Homework 2: RF Pulse Design
M229 Advanced Topics in MRI (Spring 2021)
Assigned: 4/20/2021, Due: 5/7/2021 at 5 pm by email

1. Consider a nonselective excitation in which a constant pulse of amplitude B_1 and duration τ is applied in the presence of B_0. If the carrier frequency ω is not exactly tuned to ω_0 (i.e., $\omega \neq \omega_0$), determine the resultant transverse magnetization output at time τ. Use the small tip angle solution based in the rotating frame at the carrier frequency ω.

2. Adiabatic Full Passage Pulse Design using Hyperbolic Secant Pulse Equations

Design an adiabatic RF pulse using the method outlined in class:

$$B_1(t) = A(t)e^{-i\int \omega(t')t'}$$
$$A(t) = A_0 \operatorname{sech}(\beta t)$$
$$\omega_1(t) = -\mu\beta \tanh(\beta t)$$

a) amplitude modulation function

Design an amplitude modulation function, A_t, with *sech*. Use β = 672 [rad/s] and A_0 = 0.12 [G]. Use 512 samples for the pulse duration of 10.24 ms.

```
>> beta = 672;
>> pulseWidth = 10.24; % in [ms]
>> A0 = 0.12 % in [G]
>> nSamples = 512; % the number of samples
>> dt = pulseWidth/nSamples/1000; % time step in [sec]
```

Plot the amplitude modulation function in *Gauss*.

```
>> plot(time, A_t); grid on;
>> title('Amplitude Modulation Function'); xlabel('Time (ms)'); ylabel('A(t) (G)');
```

b) frequency modulation function

Design a frequency modulation function, *w1_t*, with *tanh*. Use μ = 5 [dimensionless].

```
>> mu = 5; % [dimensionless]
```

Plot the frequency modulation function in *Hz*.

```
>> plot(time,w1_t);  grid on;
>> title('Frequency Modulation Function'); xlabel('Time (ms)'); ylabel('\omega_1(t) (Hz)');
```

c) inversion profile using Bloch simulation

Combine amplitude and frequency modulation functions:

```
>> rf_pulse = A_t .* exp(1i .* cumsum(w1_t)*dt));
```

Or, you can use phase modulation function instead:

$$\phi(t) = \mu \ln(\operatorname{sech} \beta t)$$

```
>> rf_pulse = A_t .* exp(1i .* phi);
```

Simulate the inversion profile over a sufficient range of frequency (e.g. -4,000 Hz to 4,000 Hz) using Bloch simulation. Plot the inversion profile.

```
>> plot(freq_range, mz);
>> title('Inversion Profile'); xlabel('Frequency (Hz)'); ylabel('M_z'); grid on;
```

d) inversion profiles with different B1+ variation

Simulate the inversion profiles with different B1+ variations (60% attenuated, 30% attenuated, and 150% amplified pulses). Plot the inversion profiles.

3. 2D EPI Pulse Design

Design a 2D separable EPI RF pulse using the method outlined in class.

a) gradient design

Design a blipped EPI trajectory with $k_{x,max} = k_{y,max} = 0.5$ *cycles/cm*, and nine lines ($L=9$). Assume the trapezoid ramps (τ_R) are 1/8 ms long, and the trapezoids themselves (τ) are 1 ms long. The blips are 1/4 ms long. What is the maximum amplitude of G_x and G_y?

Sample the RF and gradient waveforms at 5 us (200 samples per trapezoid and 25 samples per trapezoid ramp). Include a refocusing lobe at the end to bring the trajectory back to $k_x = k_y = 0$, using 1 ms trapezoids on x and y. Plot the gradient waveforms, G_x and G_y, with the axes labeled.

b) RF pulse design

Design the RF waveform with TBW = 4 for the subpulses, and the envelope. This will produce a 4 cm by 4 cm excited volume.

```
>> tbw = 4;
>> rf_fast = wsinc(tbw,samples);
>> rf_slow = wsinc(tbw,L);
```

Apply the “flat-top only design” (RF only played flat part), and use the RF waveform to be zero during the refocusing gradient. Scale the RF to a flip angle of 1 radian (i.e. sum(rf) = 1). Plot the RF waveform in Gauss.

c) 2D Bloch simulation

Simulate the pulse over a sufficient range (e.g., -12cm to 12cm in x and y) at on-resonance. Plot the profile as an image using

```
>> imshow(abs(mxy),[]);
```

and cross-section plots along x (M_{xy} vs. x) and y (M_{xy} vs. y)

```
>> subplot(211); plot(x,abs(mxy(:,round(length(y)/2))));
>> subplot(212); plot(y,abs(mxy(round(length(x)/2),:)));
```