M219 Principles and Applications of MRI (Winter 2024)
Homework Assignment #2 (20 points)
Assigned: 1/29/2024, Due: 2/14/2024 at 5 pm by email

E-mail a PDF (entitled M219_HW02_[Last Name].pdf). Please only submit neat and clear solutions. If your assignments are hard to read, poorly commented, or sloppy points may be deducted. As appropriate, each solution should be obtained using Matlab; provide the code.

For all problems – clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

Problem #1. (2 point) True or False (state clearly your reasoning)

T/F: Given a fixed flip angle, the larger the M the stronger the B1 field needs to be because a stronger force is required to flip a larger M.

Problem #2 (8 points) – Forced Precession in the Rotating Frame

The goal is to derive the solution for the individual magnetization components for Forced Precession in the Rotating Frame without Relaxation when the RF phase is non-zero. To do so, consider the following steps.

(a) First, define $\vec{B}_{1,rot}(t)$ from the general form of the RF pulse in the laboratory frame: (1 point)
(b) $\vec{B}_1(t) = B_1^E(t)[\cos(\omega_{RF}t + \theta)\,\hat{\imath} - \sin(\omega_{RF}t + \theta)\,\hat{\jmath}]$
(c) Then, show the steps required to define the system of differential equations in the rotating frame in the presence of $B_0\hat{k}$ and $\vec{B}_{1,rot}$. You'll want to define $\vec{B}_{eff}$ first. (1 point)
(d) Then, write the general solution for this system of differential equations. (1 point)
(e) Next, write a specific solution when $B_1^e(t) = B_1 \sqcap \left(\frac{t-\tau_p/2}{\tau_p}\right)$. (1 point)
(f) Finally, use homogeneous coordinate operators to simulate the action of this hard RF pulse for α=90° RF pulse (θ=π/4) with $B_{1,max}$ =1μT and B_0 =0.5 Gauss in the rotating frame. Plot the components of the bulk magnetization as a function of time. (2 points)
(g) Add relaxation to your simulation in the rotating frame. Use T_1=1000 ms and T_2=100 ms. Plot the components of the bulk magnetization during the application of the same RF pulse. Overlay a plot when T_1=250 ms and T_2=25 ms. (2 points)

Problem #3. Nishimura 5.2 (2 points):

Design a 2D imaging sequence that uses a k-space trajectory entirely different from 2DPR and 2DFT. Draw a labeled timing diagram showing the gradient waveforms applied and the resultant k-space trajectories. Assume idealized excitations. If a set of FID signals are acquired, indicate how the pulse sequence varies from measurement to measurement. Discuss strategies for image reconstruction with your measurement set.

Problem #3. Nishimura 5.5 (a) (2 points):

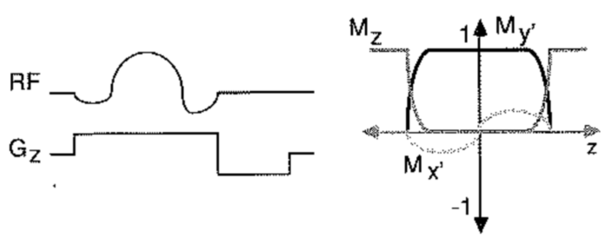
Consider an impulse object at the origin that is precessing at a slightly higher frequency than expected; i.e., at frequency $f_0 + \Delta f$. However, the demodulation of this signal down to baseband is still based on f_0.

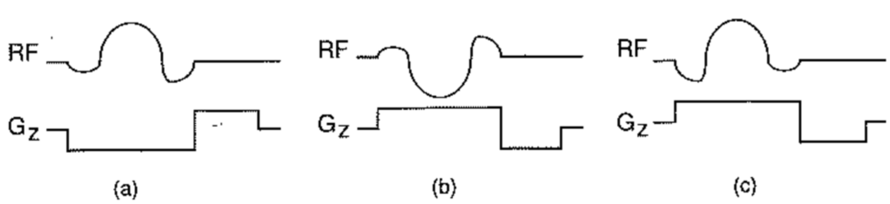
(a) If using a double-sided 2DFT sequence (Fig 5.11), describe the nature of the resultant measurements in k-space. At what position will the impulse be reconstructed if the readout gradient amplitude is G_x? For comparison, it may help to consider first the case where the impulse is precessing at f_0.

Problem #5. Nishimura 6.3 (6 points):
Consider the selective excitation pulse $B_1(t)$ plotted below, applied along *x'* and in the precence of G_z. The components of the distribution $M_{rot}(z)$ following this excitation are also plotted.

For the selective excitations shown in (a), (b), and (c), sketch the components ($M_{x'}(z)$, $M_{y'}(z)$, $M_z(z)$) of the resultant distribution $M_{rot}(z)$. Although the tip angle is clearly 90°, use the ramifications of the small tip-angle solution to guide your modifications to the slice profile.

For each of the three cases, comment on whether the results would still hold true if the small tip-angle approximation (which allows Fourier analysis of the pulse) was not assumed.





(2 points for each case)