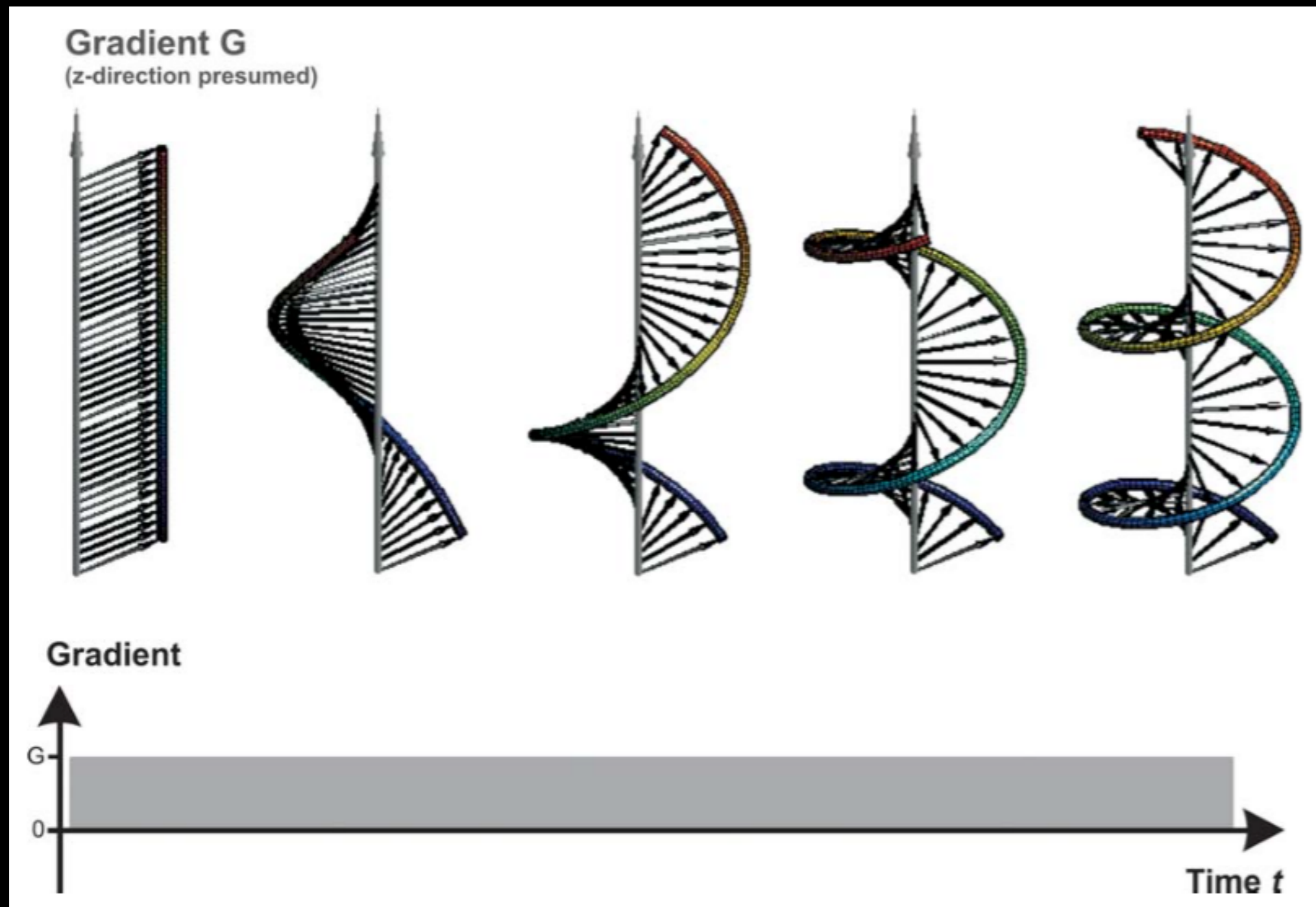# Gradient Dephasing

Weigel, JMRI 2015;41:266–295.

Gradient Dephasing

Gradient Dephasing

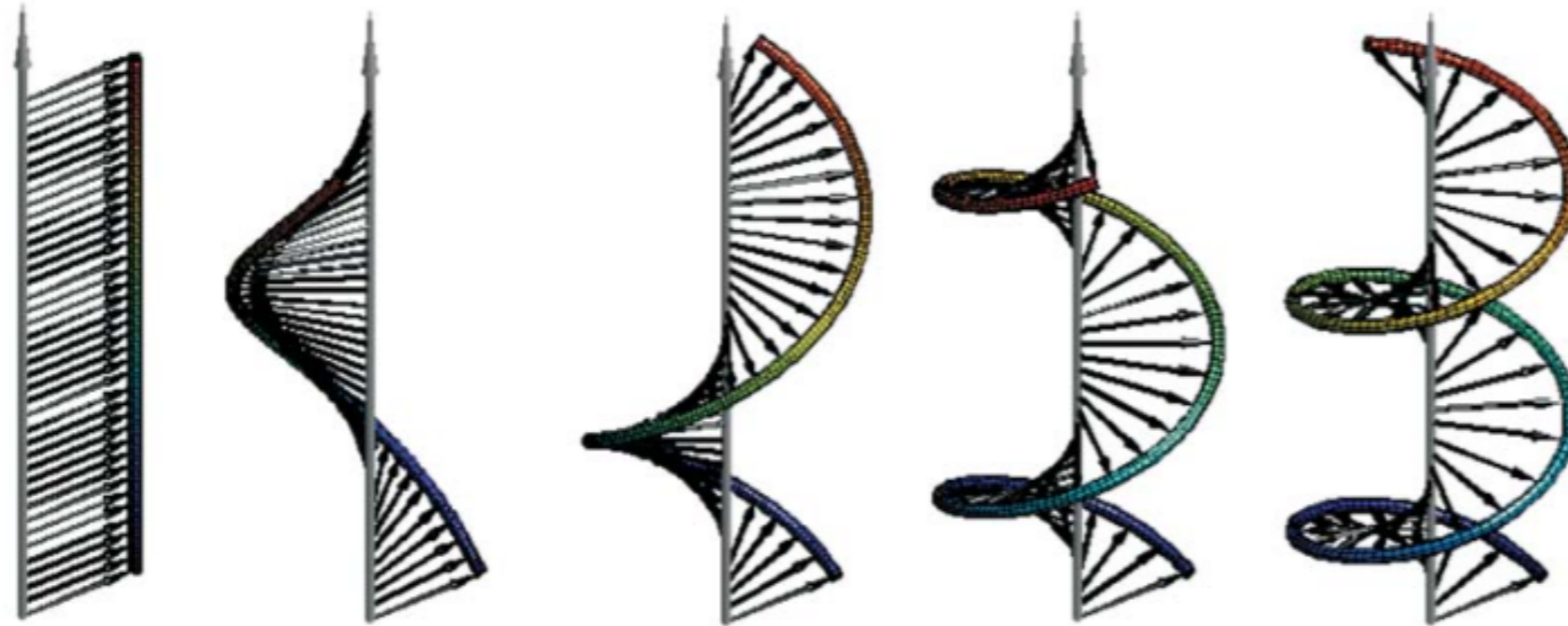*Weigel, JMRI 2015;41:266–295.*

Gradient Dephasing

Gradient G
(z-direction presumed)

$\tilde{F}_+(0)$ $\tilde{F}_+(k_1)$ $\tilde{F}_+(k_2)$ $\tilde{F}_+(k_3)$ $\tilde{F}_+(k_4)$
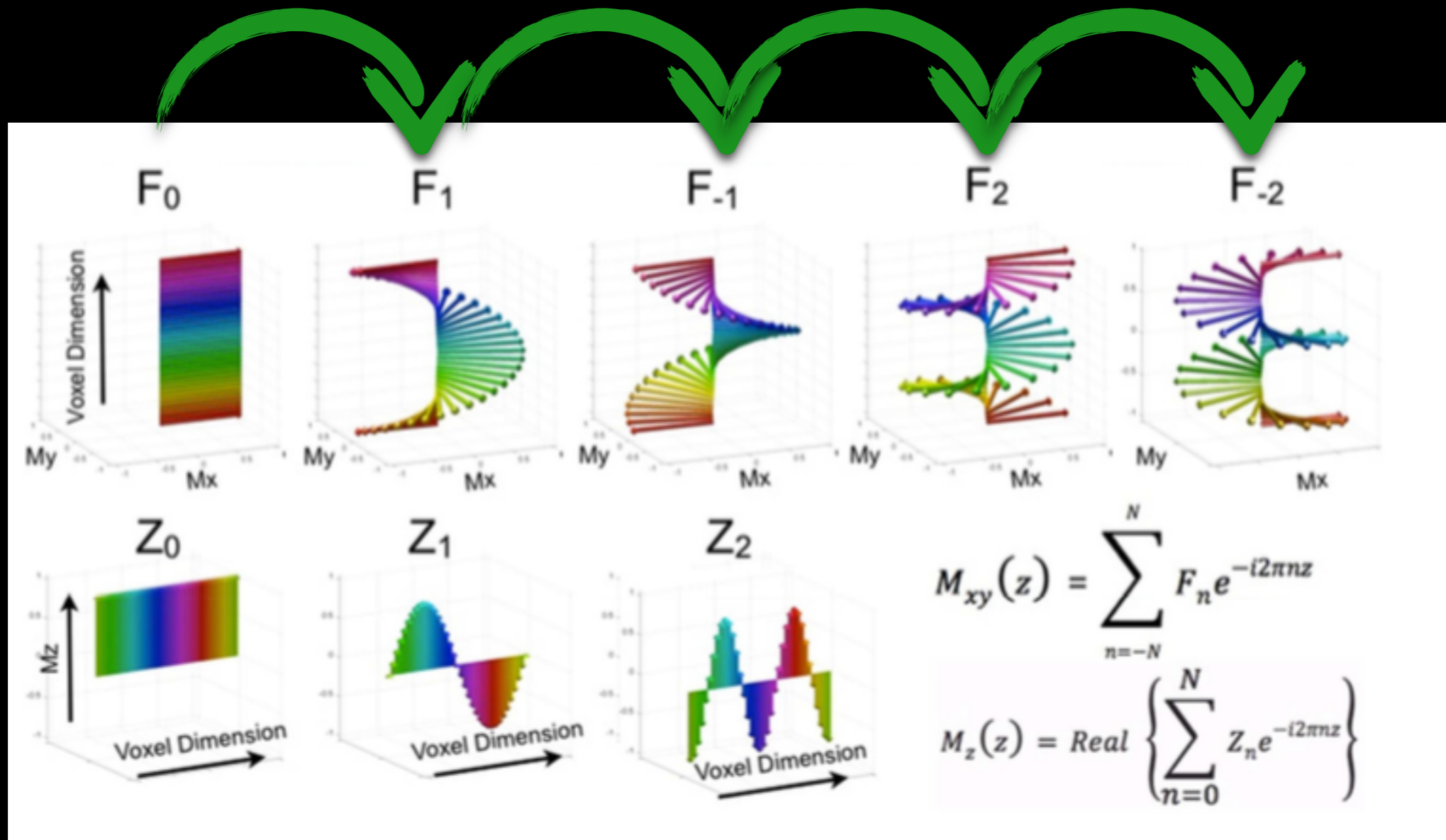
k

k_4
k_3
k_2
k_1
0

slope ~ gradient G

Time t

Gradient

G

0

Time t
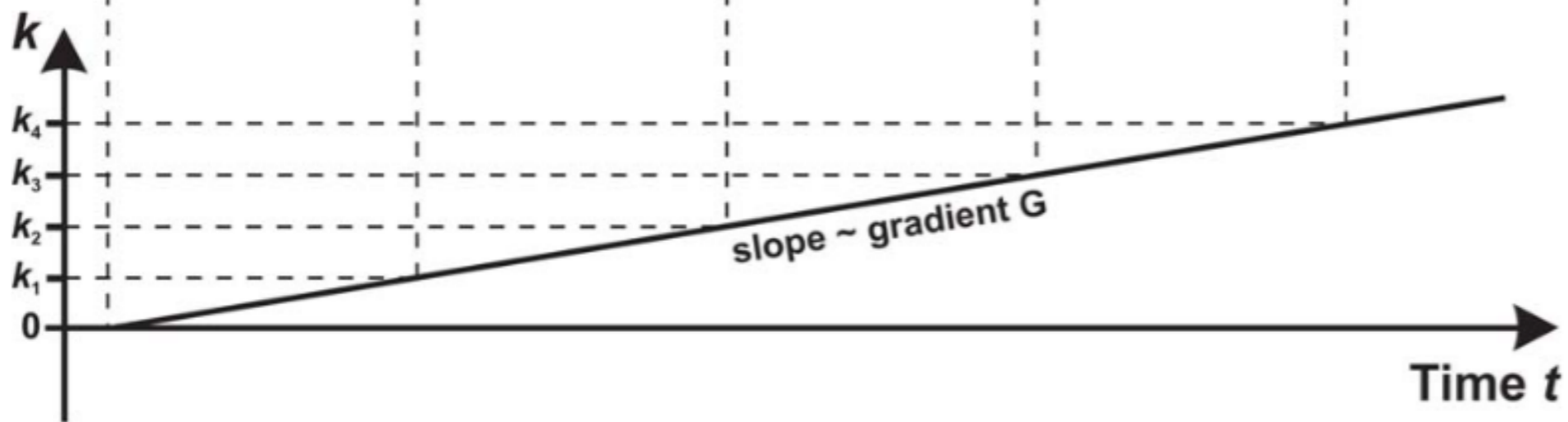
Gradient Dephasing
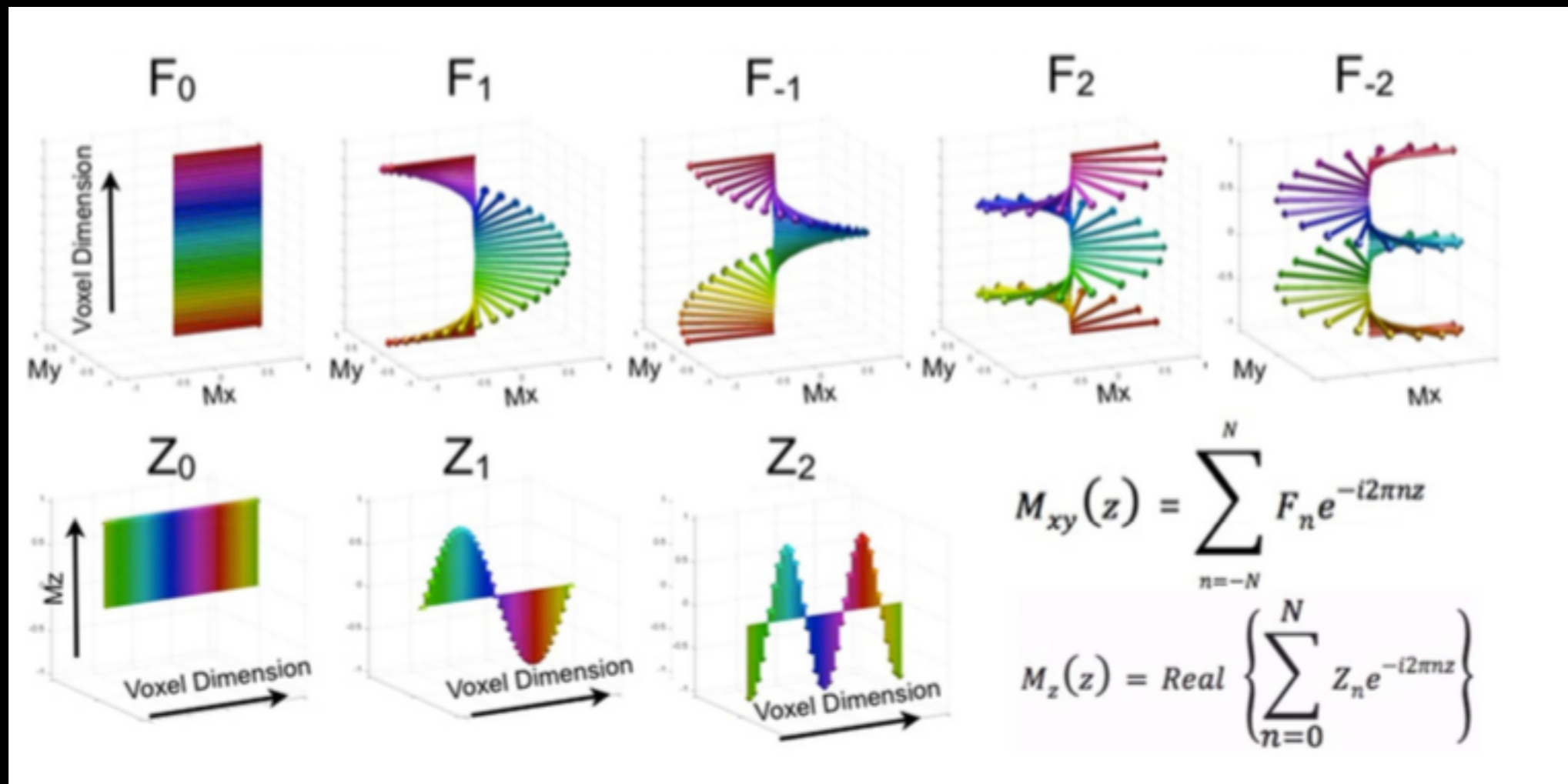
"Discrete" Gradient Dephasing

transition between states

$$M_{xy}(z) = \sum_{n=-N}^{N} F_n e^{-i2\pi nz}$$

$$M_z(z) = Real\left\{\sum_{n=0}^{N} Z_n e^{-i2\pi nz}\right\}$$

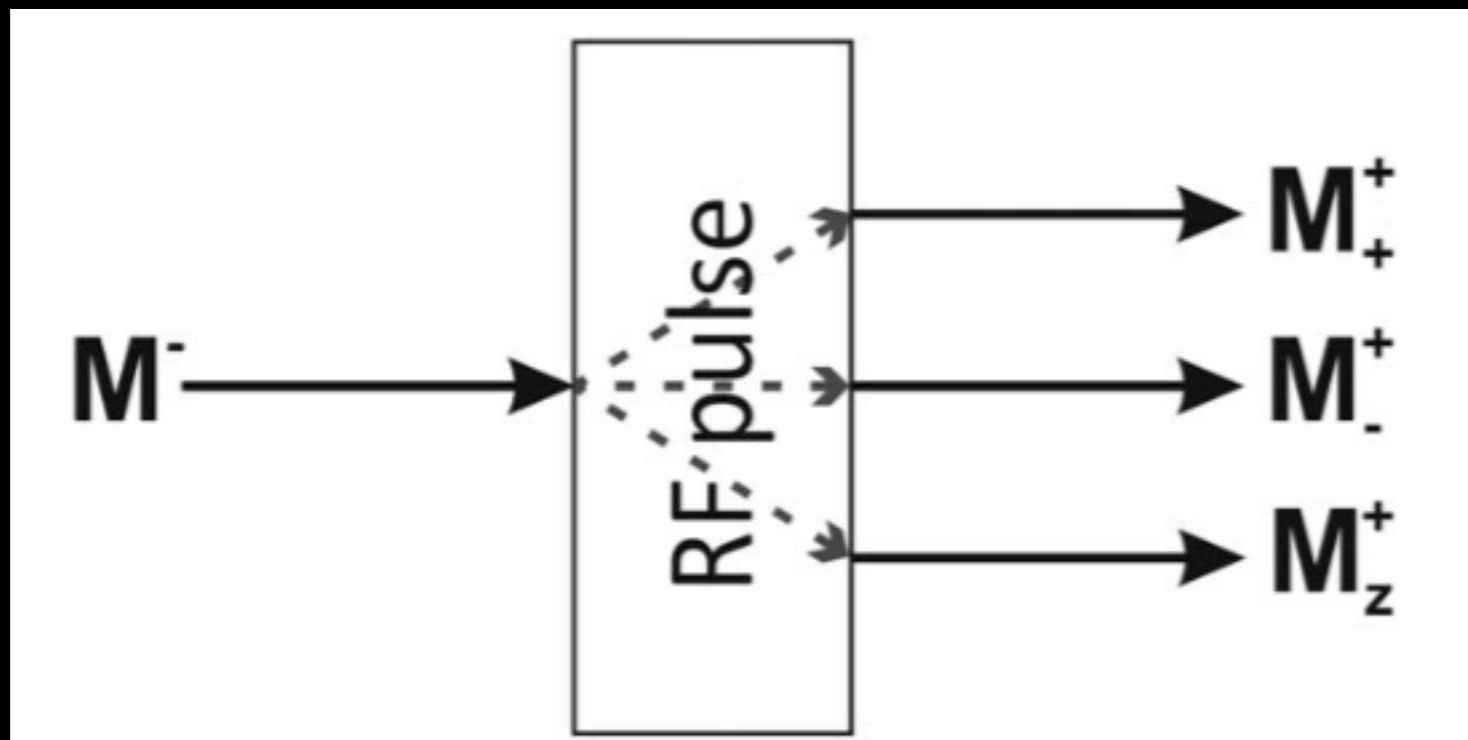*k* is the number of twists/cycles across a voxel

RF Pulse

- Woessner Decomposition
  magnetization after an RF pulse can be regarded as
  a composition of 3 components:
  - transversal component that is unaffected (0°-pulse)
  - transversal component that is refocused (180°-pulse)
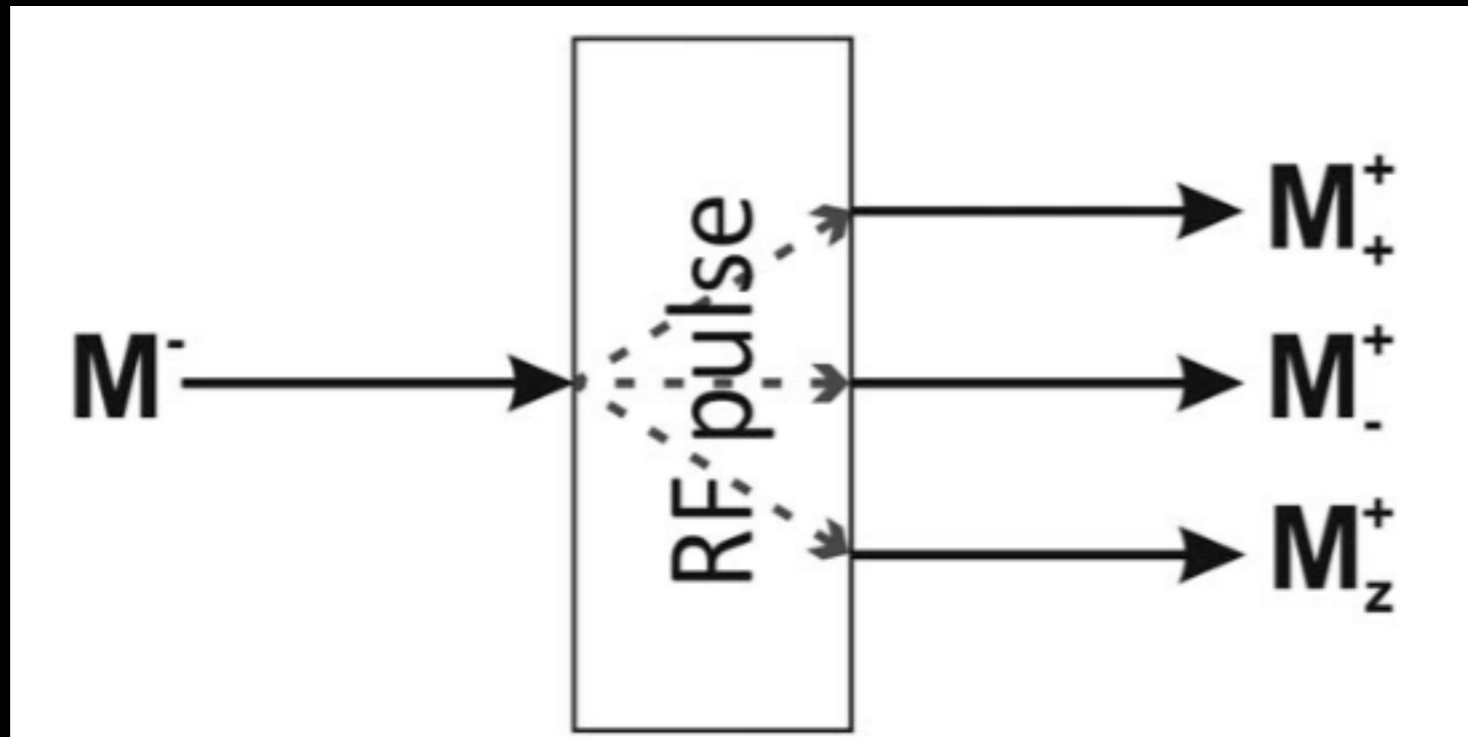 - a longitudinal component

rephasing

dephasing
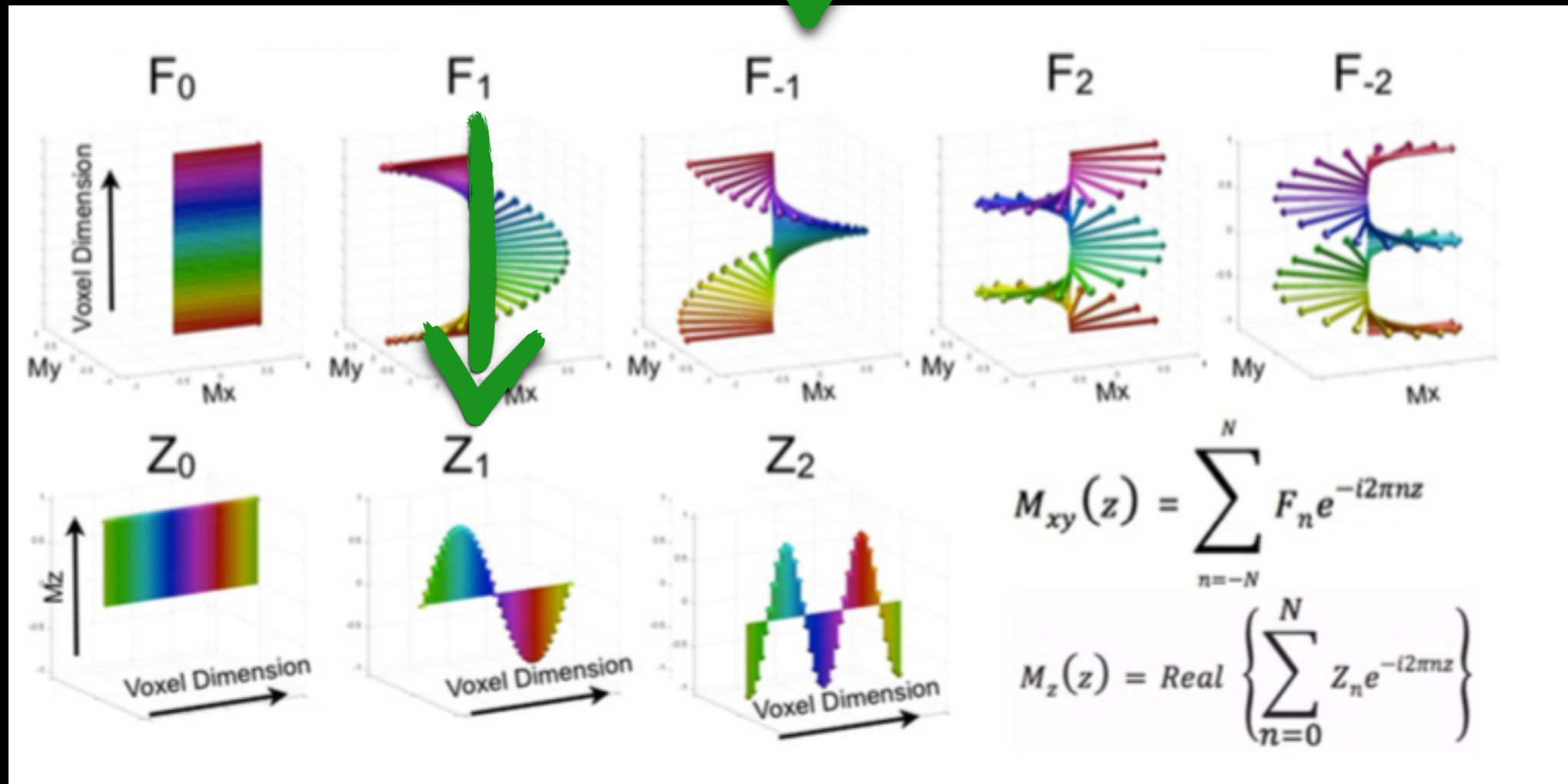
longitudinal

Woessner DE. J Chem Phys 1961; 34: 2057–2061.

RF Pulse

- The RF pulse operator splits any given EPG state with dephasing order k into 3 different new states:
  - a transversal state with identical k
  - a transversal state with inverted k
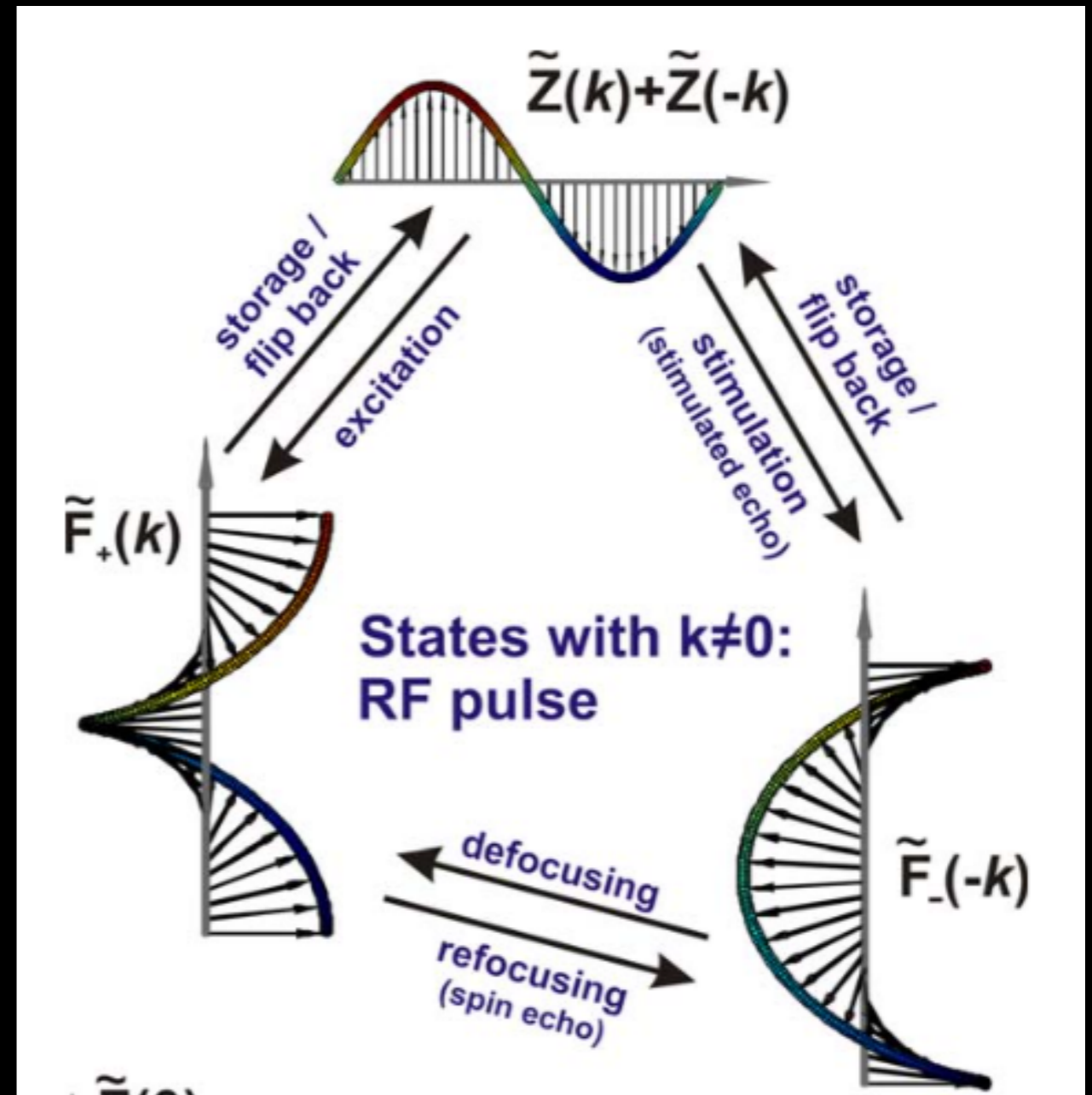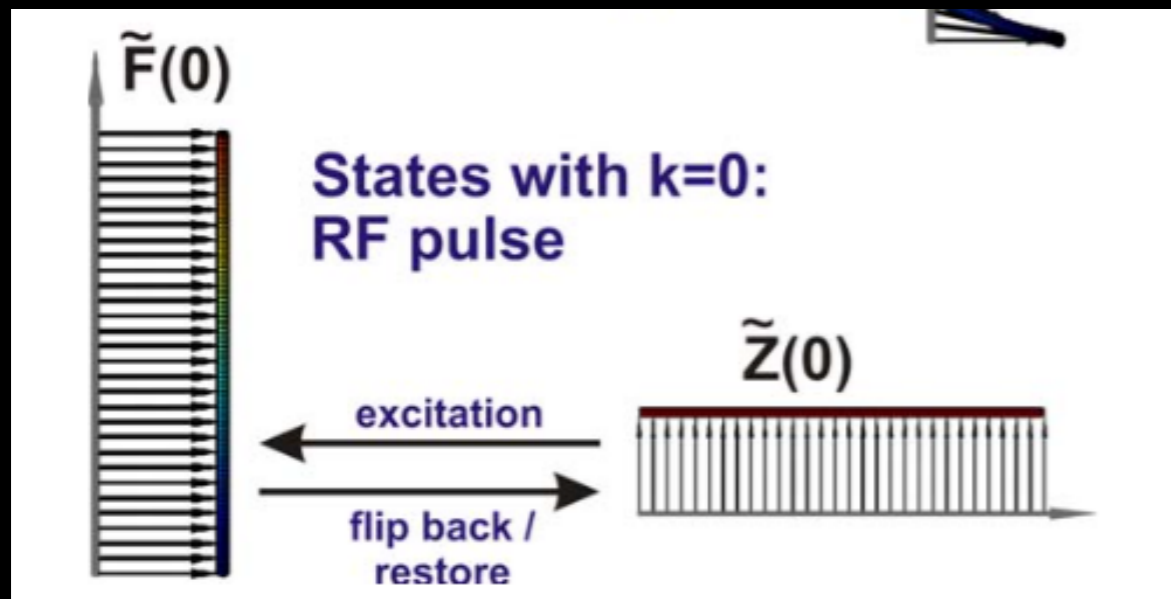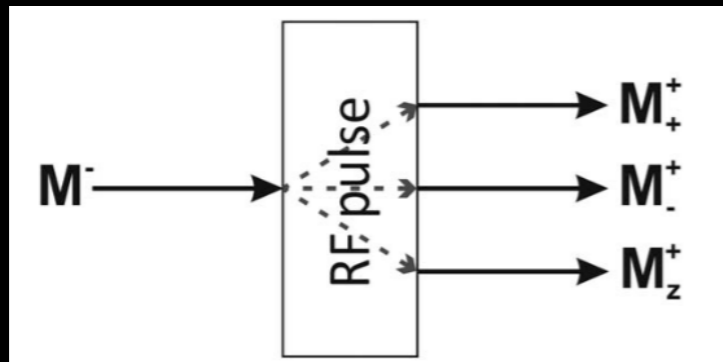 - a longitudinal state with identical k
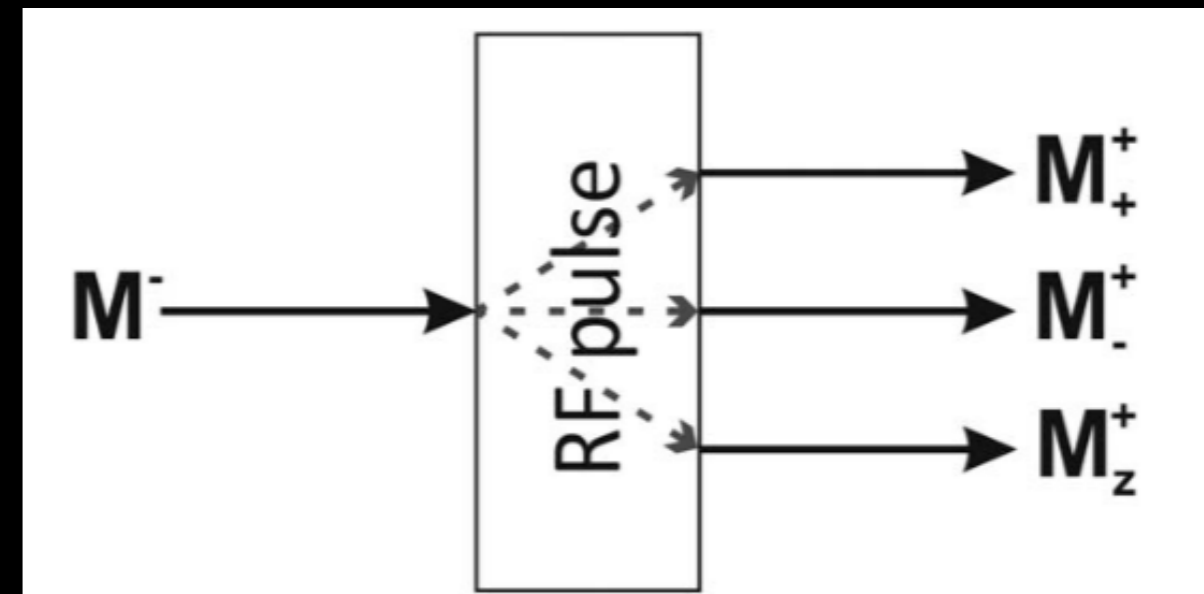
RF Pulse

mixes *F* and *Z* states!

RF Pulse

EPG Concept Summary

Fourier based configuration states

RF pulse partitioning

$$M_{xy}(z) = \sum_{n=-N}^{N} F_n e^{-i2\pi nz}$$

$$M_z(z) = Real\left\{\sum_{n=0}^{N} Z_n e^{-i2\pi nz}\right\}$$
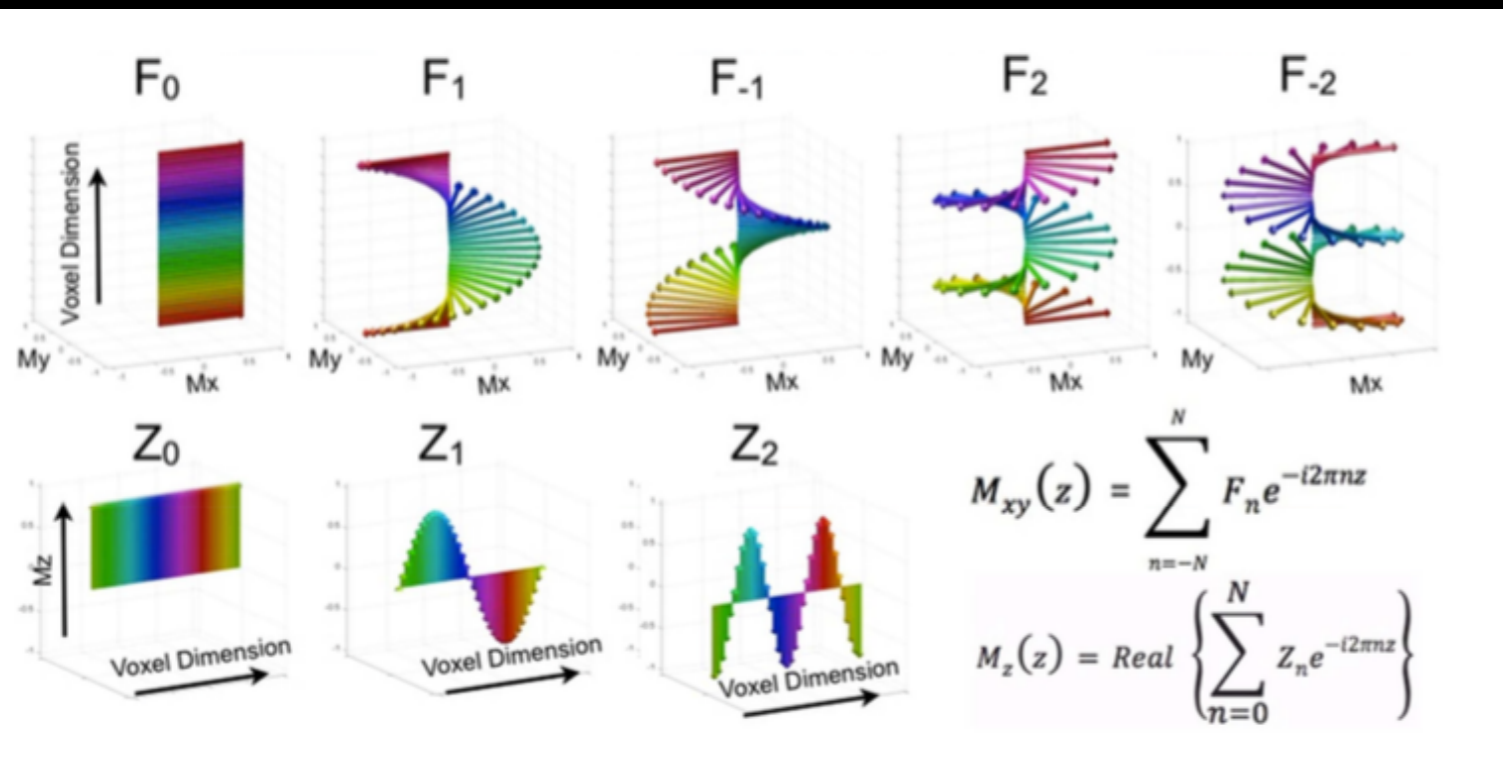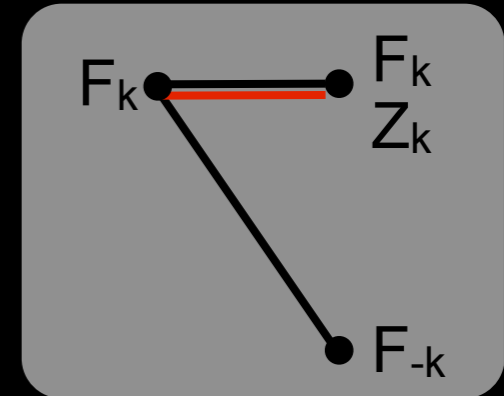
Phase graph approach that depicts the evolution of a complete isochromat ensemble.

EPG "Calculus"

- RF pulse for state *k*:
  - Produces signal in longitudinal state *k* and transverse states *k* and *-k*

F_k F_k
Z_k
F_{-k}

- Gradient dephaser for state *k*:
  - Moves transverse magnetization to *k*+1
  - Does not affect longitudinal magnetization

F_{k+1}
F_k

EPG: Spin Echo

90°   180°

T_1, T_2 T_1, T_2

F_2

F_1

F_0 Z_1

F_{-1} SE
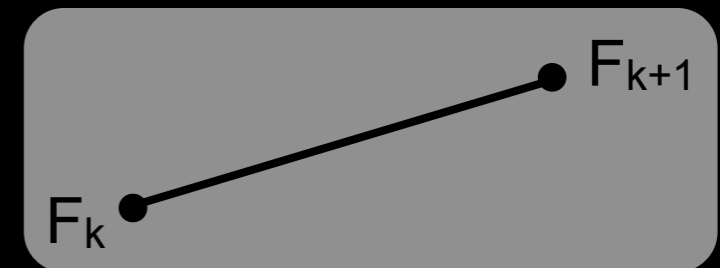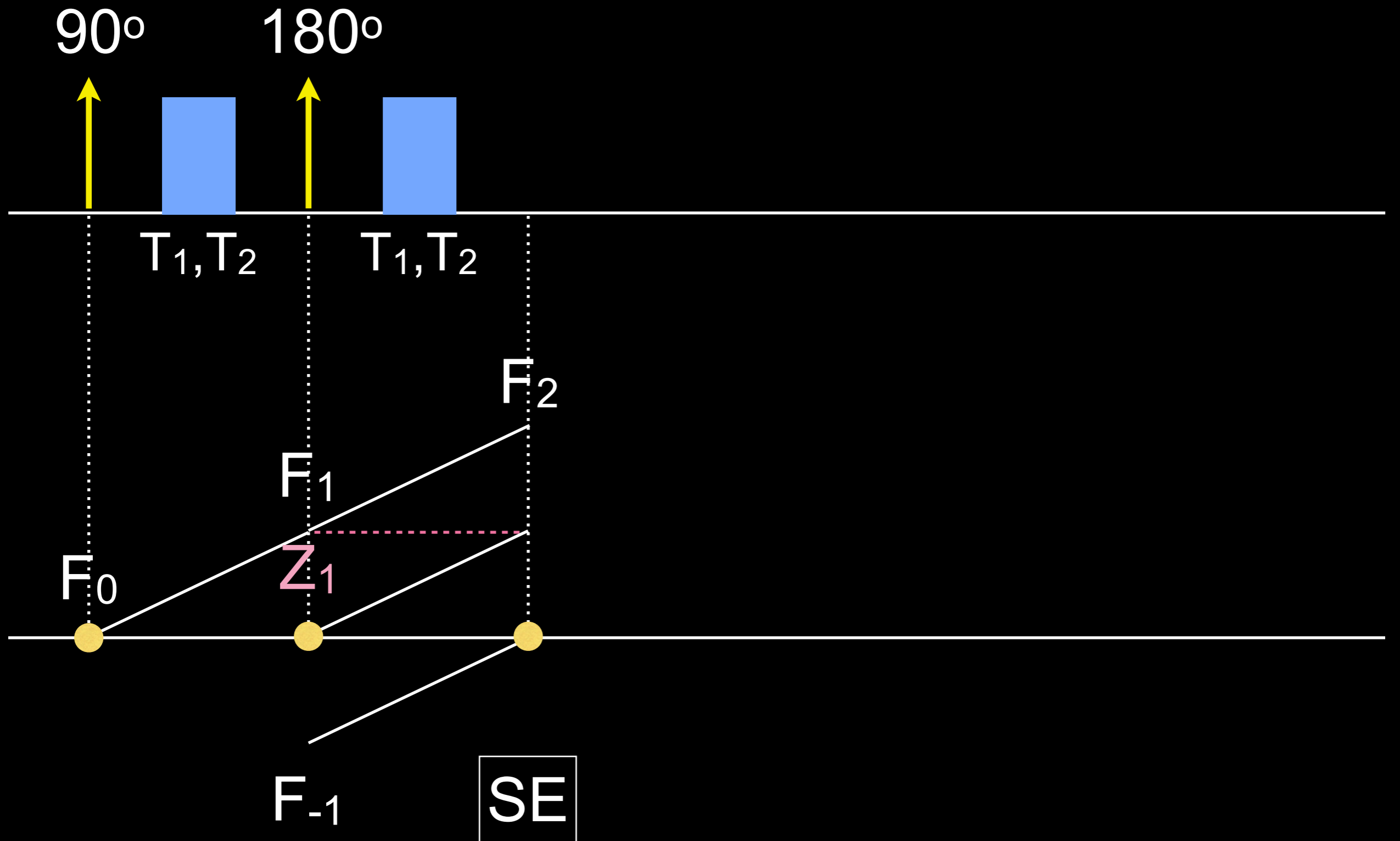
EPG: Stimulated Echo

90° θ θ

F_1

Z_1

F_0

F_{-1}

STE

EPG: 3-Pulse Experiment

Weigel, JMRI 2015;41:266–295.
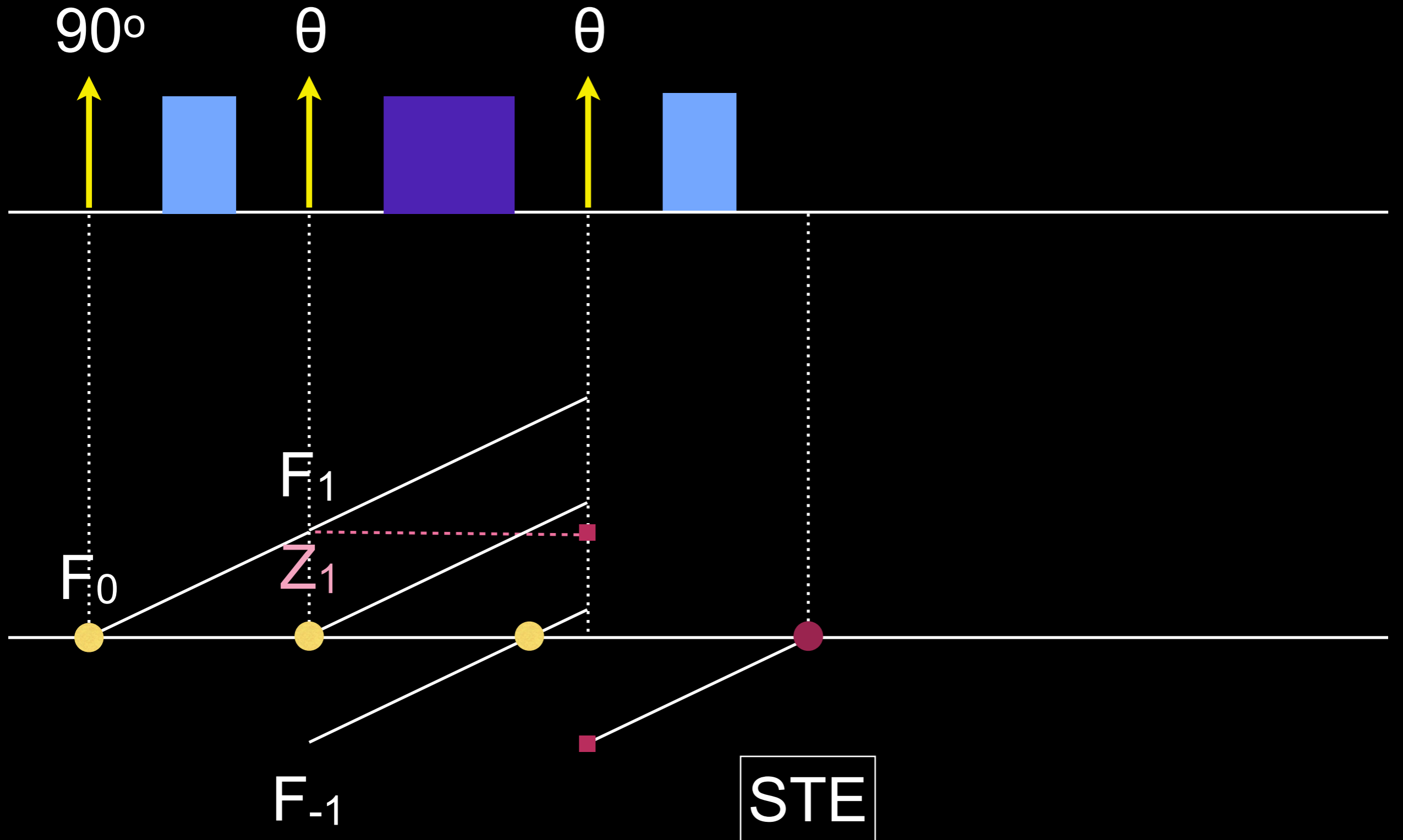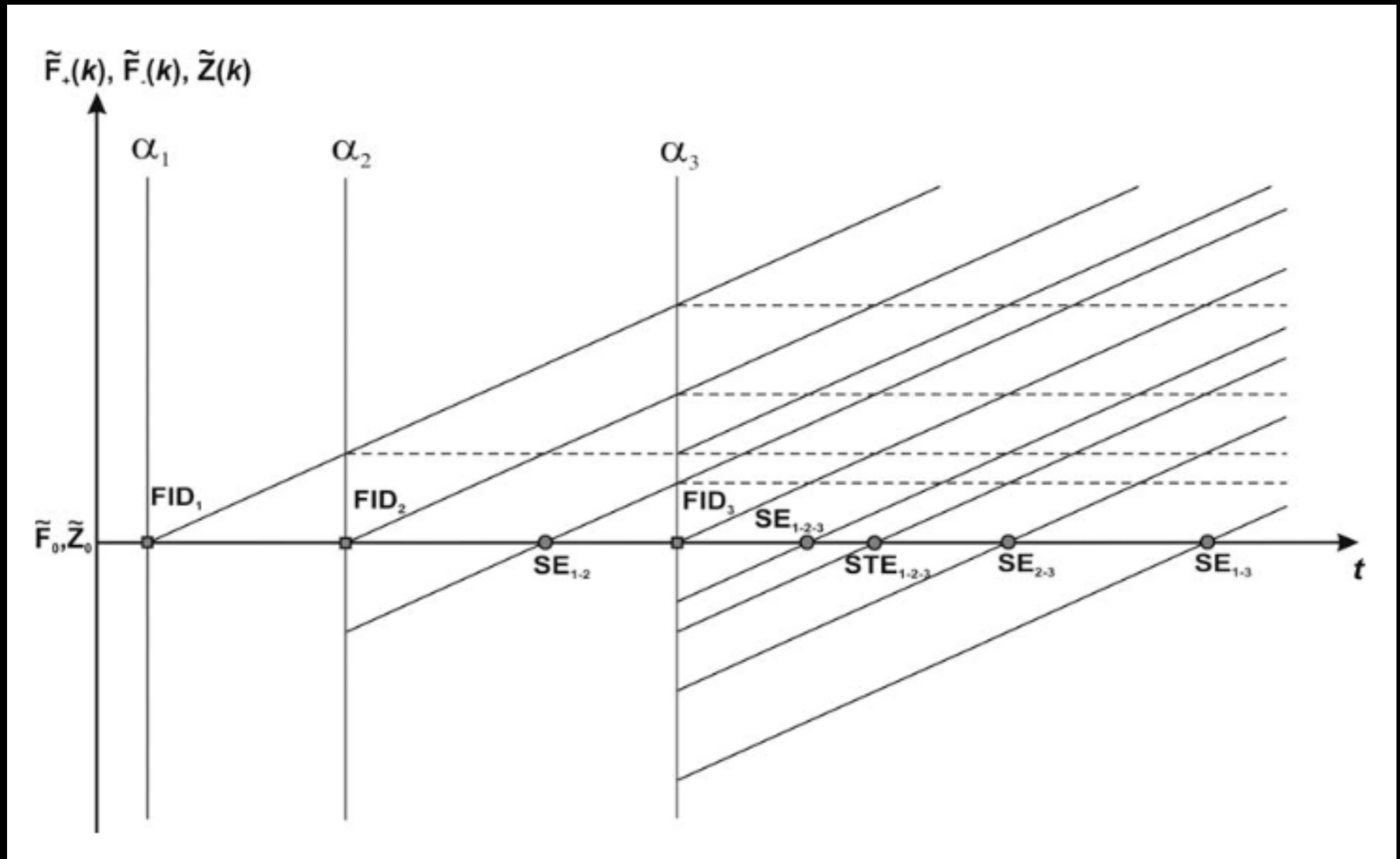
EPG: Train of Spin Echo

EPG: CPMG

F_0 = observable signal ("Echo")
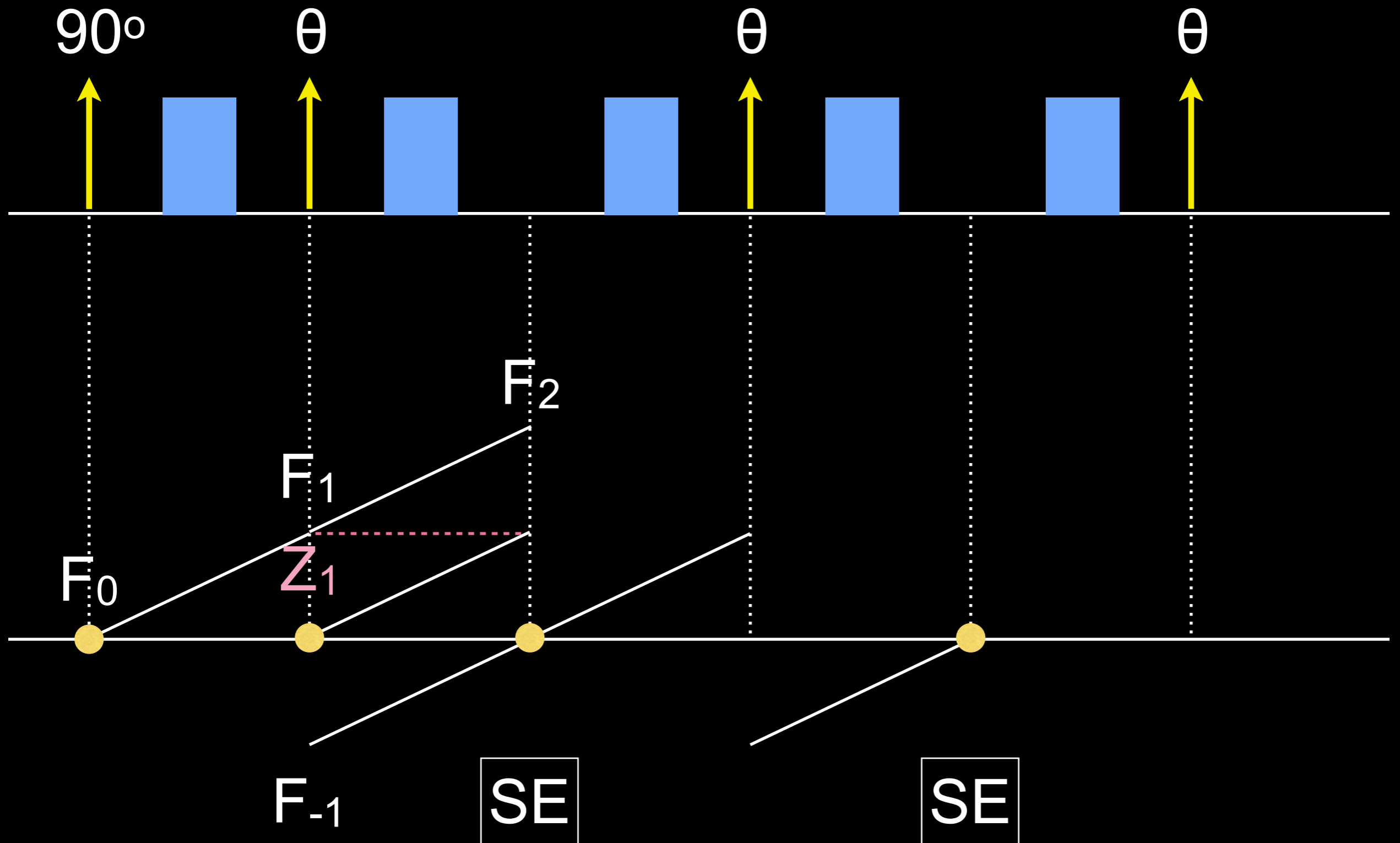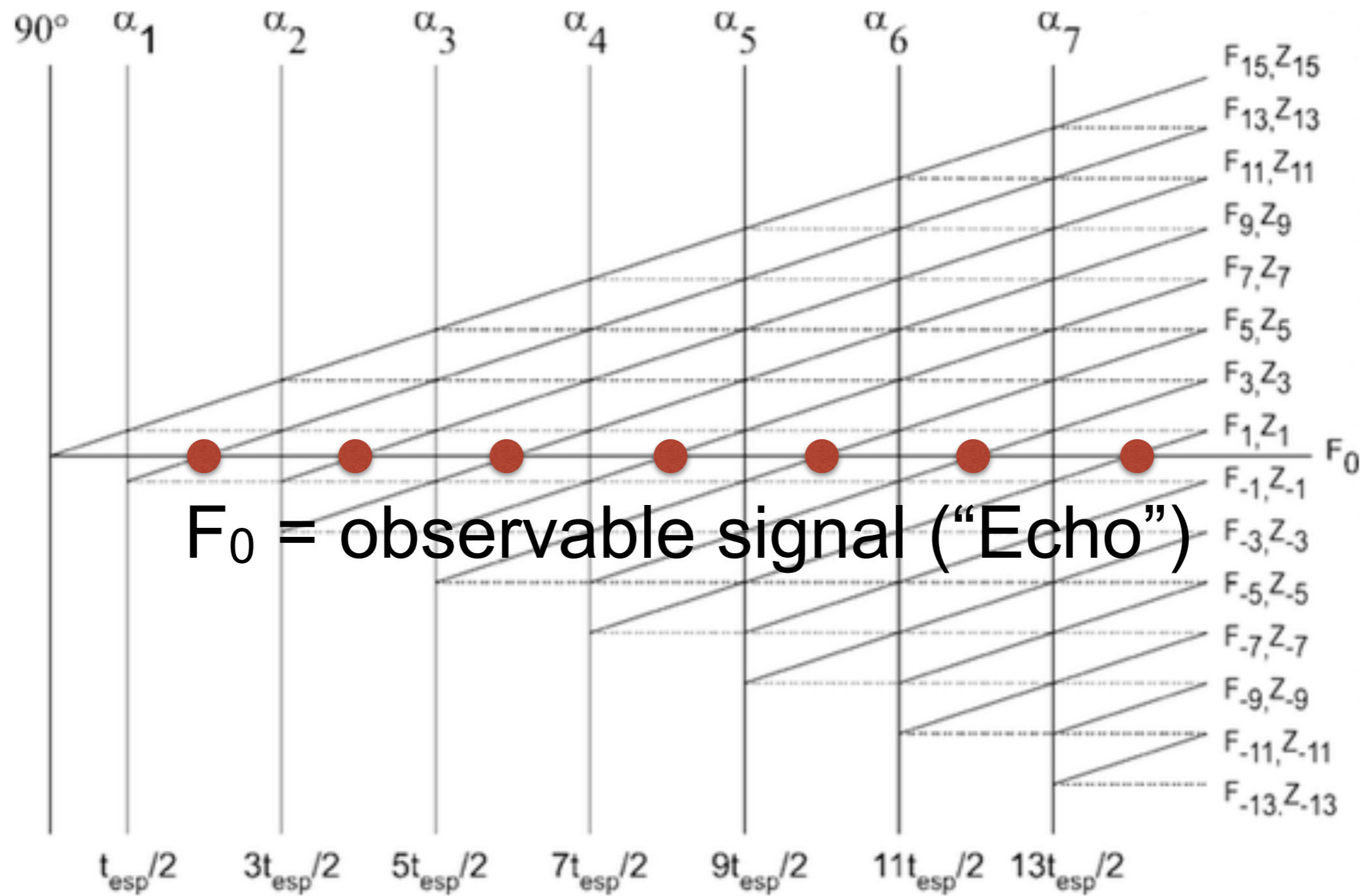
EPG: Matrix formulation

- Phase states
  - Can represent as a matrix:

$$P = \begin{bmatrix} F_0 & F_1 & F_2 \\ F_0^* & F_{-1} & F_{-2} & \cdots \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

EPG: Matrix formulation

- ## RF pulses
  - invert state (e.g., F_3 to F_{-3}) or can transfer between F and Z states
  - Simple pre-multiplication P' = RP, where R is

$$\begin{pmatrix} \cos^2 \frac{\alpha}{2} & e^{2i\phi} \sin^2 \frac{\alpha}{2} & -ie^{i\phi} \sin \alpha \\ e^{-2i\phi} \sin^2 \frac{\alpha}{2} & \cos^2 \frac{\alpha}{2} & ie^{-i\phi} \sin \alpha \\ -\frac{i}{2} e^{-i\phi} \sin \alpha & \frac{i}{2} e^{i\phi} \sin \alpha & \cos \alpha \end{pmatrix}$$

for an RF pulse with flip angle α and phase ϕ

Scheffler, Concepts in MR 1999

EPG: Matrix formulation

- Gradients (in discretized units)
  - Increase number of states by 1
  - Replace all F_k states with F_{k-1} (e.g., F_0 becomes F_1)
  - Replace F_0 using F_0^*
  - Do not change Z states

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & 0 \\ F_0^* & F_{-1} & F_{-2} & \cdots & 0 \\ Z_0 & Z_1 & Z_2 & 0 \end{bmatrix}$$

phase states grow linearly w.r.t. TSE ETL

EPG: Matrix formulation

- Relaxation
  - Transverse:
    All F states attenuated by $E_2 = \exp(-T/T_2)$
  - Longitudinal:
    All Z states attenuated by $E_1 = \exp(-T/T_1)$
    Z_0 state only has recovery of $M_0(1-E_1)$

EPG: Extensions

- Non-ideal slice profiles

- Variable RF flip angle and phase

- Motion / flow effects

- Diffusion effects
  - Weigel M, et al., JMR 2010; 205: 276-285

EPG Simulation

- Phase state propagation
  - RF pulse
  - T_1, T_2 decay
  - free precession
  - gradient pulse
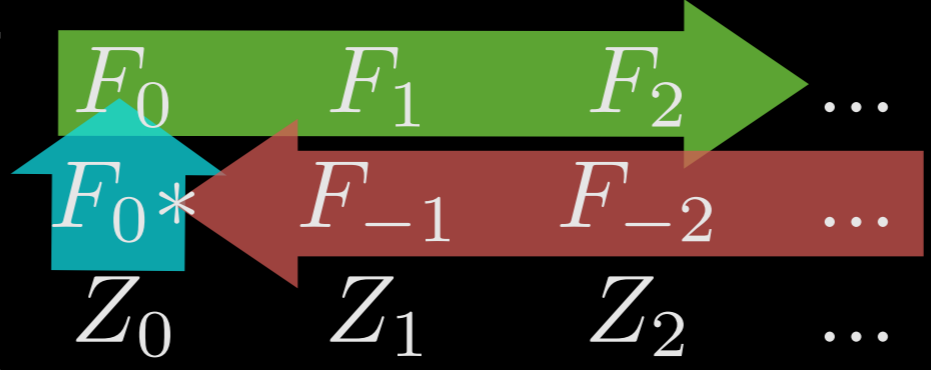
EPG Simulation

Phase states:

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & ... \\ F_0* & F_{-1} & F_{-2} & ... \\ Z_0 & Z_1 & Z_2 & ... \end{bmatrix}$$

RF pulse (θ, ϕ), P⁺ = RP:

$$R_{\{\theta,\phi\}} = \begin{bmatrix} \cos^2 \frac{\theta}{2} & e^{2i\phi} \sin^2 \frac{\theta}{2} & -ie^{i\phi} \sin\theta \\ e^{-2i\phi} \sin^2 \frac{\theta}{2} & \cos^2 \frac{\theta}{2} & ie^{-i\phi} \sin\theta \\ -\frac{i}{2} e^{-i\phi} \sin\theta & \frac{i}{2} e^{i\phi} \sin\theta & \cos\theta \end{bmatrix}$$

EPG Simulation

Gradients:

$$P = \begin{bmatrix} F_0 & F_1 & F_2 & ... \\ F_0* & F_{-1} & F_{-2} & ... \\ Z_0 & Z_1 & Z_2 & ... \end{bmatrix}$$

Relaxation:

$$F_k \rightarrow E_2 F_k$$

$$Z_k \rightarrow E_1 Z_k \qquad (k>0)$$

$$Z_0 \rightarrow E_1 Z_0 + M_0(1 - E_1)$$

EPG Simulation

- Transient state; steady state

- Different seq/tissue params

- Brian's MATLAB EPG sim code
  - will be emailed to class mailing list

EPG Simulation

- *Example*: Turbo Spin Echo
  - epg_rf.m
  - epg_grelax.m, epg_grad.m, epg_mgrad.m
  - epg_cpmg_hhw.m
  - EPGSim_CPMG_hhw.m
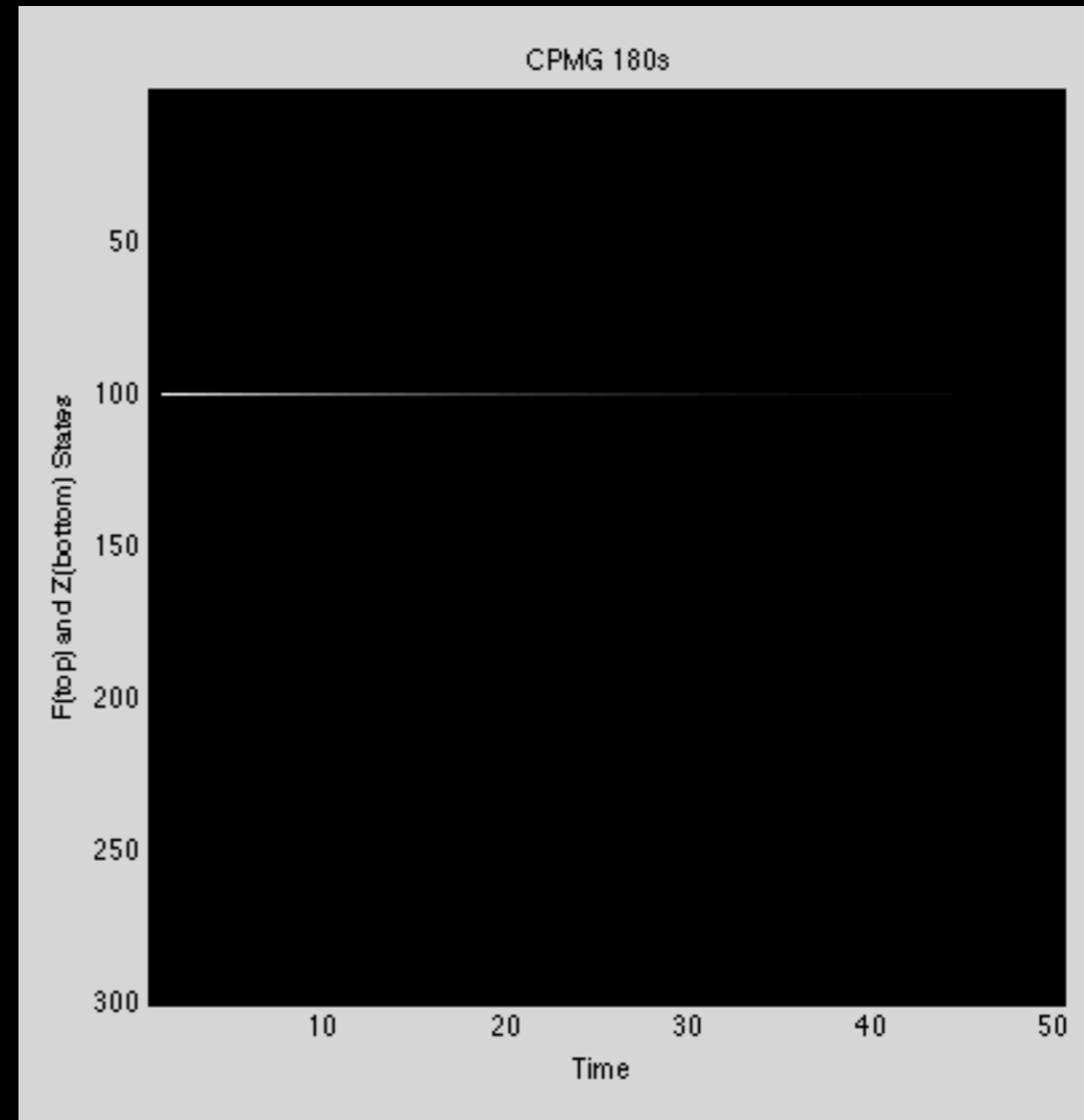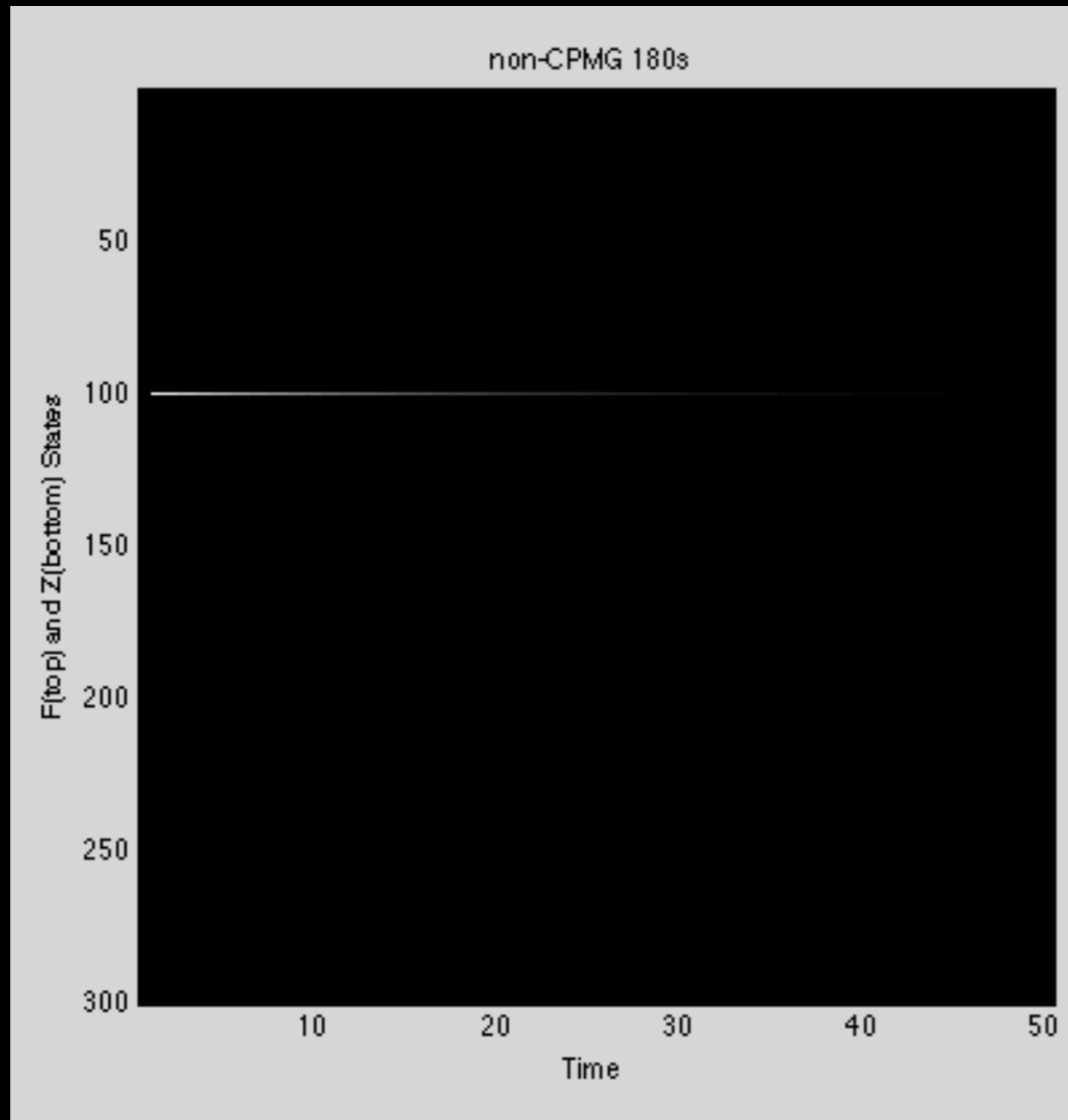
  - can look at different refocusing RF trains

EPG Simulations: TSE

- non-CPMG 180s: 90x-180x-180x-…
- CPMG 180s: 90x-180y-180y-…
- non-CPMG 120s: 90x-120x-120x-…
- CPMG 120s: 90x-120y-120y-…
- CPMG 120s +prep: 90x-150y-120y-…

EPG Simulations: TSE

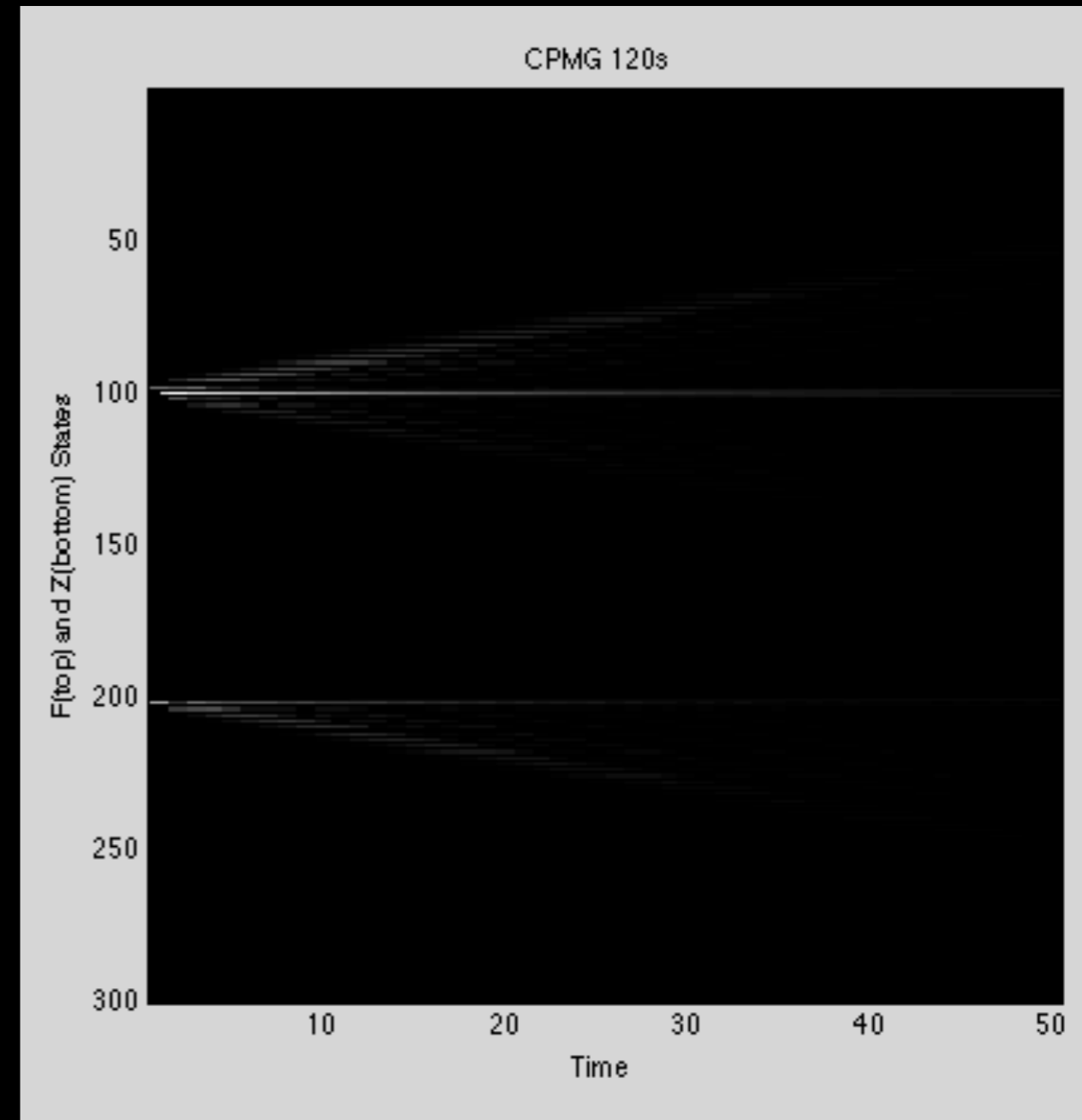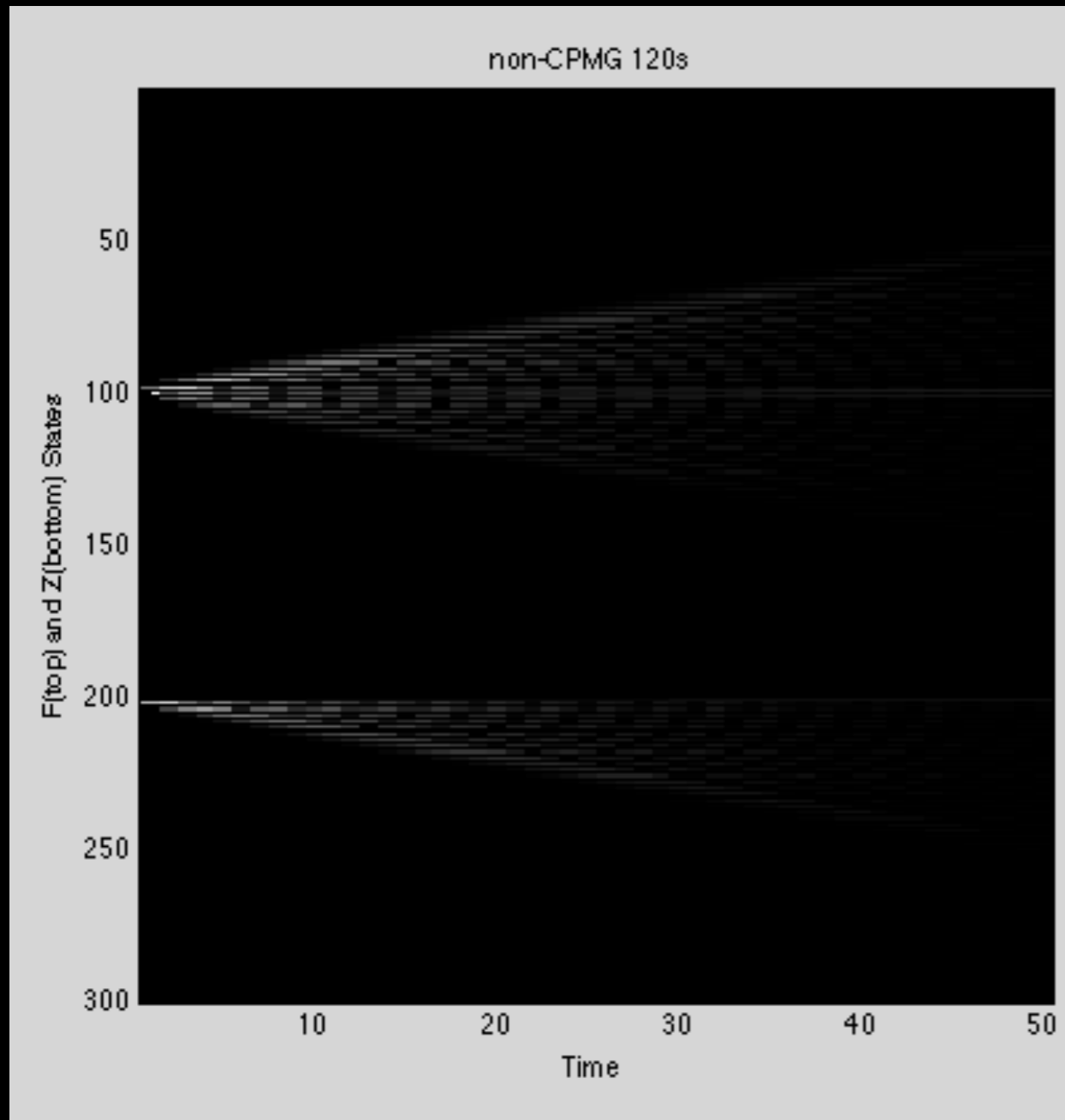non-CPMG 180s   CPMG 180s

T_1 = 1000 ms, T_2 = 100 ms, ETL = 50, ESP = 10 ms
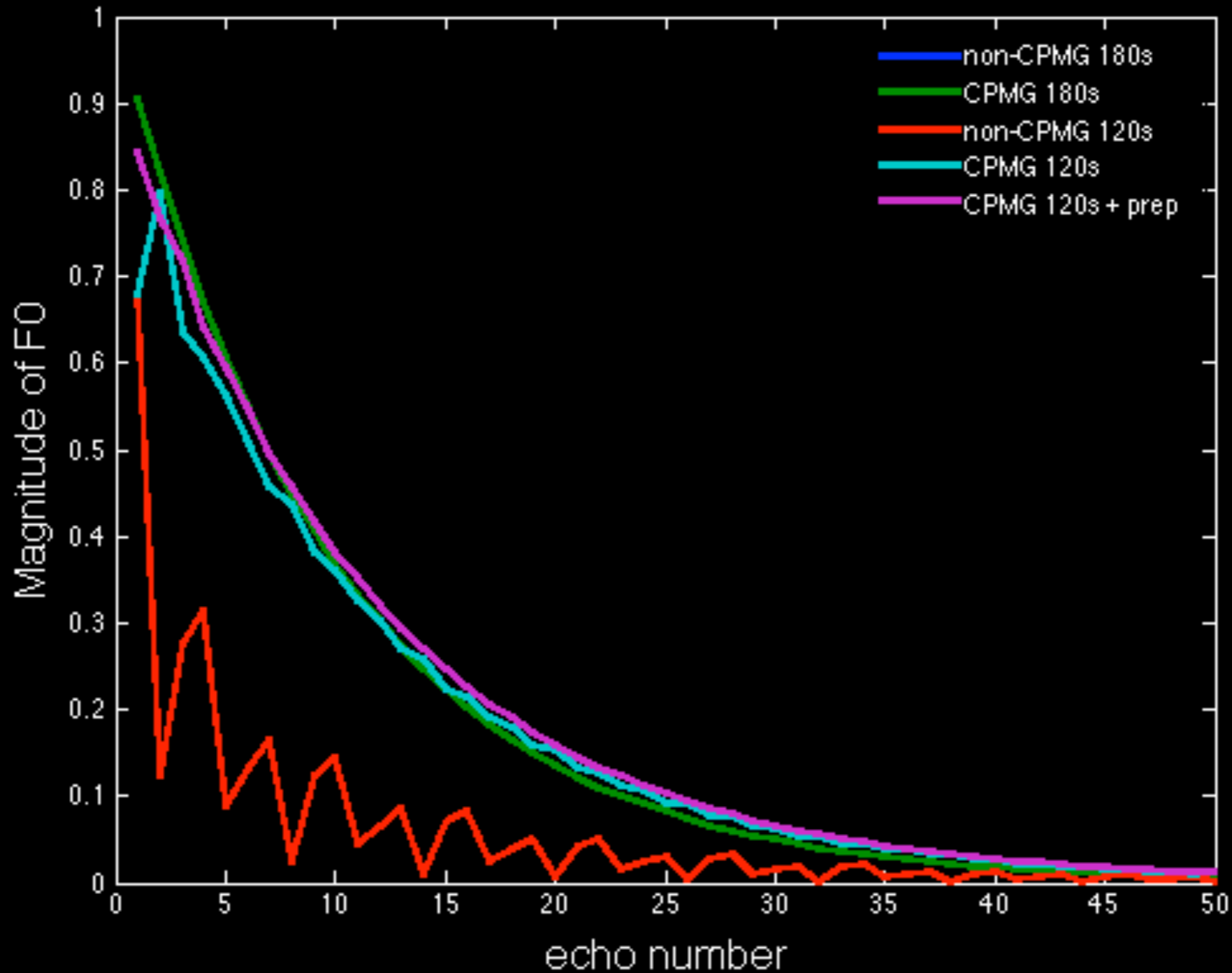
EPG Simulations: TSE

non-CPMG 120s

CPMG 120s

T_1 = 1000 ms, T_2 = 100 ms, ETL = 50, ESP = 10 ms

EPG Simulations: TSE
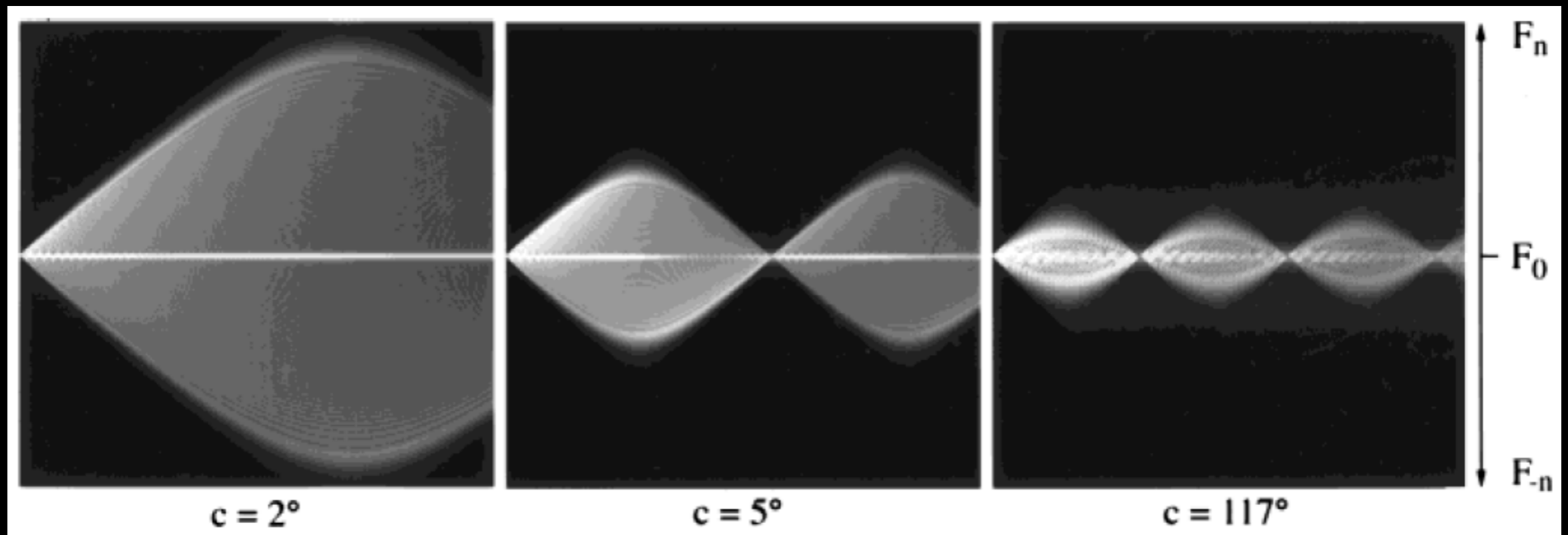
F_0 vs. echo number

T_1 = 1000 ms, T_2 = 100 ms, ETL = 50, ESP = 10 ms

EPG Simulation

- Homework 1, part 2A
  - Gradient-spoiled GRE (SSFP-FID)

EPG Simulation

- Homework 1, part 2B
  - RF-spoiled GRE

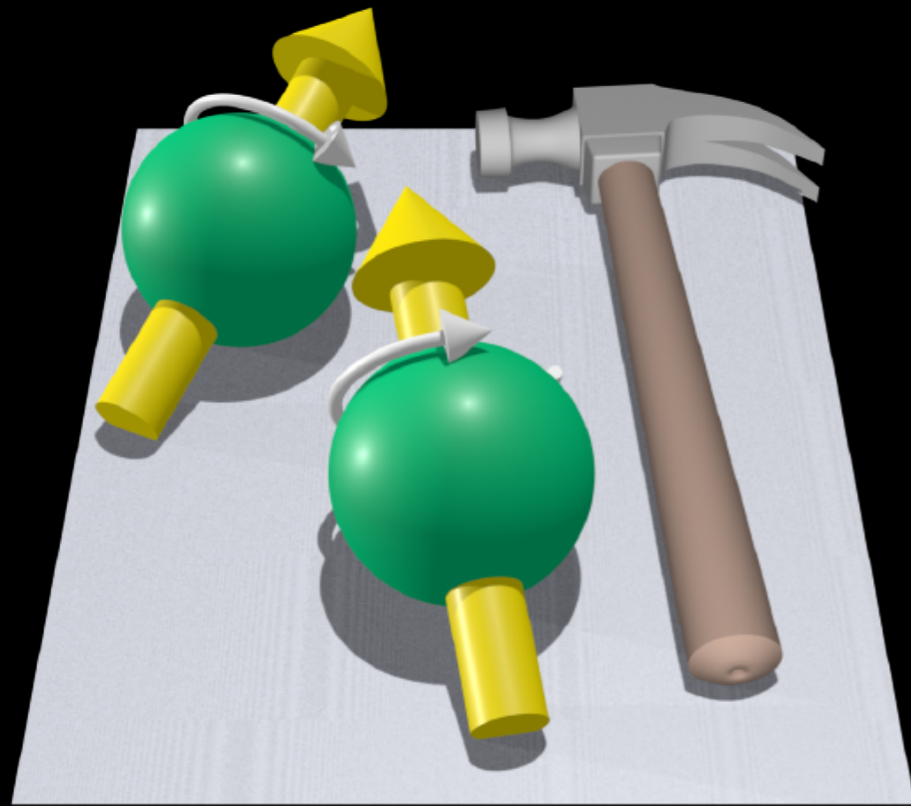*Scheffler, Concepts in MR 1999, Fig. 11*

Homework 1

- Pulse Sequence Simulations
  - 1. Bloch: Steady state comparison, bSSFP transient state and catalyzation
  - 2. EPG: SSFP-FID, RF-spoiled GRE

- Due 5 pm, Fri, 4/22 by email
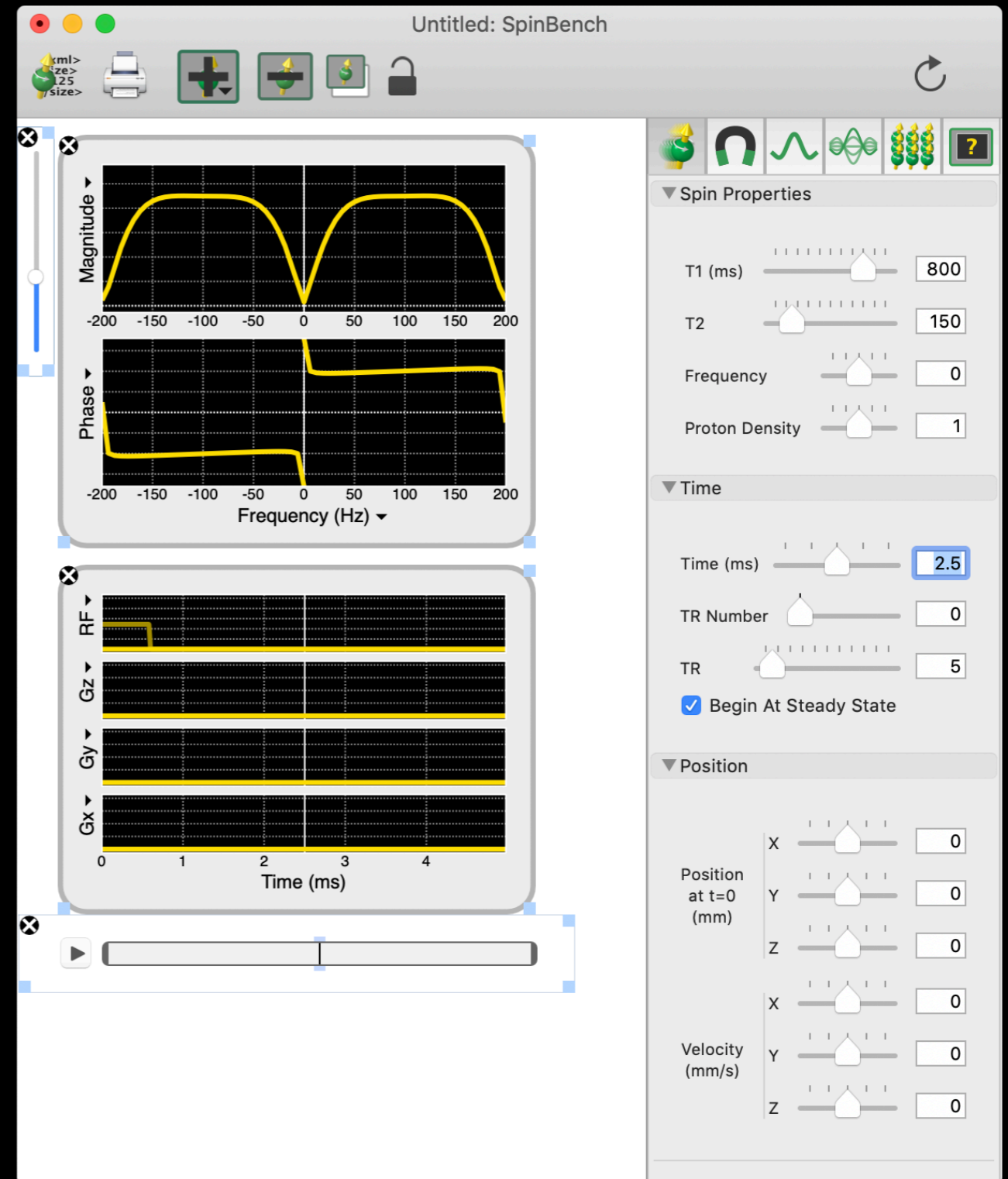 - PDF and MATLAB code

Summary

- Multiple RF pulses -> multiple echoes

- EPG analysis
  - consider groups of spins
  - explicit treatment of pathways and echoes
  - flexible and powerful
  - you can do it!

Spin Bench

https://www.heartvista.ai/spinbench

Thanks!

- Web resources
  - ISMRM 2010 Edu: Miller, Weigel
  - ISMRM 2011 Edu: Miller, Weigel

- Further reading
  - Bernstein et al., Handbook of MRI Sequences
  - Haacke et al., Magnetic Resonance Imaging
  - Scheffler, Concepts in MR 1999; 11:291-304
  - Hennig, JMR 1988; 78:397-407
  - Weigel, JMRI 2015; 41:266-295

Thanks!

- Acknowledgments
  - Brian Hargreaves's EPG slides and code
  - Kyung Sung's EPG slides
  - Isabel Dregely's EPG slides

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http://mrrl.ucla.edu/wulab