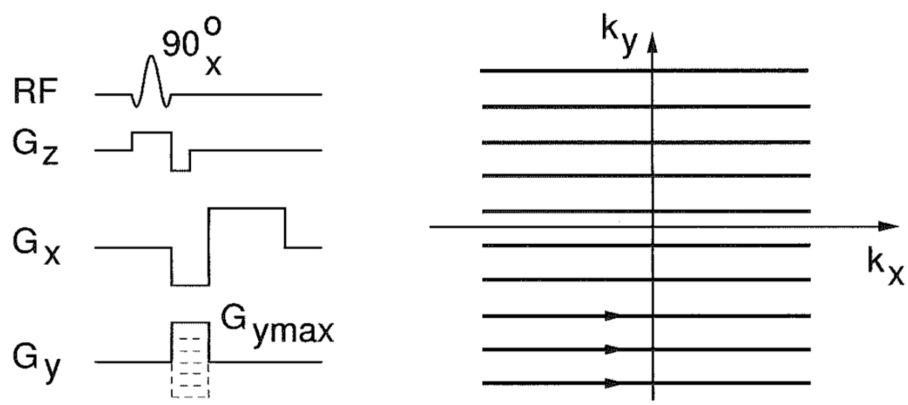
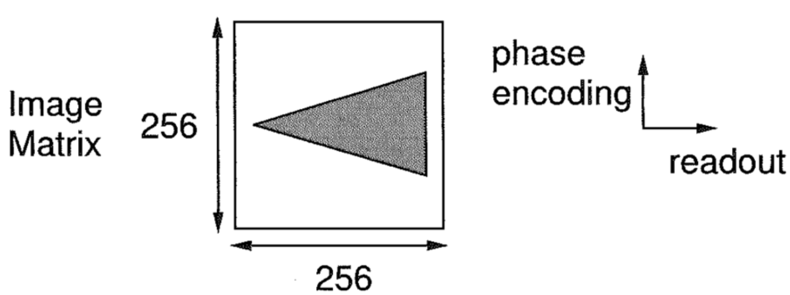
M219 Principles and Applications of MRI (Winter 2024)
Homework Assignment #3 (20 points)
Assigned: 2/14/2024, Due: 3/6/2024 at 5 pm by email

E-mail a PDF (entitled M219_HW03_[Last Name].pdf). Please only submit neat and clear solutions. If your assignments are hard to read, poorly commented, or sloppy points may be deducted. As appropriate, each solution should be obtained using Matlab; provide the code.

For all problems – clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

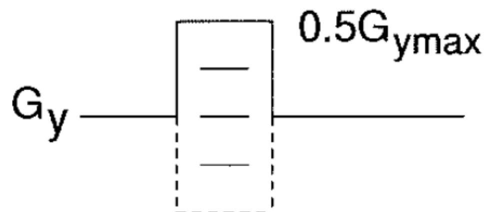
Problem #1. Nishimura 5.7 (7 points):
A 2DFT imaging sequence is executed in which k-space is acquired in a raster-line fashion, beginning with the most negative k_y phase encode and moving progressively to the adjacent phase encode. 256 phase encodes symmetrically spaced about the origin are collected, with each readout time signal sampled to 256 points after passing through a low-pass filter appropriate for the sampling rate. The “dumb” (inflexible) computer is programmed to simply take the incoming data, fill a 256 by 256 matrix from the bottom row to the top row, and then perform a 2D Inverse FFT of this matrix to reconstruct the 256 by 256 magnitude image matrix as shown below.



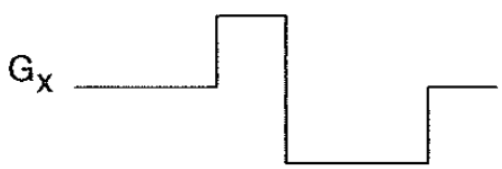


For each of the following modifications (a-g) to the 2DFT sequence described above, sketch the resultant 256 by 256 magnitude image matrix. You may clarify your answer with words if you wish.

(a) Replace G_y with (still 256 phase encodes) (1 point)



(b) Replace G_x with



(c) Cut the low-pass filter bandwidth in half but keep the same time sampling rate. (1 point)

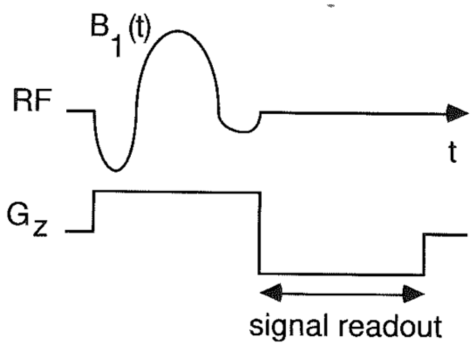
(d) Apply $90^\circ_x, -90^\circ_x, 90^\circ_x, -90^\circ_x, \cdots$ excitations as we progress through the phase encodes (see the end of section 6.1 to understand how these excitations affect the signal). (1 point)

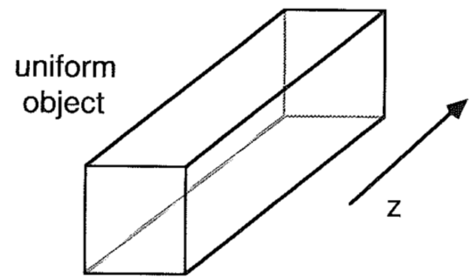
(e) Apply $90^\circ_x, 90^\circ_y, -90^\circ_x, -90^\circ_y, \cdots$ excitations as we progress through the phase encodes. (1 point)

(f) Multiply the G_x amplitude by 2. (1 point)

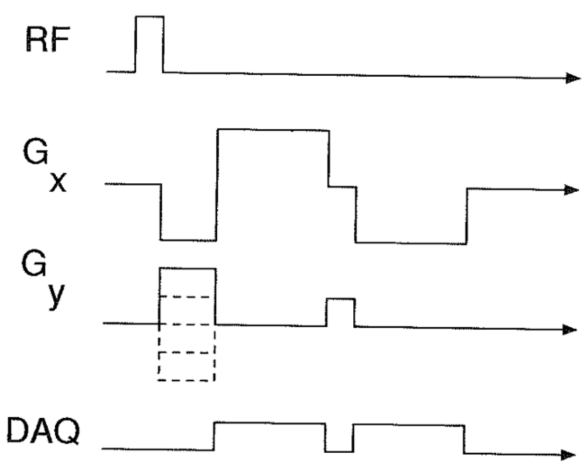
(g) Reconstruct by using a 2D FFT instead of a 2D inverse FFT. (1 point)

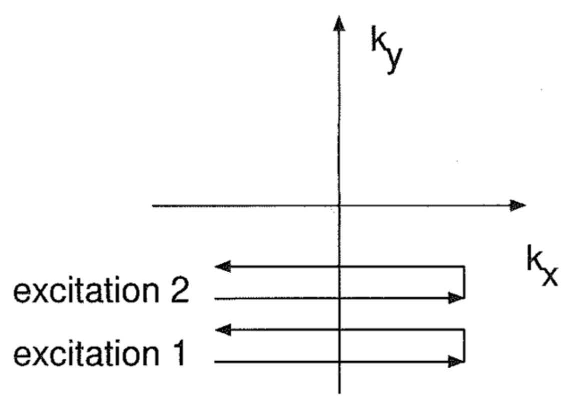
Problem #2. Nishimura 6.4 (2 points):
An RF pulse $B_1(t)$ in the presence of G_z excites a slice in a uniform object (see below). Immediately following the excitation, the slice select gradient is switched from $+G_z$ to $-G_z$ and a signal is recorded. Sketch the resultant signal that is read out during the $-G_z$ interval. Assume a small tip-angle excitation. Justify your sketch with an explanation or derivation.



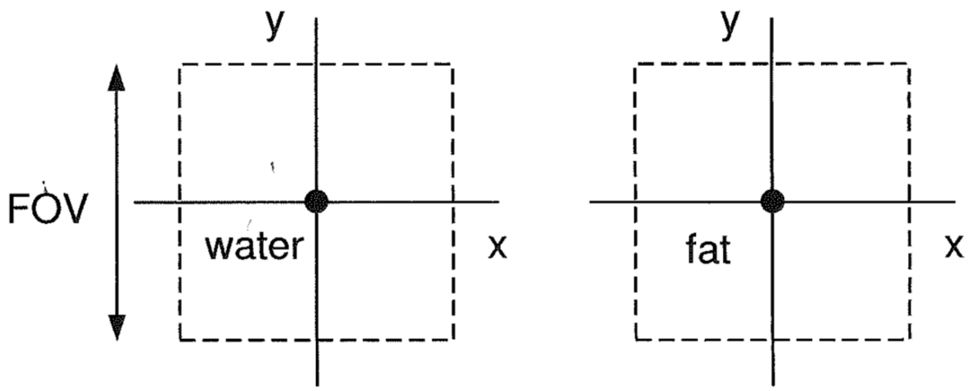


Problem #3. Nishimura 7.9 (3 points):
Consider the modified 2DFT imaging sequence shown below in which two adjacent lines in k-space are acquired per excitation. On successive excitations, the next two lines are acquired. Assume that the demodulation frequency is tune to the frequency of water and that the computer is sufficiently "smart" to place the k-space data in the proper positions. As in conventional 2DFT, the image is reconstructed using a standard 2D inverse FFT. Ignore T_2 effects.

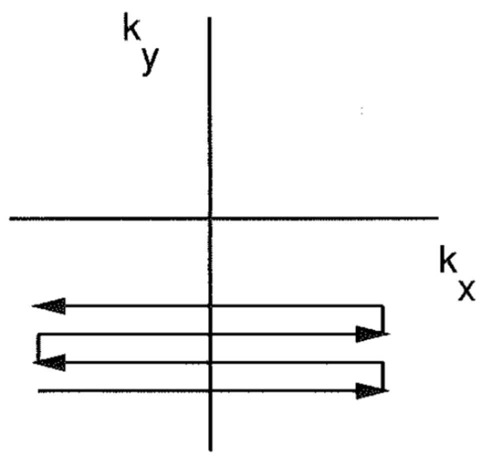




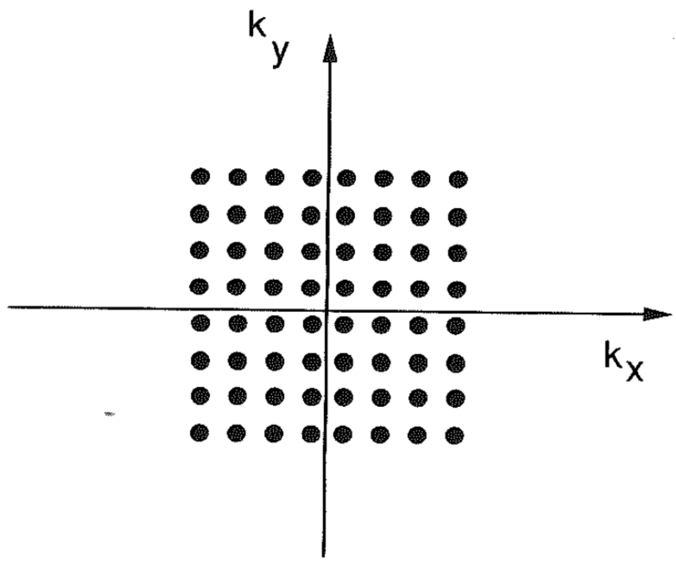
(a) Using this sequence to image the following two impulse objects, sketch the resultant magnitude image for each case. Offer explanations or analyses for your sketches. (2 points)



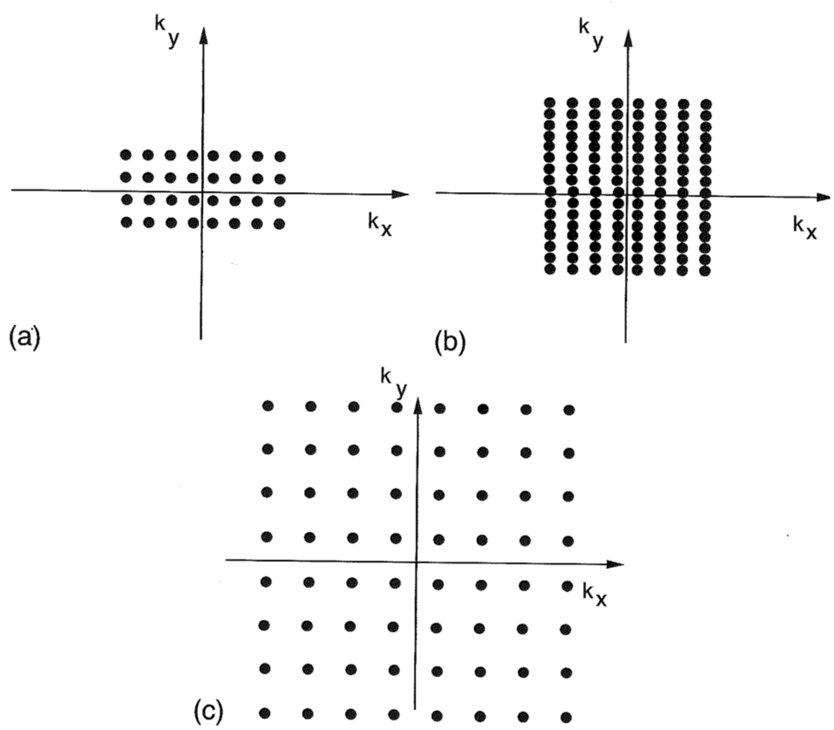
(b) Repeat part (a) for an imaging sequence that acquires 4 phase encode lines per excitation, as shown below. Under what condition(s) will this sequence produce the same images as in part (a)? (1 point)



Problem #4. Nishimura 7.10 (3 points):
A reference 2DFT sequence is as follows: 256 phase encodes, 10 ms readout duration per measurement, sampled to 256 points.
Field of view: FOV_x, FOV_y
Spatial resolution: δ_x and δ_y.
Its k-space coverage is shown below.



If this sequence produces an SNR normalized to 1, then what are the relative SNR's of the following sequences? Assume that the readout gradient amplitude does not change compared to the reference sequence. Comment on the relative FOV's spatial resolution, and signal bandwidth. (1 point for each)



Problem #5. Gradient Echo vs. Spin Echo Contrast (2 points)
The equations that describe the echo amplitude for a gradient echo and spin echo sequence are as follows:

$$A_{GRE} = \frac{\rho\left(1 - e^{-\frac{TR}{T1}}\right)}{1 - \cos(\alpha)e^{-\frac{TR}{T1}}} \sin\alpha\, e^{-TE/T_2^*}$$

$$A_{SE} = \rho\left(1 - e^{-\frac{TR}{T1}}\right) e^{-TE/T2}$$

a) Use the above equations in MATLAB to determine the TE and TR needed to generate the maximum T_1 contrast between Tissue A (ρ = 1.0, T_1=2000ms, T_2=40ms, T_2^* =25ms) and Tissue B (ρ = 1:0, T_1=500ms, T_2=40ms, T_2^* =25ms) for both a gradient echo sequence and a spin echo sequence. Assume the pulse sequences are both limited by: 5ms<TE<100ms, 10ms<TR<10,000ms. Assume α=30 for GRE and α=90 for SE. This can be done by simulating the signal amplitude for a range of TE and TR. (1 point)

b) Is it preferable to use a gradient echo or a spin echo sequence for T_1 contrast? Why? (1 point)

Problem #6. k-space and Image Space (3 points)

a) Import the provided image (heart.mat) into Matlab and render an image of the k-space magnitude (fft2.m). Hint: Use fftshift.m to ensure the dominant signals (low spatial frequencies) occur at the k-space center. (1 point)

b) Add a noisy spike artifact to a Fourier coefficient in the upper left quadrant of k-space and show the result in image space (ifft2.m). Describe the result and why this occurs. (1 point)

c) Remove (set to zero) all but the middle ten lines from the original k-space data (from the original FFT, without the noisy spike). Show the resulting k-space magnitude and the resultant image. Describe what you see. (1 point)