# RF Pulse Design:
# RF Pulses, Adiabatic Pulses

M229 Advanced Topics in MRI

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# Class Business

- Office hours
  - Holden: Fri 10-11 am
  - Shu-Fu (TA for HW1): Mon 2-4 pm
  - Timo (TA for HW2): Wed 1-3 pm
- Homework 1 is due on 4/28 (Friday)

# Outline

- Review of RF pulses
- Adiabatic passage principle
- Adiabatic inversion
- Applications of adiabatic pulses
- MATLAB demo

# Review of RF Pulses

# RF Pulses

- What do RF pulses do?

- Challenges at higher $B_0$ fields?

# Notation and Conventions

$$\vec{B} = B_0\hat{k} + B_1(t)[\cos\omega t\hat{i} - \sin\omega t\hat{j}]$$

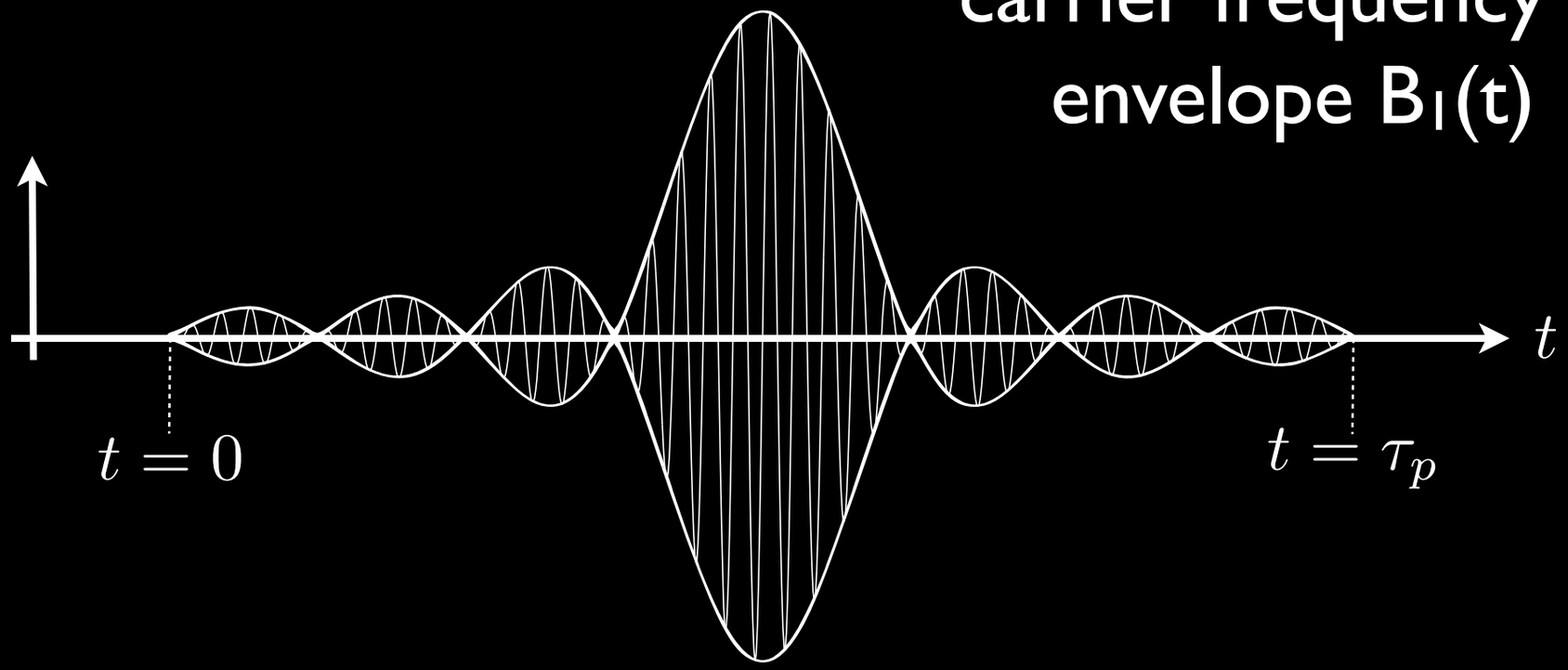
- ω = carrier frequency
- $\omega_0$ = resonant frequency
- $B_1(t)$ = complex valued envelope function

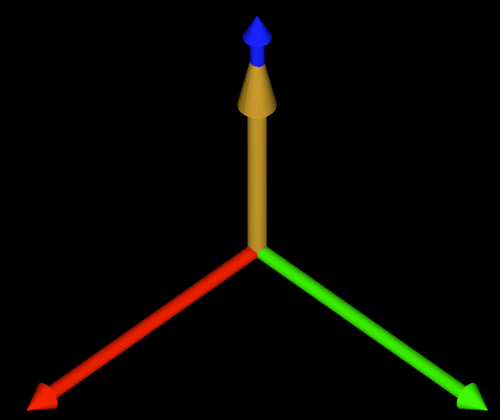
# RF Pulse - Excitation

$$\vec{B} = B_0\hat{k} + B_1(t)[\cos\omega t\hat{i} - \sin\omega t\hat{j}]$$

$$B_1(t) \cdot [\cos(\omega t)\hat{i} - \sin(\omega t)\hat{j}]$$

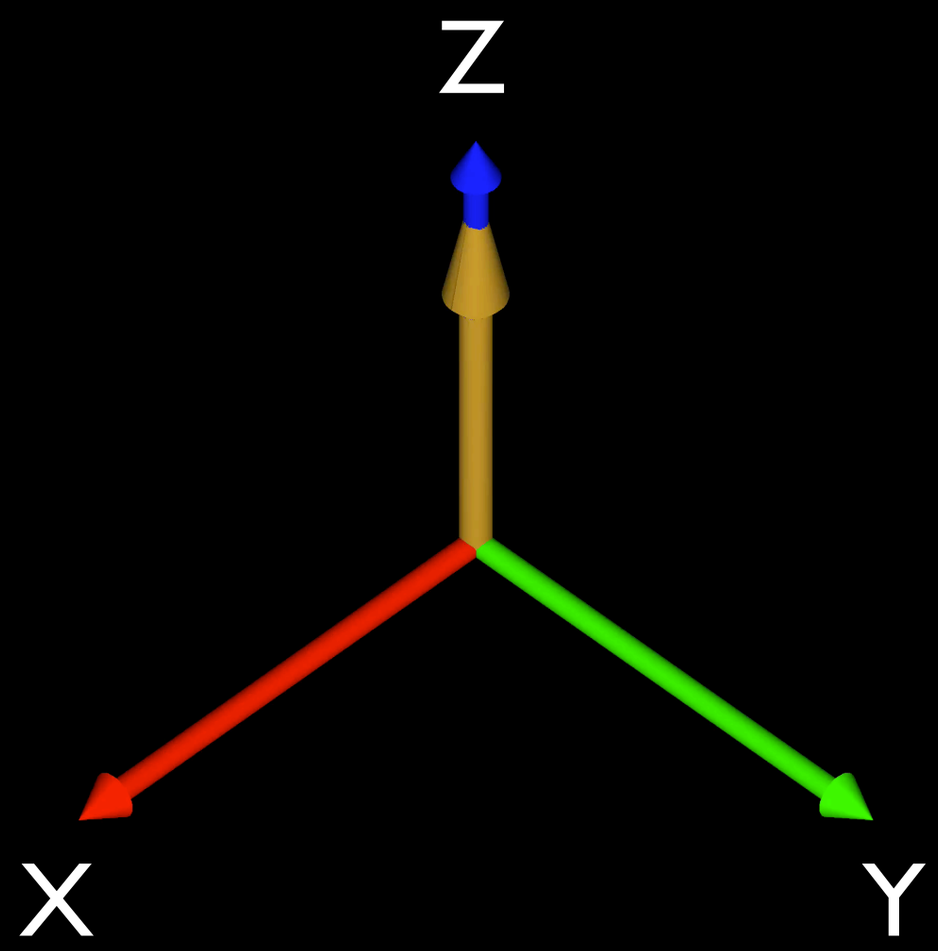
carrier frequency $\omega$
envelope $B_1(t)$



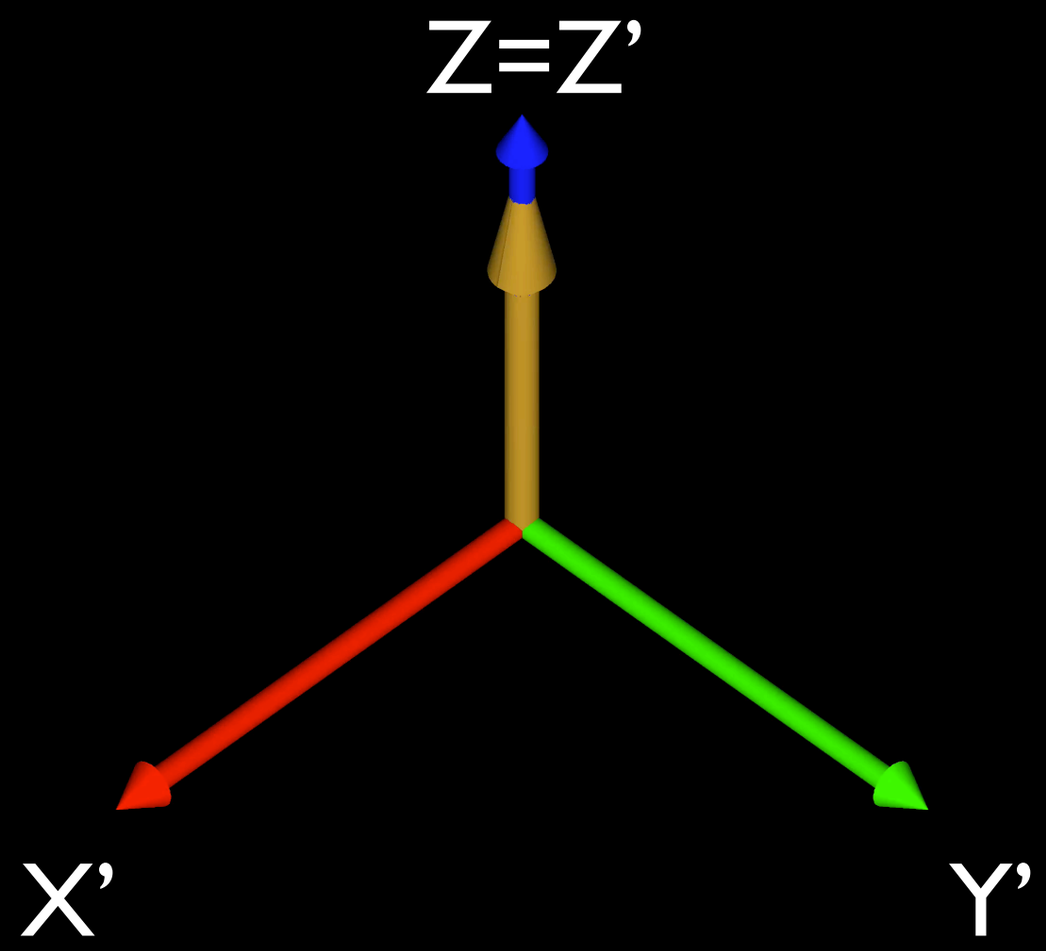



# Lab vs. Rotating Frame

- The rotating frame simplifies the mathematics and permits more intuitive understanding.




Laboratory Frame




Rotating Frame

# Rotating Frame

## Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix}$$

$$B_{z'} \equiv B_z$$

$$M_{z'} \equiv M_z$$

$$\vec{M}_{lab}(t) = R_Z(w_0 t) \cdot \vec{M}_{rot}(t)$$

$$\vec{B}_{lab}(t) = R_Z(w_0 t) \cdot \vec{B}_{rot}(t)$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma\vec{B} \quad \Rightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma\vec{B}_{eff}$$

# Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{w}_{rot}}{\gamma}$ fictitious field

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix}$$

# Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{w}_{rot}}{\gamma}$$

$$\vec{B}_{lab} = \begin{pmatrix} B_1(t)\cos\omega_0 t \\ B_1(t)\sin\omega_0 t \\ B_0 \end{pmatrix} \qquad \vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ B_1(t) \\ B_0 \end{pmatrix}$$

Assume real-valued $B_1(t)$

$$\vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix}$$

To the board ...

# Bloch Equation with Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix} \rightarrow \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$$

# Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

To the board ...

# $B_1$ Variations

- In MRI, the $B_1$ field is not always uniform across the imaging volume
- $B_1$ inhomogeneity can cause:
  - Image shading
  - Incomplete saturation (e.g. in fat suppression)
  - Incomplete inversion (e.g. CSF suppression, myocardium suppression in cardiac scar imaging)
  - Inaccurate/imprecise quantification in $T_1$ mapping

# $B_1$ Variations

- It is highly desirable if we can excite tissue homogeneously and produce a uniform flip angle throughout

→ **Adiabatic Pulses!**

*"Adiabatic pulses are a special class of RF pulses that can excite, refocus or invert magnetization vectors uniformly, even in the presence of a spatially nonuniform $B_1$ field."*

# Adiabatic Passage Principle

# Adiabatic Pulses

- A special class of RF pulses that can achieve uniform flip angle
- Flip angle is independent of the applied $B_1$ field

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Slice profile of an adiabatic pulse is obtained using Bloch simulations
- Can be used for excitation, inversion and refocusing

# Adiabatic vs. Non-Adiabatic Pulses

**Adiabatic Pulses:**

$$\theta \neq \int_0^T B_1(\tau)d\tau$$

- Amplitude and frequency/phase modulation
- Long duration (8-12 ms)
- Higher $B_1$ amplitude (>12 μT)
- Generally NOT multi-purpose (inversion pulse cannot be used for refocusing, etc.)

**Non-Adiabatic Pulses:**

$$\theta = \int_0^T B_1(\tau)d\tau$$

- Amplitude modulation, constant carrier frequency (constant phase)
- Short duration (0.3 ms to 1 ms)
- Lower $B_1$ amplitude
- Generally multi-purpose

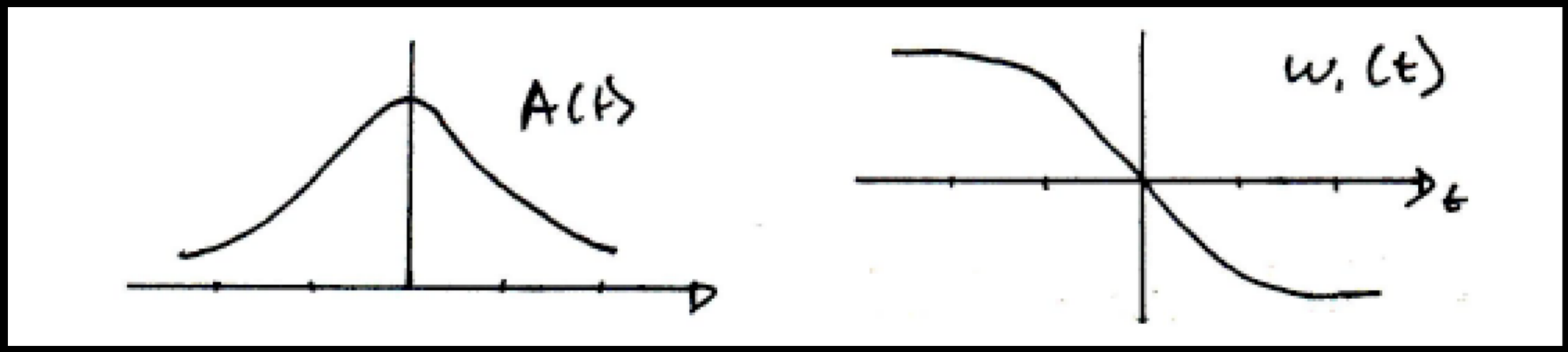
# Adiabatic Pulses

- Frequency modulated pulses:

$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

envelope

frequency sweep




- Or phase modulation:

$$B_1(t) = A(t)\exp^{-i\phi(t)}$$

# Bloch Equation (at on-resonance)

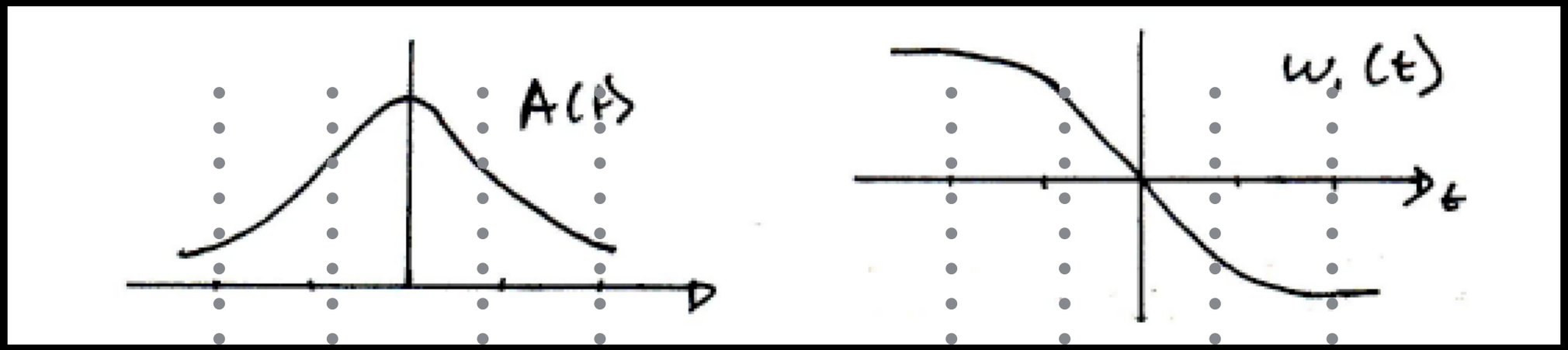
$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + \frac{\omega_1(t)}{\gamma} \end{pmatrix}$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega_1(t) & 0 \\ -\omega_1(t) & 0 & \gamma A(t) \\ 0 & -\gamma A(t) & 0 \end{pmatrix} \vec{M}$$

# Magnetization Plot

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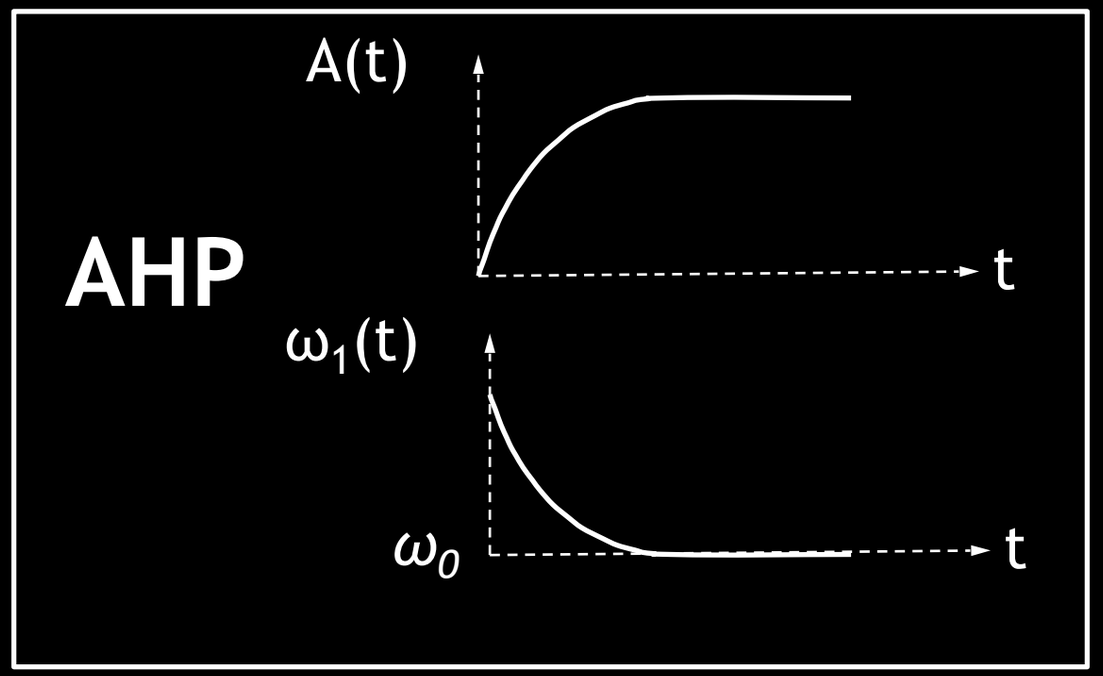
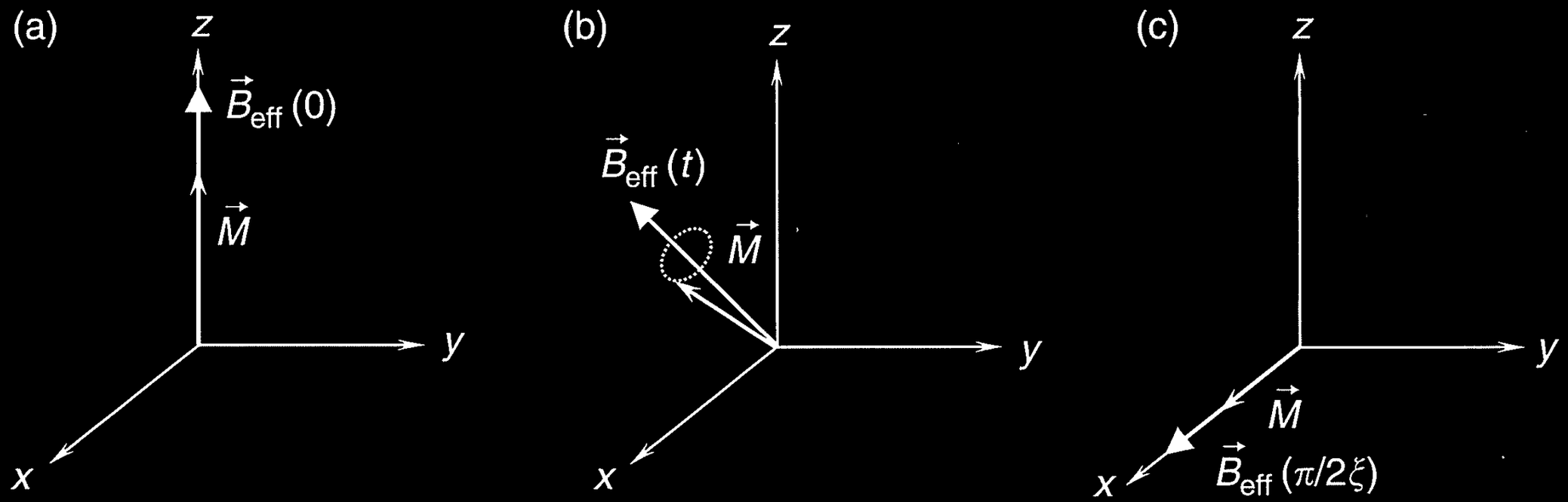
To the board ...

# Adiabatic Excitation



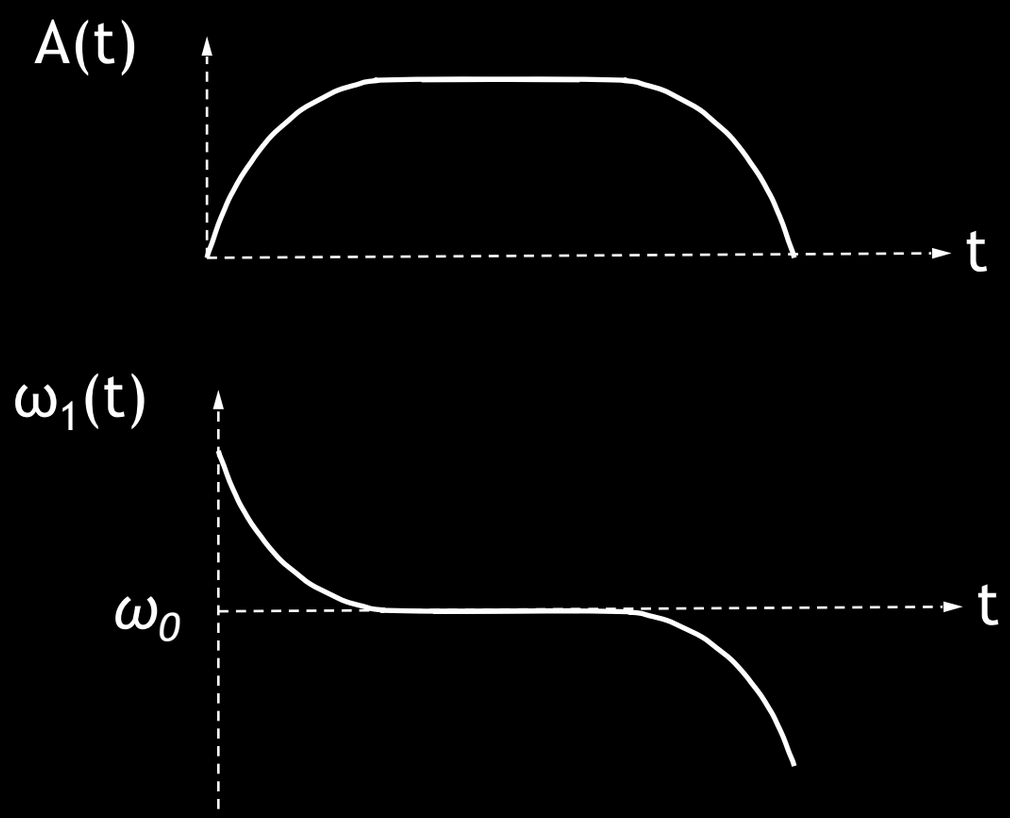




- At the end of the pulse, all the magnetization is in the transverse plane → so we have adiabatic excitation!
- This is also called an **adiabatic half passage (AHP)**

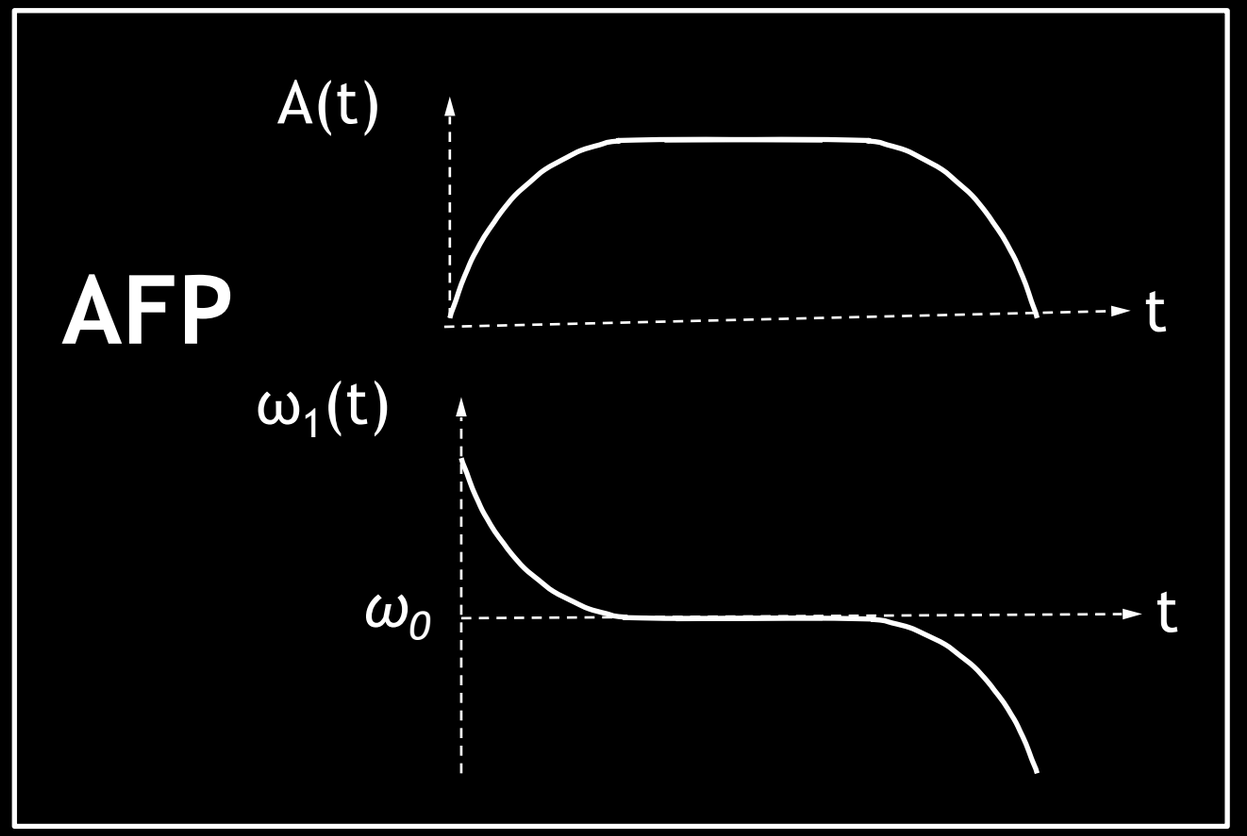
# Adiabatic Inversion

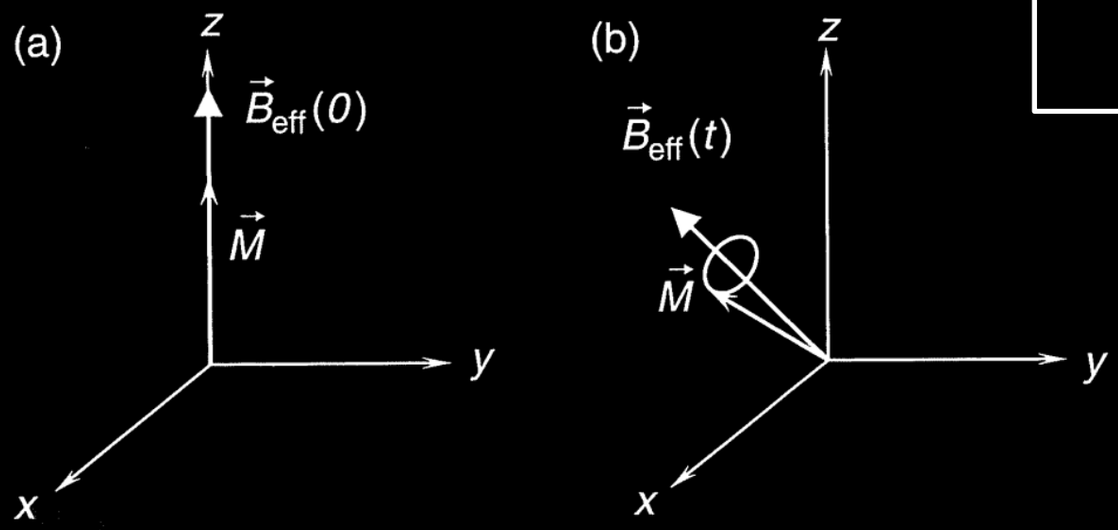
- An adiabatic inversion requires an adiabatic full passage (AFP) pulse:

**AFP**



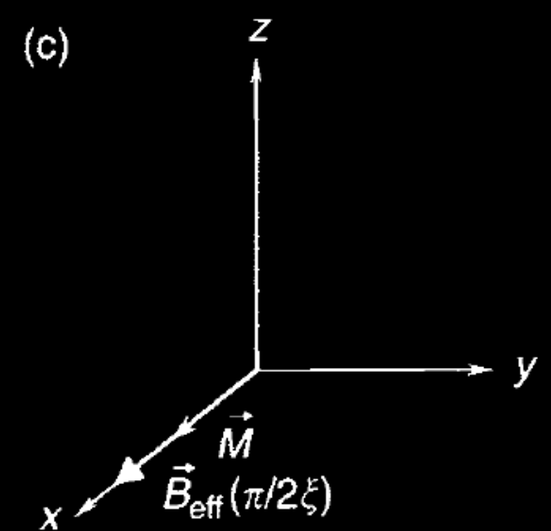
# Adiabatic Inversion

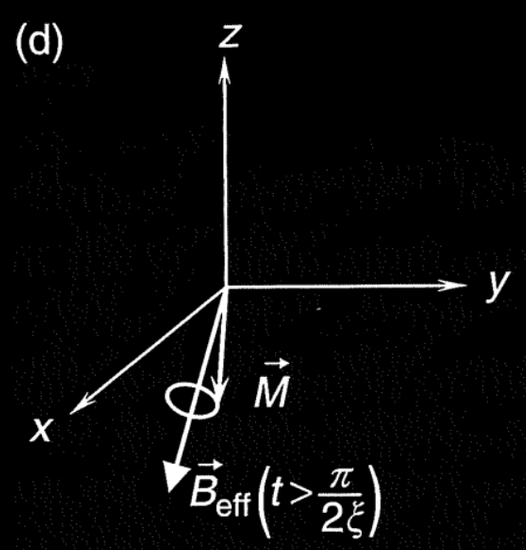


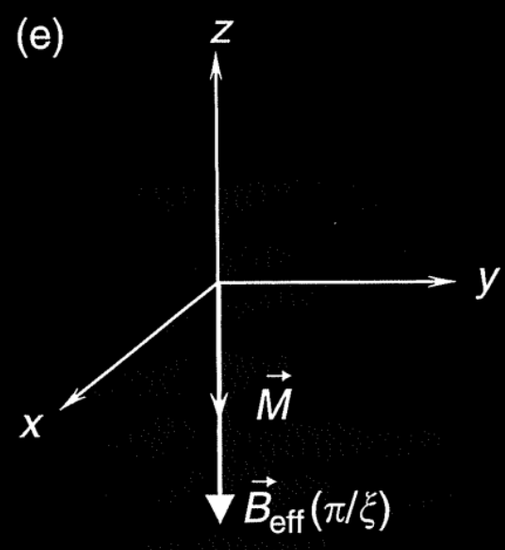









# Adiabatic Inversion

# Design of Adiabatic Inversion

- General equation for an adiabatic pulse:

$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

- Many different types of adiabatic pulses can be designed by choosing different amplitude and frequency modulation functions
- The most famous one is…

**The Hyperbolic Secant Inversion Pulse!**

# Hyperbolic Secant Pulse Equations

$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

where

$$A(t) = A_0 sech(\beta t)$$

$$\omega_1(t) = -\mu\beta tanh(\beta t)$$

$A_0$: peak amplitude (μT)

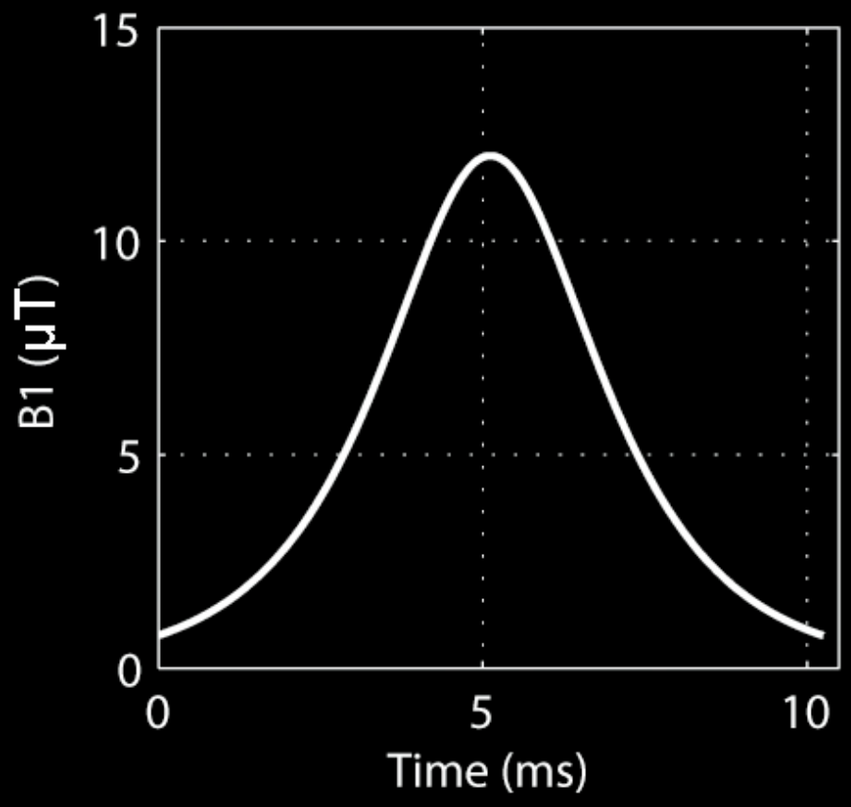
β: frequency modulation parameter (rad/s)

μ: phase modulation parameter (dimensionless)

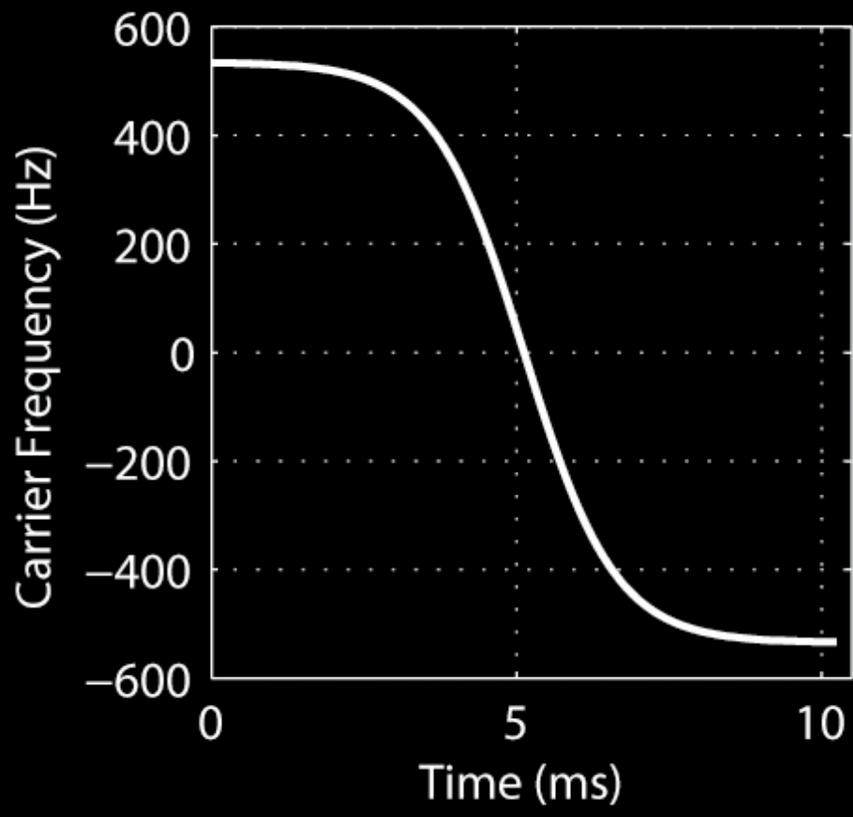
*Handbook of MRI Pulse Sequences, Ch 6.2, pp 194-195*

# Hyperbolic Secant Pulse Example

Amplitude Modulation, A(t)




Frequency Modulation, $\omega_1(t)$




**Pulse Parameters:**

$A_0$ = 12 μT

μ = 5

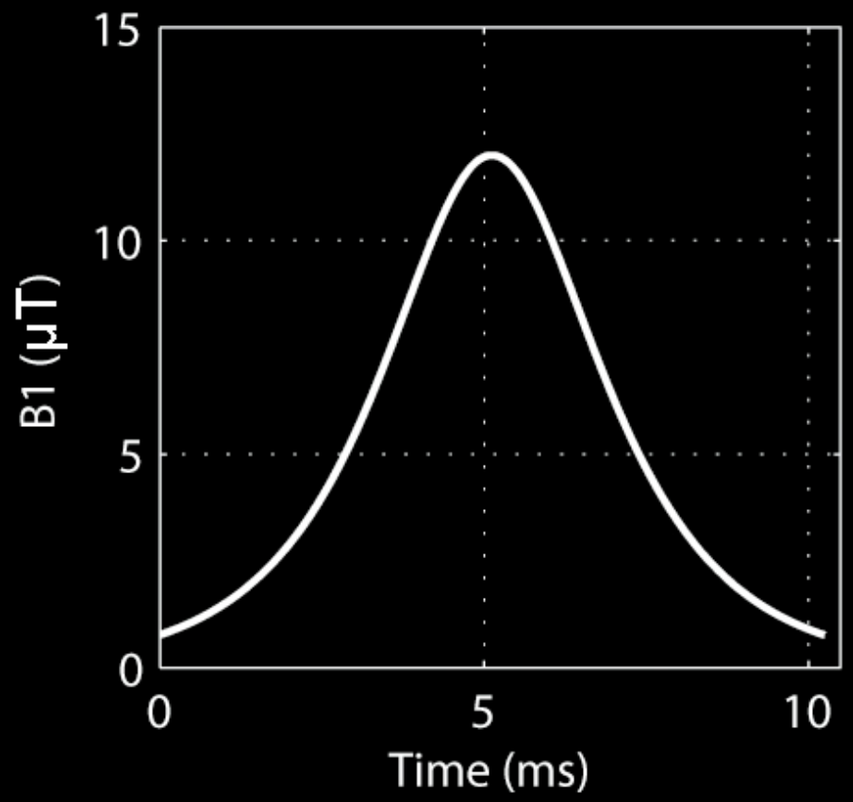
β = 672 rad/s

Duration = 10.24 ms

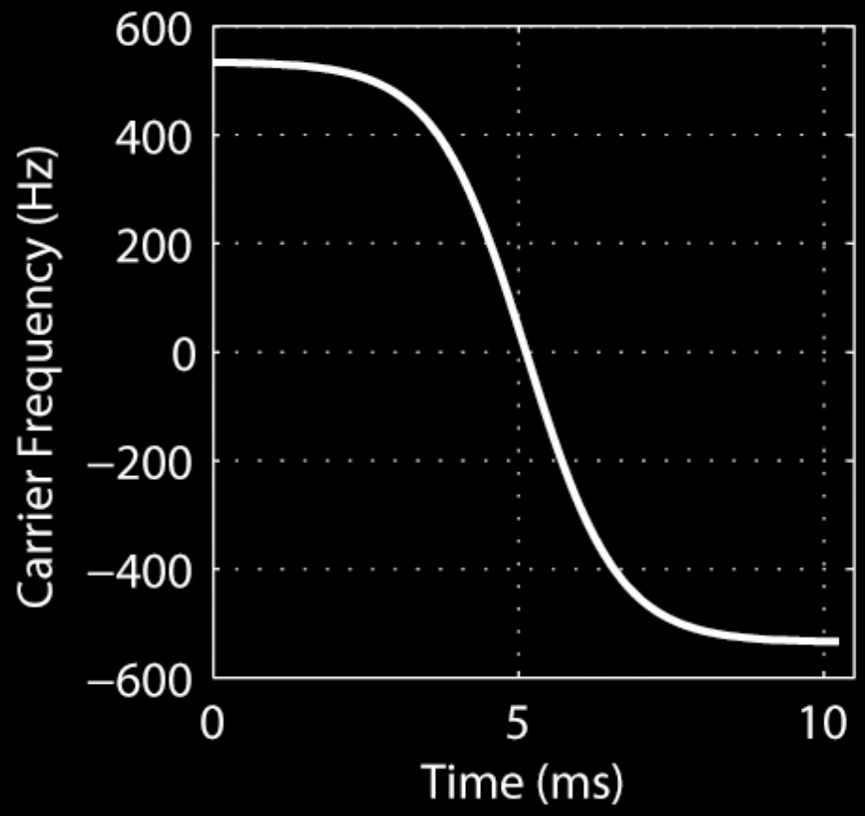
# Comparing Hyperbolic Secant with an AFP Example

**Hyperbolic Secant Pulse**

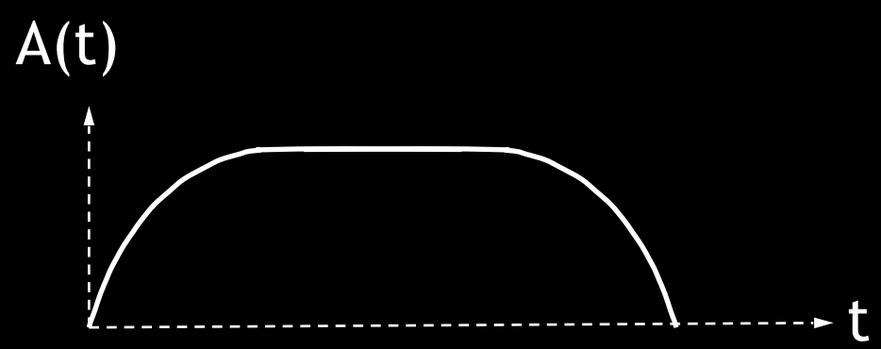
Amplitude Modulation, A(t)

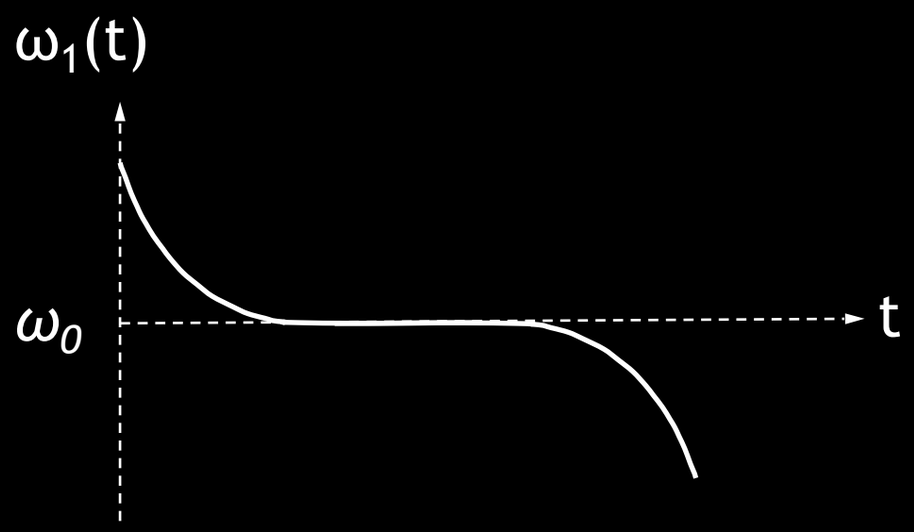



Frequency Modulation, $\omega_1(t)$




**General Adiabatic Full Passage pulse**



# Some Examples of Other Adiabatic Inversion Pulses

| Pulse Name | A(t) | $\omega_1(t)$ |
|---|---|---|
| Lorentz | $\frac{1}{1+\beta\tau^2}$ | $\frac{\tau}{1+\beta\tau^2}+\frac{1}{\sqrt{\beta}}\tan^{-1}(\sqrt{\beta}\tau)$ |
| HS | $\operatorname{sech}(\beta\tau)$ | $\frac{\tanh(\beta\tau)}{\tanh(\beta)}$ |
| Gauss[c] | $\exp\left(-\frac{\beta^2\tau^2}{2}\right)$ | $\frac{\operatorname{erf}(\beta\tau)}{\operatorname{erf}(\beta)}$ |
| Hanning | $\frac{1+\cos(\pi\tau)}{2}$ | $\tau+\frac{4}{3\pi}\sin(\pi\tau)\left[1+\frac{1}{4}\cos(\pi\tau)\right]$ |
| HSn[c] ($n=8$) | $\operatorname{sech}(\beta\tau^n)$ | $\int \operatorname{sech}^2(\beta\tau^n)\,d\tau$ |
| Sin40[d] ($n=40$) | $1-\left\lvert\sin^n\left(\frac{\pi\tau}{2}\right)\right\rvert$ | $\tau-\int \sin^n\left(\frac{\pi\tau}{2}\right)\left(1+\cos^2\left(\frac{\pi\tau}{2}\right)\right)d\tau$ |

*Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, vol. 10, p423*

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| HS | $\mathrm{sech}(\beta\tau)$ | $\frac{\tanh(\beta\tau)}{\tanh(\beta)}$ |
| $\mathrm{HSn}^c$ $(n=8)$ | $\mathrm{sech}(\beta\tau^n)$ | $\int \mathrm{sech}^2(\beta\tau^n)\,d\tau$ |
| $\mathrm{Sin40}^d$ $(n=40)$ | $1-\left\lvert \sin^n\left(\frac{\pi\tau}{2}\right)\right\rvert$ | $\tau-\int \sin^n\left(\frac{\pi\tau}{2}\right)\left(1+\cos^2\left(\frac{\pi\tau}{2}\right)\right)d\tau$ |

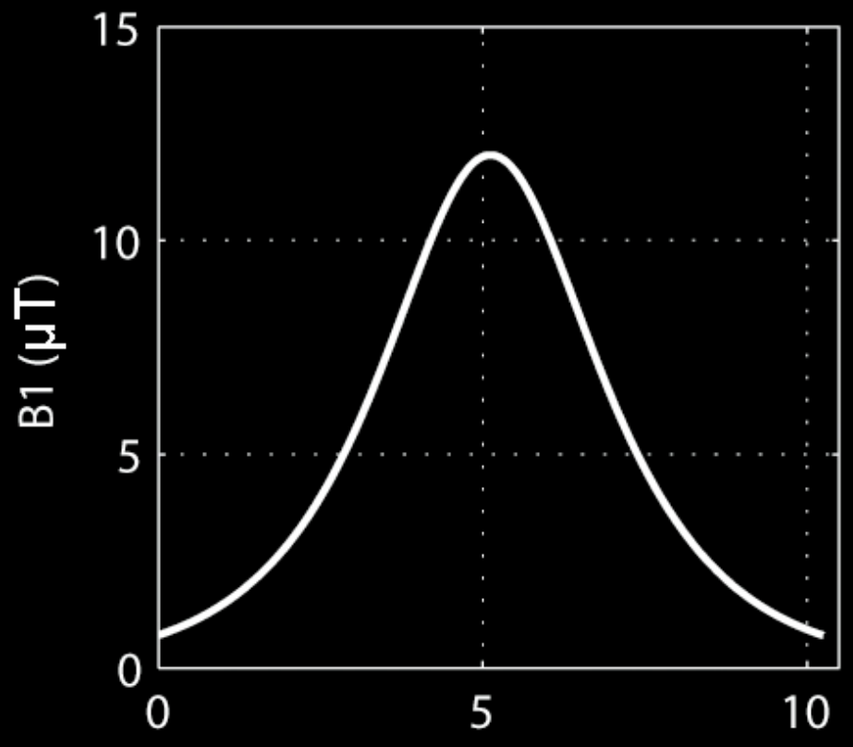
**The shape of the inversion profile depends on the choice A(t) and $\omega_1(t)$!**

*Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, vol. 10, p423*

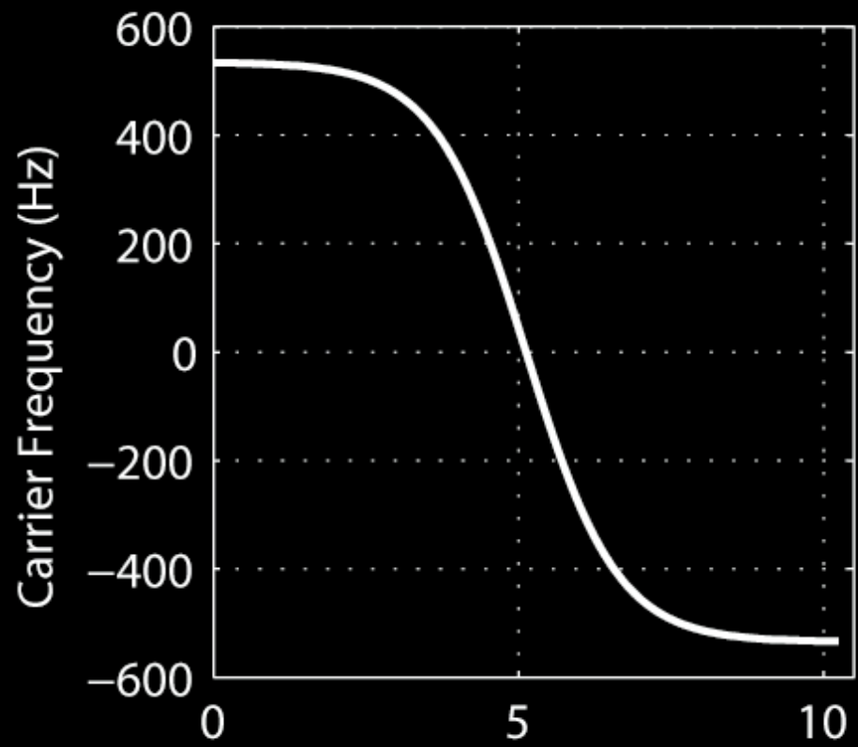
# What Will Inversion Profile Look Like?

Hyperbolic Secant Pulse

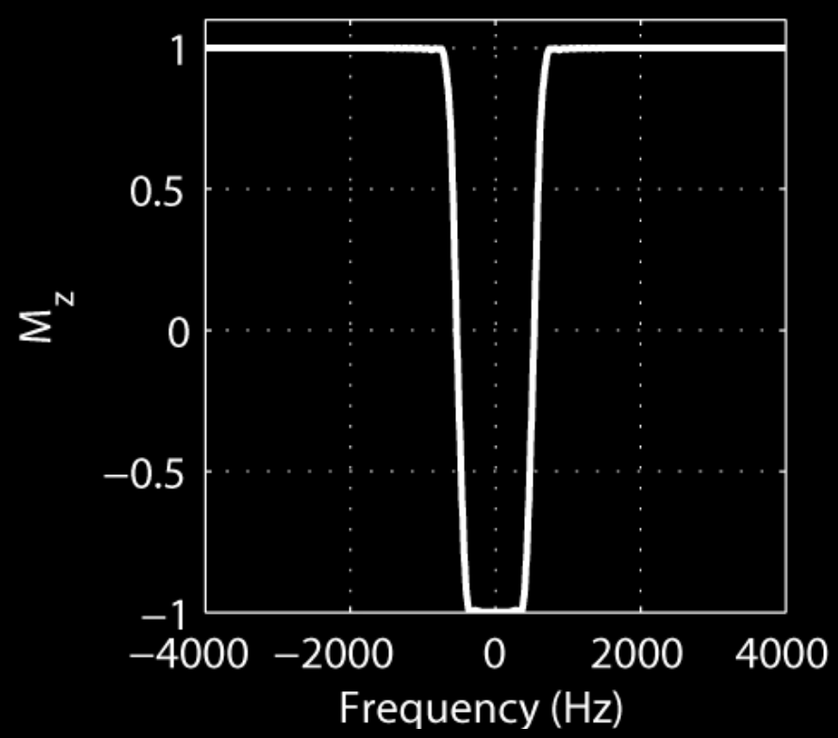
## Amplitude Modulation, A(t)




## Frequency Modulation, $\omega_1(t)$




## Inversion Profile



# Inversion Profiles

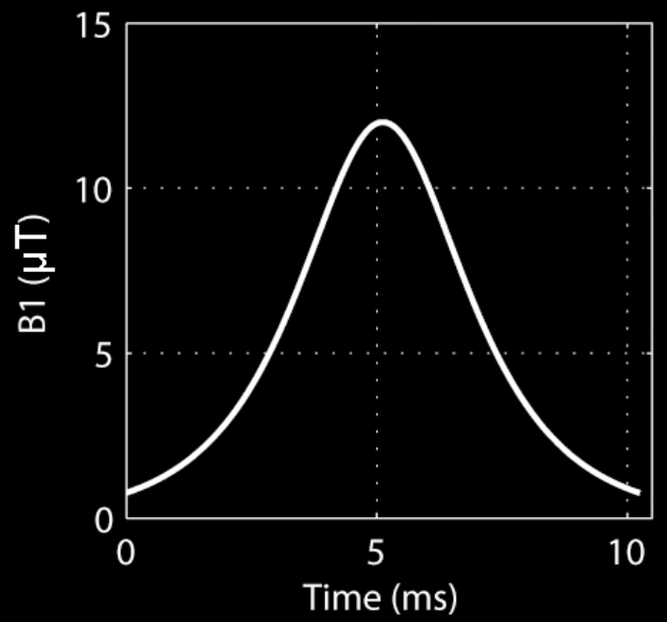
- The inversion profile typically calculated using Bloch simulation of the RF pulse (will be covered later) shows us the inversion efficiency and RF bandwidth
- The inversion efficiency depends strongly on the $B_1$ amplitude (as well as pulse duration, $T_1$, $T_2$ and pulse shape)
- For the hyperbolic secant pulse,

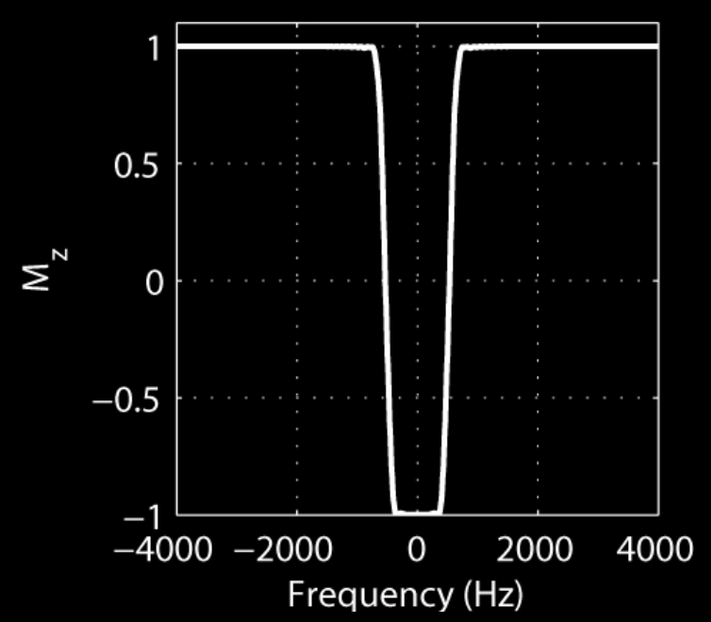
RF spectral bandwidth = $\mu\beta$

$B_{1max}$ >> $(\beta\sqrt{\mu})/\gamma$ ($B_1$ threshold for adiabaticity)

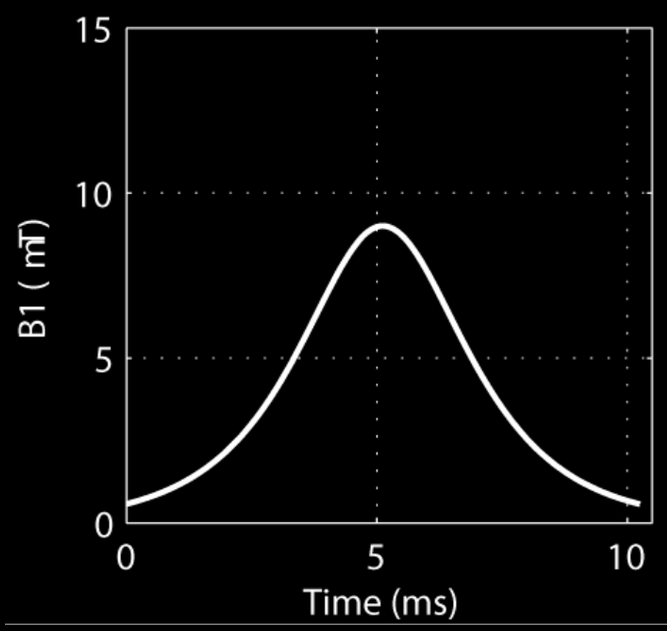
# Hyperbolic Secant: Adiabatic Property

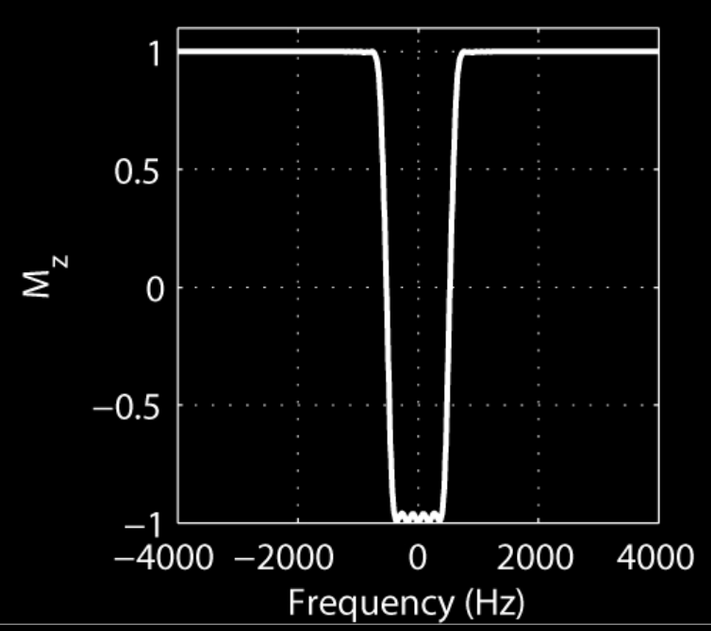
**Original Pulse (100%)**
$B_{1max}$ = 12 μT



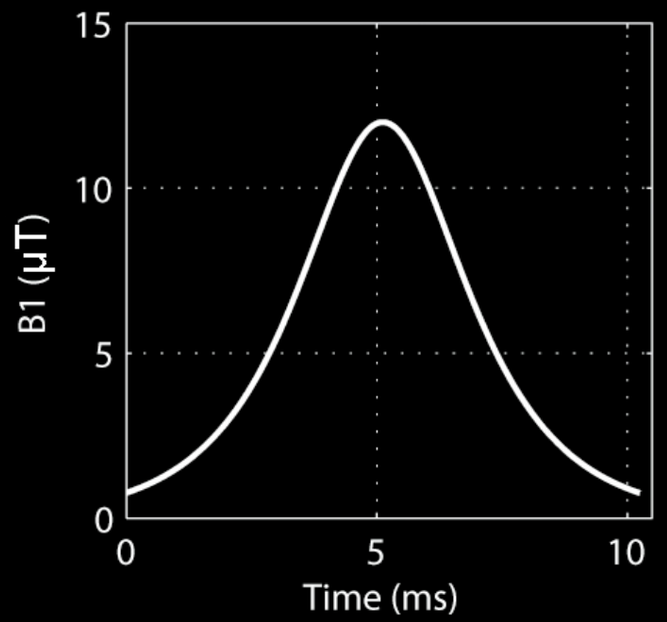

**75% Attenuated Pulse**
$B_{1max}$ = 9 μT

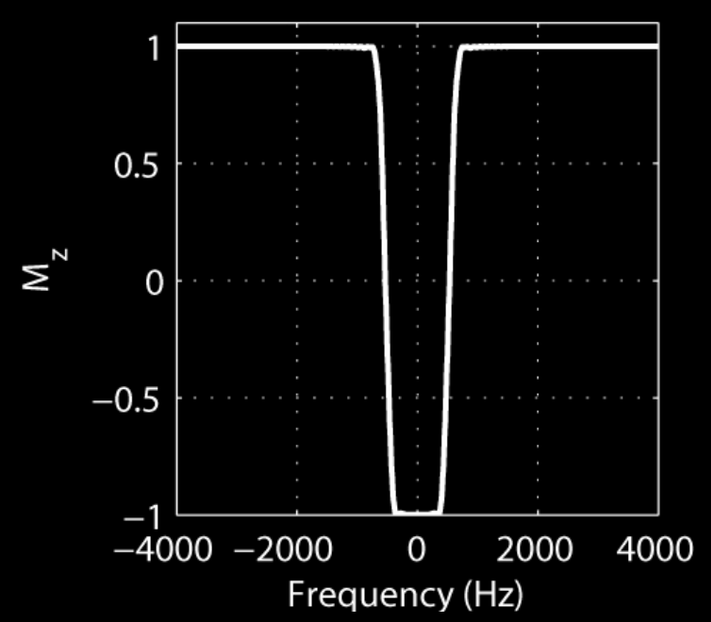


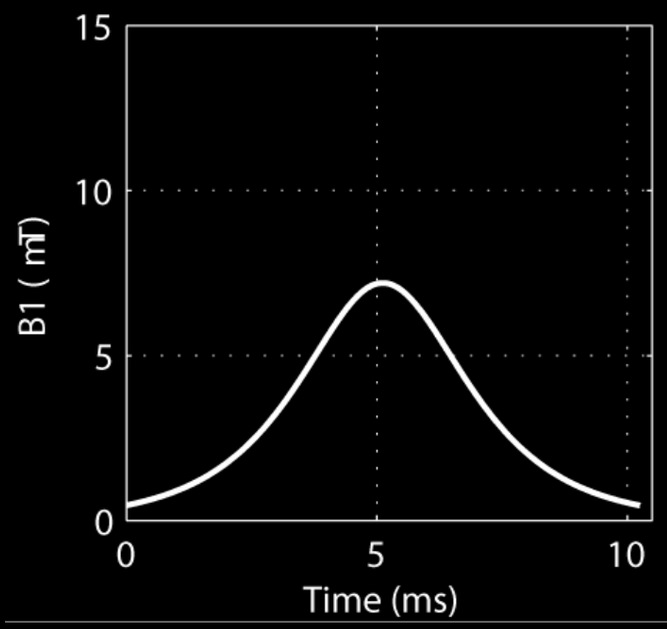
# Hyperbolic Secant: Adiabatic Property

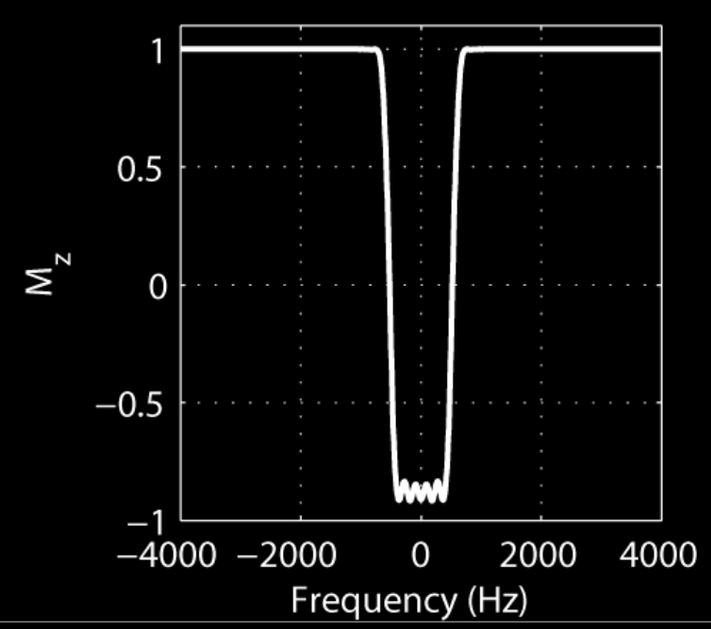
**Original Pulse (100%)**
$B_{1max}$ = 12 μT



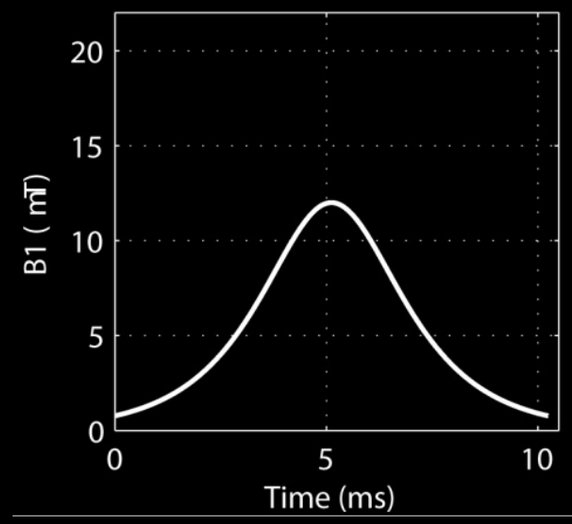

**60% Attenuated Pulse**
$B_{1max}$ = 7.2 μT

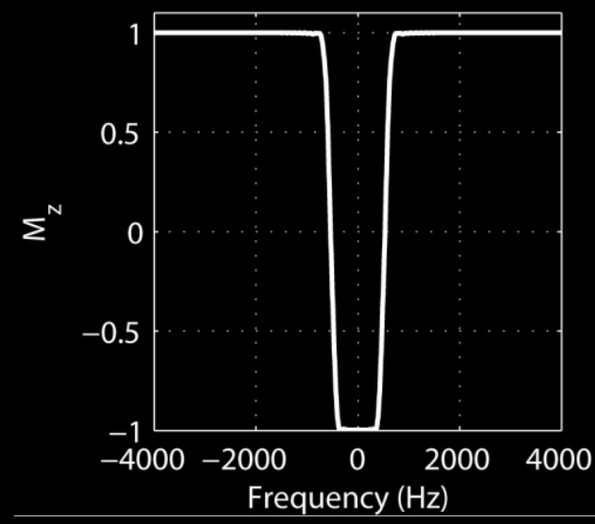


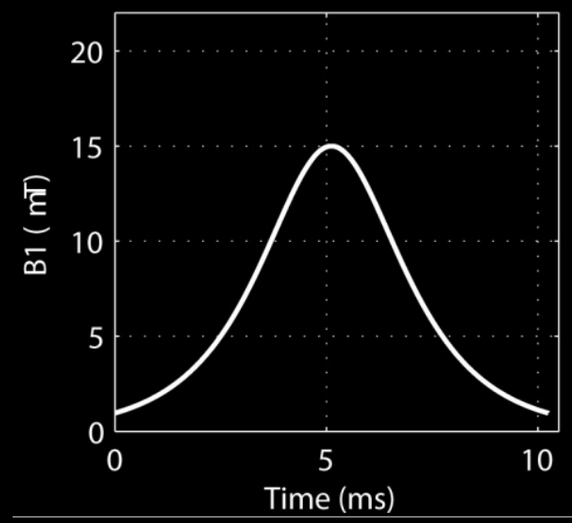

**$B_1$ Threshold ≈ 6 μT**

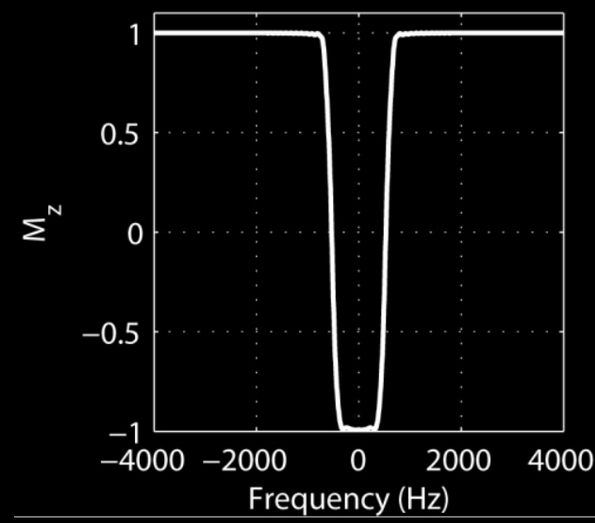
## Original Pulse (100%)
$B_1$ = 12 μT



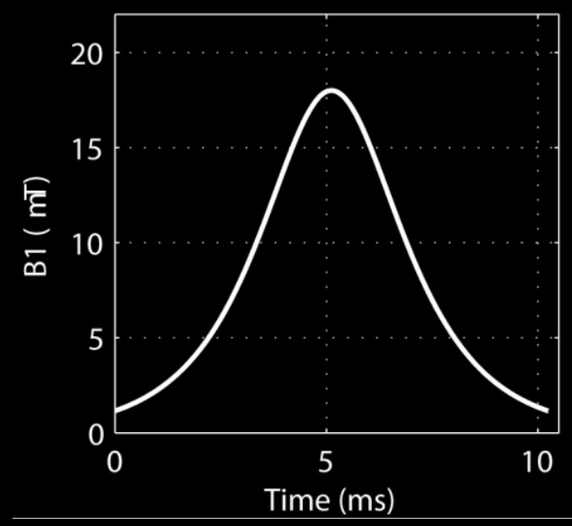



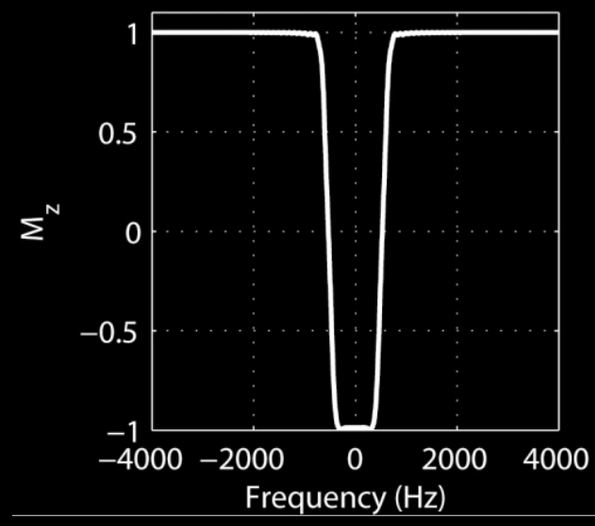

## 125% Amplified Pulse
$B_1$ = 15 μT




## 150% Amplified Pulse
$B_1$ = 18 μT



# Comments

- Many envelope/modulation functions work
- If a range of adiabaticity is required, optimization can help reduce pulse length
- Hyperbolic secant needs to be truncated, which can affect the overall performance

# Applications of Adiabatic Pulses

# Adiabatic Pulses

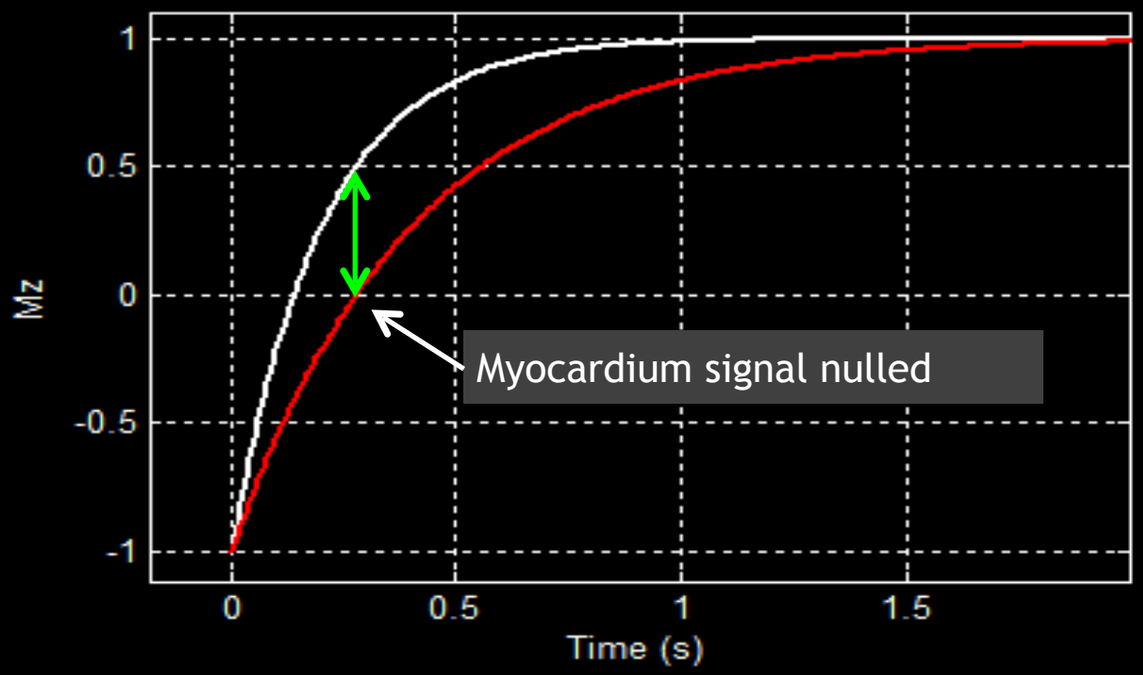
- Fat suppression (STIR)
- CSF suppression (FLAIR)
- Myocardium suppression in cardiac scar imaging (LGE)
- Black blood cardiac imaging (DIR TSE)
- $T_1$ Mapping

# Late Gadolinium Enhancement (LGE)

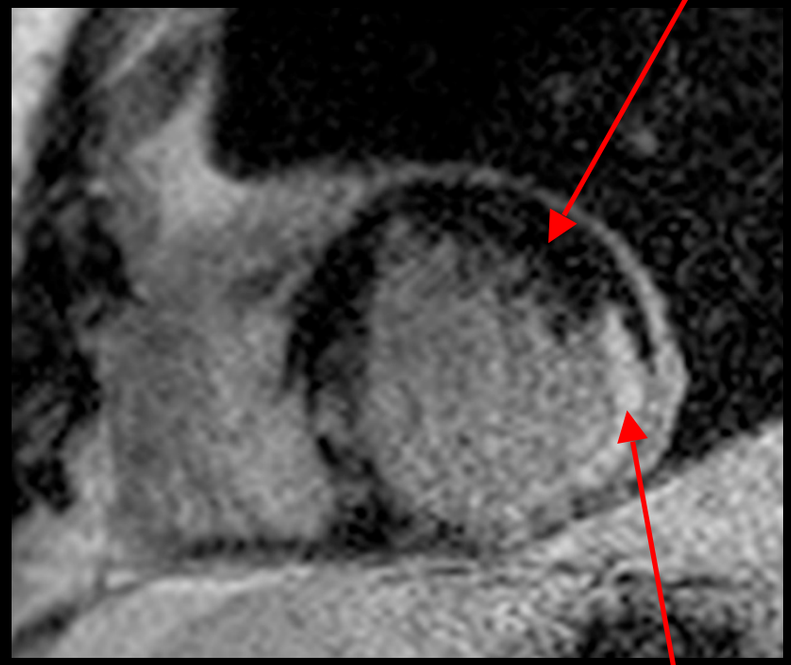
- Gold standard for detection of scar/myocardial fibrosis
- Spoiled gradient echo (SPGR) sequence with an inversion pulse (inversion recovery SPGR)
  - Inversion pulse is usually hyperbolic secant pulse
  - Healthy myocardium is nulled with the inversion pulse
  - Scar tissue (which has shorter $T_1$ than healthy tissue) appear bright

- The conventional LGE sequence uses an RF-spoiled gradient echo (FLASH) readout with an inversion recovery (IR) pulse as a preparation pulse
- The readout is acquired at a time after inversion at which the healthy myocardium signal reaches zero

Inversion recovery curves of postcontrast scar (white) and myocardium (red)



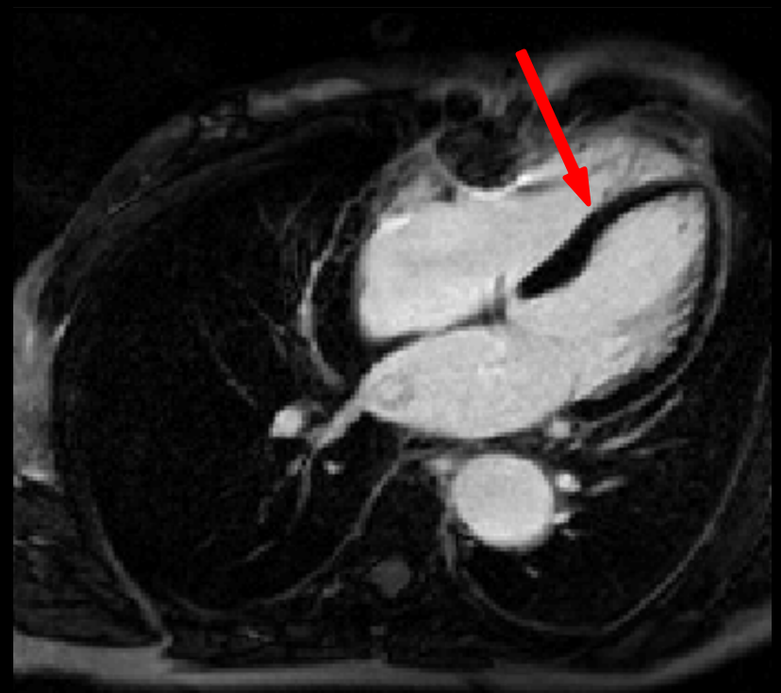

Nulled signal from healthy myocardium



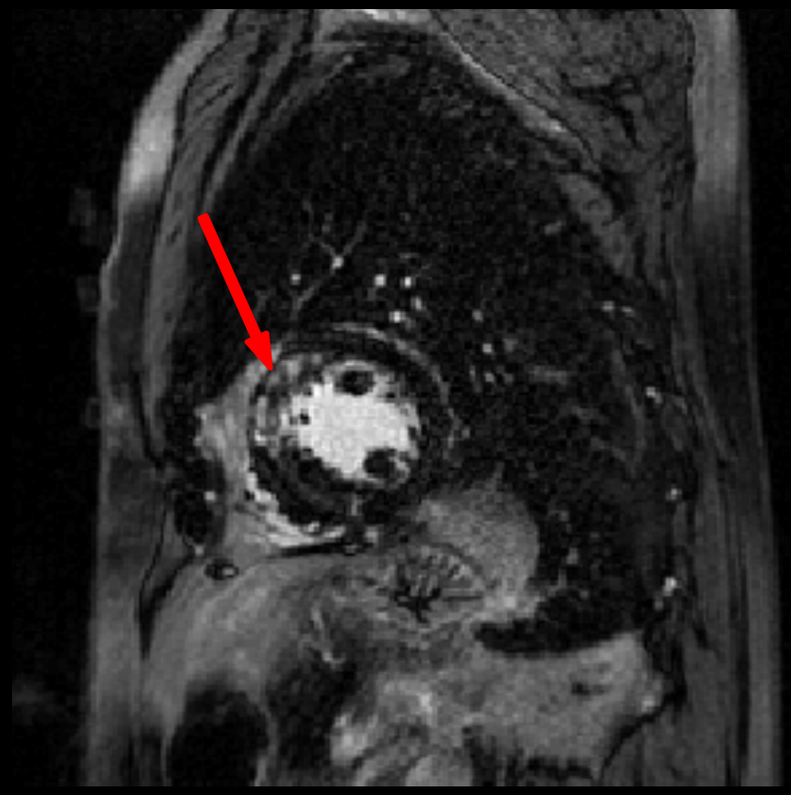
Hyper-enhanced scar region

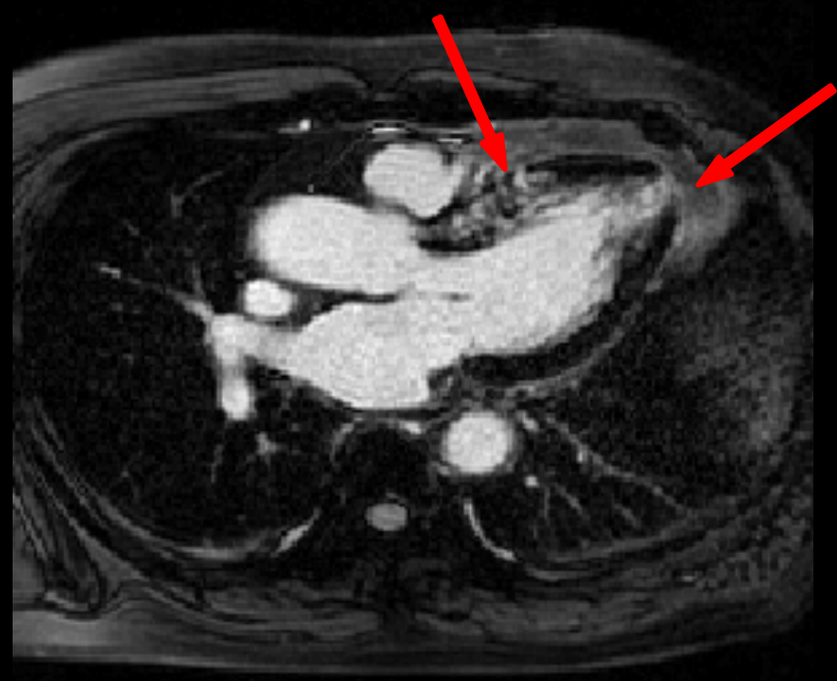
# Clinical Example

Patient with healthy myocardium



Patient with scar tissue

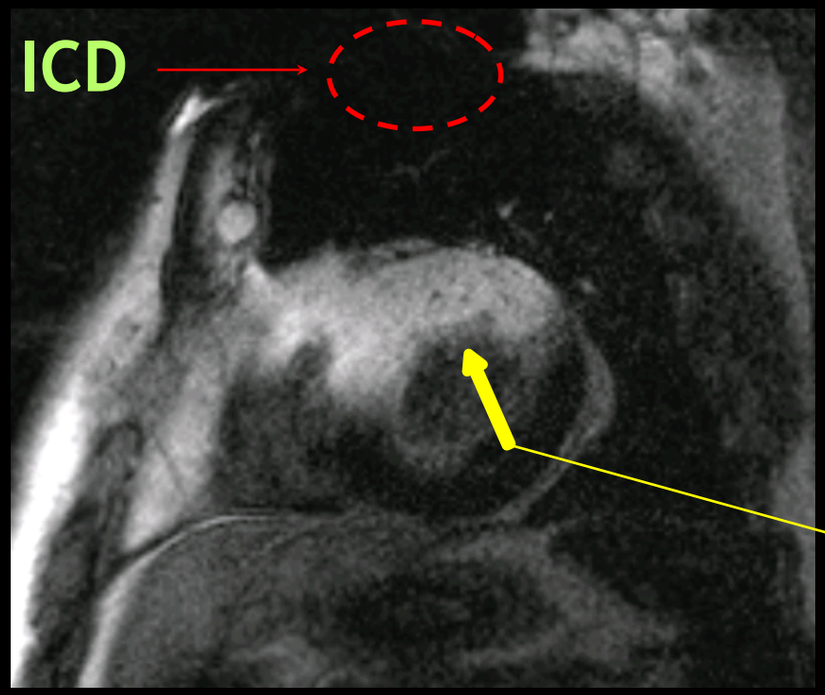


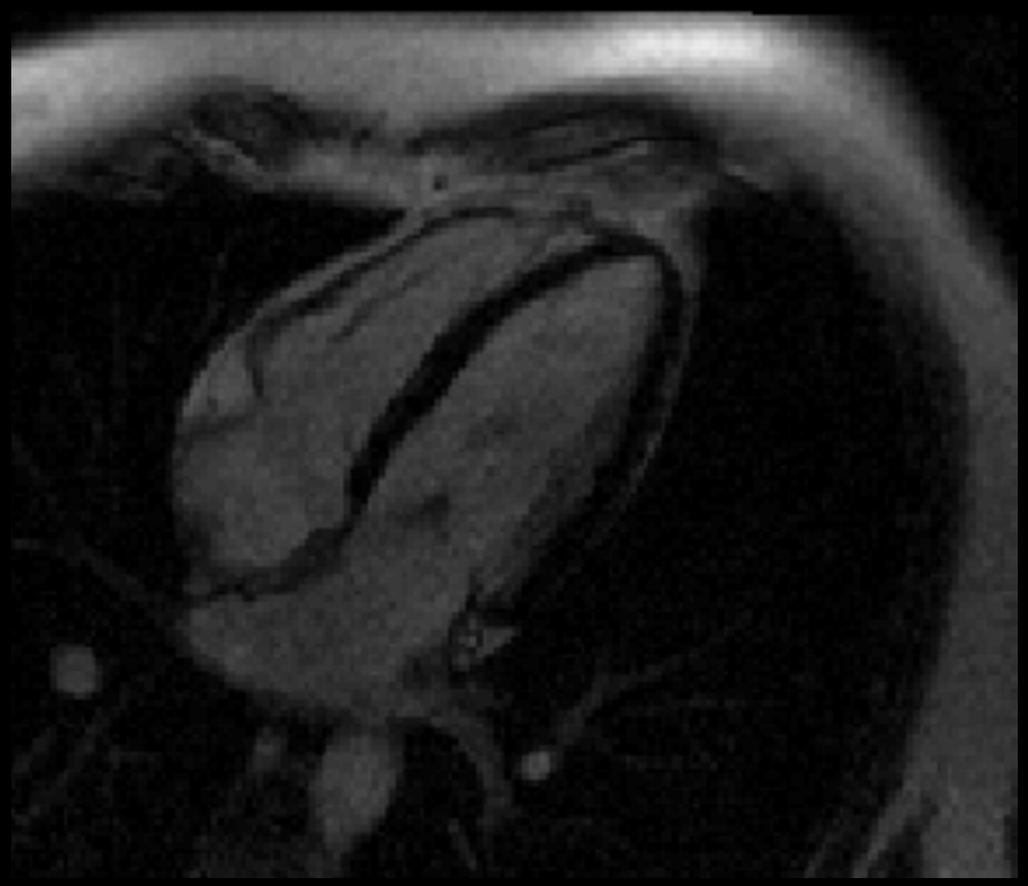


# Clinical Example

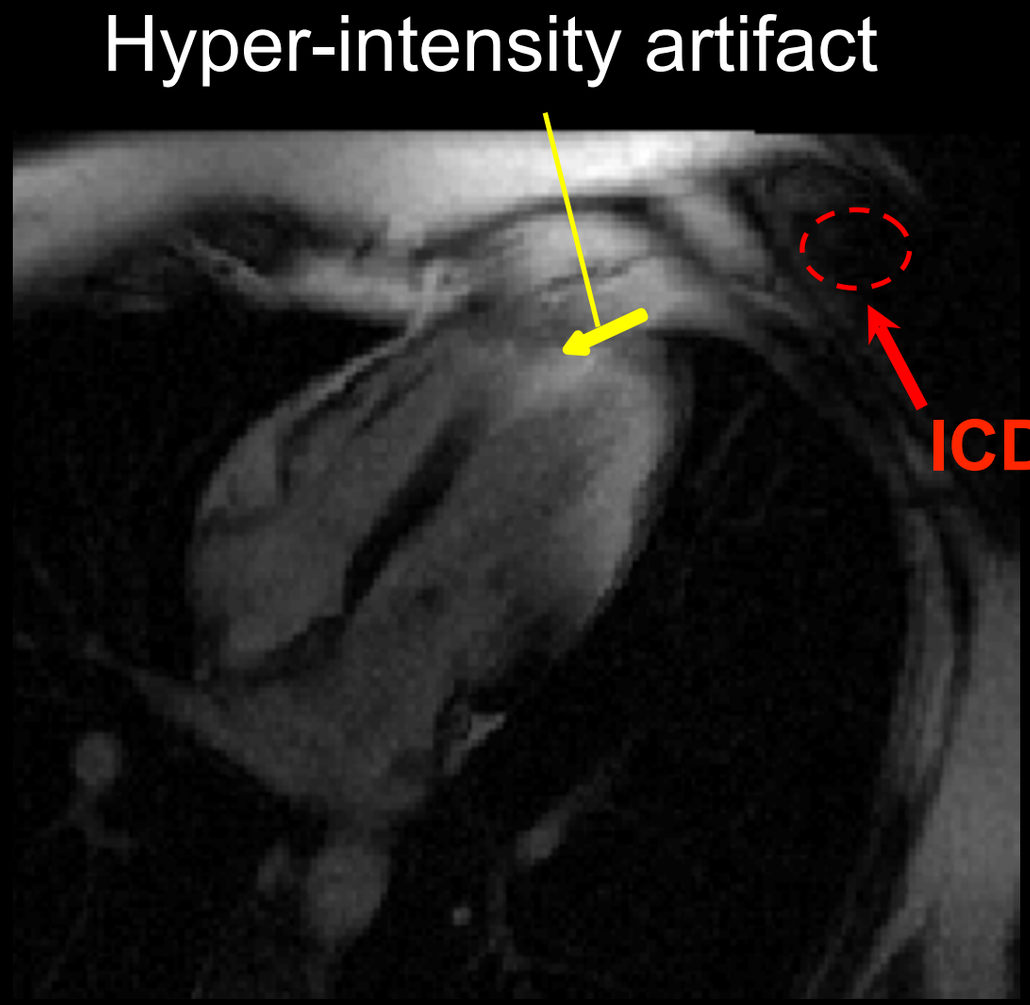
Late Gadolinium Enhancement (LGE) in patients with implantable cardiac devices

- Presence of an implantable cardiac device in the patients produces an interesting off-resonance artifact



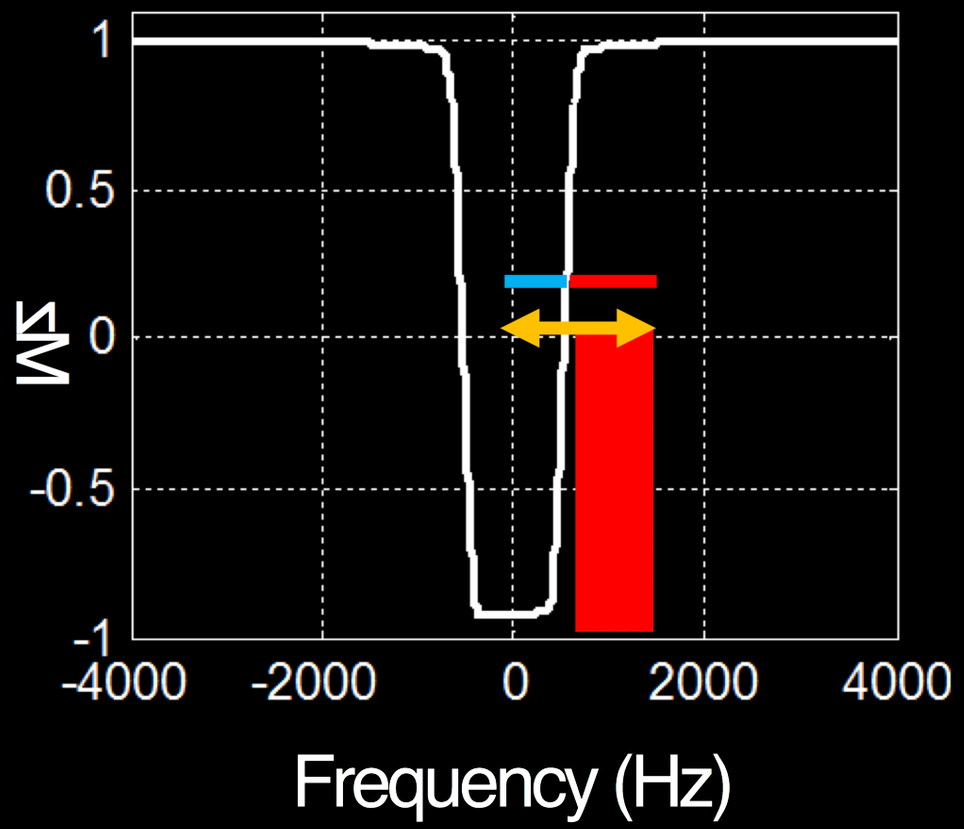


**Conventional IR LGE Image**



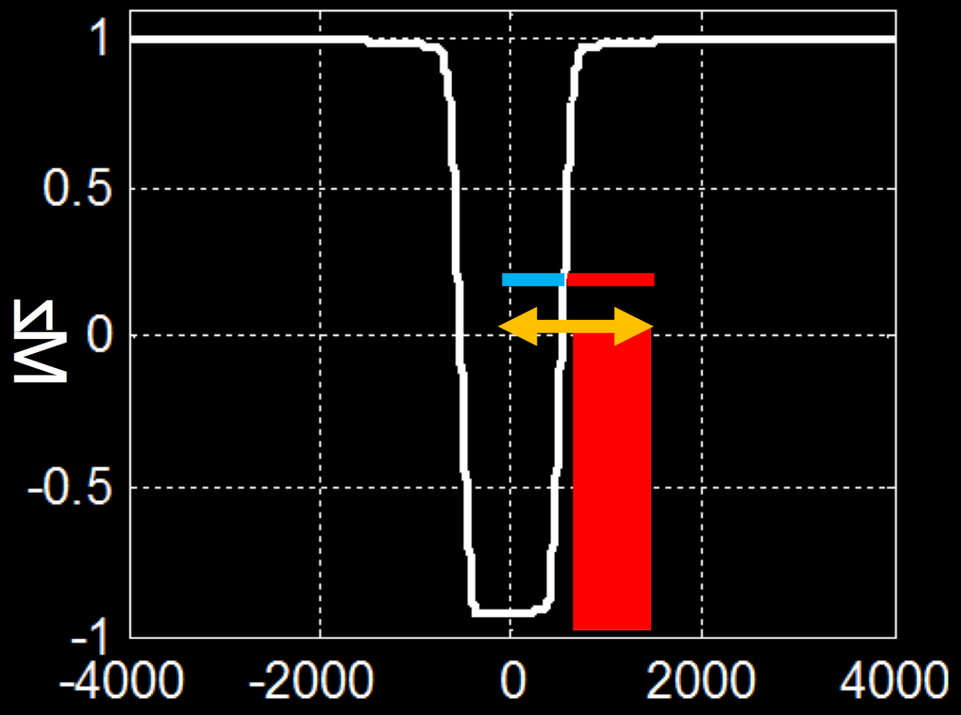

**Conventional IR LGE Image**

# Cause of Artifact

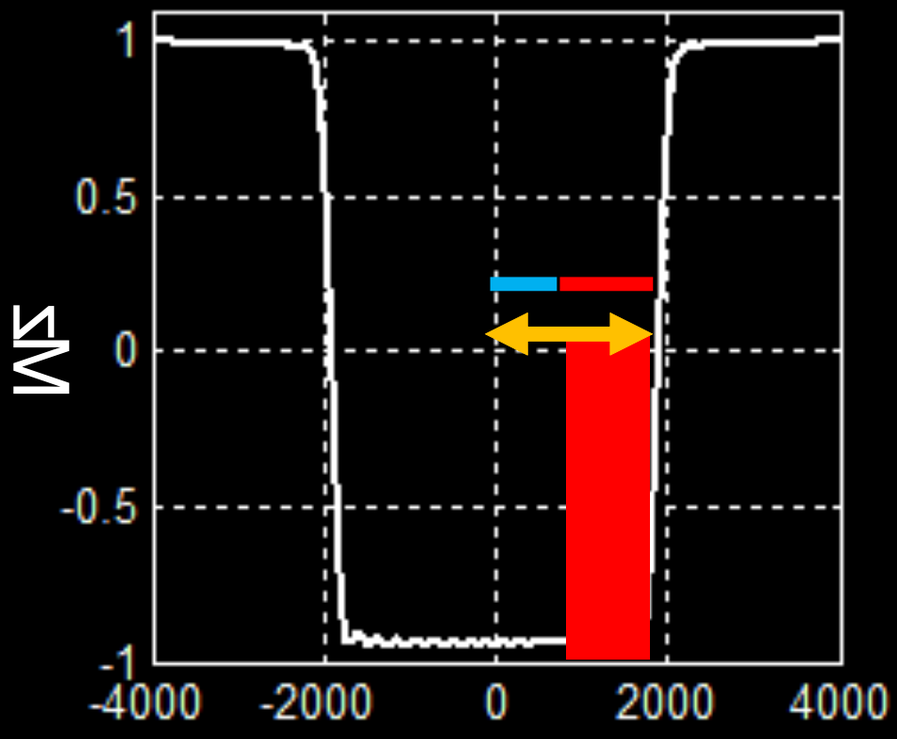



Longitudinal magnetization produced
by conventional IR pulse
BW = 1.1 kHz

# Solution: Increase Bandwidth of Inversion Pulse

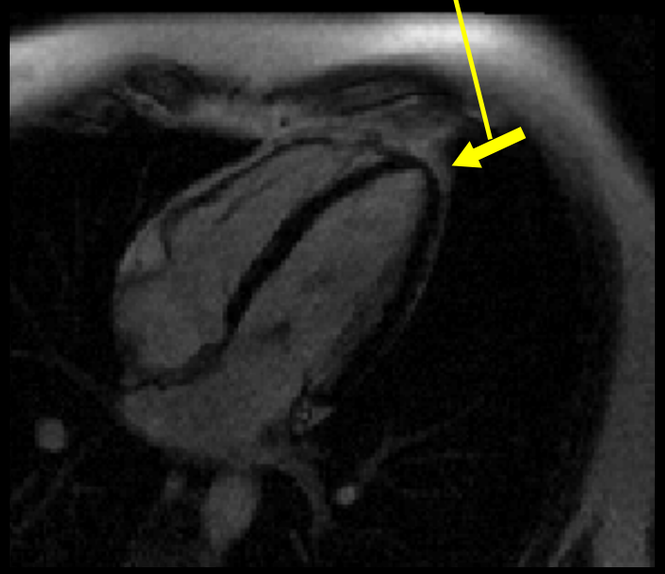



Longitudinal magnetization produced by conventional IR pulse
BW = 1.1 kHz

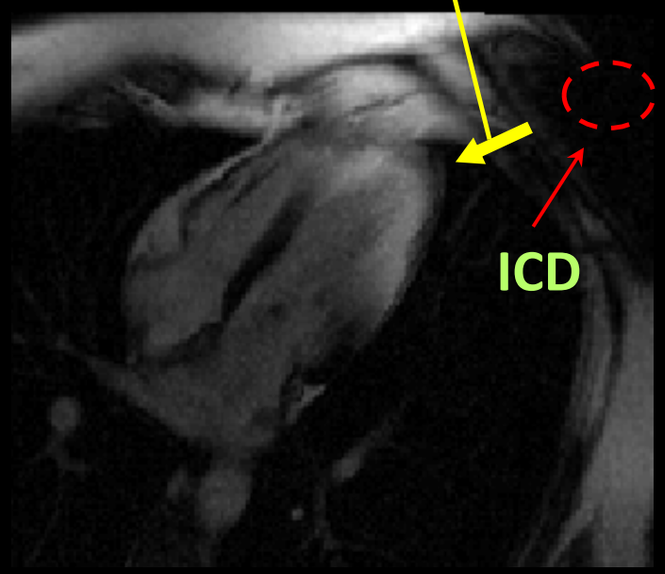



Longitudinal magnetization produced by wideband IR pulse
BW = 3.8 kHz

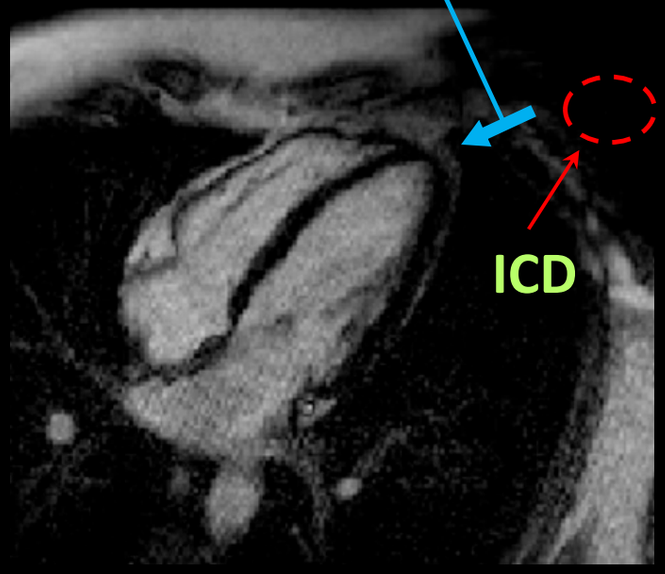
No artifact (no ICD)



Hyper-intensity artifact




**Hyper-intensity artifact corrected**




**Conventional IR LGE Image**

**Conventional IR LGE Image**

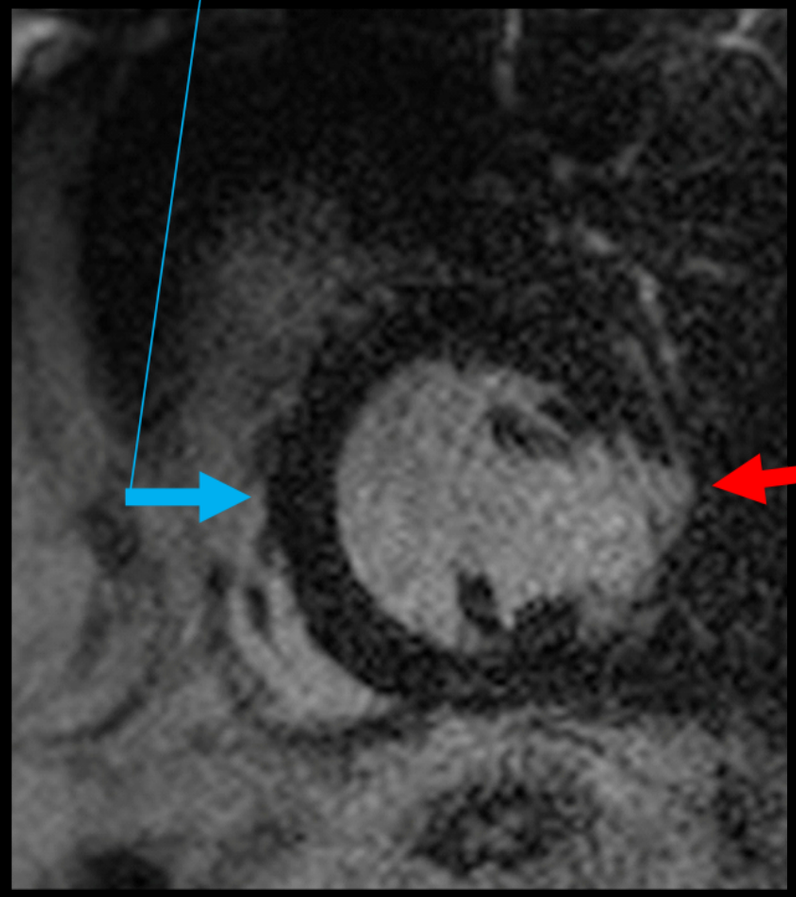
**Wideband IR LGE Image**

**Hyper-intensity artifacts**

Antero-lateral scar difficult to diagnose

**Artifacts eliminated**

Antero-lateral scar clearly visible



# MATLAB Demo

```
%%% User inputs:
mu = 5;    % Phase modulation parameter [dimensionless]
beta1 = 672;    % Frequency modulation parameter [rad/s]
pulseWidth = 10.24;    % RF pulse duration [ms]
A0 = 0.12;    % Peak B1 amplitude [Gauss].

%%%%%%

nSamples = 512;          % number of samples in the RF pulse
dt = pulseWidth/nSamples/1000;  % time step, [seconds]
tim_sech = linspace(-pulseWidth/2,pulseWidth/2,nSamples)./1000';
% time scale to calculate the RF waveforms in seconds.

% Amplitude modulation function B1(t):
B1 = A0.* sech(beta1.*tim_sech);

% Carrier frequency modulation function w(t):
w = -mu.*beta1.*tanh(beta1.*tim_sech)./(2*pi);
% The 2*PI scaling factor at the end converts the unit from rad/s to Hz

% Phase modulation function phi(t):
phi = mu .* log(sech(beta1.*tim_sech));

% Put together complex RF pulse waveform:
rf_pulse = B1 .* exp(1i.*phi);

% Generate a time scale for the Bloch simulation:
tim_bloch = [0:(nSamples-1)]*dt;
```

Figure 1

File Edit View Insert Tools Desktop Window Help

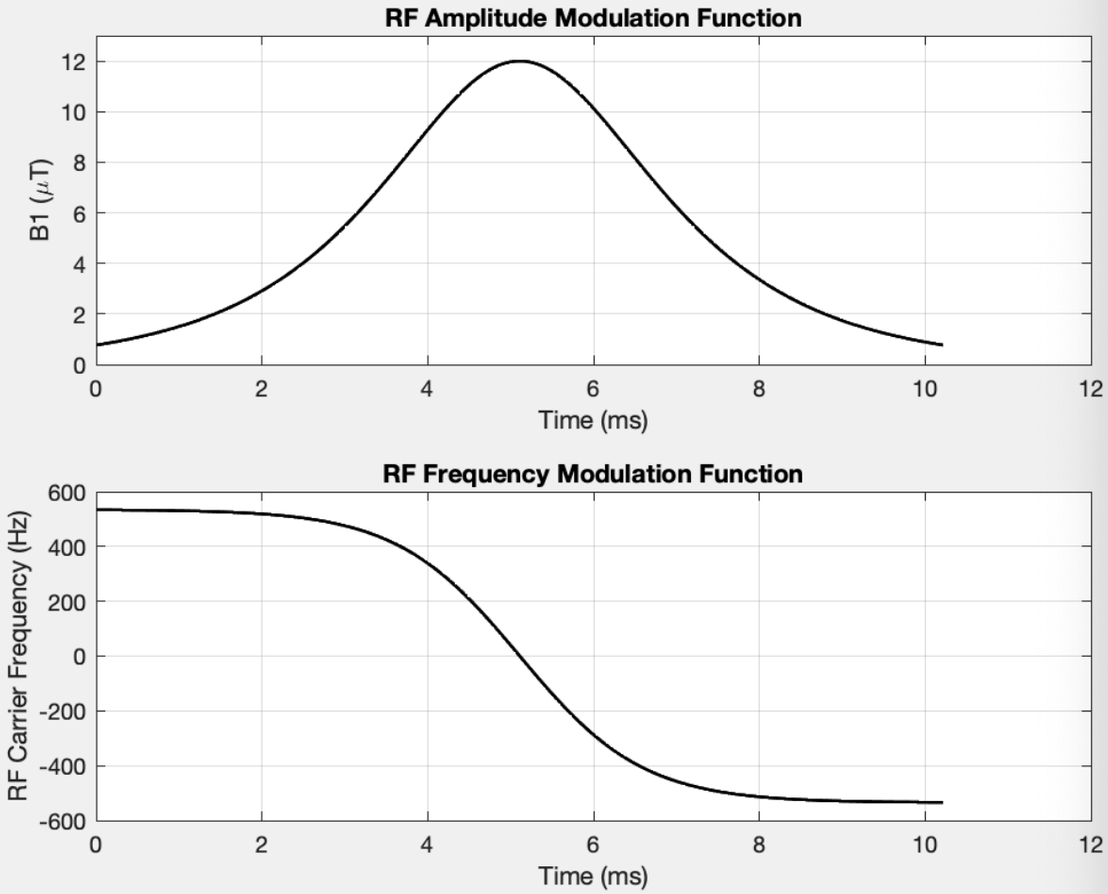
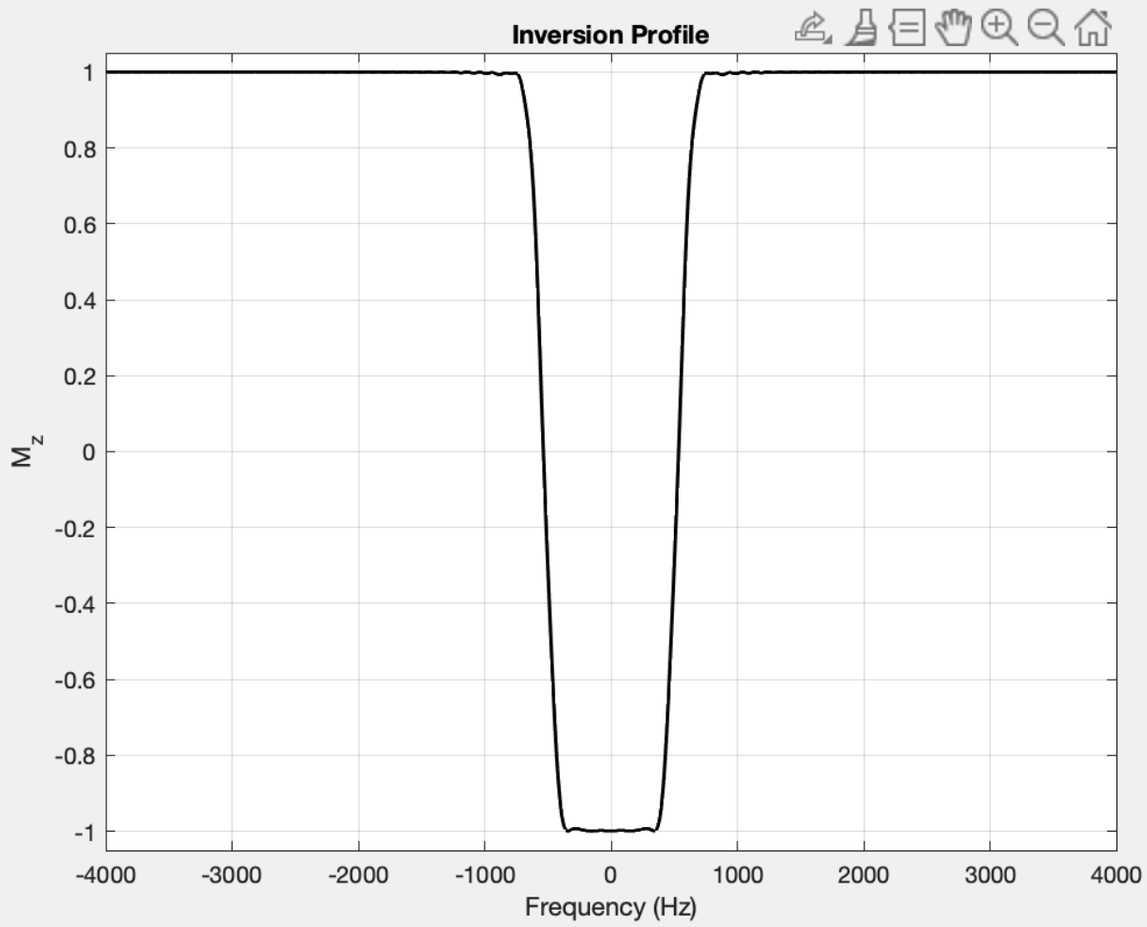



Figure 2

File Edit View Insert Tools Desktop Window Help



# Thank You!

- Further reading
  - Read "Adiabatic Refocusing Pulses" p.200-212
  - Tannus et al., “Adiabatic Pulses”, NMR in Biomedicine, Vol. 10, 423-434 (1997)

- Acknowledgments
  - John Pauly’s EE469B (RF Pulse Design for MRI)
  - Shams Rashid
  - Kyung Sung

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