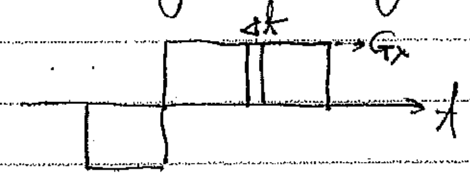
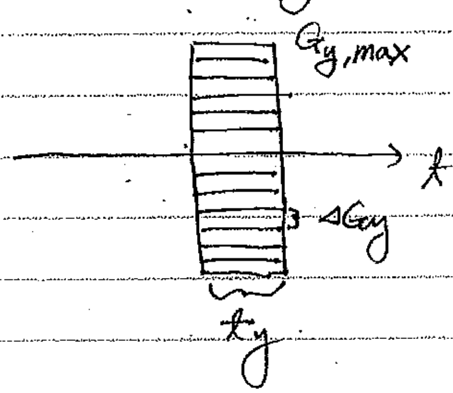
* Spatial resolution along "frequency encoding" & "phase encoding" direction

\- frequency encoding




$$W_{kx} = N_{read} \times \Delta k_x$$
$$= N_{read} \cdot \frac{\gamma}{2\pi} G_x \Delta t$$
$$= \frac{\gamma}{2\pi} G_x T_{read}$$

\- Phase encoding




$$W_{ky} = N_{PE} \times \Delta k_y$$
$$= N_{PE} \cdot \frac{\gamma}{2\pi} \Delta G_y \cdot t_y$$
$$= \frac{\gamma}{2\pi} 2 G_{y,max} \cdot t_y$$

①

* $TBW = 4$ , Pulse duration = 1 ms

$\Rightarrow BW = 4\,kHz$

$$f = -\frac{\gamma}{2\pi} G_z \cdot z$$

4.257 kHz/G

to make $\Delta z = 1$ cm

$$G_z = \frac{\Delta f}{\Delta z} \cdot \frac{1}{\frac{\gamma}{2\pi}} = \frac{4}{1} \cdot \frac{1}{4.257}$$

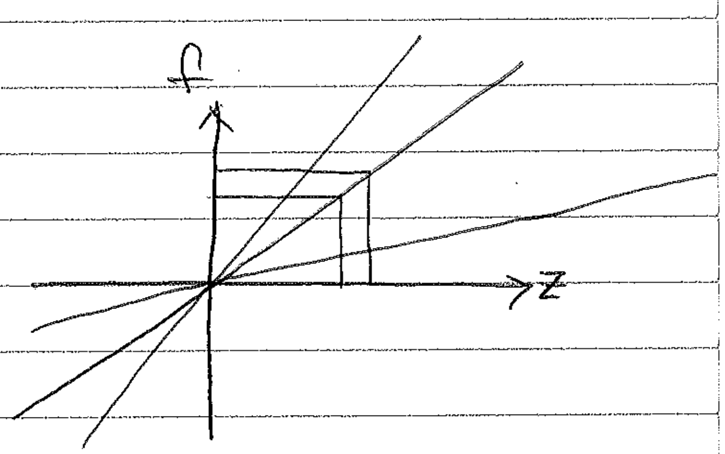
* $TBW = 16$ , pulse duration = 1 ms

$\Rightarrow BW = 16\,kHz$

To make $\Delta z = 1$ cm

$$G_z = \frac{16}{1} \cdot \frac{1}{4.257}$$

* $\Delta f = -\frac{\gamma}{2\pi} G_z \cdot \Delta z$



* Full analytical solution (5)

$$\frac{d\vec{M}}{dt} = \begin{bmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{bmatrix} \vec{M}$$

$M_r = M_x + i M_y \leftarrow$ phasor representation

$$\frac{dM_r}{dt} = -i\, \omega(z)\, M_r + i\, \omega_1(t)\, M_z$$

$$\frac{dM_z}{dt} = -i\, \omega_1(t)\, M_y$$

Solution $\Longrightarrow$ very difficult $\Longrightarrow$ using approximation

$\sin\theta \simeq \theta$

$\cos\theta \simeq 1$

$M_z \simeq M_0 \simeq$ constant

$$\Rightarrow \frac{dM_z}{dt} = 0$$

$$\frac{dM_r}{dt} = -i\, \omega(z)\, M_r + i\, \omega_1(t)\, M_0$$

---

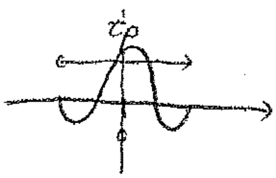
$$\frac{dy}{dx} + p(x)\,y = q(x)$$

$$y = \frac{\int u(x)\, q(x)\, dx}{u(x)}, \quad \text{where} \quad u(x) = \exp\left\{\int p(x)\, dx\right\}$$

$$\underset{x}{\frac{d M_r^{\,y}}{dt}} + \underbrace{i\,w(z)}_{p(x)} M_r^{\,y} = \underbrace{i\,W_1(t)\,M_0}_{q(x)}$$

$$M_r(t,z) = i\cdot M_0\, e^{-iw(z)\cdot t}\int_0^t W_1(\tau)\, e^{-iw(z)\tau}\, d\tau$$

Assume, the RF pulse is symmetric and peaks at $t=0$ such that the pulse ends at $t = \tau_p/2$, $\tau_p$ is the length of the RF pulse $\longrightarrow$ let $\tau' = \tau - \tau_p/2$



$$M_r(\tau_p) = i\,M_0\, e^{-\frac{i w(z)\tau_p}{2}} \int_{-\tau_p/2}^{\tau_p/2} W_1\left(t+\frac{\tau_p}{2}\right) e^{-iw(z)t}\, dt$$

assuming
$$W_1\left(t+\frac{\tau_p}{2}\right) = \begin{cases} 0 & \text{for } |t| > \frac{\tau_p}{2} \\ W_1\left(t+\frac{\tau_p}{2}\right) & \text{for } |t| < \frac{\tau_p}{2} \end{cases}$$

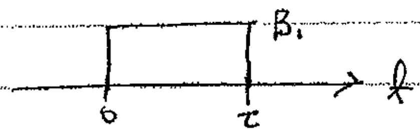
$$\Rightarrow M_r(\tau,z) = i\,M_0\, e^{-i\,w(z)\,\tau/2}\, \mathcal{F}_{1D}\left\{W_1\left(t+\frac{\tau}{2}\right)\right\}\Bigg|_{f=-f(z) = -(\gamma/2\pi)\,G_z\cdot z}$$

$$* \quad |M_{xy}(\tau, z)| = M_0 \, \mathcal{F}_{1D}\left\{ W_1\left(t + \frac{\tau}{2}\right) \right\} \Big|_{f = -\frac{\gamma}{2\pi} G_z z}$$

* Small tip-angle example

- consider a rectangular RF pulse (duration of $\tau$)

$$B_1(t) = B_1 \cdot \Pi\left(\frac{t - \tau/2}{\tau}\right)$$

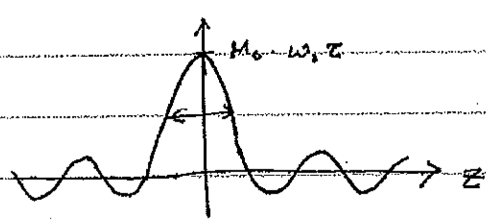



- $$M_r(\tau, z) = i M_0 e^{-i\omega(z)\tau/2} \mathcal{F}_{1D}\left\{\omega_1\left(t + \frac{\tau}{2}\right)\right\}\Big|_{f = \frac{-\gamma}{2\pi} G_z \cdot z}$$

$$\omega_1\left(t + \frac{\tau}{2}\right) = \underbrace{\gamma \cdot B_1}_{\omega_1} \cdot \Pi\left(\frac{t}{\tau}\right)$$

$$\mathcal{F}_{1D}\left\{\Pi\left(\frac{t}{\tau}\right)\right\} = \tau \, \mathrm{sinc}(\tau \cdot f)$$

$$\Rightarrow M_r(\tau, z) = i M_0 \underbrace{e^{-i\omega(z)\tau/2}}_{\text{canceled out by the refocusing pulse}} \cdot \omega_1 \tau \, \mathrm{sinc}\left(\tau \cdot \frac{\gamma}{2\pi} G_z \cdot z\right)$$




$$\Delta z = \frac{1}{\frac{\tau}{2\pi} \gamma \cdot G_z}$$