Spatial Localization - I

Daniel B. Ennis, Ph.D.

Magnetic Resonance Research Labs

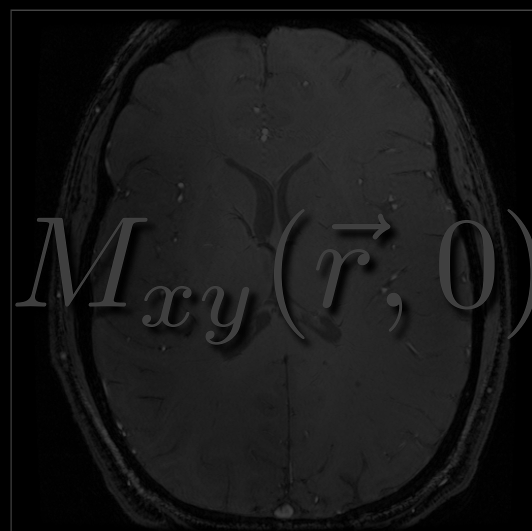
David Geffen School of Medicine

UCLA Radiology

The MRI Signal Equation

Daniel B. Ennis, Ph.D.
Magnetic Resonance Research Labs

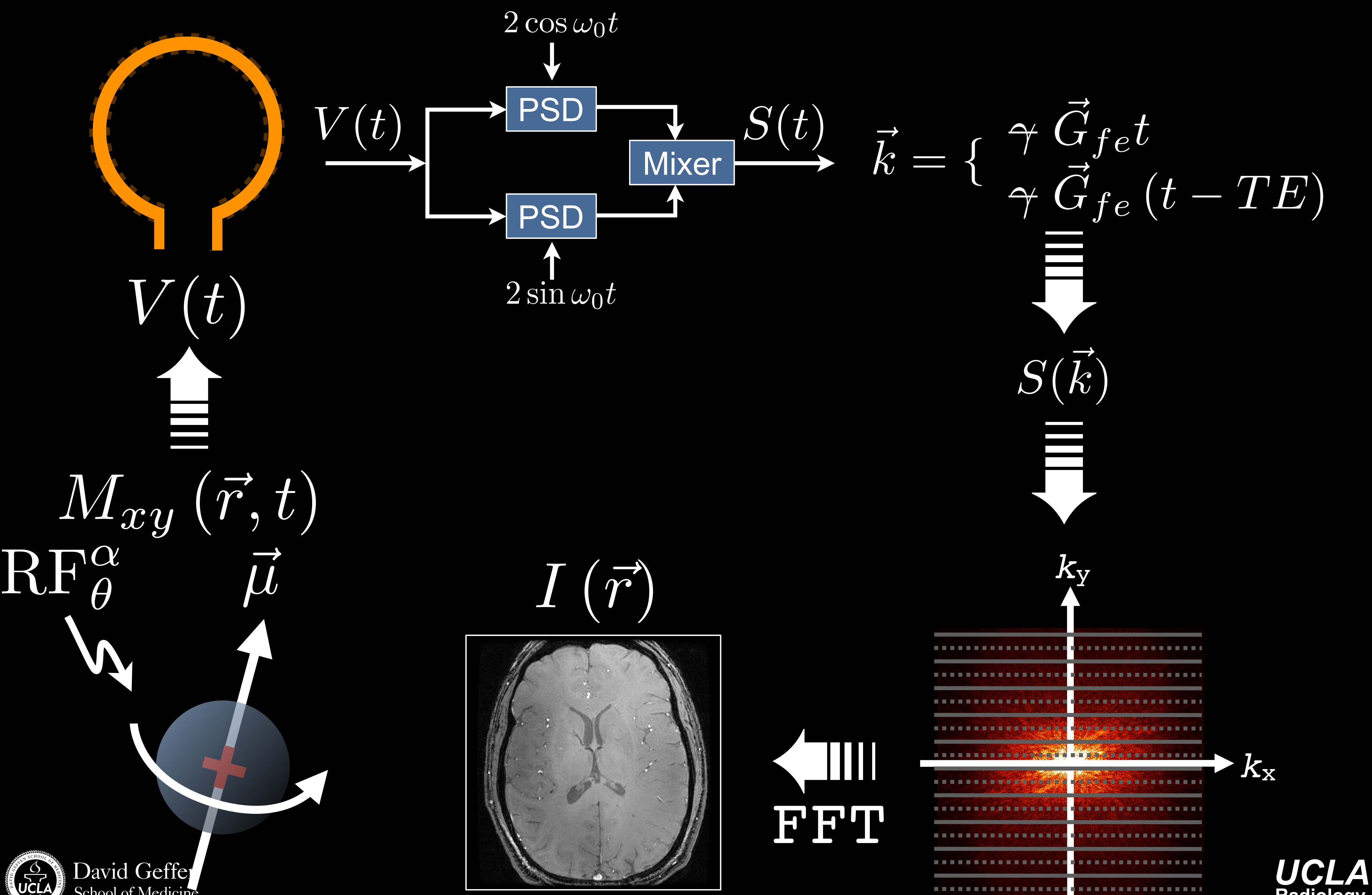
$$S(\vec{k}) = \int_{object} M_{xy}(\vec{r},0)\; e^{-i2\pi\vec{k}\cdot\vec{r}}\; d\vec{r}$$



David Geffen School of Medicine

UCLA Radiology

Signals in MRI



David Geffen School of Medicine

UCLA Radiology

Lecture #9 Summary

$$V(t) = -\frac{\partial \Phi(t)}{\partial t} = -\frac{\partial}{\partial t}\int_{object} \vec{B}(\vec{r}) \cdot \vec{M}(\vec{r},t)\, d\vec{r}$$

Coil Sensitivity

Bulk Magnetization

$\frac{dM_z}{dt} \approx 0$

$$B_{r,xy}(\vec{r}) = |B_{r,xy}(\vec{r})| e^{-i\phi_r(\vec{r})}$$

Free Precession $M_{xy}(\vec{r},t) = |M_{xy}(\vec{r},0)| e^{-\frac{t}{T_2}} e^{-i\omega_0 t} e^{-i\phi_{RF}}$

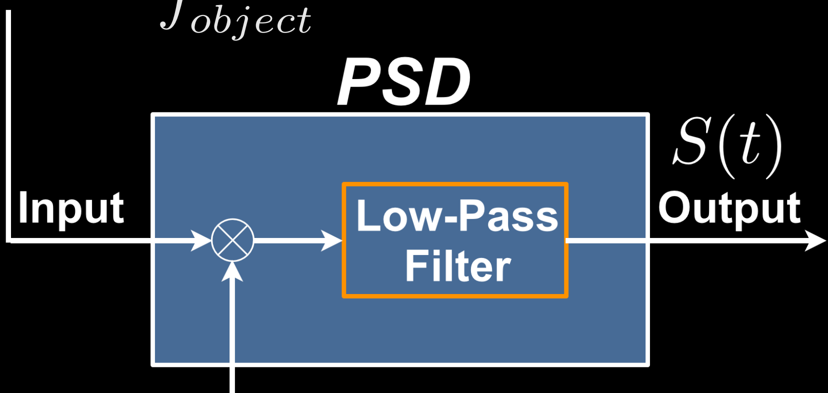
Lots of trigonometry and algebra…

$$V(t) = \int_{object} \omega(\vec{r}) |B_{r,xy}(\vec{r})| |M_{xy}(\vec{r},0)| e^{-\frac{t}{T_2(\vec{r})}} \cos\left(-\omega(\vec{r})t + \phi_e(\vec{r}) - \phi_r(\vec{r}) + \frac{\pi}{2}\right) d\vec{r}$$

High frequency voltage signal.

Lecture #12 Summary

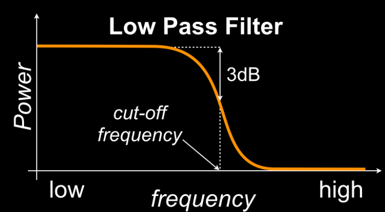
$$V(t) = \int_{object} \omega(\vec{r})|B_{r,xy}(\vec{r})||M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2(\vec{r})}} \cos\left(-\omega(\vec{r})t + \phi_e(\vec{r}) - \phi_r(\vec{r}) + \frac{\pi}{2}\right) d\vec{r}$$



$$V_{PSD}^{\cos}(t) = \omega_0 \int_{object} |B_{r,xy}(\vec{r})||M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2(\vec{r})}} \cos\left(-\Delta\omega(\vec{r})t + \phi_e(\vec{r}) - \phi_r(\vec{r}) + \frac{\pi}{2}\right) d\vec{r}$$

-or-

$$V_{PSD}^{\sin}(t) = \omega_0 \int_{object} |B_{r,xy}(\vec{r})||M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2(\vec{r})}} \sin\left(-\Delta\omega(\vec{r})t + \phi_e(\vec{r}) - \phi_r(\vec{r}) + \frac{\pi}{2}\right) d\vec{r}$$



Spatial Frequency Encoding

$$\begin{aligned} S(t) &= V_{PSD}^{\cos} + iV_{PSD}^{\sin} \\ &= \omega_0 e^{i\pi/2} \int_{Object} B_{r,xy}^{*}(\vec{r}) M_{xy}(\vec{r},0) e^{-i\Delta\omega(\vec{r})t} d\vec{r} \end{aligned}$$

$$\Delta\omega(\vec{r})t = \gamma\vec{G}\cdot\vec{r} = 2\pi\vec{k}\cdot\vec{r}$$

Definition of k-space

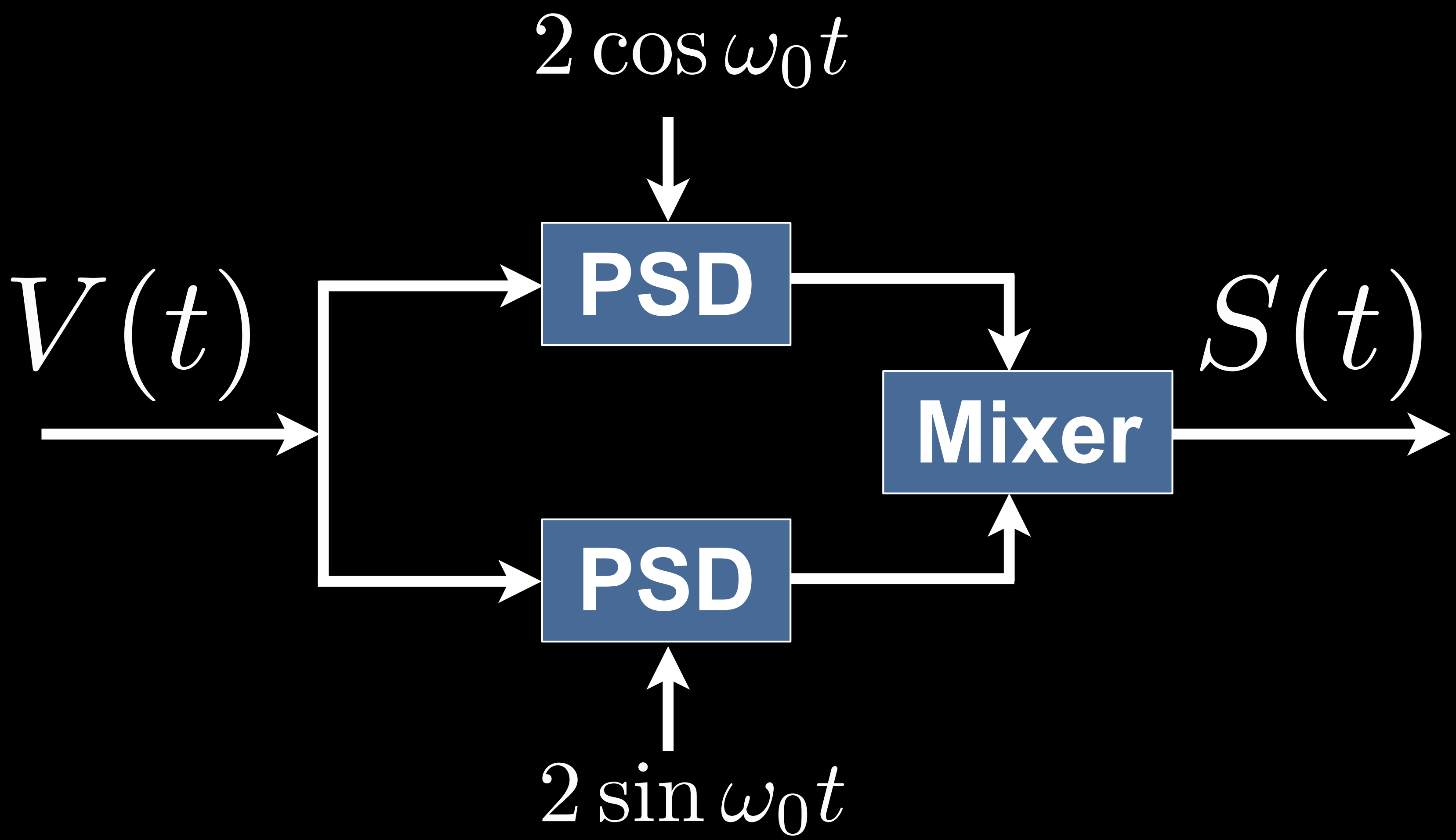
$$S(\vec{k}) = \int_{Object} M_{xy}(\vec{r},0) e^{-i2\pi\vec{k}\cdot\vec{r}} d\vec{r}$$

MRI Signal Equation

Quadrature Detection

$$V^{c}_{psd}(t) \text{ and } V^{s}_{psd}(t) \rightarrow S(t)$$

Quadrature Detection

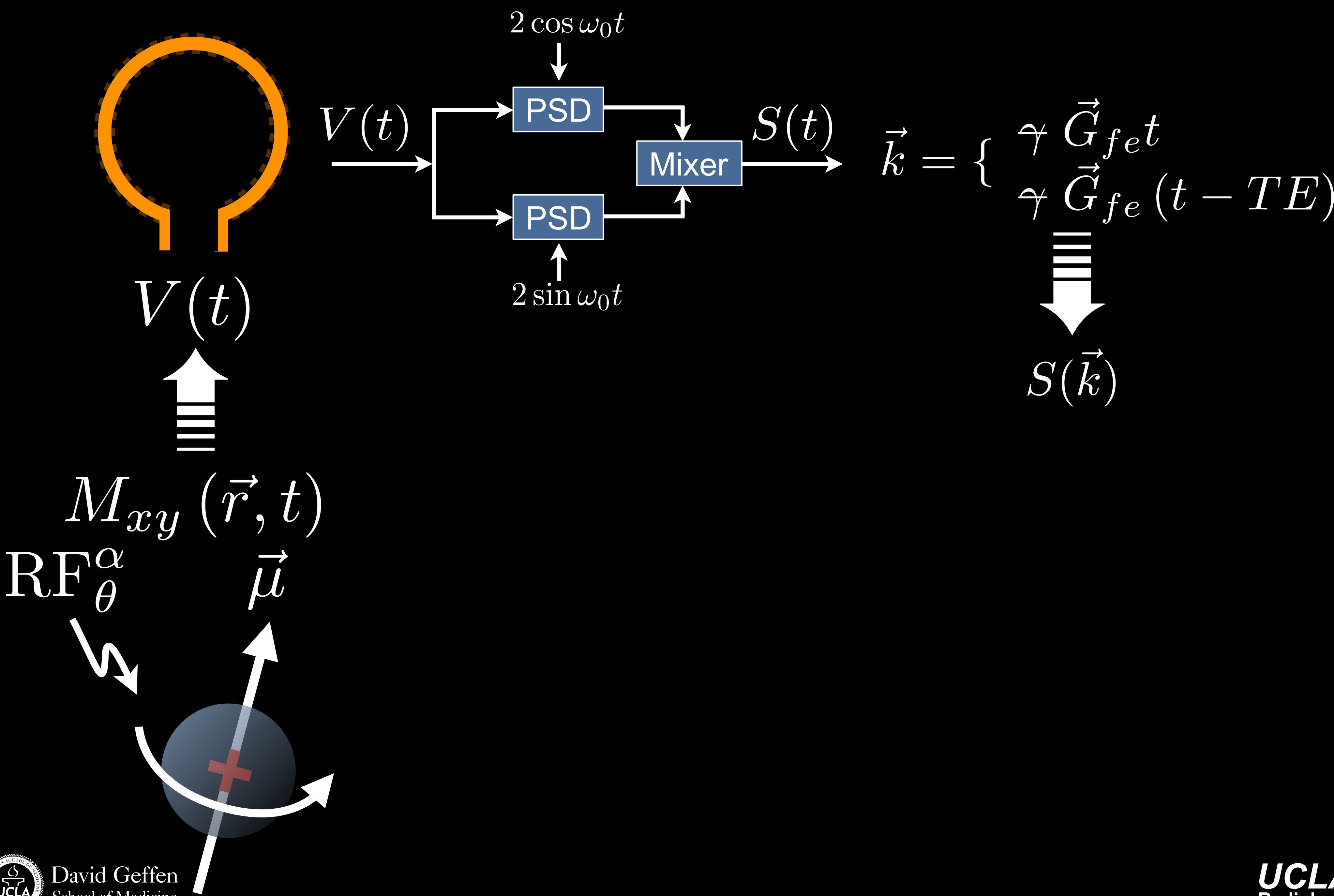


To The Board...

Phase Sensitive Detection

$$S\left(t\right) \text{ to } S\left(\vec{k}\right)$$

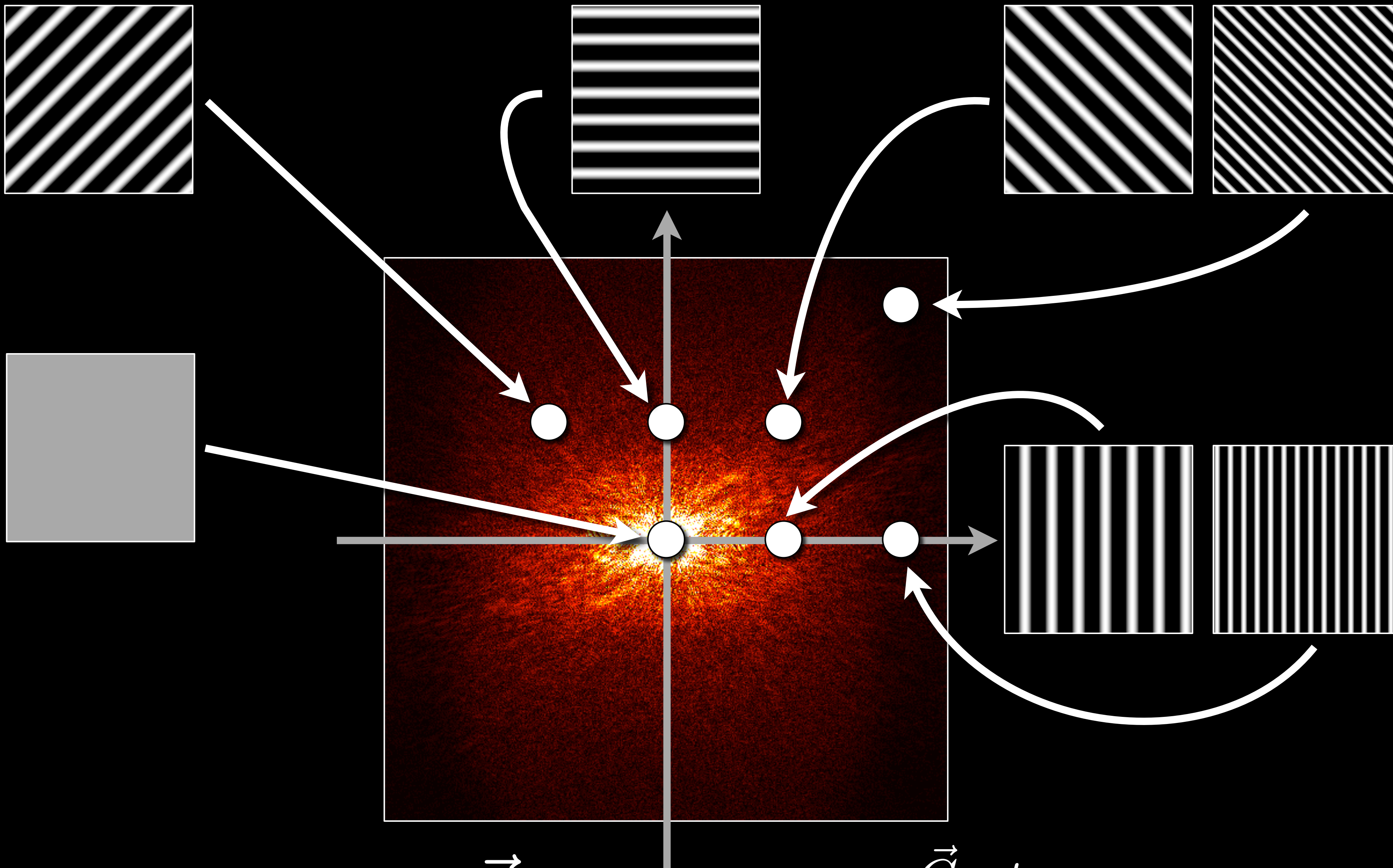
Signals in MRI



How does S(t) relate to S(k)?

To The Board...

k-space



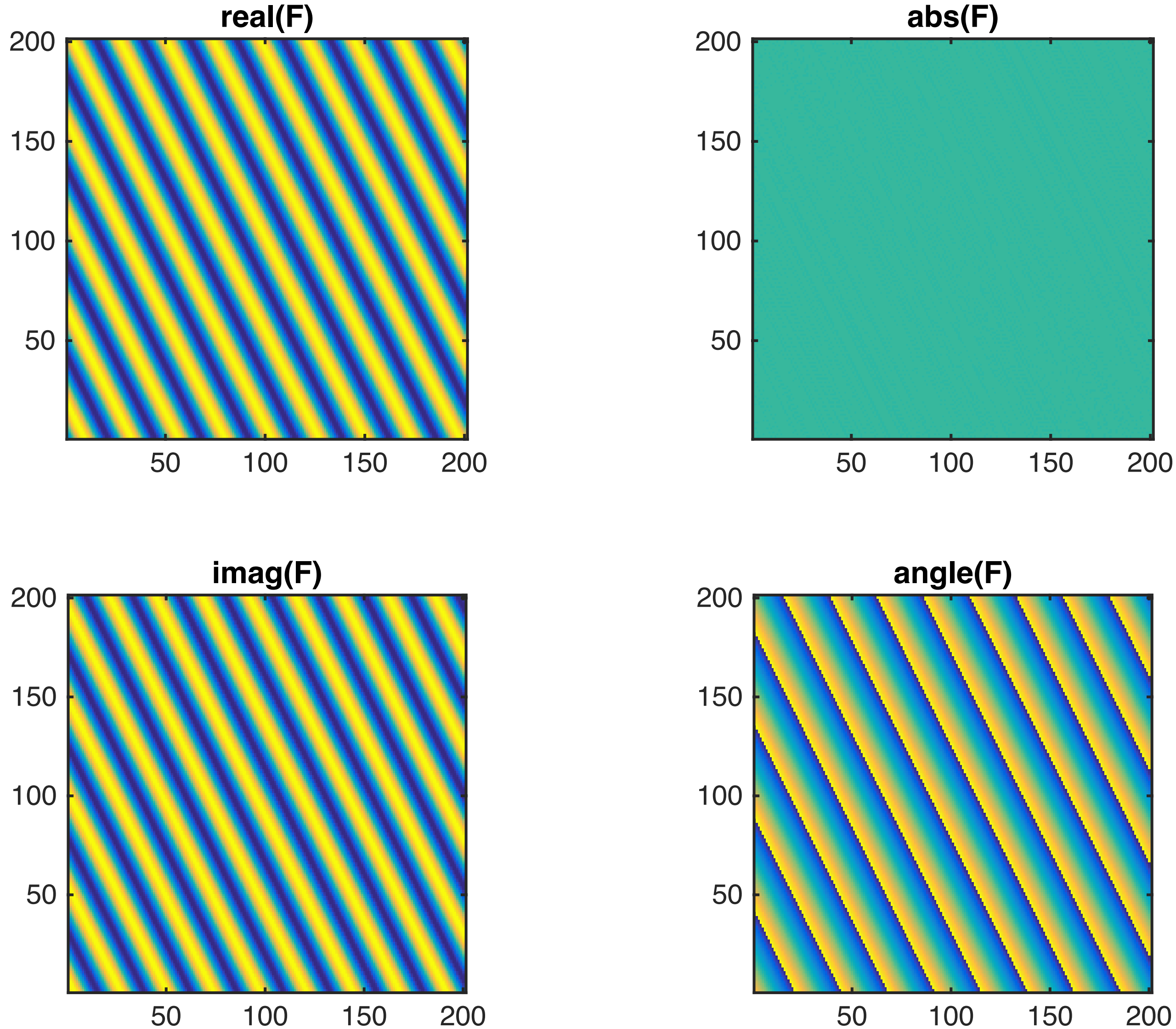
$$e^{-i2\pi\vec{k}\cdot\vec{r}}$$

$$\vec{k} = \begin{cases} \bar{\gamma}\,\vec{G}_{fe}\,t \\ \bar{\gamma}\,\vec{G}_{fe}\,(t - TE) \end{cases}$$

k-space

```
%% Define and display some Fourier sampling functions...
gamma_bar=4257.7480;        % Gyromagnetic ratio, [Hz/G]
Gx=1;                       % [Gauss/cm]
Gy=1;                       % [Gauss/cm]
dt=1.0e-3;                  % [s]

kx=gamma_bar*Gx*dt;         % Kx-space component
ky=gamma_bar*Gy*dt;         % Ky-space component

[X,Y]=ndgrid(-1:0.01:1,-1:0.01:1); % Define some positions in space [cm]

F=exp(-1i*2*pi*(kx*X+ky*Y)); % Fourier sampling functions

%% Display the sampling function
figure; hold on;
subplot(2,2,1);
  imagesc(real(F));
  title('real(F)'); axis image xy;
subplot(2,2,3);
  imagesc(imag(F));
  title('imag(F)'); axis image xy;
subplot(2,2,2);
  imagesc(abs(F));
  title('abs(F)'); axis image xy;
subplot(2,2,4);
  imagesc(angle(F));
  title('angle(F)'); axis image xy;
```

k-space

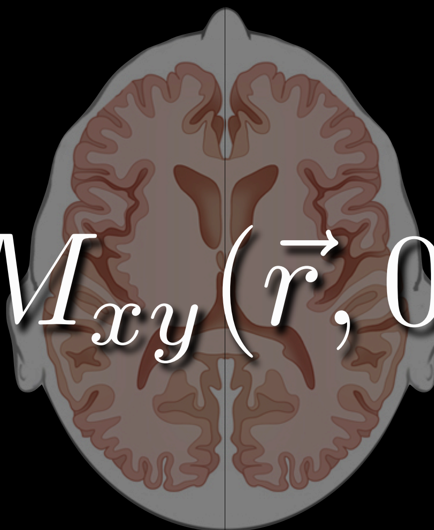


David Geffen School of Medicine

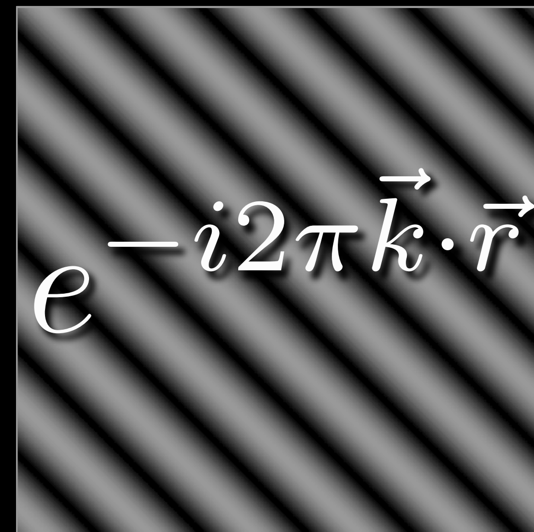
UCLA Radiology

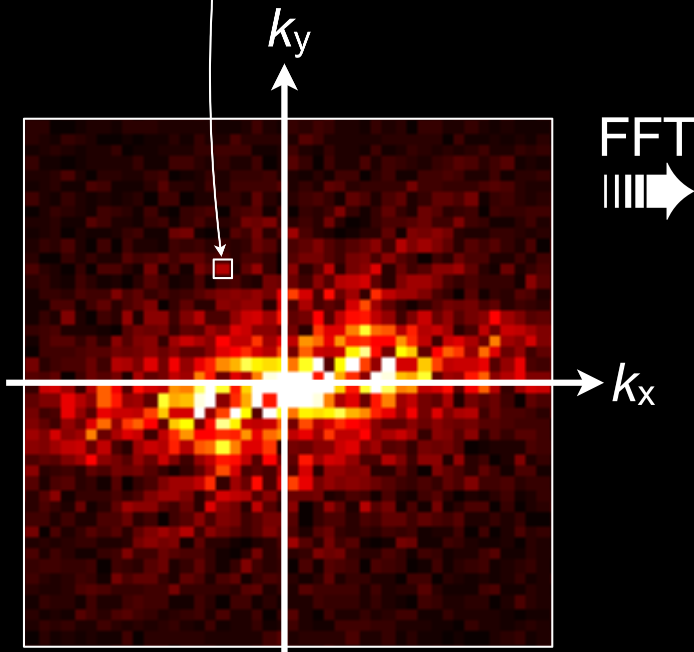
MRI Signal Equation

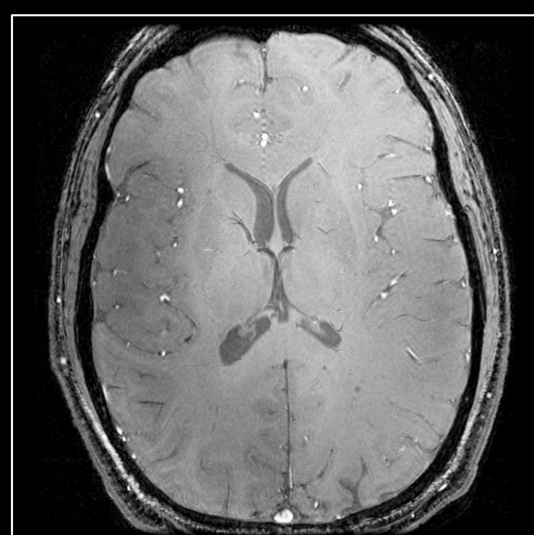
$$S(\vec{k}) = \int M_{xy}(\vec{r}, 0)\; e^{-i2\pi \vec{k}\cdot\vec{r}}\; d\vec{r}$$



$object$







Lecture #9 Learning Objectives

- Understand that SE and GRE control image contrast at the echo time.
- Appreciate that gradients move us through k-space.
- Describe how to calculate scan time.
- Explain the concept of “coil sensitivity.”
- Explain why MRI is not directly sensitive to M_z.
- Understand the role of phase sensitive detection.
- Describe the importance of quadrature detection.
- Be able to define the MRI signal equation and each term.

Spatial Localization - I

Daniel B. Ennis, Ph.D.
Magnetic Resonance Research Labs

David Geffen
School of Medicine

UCLA
Radiology

Lecture #10 - Learning Objectives

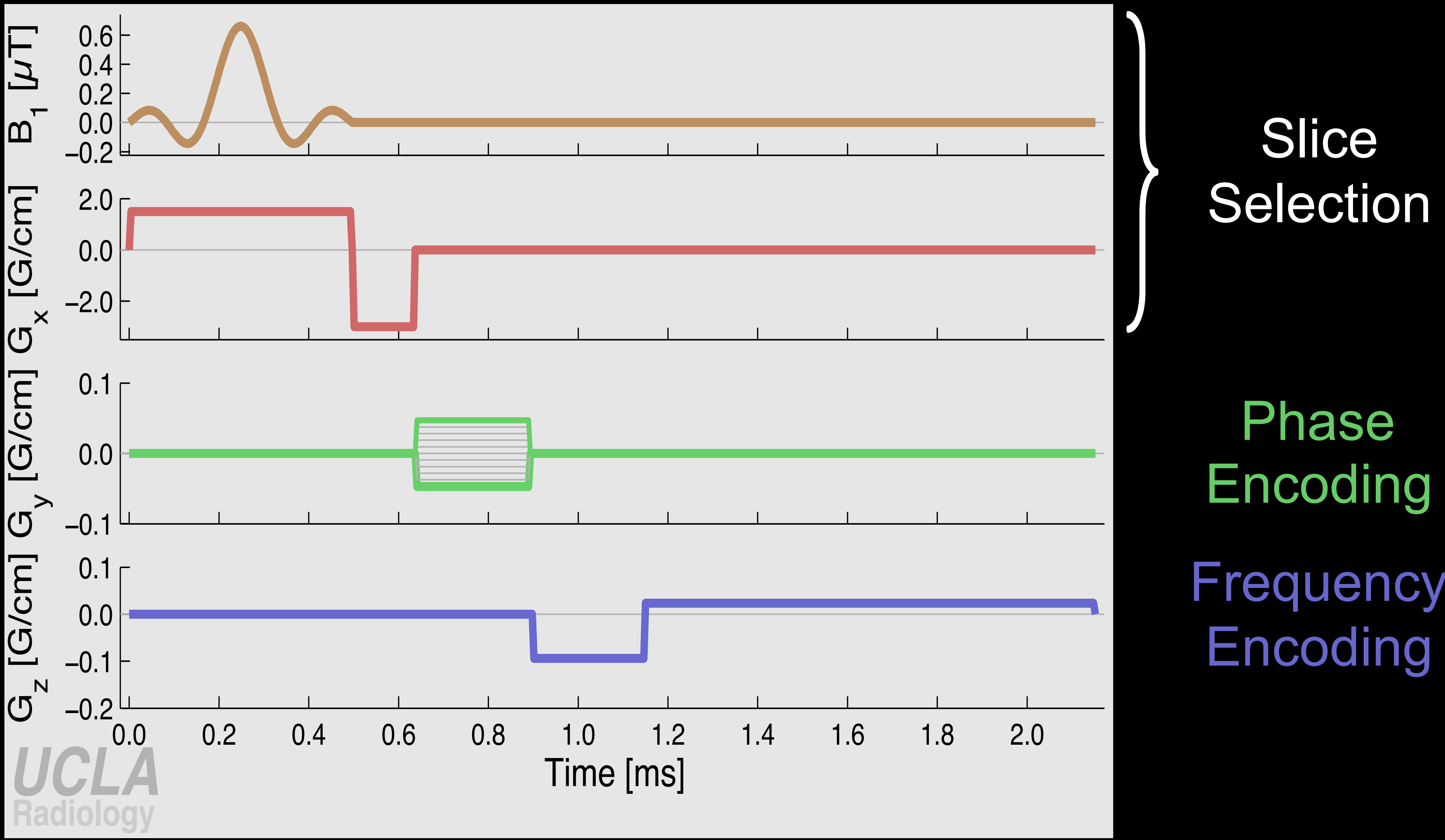
- **Describe the three steps required for spatial localization.**
- **Be able to explain the role of RF and gradients during slice selection.**
- **Learn to define B_{eff} for various combinations of B-fields.**
- **Identify the complexity of the Bloch equations for forced precession in the presence of a gradient field.**
- **Understand the small tip angle approximation.**
- **Appreciate that the small tip angle approximation works for intermediate flip angles!**
- **Understand what truncation artifacts are and one way to reduce them.**

Spatial Localization

Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.

3 Steps for Spatial Localization

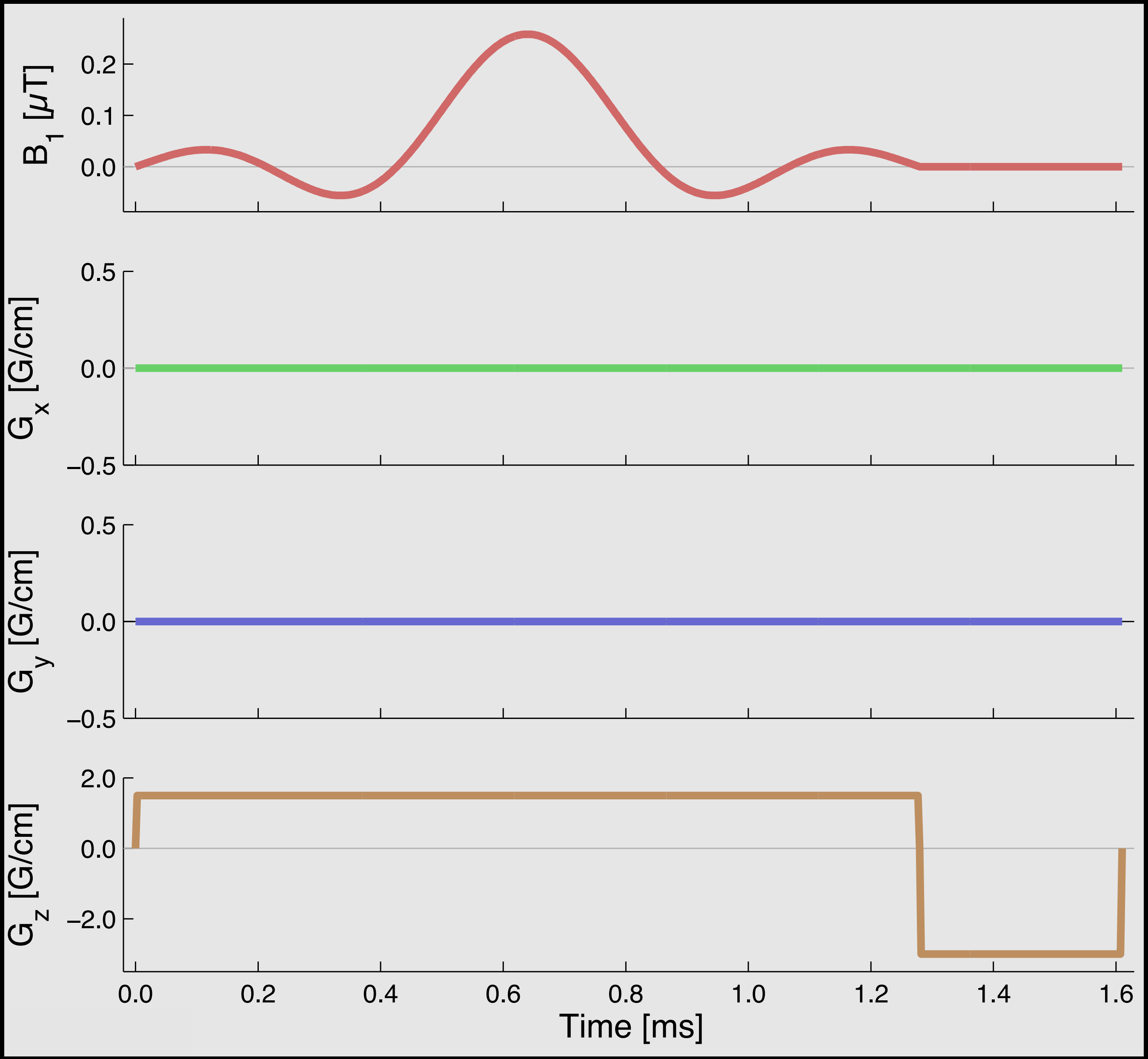


Pulse Sequence Diagram - Timing diagram of the RF and gradient events that comprise an MRI pulse sequence.

Slice Selection

- Consists of:
  - Slice selection gradient
    - Constant magnitude
  - RF (B_1) Pulse
    - Contains frequencies matched to slice of interest
  - Slice re-phasing gradient
    - Increases SNR
    - Re-phases spins within slice
    - AKA “slice refocusing gradient”
- Permits exciting the slice of interest.

Slice Selection



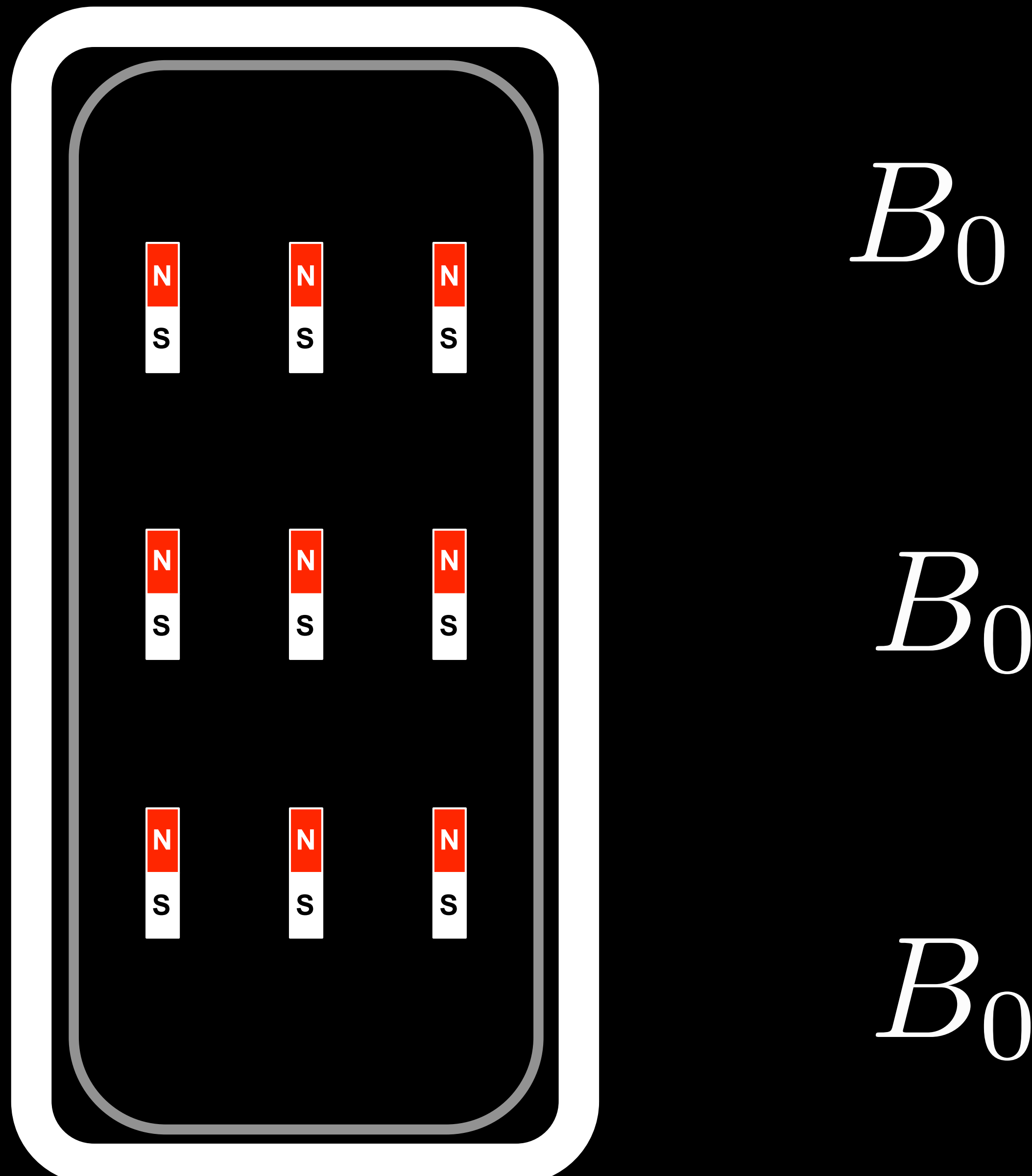
Tuned to frequencies (slice) of interest.

Creates a range of frequencies.

Re-phases spins in slice to increase signal-to-noise.

Slice selection requires RF and a gradient.

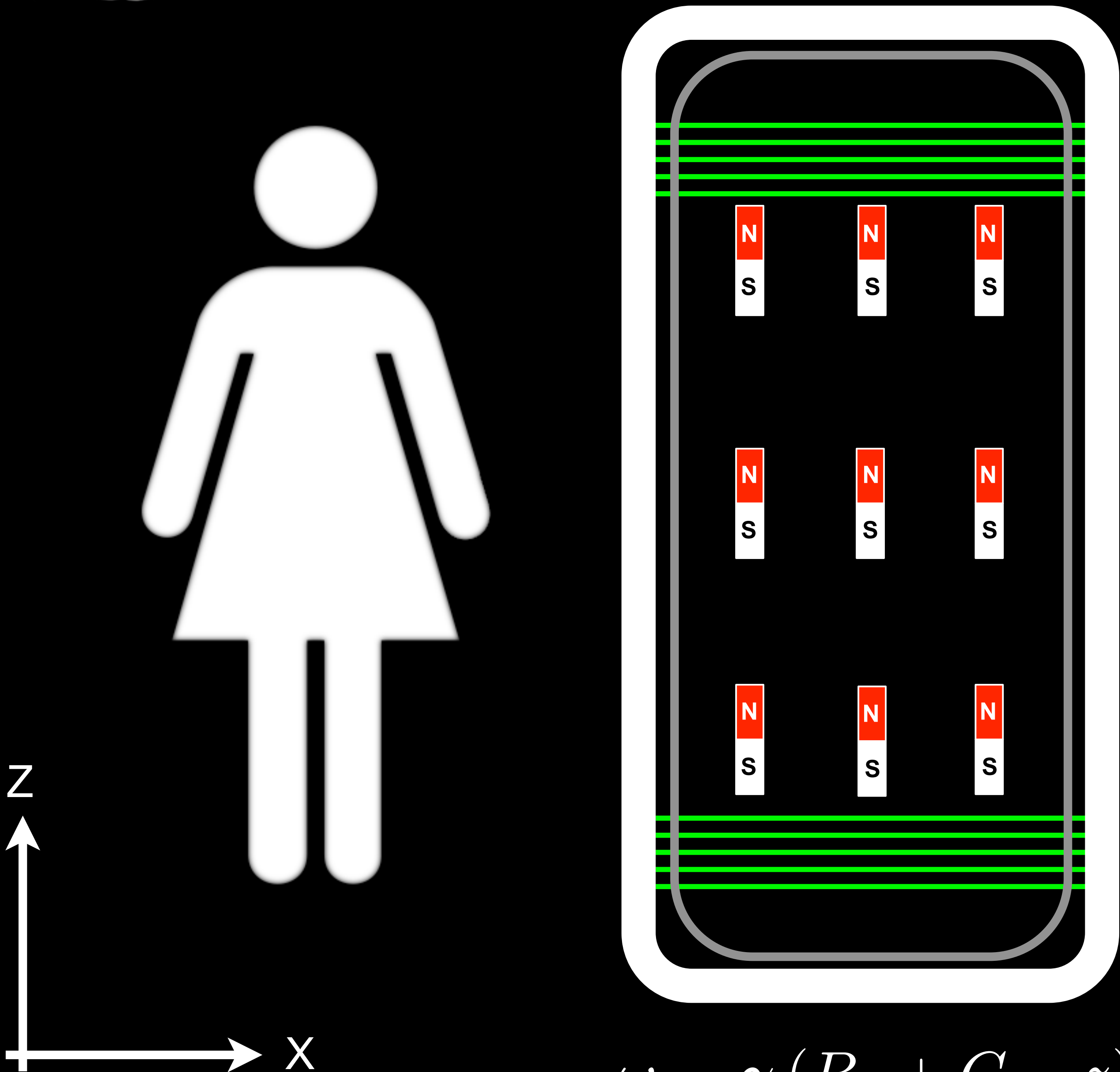
No Gradients Turned On



$$\omega = \gamma B_0$$

Everything precesses at the Larmor frequency.

Z-Gradients is ON



$$B_0 + \delta B_0$$

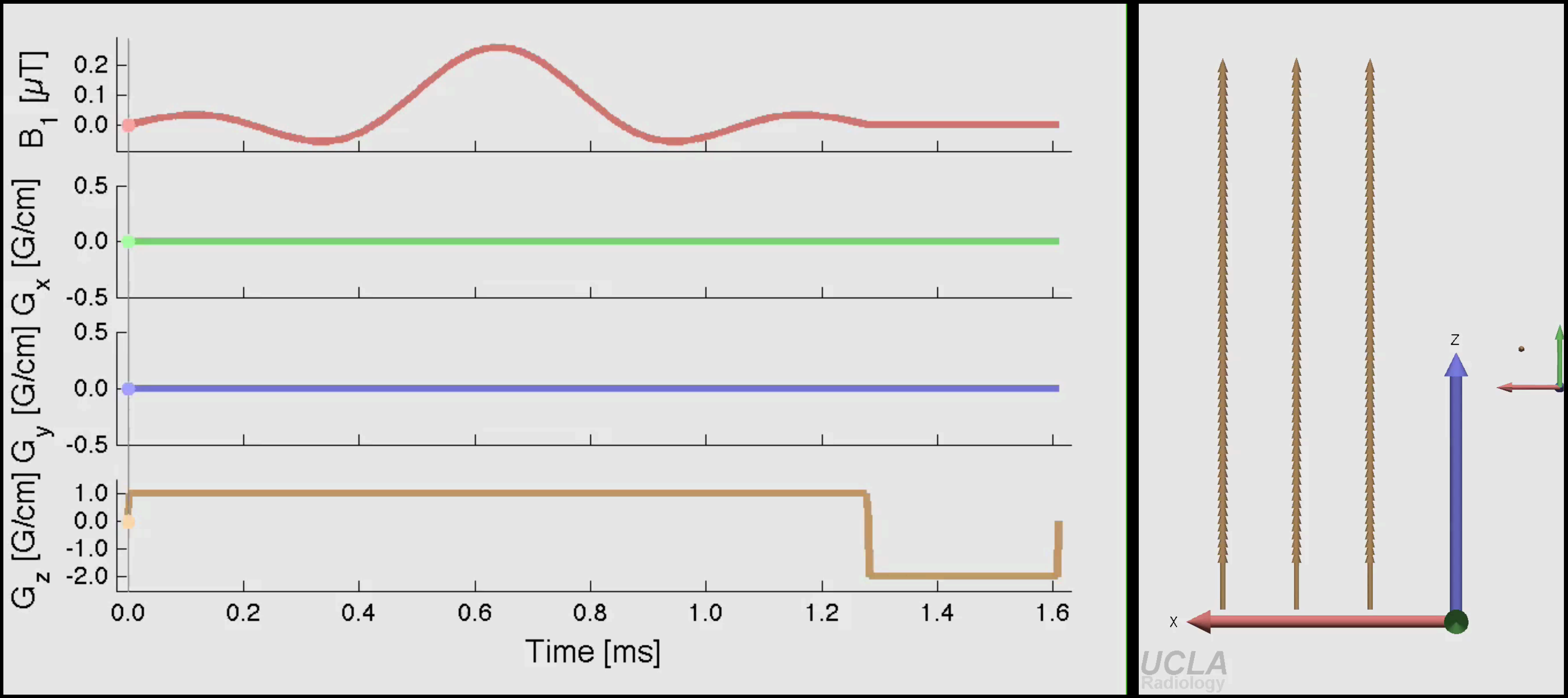
$$B_0$$

$$B_0 - \delta B_0$$

$$\omega = \gamma \left(B_0 + G_z \cdot z \right)$$

This frequency excites a slice at position *z* when G_z is turned on.

Slice Selection & Rephasing

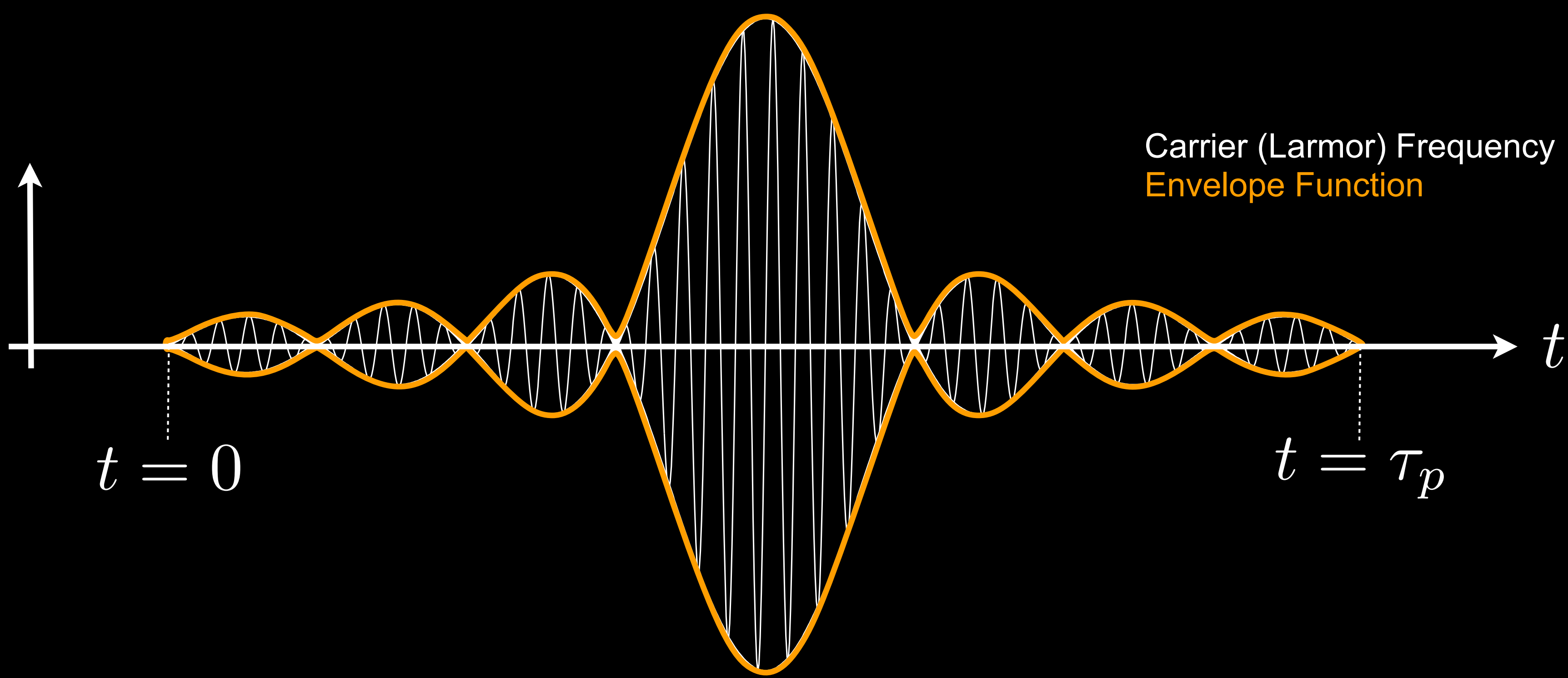


Excitation Pulses

Sinc Envelope Function

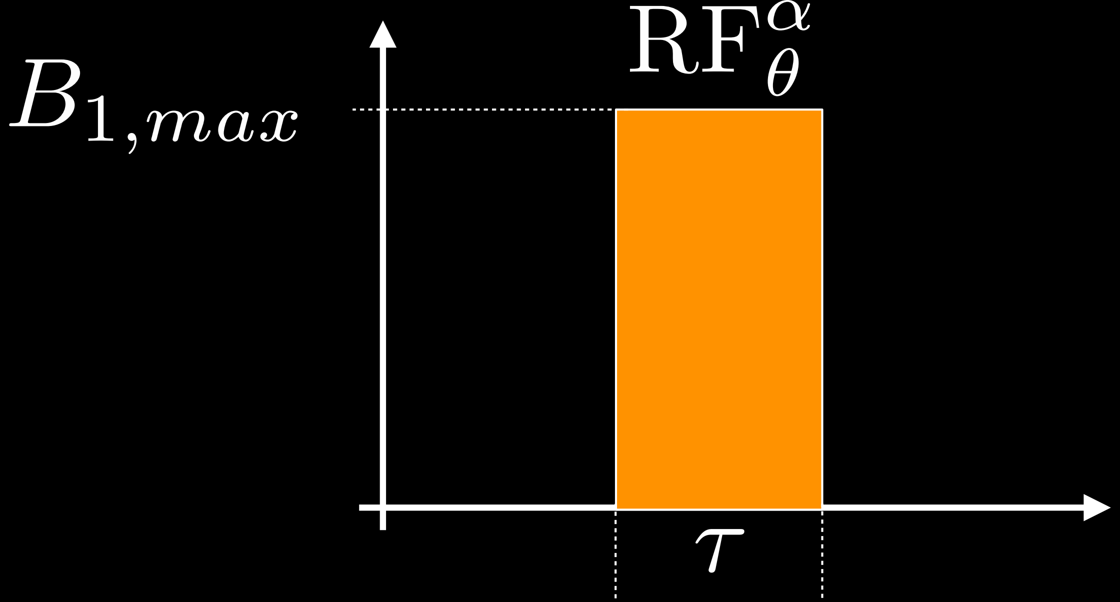
$$B_1(t) = B_1^e(t)\left[\cos(\omega_{RF}t)\,\hat{i} - \sin(\omega_{RF}t)\,\hat{j}\right]$$ Laboratory Frame

$$B_1^e(t) = \begin{cases} B_1 \mathrm{sinc}\left[\pi f_\omega (t - \tau_p/2)\right], & 0 \le t \le \tau_p \\ 0, & otherwise \end{cases}$$



SINC functions are used to excite a narrow band of frequencies.

How to determine α?

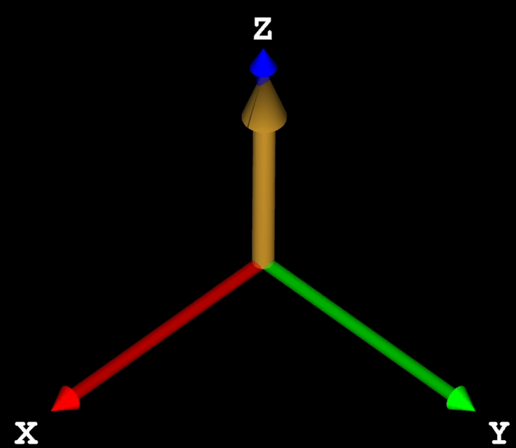


$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t)\, dt$$

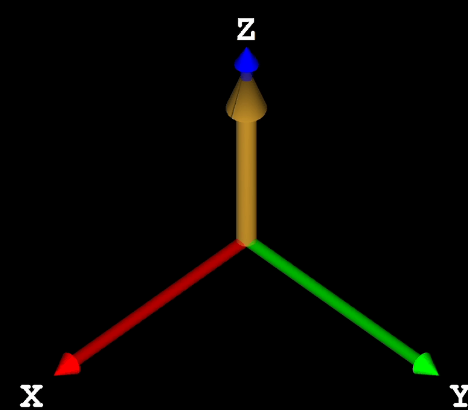
Rules: 1) Specify α [radians]
2) Use $B_{1,max}$ if we can
3) Shortest duration pulse

Excitation Pulses

- **Tip M_z into the transverse plane**
- **Typically 200μs to 5ms**
- **Non-uniform across slice thickness**
  - Imperfect slice profile
- **Non-uniform within slice**
  - Termed B_1 inhomogeneity
  - Non-uniform signal intensity across FOV



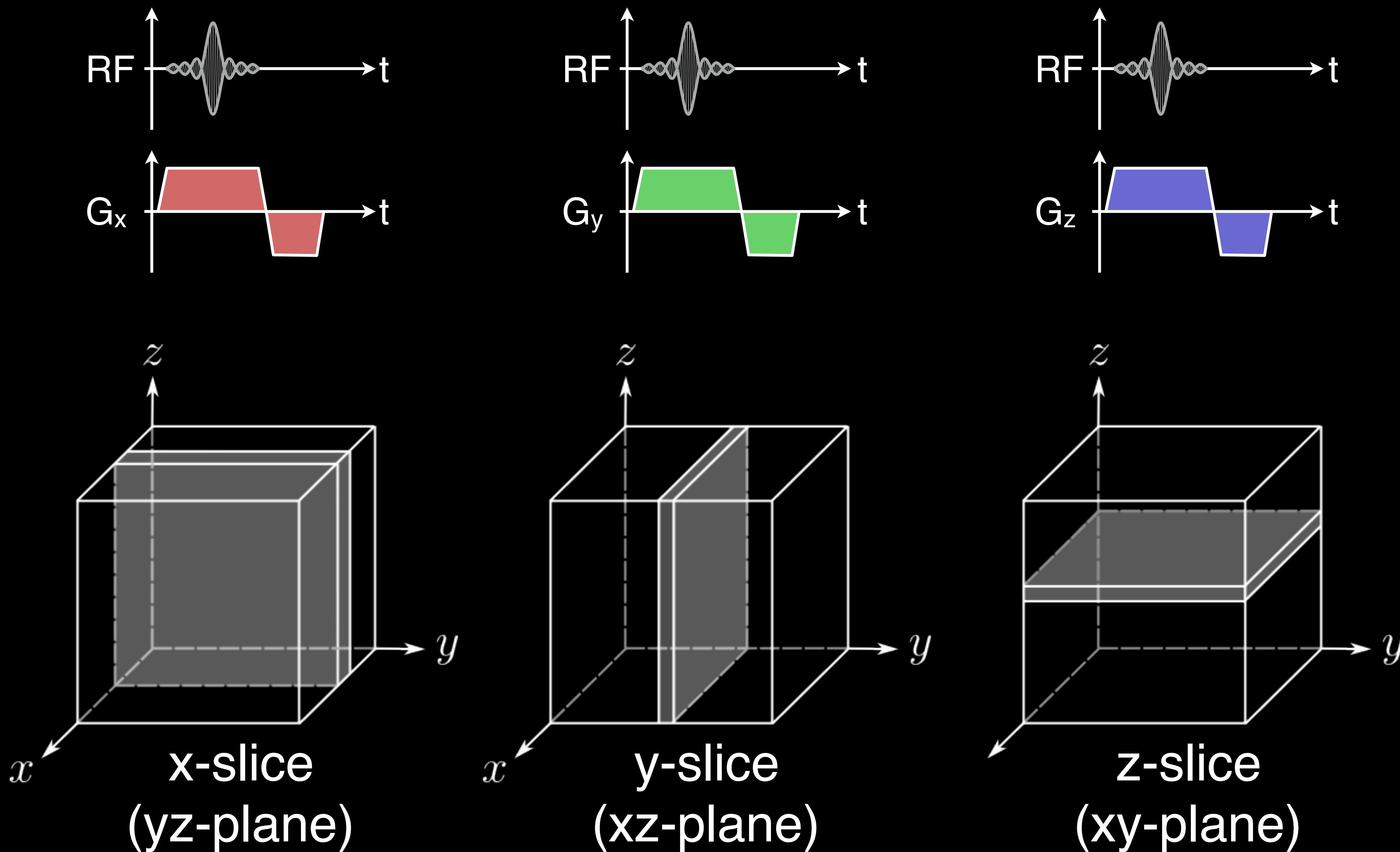
90° Excitation Pulse



Small Flip Angle Pulse

Slice Selective Excitation

Slice Selective Excitation



Slice selection requires a simultaneous RF pulse and gradient.

Gradient Components & Vectors

$$B_{G,z}(x) = G_x x$$ x-gradient — Freq. Encode

$$B_{G,z}(y) = G_y y$$ y-gradient — Phase Encode

$$B_{G,z}(z) = G_z z$$ z-gradient — Slice Select

The magnetic field at a position depends on the magnitude of the applied gradient.

B_0 and Gradients

$$
\begin{aligned}
B_{G,z}\vec{k} &= (G_x x + G_y y + G_z z)\vec{k} \\
&= (\vec{G} \cdot \vec{r})\vec{k}
\end{aligned}
$$

Total applied gradient field.

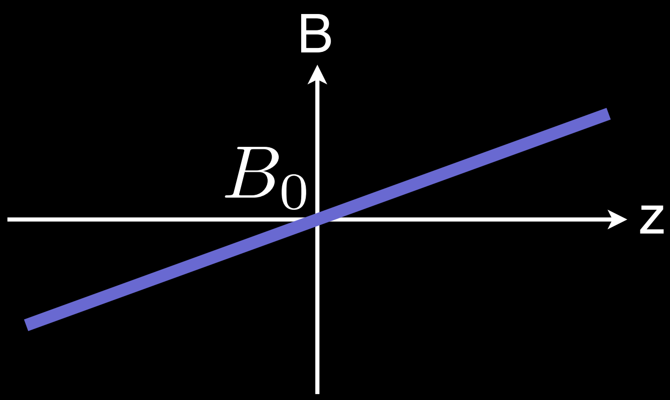
$$
\begin{aligned}
\vec{B}\left(\vec{r}, t\right) &= \left(B_0 + B_{G,z}\right)\vec{k} \\
&= \left(B_0 + \vec{G}\left(t\right) \cdot \vec{r}\right)\vec{k}
\end{aligned}
$$

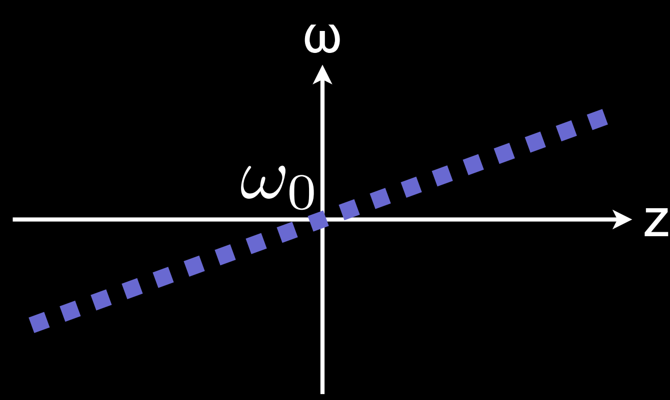
Total applied magnetic field.

Gradients

- **Gradients produce a spatial distribution of frequencies**

$$\vec{B}(z) = (B_0 + G_z \cdot z)\,\hat{k} \qquad \vec{\omega}(z) = -\gamma \vec{B}(z) = -\gamma\,(B_0 + G_z \cdot z)\,\hat{k}$$

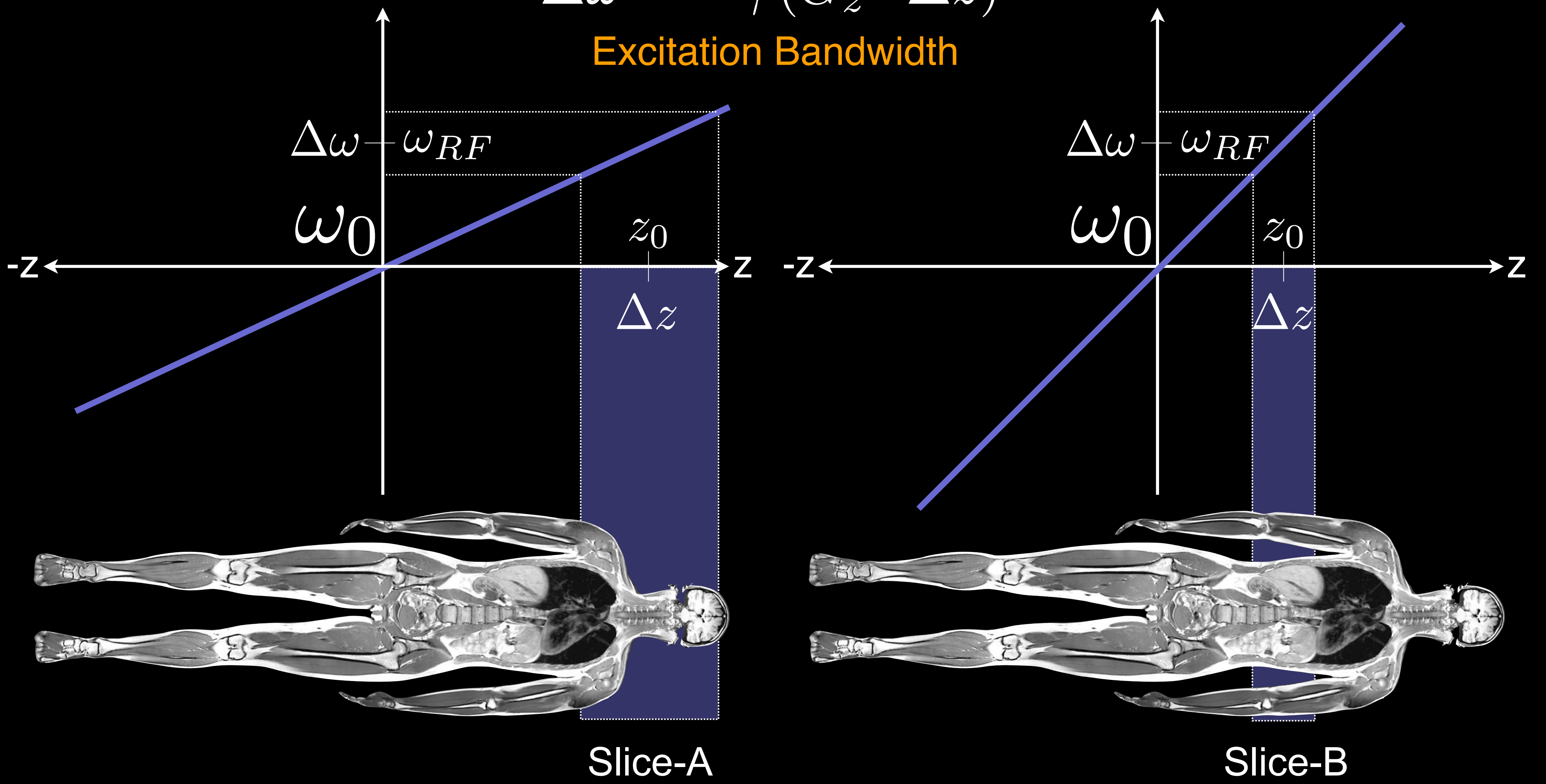




Gradients create a direct correspondence between frequency and spatial position.

Slice Selective Excitation

$$\Delta\omega = -\gamma\left(G_z \cdot \Delta z\right)$$



How do you move the slice along ±z?

Compare $\Delta\omega$ and ω_{RF} for Slice-A and Slice-B.

Do we usually acquire $\omega_{RF} > \omega_0$?

Selective Excitation

- **What factors control slice selection?**

$$B_1^e(t)$$

Pulse envelope function

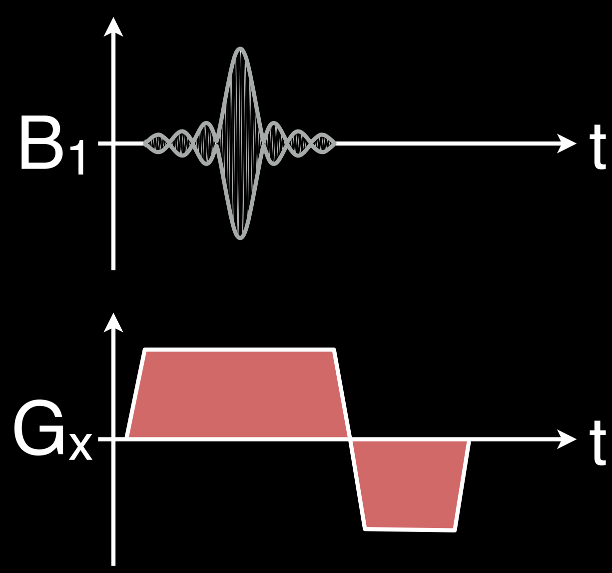
(e.g. $B_{1,max}$ and $\Delta\omega$)

$$\omega_{RF}$$

Excitation carrier frequency

$$\vec{G}$$

Gradient amplitude



Forced Precession with a Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff}(z,t) = \begin{bmatrix} B_1(t) \\ 0 \\ \cancel{B_0 +} \, G_z \cdot z \, \cancel{- \frac{\omega_{RF}}{\gamma}} \end{bmatrix}$$

Effective B-Field in the Rotating Frame

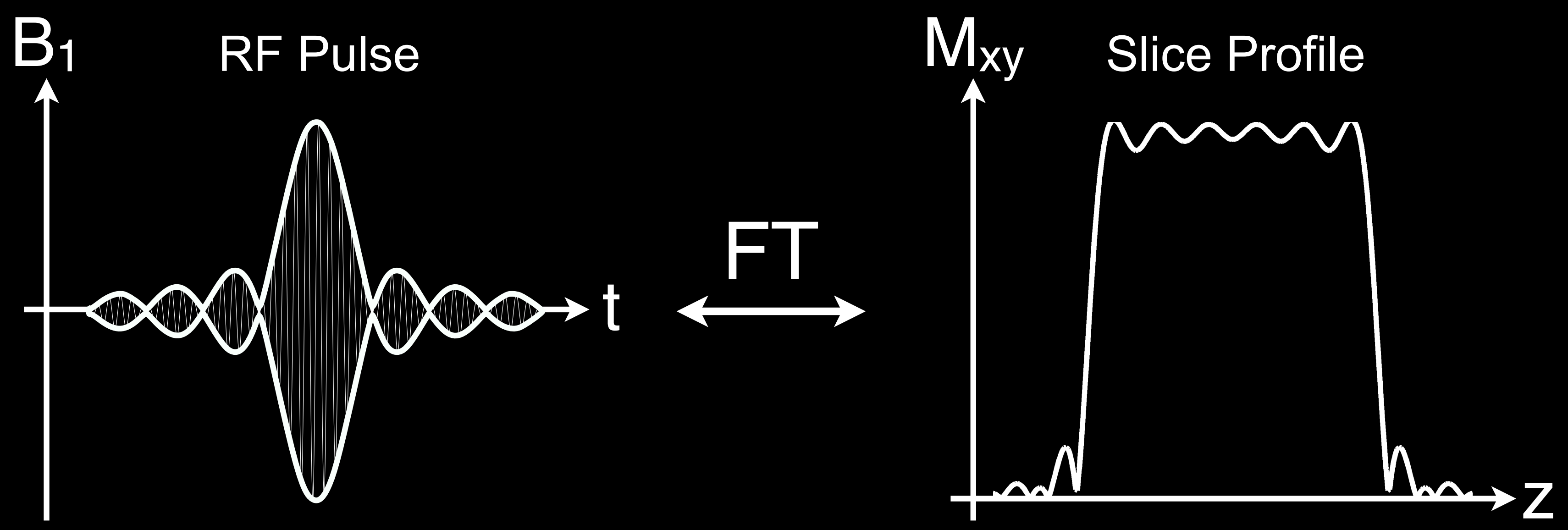
$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \gamma B_1(t) & 0 & \gamma G_z \cdot z \end{vmatrix} \Rightarrow \begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \omega_1(t) & 0 & \omega(z) \end{vmatrix}$$

Coupled system of differential equations!

To The Board…

Slice Selective Excitation

- What is the ideal slice profile?
- Changing the shape (envelope function) of the pulse affects the *excitation bandwidth* of excitation.
- How do we know which shape to use?
  - Small Tip Angle Approximation
    - ➡ Slice profile depends on the FT of the shape.



Small Tip Angle Approximation

Small Tip Approximation

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \omega_1(t) & 0 & \omega(z) \end{vmatrix}$$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

Solving a first order linear differential equation:

$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\omega(z)(t-s)} ds$$

To the board ...

Summary for Small Tip

Assuming carrier frequency = resonance frequency

$$\omega_{RF} = \omega_0$$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix}$$

$M_z \approx M_0$ small tip-angle approximation

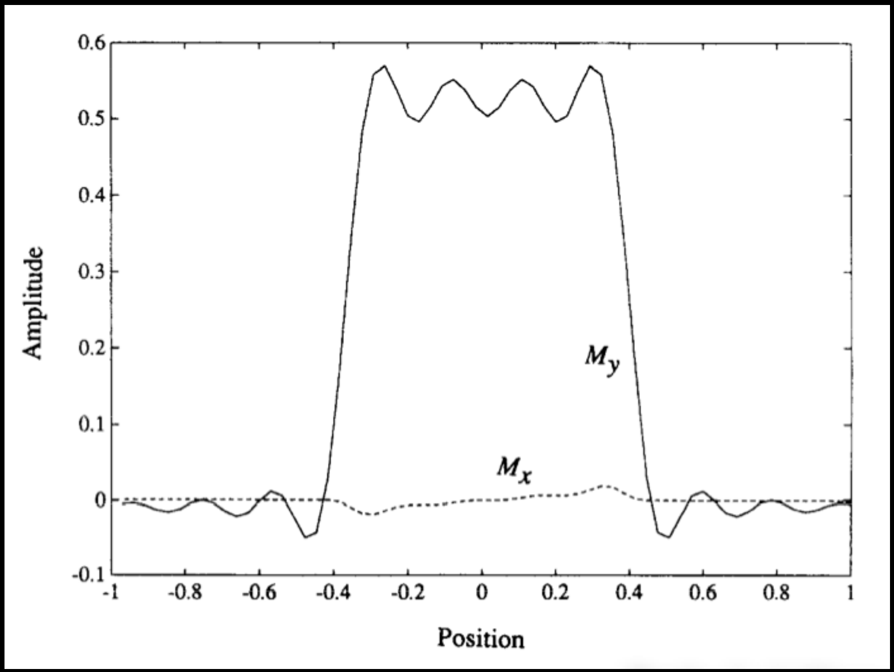
$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$

Small Tip Approximation

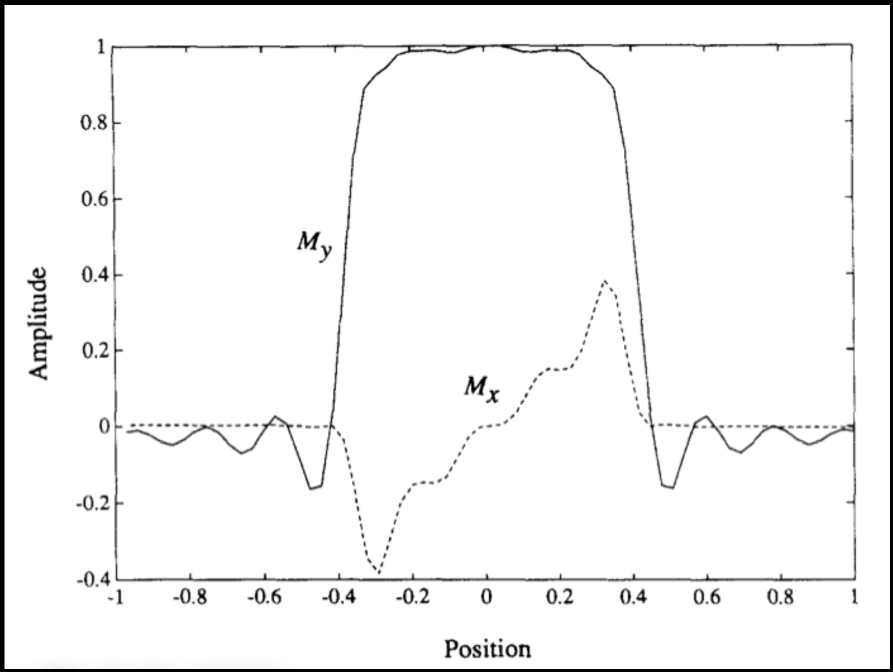
1. The excitation profile, within the small angle approximation, is just the Fourier transform of the pulse.
2. Remember that the Bloch equations are non-linear and thus cannot be expected to behave linearly.
3. The approximation works surprisingly well even for flip angles up to 90°!

Shaped Pulses

30°



90°



Pauly, J. *J. Magn. Reson.* **81** 43-56 (1989)

The small flip angle approximation still works reasonably well for flip angles that aren't necessarily "small".

Truncation Artifacts

In MRI we want pulses to be as short as possible:

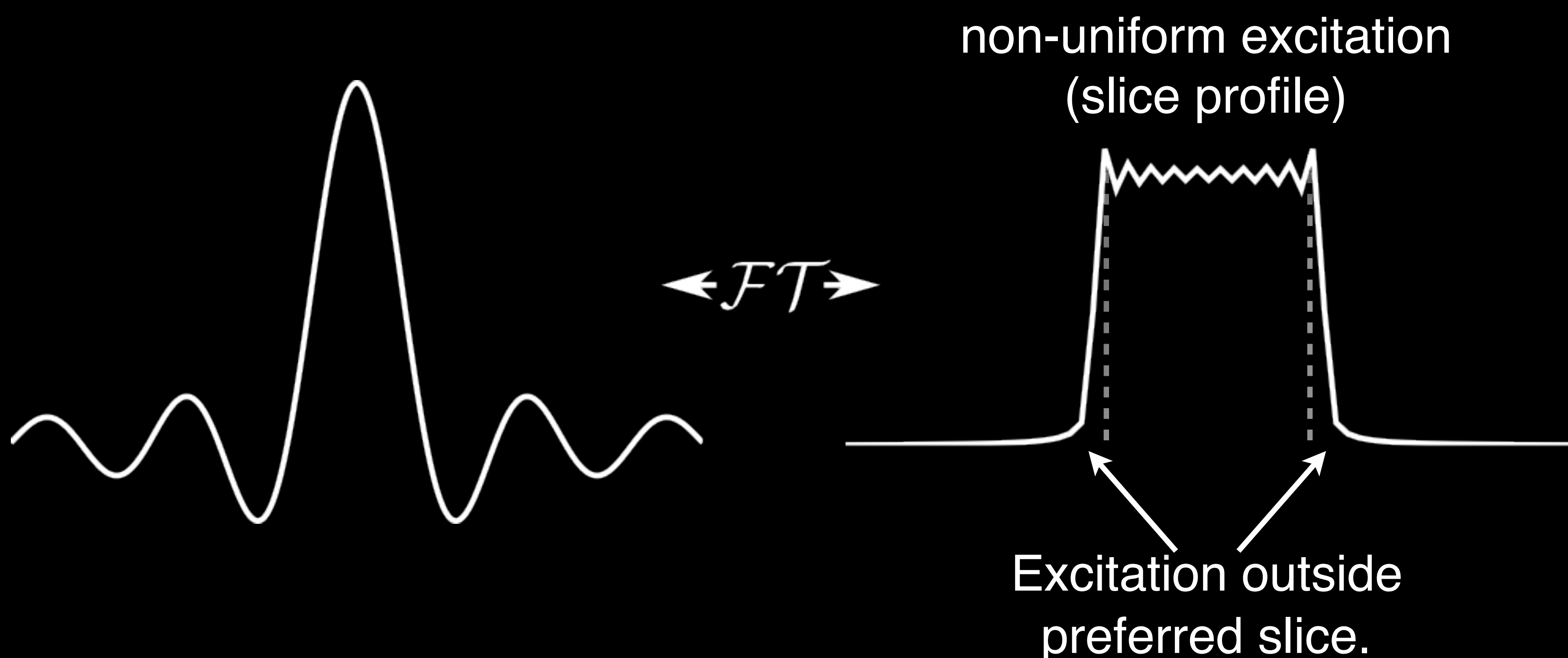
1) To avoid relaxation effects.
2) To improve scan efficiency.

The ***sinc*** function is defined over all time, which is impractical in any experiment.

The ***sinc*** pulse needs to be truncated to be appropriate for clinical scans.

Truncation Artifacts

What happens when we truncate our pulses?



Deviations from the ideal slice profile
are known as truncation artifacts.

Reducing Truncation Artifacts

Alternative Pulse Shapes

$$B_x(t) = A \exp\left[-a(t-\tau/2)^2\right]$$ Gaussian

Reduced side-lobes, but not as flat of a slice profile.

Thanks

Daniel B. Ennis, Ph.D.
ennis@ucla.edu
310.206.0713 (Office)
http://mrrl.ucla.edu

Peter V. Ueberroth Bldg.
Suite 1417, Room C
10945 Le Conte Avenue