* Hermitian Symmetry

① Real valued f

$$f(x) = f_e(x) + f_o(x)$$

$$\mathcal{FT}\{f(x)\} = \mathcal{FT}\{f_e(x)\} + \mathcal{FT}\{f_o(x)\}$$

$$\Rightarrow F(f) = Re\{F_e(f)\} + Im\{F_o(f)\}$$

$$\bar{F}(f) = Re\{F_e(f)\} \underbrace{- Im\{F_o(f)\}}$$

$$+ Im\{\underbrace{-F_o(f)}_{F_o(-f)}\}$$

$$= F(-f)$$

$$\therefore F(f) = \bar{F}(-f)$$ conjugate symmetry

② Imaginary valued f

$$\mathcal{FT}\{f(x)\} = \mathcal{FT}\{f_e(x)\} + \mathcal{FT}\{f_o(x)\}$$

$$\Rightarrow F(f) = Re\{F_o(f)\} - Im\{F_e(f)\}$$

$$\bar{F}(f) = Re\{F_o(f)\} + Im\{F_e(f)\}$$

$$= -Re\{F_o(-f)\} + Im\{F_e(-f)\}$$

$$= -F(-f)$$

$$\therefore F(f) = -\bar{F}(-f)$$ conjugate antisymmetry