M219 Principles and Applications of MRI (Winter 2024)
Homework Assignment #1 (20 points)
Assigned: 1/15/2024, Due: 1/29/2024 at 5 pm by email

E-mail a PDF (entitled M219_HW01_[Last Name].pdf). Please only submit neat and clear solutions. If your assignments are hard to read, poorly commented, or sloppy points may be deducted. As appropriate, each solution should be obtained using Matlab; provide the code.

For all problems – clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

Problem #1. Nishimura 2.6 (2 points):
A vector ***M*** starting at $[0\ 0\ 1]^T$ is rotated about the x axis by 90°, then about z by some angle θ, and finally about x again by 90°. Determine the resulting vector as a function of θ.

Problem #2. Nishimura 2.7 (2 points):
If the M vector starts at $[0\ 0\ 1]^T$, specify a rotation axis in the *xz* plane and rotation angle α such that the vector is rotated to $[1\ 0\ 0]^T$. Use Eq. 2.39 to verify that the rotation matrix is correct.

Problem #3. Nishimura 4.1 (2 points):
Let the total number of spins per unit volume be $N = n_+ + n_-$.

(a) Use Eq. 4.5 to find an expression for $n_+ - n_-$ in terms of N and the physical parameters (1 point).

(b) If $I_z = ½$ and kT is large compared to the energy difference between n_+ and n_-, simplify your expression from part (a) (1 point).

Problem #4. Nishimura 4.4 (4 points):
Consider two materials *A* and *B* with the same M_0 but with relaxation time constants (T_{1A}, T_{2A}) and (T_{1B}, T_{2B}) respectively. Let $\Delta S_{xy}(t) = M_{xyA}(t) - M_{xyB}(t)$ be the difference in transverse magnetization, and $\Delta S_z(t) = M_{zA}(t) - M_{zB}(t)$ be the difference in longitudinal magnetization. Assume a 90° excitation.

(a) Find an expression for the time that maximizes $|\Delta S_{xy}|$ (1 point).

(b) Find an expression for the time that maximizes $|\Delta S_z|$ (1 point).

(c) Evaluate the expressions from parts (a) and (b) if the two materials are white brain matter and gray brain matter (at $B_0 = 1T$) (2 point).

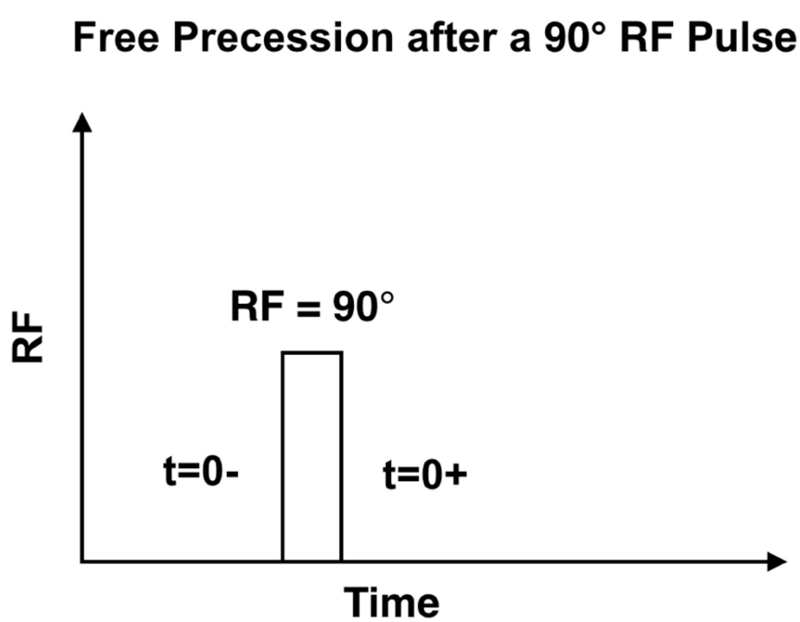
Problem #5 (6 points) – Free Precession in the Laboratory Frame Without Relaxation

(a) If the M vector starts at $[0\ 0\ M_z]^T$, write an expression for each component $M_x(0_-)$, $M_y(0_-)$, and $M_z(0_-)$ immediately prior to a 90 degree RF pulse as shown in the included figure. (1 point)

(b) Write an expression for each component $M_x(0_+)$, $M_y(0_+)$, and $M_z(0_+)$ immediately after a 90 degree RF pulse. Explain what RF phase you used. (1 point)

(c) Write a general expression (include the RF phase!) as a function of time for each component $M_x(t)$, $M_y(t)$, and $M_z(t)$ beginning immediately after the 90° RF pulse. (2 point)

(d) Plot all three components of your answer from (C) in MATLAB. Assume B_0 = 1.5T. Remember to make your simulation time-steps sufficiently small to observe the time varying magnetization. (2 point)



Problem #6 (4 points) – RF Pulses

Design the shortest possible inversion RF half sin-pulse (*i.e.* sin on [0, π]) for ^{31}P at 1.5T that does not exceed the RF amplifier's ability to output $B_{1,max} = 25\mu T$. In doing so, fully specify the variables in the following equation:

$$B_1(t) = B_1^e(t)e^{-i\omega_{RF}t}$$

Plot the components of the resultant B_1 pulse amplitude in the laboratory and rotating frames as a function of time. (2 point)

Now design the shortest possible saturation RF half sin-pulse (*i.e.* sin on [0, π]) for ^{19}F at 3.0T that does not exceed the same RF amplifier's ability to output $B_{1,max}$ = 25μT. Plot the components of the resultant B_1 pulse amplitude in the laboratory frame as a function of time. (2 point)