$$f(x) = f_e(x) + f_o(x) = \frac{1}{2}\left(f(x)+f(-x)\right) + \frac{1}{2}\left(f(x)-f(-x)\right)$$

$$\mathcal{F}\{f(x)\} = F(f) = F_e(f) + F_o(f)$$

$$= \frac{1}{2}\left[F(f)+F(-f)\right] + \frac{1}{2}\left[F(f)-F(-f)\right]$$

* $f(x) = f_e(x)$

$$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f_e(x)\cos(2\pi x f)\,dx$$

$\downarrow$

even & real

$$= Re\{F_e(f)\}$$

* $f(x) = f_o(x)$

$$\mathcal{F}\{f(x)\} = -j\int_{-\infty}^{\infty} f_o(x)\sin(2\pi x f)\,dx$$

$\Downarrow$

$$= Im\{F_o(f)\}$$

odd & Imaginary

* Hermitian Symmetry

① Real valued f

$$f(x) = f_e(x) + f_o(x)$$

$$\mathcal{FT}\{f(x)\} = \mathcal{FT}\{f_e(x)\} + \mathcal{FT}\{f_o(x)\}$$

$$\Rightarrow F(f) = Re\{F_e(f)\} + Im\{F_o(f)\}$$

$$\overline{F}(f) = Re\{F_e(f)\} \underbrace{- Im\{F_o(f)\}}_{+ Im\{\underbrace{-F_o(f)}_{F_o(-f)}\}}$$

$$= F(-f)$$

$$\therefore F(f) = \overline{F}(-f)$$ conjugate symmetry

② Imaginary valued f

$$\mathcal{FT}\{f(x)\} = \mathcal{FT}\{f_e(x)\} + \mathcal{FT}\{f_o(x)\}$$

$$\Rightarrow F(f) = Re\{F_o(f)\} - Im\{F_e(f)\}$$

$$\overline{F}(f) = Re\{F_o(f)\} + Im\{F_e(f)\}$$

$$= -Re\{F_o(-f)\} + Im\{F_e(-f)\}$$

$$= -F(-f)$$

$$\therefore F(f) = -\overline{F}(-f)$$ conjugate antisymmetry