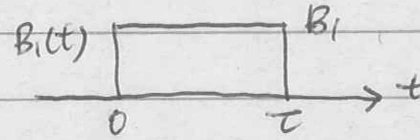
M229 Lecture 6 2023.4.20

* Small Tip Approximation

$$M_r(\tau, z) = i M_0 e^{-i\omega(z)\tau/2} \cdot FT_{1D}\{\omega_1(t+\tfrac{\tau}{2})\}\Big|_{f=-(\gamma/2\pi)G_z \cdot z}$$

Simple $B_1(t) = B_1 \cdot \Pi\left(\frac{t-\frac{\tau}{2}}{\tau}\right)$



$$\omega_1(t+\tfrac{\tau}{2}) = \gamma \cdot B_1 \cdot \Pi\left(\frac{t}{\tau}\right)$$

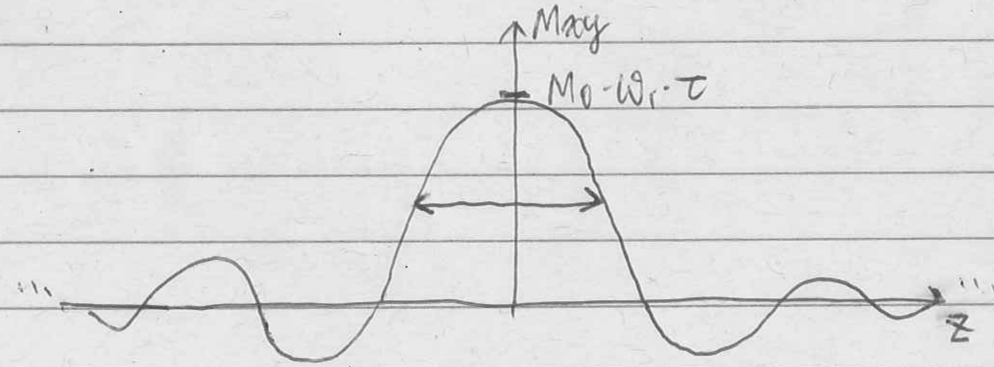
$$FT_{1D}\left\{\Pi\left(\frac{t}{\tau}\right)\right\} = \tau \cdot \mathrm{sinc}(\tau f) = \tau \cdot \mathrm{sinc}\left(\tau \cdot \frac{\gamma}{2\pi} G_z \cdot z\right)$$

$$\rightarrow M_{xy}(z) = i M_0 \underline{e^{-i\omega(z)\cdot\tau/2}} \cdot \omega_1 \cdot \tau \cdot \mathrm{sinc}\left(\tau \cdot \frac{\gamma}{2\pi} G_z \cdot z\right)$$

Note: τ is at the end of the RF pulse

extra phase can be rephased using a G_z gradient

$$M_{xy}(z) \propto \mathrm{sinc}\left(\tau \cdot \frac{\gamma}{2\pi} G_z \cdot z\right)$$

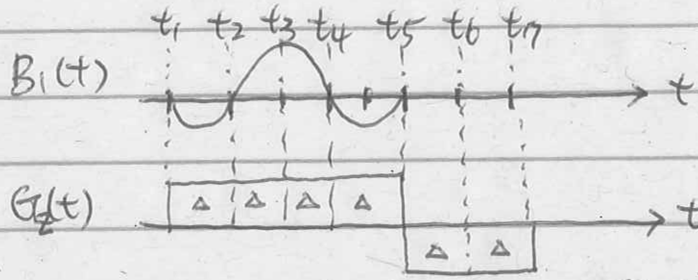


What if we intend to excite a rectangular slice?

* Excitation k-Space

$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s)\, e^{i2\pi \underline{\vec{k}(s,t)}\cdot\vec{r}}\, ds$$

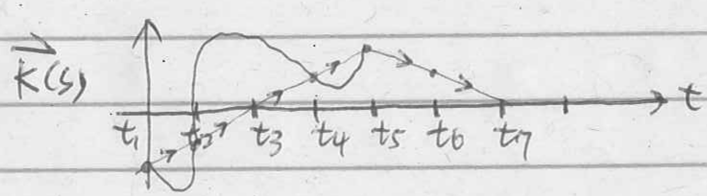
where $\vec{k}(s,t) = -\frac{\gamma}{2\pi}\int_{s}^{t} \vec{G}(\tau)\, d\tau$

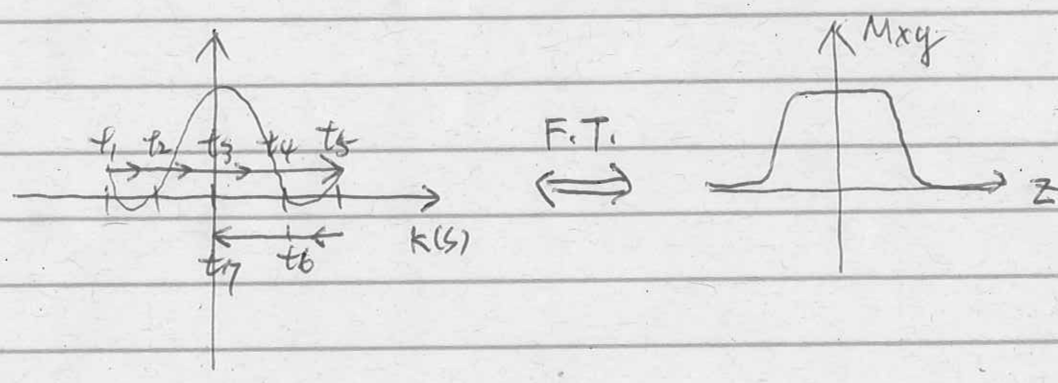


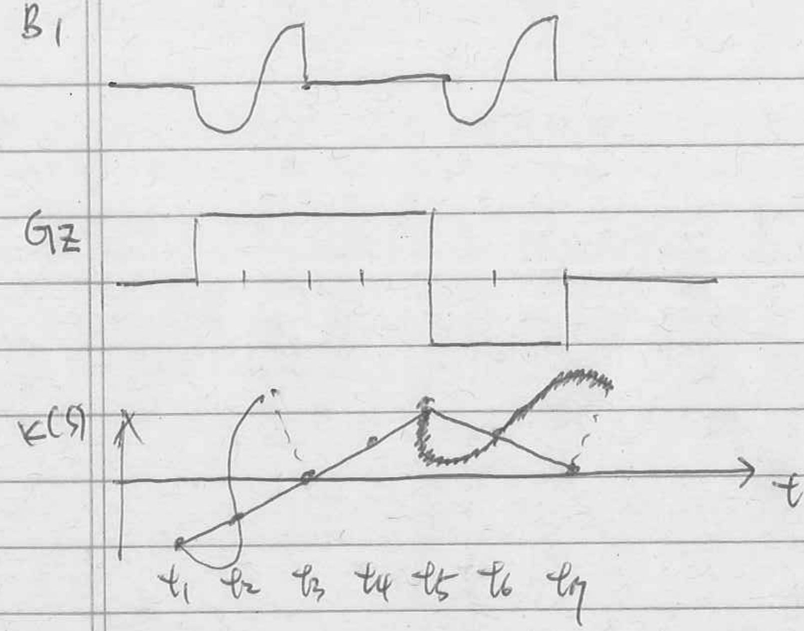
end of pulse is $t = t_7$

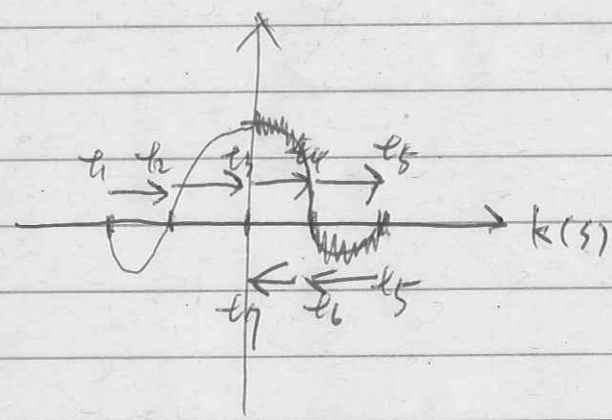
at $s = t_7$, $\vec{k}(s) = 0 = -\frac{\gamma}{2\pi}\int_{t_7}^{t_7} \vec{G}(\tau)\, d\tau$

t_6,	$+\Delta$
t_5,	$+2\Delta$
t_4,	$+2\Delta - \Delta = \Delta$
t_3,	0
t_2,	$-\Delta$
t_1,	-2Δ






B1
Gz
k(s)
t
t1 t2 t3 t4 t5 t6 t7


t1 t2 t3 t4 t5
k(s)
t7 t6 t5

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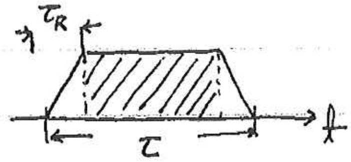
Gradient design for 2D EPI RF pulses

* Readout lobes (G_x)



ex> $\tau = 1\ ms$, $\tau_R = 1/4\ ms$

$$2\, k_{x,max} = \frac{\gamma}{2\pi} (\tau - 2\tau_R)\, G_{max}$$

$$= 4.257\ KHz/G \cdot \frac{1}{2}\ ms \cdot 4\ G/cm$$

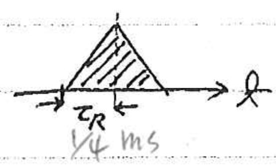
$$= 8.514\ cycles/cm$$

$$\Delta X = \frac{TBW}{2\, k_{x,max}} = \frac{1}{8.514\ cycles/cm} \cong 0.12\ cm \quad (TBW = 1)$$

with a TBW = 4 pulse (Typical)

$$4 \cdot \Delta x \cong 0.47\ cm$$

* Blips (G_y)



$$\Delta k_y = \frac{\gamma}{2\pi} \frac{1}{2} \cdot 2\tau_R\ G_{max}$$

$$= 4.257\ kHz/G \cdot \frac{1}{4}\ ms \quad 4\ G/cm$$

$$= 4.257\ cycles/cm$$

Assume $L = 11$ (k-space lines)

$$2\, k_{y,max} = (L-1)\Delta k_y = 42\ cycles/cm$$

$$\Delta y = \frac{TBW = 1}{2 k_{y,max}} = 0.024\ cm$$

$$FOV = \frac{1}{\Delta k_y} = 0.23\ cm$$