
Fast Imaging Trajectories: Non-Cartesian Sampling (1)

M229 Advanced Topics in MRI

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2024.04.25

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Class Business

- Office hours
- Homework 1 being graded
- Homework 2 due 4/29 Mon by 5 pm
- Final project
 - Proposal due 5/10 Fri by 5 pm
 - Abstract due 6/7 Fri by 5 pm
 - Presentations and Q&A on 6/11 Tue 10-12

Outline

- Review of k-space sampling (2DFT)
- Radial
- Concentric rings

MR Signal Equation

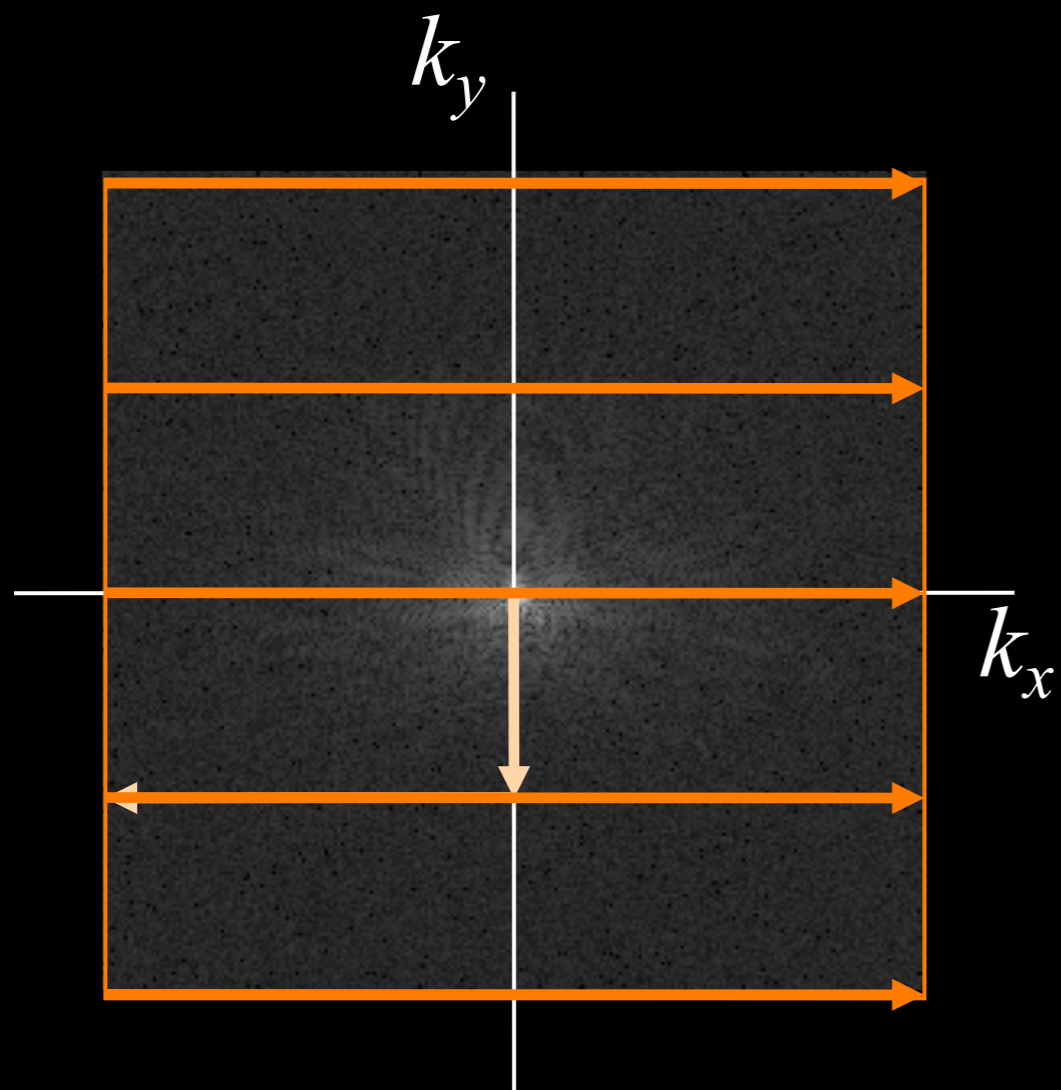
$$s(t) = \iint_{X,Y} M(x, y) \cdot \exp(-i2\pi \cdot [k_x(t)x + k_y(t)y]) dx dy$$

$$= m(k_x(t), k_y(t))$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t, \quad k_y(t) = \frac{\gamma}{2\pi} G_y t$$

$$m = \mathcal{FT}(M(x, y))$$

k-Space Sampling



set of $s(t)$ covers $m(k_x, k_y)$

Pulse Sequence Diagram

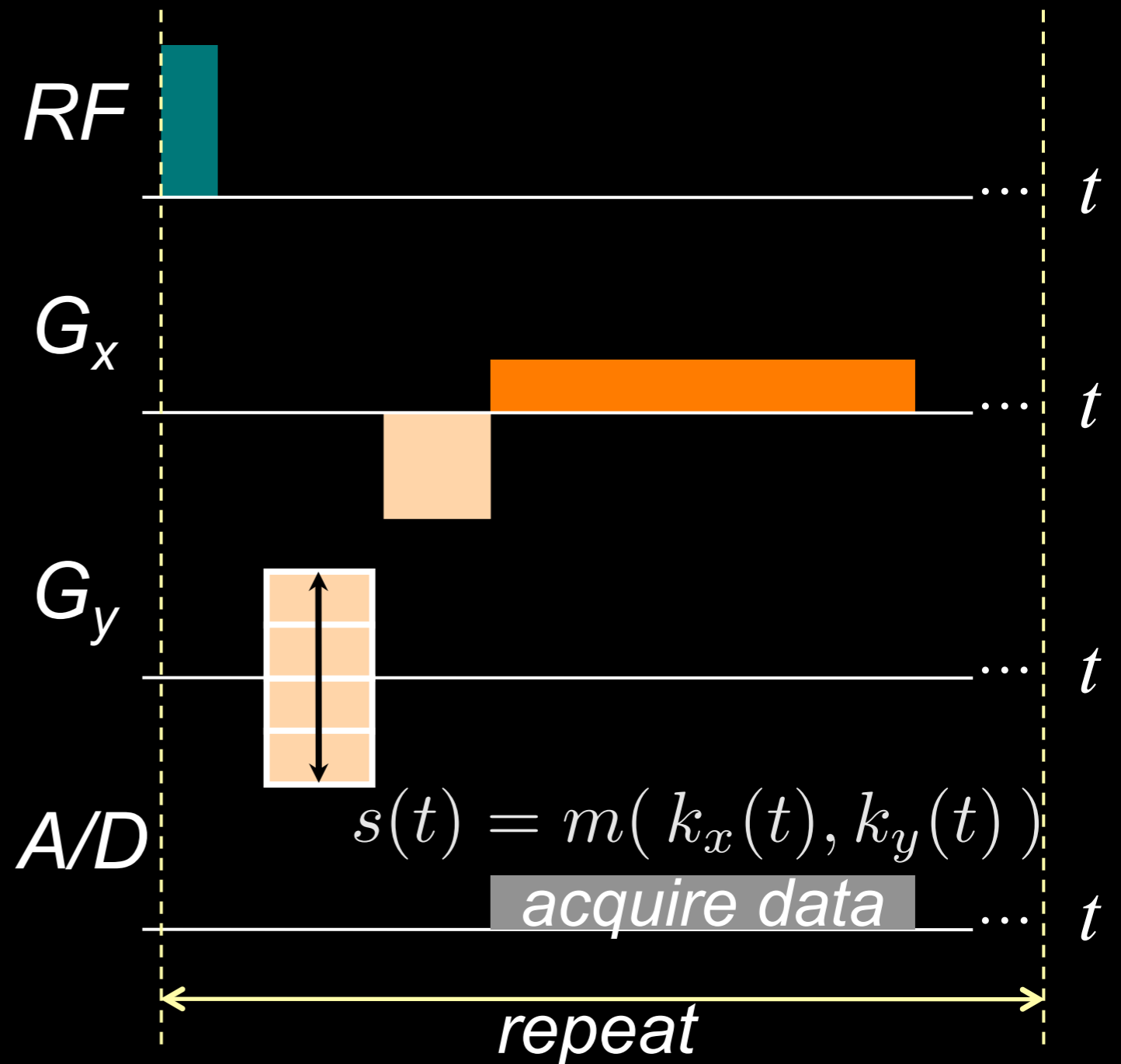
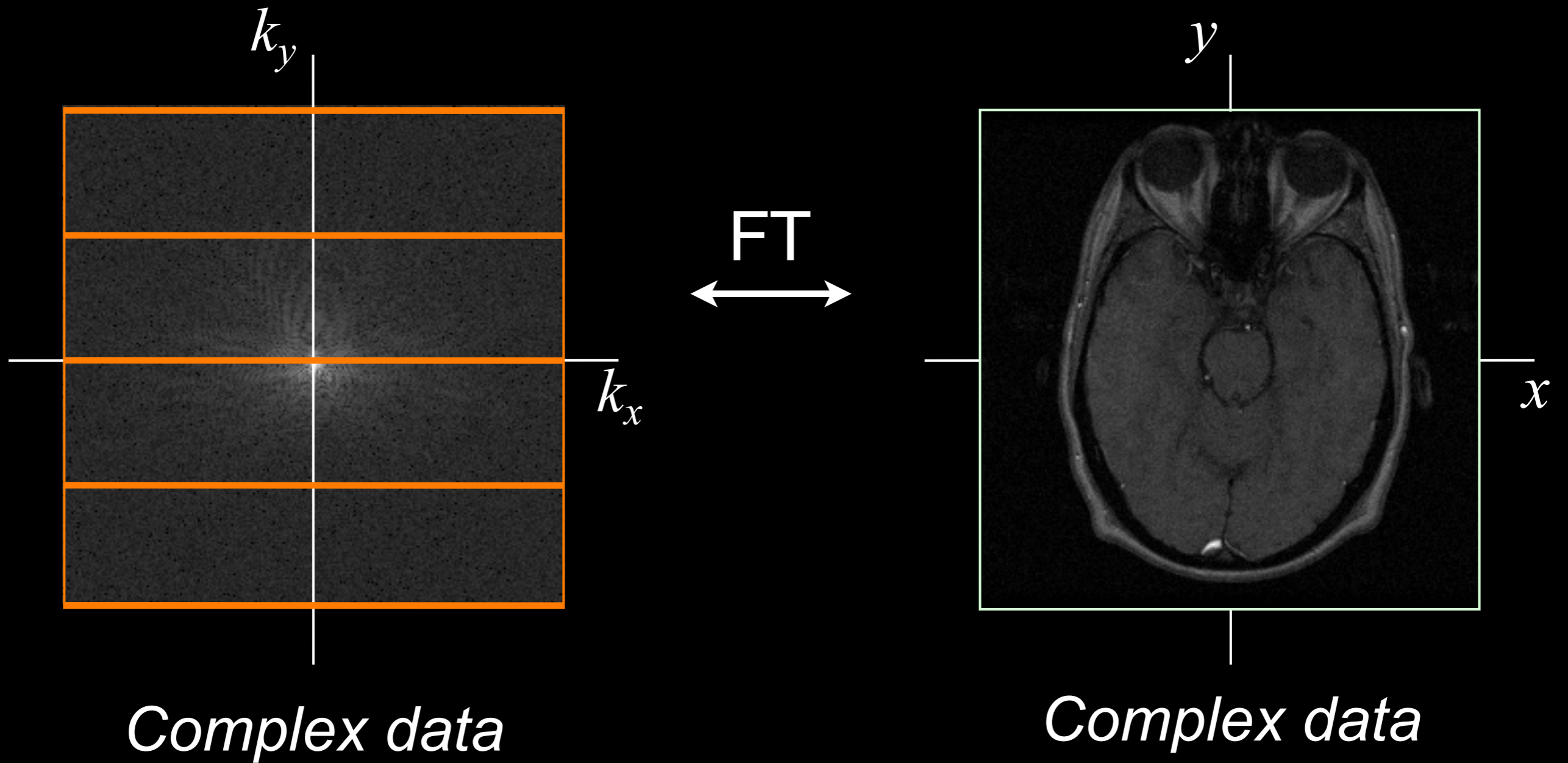
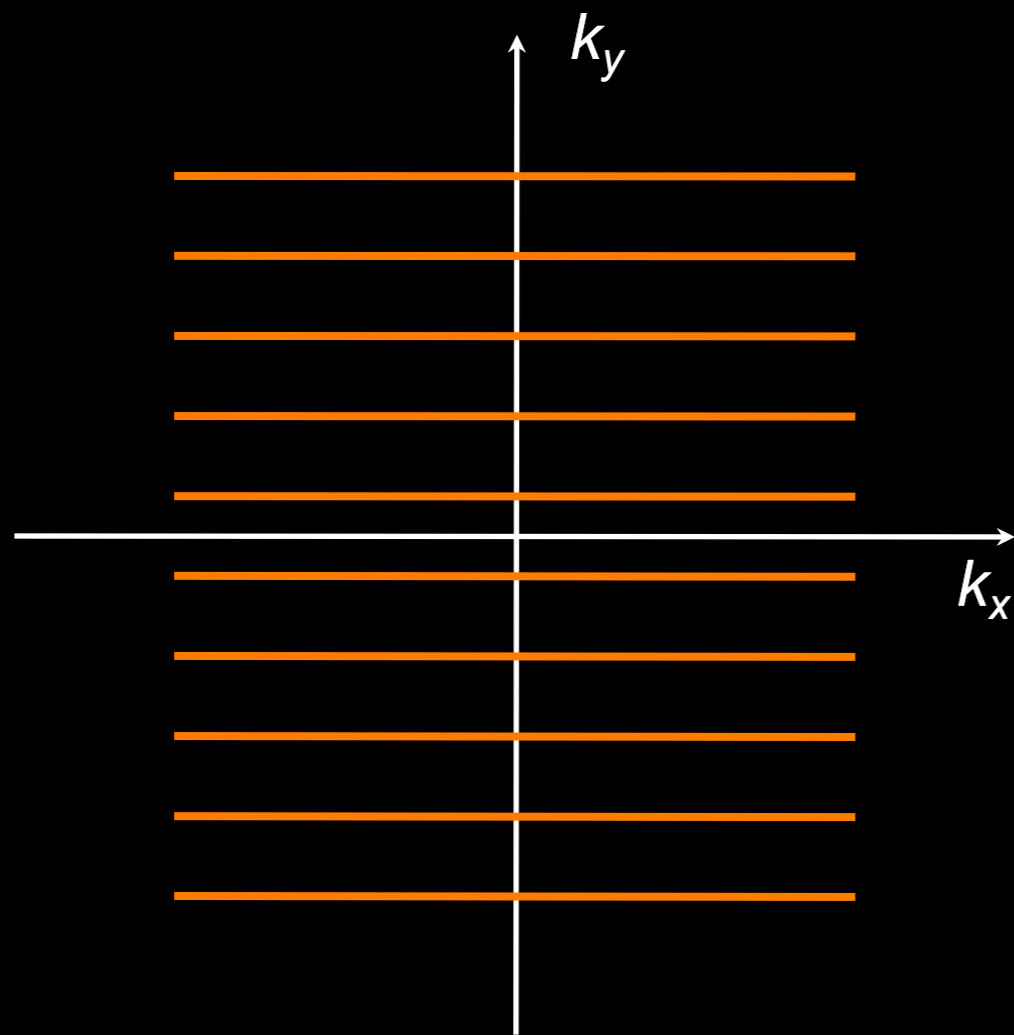


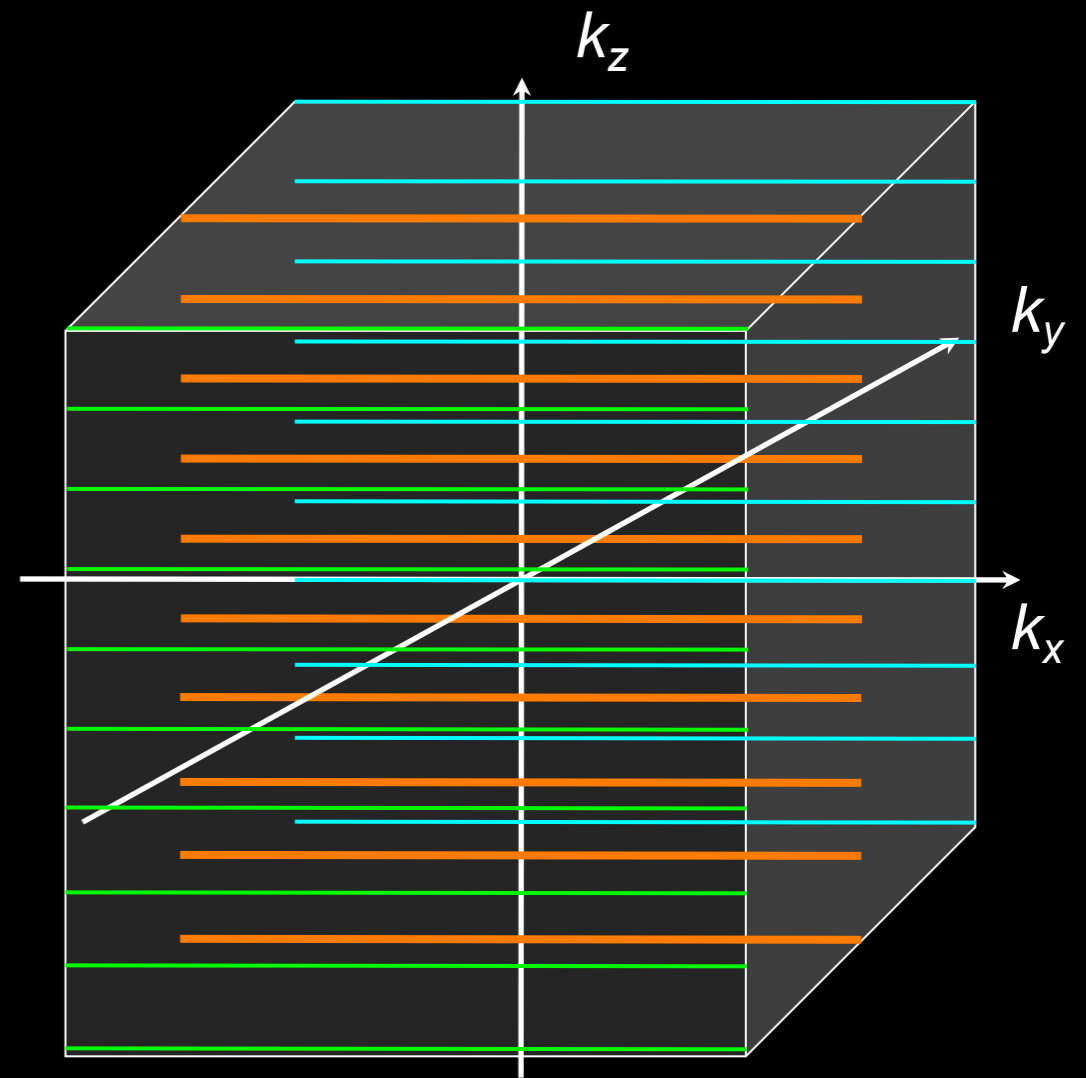
Image Reconstruction



Cartesian Sampling



Cartesian 2DFT



Cartesian 3DFT

MR Signal Equation

$$s(t) = \iint_{X,Y} M(x, y) \cdot \exp(-i2\pi \cdot [k_x(t)x + k_y(t)y]) dx dy$$

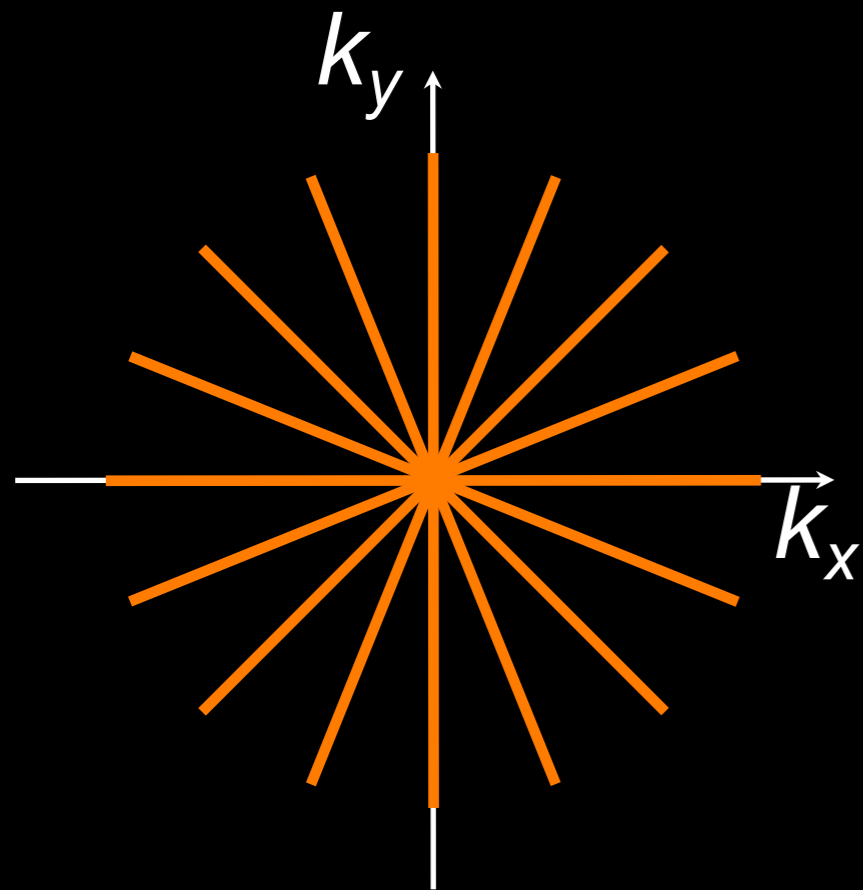
$$= m(k_x(t), k_y(t))$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t, \quad k_y(t) = \frac{\gamma}{2\pi} G_y t$$

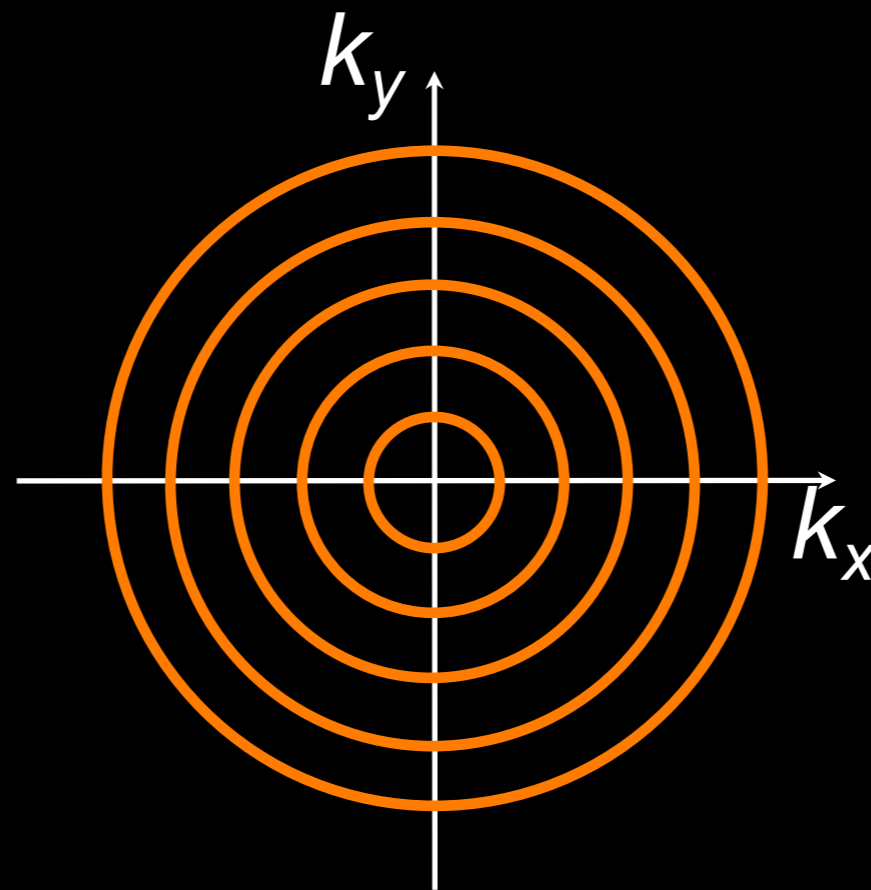
$$m = \mathcal{FT}(M(x, y))$$

$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau, \quad k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau$$

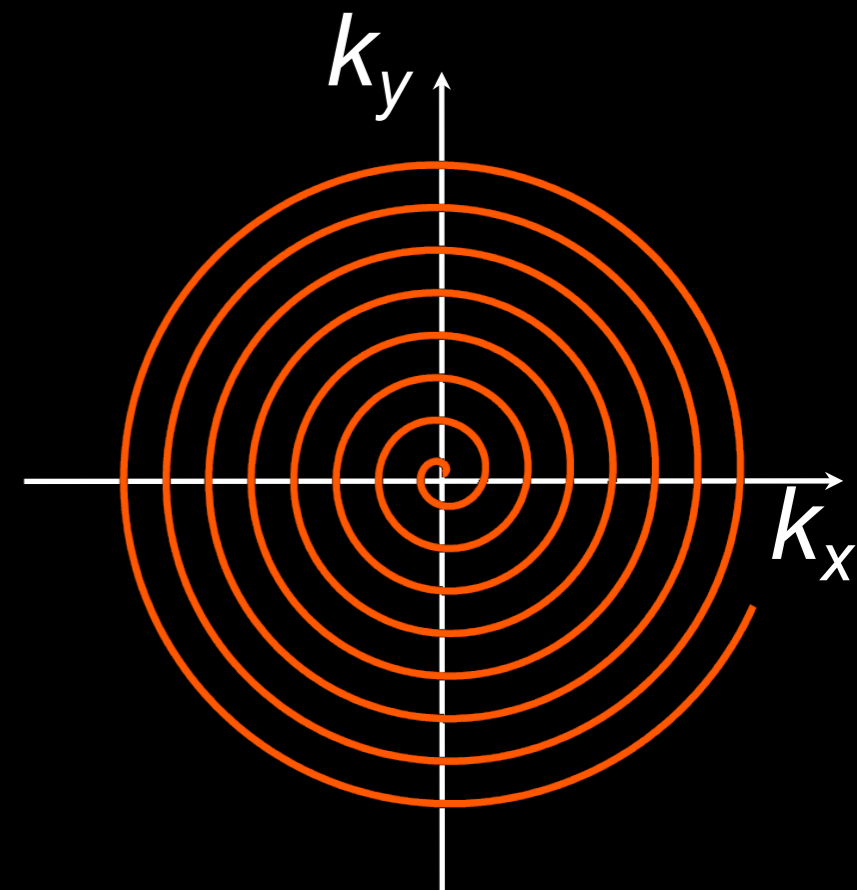
Non-Cartesian Sampling



2D Radial



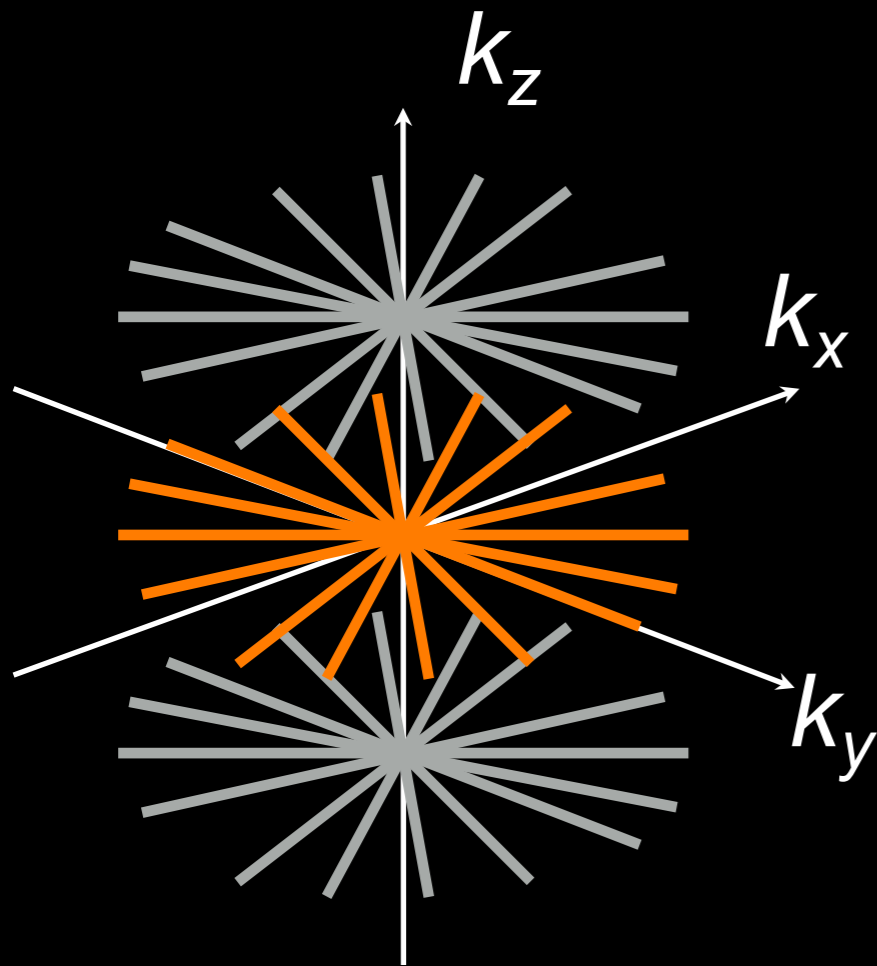
2D Concentric Rings



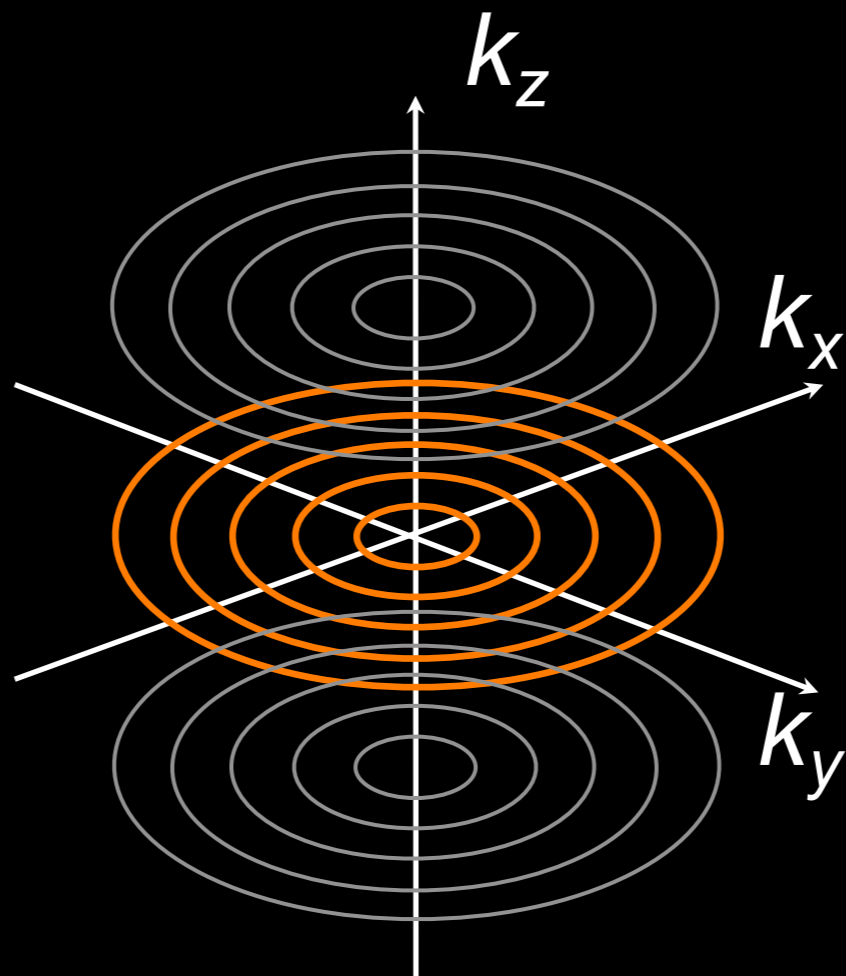
2D Spiral

and much more ...

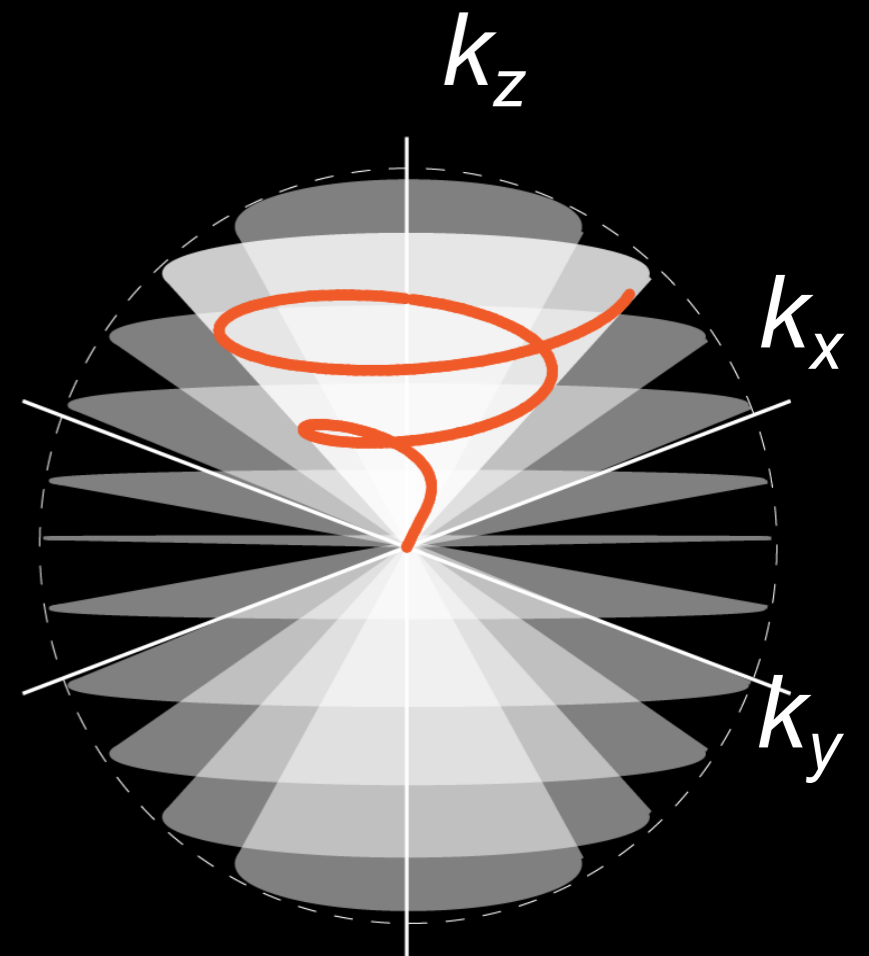
Non-Cartesian Sampling



3D Stack of Stars



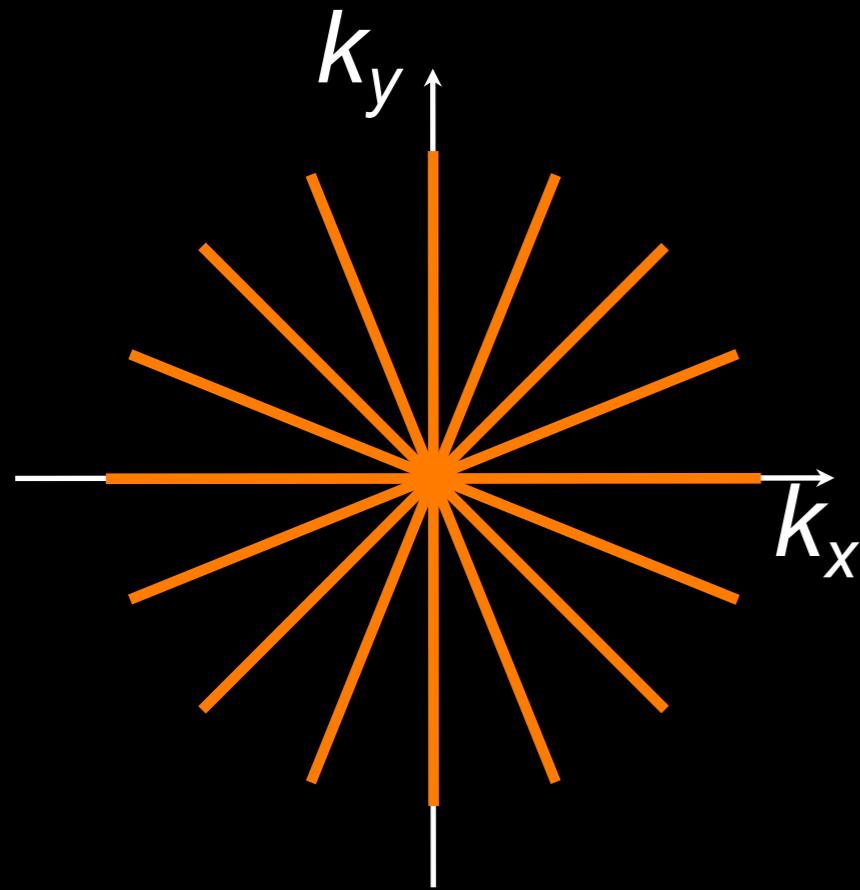
3D Stack of Rings



3D Cones

and much more ...

Radial



The original MRI trajectory!

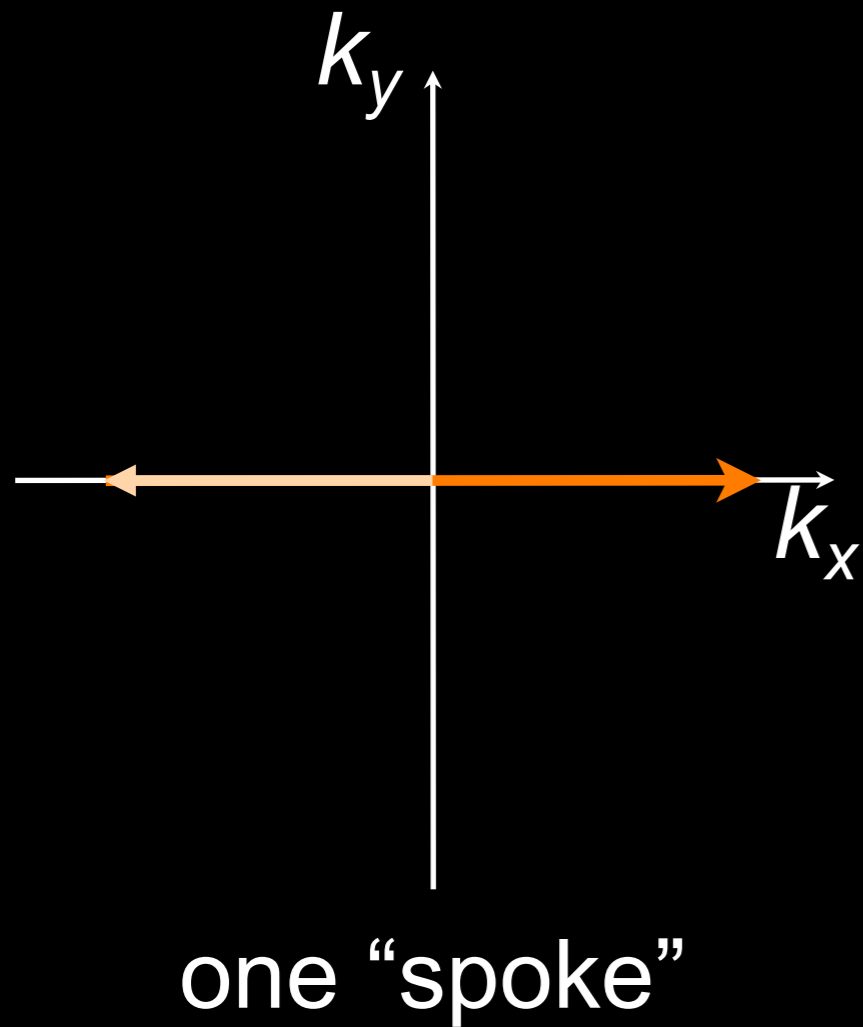
- *Lauterbur, Nature 1973*

Samples k-space on a polar grid

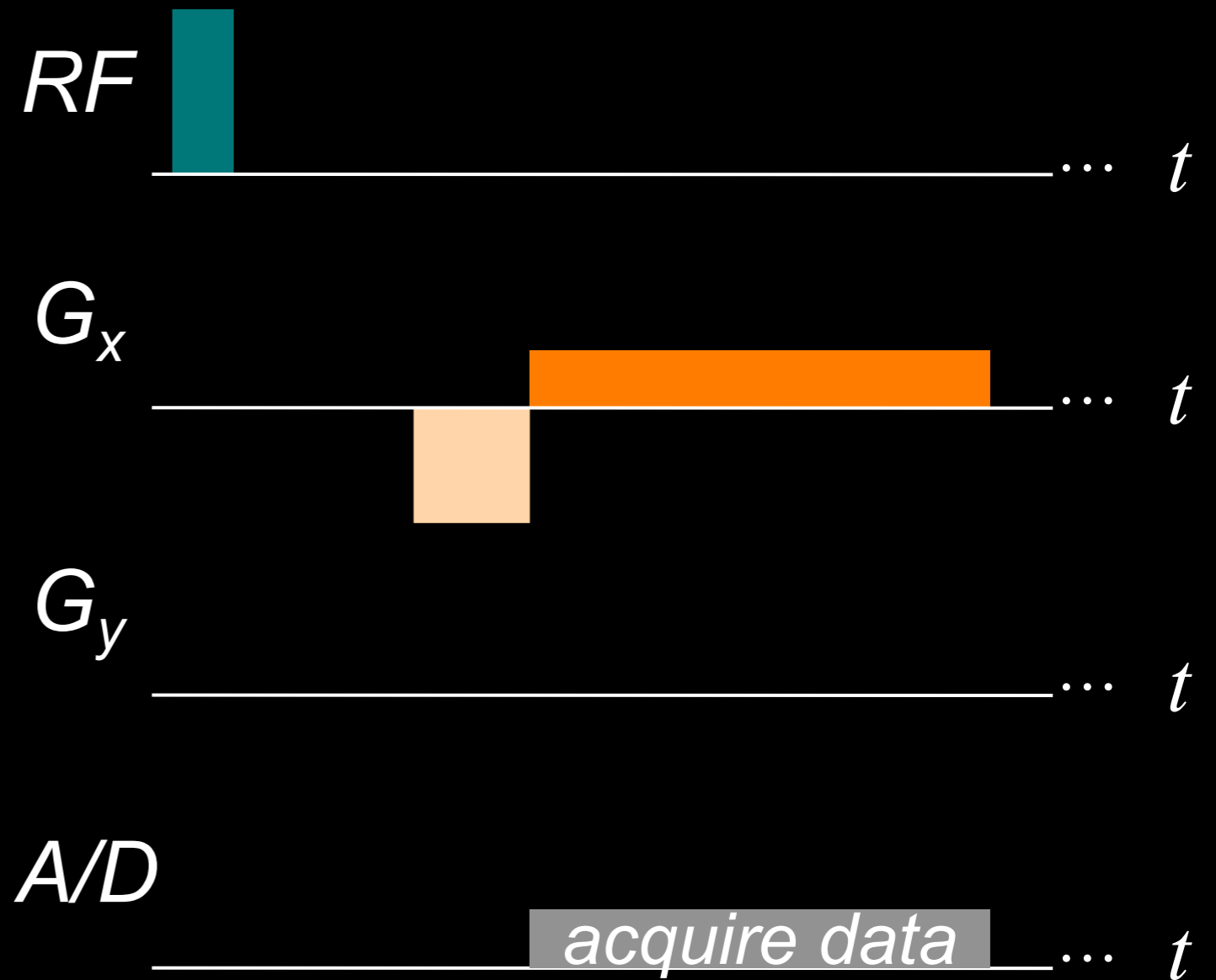
- “Spokes” correspond to projections

- Projection reconstruction (2DPR)

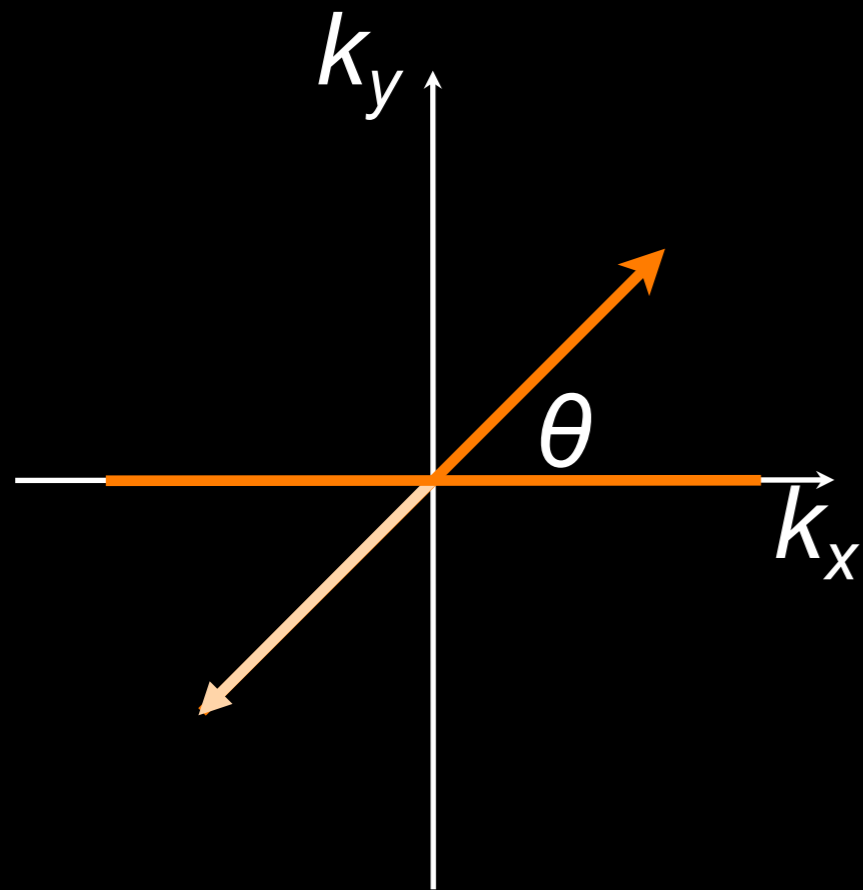
Radial: Gradient Design



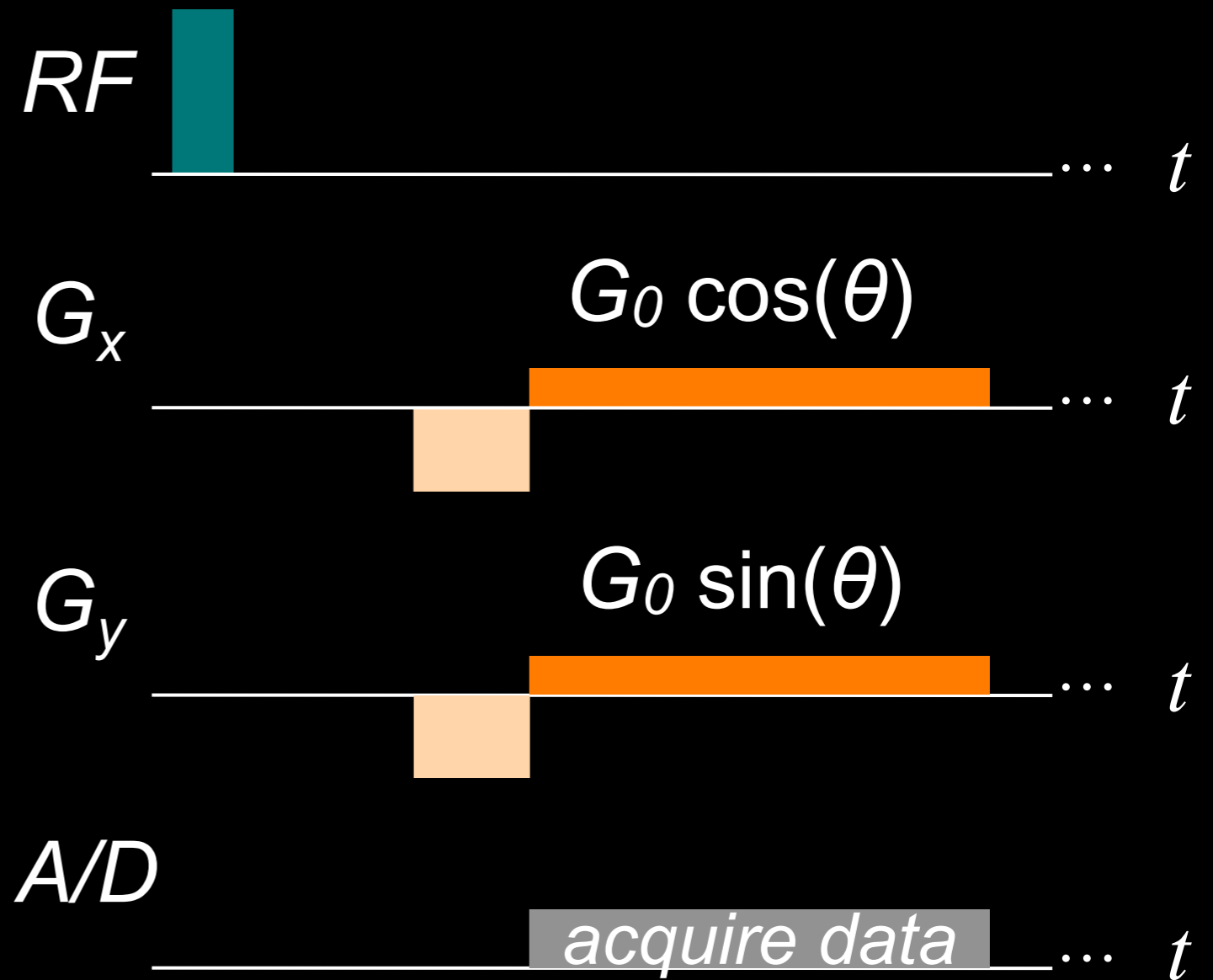
Pulse Sequence Diagram



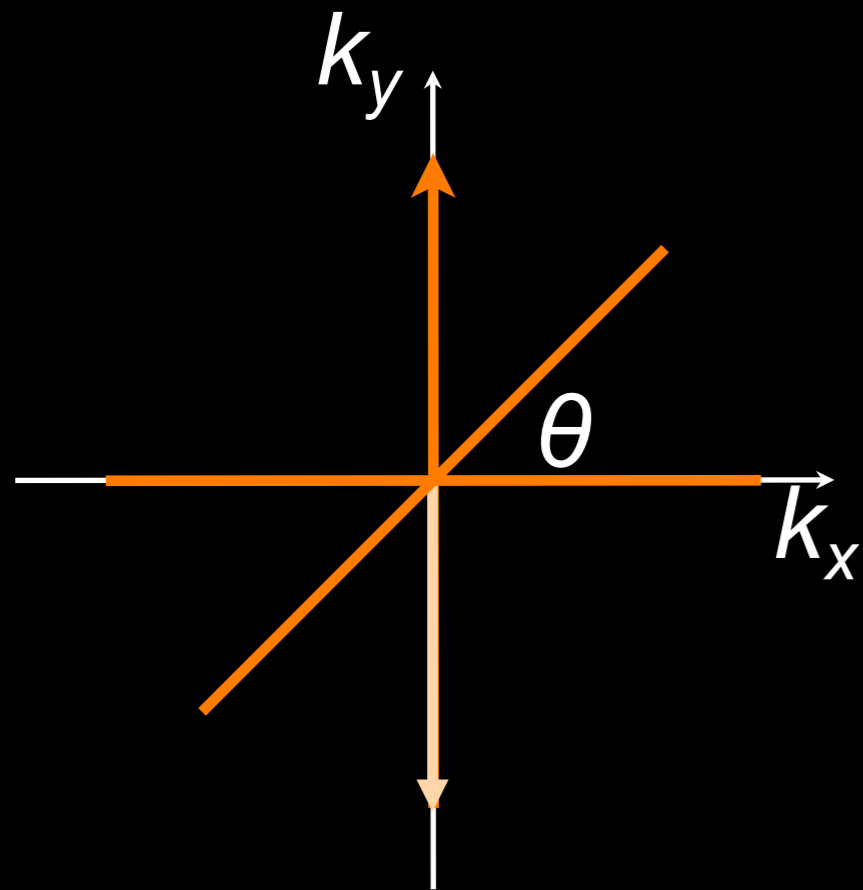
Radial: Gradient Design



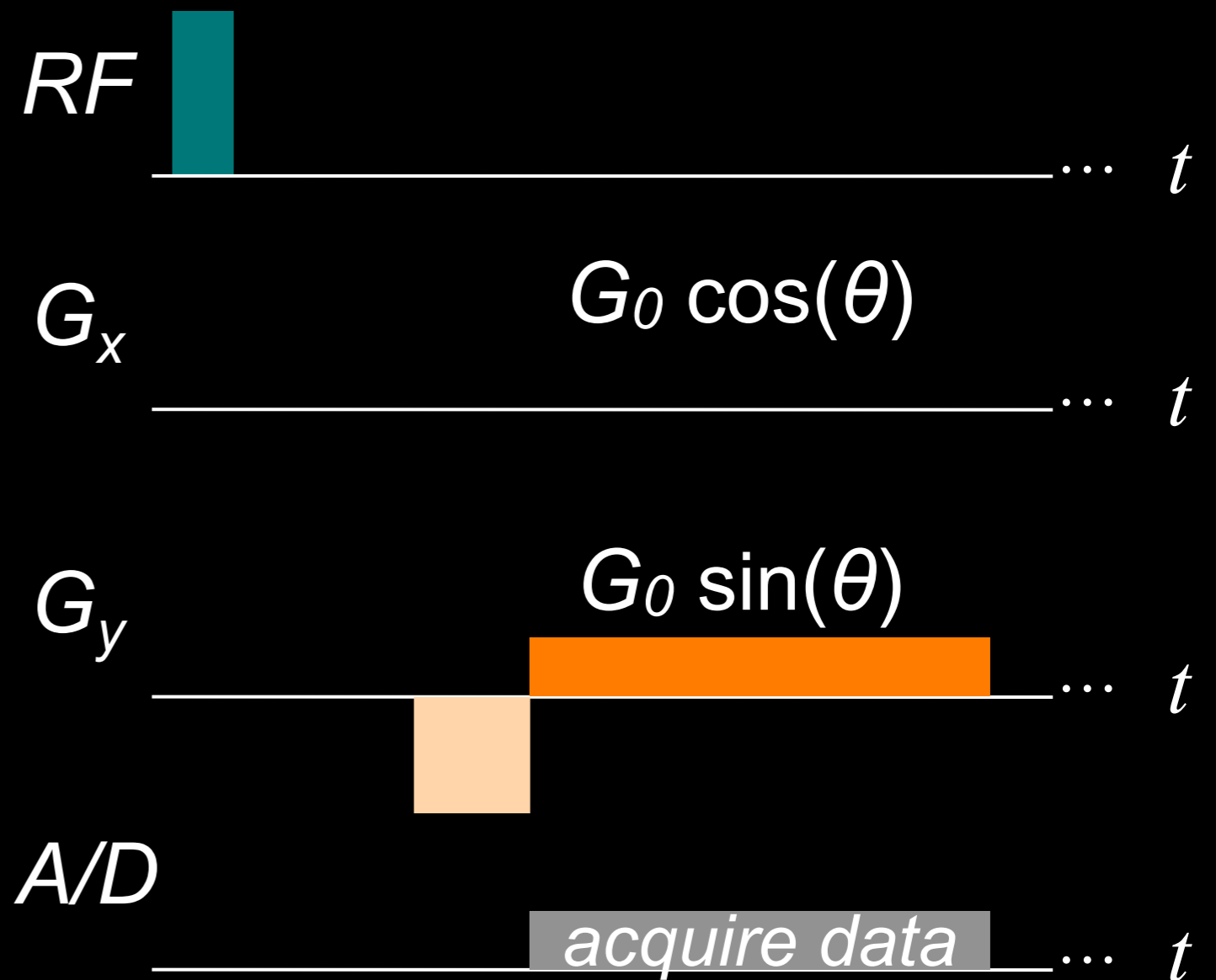
Pulse Sequence Diagram



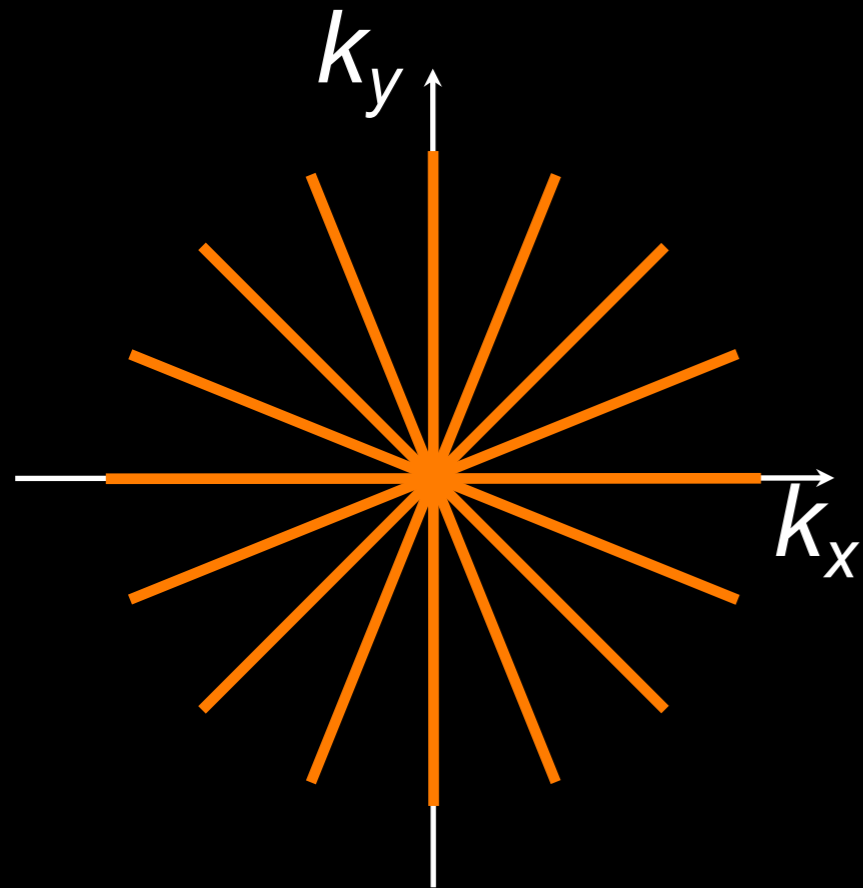
Radial: Gradient Design



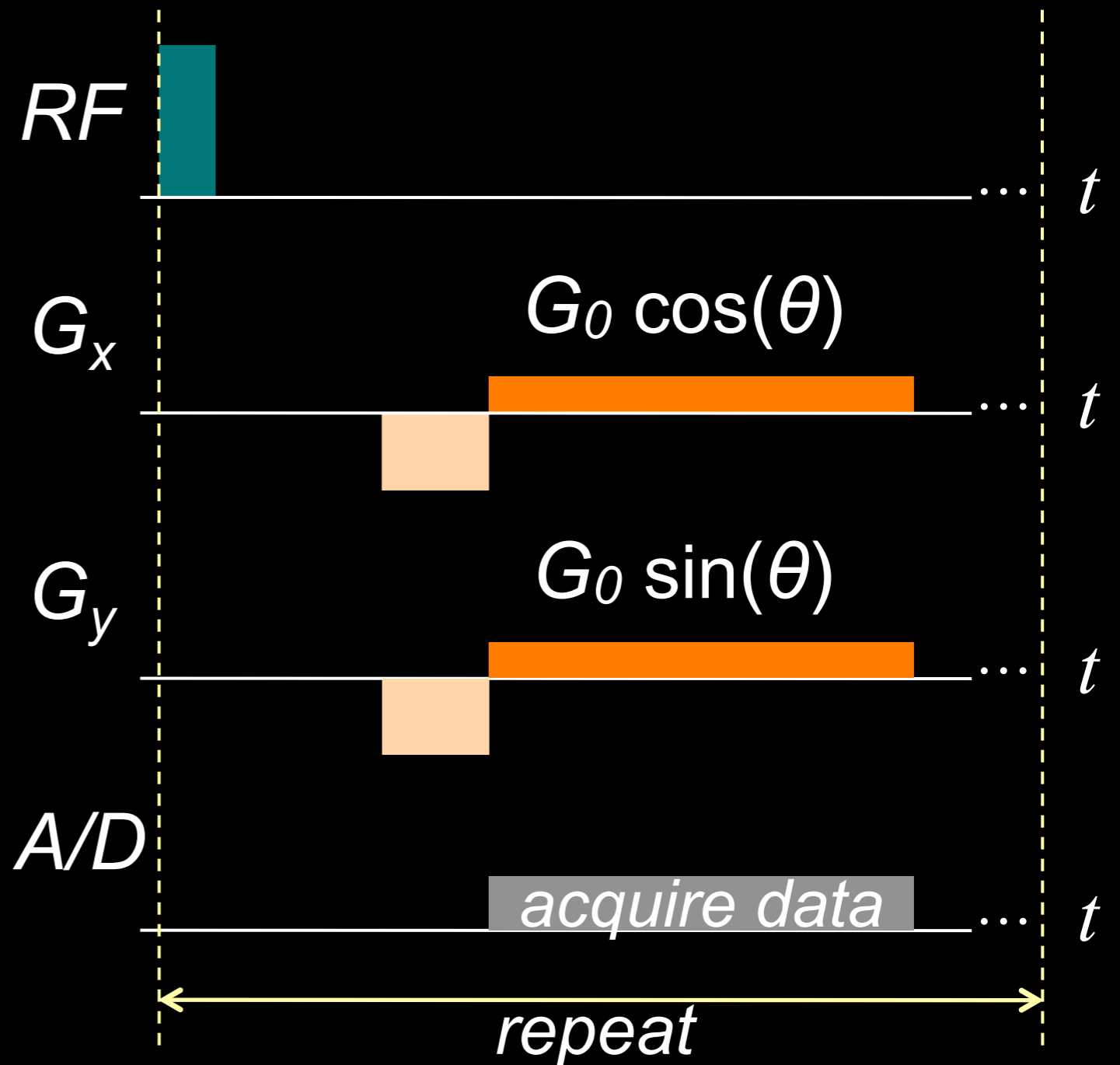
Pulse Sequence Diagram



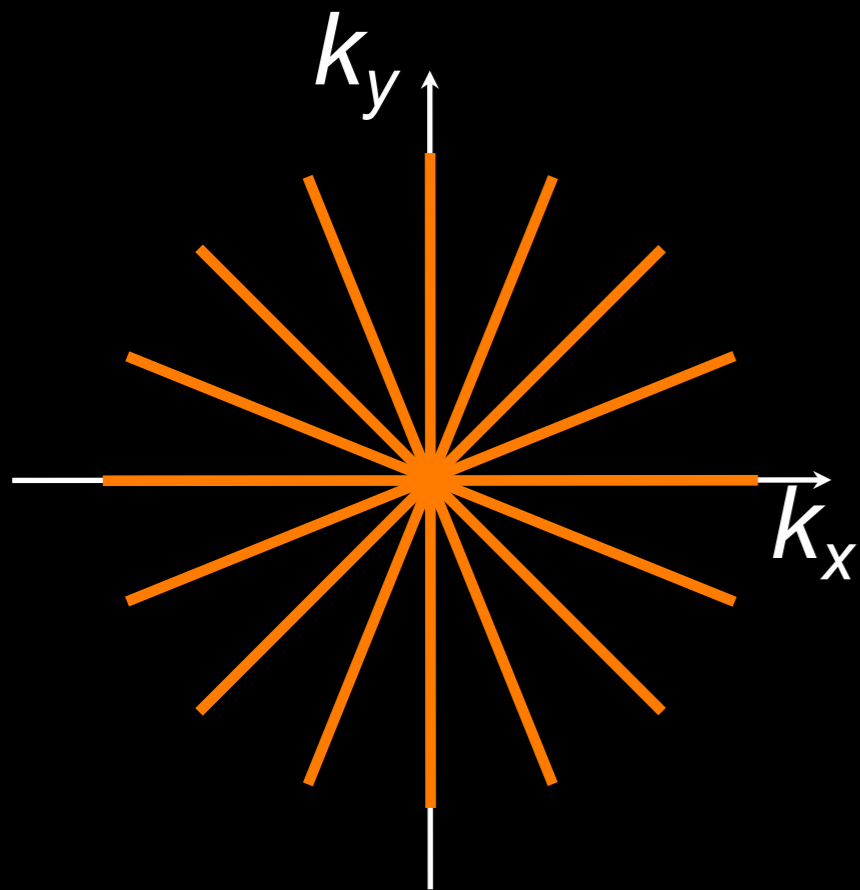
Radial: Gradient Design



Pulse Sequence Diagram



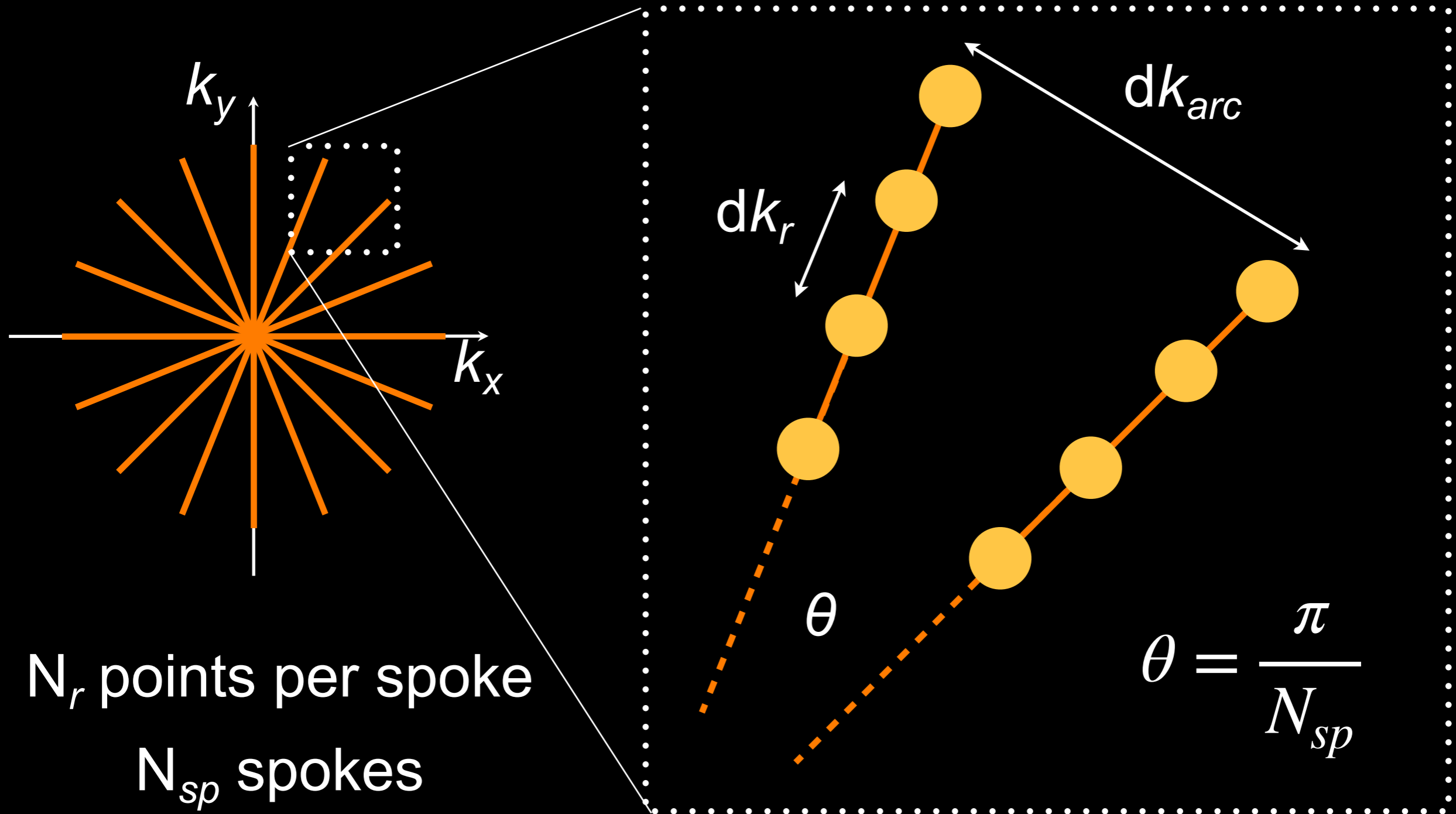
Radial: Sampling Requirements



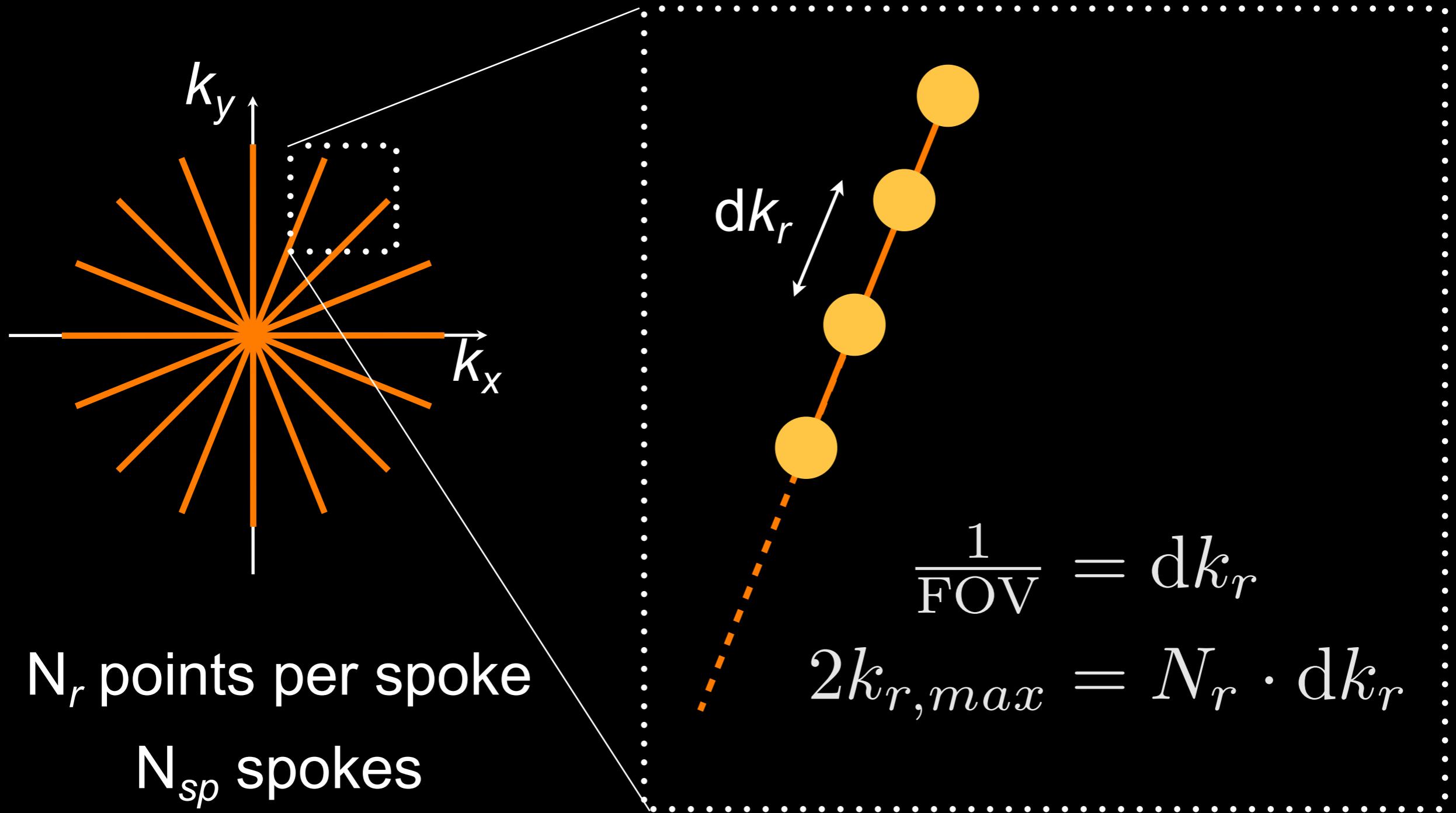
N_r points per spoke

N_{sp} spokes

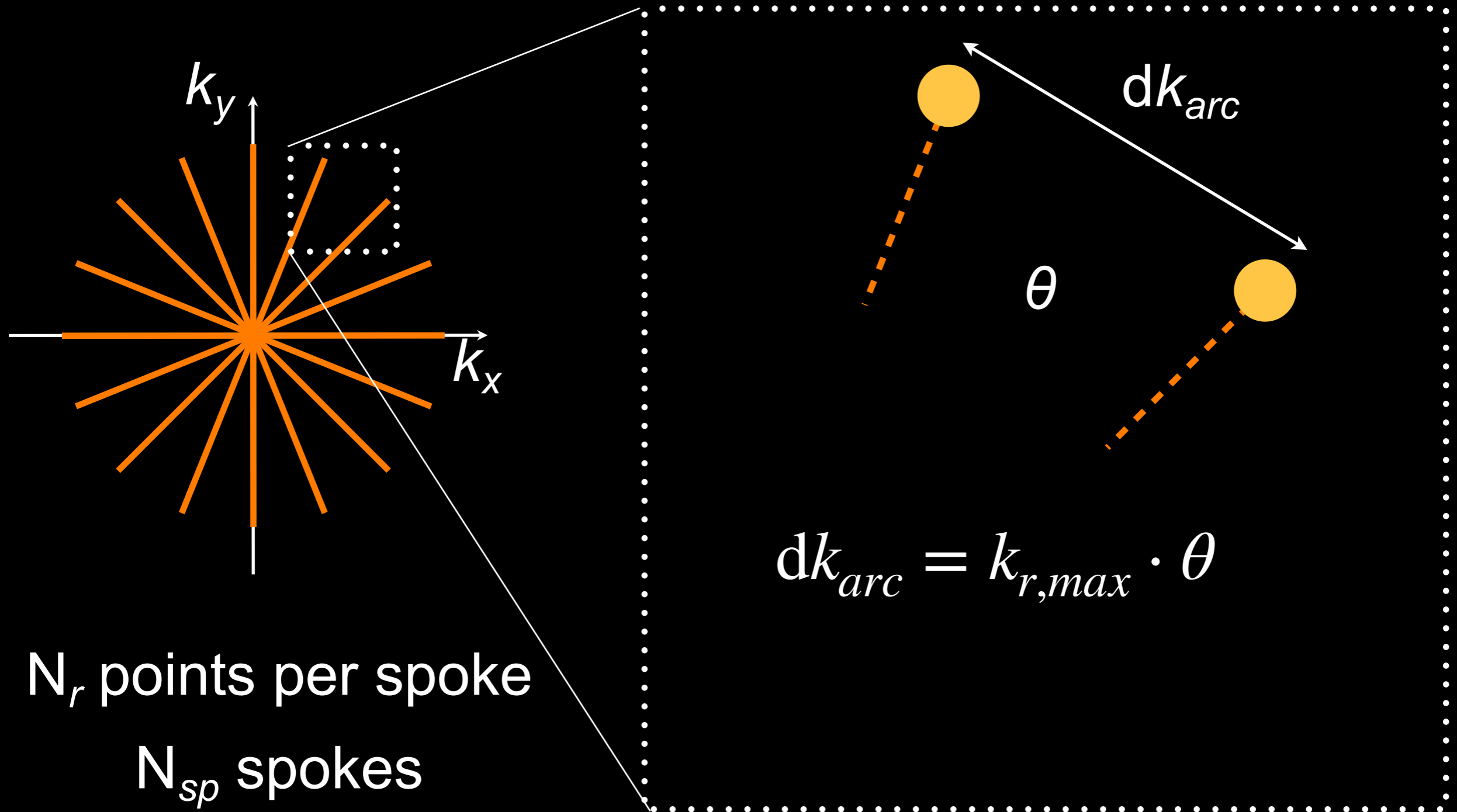
Radial: Sampling Requirements



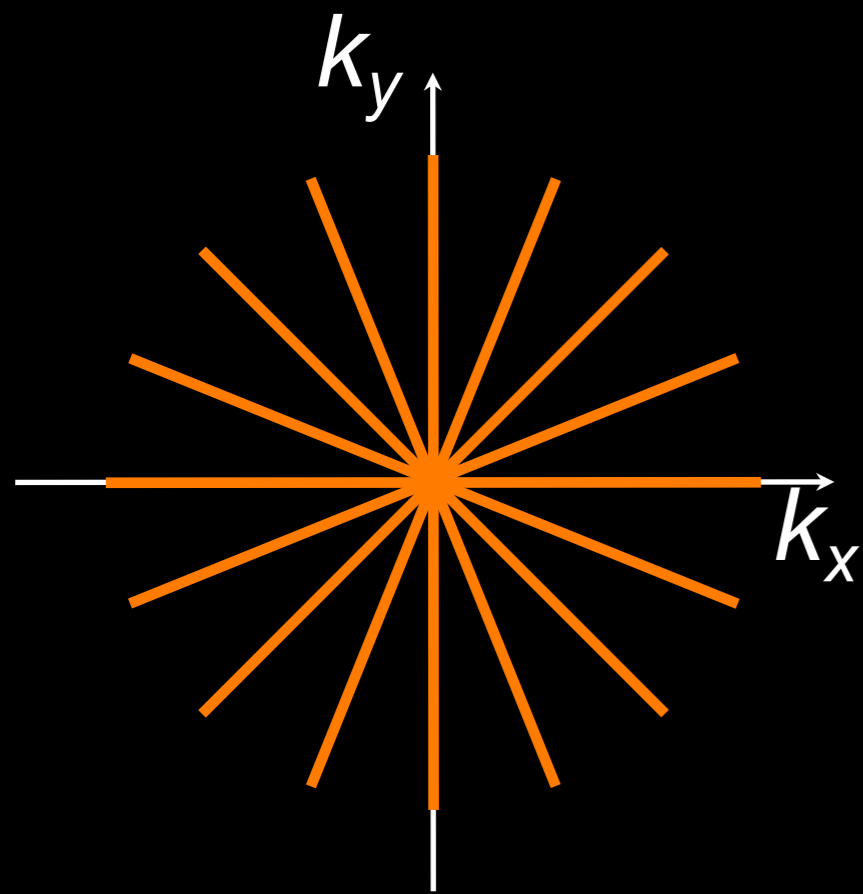
Radial: Sampling Requirements



Radial: Sampling Requirements



Radial: Sampling Requirements



N_r points per spoke

N_{sp} spokes

To satisfy Nyquist at edges of k-space:

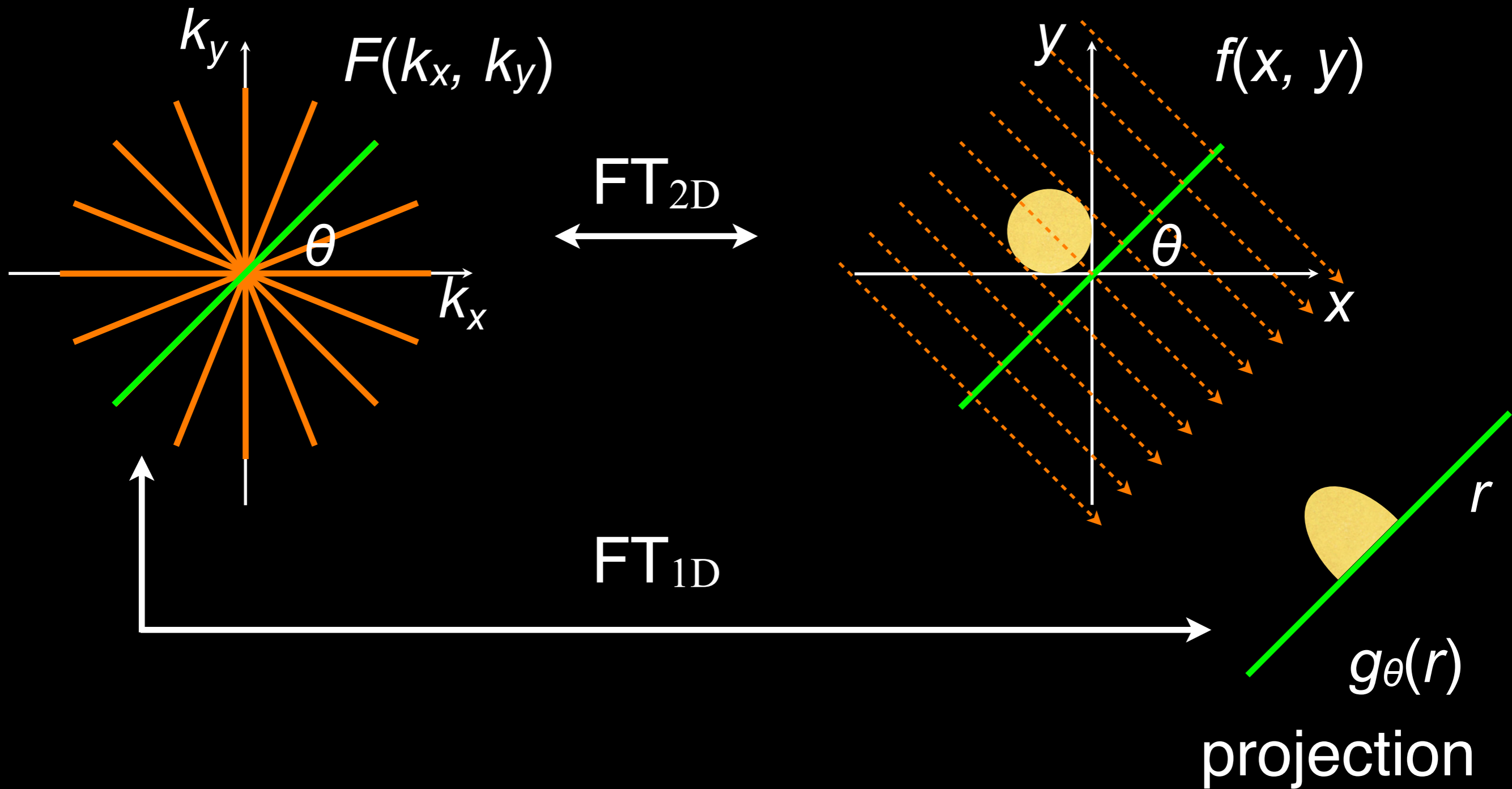
$$dk_{arc} = \left(\frac{N_r}{2} \cdot dk_r \right) \cdot \frac{\pi}{N_{sp}} \leq dk_r$$

$$N_{sp} \geq \frac{\pi}{2} \cdot N_r$$

Example: $N_r = 256$, $N_{sp} = 403$

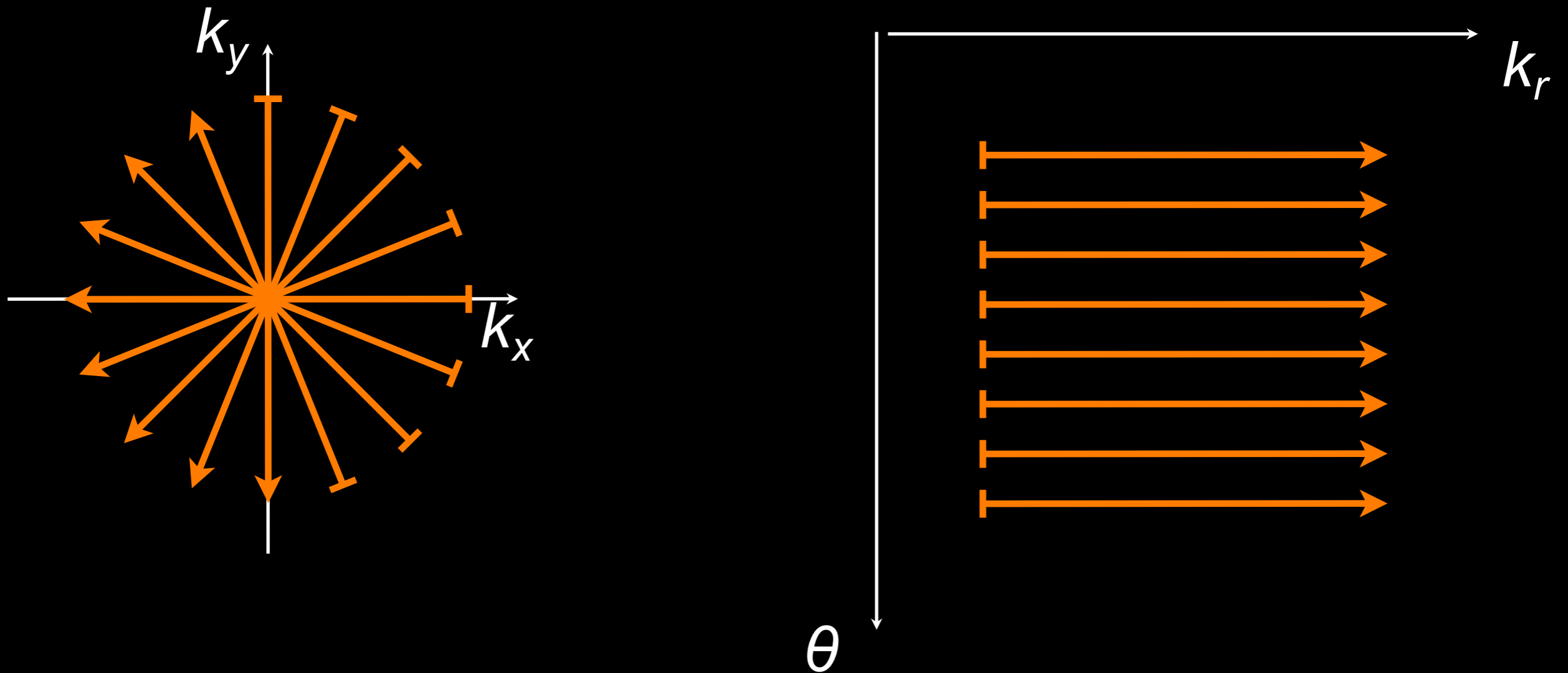
Radial: Image Reconstruction

Central Section Theorem



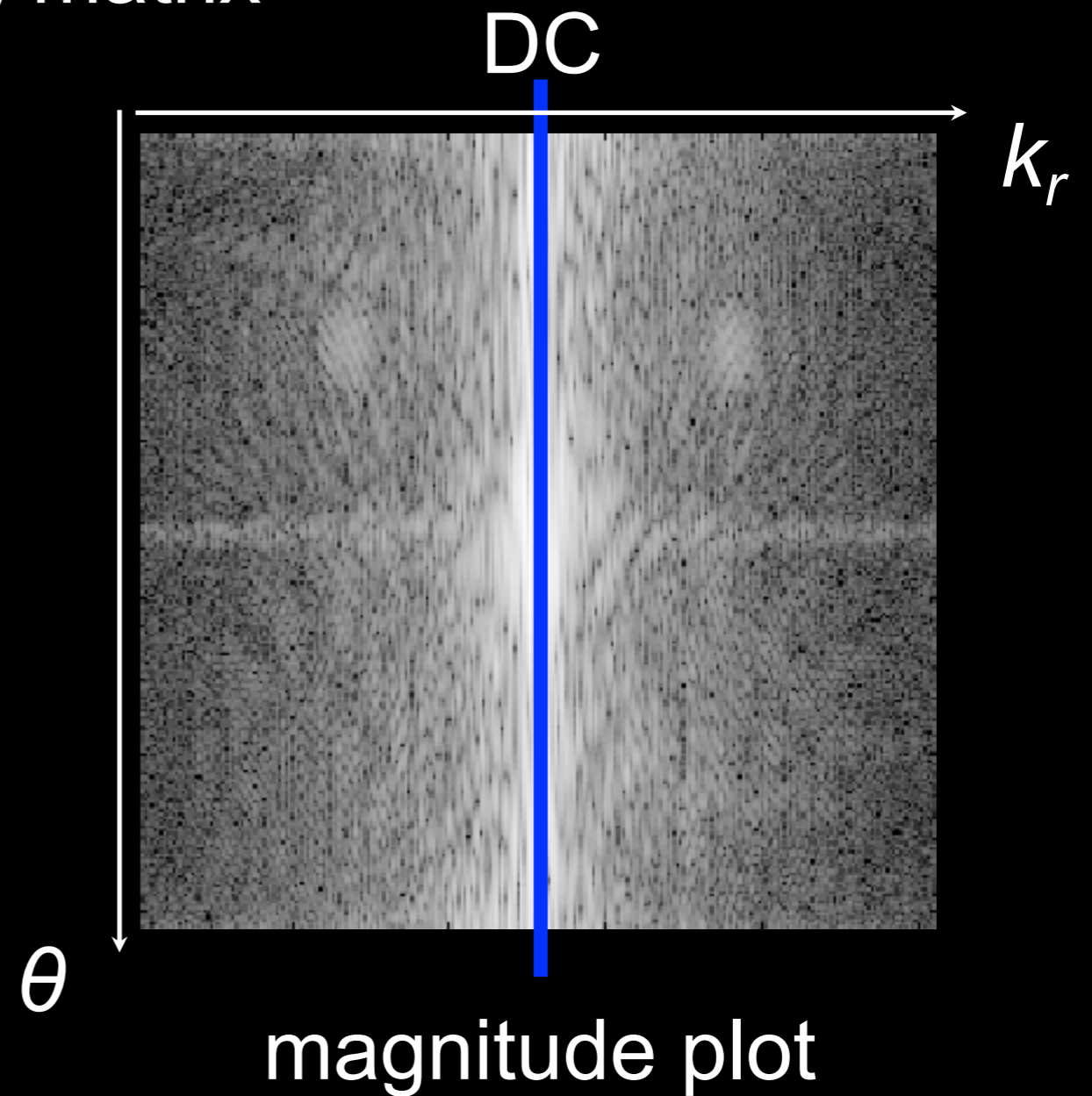
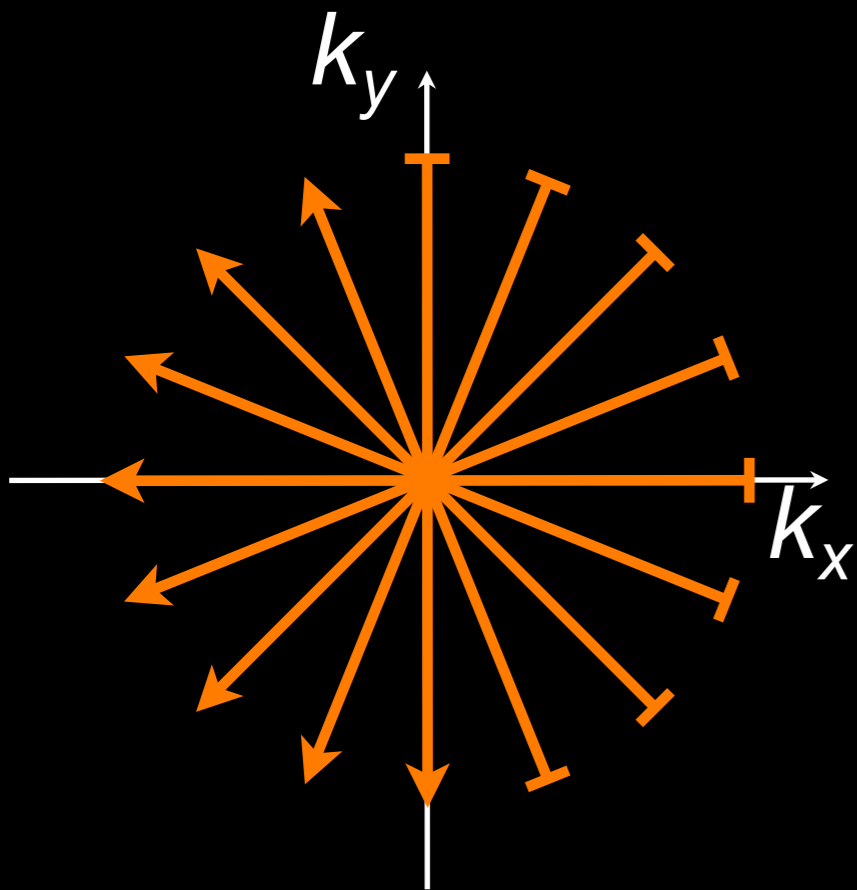
Radial: Image Reconstruction

Collect spokes into (k_r, θ) matrix \rightarrow



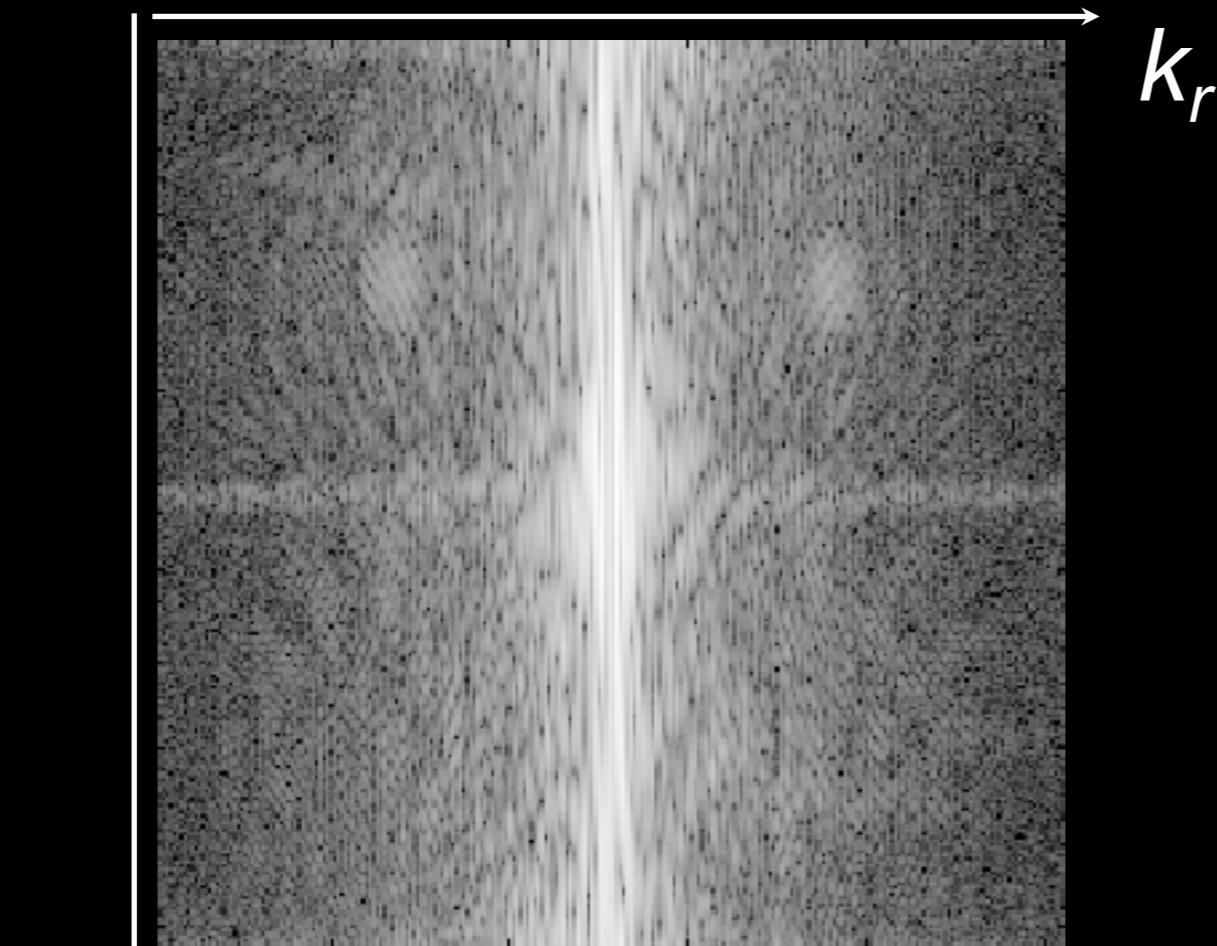
Radial: Image Reconstruction

Collect spokes into (k_r, θ) matrix \rightarrow

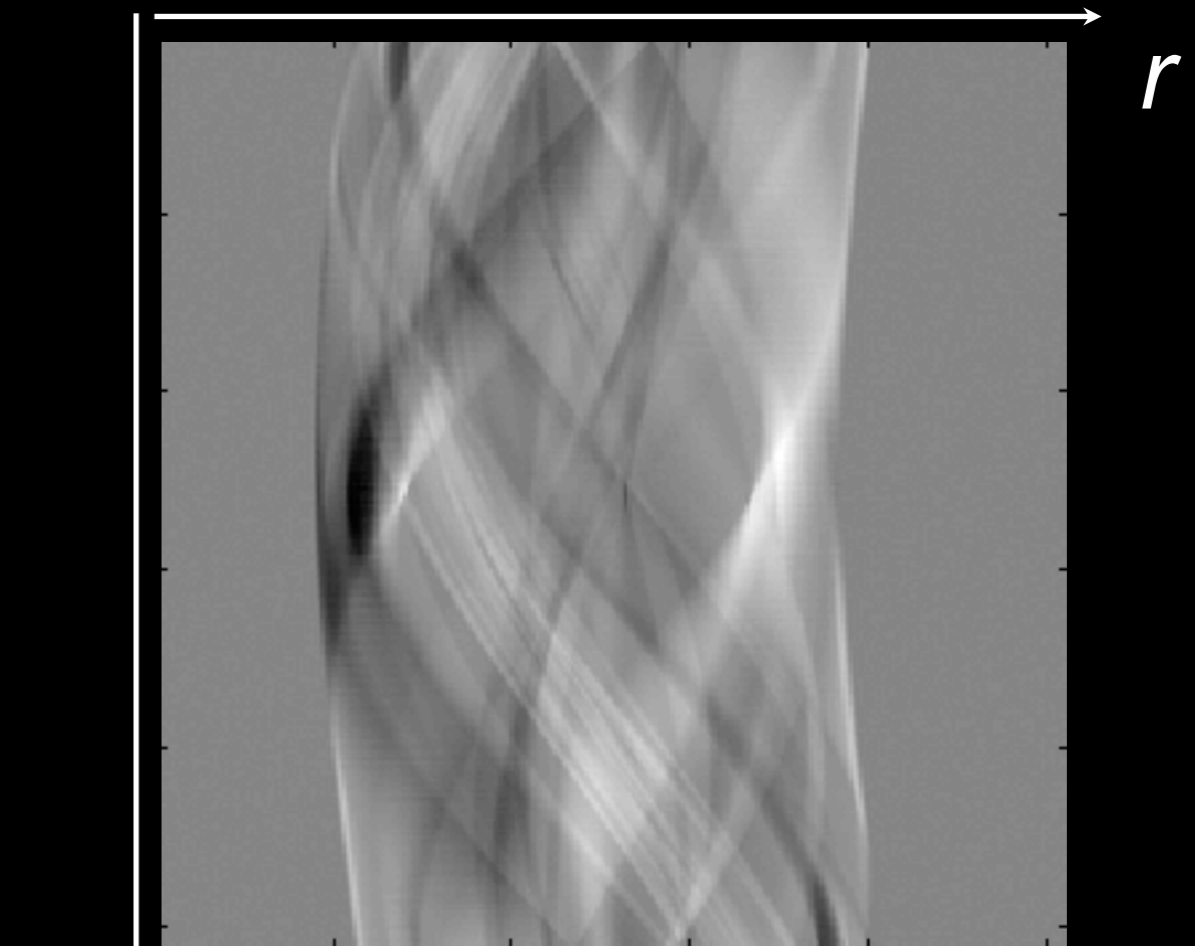


Radial: Image Reconstruction

1DFT of each spoke along $k_r \rightarrow$ "Sinogram"



magnitude plot

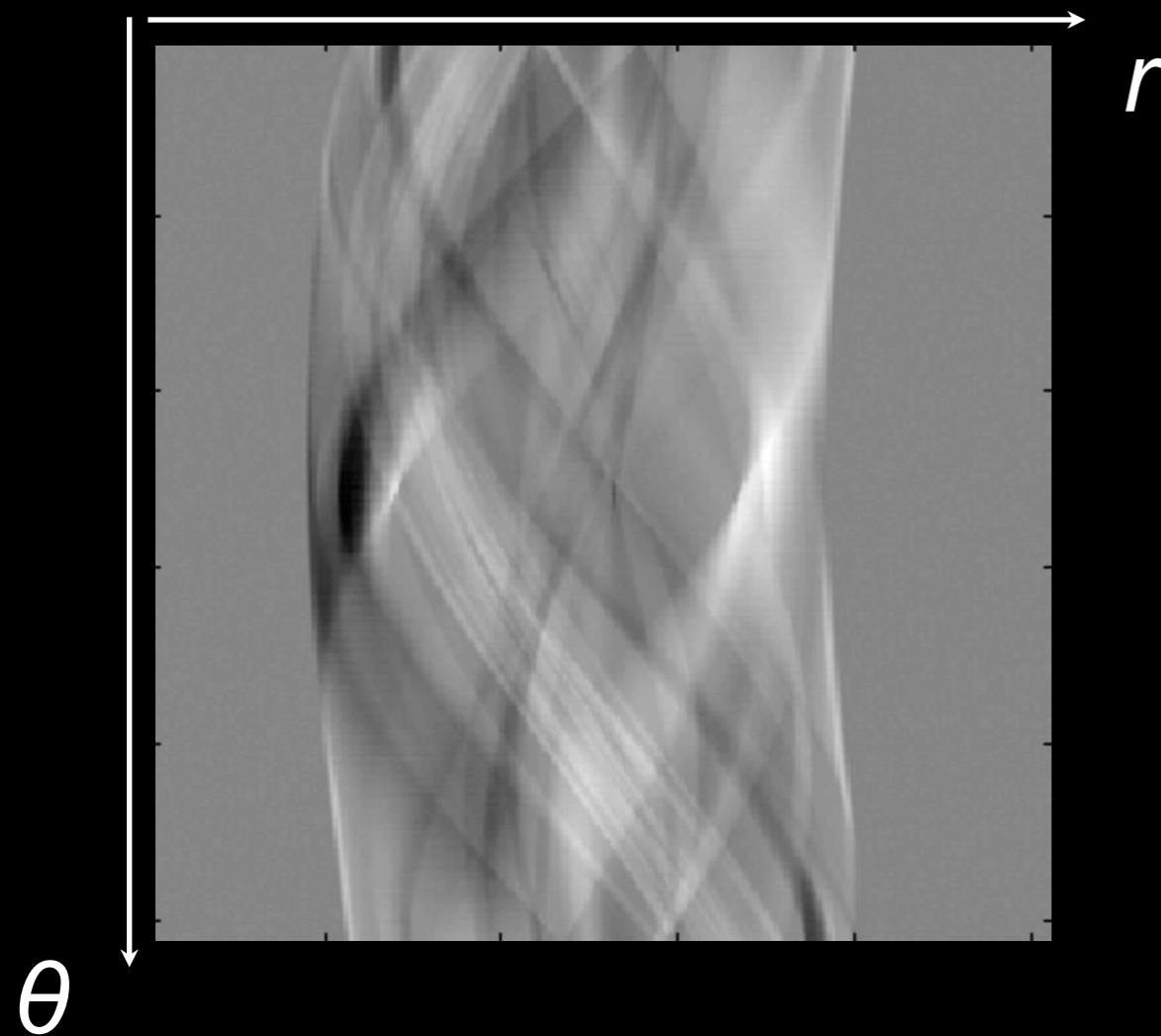


real channel

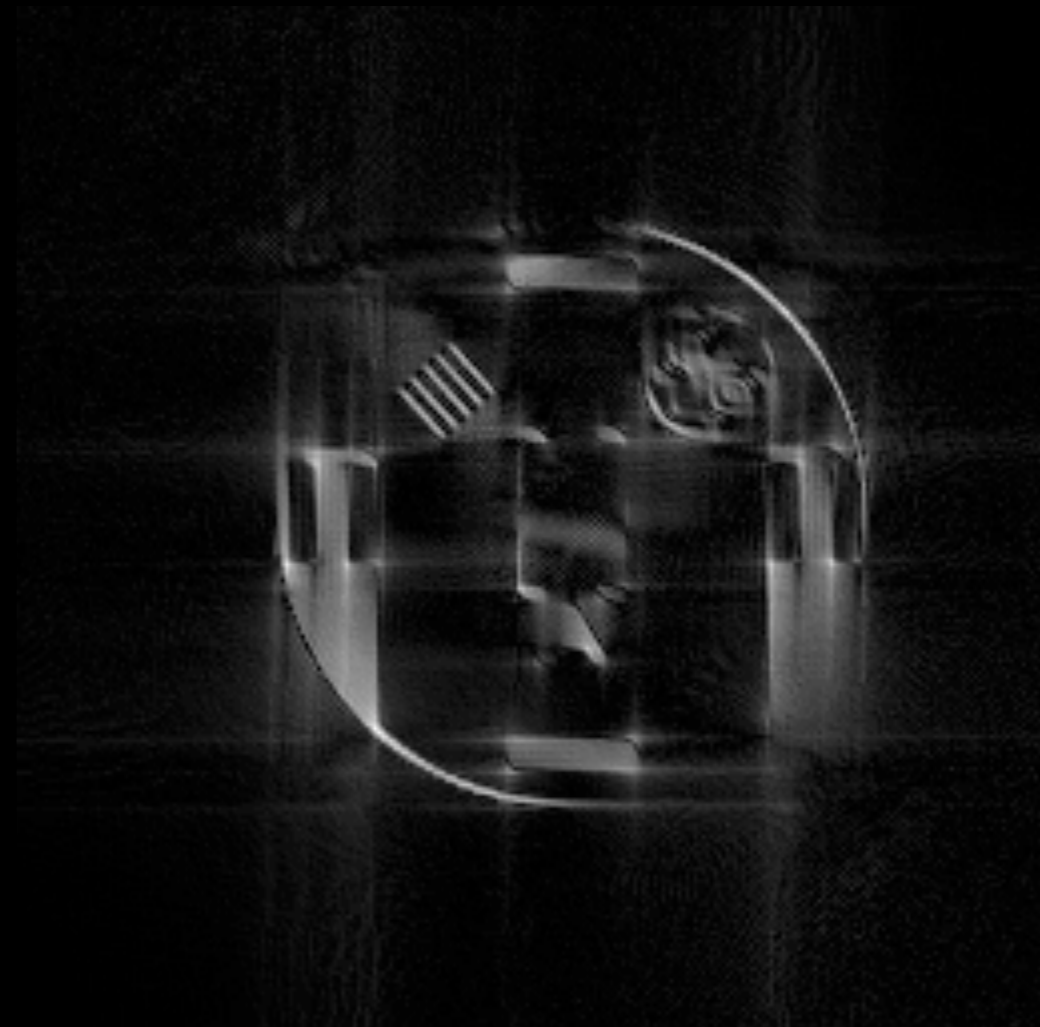
Radial: Image Reconstruction

Filtered back projection \rightarrow

Image



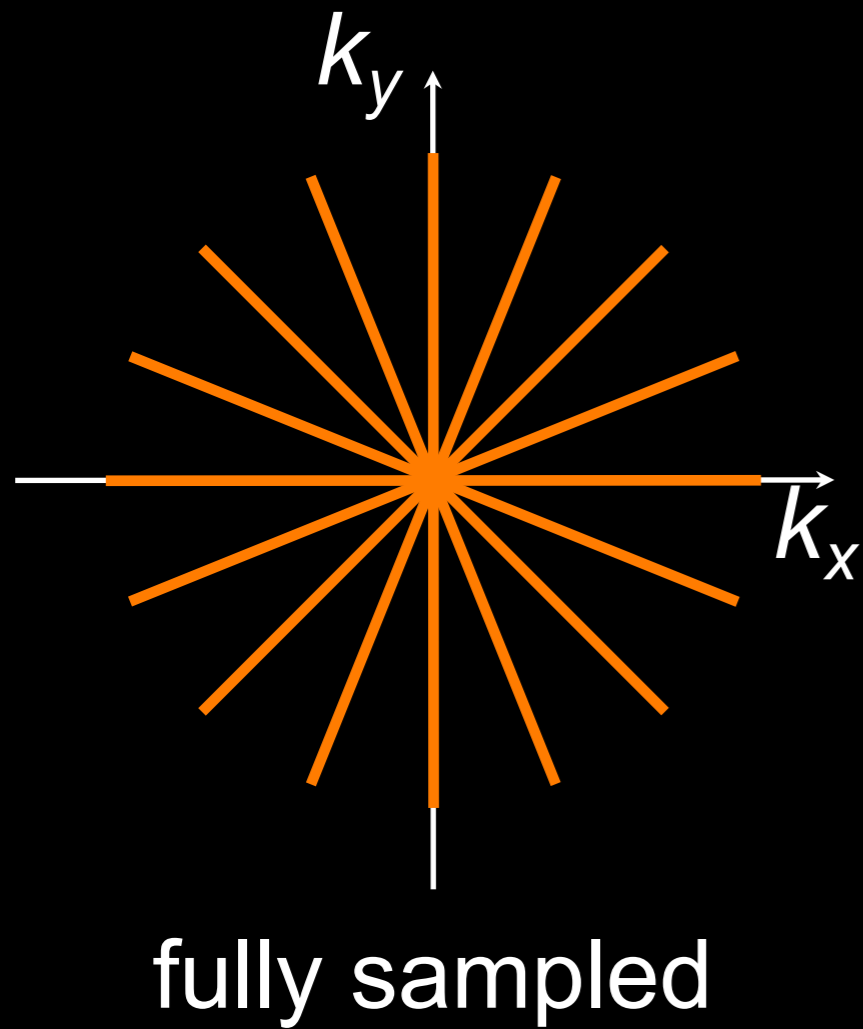
real channel



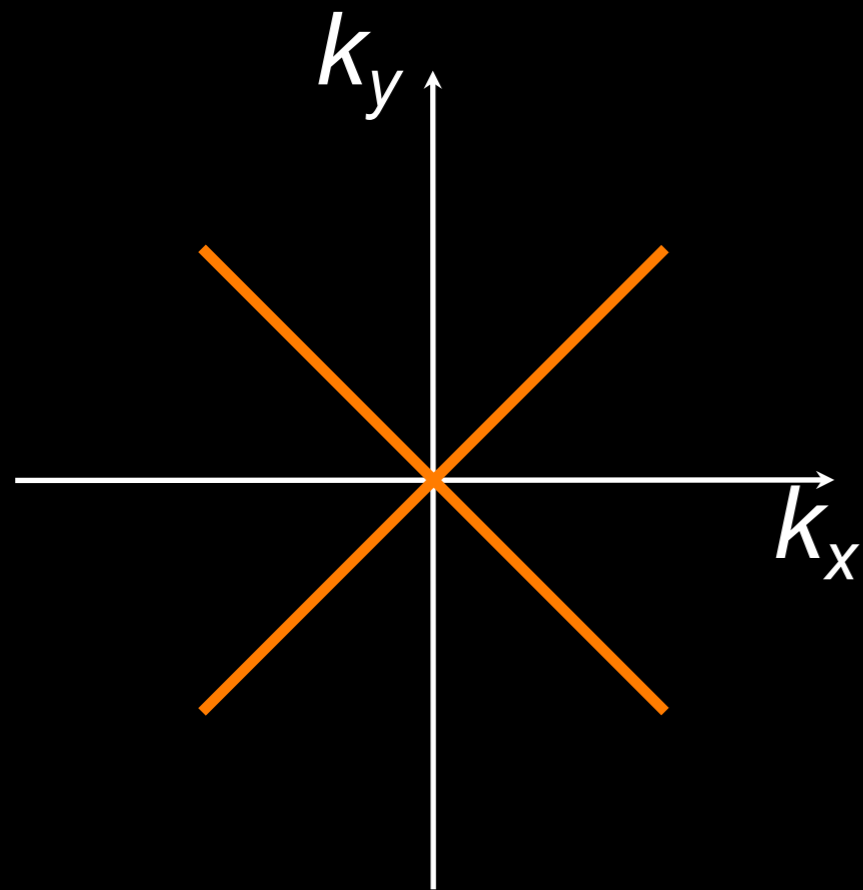
magnitude

alternatively, can use "gridding" reconstruction

Radial: Undersampling



Radial: Undersampling

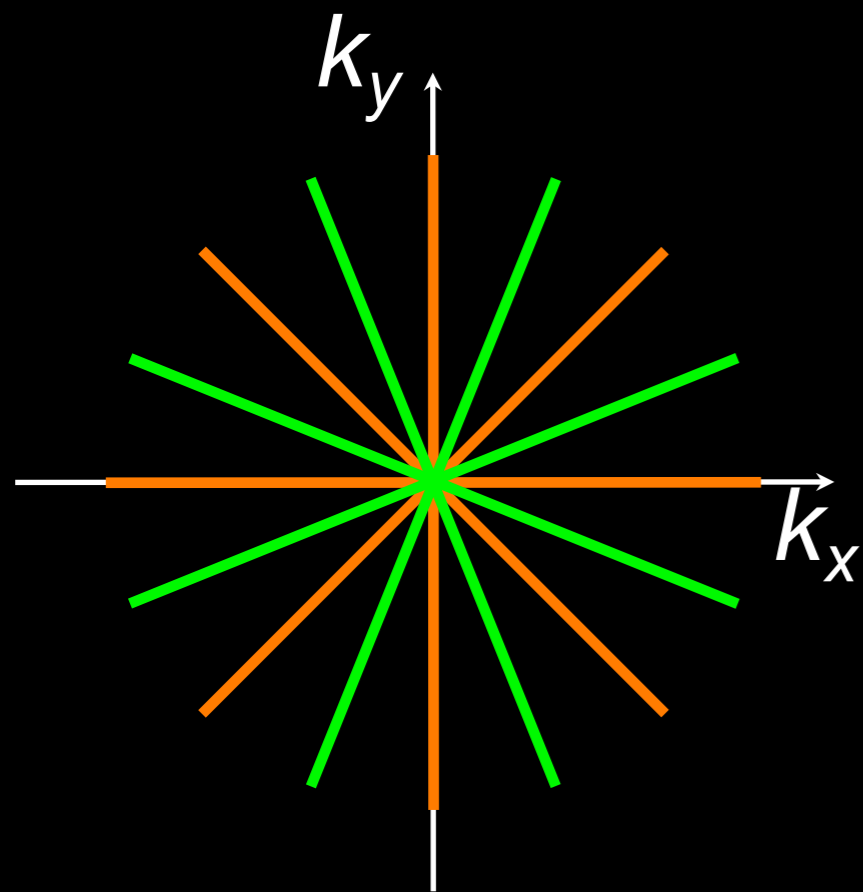


undersampled



streaking artifacts

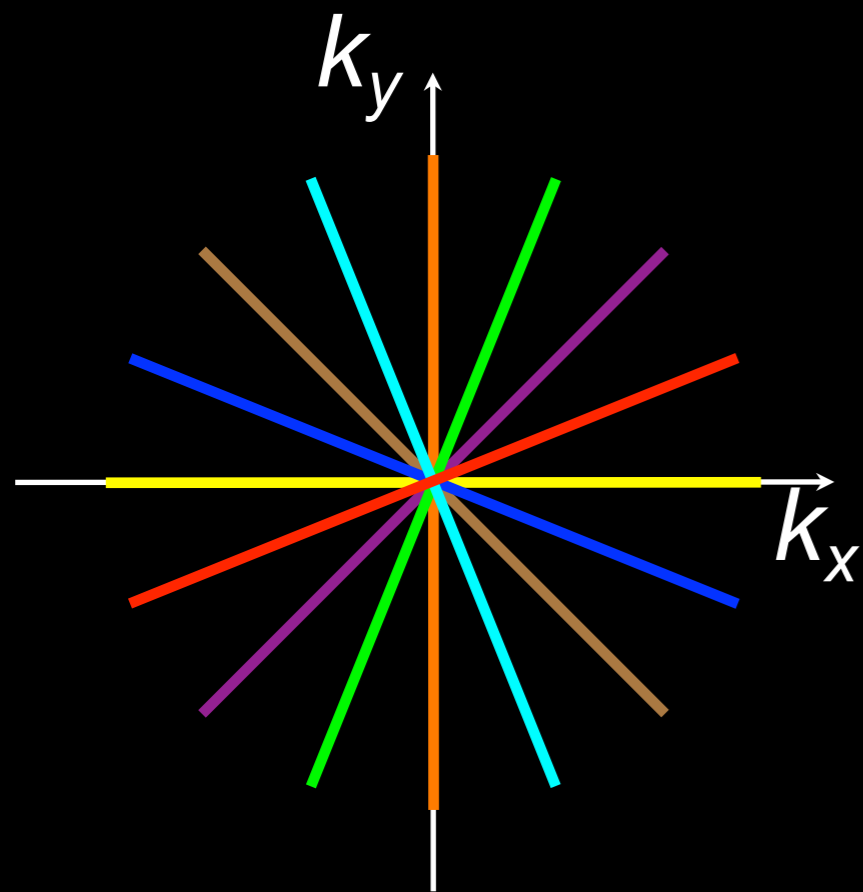
Radial: Acq Ordering



interleaved acq



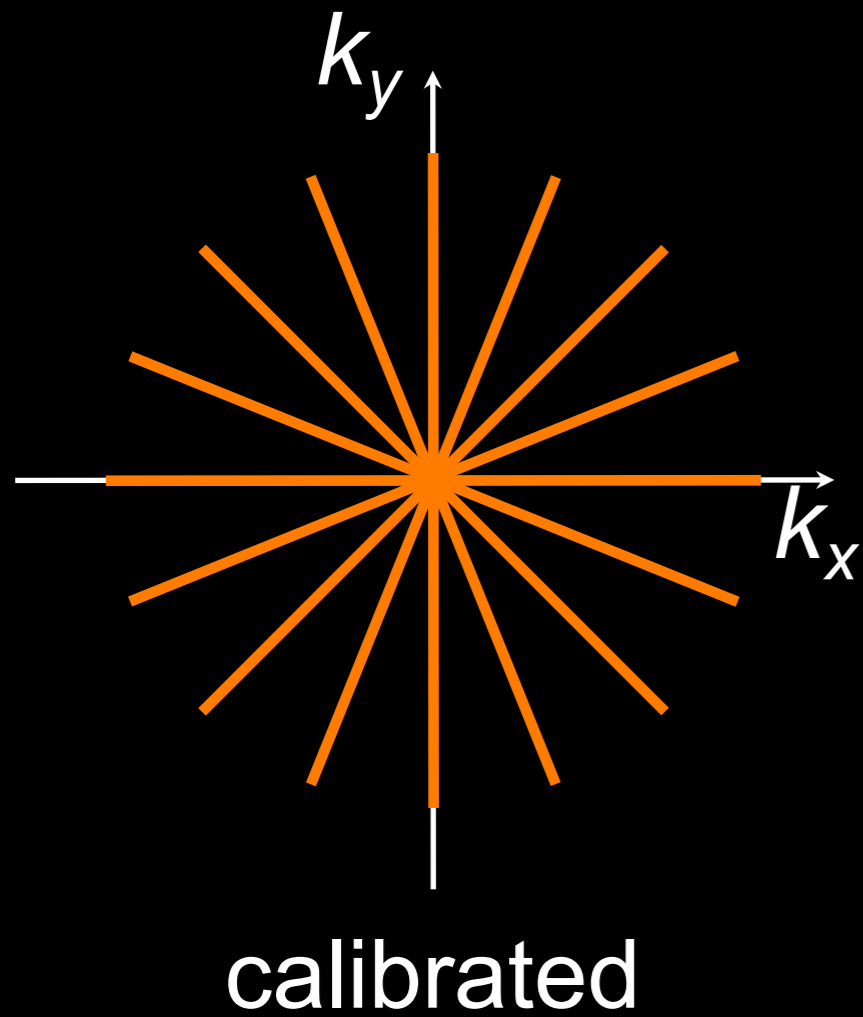
Radial: Acq Ordering



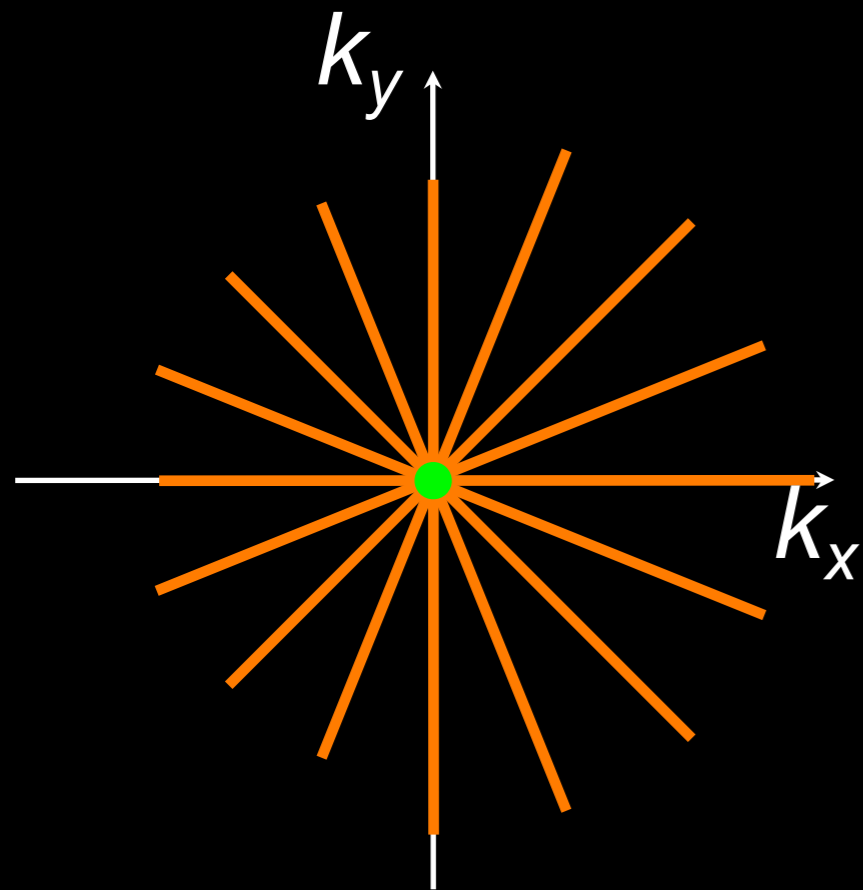
random acq



Radial: Gradient Delays

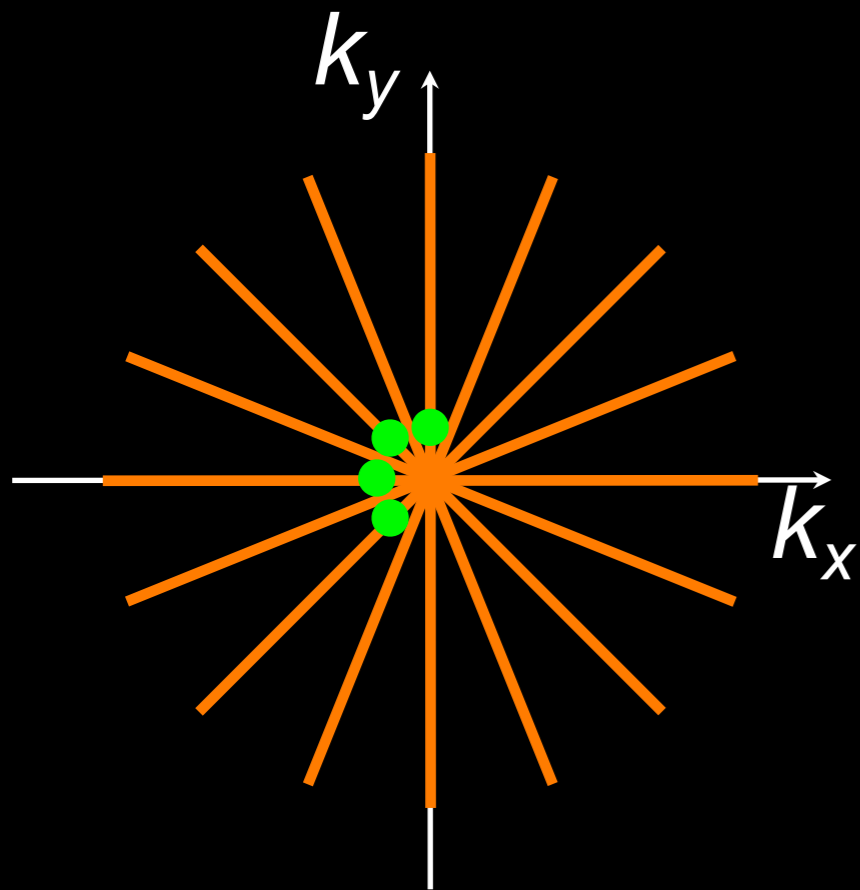


Radial: Gradient Delays

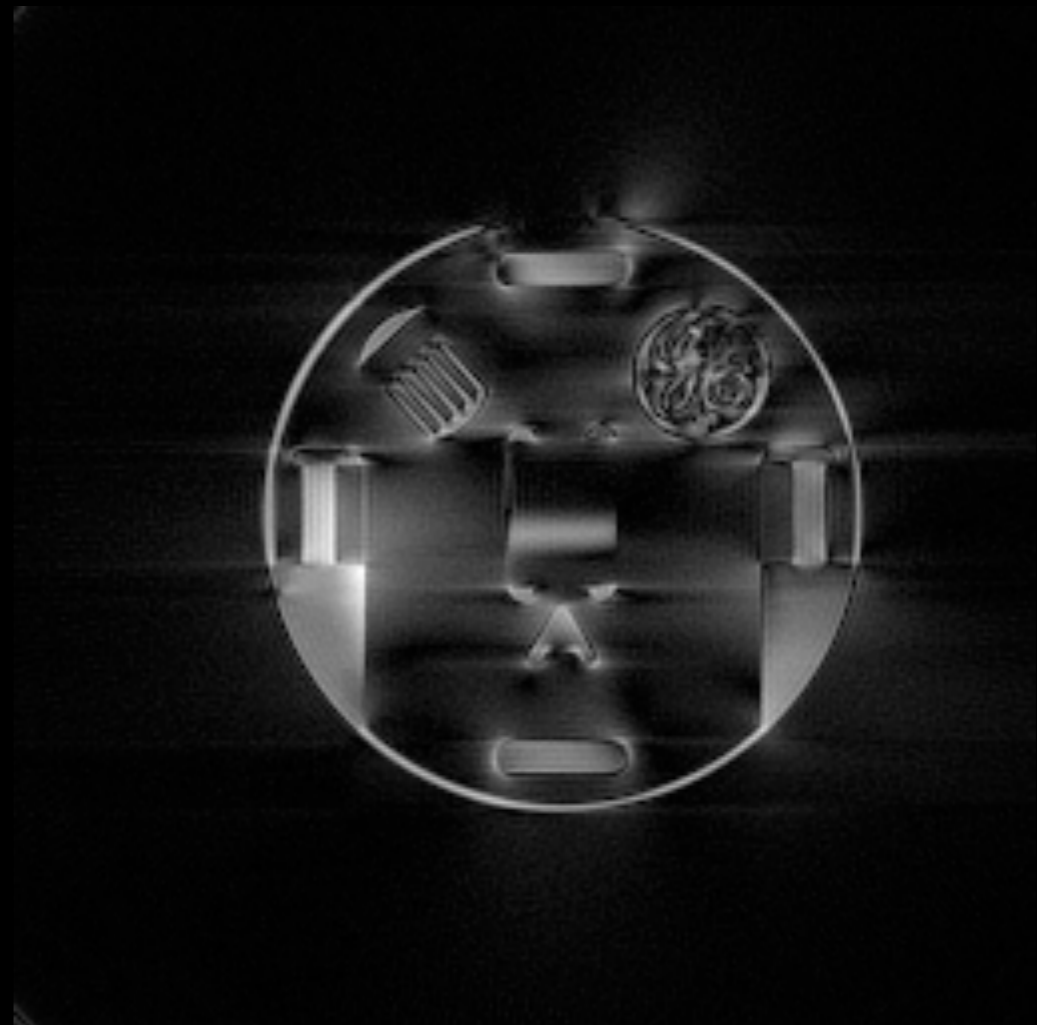


grad-acq delay

Radial: Gradient Delays

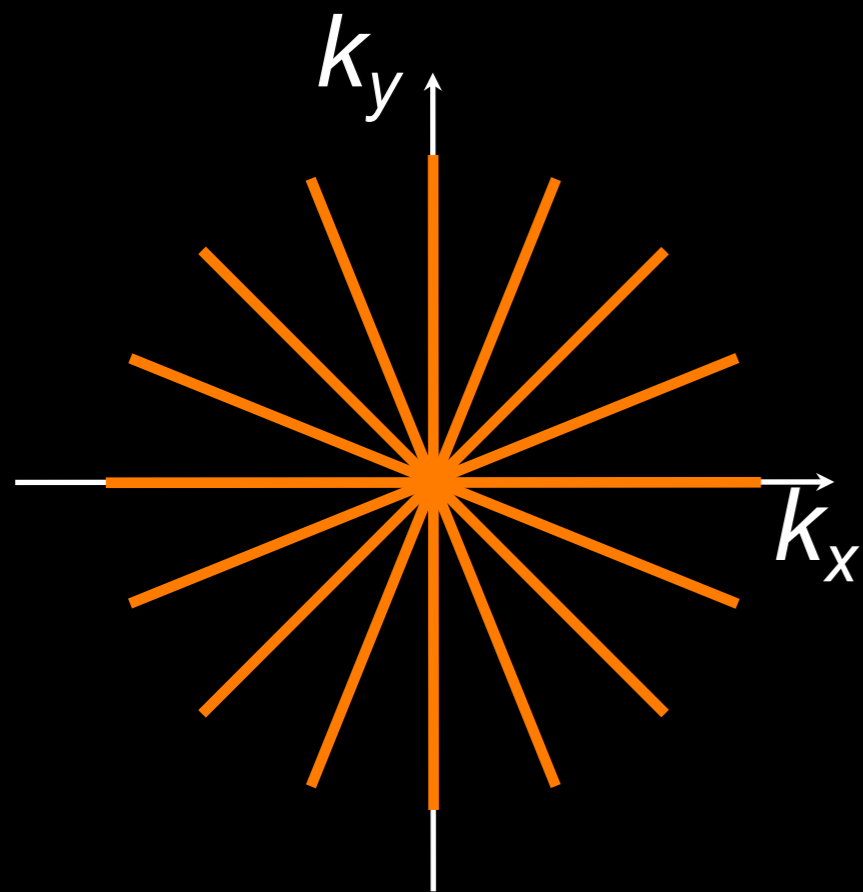


recon unaware of delays
mis-aligned DC



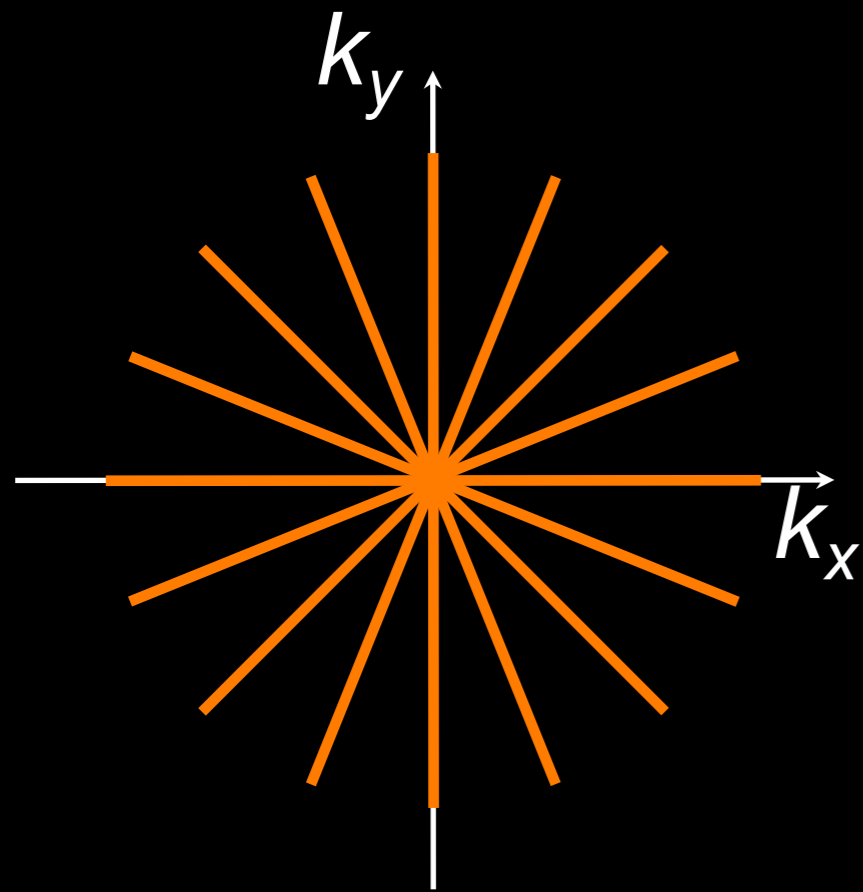
misalignment artifacts

Radial: Off-resonance Effects



on resonance

Radial: Off-resonance Effects



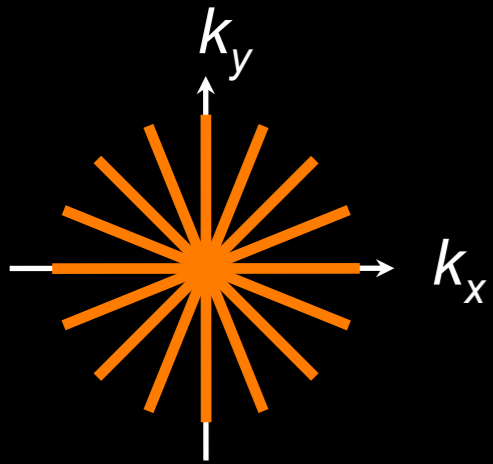
+200 Hz globally



off-res blurring

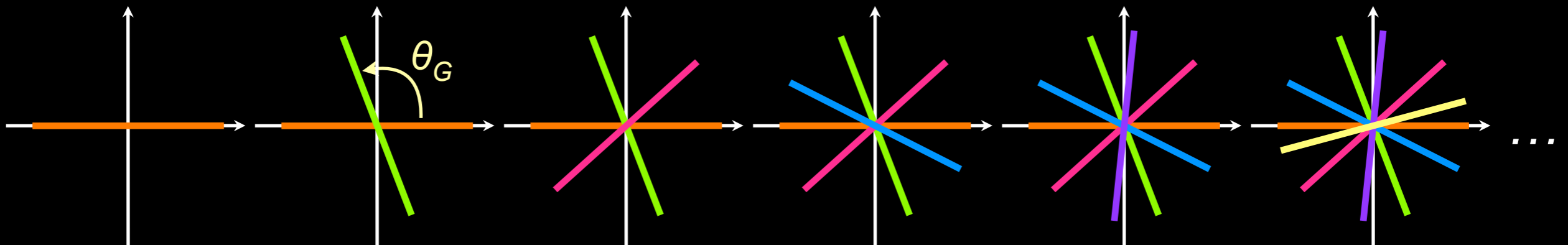
Radial: Real-time MRI

2D Radial MRI



- Robust to motion (oversample center of k-space)
- Can tolerate a lot of undersampling

Golden Angle Ordering



- Almost uniform sampling of $k-t$ space
- Flexible choice of temporal frame location and width

Radial: Real-time MRI

Radial FLASH

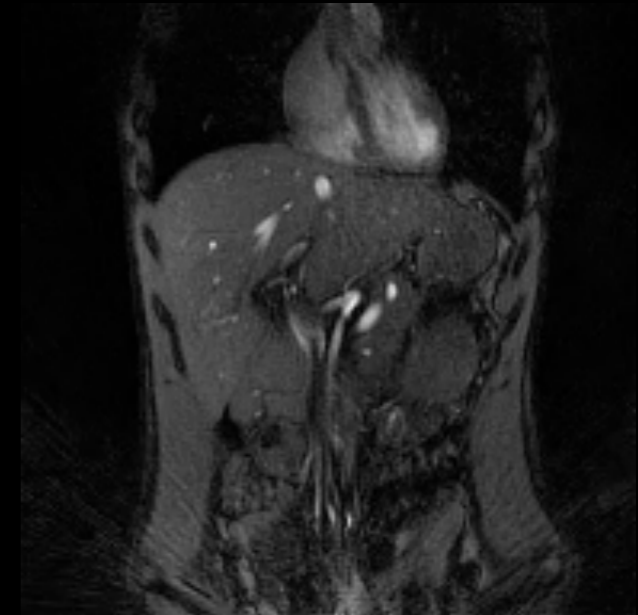
- golden-angle ordering
- 192 x 192 matrix
- TR = 3.1 ms
(1 spoke per TR)
- 3.0 T

Reconstruction

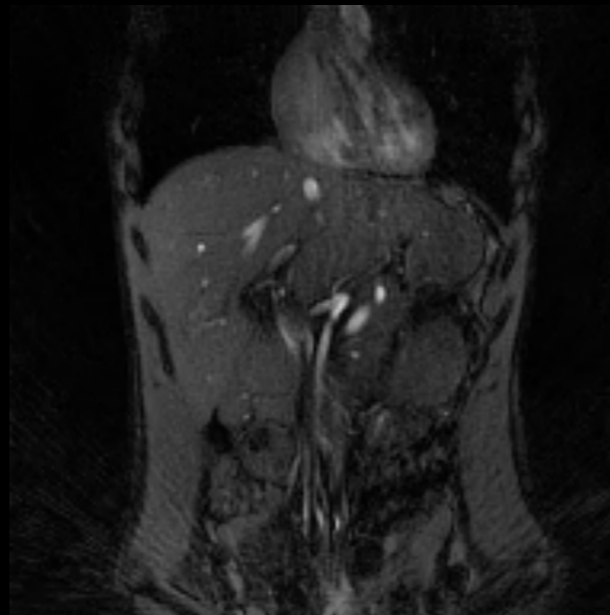
- sliding window of 20 TRs
(display at 16 frames/sec)
- **parallel imaging (SPIRiT)**
(300 spokes for Nyquist)



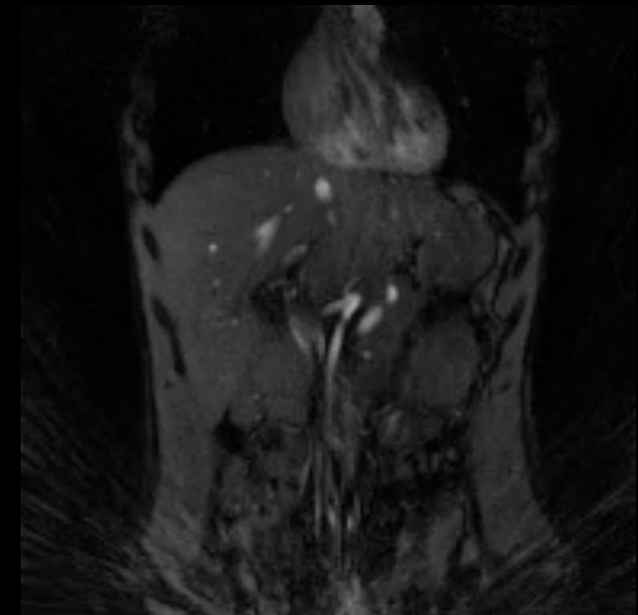
255 spokes/frame
(791 ms/frame)



144 spokes/frame
(446 ms/frame)



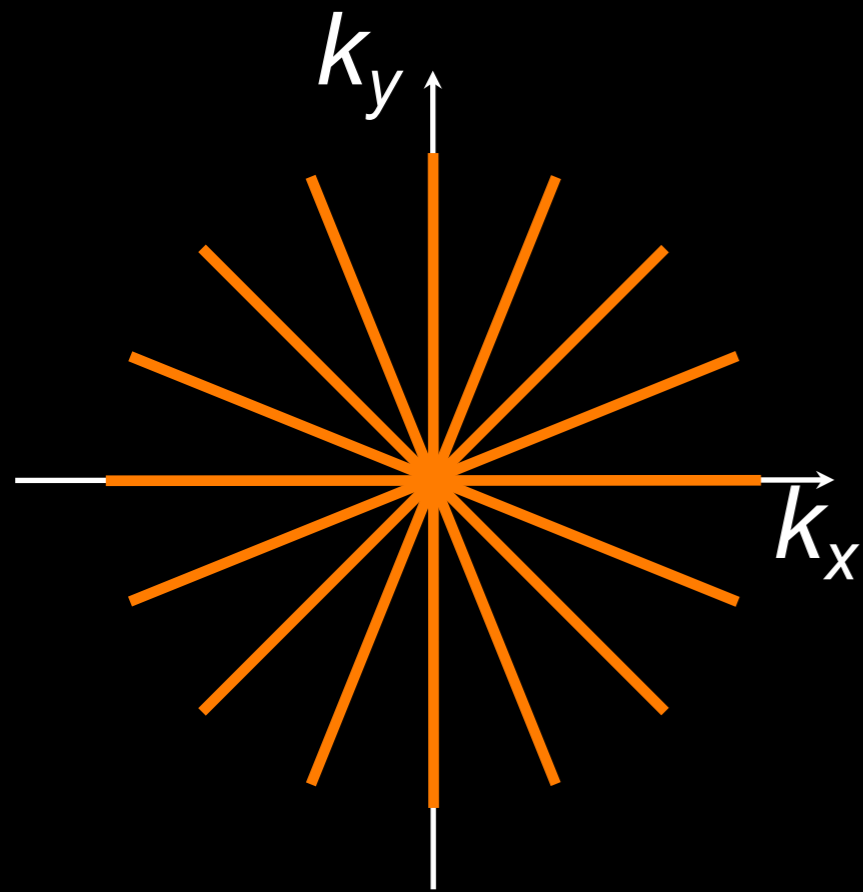
89 spokes/frame
(276 ms/frame)



55 spokes/frame
(171 ms/frame)

courtesy of Samantha Mikael

Radial: Pros and Cons



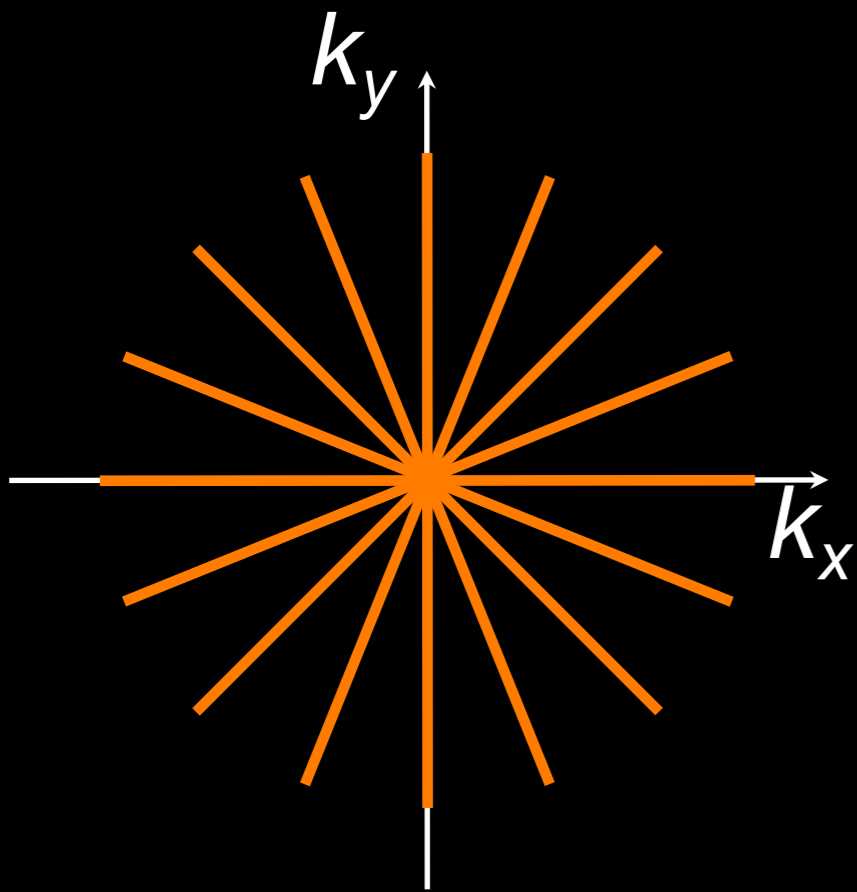
Pros

- Robust to motion (get DC every TR)
- Can tolerate a lot of undersampling
- Half-spoke PR has very short TE

Cons

- SNR penalty (non-uniform density)
- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

Radial: Extensions



3D stack of stars

3D koosh ball

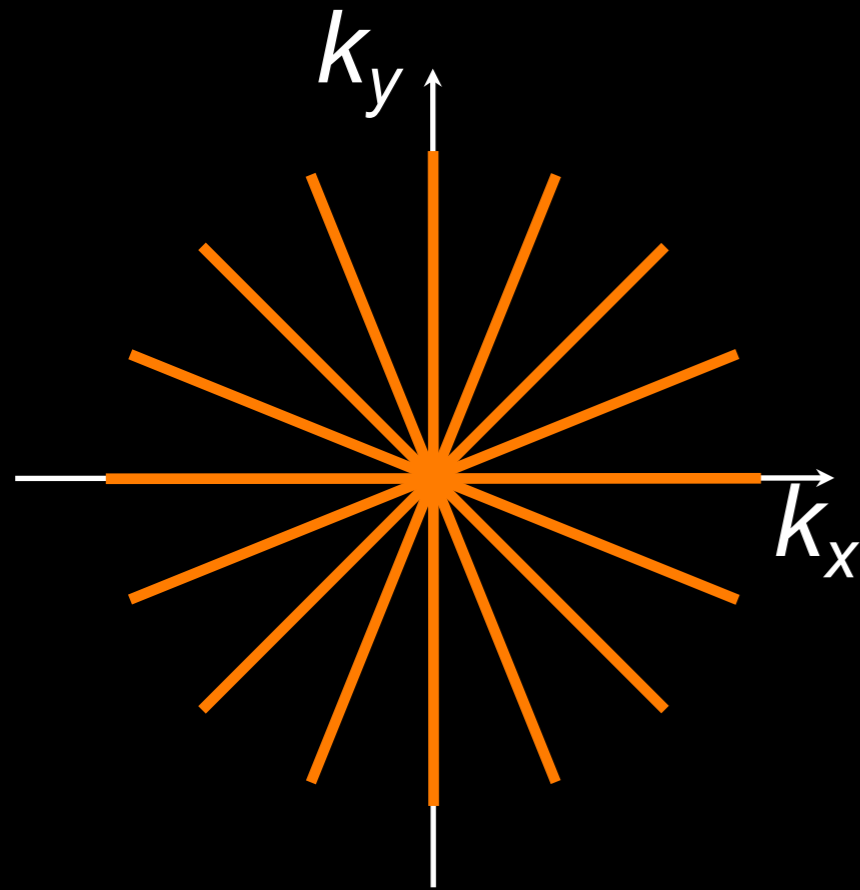
Multiple spokes per TR

Golden angle ordering

Parallel imaging

Partial Fourier

Radial: Applications



Fast imaging

- Cardiac MRI

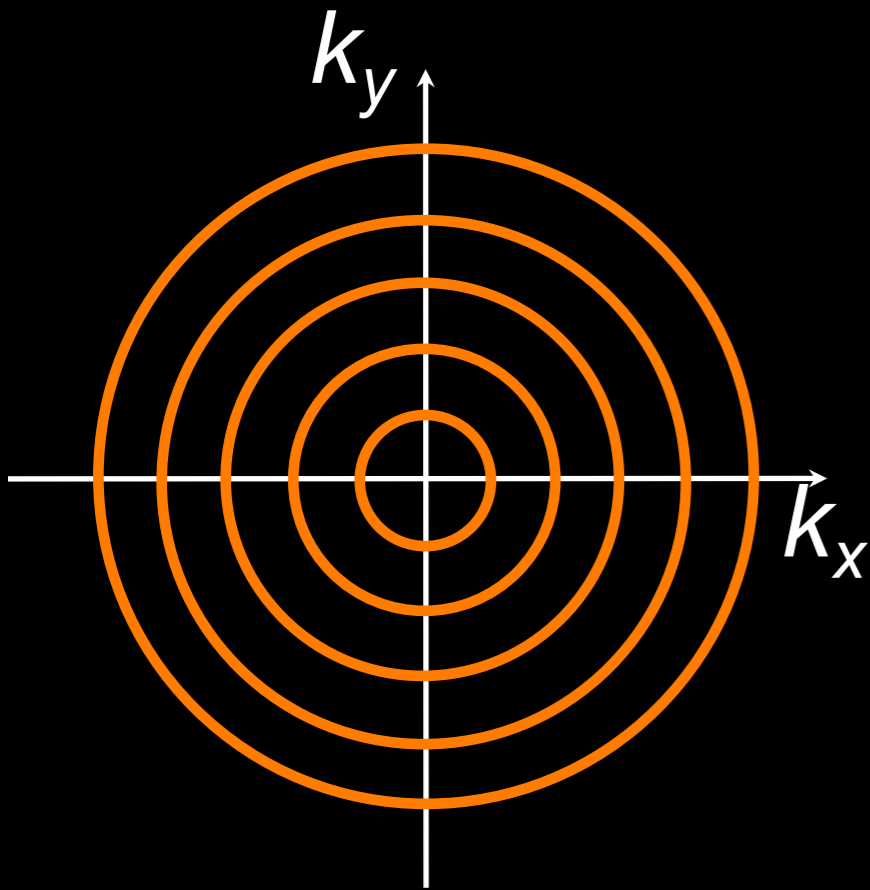
Improve motion robustness

- Cardiac MRI
- Abdominal MRI

Ultra-short TE (UTE) imaging

- Musculoskeletal MRI
- Lung MRI

Concentric Rings

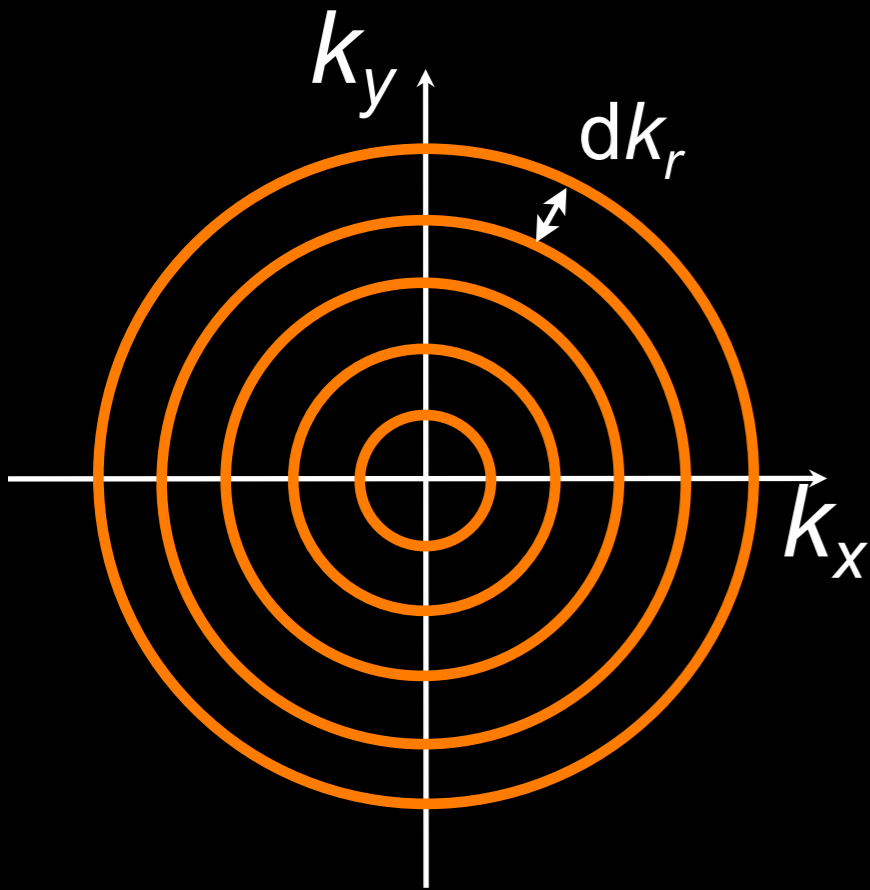


Non-rectilinear sampling!

Samples k-space on a polar grid

- “dual” of radial sampling
- shares some properties of 2DPR
- exhibits distinct characteristics

Rings: Sampling Requirements



N concentric rings

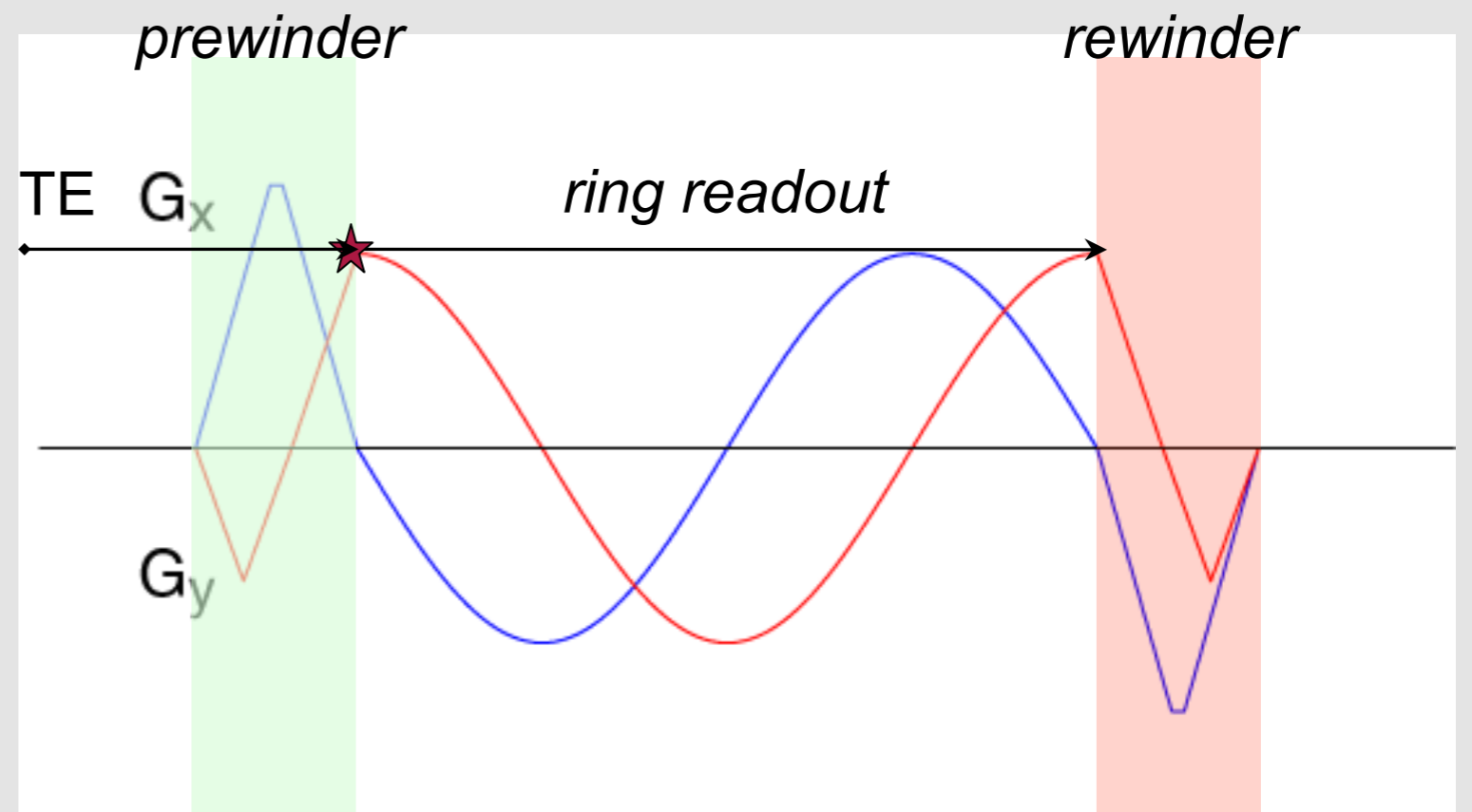
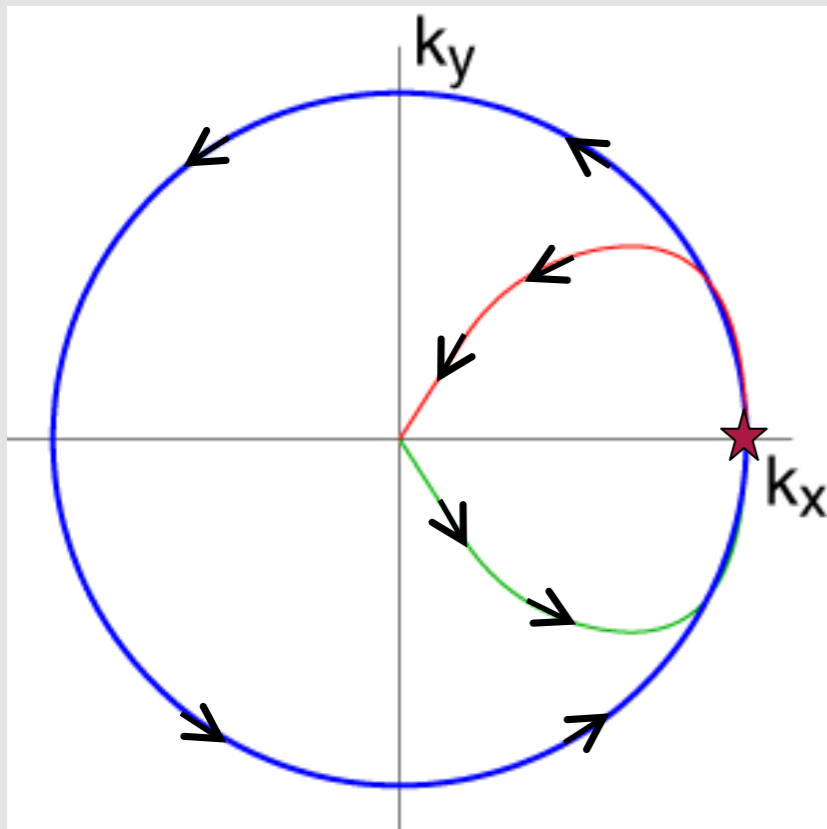
uniform spacing of dk_r

$$\frac{1}{\text{FOV}} = dk_r$$

$$k_{r,max} = (N - 1) \cdot dk_r$$

Subject to hardware limits

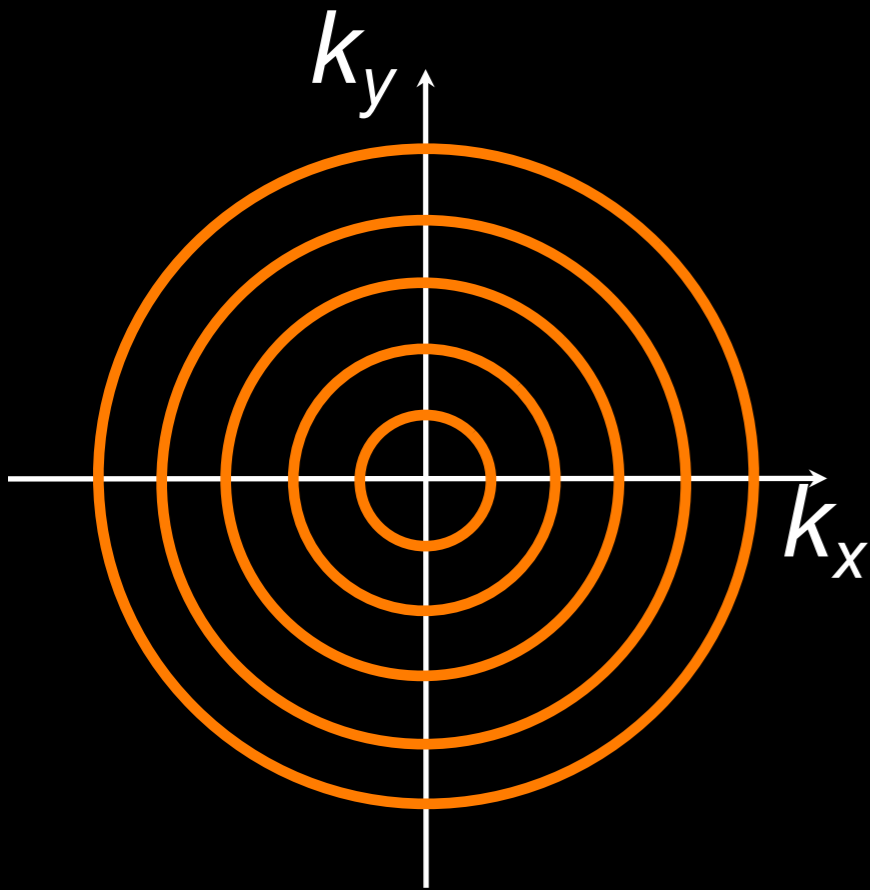
Rings: Gradient Design



Scale down gradients for outermost ring

- Sampling density identical to 2DPR
- Robust to gradient delays & timing errors

Rings: Scan Time



For an $M \times M$ image,

need $N = M/2$ rings

Scan time = $(M/2) \times TR_{\text{ring}}$

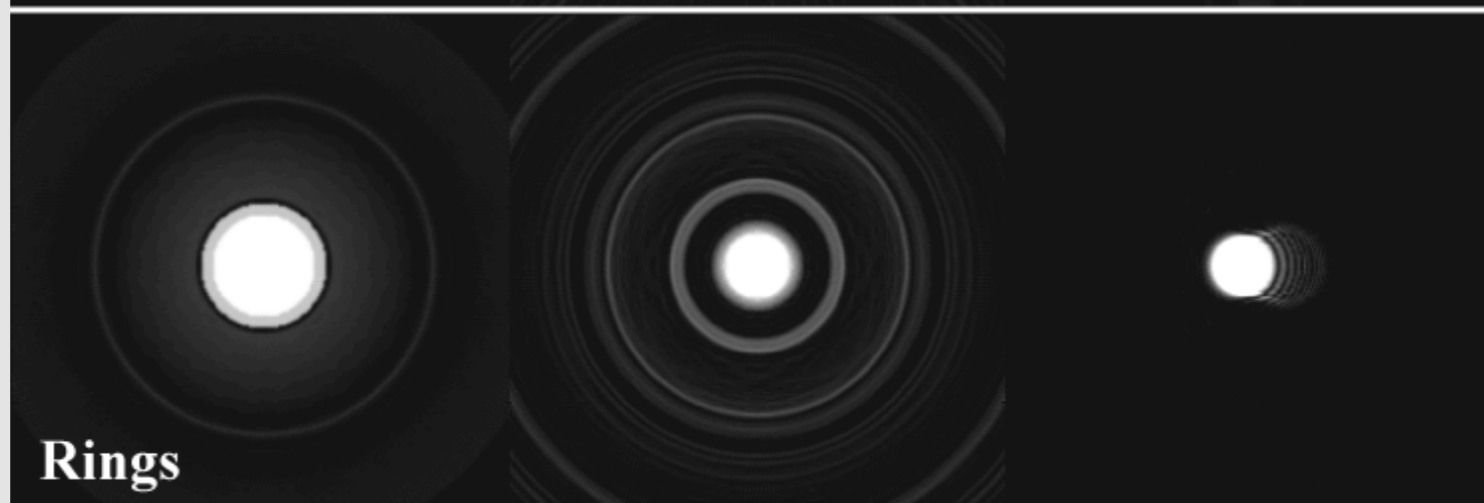
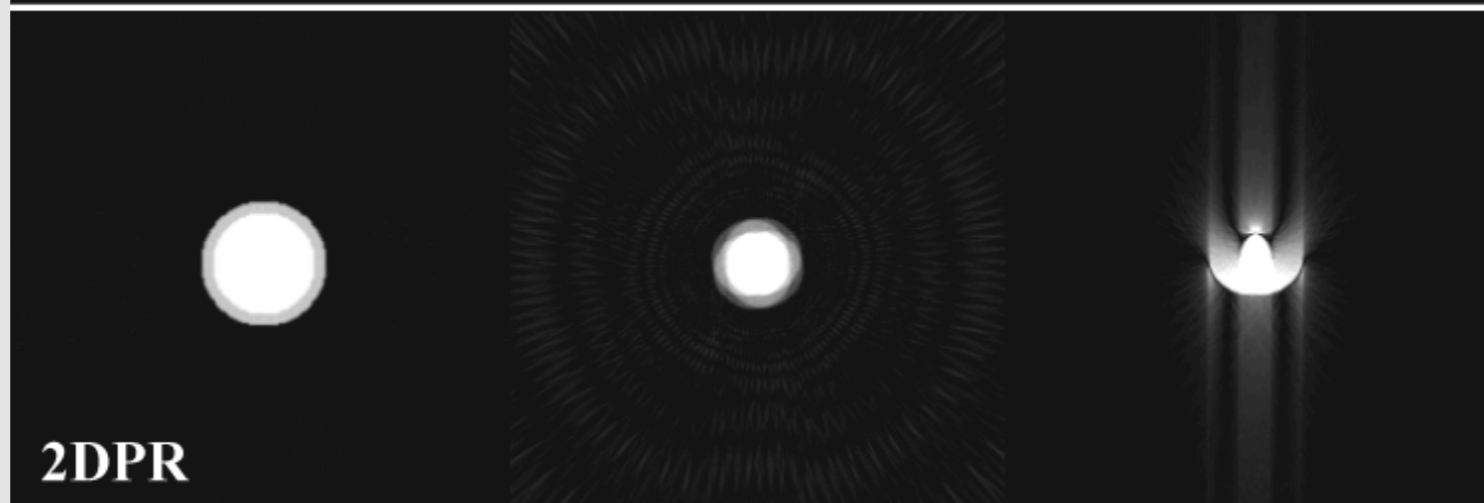
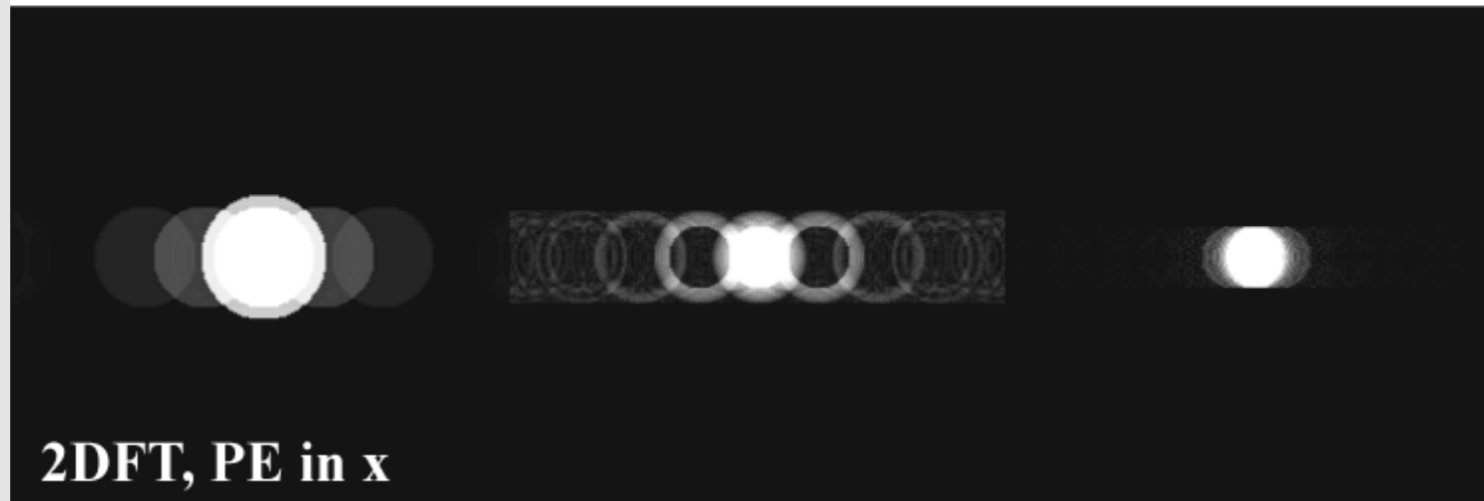
Compare with 2DFT:

Scan time = $M \times TR_{\text{line}}$

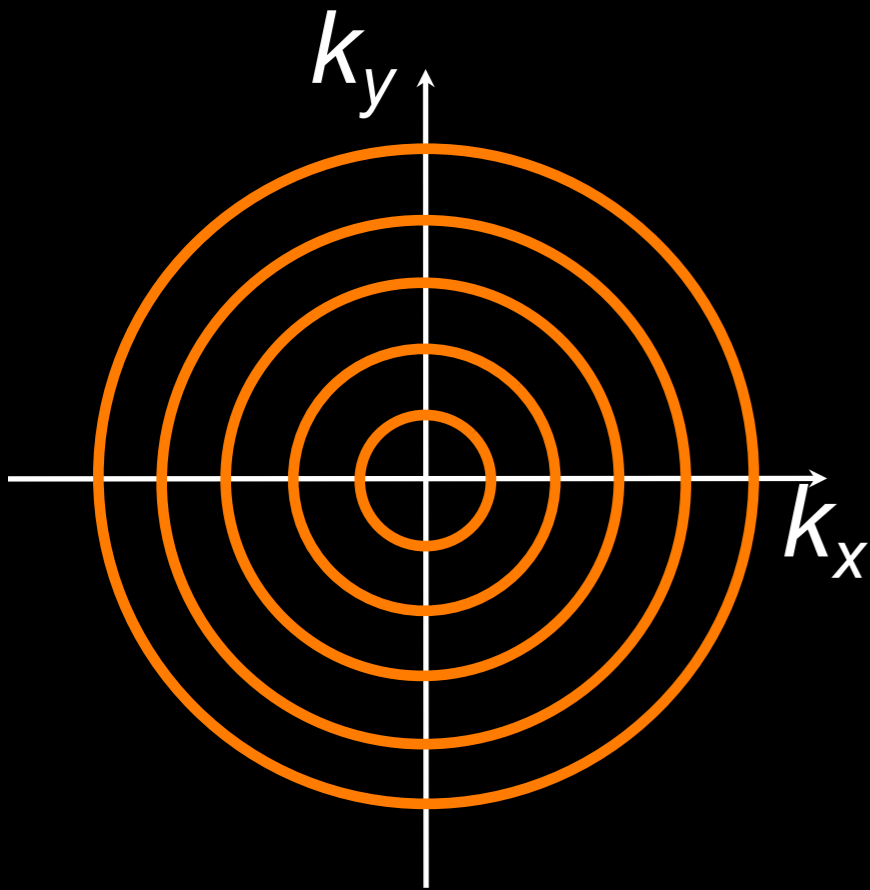
Rings offer $\sim 2x$ acceleration

Rings: Motion and Flow

a. Pulsatile flow b. Contraction/Dilation c. Linear drift



Rings: Image Reconstruction



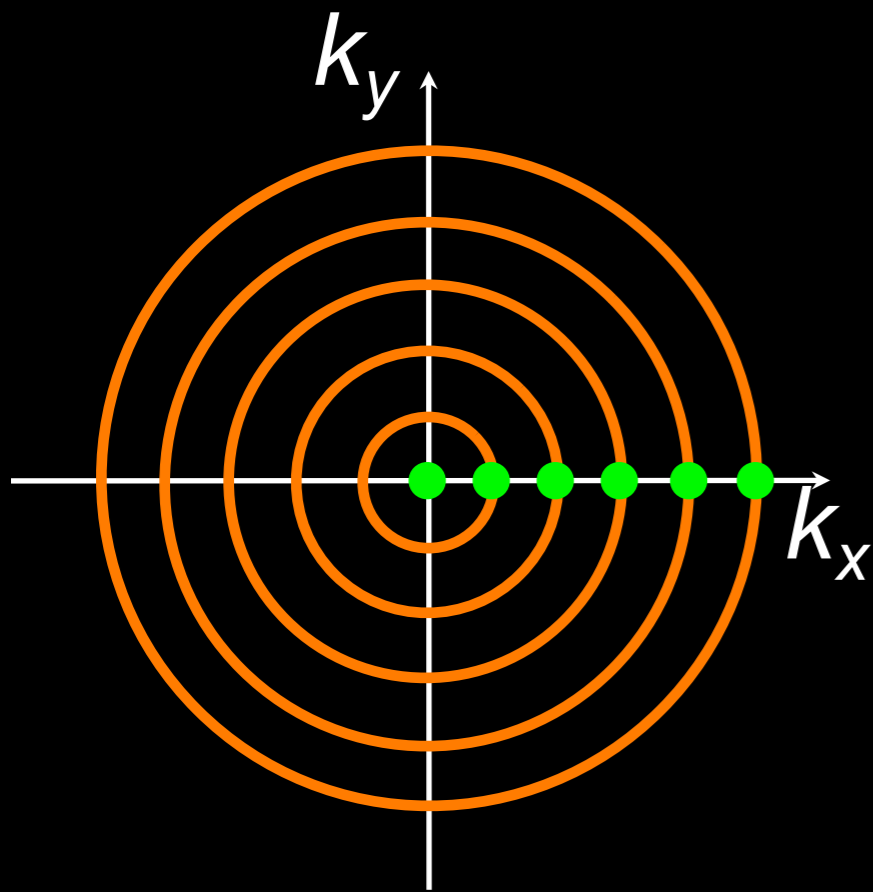
Reformat into spokes

- filtered back projection

Resample onto Cartesian grid

- “gridding” reconstruction

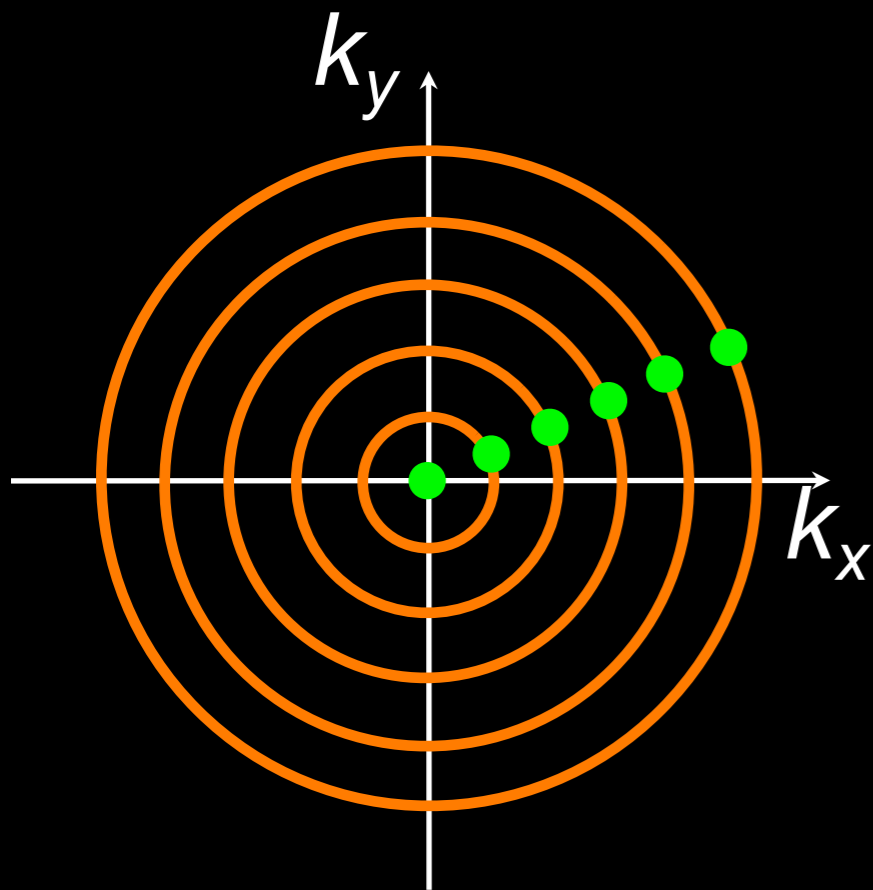
Rings: Gradient Delays



calibrated



Rings: Gradient Delays

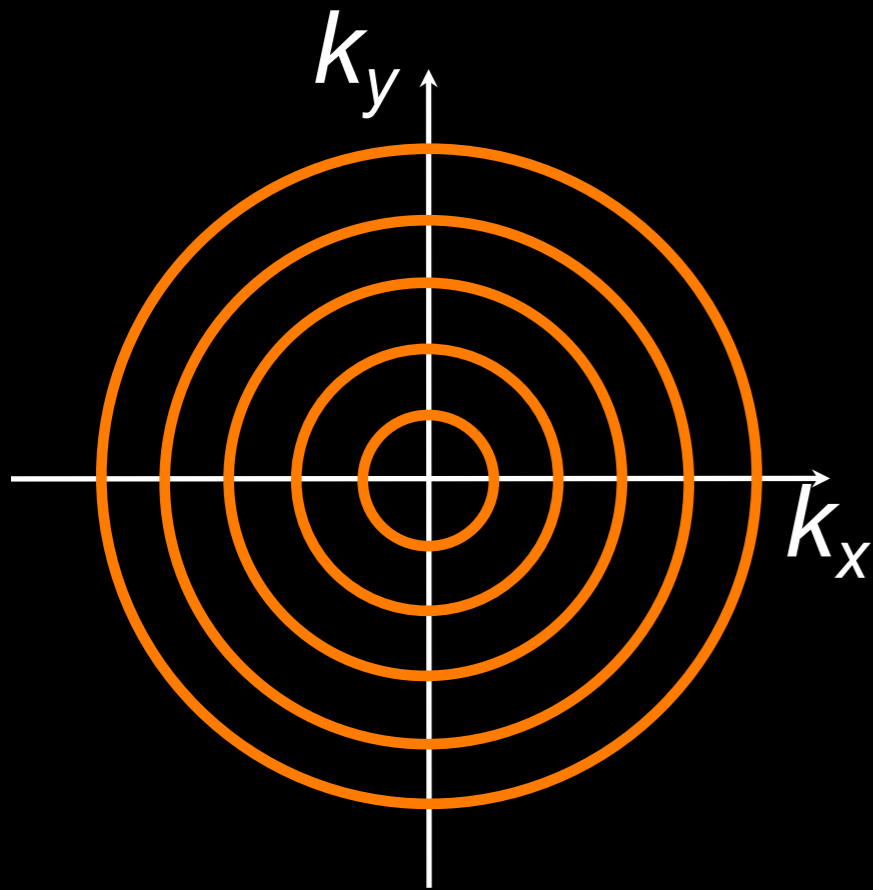


grad-acq delay

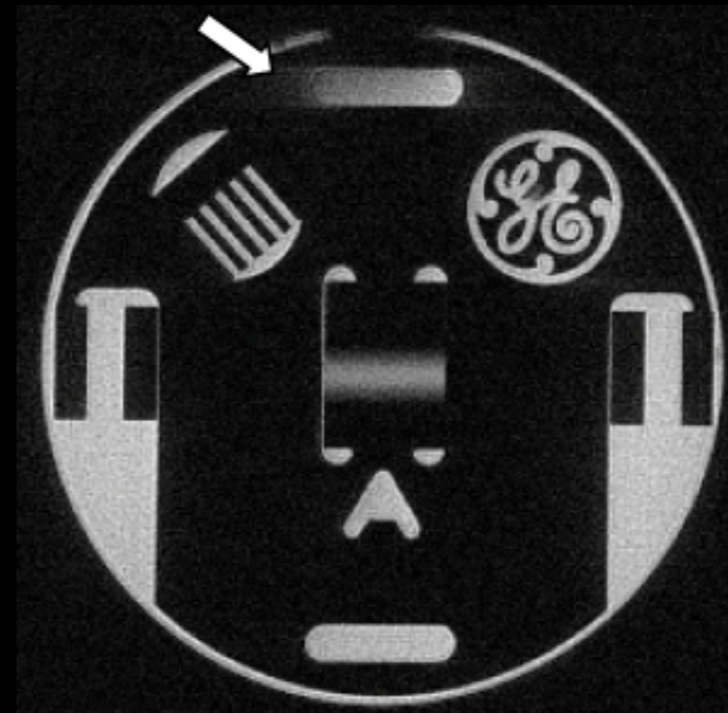


in-plane rotation

Rings: Off-resonance Effects

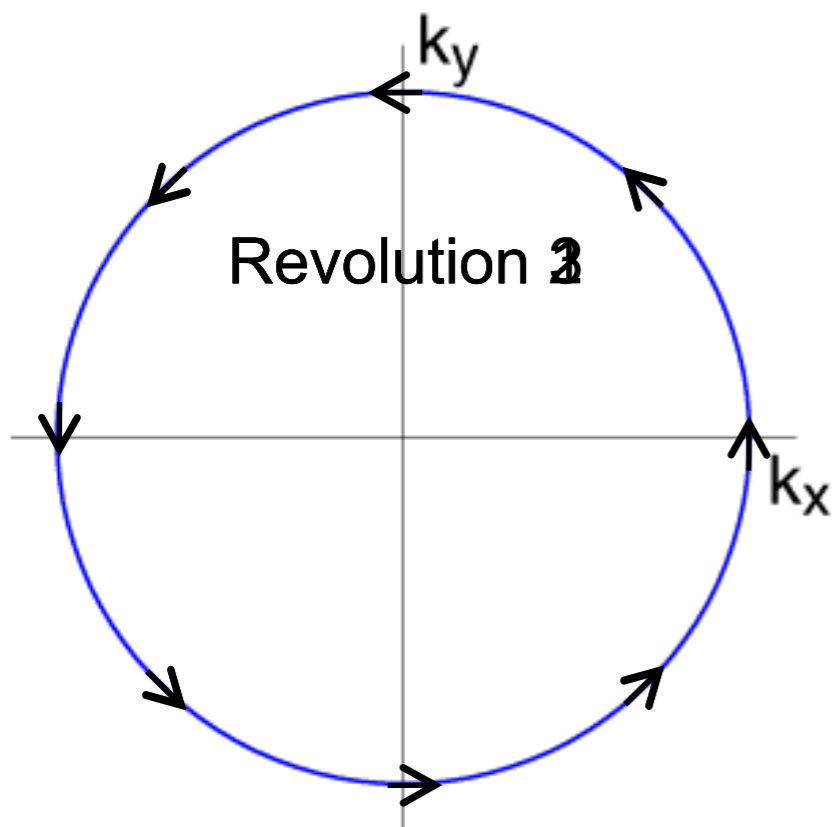


w/spatially
varying off-res

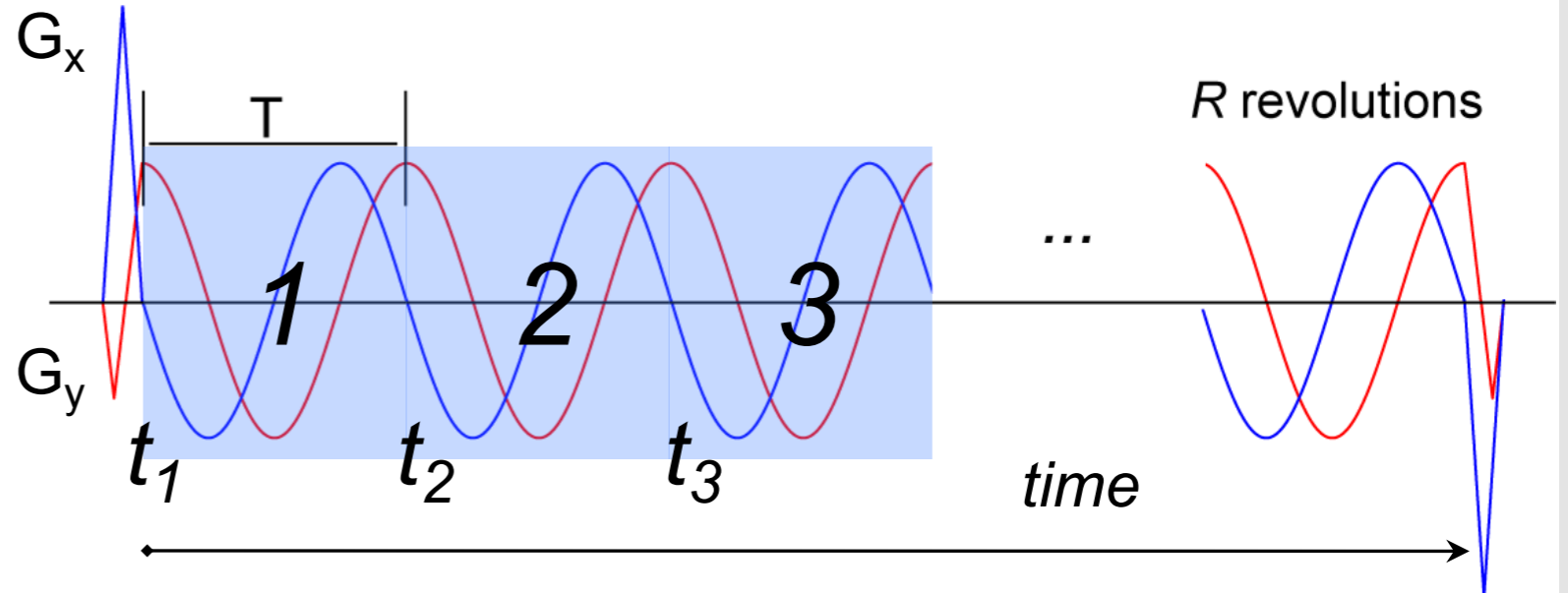


off-res blurring

Rings: Resolving Off-Res Effects



Retracing gradients for spectral encoding



Encodes (k_x , k_y , *time*) simultaneously

- Resolve off-resonance effects
- "Spectral" encoding

Rings: Resolving Off-Res Effects

Concentric Rings with 2 Revolutions / TR



Regular recon

Field map

ORC image

Rings: Resolving Off-Res Effects

Concentric Rings with 3 Revolutions / TR



Field map



Water image

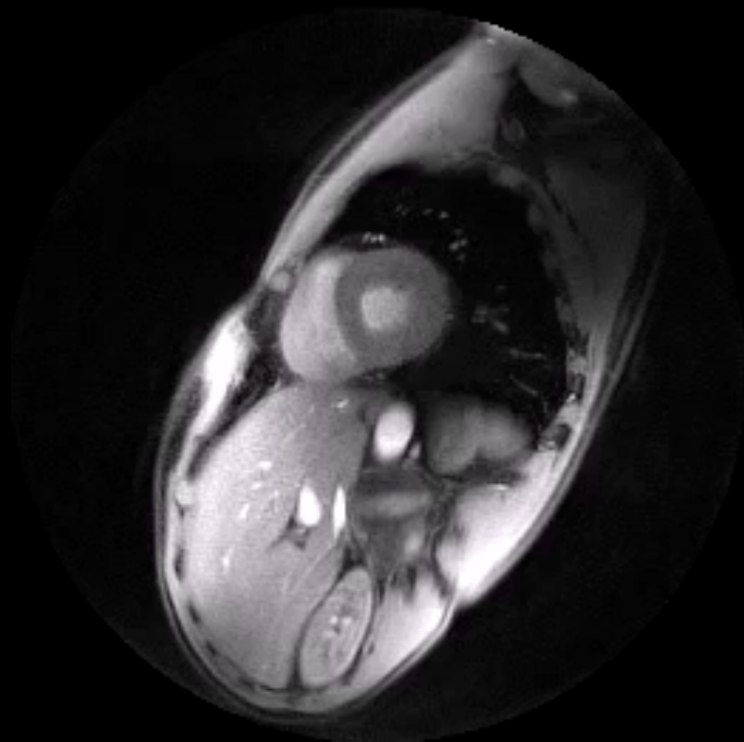


Fat image

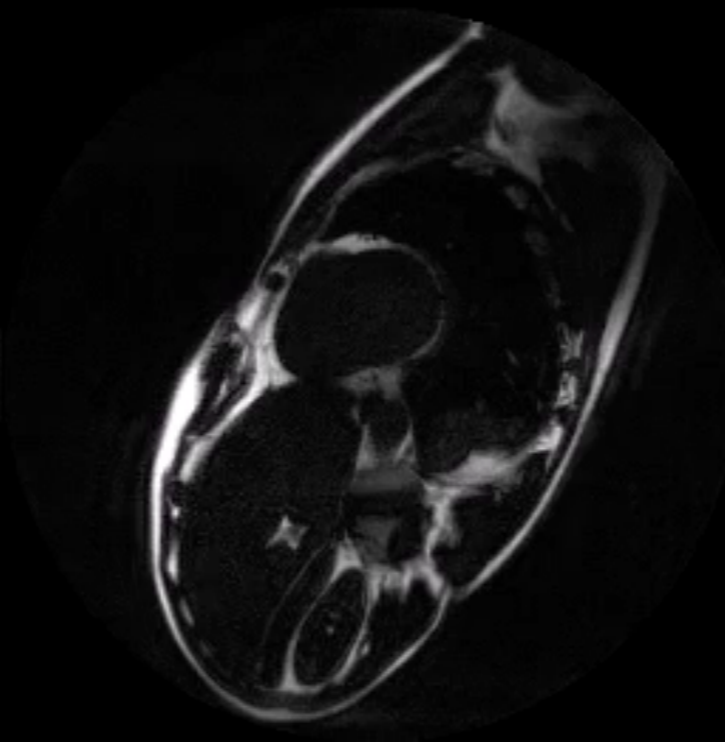
Rings: Resolving Off-Res Effects

1.5 T, 2D GRE, Cardiac F/W Cine

13-HB BH scan (with add'l 3-fold *k-t* BLAST acceleration)



Water



Fat

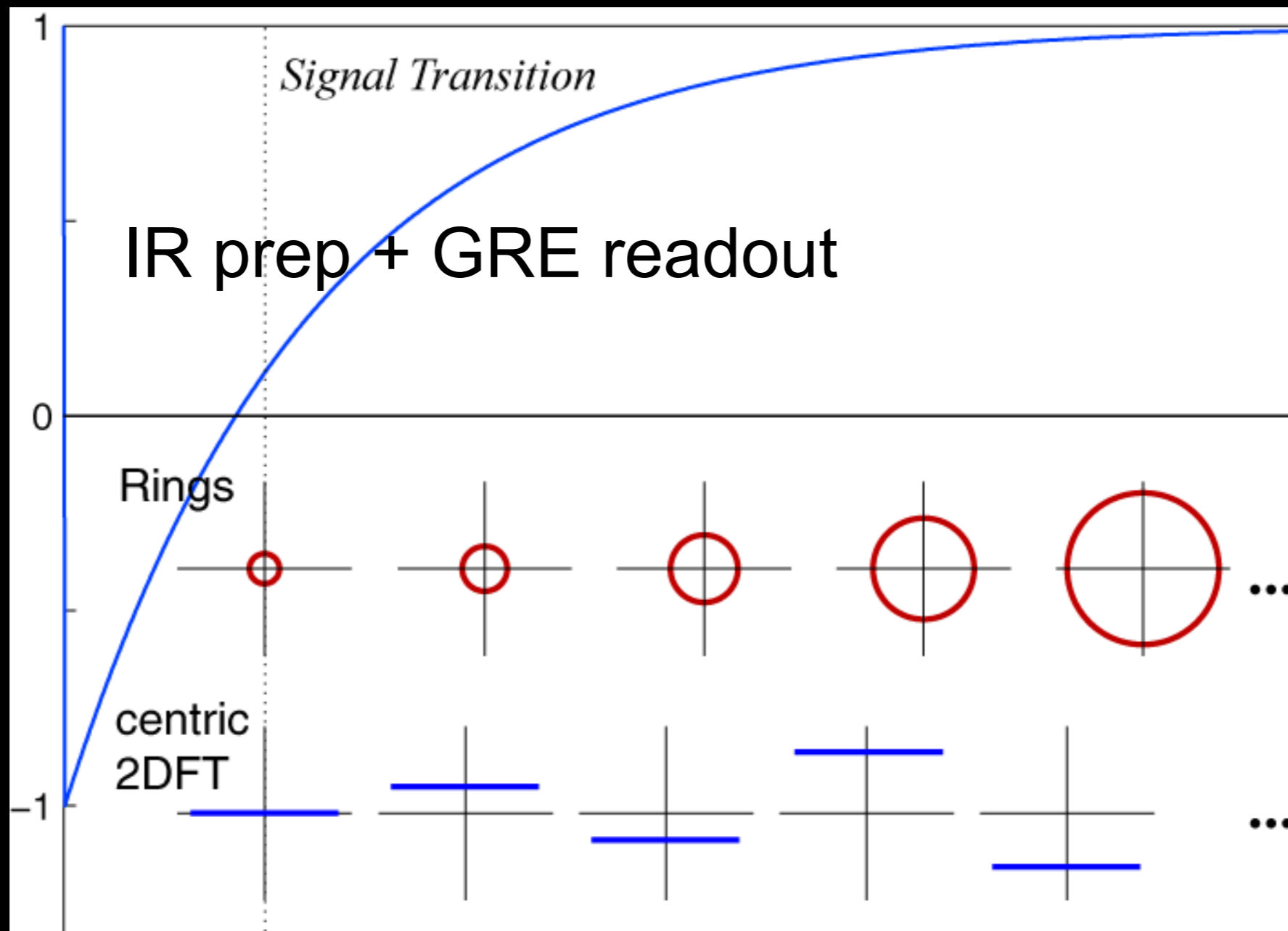


Combined

Rings: Magnetization-Prepared MRI

Inherent 2D centric ordering

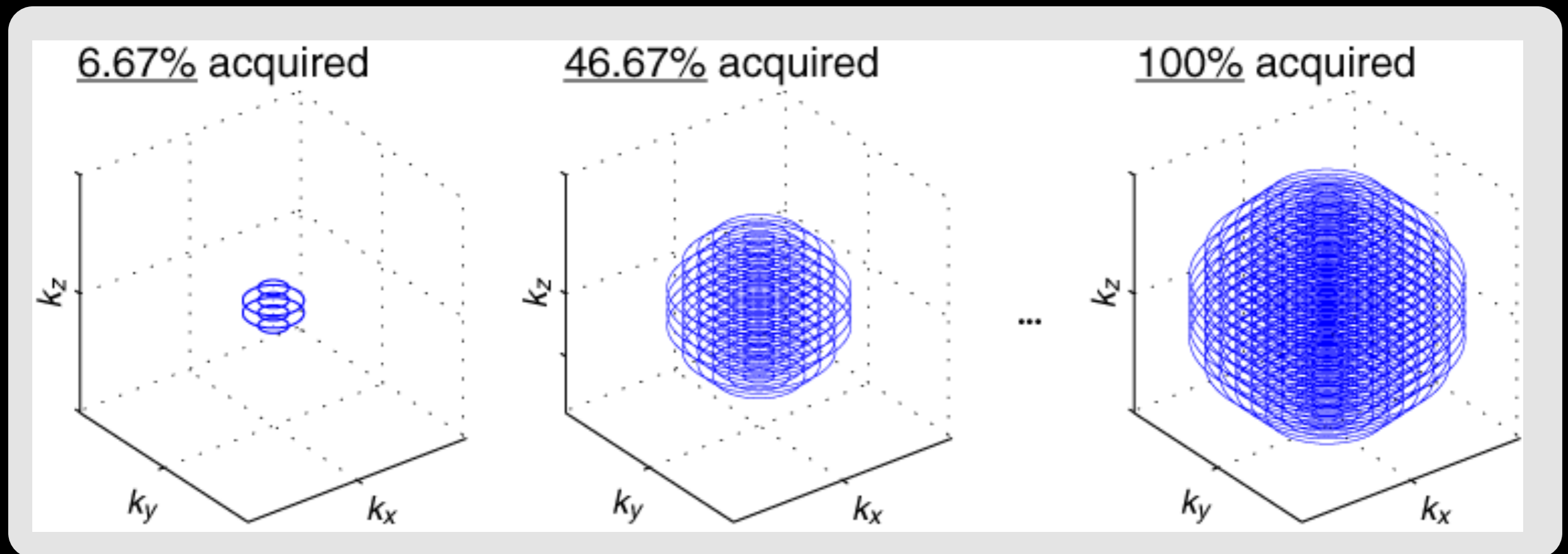
- improved mag-prep contrast and k-space weighting



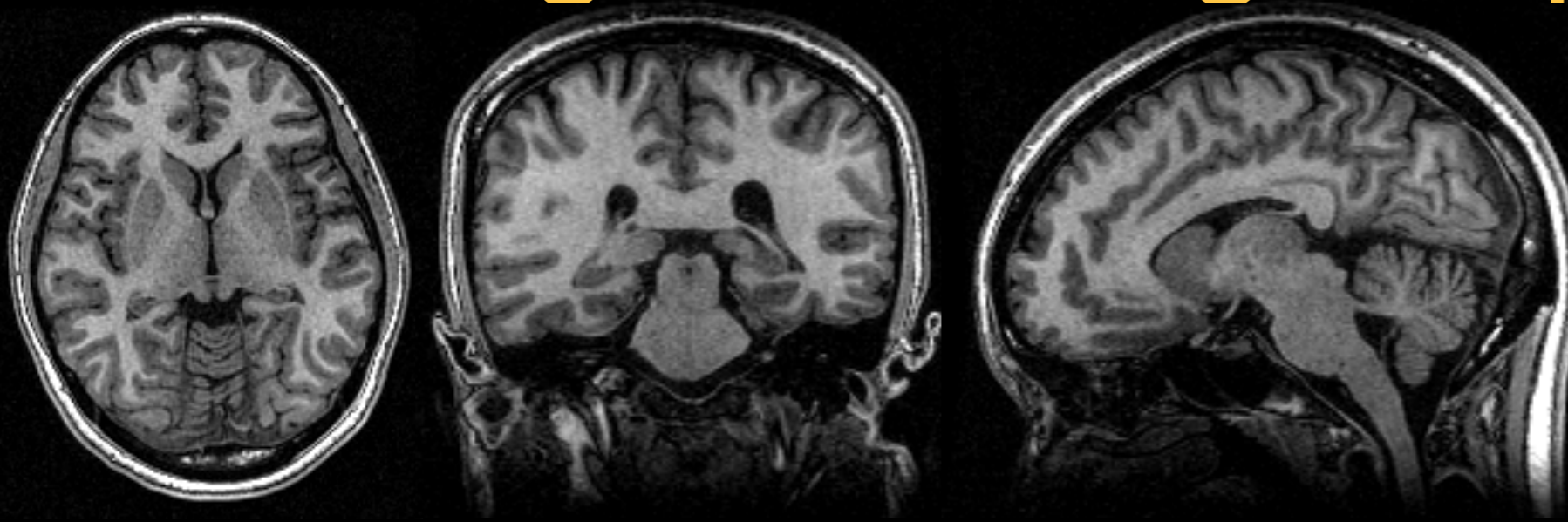
Rings: 3D Mag-Prep MRI

Fully 3D centric ordering

- improved mag-prep contrast and k-space weighting
- spherical k-space coverage saves time



Rings: 3D Mag-Prep MRI



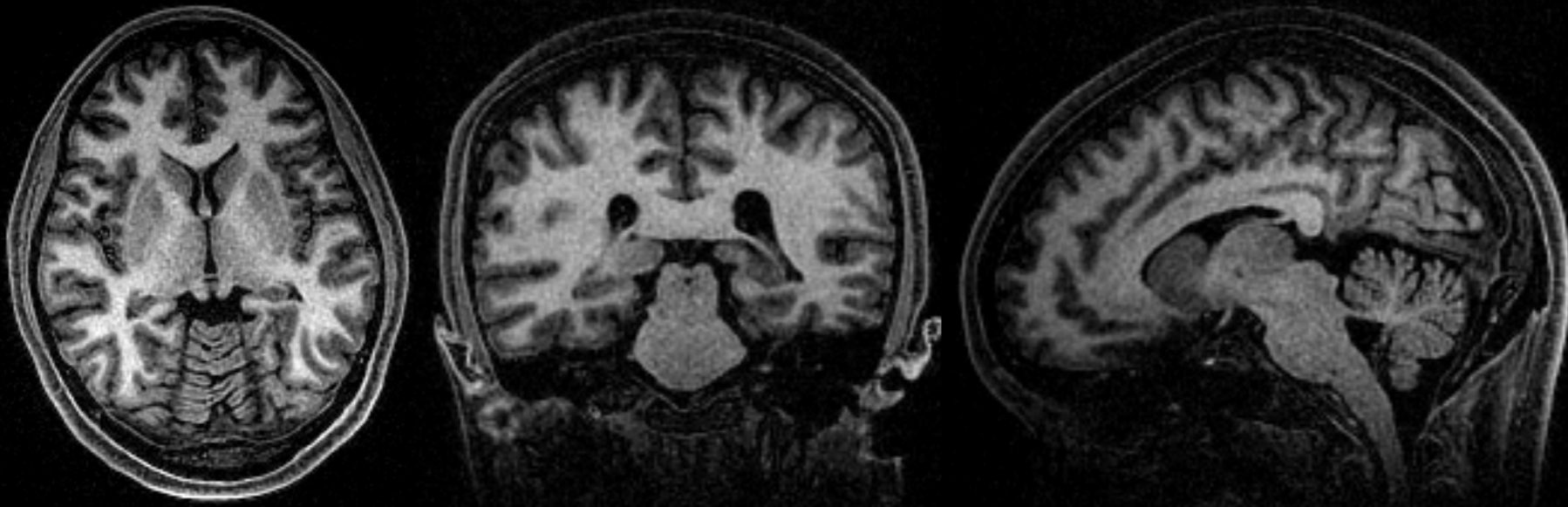
Product 3DFT

TI/TD = 600/----- ms

9 min 34 s

SNR_{WM} 24.07

CNR_{GW} 8.86



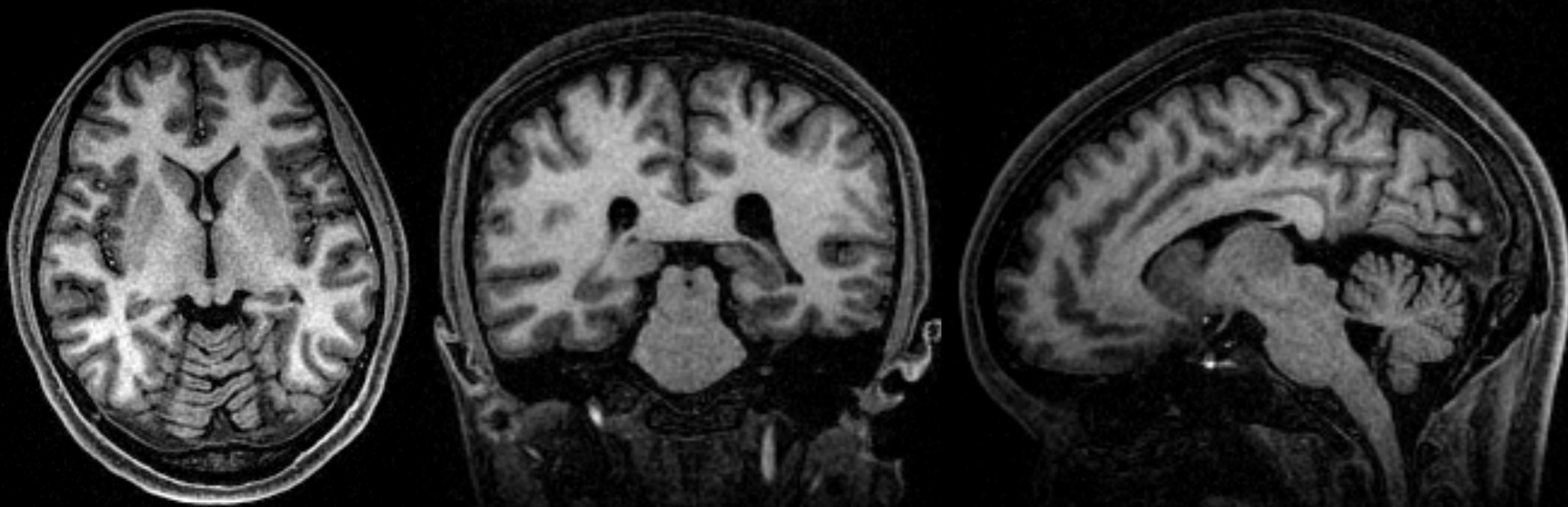
3D Rings, Protocol A

TI/TD = 600/----- ms

4 min 52 s

SNR_{WM} 25.78

CNR_{GW} 12.05



3D Rings, Protocol B

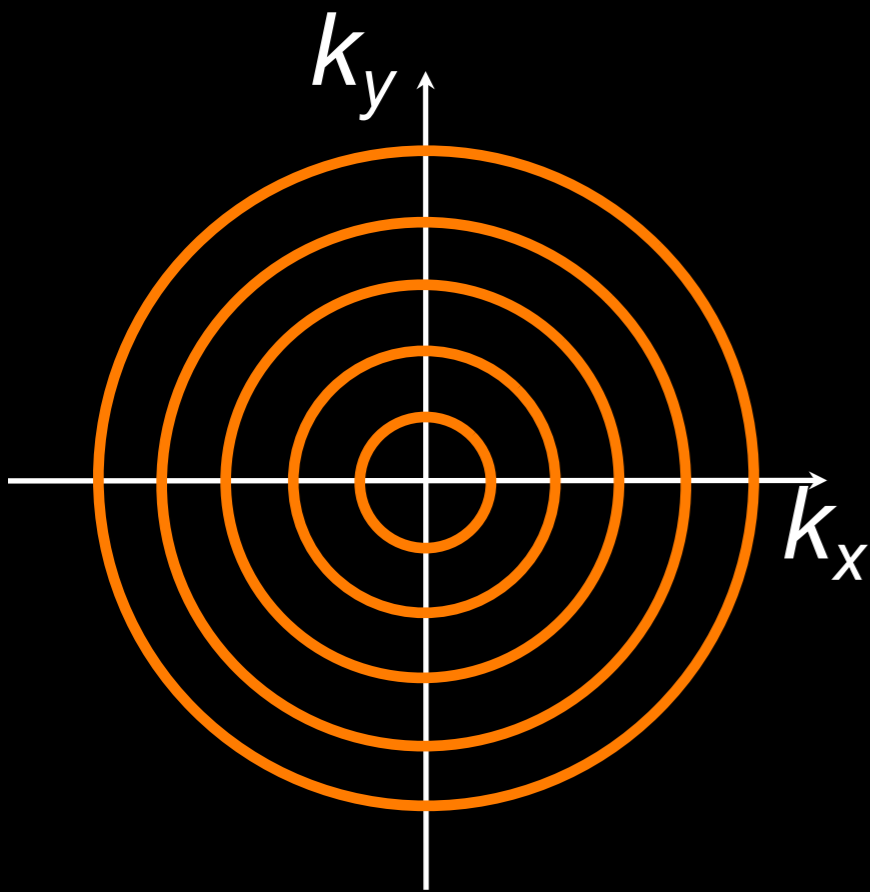
TI/TD = 900/600 ms

7 min 00 s

SNR_{WM} 33.46

CNR_{GW} 16.19

Rings: Pros and Cons



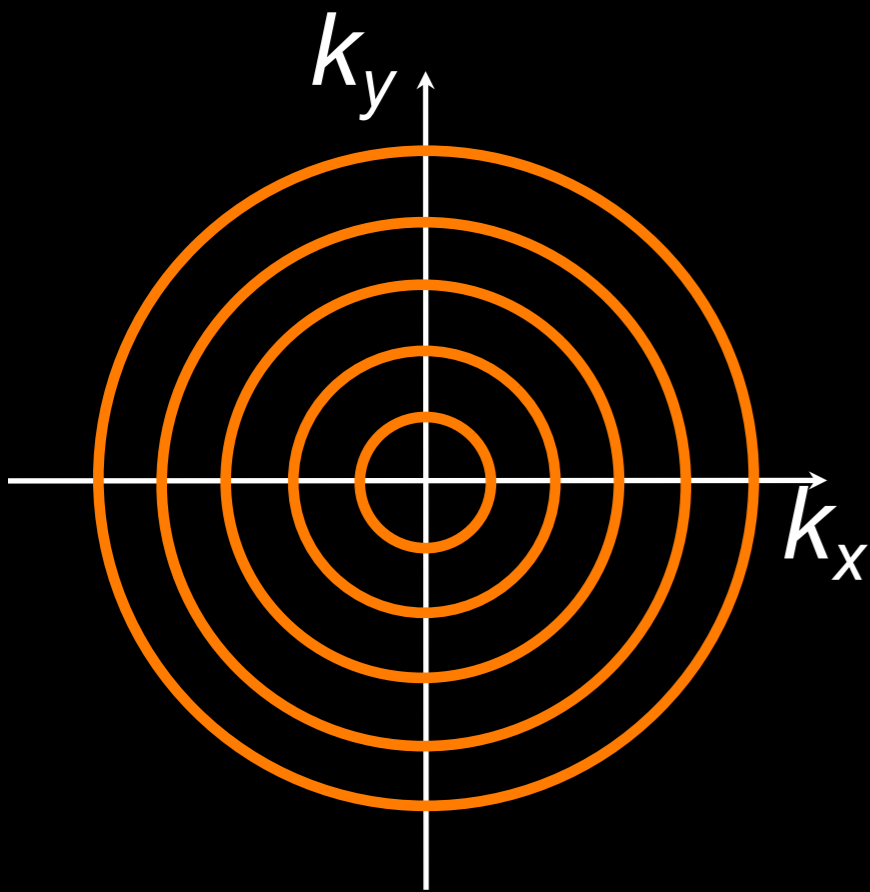
Pros

- 2x reduction in #TRs (vs. Cartesian)
- Favorable motion/flow properties
- Robust to gradient delays
- Efficient spatial/spectral encoding
- Effective for mag-prep MRI

Cons

- SNR penalty (non-uniform density)
- Scale-down design not optimal

Rings: Extensions



Variable density sampling

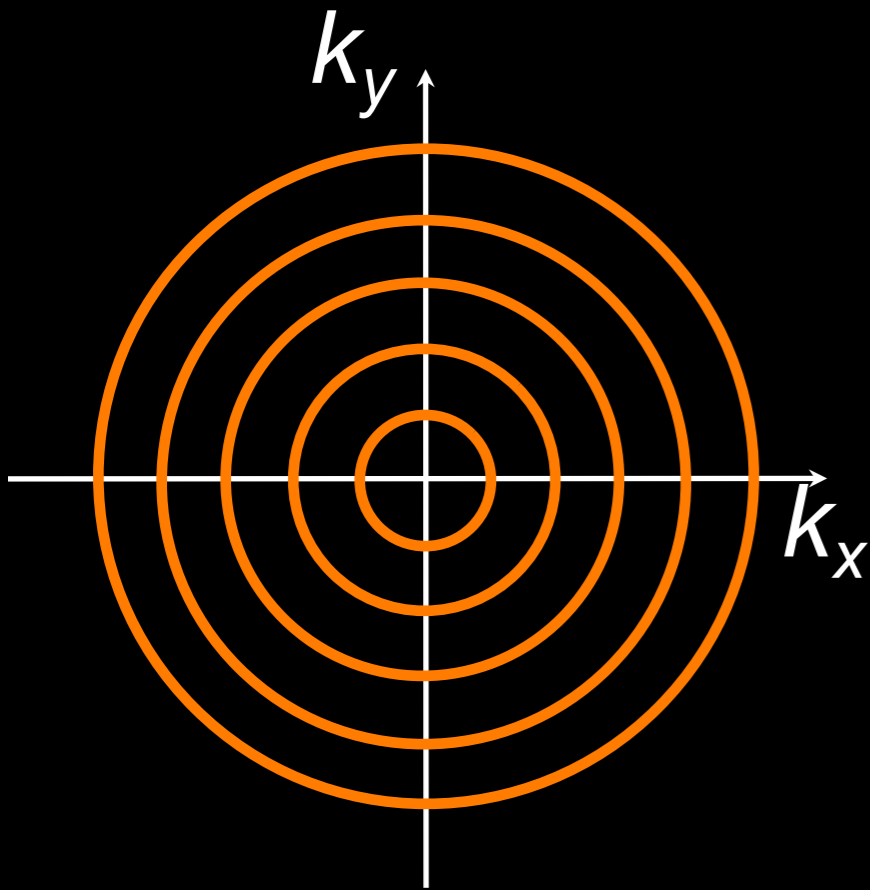
Multiple rings per TR

3D concentric cylinders

Parallel imaging

Partial Fourier

Rings: Applications



Fast imaging

- Cardiac MRI

Chemical shift imaging

- Fat/water separation
- MR spectroscopic imaging

Mag-prep imaging

- Neuro MRI
- Non-con MR angiography (MRA)
- Contrast-enhanced MRA

Non-Cartesian Sampling

- **Benefits**

- Reduced scan time
- Robustness to motion and flow
- Short echo time

- **Applications**

- Dynamic MRI
- Real-time MRI
- Cardiovascular MRI
- Short-TE MRI

- **Challenges**

- Hardware performance
- Gradient fidelity
- Off-resonance effects
- Implementation

- **Challenges addressed**

- **On-going research**

- **Use judiciously!**

Thanks!

- Further reading
 - Bernstein et al., Handbook of MRI Sequences
- Next time
 - Spiral, 3D Non-Cartesian trajectories
 - Gridding reconstruction
 - Trajectory measurement
 - Off-resonance correction

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<http://mrrl.ucla.edu/wulab>