MRI Systems II – Nuclear Precession and B1

M219 - Principles and Applications of MRI

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Course Overview

- Course website
  - https://mrrl.ucla.edu/pages/m219

- 2023 course schedule
  - https://mrrl.ucla.edu/pages/m219_2023

- Assignments
  - Homework #1 due on 1/30

- Office hours, Fridays 10-12pm
  - In-person (Ueberroth, 1417B)
  - Zoom is also available

Main Field (B₀) - Principles

- B₀ is a strong magnetic field
  - >1.5T
  - Z-oriented

$$\vec{B}_0 = B_0 \vec{k}$$

- B₀ generates bulk magnetization ($\vec{M}$)
  - More B₀, more

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

- B₀ forces $\vec{M}$ to precess
  - Larmor Equation

$$\omega = \gamma B$$

To the board

Equation of Motion for the Bulk Magnetization

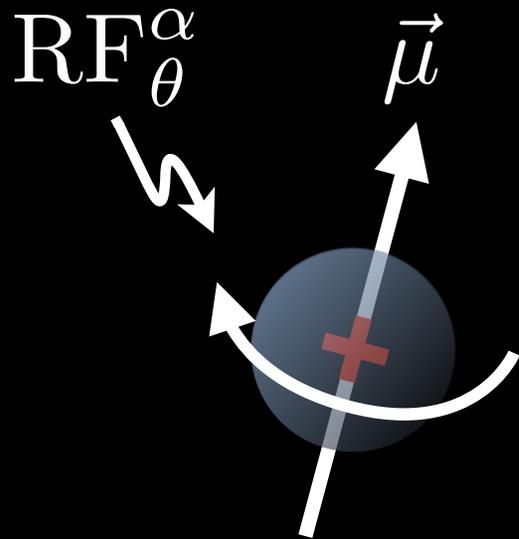$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma\vec{B}$$

Equation of motion for an ensemble of spins
[Classical Description]

What is a general solution?

The *equation of motion* describes the bulk magnetization "behavior" in the presence of a B-field.
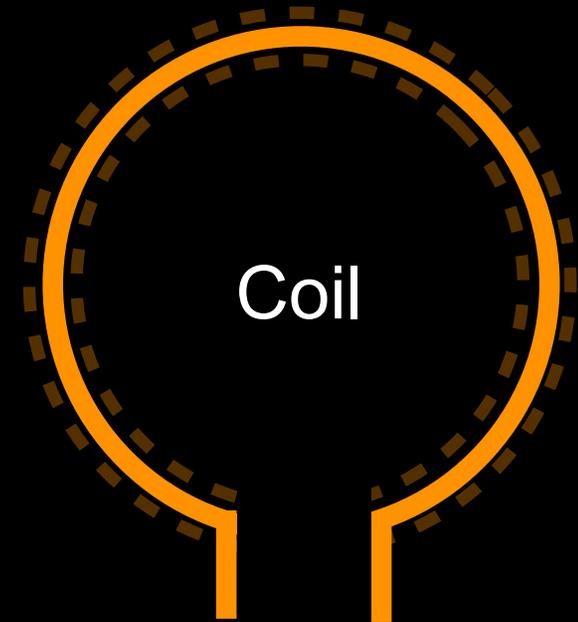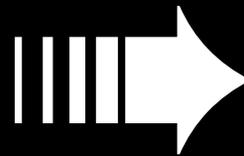
To the board

Signal Reception

$\mathrm{RF}_\theta^\alpha$
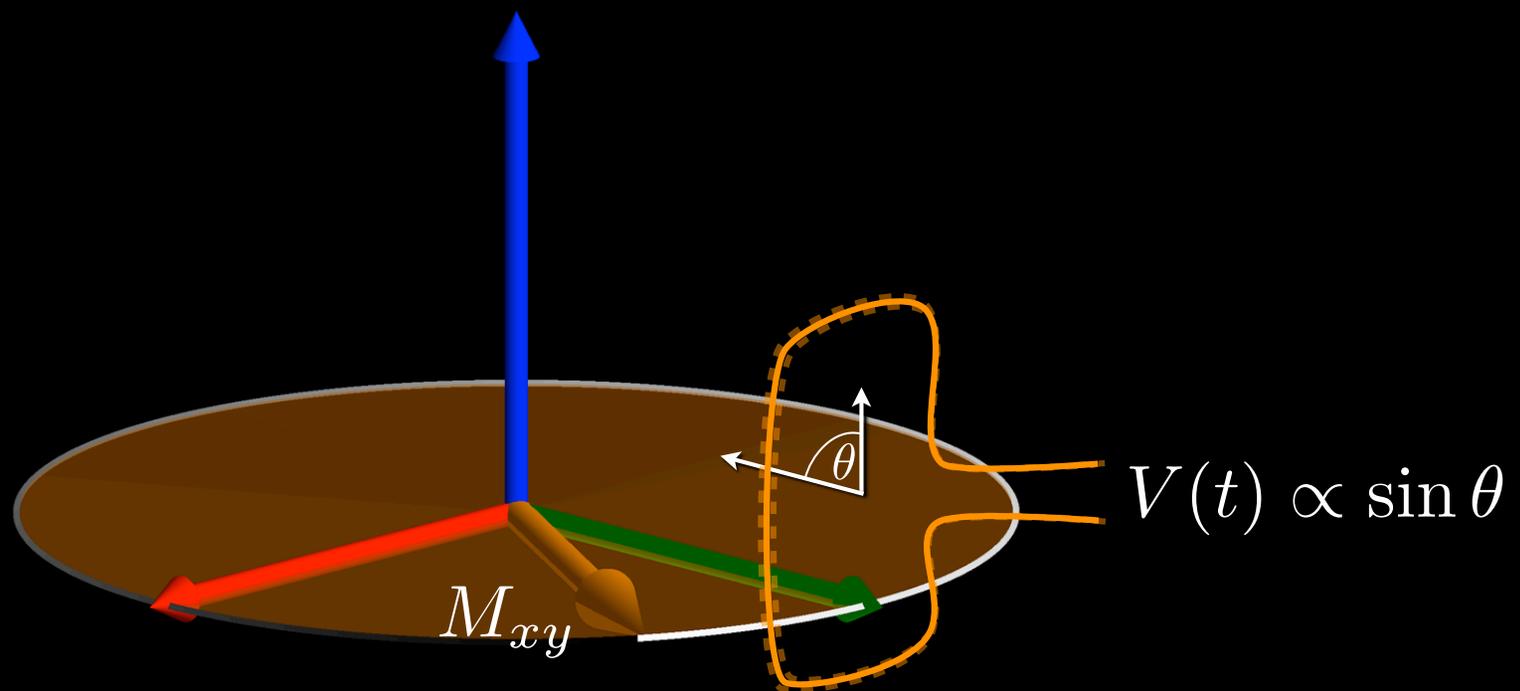
$\vec{\mu}$

Faraday's Law
of Induction

Coil

$M_{xy}\left(\vec{r}, t\right)$

$V(t)$

NMR Signal Detection

- **Coil only detects M$_{xy}$**
- **Coil does *not* detect M$_z$**
- **Coil must be properly oriented**

$$V(t) \propto \sin\theta$$

$$M_{xy}$$

How does RF alter $\vec{M}$?

$$\vec{B}_1\left(t\right)$$

Generating B$_1$-Fields

MRI Hardware

Y-grad

X-grad

Cryostat

Z-grad

Body Tx/Rx Coil (B_1)

Main Coil (B_0)

Image Adapted From: http://www.ee.duke.edu/~jshorey
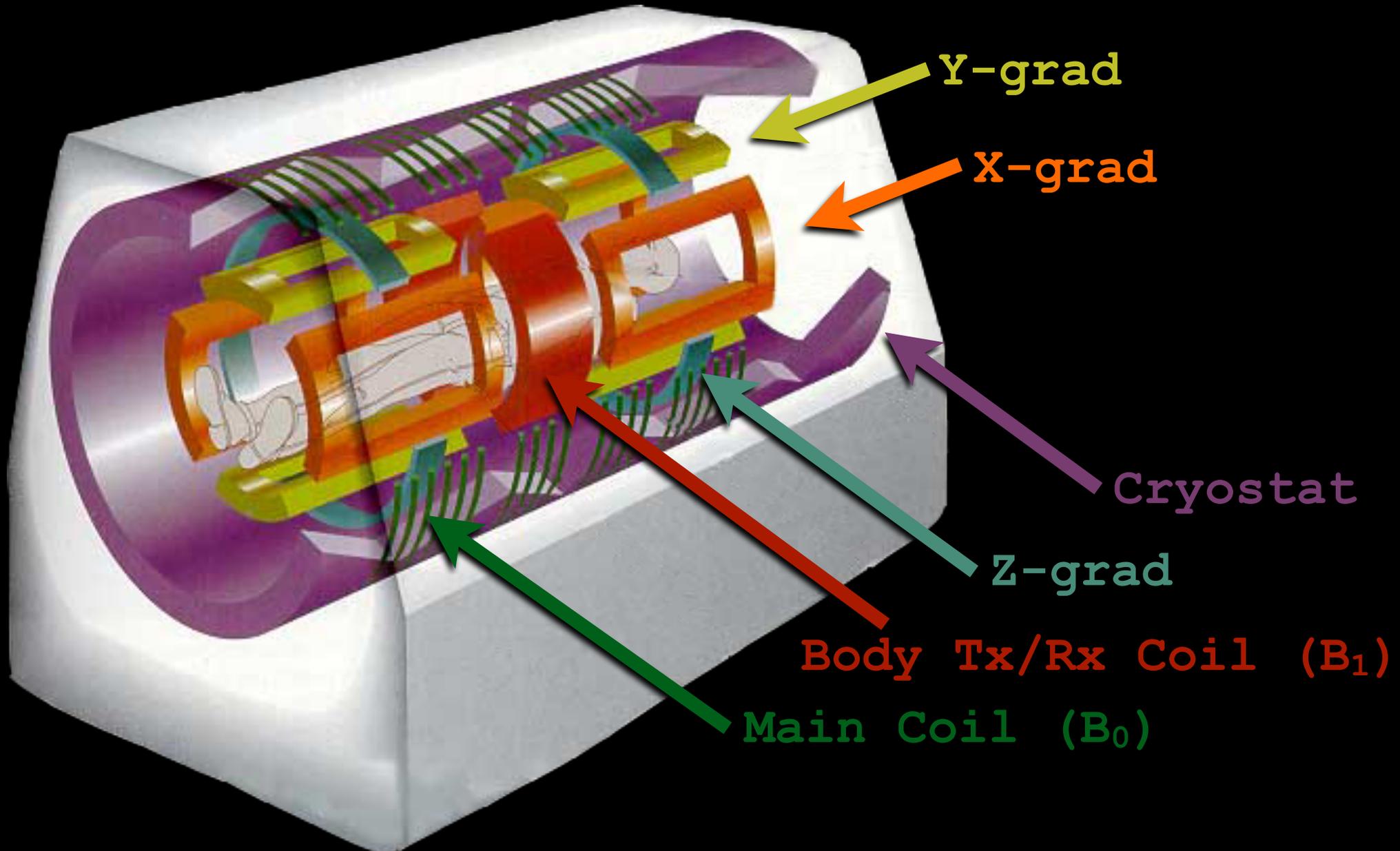
RF Shielding

- RF fields are close to FM radio
  - ^1H @ 1.5T $\Rightarrow$ 63.85 MHz
  - ^1H @ 3.0T $\Rightarrow$ 127.71 MHz
  - KROQ $\Rightarrow$ 106.7 MHz

- Need to shield local sources from interfering

- Copper room shielding required
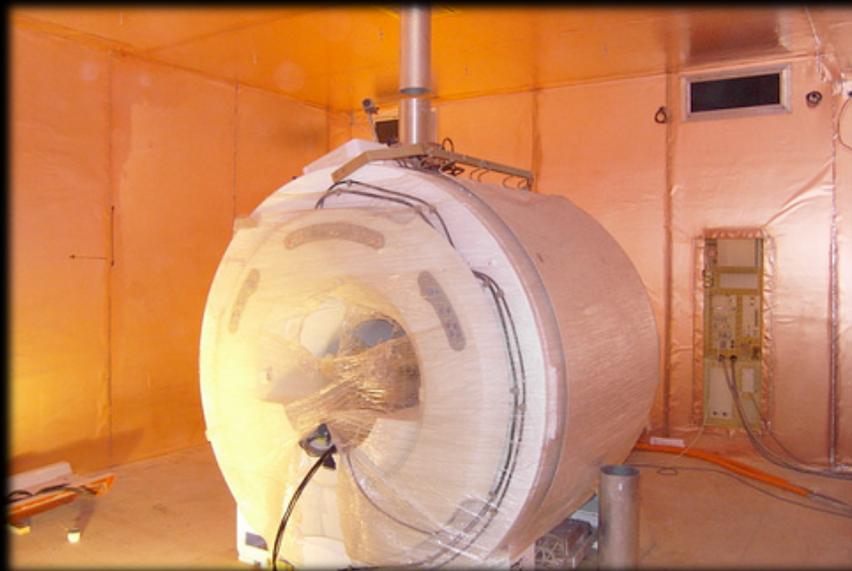
RF Birdcage Coil

- **Most common design**

- **Highly efficient**
  - Nearly all of the fields produced contribute to imaging

- **Very uniform field**
  - Especially radially
  - Decays axially
  - Uniform sphere if L≈D

- **Generates a "quadrature" field**
  - Circular polarization

Body Tx/Rx Coil (B$_1$)

CCW (lefthand)

CW (righthand)

http://mri-q.com/birdcage-coil.html
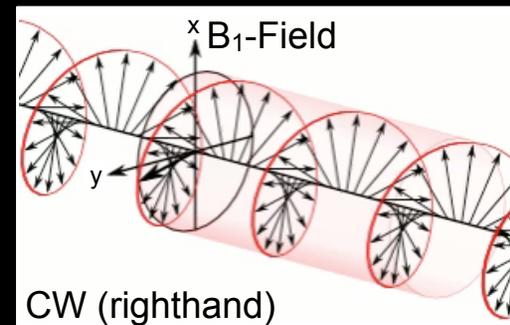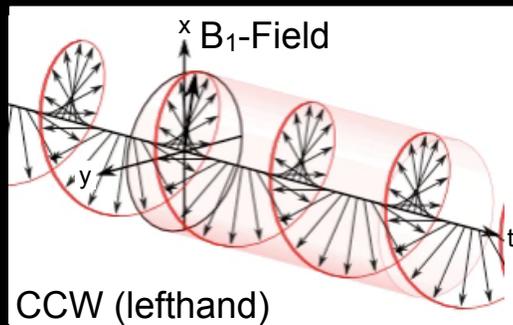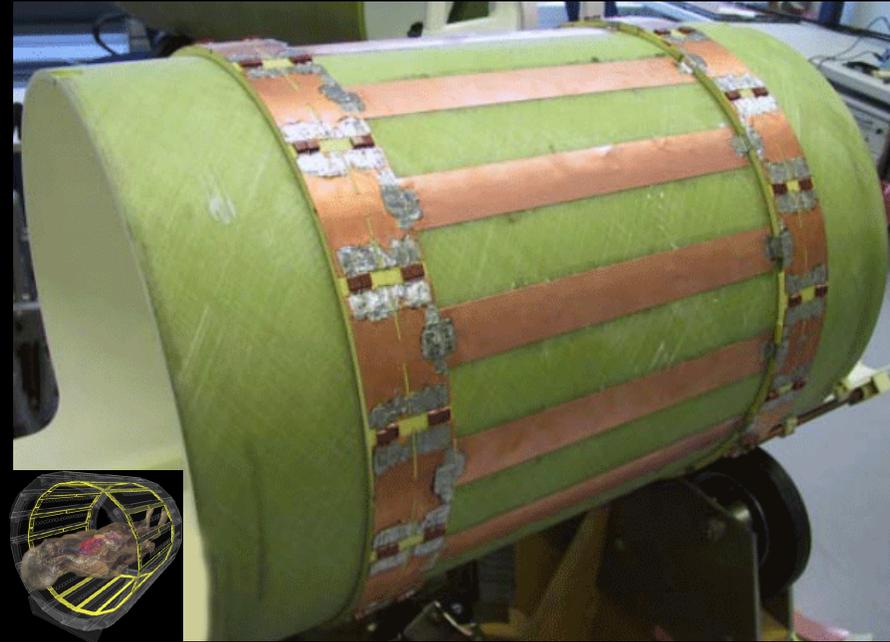
B$_1$ Field - RF Pulse

- B$_1$ is a
  - radiofrequency (RF)
    - 42.58MHz/T (63MHz at 1.5T)
  - short duration pulse (~0.1 to 5ms)
  - small amplitude
    - <30 µT
  - circularly polarized
    - rotates at Larmor frequency
  - magnetic field
  - perpendicular to B$_0$
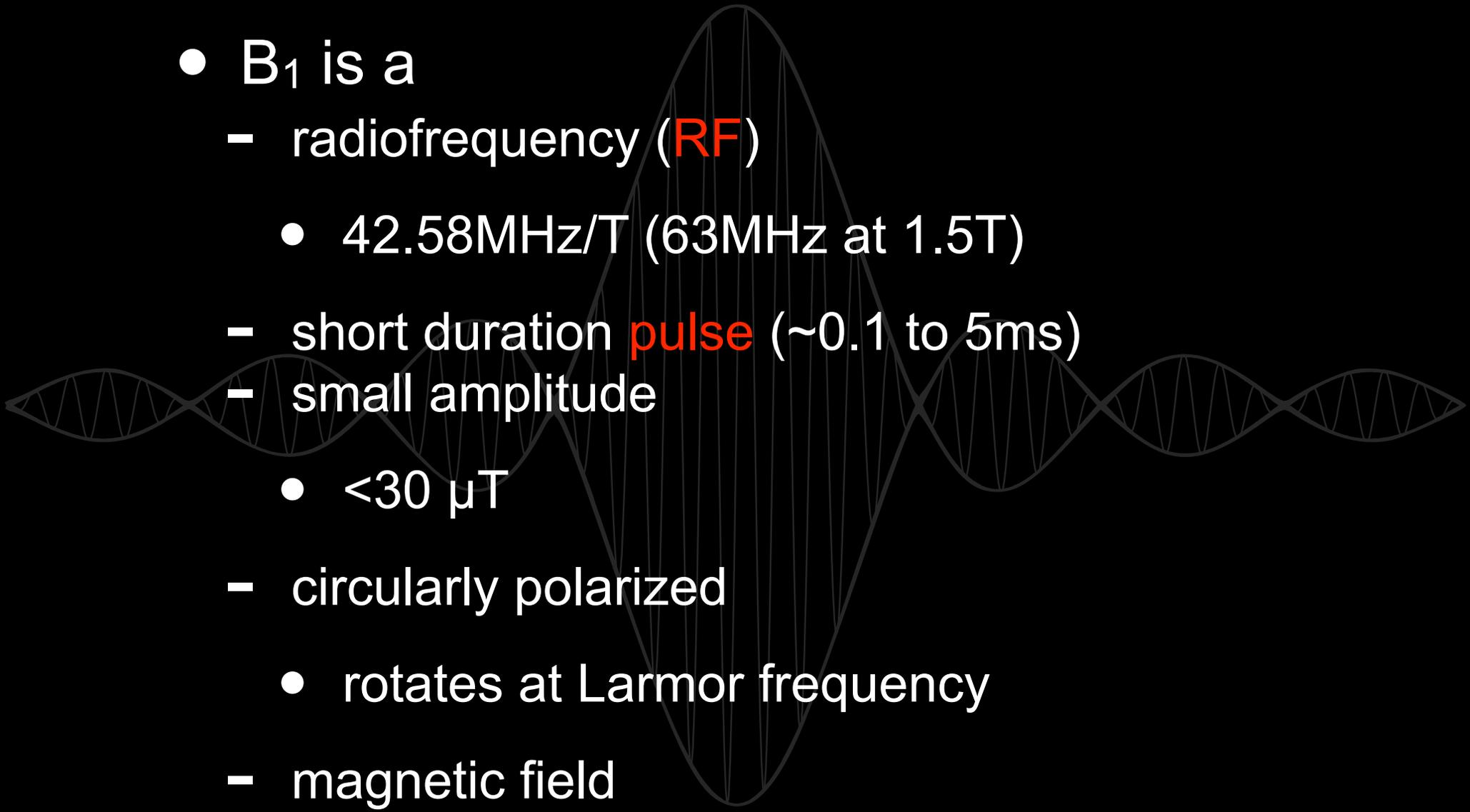
Basic RF Pulse

$$\vec{B} = \vec{B}_0 + \vec{B}_1(t)$$

$$\vec{B}_1(t) = B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$B_1^e(t)$ **pulse envelope function**

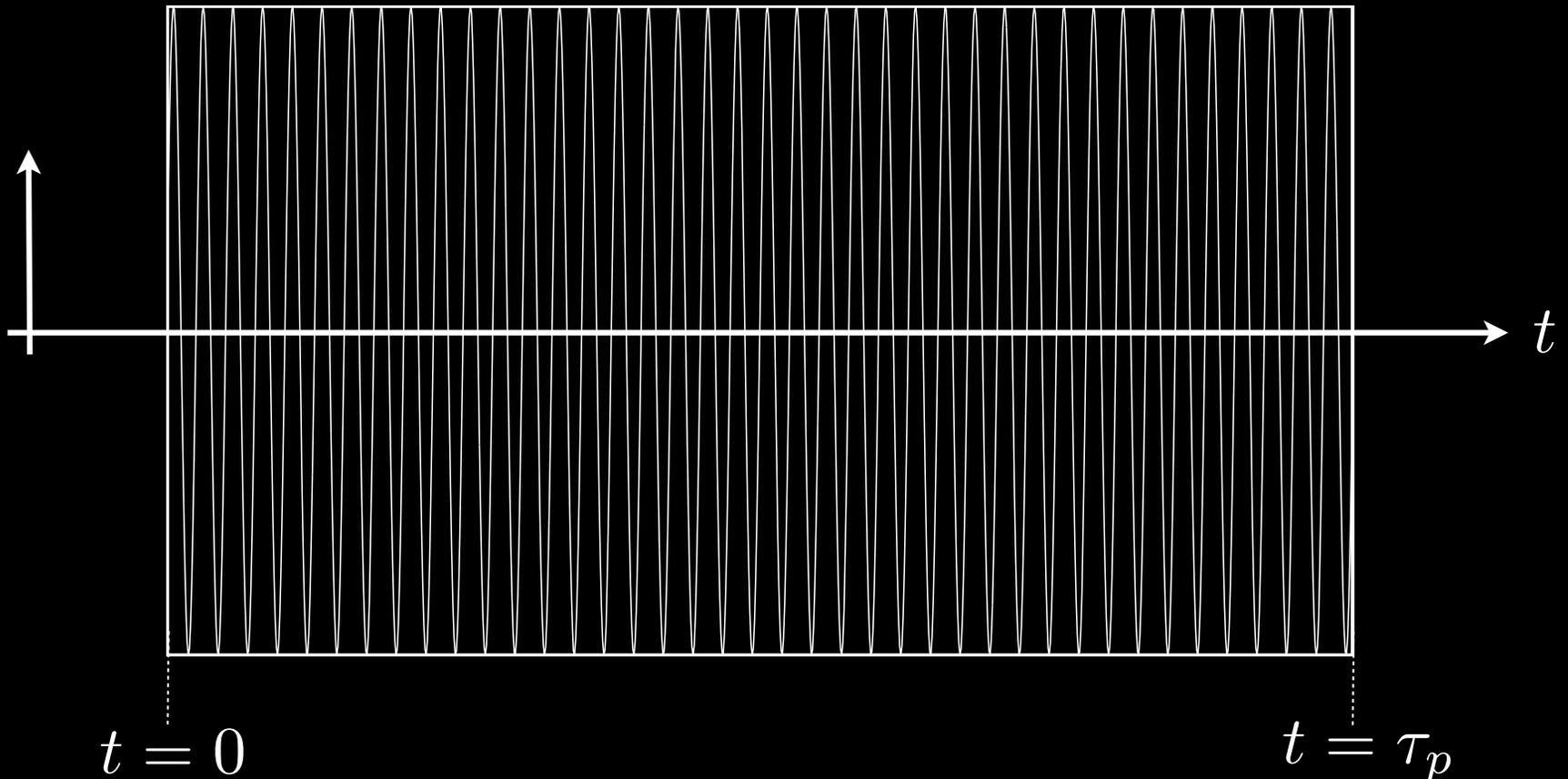ω_{RF} **excitation carrier frequency**

θ **initial phase angle**

B_1 is perpendicular to B_0.

$$\vec{B}_0 = B_0\hat{k}$$

Rect Envelope Function

$$B_1^e(t) = B_1 \sqcap \left(\frac{t - \tau_p/2}{\tau_p} \right) = \begin{cases} B_1, & 0 \leq t \leq \tau_p \\ 0, & otherwise \end{cases}$$

$t = 0$ $t = \tau_p$

Sinc Envelope Function

$$B_1^e\left(t\right) = \begin{cases} B_1\mathrm{sinc}\left[\pi f_\omega\left(t - \tau_p/2\right)\right], & 0 \leq t \leq \tau_p \\ 0, & otherwise \end{cases}$$

$t = 0$

$t = \tau_p$

t
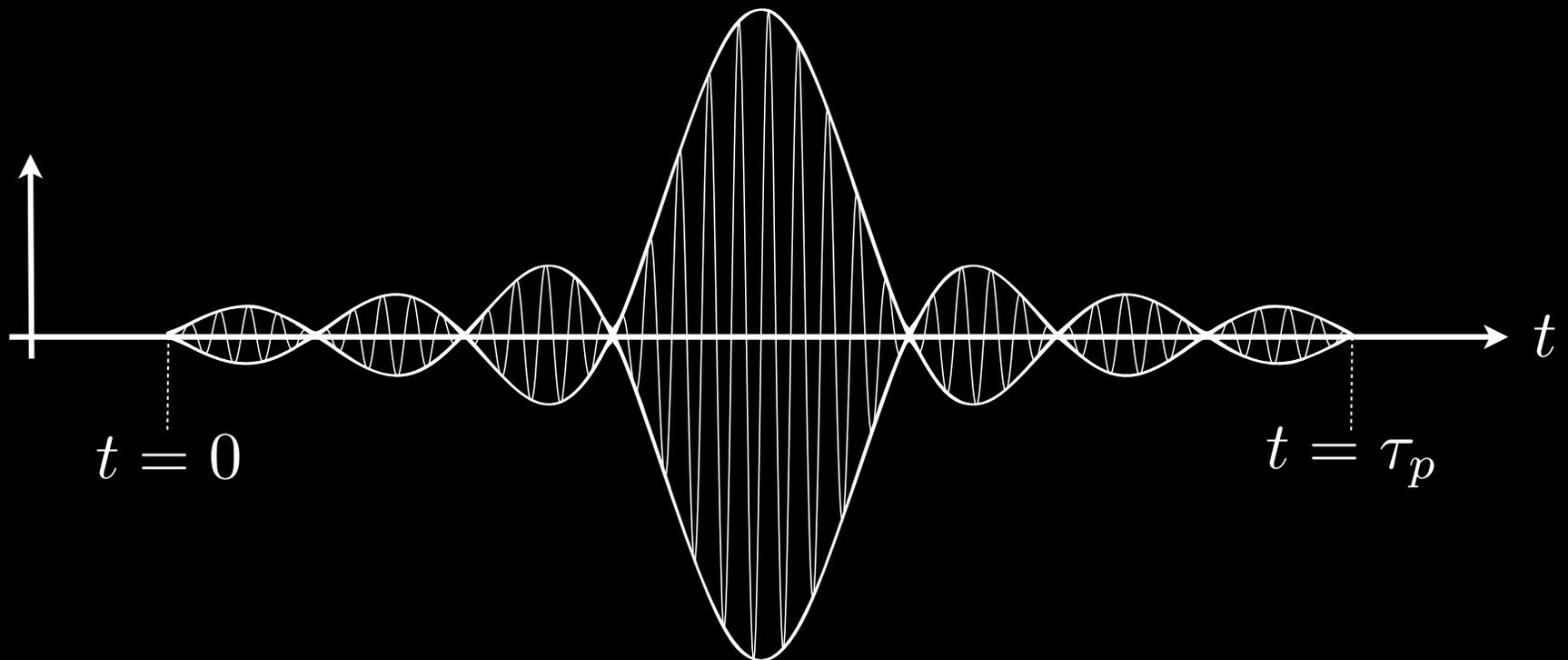
Rotating Frame

Lab vs. Rotating Frame

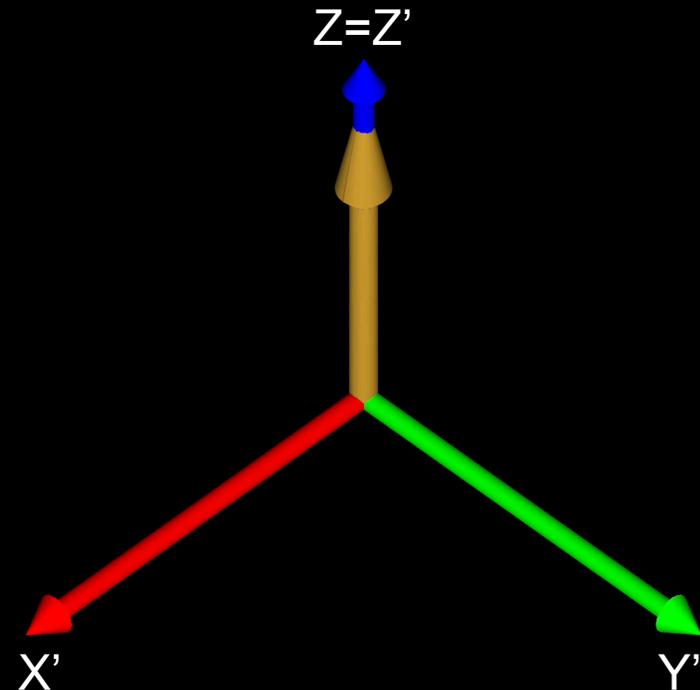- **The rotating frame simplifies the mathematics and permits more intuitive understanding.**

90° RF (Laboratory Frame)
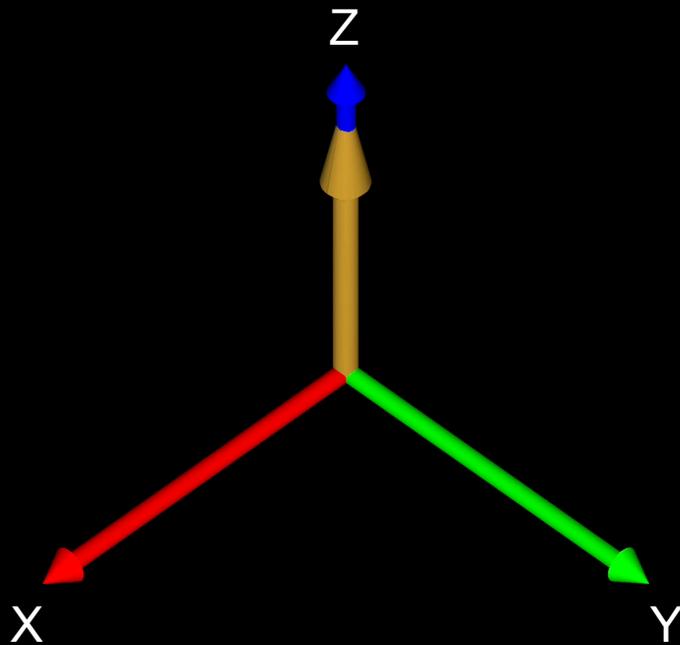
90° RF (Rotating Frame)

Z

Z=Z'

X

Y

X'

Y'

Spins Precess

Observer Precesses

Note: Both coordinate frames share the same z-axis.

Combined B_0 & B_1 Effects

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

$$= \vec{M} \times \gamma \left(\vec{B}_0 + \vec{B}_1 \right)$$

Relationship Between Lab and Rotating Frames

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma\vec{B}$$

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{aligned} B_{z'} &\equiv B_z \\ M_{z'} &\equiv M_z \end{aligned}$$

$$\overrightarrow{M}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{M}_{rot}(t)$$

Bulk magnetization components in the rotating frame.

$$\overrightarrow{B}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{B}_{rot}(t)$$

Applied B-field components in the rotating frame.

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma\vec{B} \qquad \Longrightarrow \qquad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma\vec{B}_{eff}$$

Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma\vec{B}$$

Equation of motion for an ensemble of spins (isochromats).
[Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma\left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}\right)$$

Equation of motion for an ensemble of spins (isochromats).
[Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of *B0*.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\vec{B}(t) = B_0\hat{k} + B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$$\vec{B}_{lab}(t) = \begin{pmatrix} B_1^e(t)\cos(\omega_{RF}t + \theta) \\ -B_1^e(t)\sin(\omega_{RF} + \theta) \\ B_0 \end{pmatrix} \qquad \vec{B}_{rot}(t) = \begin{pmatrix} B_1^e(t)\cos\theta \\ -B_1^e(t)\sin\theta \\ B_0 \end{pmatrix}$$

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that
M experiences in the
rotating frame.

Fictitious field that demodulates
the apparent effect of *B0.*

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

Assume no RF phase ($\theta = 0$)

$$\overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

$$\overrightarrow{B}_{eff}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \quad \dfrac{\omega_{RF}}{\gamma} \end{pmatrix}$$

Questions?

- Related reading materials
  - Nishimura Chap 3 and 4

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