M219, Winter 2018

Homework Assignment #3

Due Thursday, February 22nd by 10pm

To submit the assignment, e-mail `DEnnis@mednet.ucla.edu` a PDF entitled `M219_HW03_[First Initial]_[Last Name].pdf` (*e.g.* `M219_HW03_D_Ennis.pdf`). Please only submit neat and clear solutions. Late assignments will be discounted by $e^{-t/\tau}$, where $\tau = 72$ hours.

For all problems – Clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually. As appropriate, each solution should be obtained using Matlab. Please comment and submit your code using the `publish.m` function for each problem.

1 Gradient Echo vs. Spin Echo Contrast (3 points)

The equations that describe the echo amplitude for a gradient echo and spin echo sequence are as follows:

$$A_{GRE} = \frac{\rho\left(1 - e^{-\frac{\mathrm{TR}}{\mathrm{T_1}}}\right)}{1 - \cos\alpha e^{-\frac{\mathrm{TR}}{\mathrm{T_1}}}} \sin\alpha e^{-TE/T_2^*} \tag{1}$$

$$A_{SE} = \rho\left(1 - e^{-\frac{\mathrm{TR}}{\mathrm{T_1}}}\right) e^{-TE/T_2} \tag{2}$$

A. Using the above equations in MATLAB to determine the TE and TR needed to generate the maximum T_1 contrast between Tissue A ($\rho = 1.0$, T_1=2000ms, T_2=40ms, T_2^*=25ms) and Tissue B ($\rho = 1.0$, T_1=500ms, T_2=40ms, T_2^*=25ms) for both a gradient echo sequence and a spin echo sequence. Assume the pulse sequences are both limited by: 5ms≤TE≤100ms, 10ms≤TR≤10,000ms. Assume α=30° for GRE and α=90° for SE. This can be done by simulating the signal amplitude for a range of TE and TR.

B. Is it preferable to use a gradient echo or a spin echo sequence for T_1 contrast? Why?

2 Slice Selection (3 points)

A. If a gradient of $G_z = 8G/cm$ is applied and we want to excite a slice that is 3mm thick at isocenter what should be the center frequency (ω) and bandwidth ($\Delta\omega$) of the RF pulse for ^1H on a 3.0T scanner?

B. Define ω and $\Delta\omega$ for a slice that is +30mm from isocenter in the z-direction and 3mm thick.

C. Redesign the RF pulse from Part A to excite 31P ($\gamma = 17.235 MHz/T$) at isocenter. What is the new center frequency (ω)? With the same bandwidth ($\Delta\omega$) and G_z (8G/cm), what is the new slice thickness?

3 k-space and Image Space (4 points)

A. Import the provided image (heart.mat) into Matlab and render an image of the k-space magnitude (fft2.m).

   Hint: Use fftshift.m to ensure the dominant signals (low spatial frequencies) occur at the k-space center.

B. Add a noisy spike artifact to a Fourier coefficient in the upper left quadrant of k-space and show the result in image space (ifft2.m). Describe the result and why this occurs.

C. Remove (i.e set to zero) all but the middle ten lines from the original k-space data (from the original FFT, without the noisy spike). Show the resulting k-space magnitude and the resultant image. Describe what you see.

D. Rotate the image by -45^o (J = imrotate(I,-45,'bilinear','crop');). Show the resulting k-space magnitude and the resultant image. Describe what you see.

E. Remove (set to zero) every fourth line of the k-space data from the original FFT (without the noisy spike). Show the resulting k-space magnitude and the resultant image. Describe what you see.

4 k-space Sampling (5 points)

The bandwidth (Δf) of a rectangular readout gradient with amplitude G for an arbitrary field of view (FOV) is given by:

$$\Delta f = \gamma \cdot G \cdot FOV \tag{3}$$

If the signal is read out discretely with this gradient across N-points devoting a dwell-time δt to each point in k-space, the k-space increment is given by:

$$\Delta k = \gamma \cdot G \cdot \Delta t \tag{4}$$

A. What is the spatial resolution Δx of this readout given these general expressions?

B. Combine the result of 4A with the given expressions to rewrite a simplified expression for Δf.

C. Let N = 128, Δx = 2mm. On a 3.0T scanner, calculate Δf and Δt for G = 20mT/m and G = 40mT/m. Does increasing G increase or decrease Δt?

D. How many cycles of precession are captured for each of the readout gradients from 4C within a single dwell time Δt for water at 3.0T?