Fast Imaging, Advanced Image Reconstruction

M219 Principles and Applications of MRI

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2024.02.14

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Review: Basic Recon

Image Reconstruction

$$S = \mathcal{T}\left\{I\right\}$$

Data Consistency Constraint

Measured Signal

Spatial Information Encoding Scheme (Fourier Transform)

Image Function

$$I = \mathcal{T}^{-1}\left\{S\right\}$$

The Fourier Transform

$$S(\vec{k}) = \int_{-\infty}^{+\infty} I\left(\vec{r}\right) e^{-i2\pi \vec{k}\cdot\vec{r}} d\vec{r}$$

MRI Signal Equation

$$S(\vec{k}) \xleftrightarrow{\mathcal{F}} I(\vec{r})$$

$$S(k_x) = \int_{-\infty}^{+\infty} I\left(x\right) e^{-i2\pi (k_x x)} dx$$ 1D

$$S(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I\left(x, y\right) e^{-i2\pi (k_x x + k_y y)} dx dy$$ 2D

$$S(k_x, k_y, k_z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I\left(x, y, z\right) e^{-i2\pi (k_x x + k_y y + k_z z)} dx dy dz$$ 3D

Finite Sampling

$S(k)$ is measured at $k \in \mathcal{D}$

$$\mathcal{D} = \{n\Delta k, -N/2 \leq n \leq +N/2\}$$

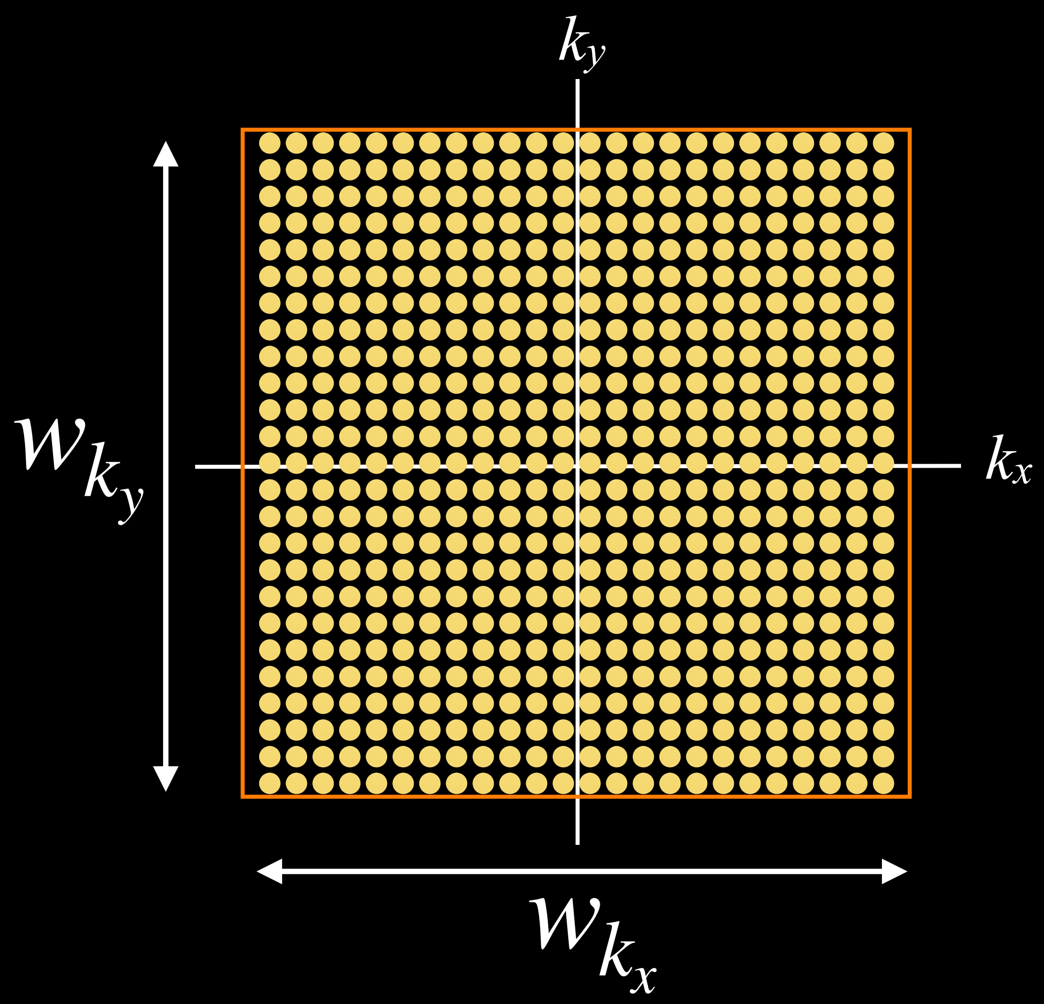
Fourier Step-size

Number of Sample Points

$$I(x) = \Delta k \sum_{n=-N/2}^{N/2-1} S[n] e^{i2\pi n \Delta k x}, \ \ |x| < \frac{1}{\Delta k}$$ Eqn. 6.20

This is the fundamental image reconstruction equation for MRI.

Sampling Considerations



$$\Delta k_x = \frac{1}{FOV_x} \qquad \Delta k_y = \frac{1}{FOV_y}$$

$$w_{k_x} = \frac{1}{\Delta x} \qquad w_{k_y} = \frac{1}{\Delta y}$$

Review Sampling Theorem

Review Lectures 9/10 Spatial Localization

Noise Considerations

- Signal-to-Noise Ratio (SNR)
  - A fundamental measure of image quality
  - $SNR \triangleq \dfrac{signal\ amplitude}{\sigma\ of\ noise}$
  - $SNR_{dB} = 20 \cdot log(SNR)$

Nishimura Ch. 7.5

Noise Considerations

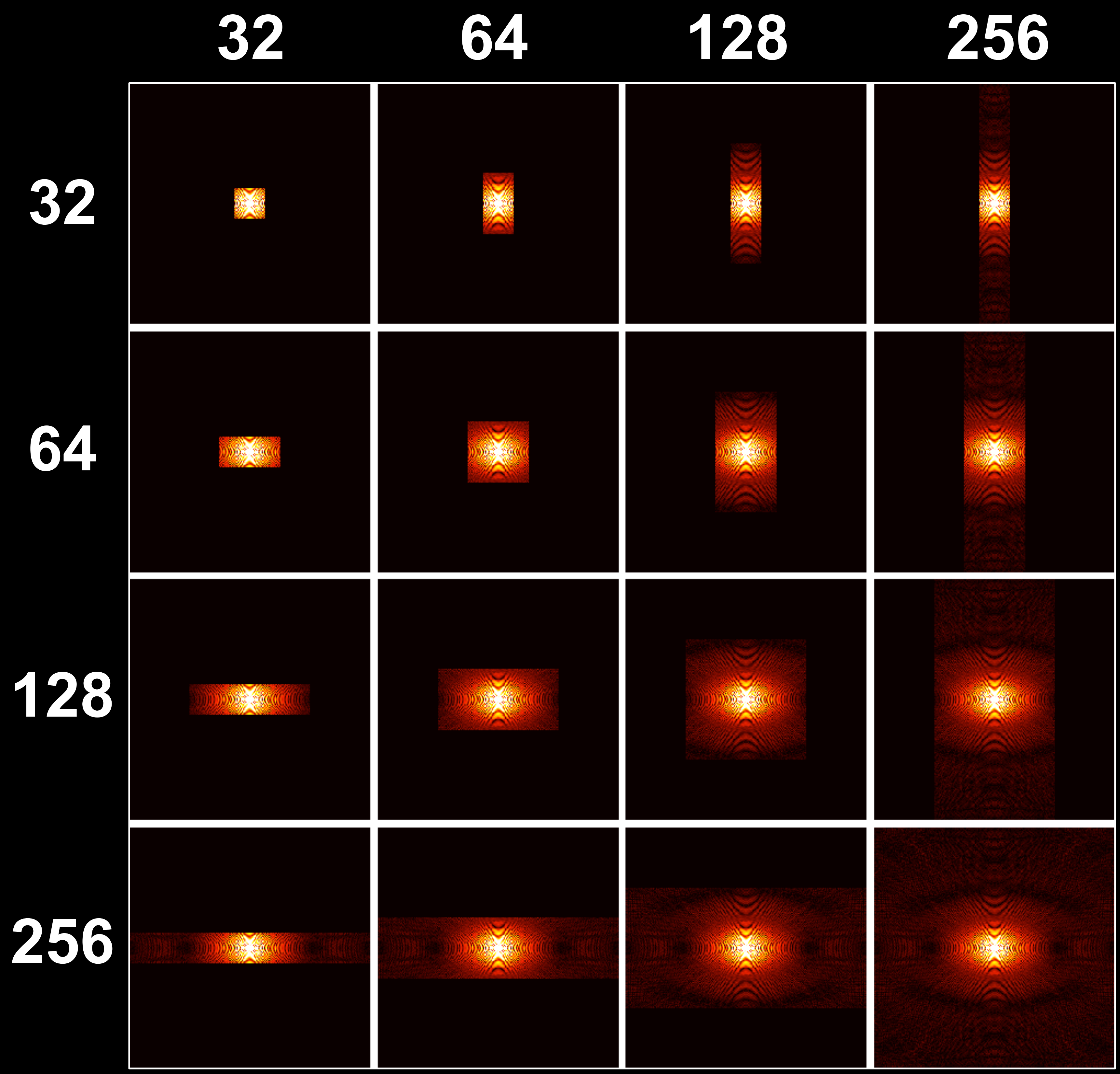
- Summary of Acquisition Time Effects
  - $SNR \propto \sqrt{N_{ave} \cdot T_{read}}$
  - $SNR \propto \sqrt{measurement\ time}$

- Effect of Spatial Resolution
  - $SNR \propto (\delta_x)(\delta_y)(\delta_z)$

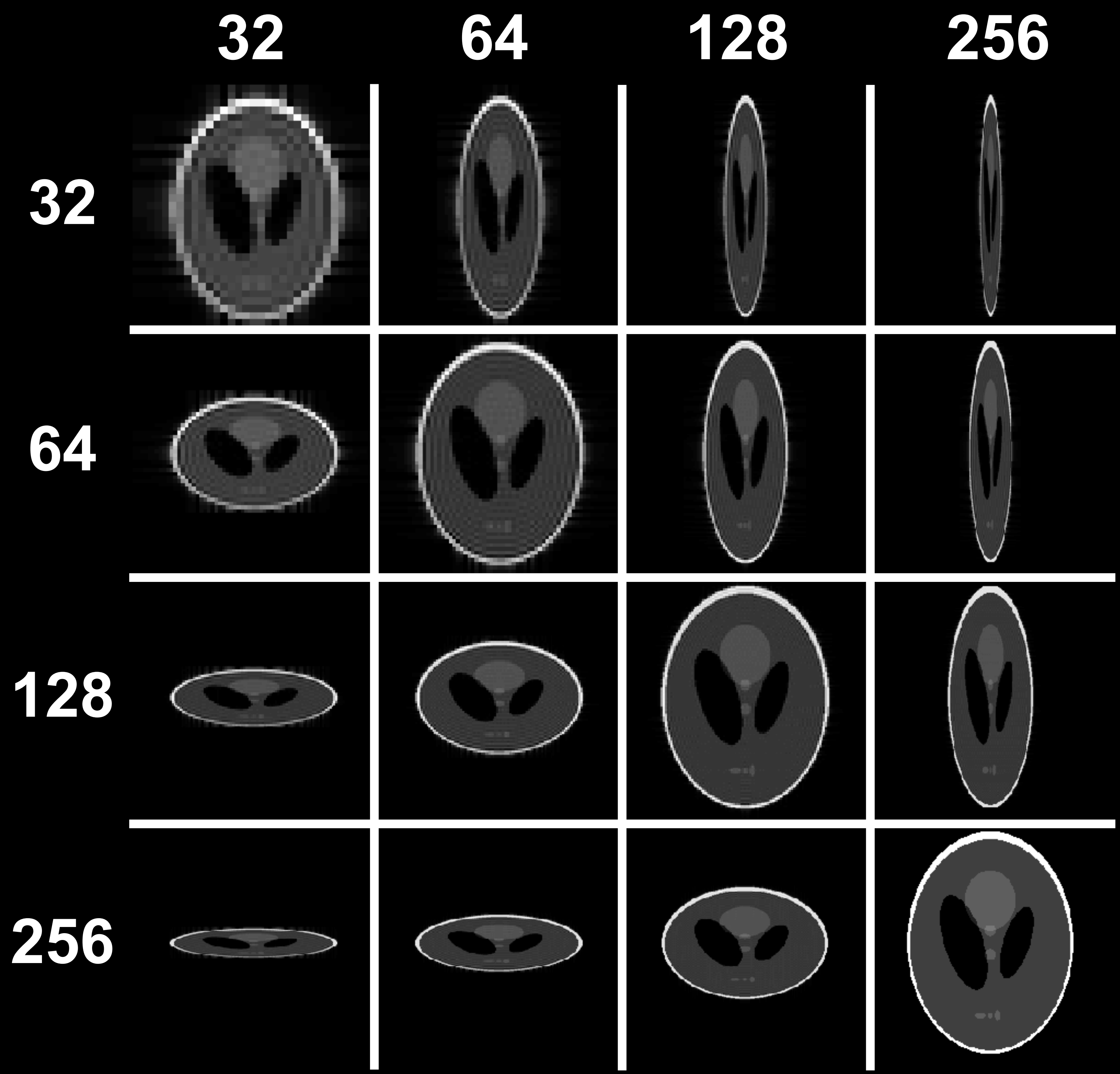
- Other factors
  - $SNR \propto f(\rho, T_1, T_2, \ldots)$

Nishimura Ch. 7.5

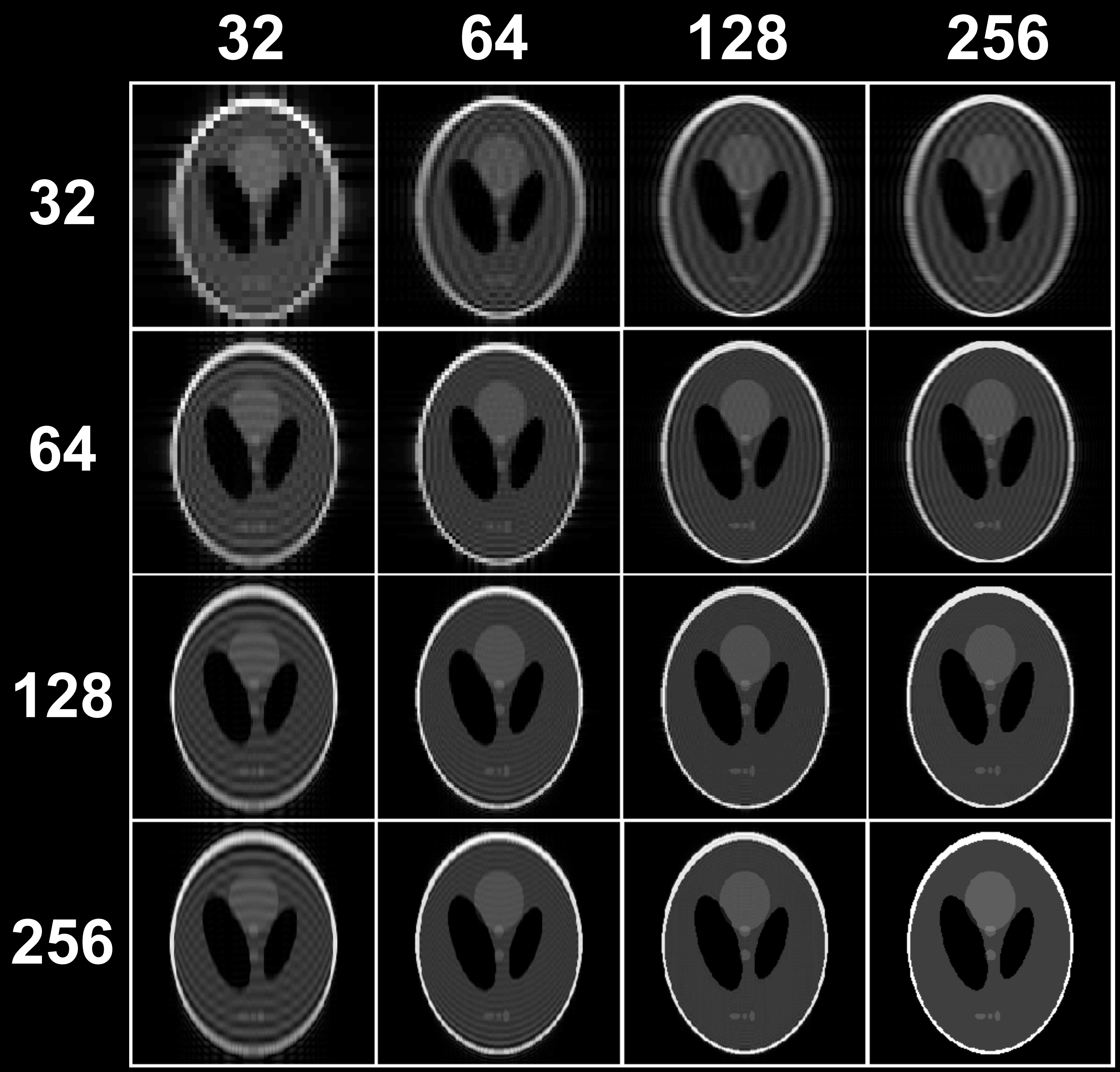
Gibb's Ringing



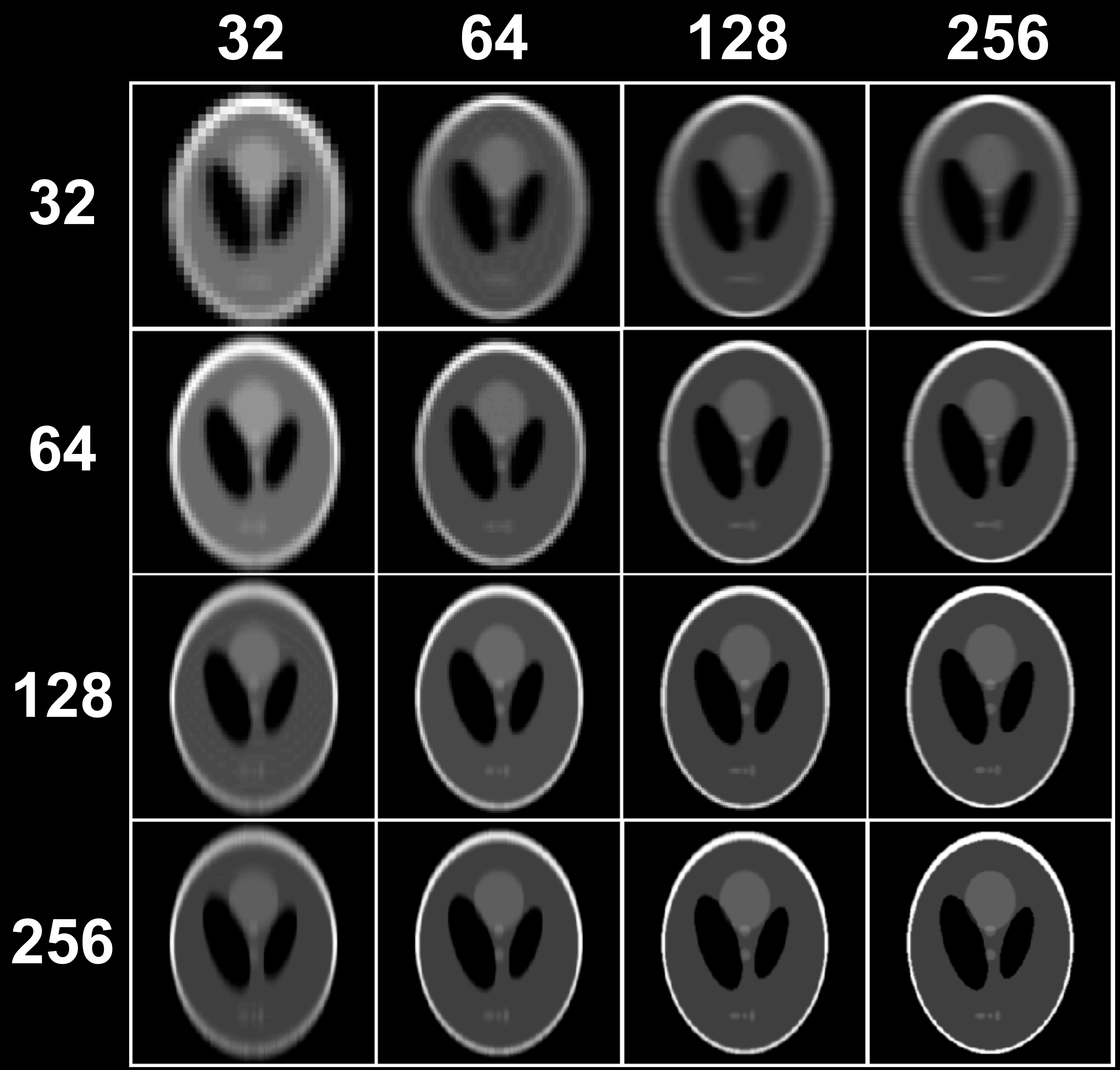
Gibb's Ringing



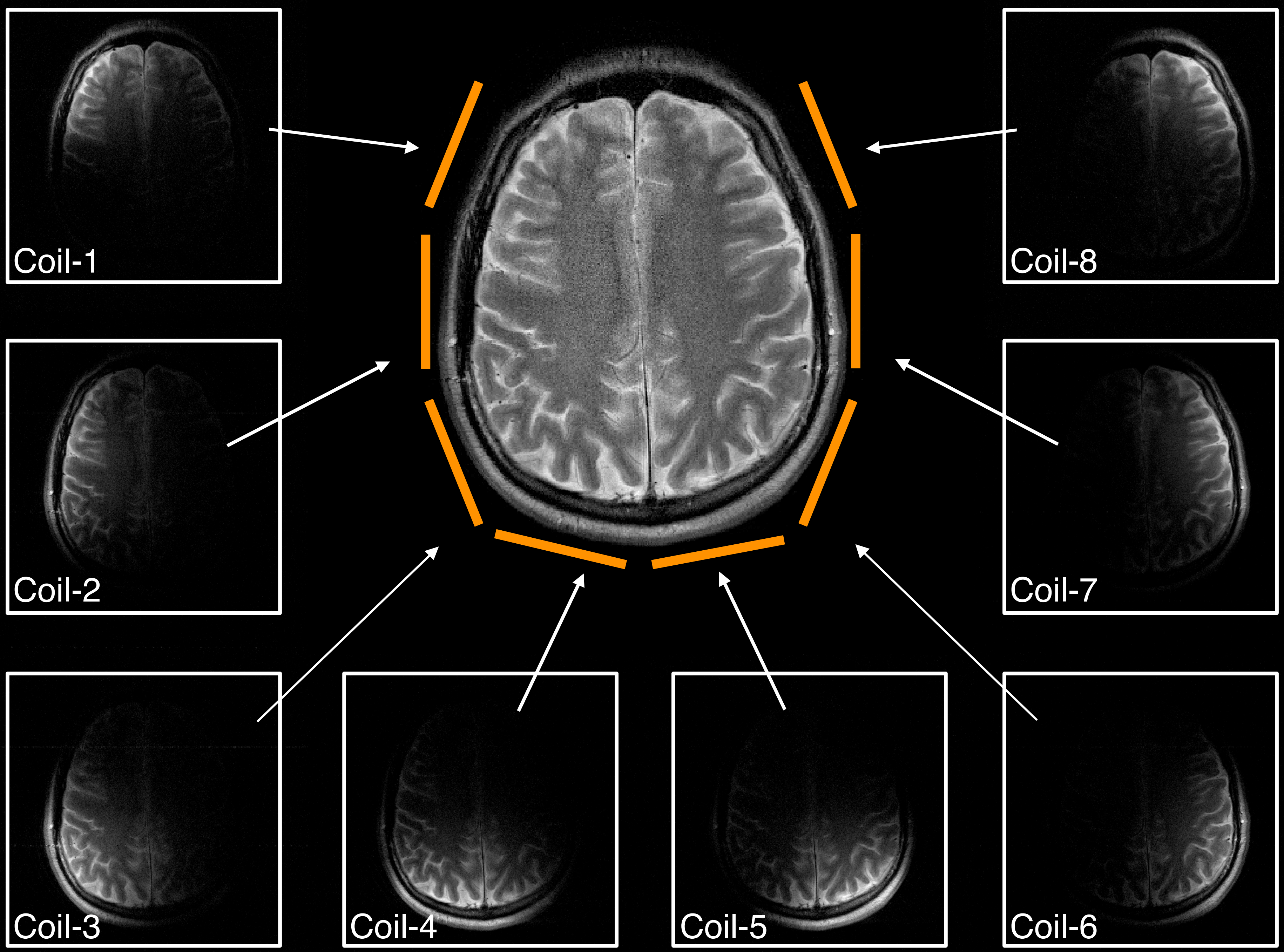
Zero-Pad



Hamming Window & Zero-Pad



Multi-Coil Reconstruction



Each coil element (channel) has a unique sensitivity profile – $\vec{B}_r\left(\vec{r}\right)$

Outline

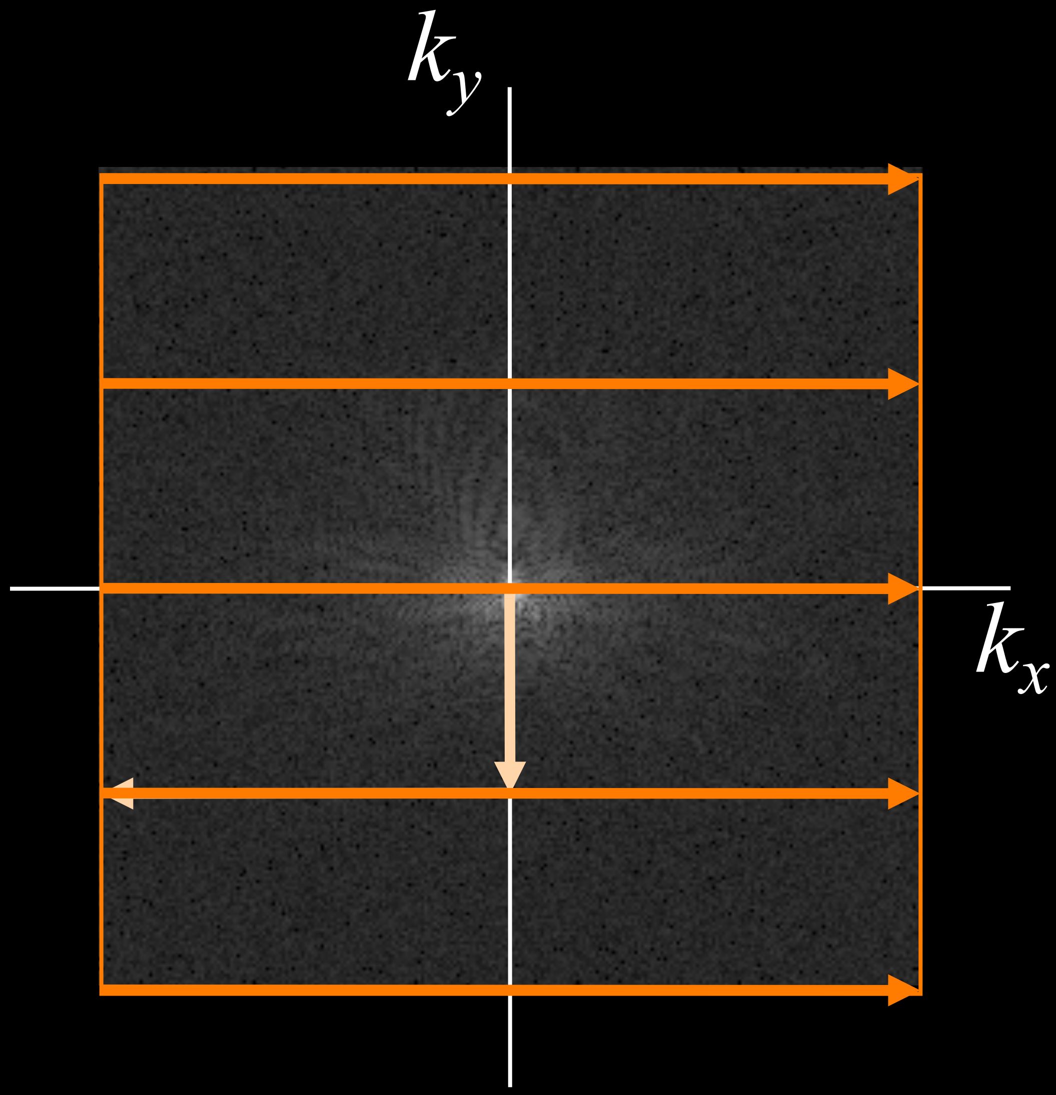
- Fast Imaging
  - Non-Cartesian MRI
  - Echo-planar imaging (EPI)
- Advanced MR Image Reconstruction
  - Parallel imaging
  - Compressed sensing

Overview

- Motivation
  - MRI is relatively slow; need to accelerate
- Strategies
  - Efficient pulse sequences
  - Fast k-space sampling trajectories
  - Data undersampling + advanced recon
- Many challenges and trade-offs
- Key drivers for MRI research

Fast Imaging

k-Space Sampling



set of $s(t)$ covers $m(k_x, k_y)$

Pulse Sequence Diagram

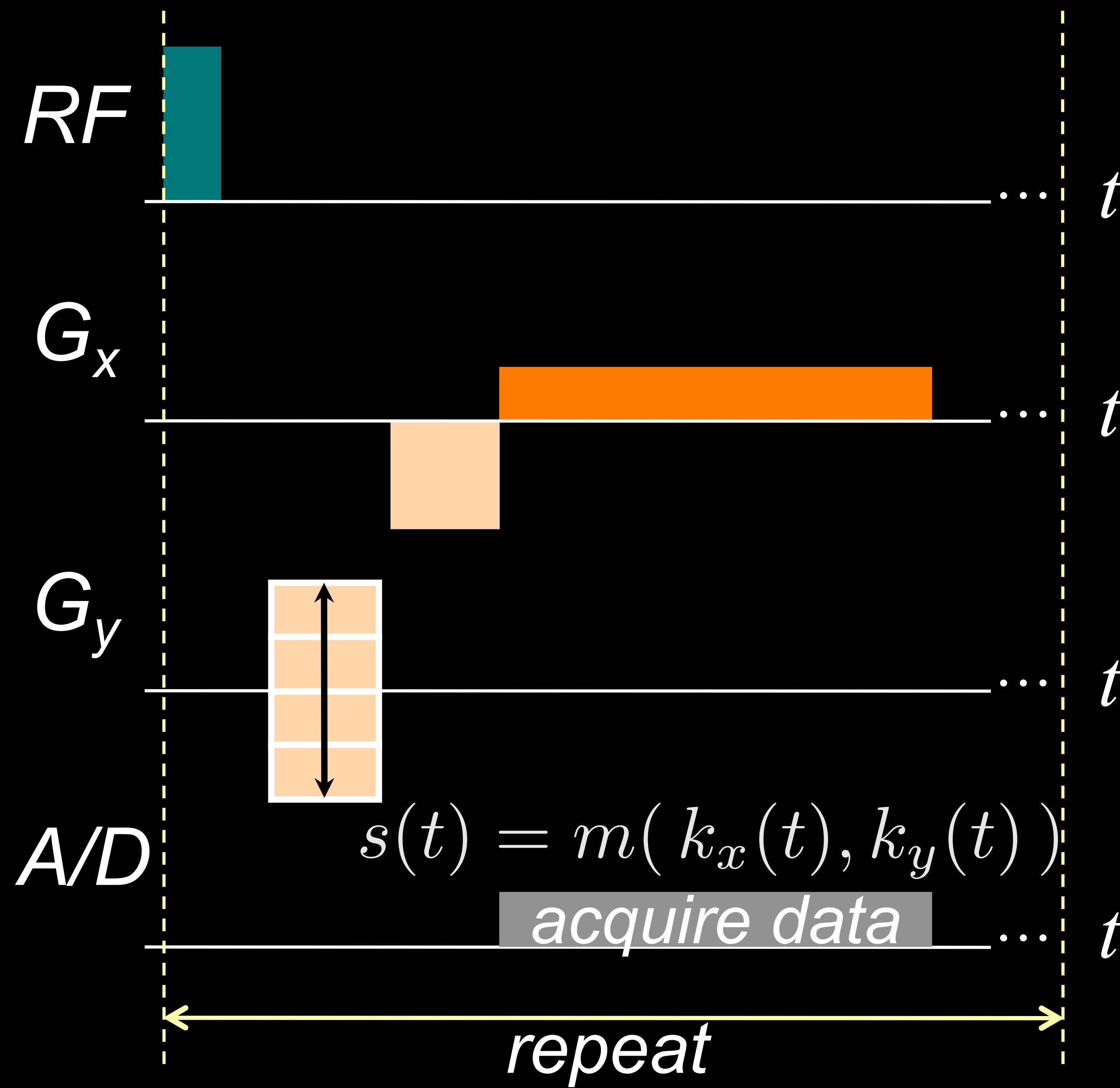
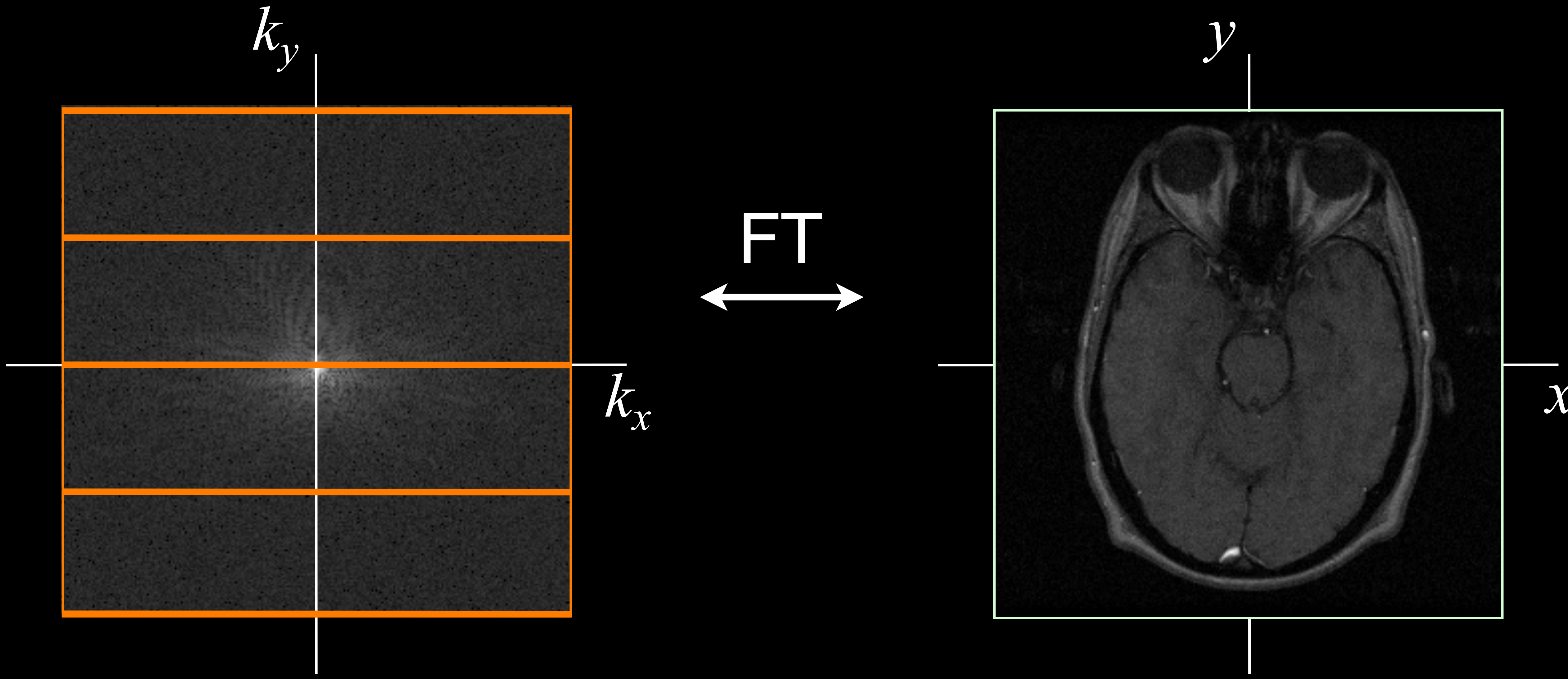


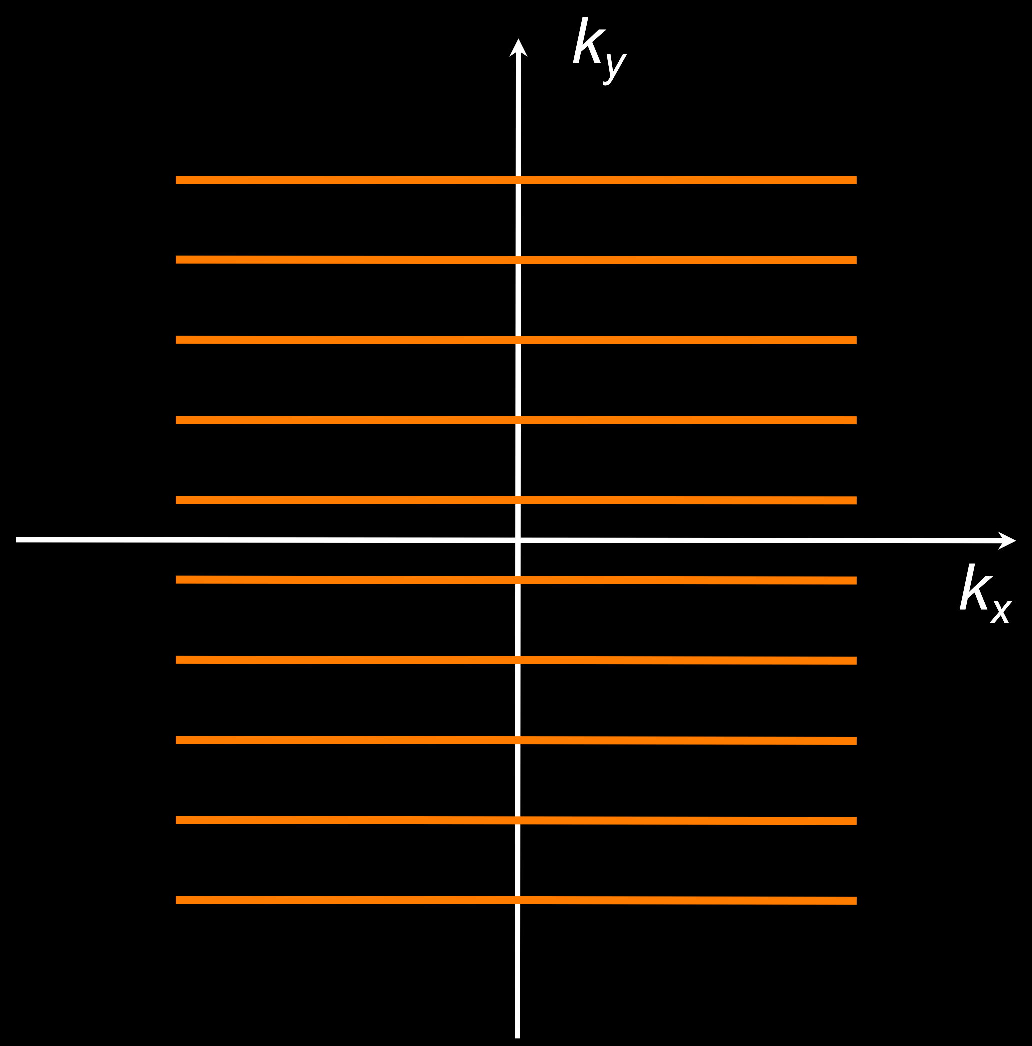
Image Reconstruction



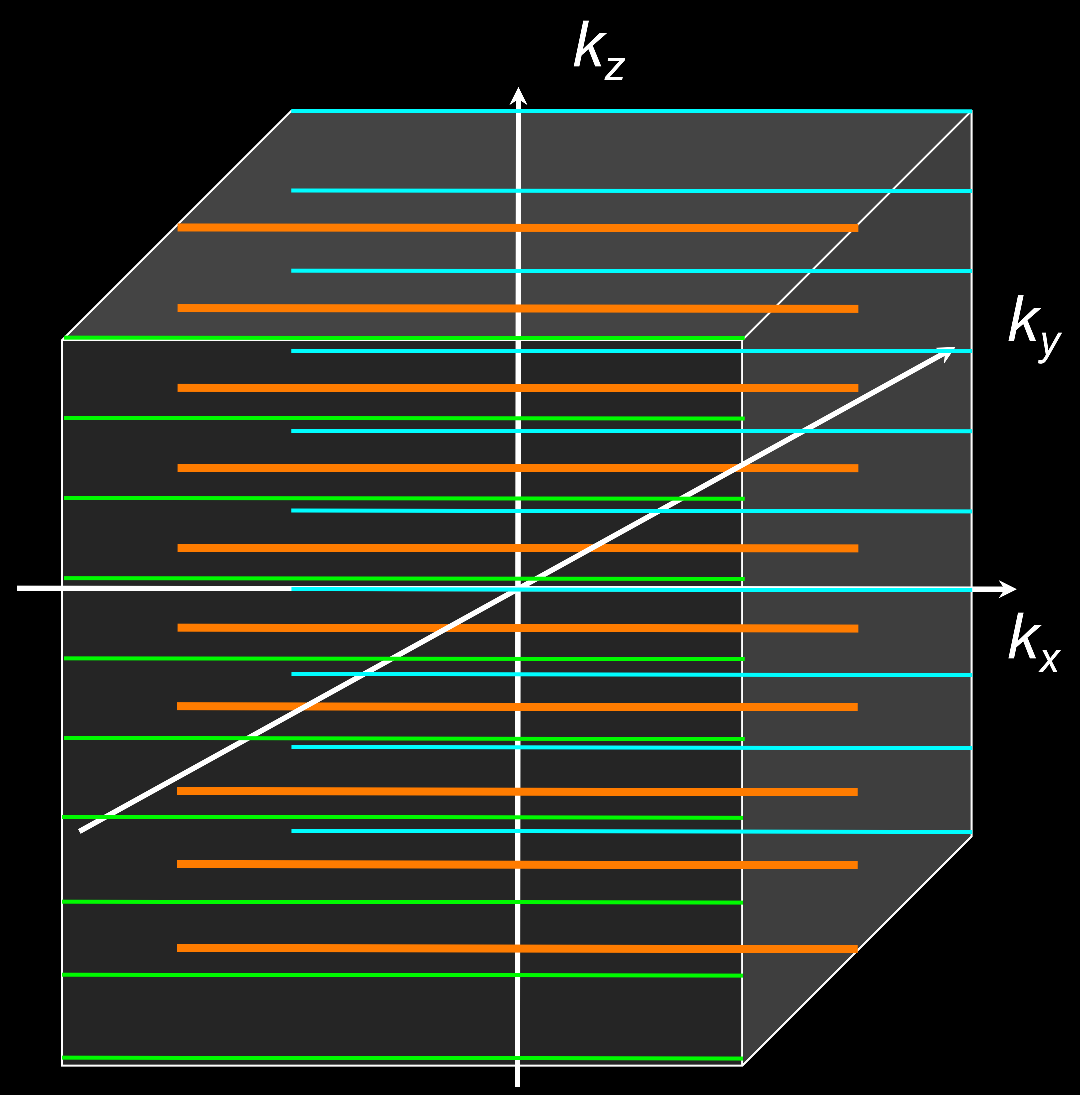
Complex data

Complex data

Cartesian Sampling



Cartesian 2DFT



Cartesian 3DFT

MR Signal Equation

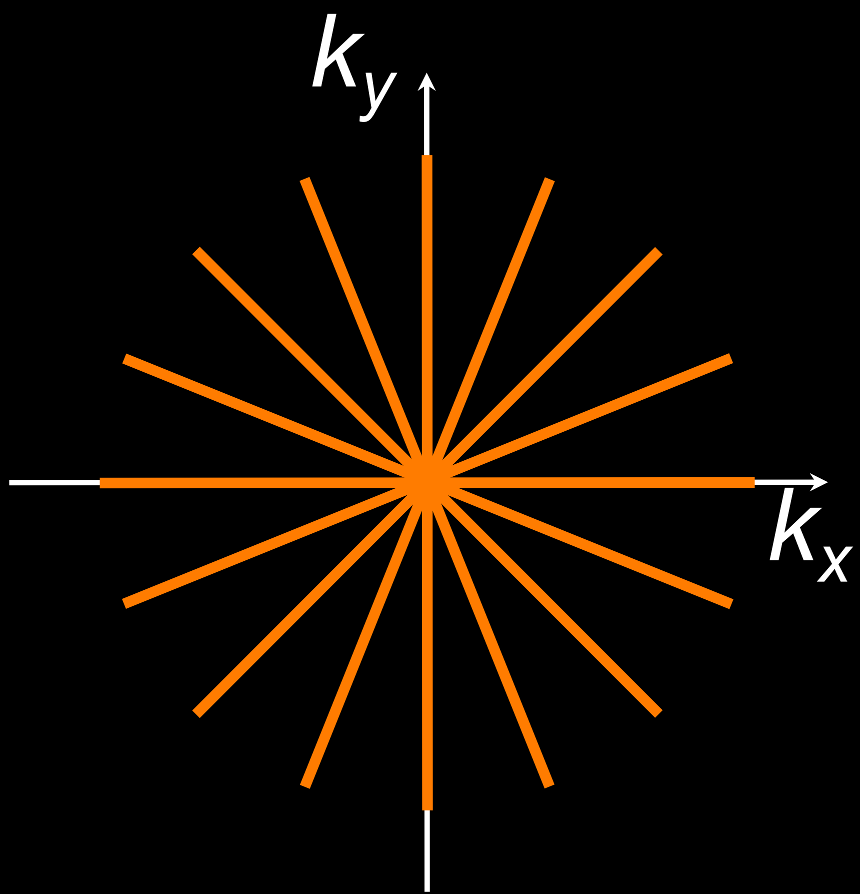
$$s(t) = \iint_{X,Y} M(x,y) \cdot \exp(-i2\pi \cdot [k_x(t)\,x + k_y(t)\,y])\,\mathrm{d}x\,\mathrm{d}y$$

$$= m(\,k_x(t), k_y(t)\,) \qquad \boxed{k_x(t) = \frac{\gamma}{2\pi} G_x t,\; k_y(t) = \frac{\gamma}{2\pi} G_y t}$$

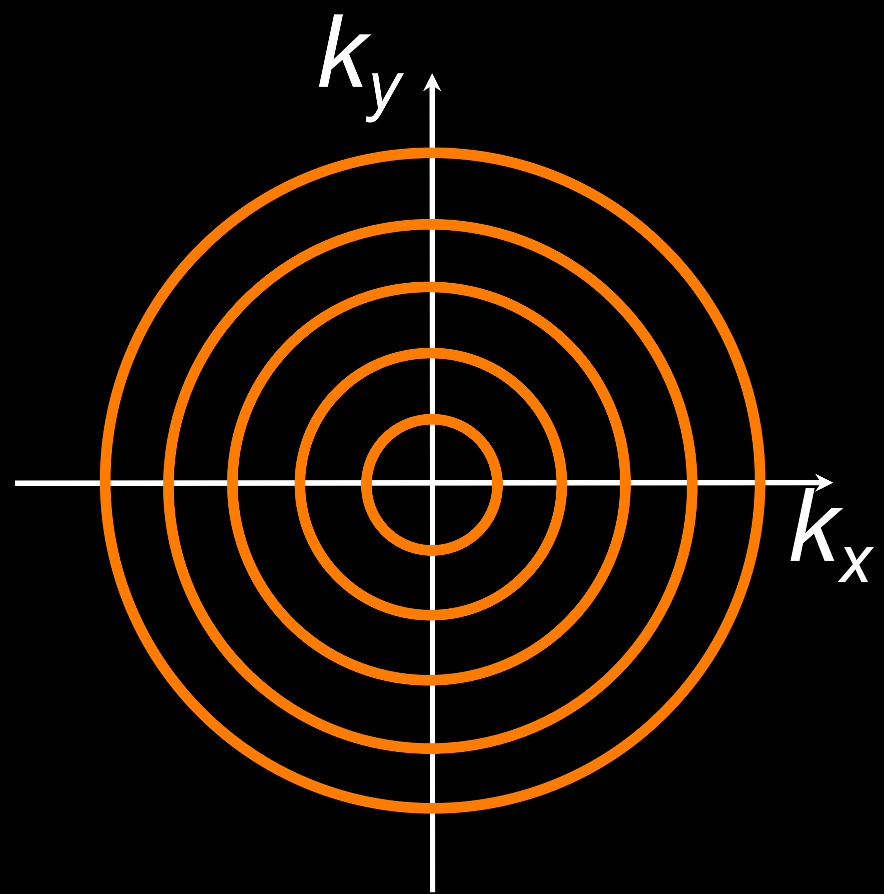
$$m = \mathcal{FT}(\,M(x,y)\,)$$

$$\boxed{k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau)\,\mathrm{d}\tau,\; k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau)\,\mathrm{d}\tau}$$

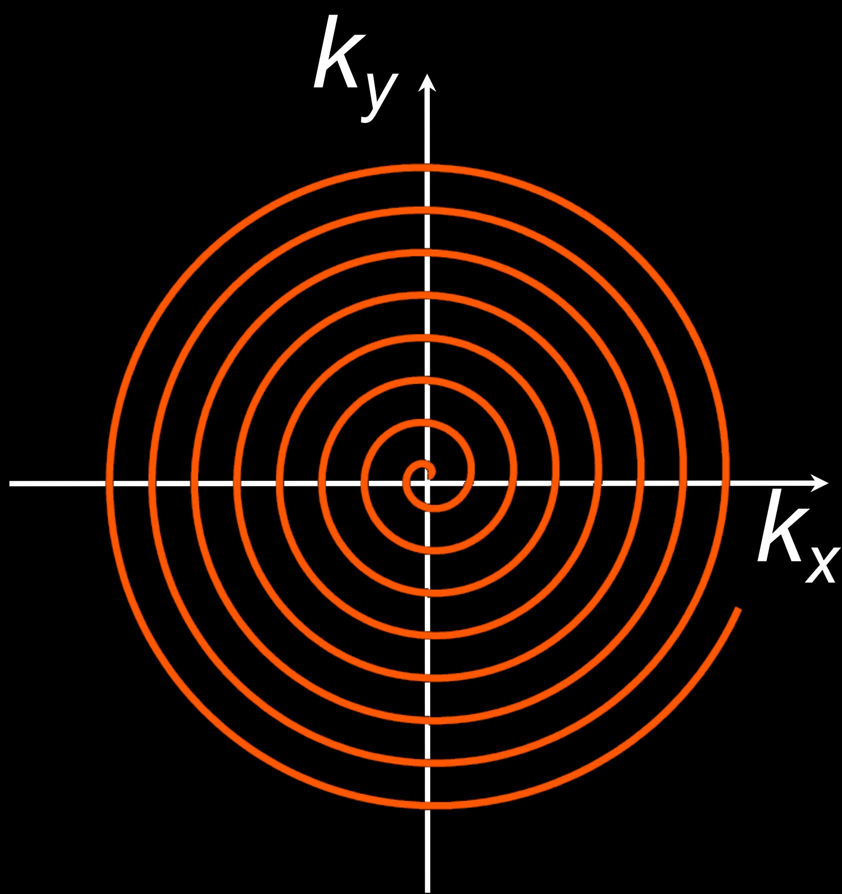
Non-Cartesian Sampling



2D Radial



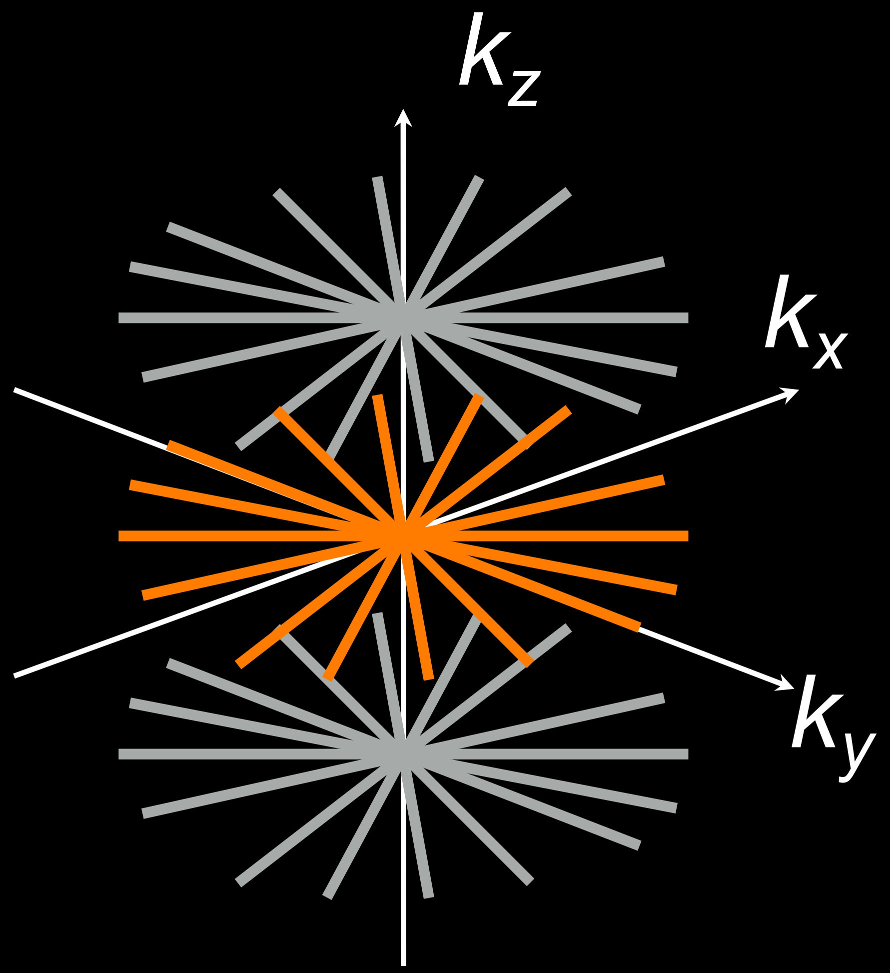
2D Concentric Rings



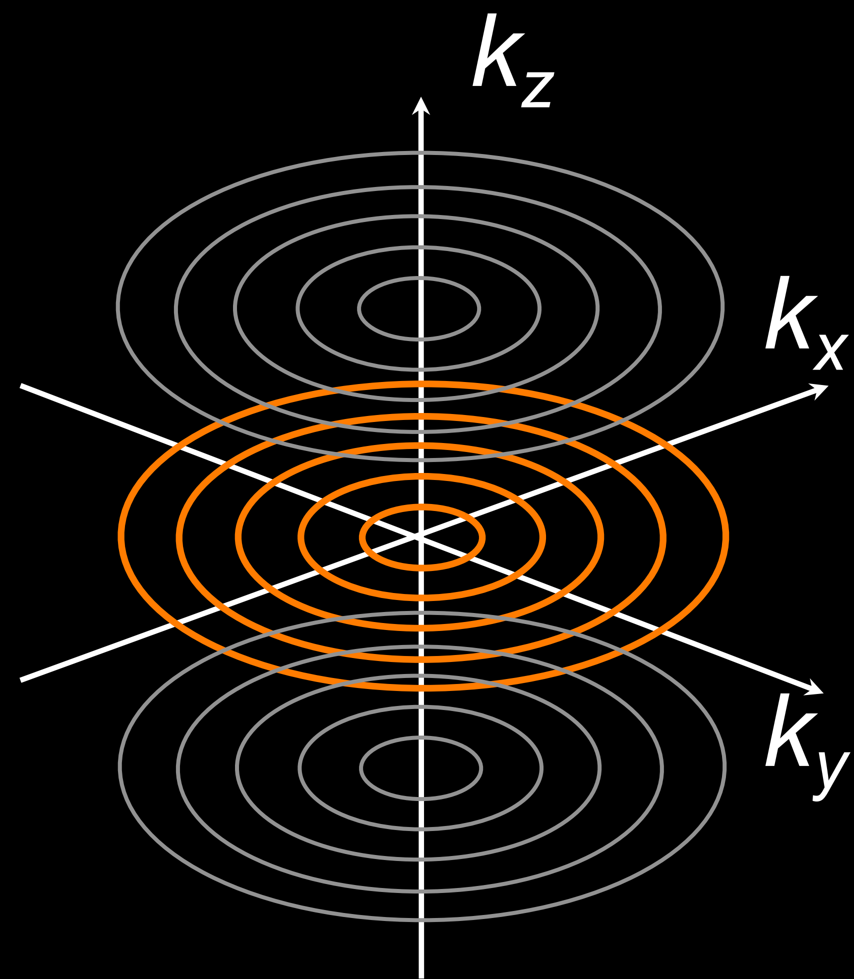
2D Spiral

and much more ...

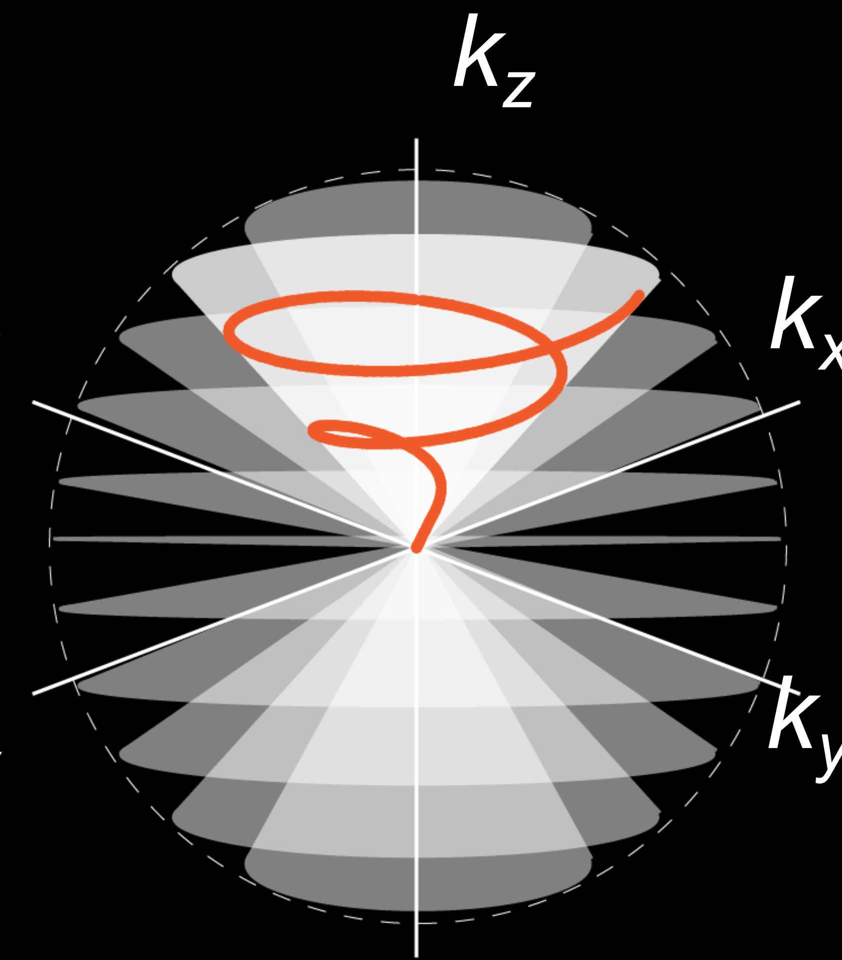
Non-Cartesian Sampling



3D Stack of Stars



3D Stack of Rings



3D Cones

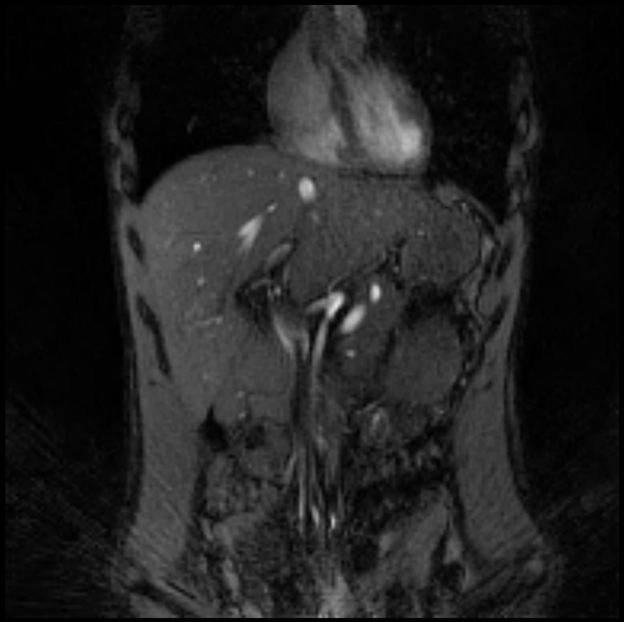
and much more ...

Radial: Real-time MRI

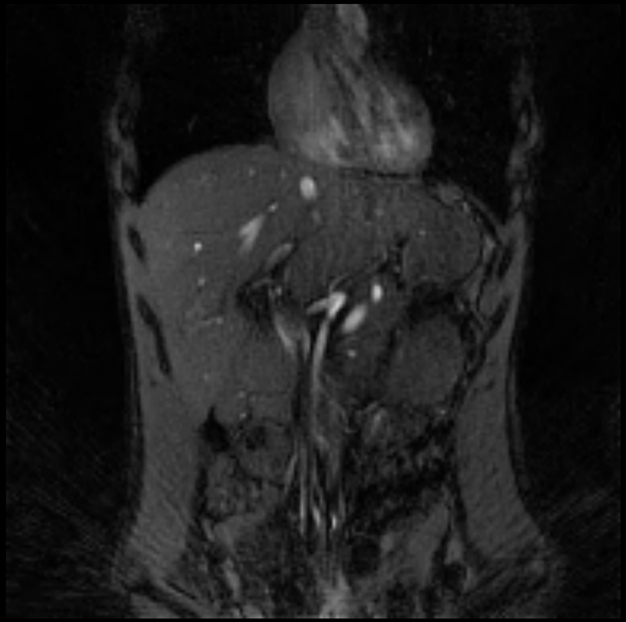
<u>Radial FLASH</u>
- golden-angle ordering
- 192 x 192 matrix
- TR = 3.1 ms
  (1 spoke per TR)
- 3.0 T

<u>Reconstruction</u>
- sliding window of 20 TRs
  (display at 16 frames/sec)
- parallel imaging (SPIRiT)
  (300 spokes for Nyquist)

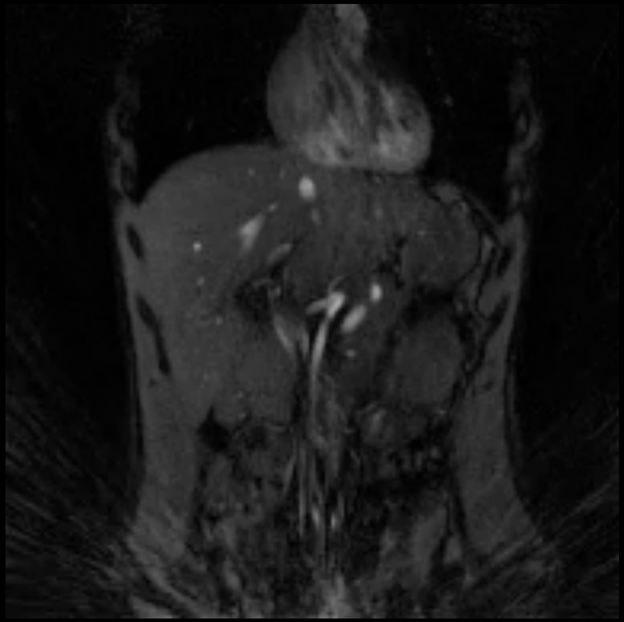
255 spokes/frame
(791 ms/frame)



144 spokes/frame
(446 ms/frame)



89 spokes/frame
(276 ms/frame)



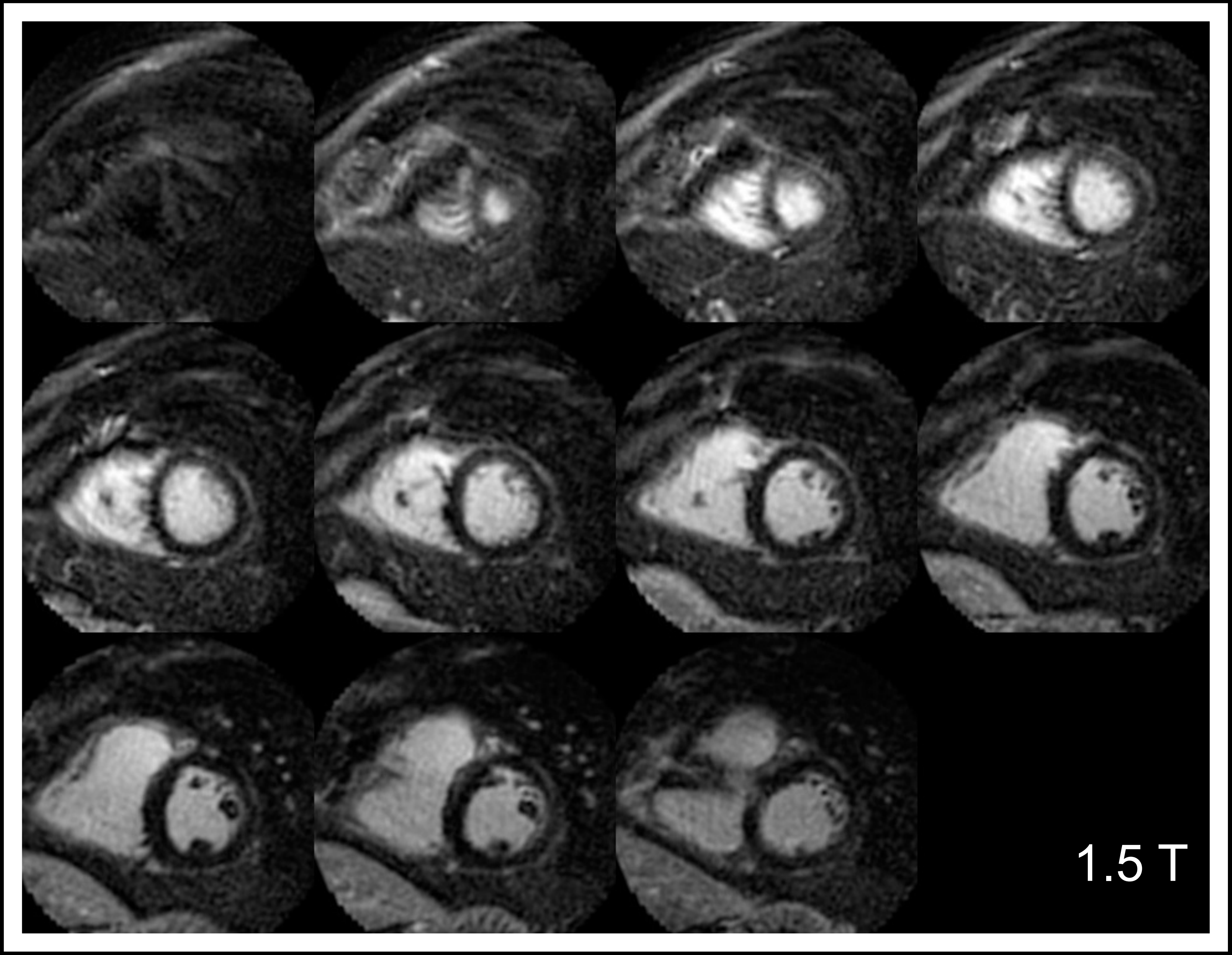
55 spokes/frame
(171 ms/frame)

courtesy of Samantha Mikaiel

Spirals: 3D LGE MRI

<u>3D Spiral IR-GRE</u>
- 6-interleaf VD spiral
- 7.5-ms readout
- 90 x 90 x 11 matrix
- outer volume suppr
- water-only RF exc
- TR = 15.48 ms
- 8-HB BH scan

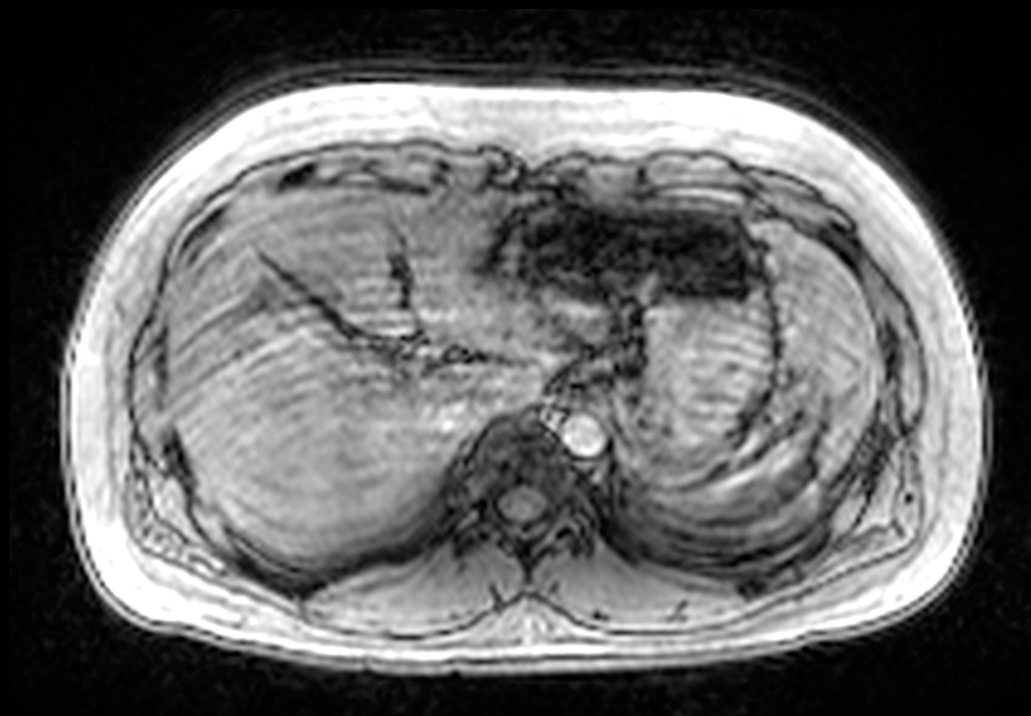
<u>Reconstruction</u>
- SPIRiT (R = 2)
- ~5-sec recon



courtesy of Joelle Barral & Juan Santos (HeartVista)

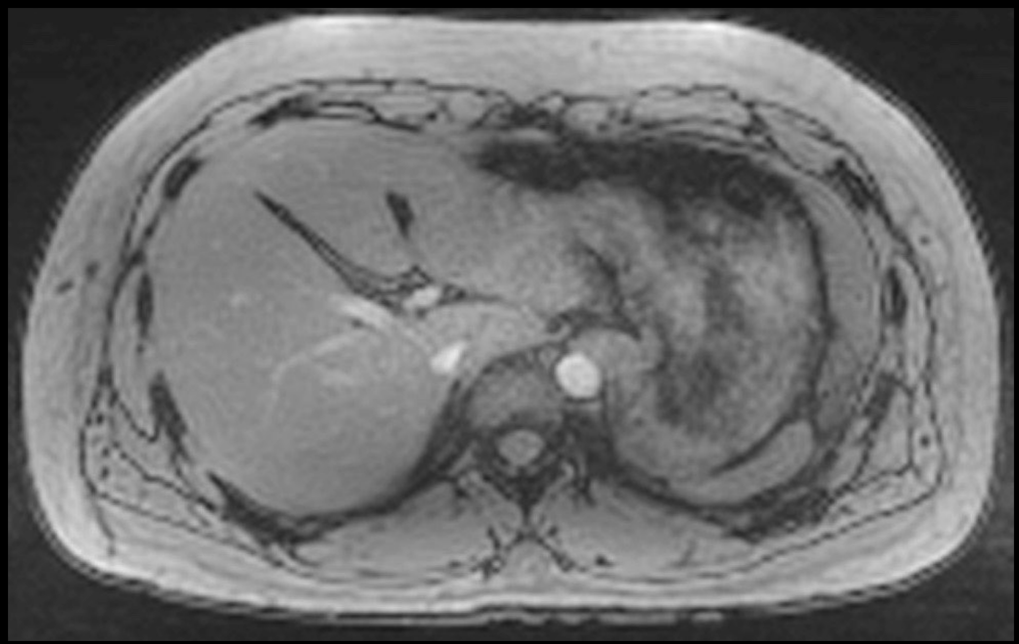
3D Stack-of-Radial: Liver MRI

3D Cartesian MRI

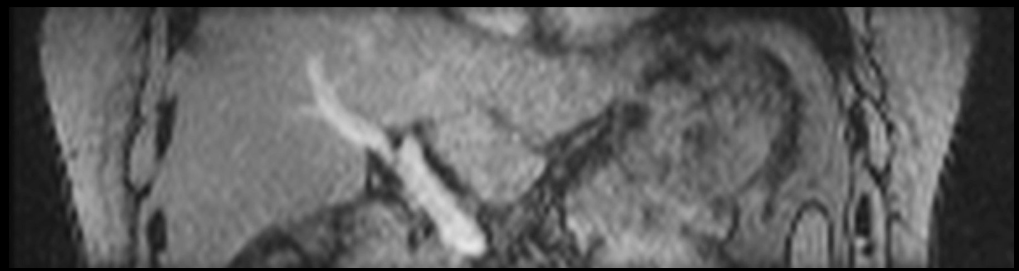


Insufficient breath-holding

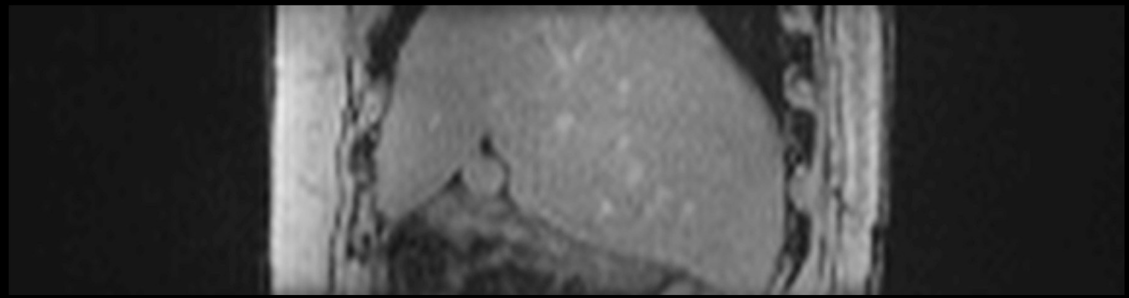
Free-breathing 3D Stack-of-Radial MRI



Axial



Coronal



Sagittal

courtesy of Tess Armstrong

3D Radial: Coronary MRA

Contrast-Enhanced MRA at 3.0T

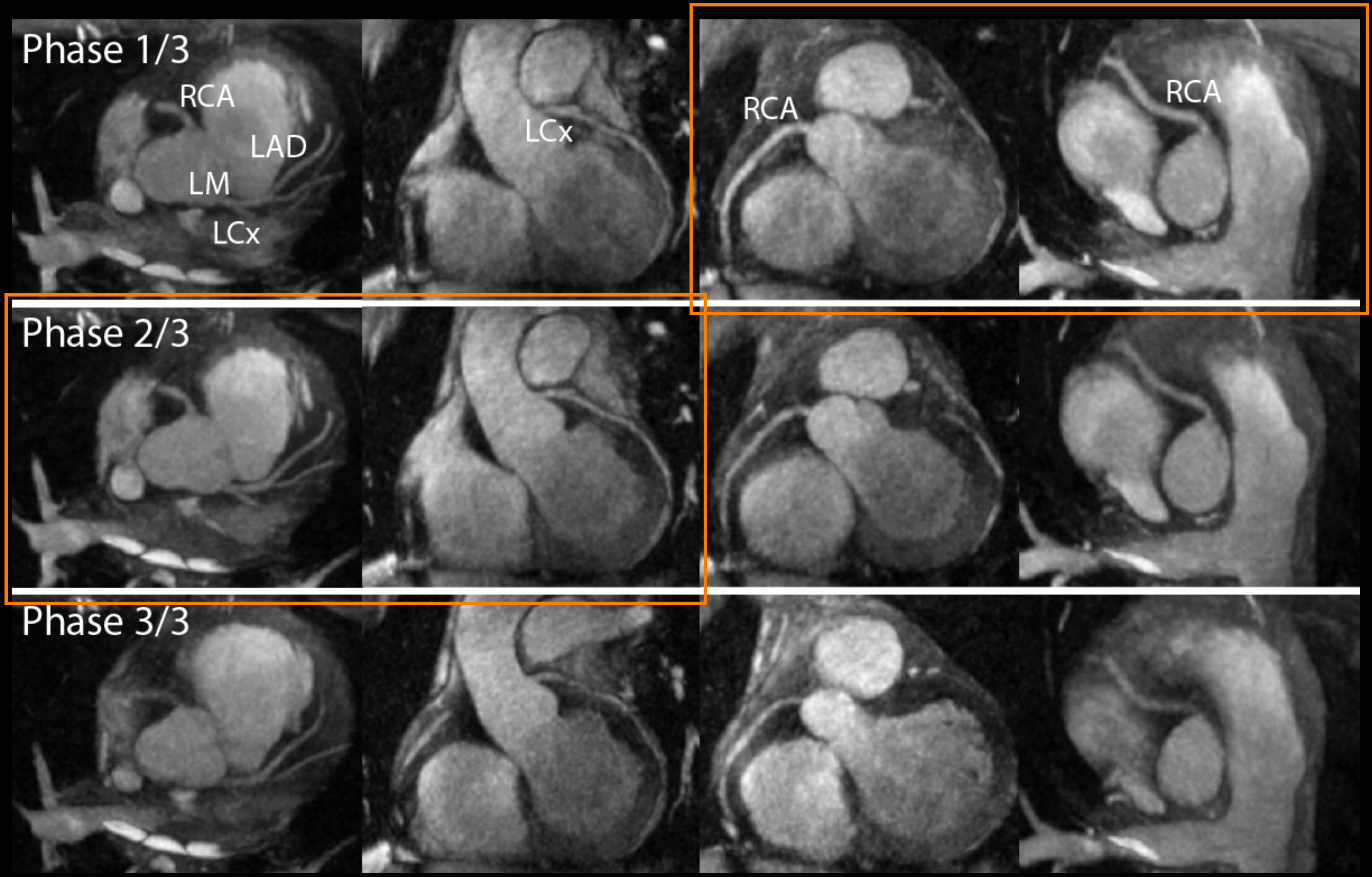
ECG-gated, fat-saturated, inversion-recovery prepared spoiled gradient echo sequence

$(1.0\ \text{mm})^3$ spatial resolution, 1D self navigation, CG-SENSE recon, 5.4 min scan time

courtesy of Debiao Li and J Pang (Cedars-Sinai)

3D Cones: Coronary MRA

Multi-Phase Thin-Slab MIP Reformats



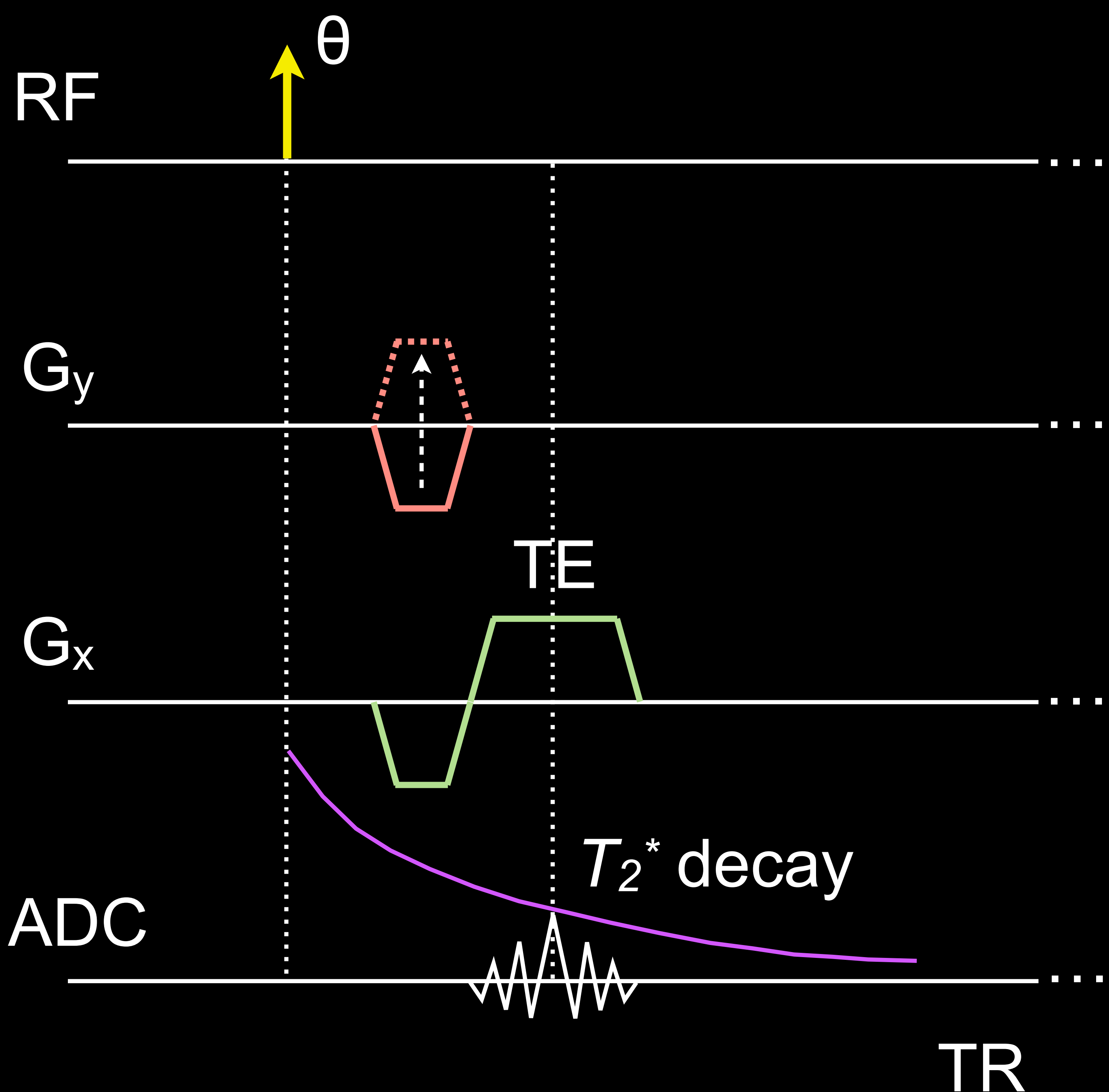
Wu HH et al., MRM 2013; 69: 1083-1093

Echo-Planar Imaging

- Echo-Planar Imaging (EPI)[1]
- Ultra-fast imaging (<100 ms/frame)
- Imperfections and artifacts
- Ongoing topic of rapid MRI research

[1] *Mansfield P, J Phys C: Solid State Phys 1977*

Gradient Echo



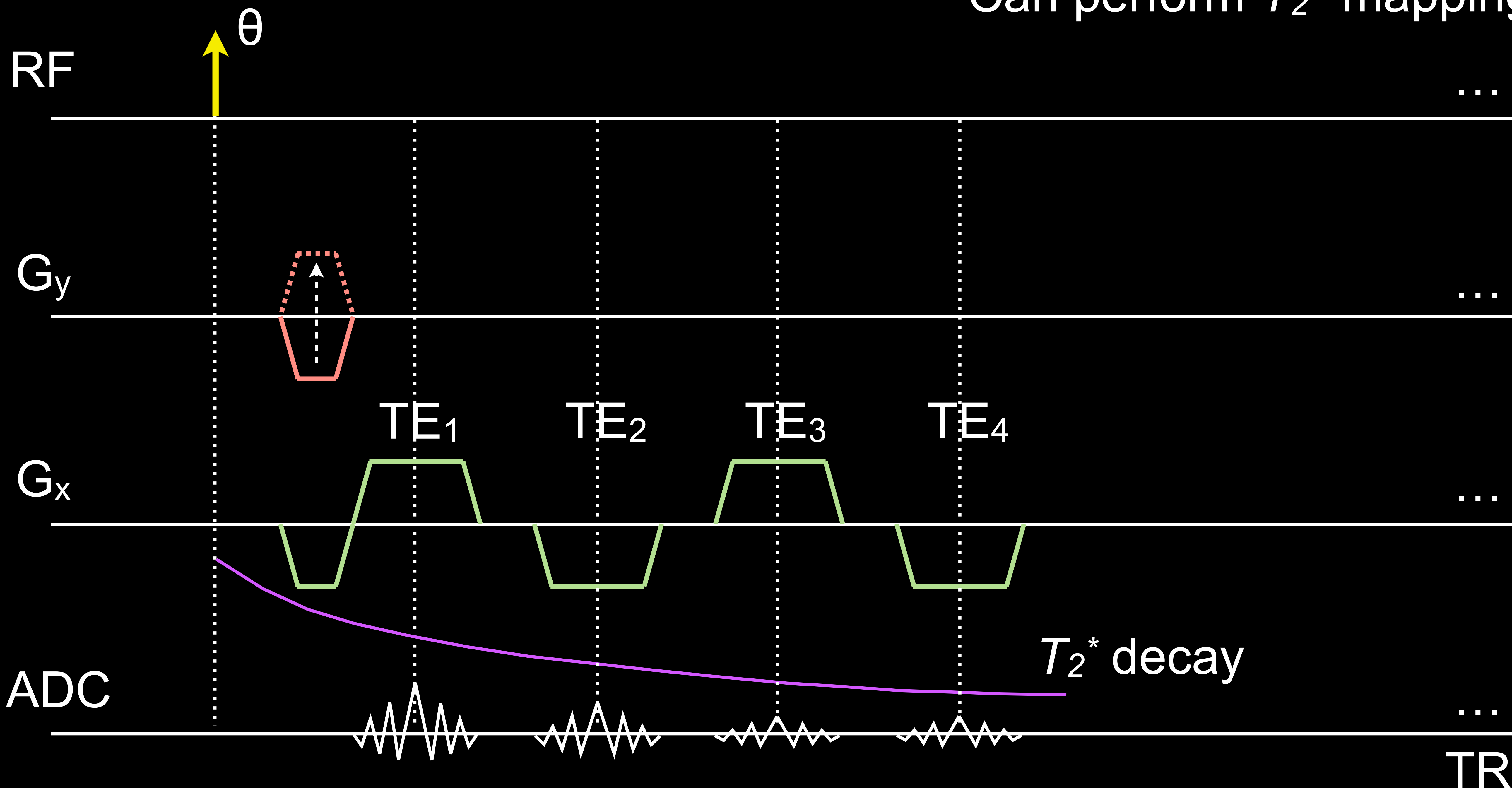
- Utilization of transverse magnetization
  - With T_s = 8 μs and N_x = 128, T_{acq} = 1.024 ms
  - <2% of T_2^* in brain at 3 T![1]
- Scan time
  - $T_{GRE} = N_{pe} \times TR$
  - TR = 10 ms, N_{pe} = 256: T_{GRE} = 2.56 sec

[1]*Peters, et al., Proc ISMRM 2006*

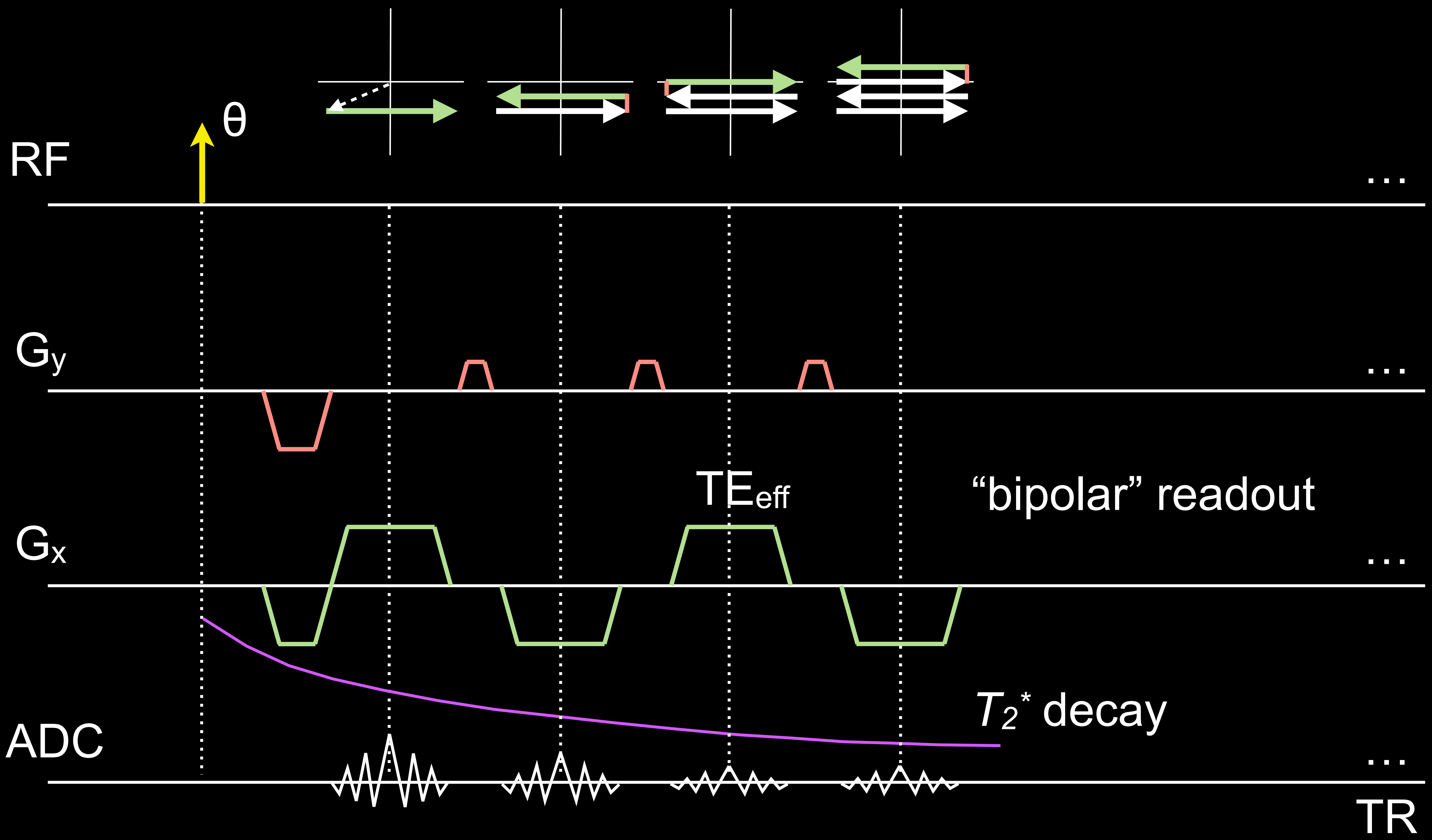
Multi-echo Gradient Echo

ΔTE can be non-uniform

Can perform T_2* mapping



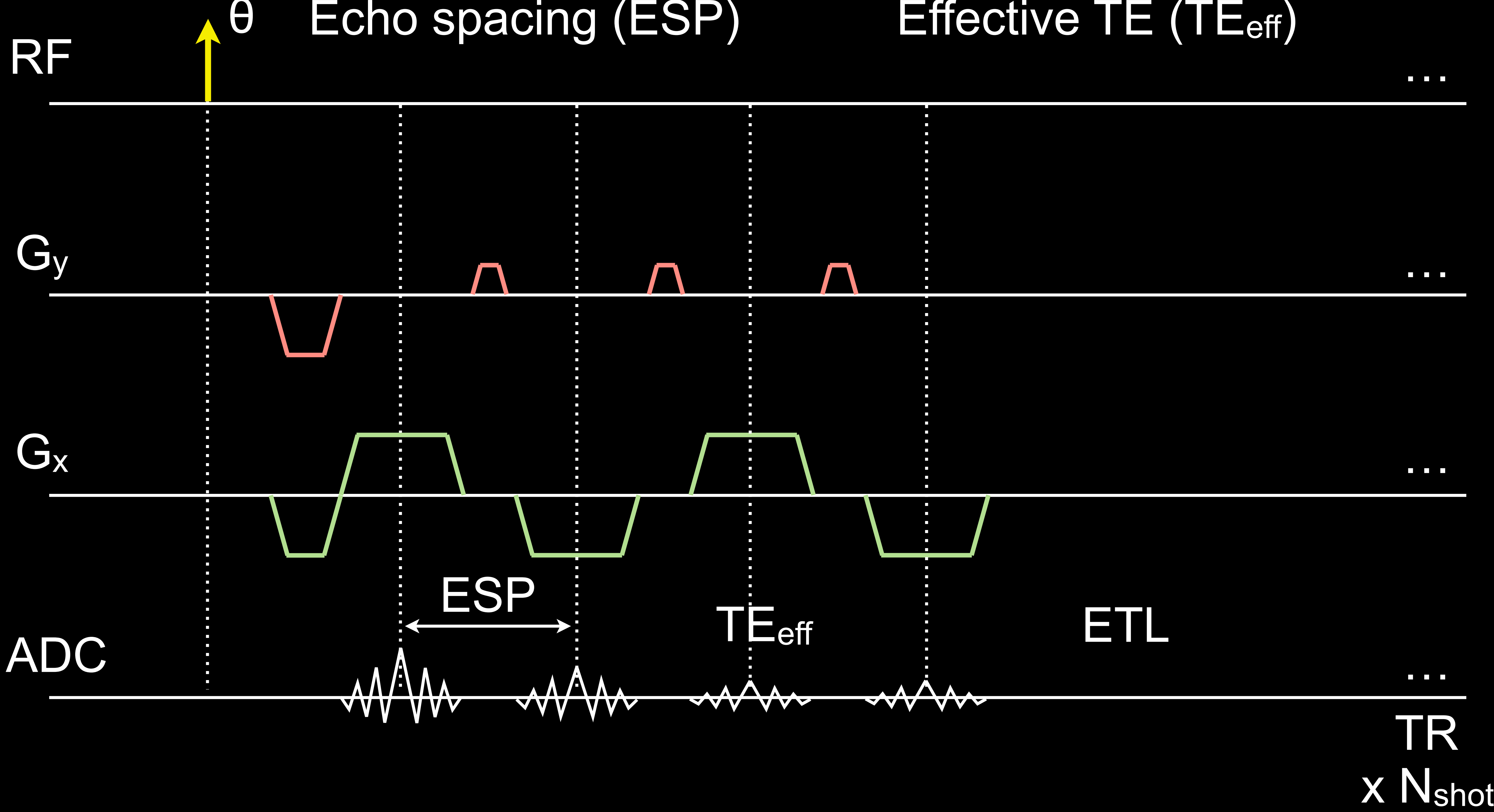
Gradient-Echo EPI



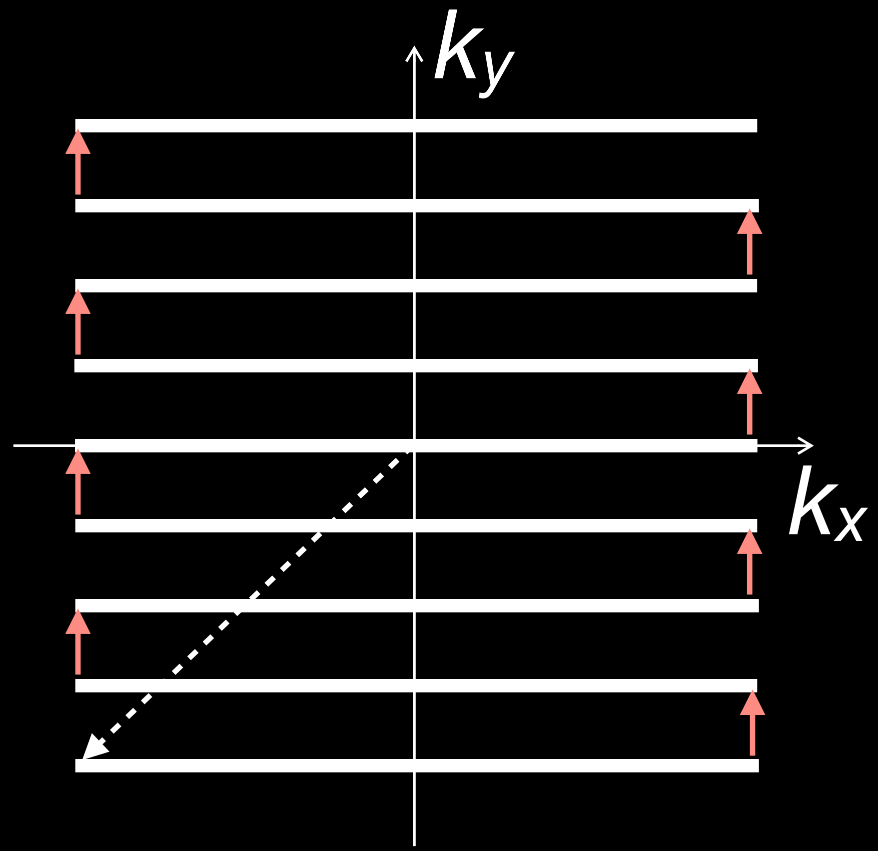
EPI Sequence Parameters

Echo train length (ETL) Number of shots (N_{shot})

Echo spacing (ESP) Effective TE (TE_{eff})



EPI k-Space Sampling



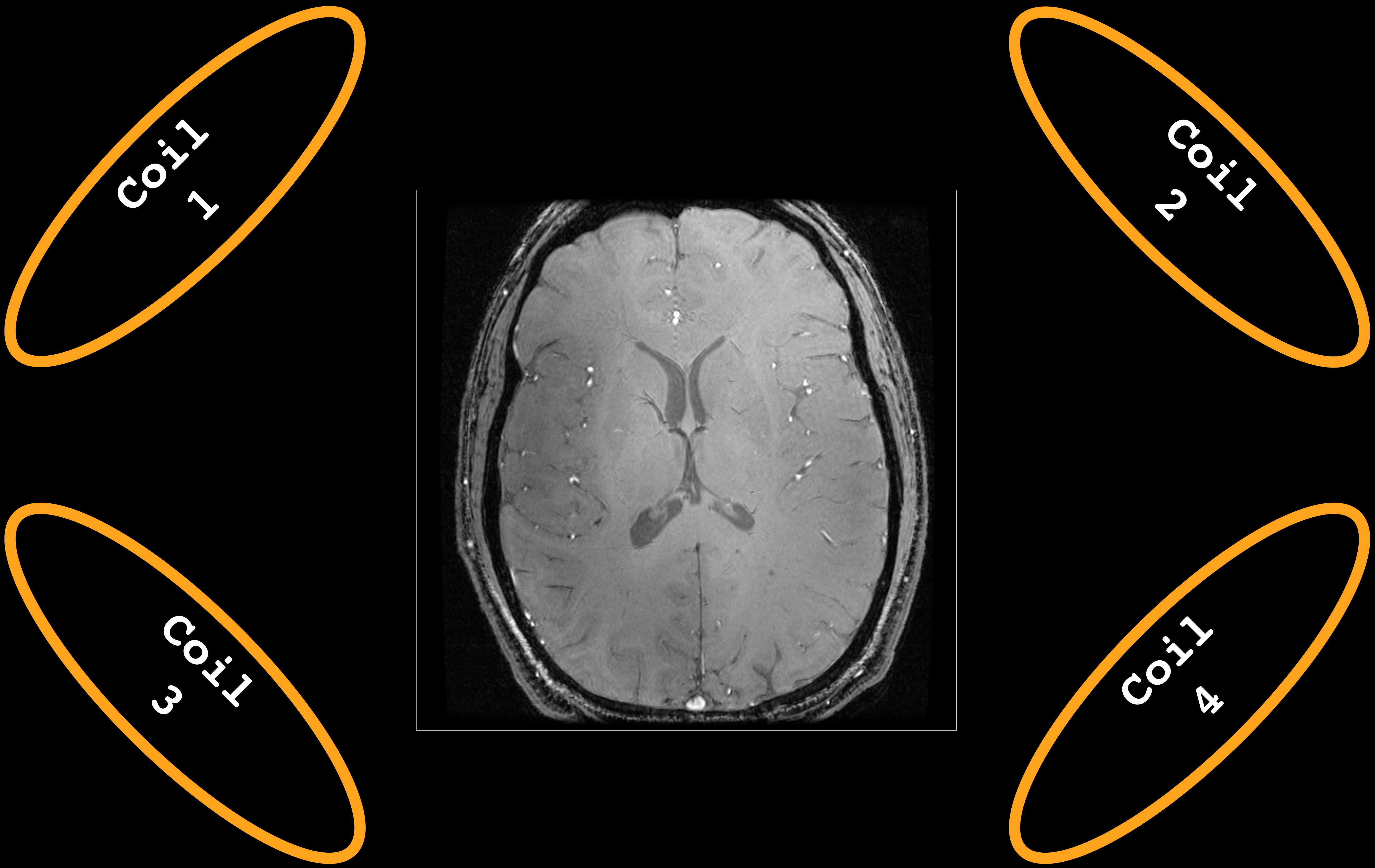
- ETL can be 4-64 or higher
  - Limited by T_2* decay, off-resonance effects
  - aka “EPI factor”
- ESP typically ~1 ms
  - Must accommodate RF, gradients, ADC
  - Short ESP facilitates high ETL

Fast Sampling Trajectories

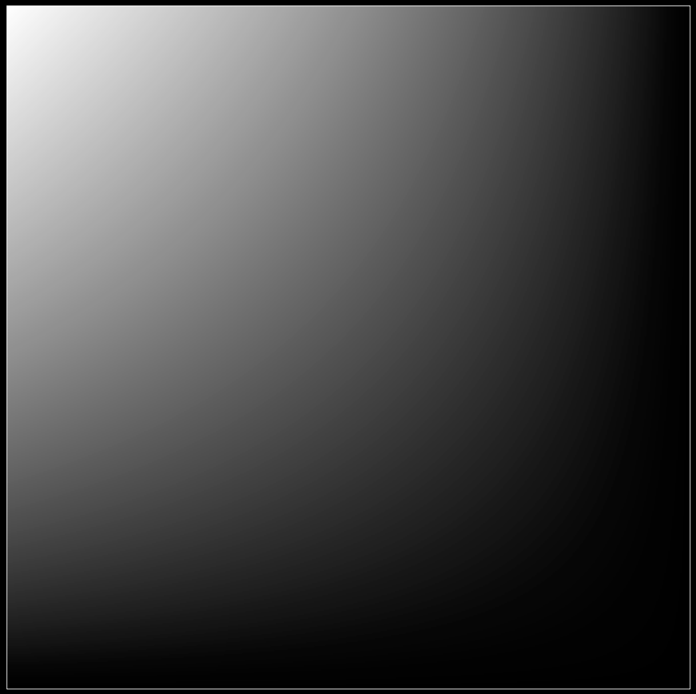
- Benefits
  - Reduced scan time
  - Robustness to motion and flow
  - Short echo time
- Applications
  - Dynamic MRI
  - Real-time MRI
  - Cardiovascular MRI
  - Short-TE MRI
- Challenges
  - Hardware performance
  - Gradient fidelity
  - Off-resonance effects
  - Design and implementation
- Challenges addressed
- On-going research
- Use judiciously!

Parallel Imaging

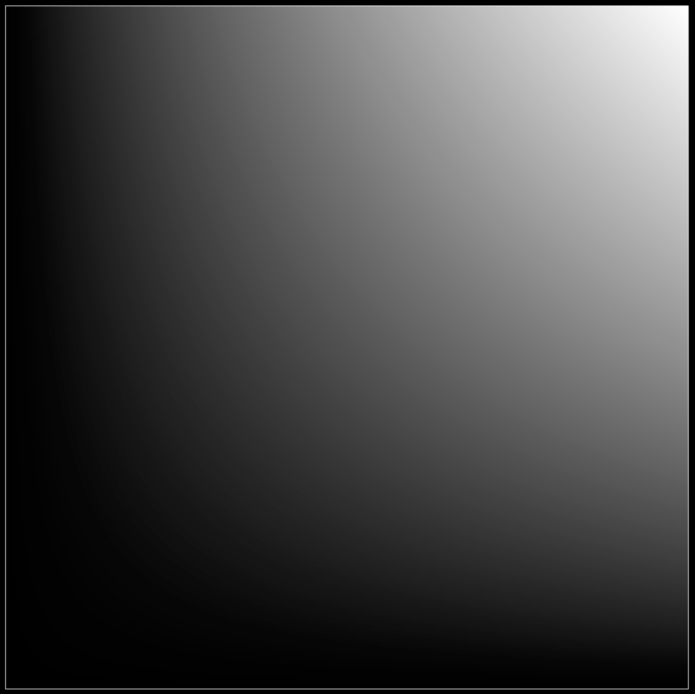
Multi-coil Arrays

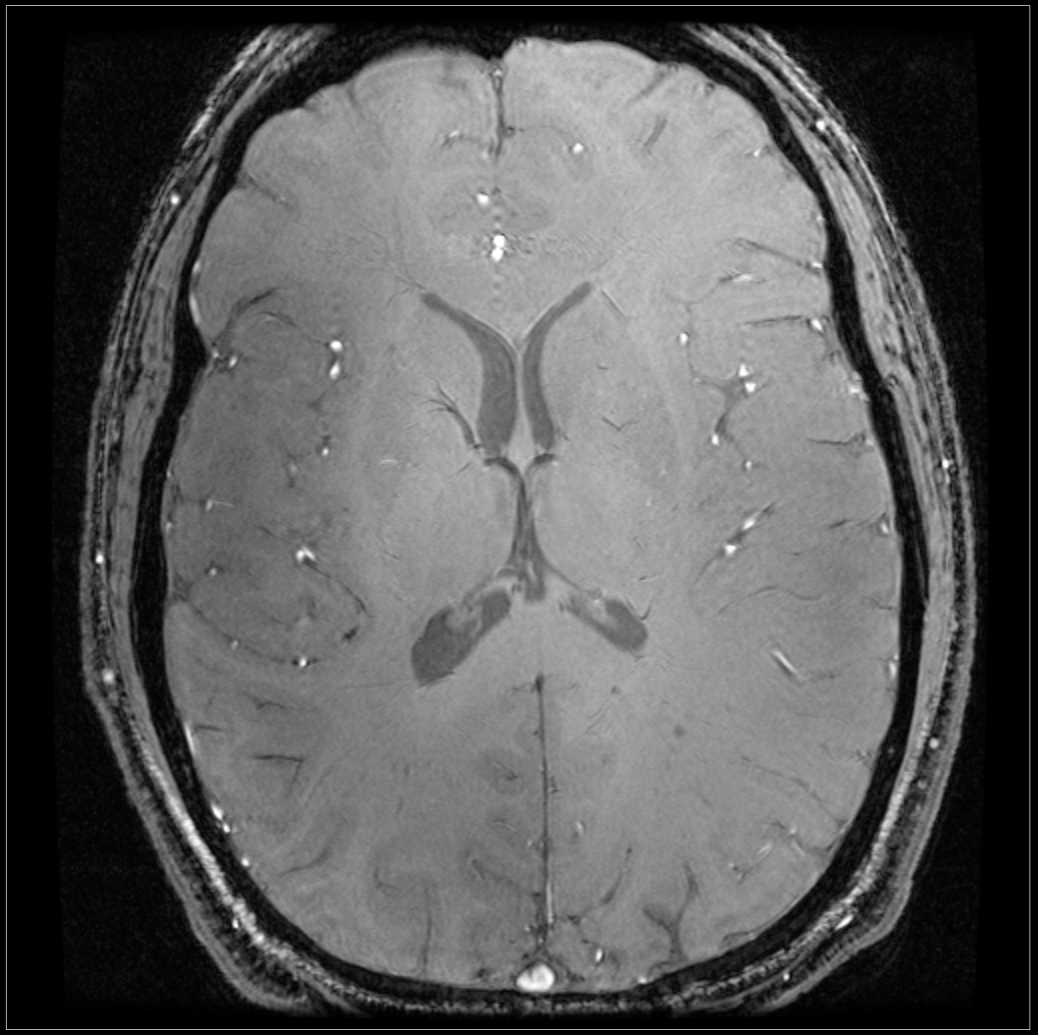


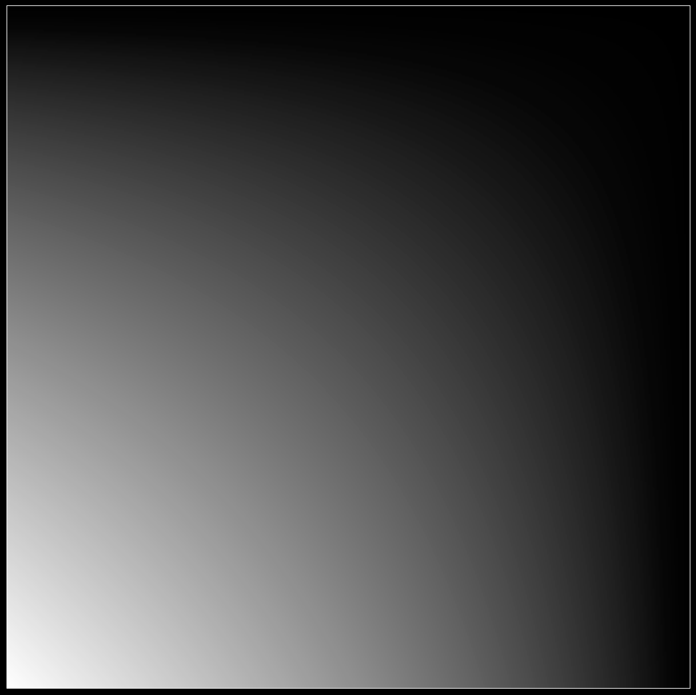
Multi-coil Sensitivity

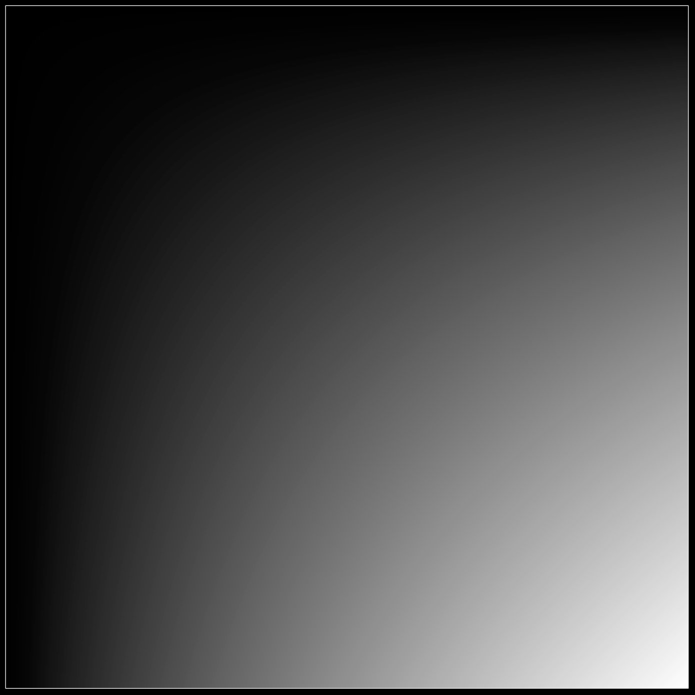


$$\left\| \vec{B}\left(\vec{r}\right) \right\|$$



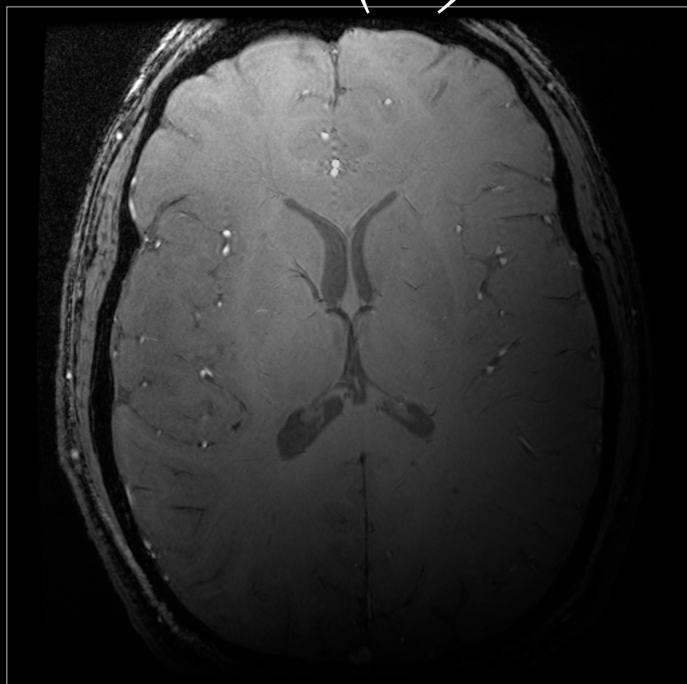




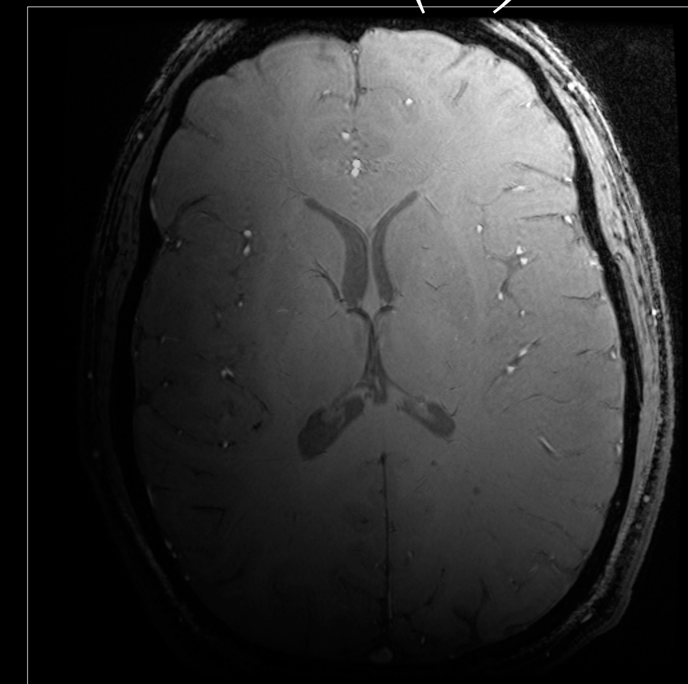


Multi-coil Images

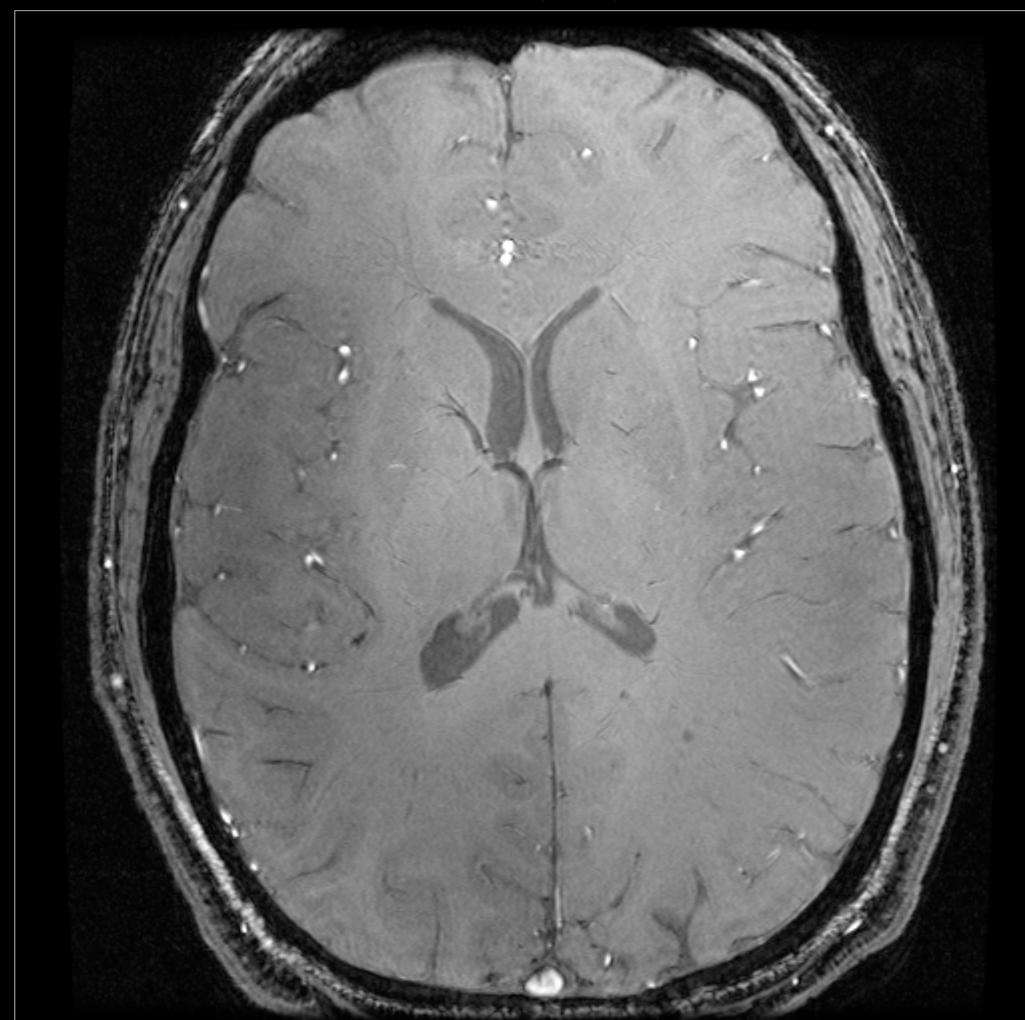
$m_1(x)$



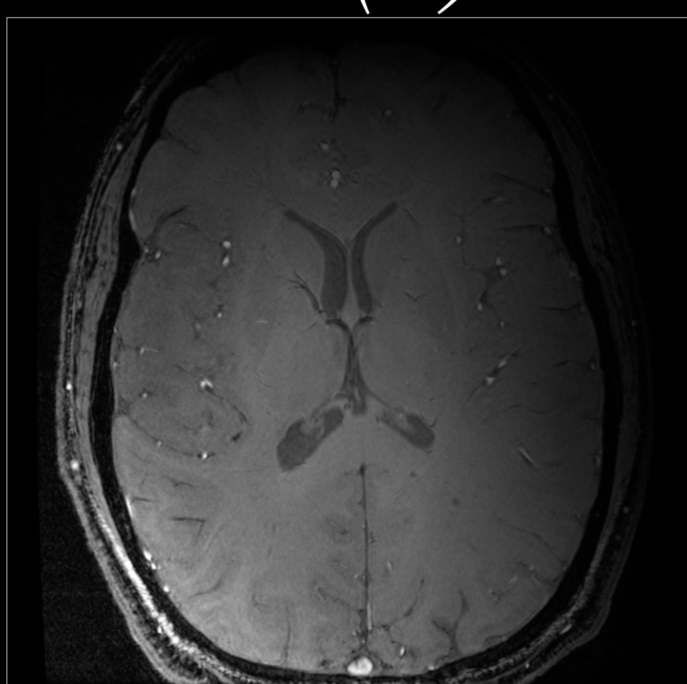
$m_2(x)$



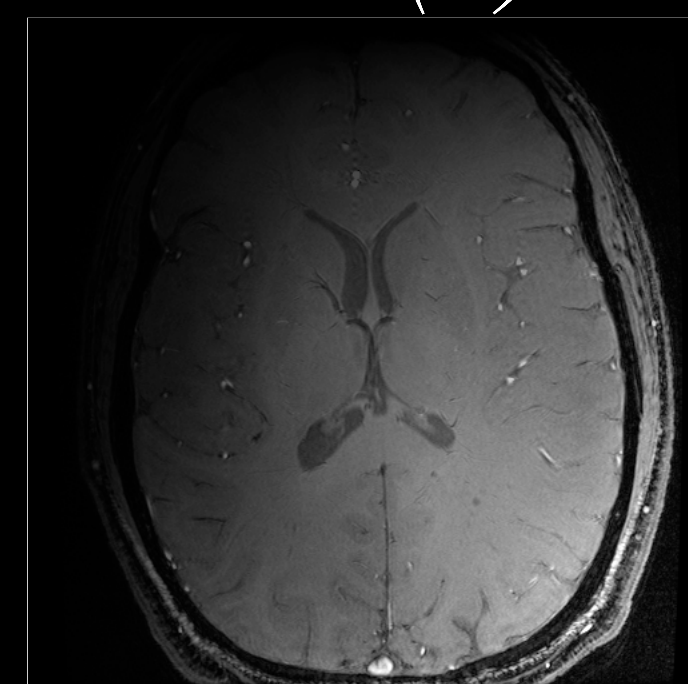
$m_s(x)$



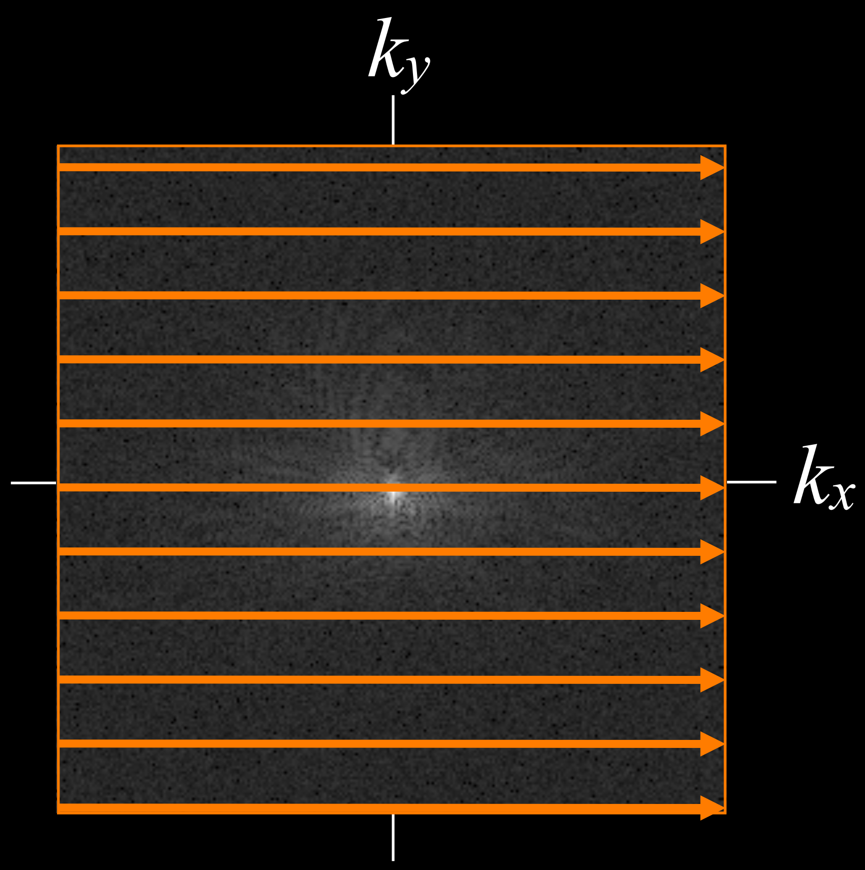
$m_3(x)$

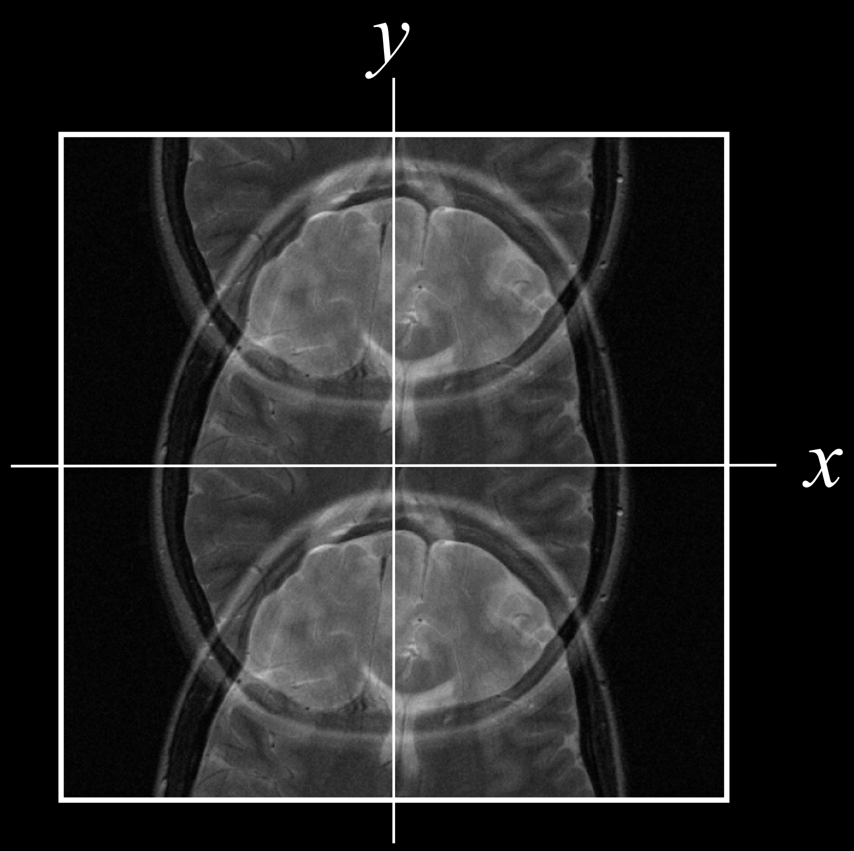


$m_4(x)$

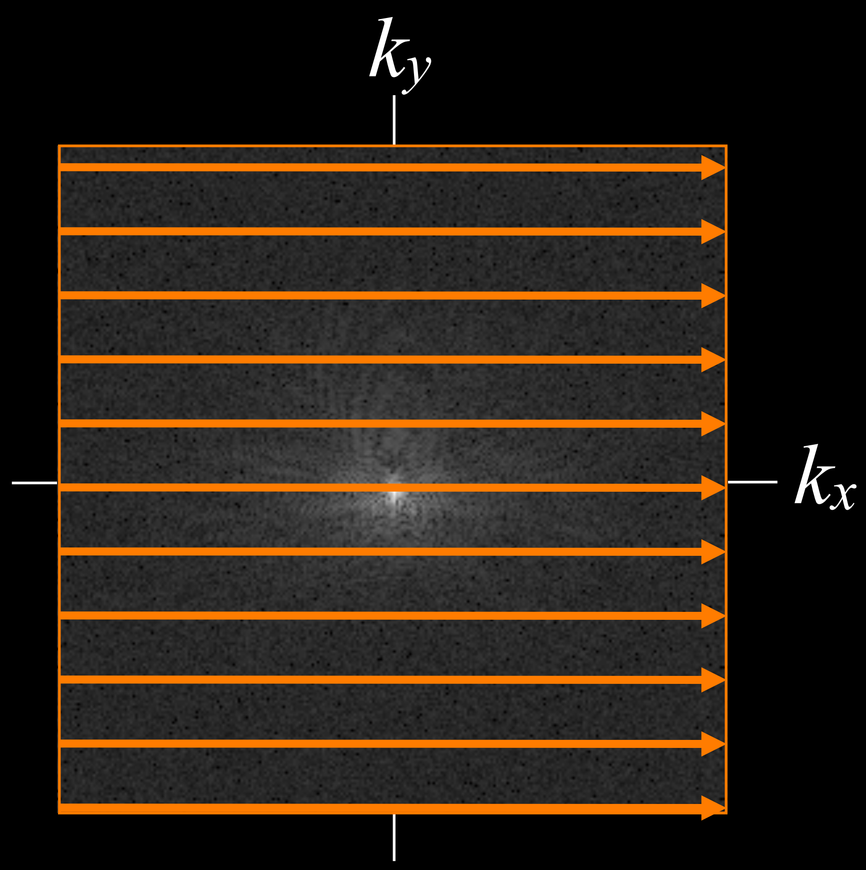


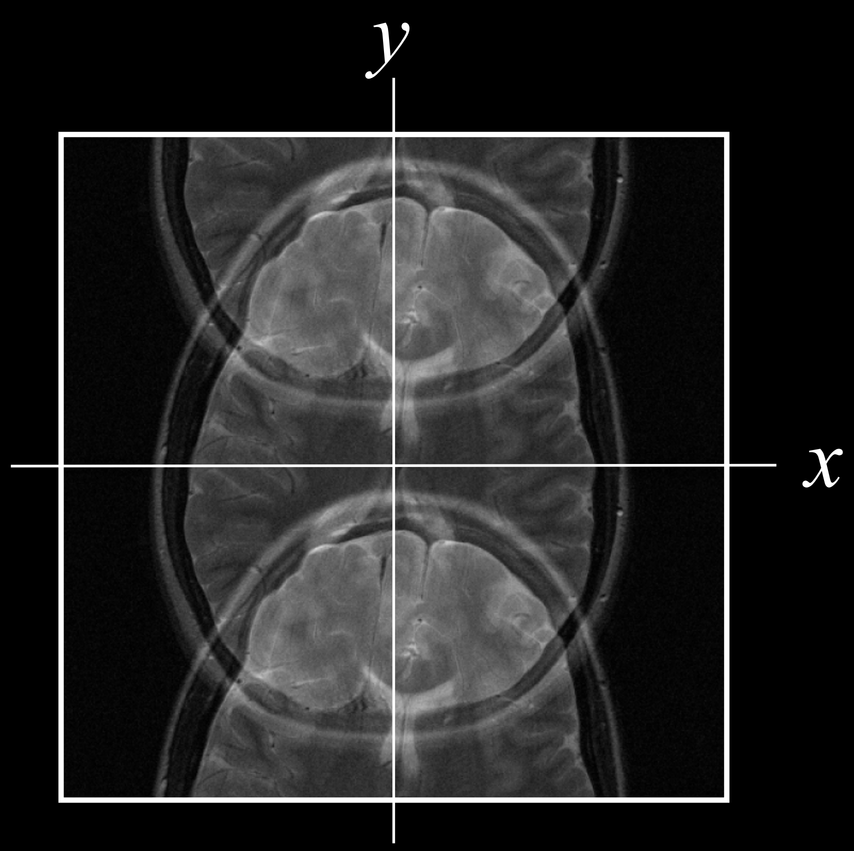
Accelerate Imaging with Array Coils





Accelerate Imaging with Array Coils





- Parallel Imaging
  - Coil elements provide some localization
  - Undersample in k-space, producing aliasing
  - Sort out in reconstruction

Parallel Imaging

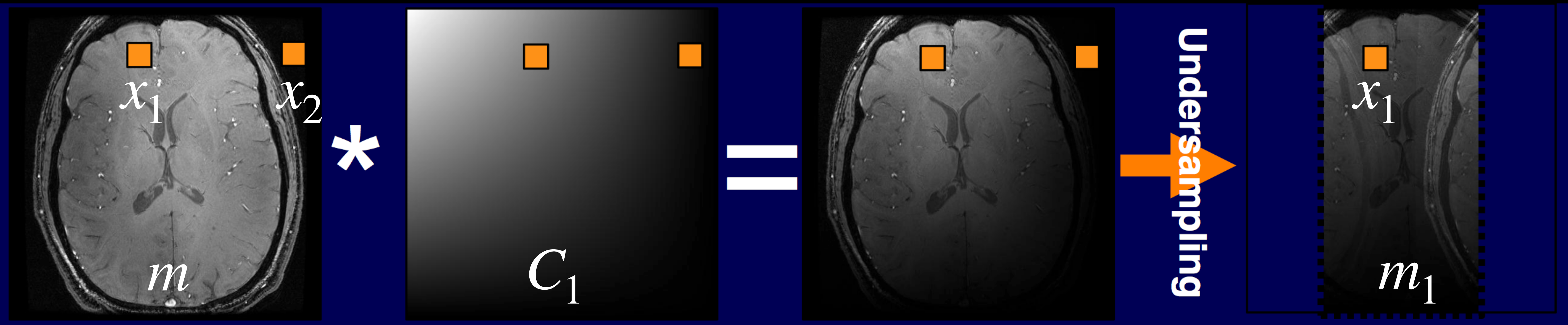
- Many approaches:
  - Image domain - SENSE
  - k-space domain - SMASH, GRAPPA
  - Hybrid - ARC

- We will introduce one:
  - SENSE: optimal if you know coil sensitivities

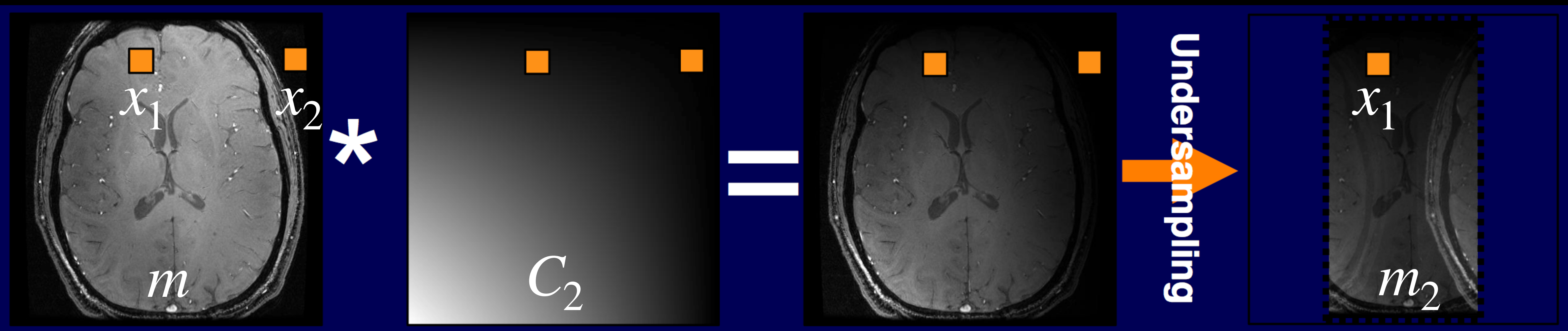
Pruessmann et al. MRM 1999
https://pubmed.ncbi.nlm.nih.gov/10542355/

Cartesian SENSE

$$m_1(\vec{x_1}) = C_1(\vec{x_1})m(\vec{x_1}) + C_1(\vec{x_2})m(\vec{x_2})$$



$$m_2(\vec{x_1}) = C_2(\vec{x_1})m(\vec{x_1}) + C_2(\vec{x_2})m(\vec{x_2})$$



$$\underbrace{\begin{pmatrix} m_1(\vec{x_1}) \\ m_2(\vec{x_1}) \\ \cdot \\ \cdot \\ \cdot \\ m_L(\vec{x_1}) \end{pmatrix}}_{\text{Aliased Images}} = \underbrace{\begin{pmatrix} C_1(\vec{x_1}) & C_1(\vec{x_2}) \\ C_2(\vec{x_1}) & C_2(\vec{x_2}) \\ \cdot & \\ \cdot & \\ \cdot & \\ C_L(\vec{x_1}) & C_L(\vec{x_2}) \end{pmatrix}}_{\text{Sensitivity at Source Voxels}} \underbrace{\begin{pmatrix} m(\vec{x_1}) \\ m(\vec{x_2}) \end{pmatrix}}_{\text{Source Voxels}} + \begin{pmatrix} n_1(\vec{x_1}) \\ n_2(\vec{x_1}) \\ \cdot \\ \cdot \\ \cdot \\ n_L(\vec{x_1}) \end{pmatrix}$$

OR

$$\underbrace{m_s}_{L \times 1} = \underbrace{C}_{L \times 2}\overset{2 \times 1}{m} + \underbrace{n}_{L \times 1}$$

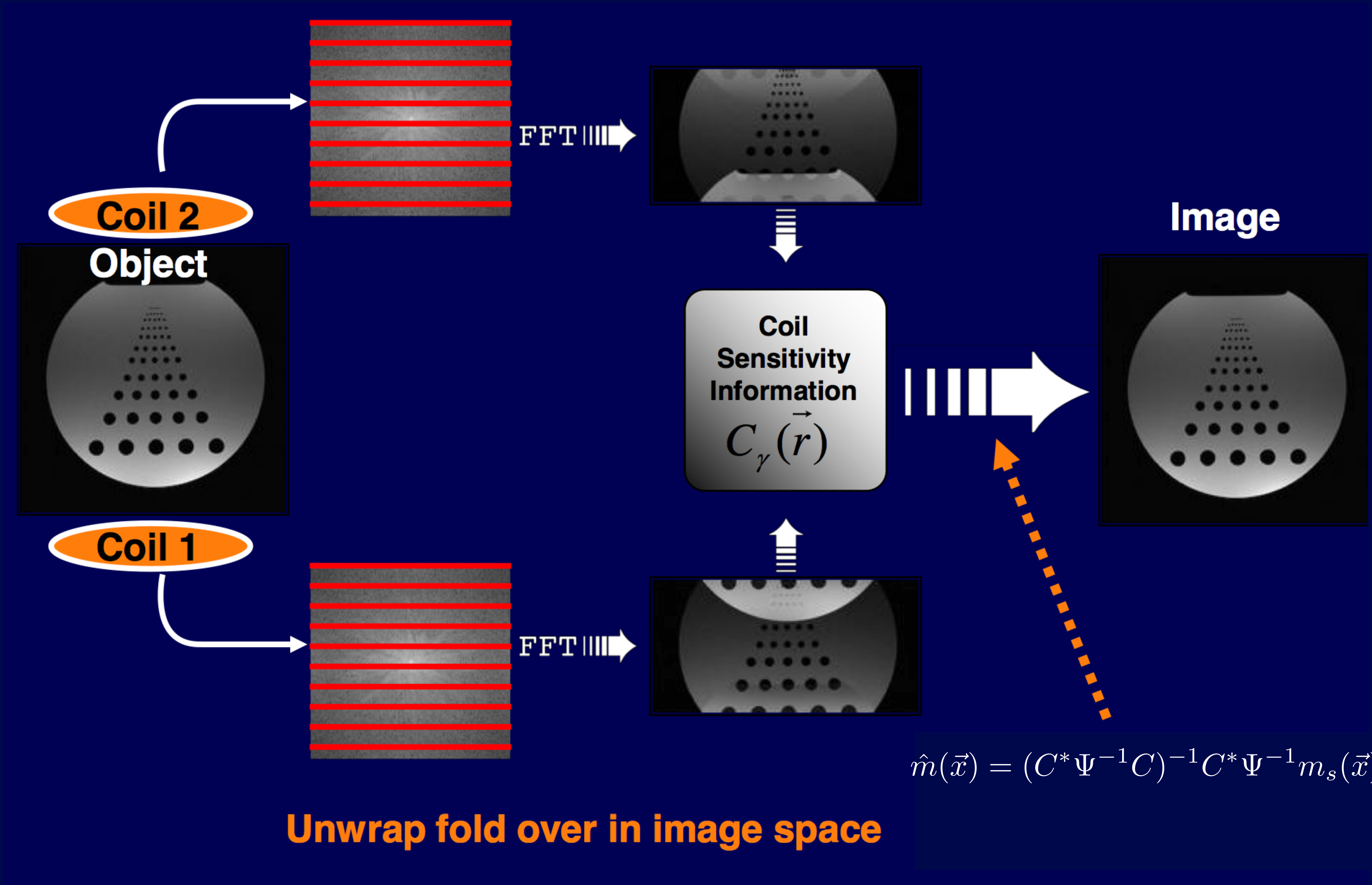
$$\hat{m}(\vec{x}) = \underbrace{(C^*\Psi^{-1}C)}_{2 \times 2}{}^{-1}\underbrace{C^*\Psi^{-1}}_{2 \times L}\underbrace{m_s(\vec{x})}_{L \times 1}$$

L aliased reconstruction resolves 2 image pixels

For an N x N image, we solve (N/2 x N)
2 x 2 inverse systems

For an acceleration factor R, we solve (N/R x N)
R x R inverse systems

SENSE Reconstruction

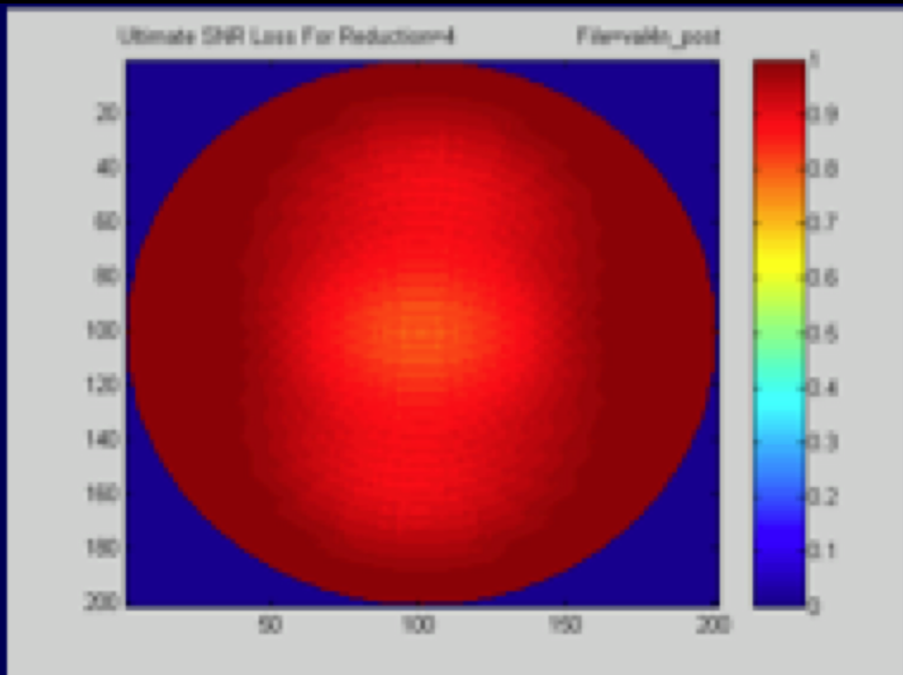


SNR Cost

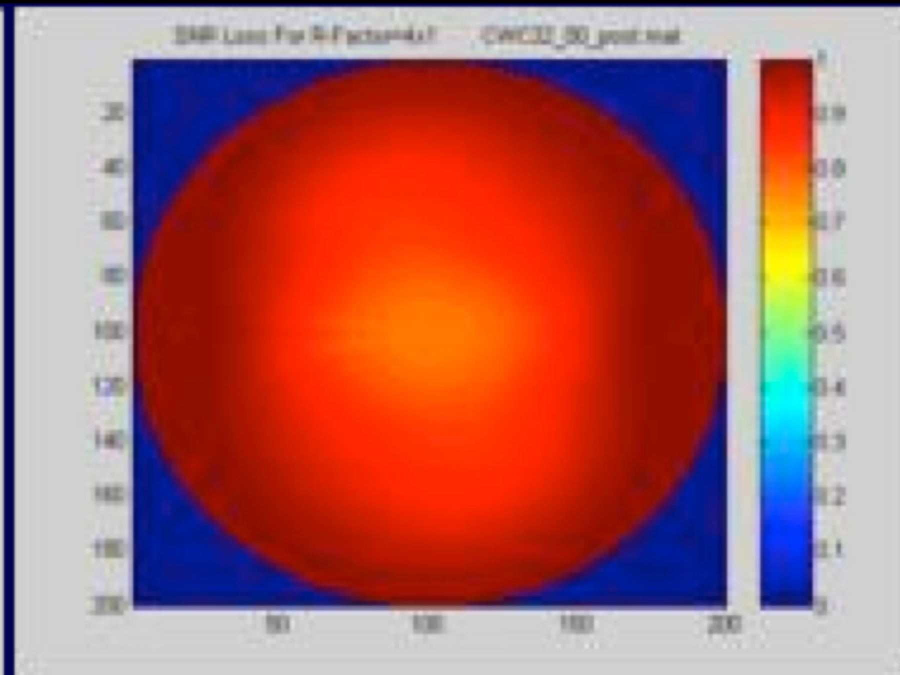
- How large can R be?
- Two SNR loss mechanisms
  - Reduced scan time
  - Condition of the SENSE decomposition
- SNR Loss

$$SNR_{SENSE} = \frac{SNR}{\underbrace{g}_{\text{Geometry Factor}}\underbrace{\sqrt{R}}_{\text{Reduced Scan Time}}}$$

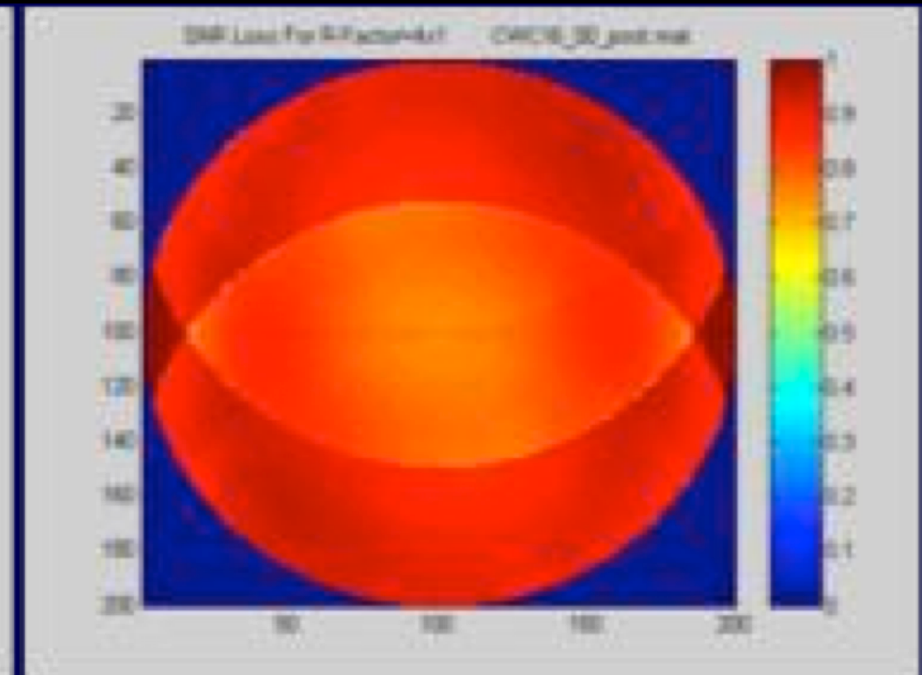
1/g-factor Map for R=4



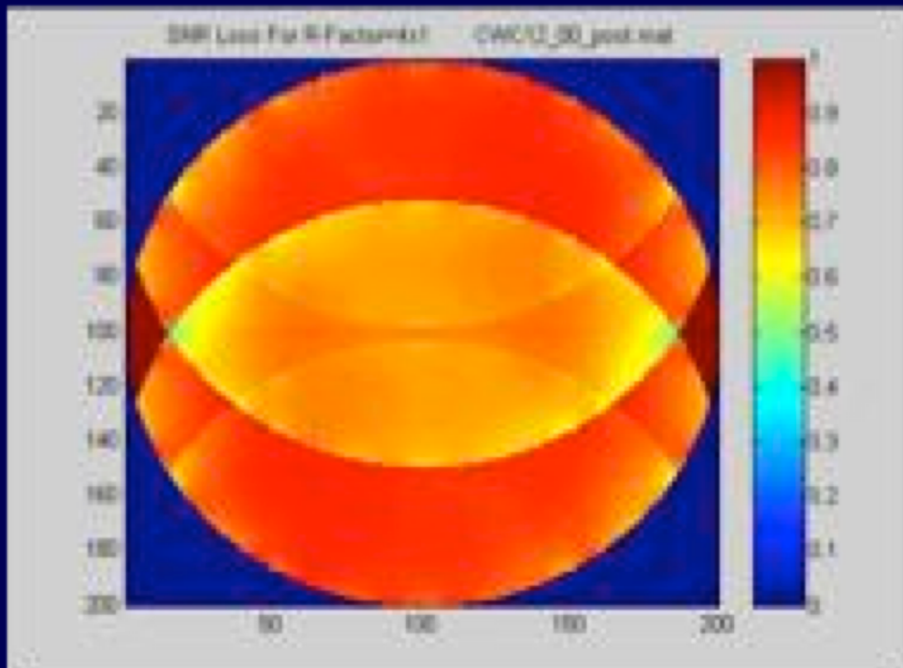
∞ elements



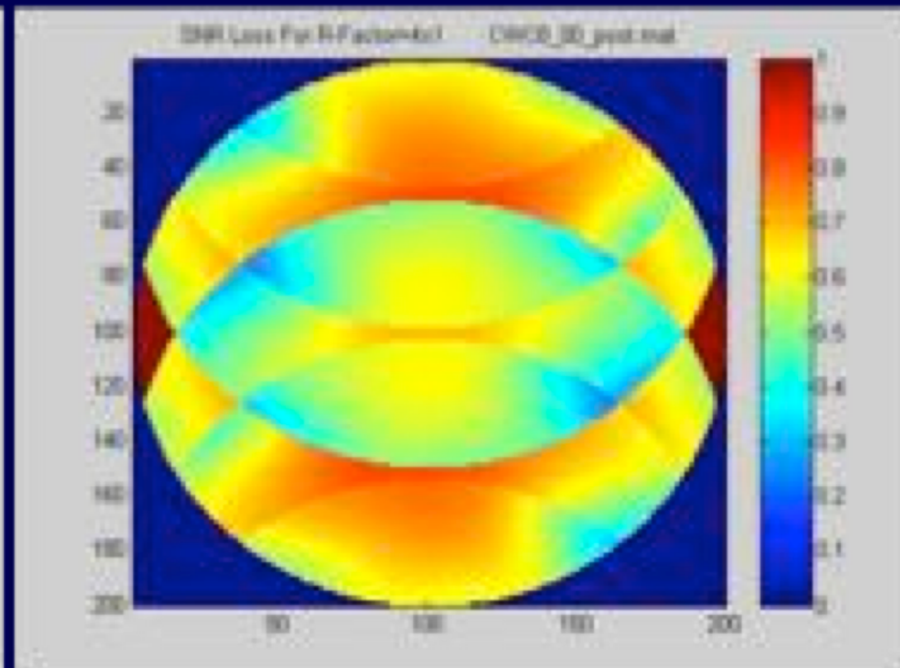
32 elements



16 elements



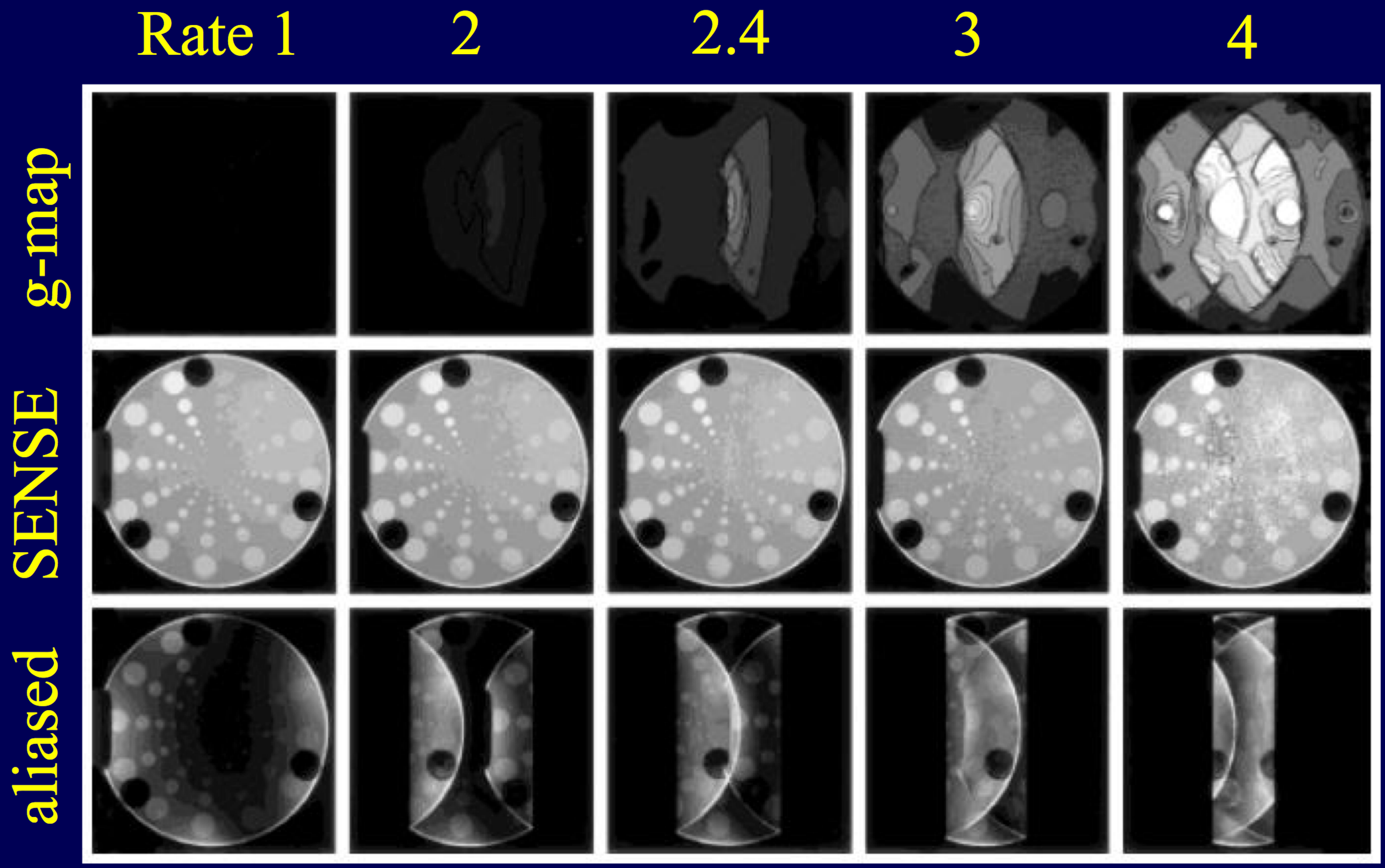
12 elements



8 elements

Relative SNR Scale

g-factor and its impact on images



Parallel Imaging

- Utilizes coil sensitivities to increase the speed of MRI (typical R=2-4)
- Cases for parallel imaging
  - Higher patient throughput
  - Real-time imaging/Interventional imaging
  - Motion suppression
- Cases against parallel imaging
  - Low SNR applications

Compressed Sensing (CS)

What is CS?

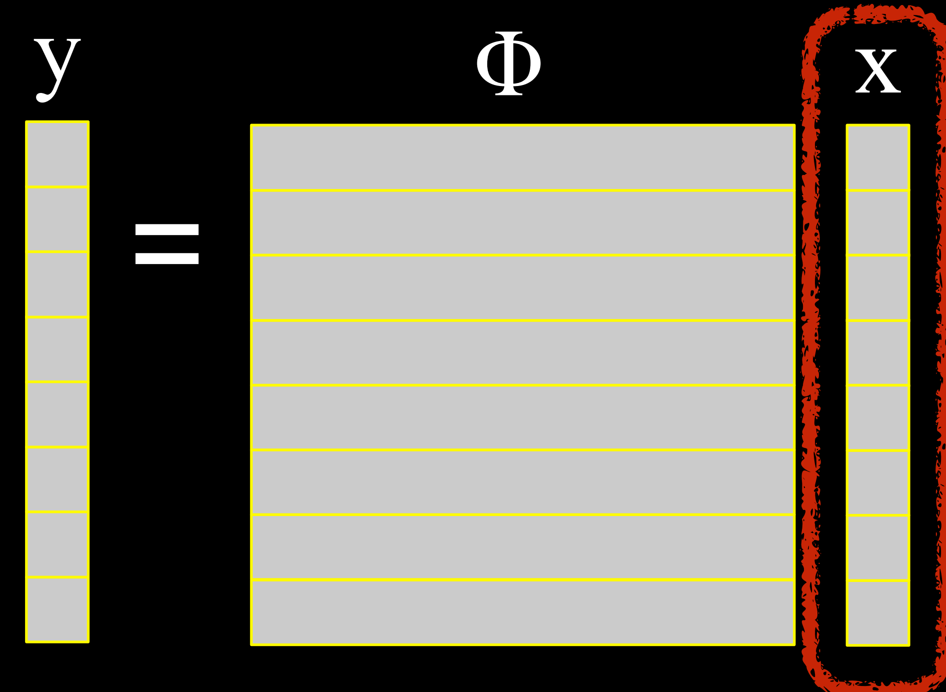
- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

Donoho, IEEE TIT, 2006
Candes et al., Inverse Problems, 2007

What is CS?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

8 Equations
8 Unknowns

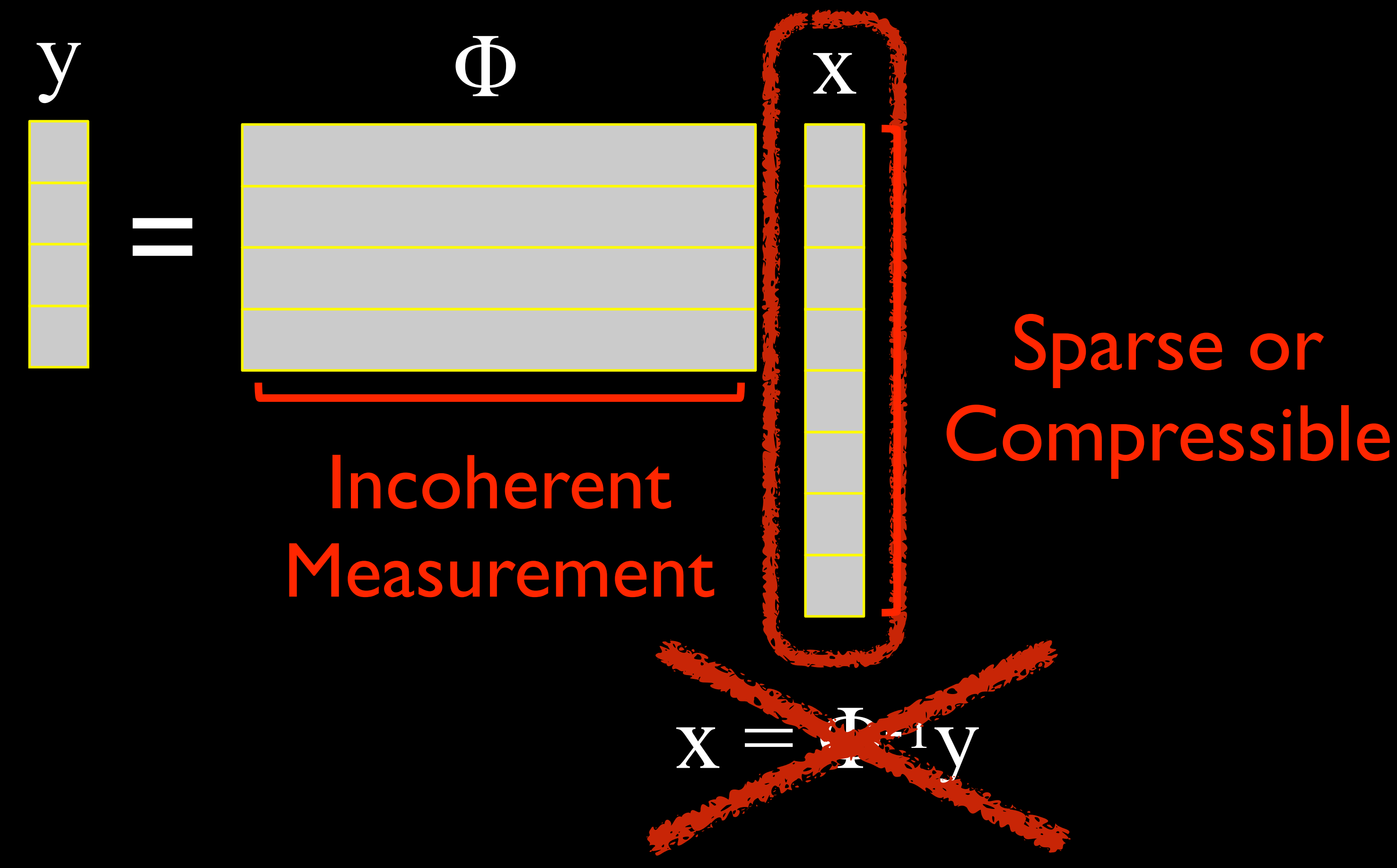


$$x = \Phi^{-1} y$$

What is CS?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

4 Equations
8 Unknowns



What is CS?

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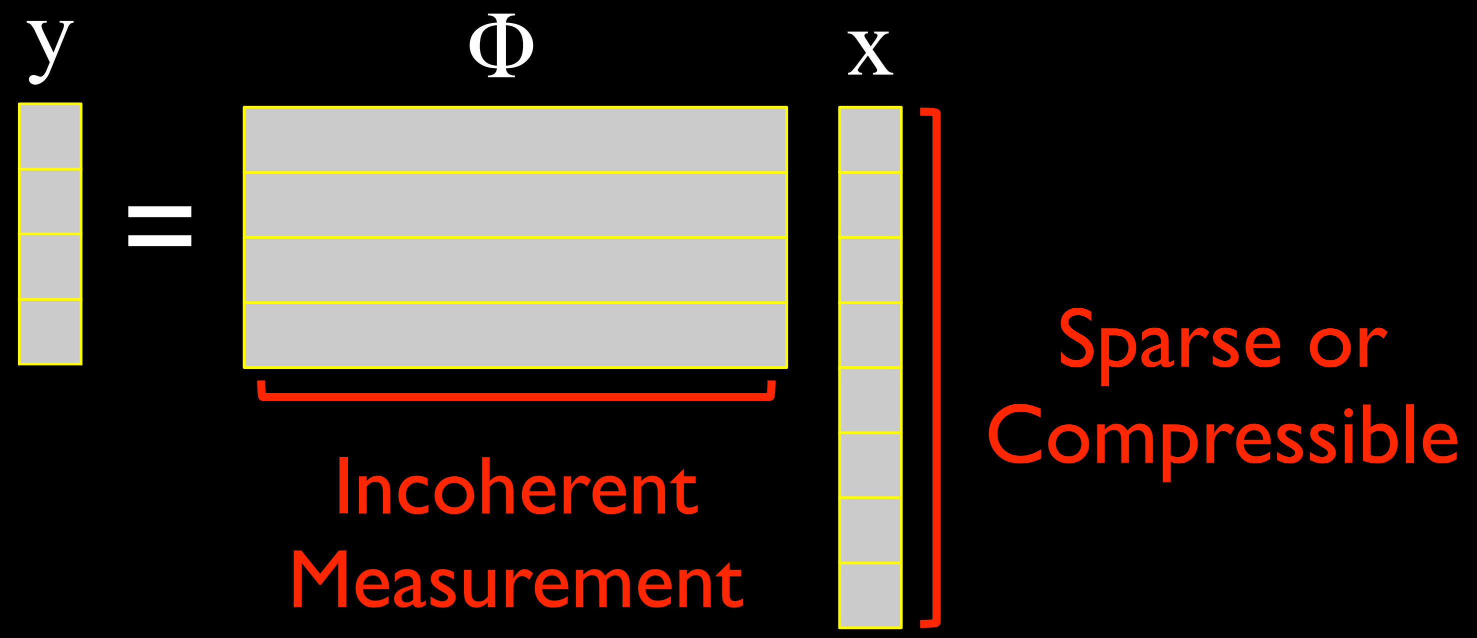
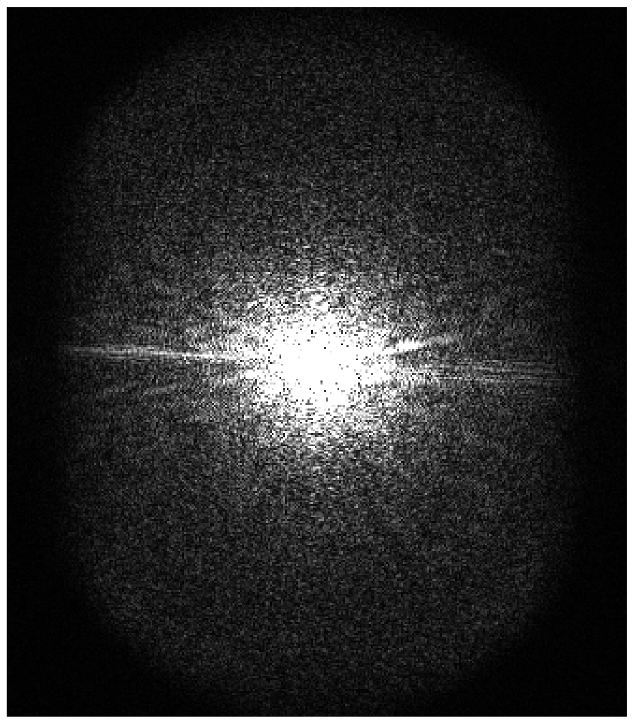
4 Equations
8 Unknowns



We still can find 8 unknowns!

Compressed Sensing MRI

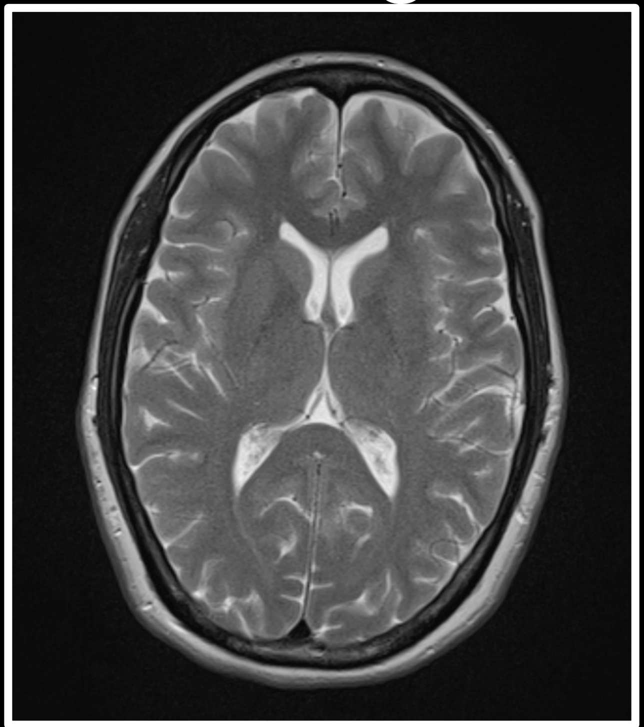
k-space



Inverse Fourier Transform Φ^{-1}

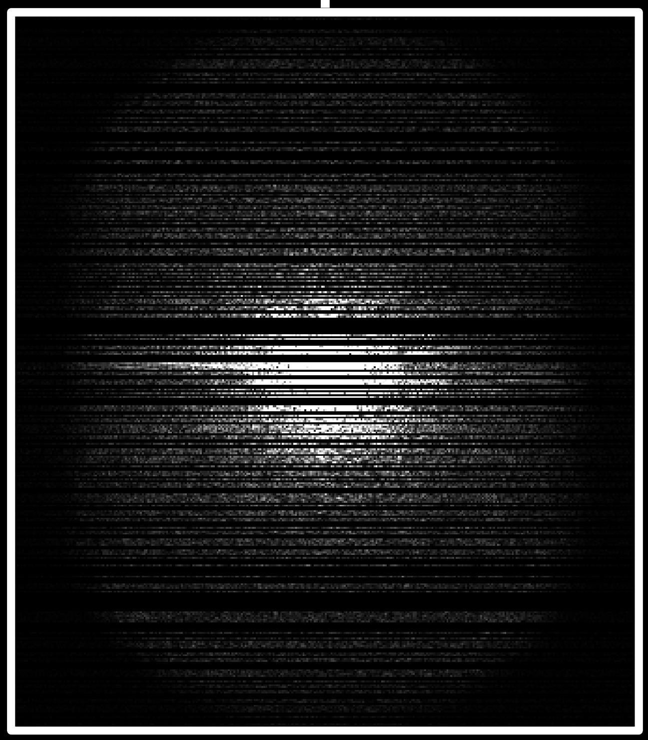
$$x = \Phi^{-1} y$$

Image



Compressed Sensing MRI

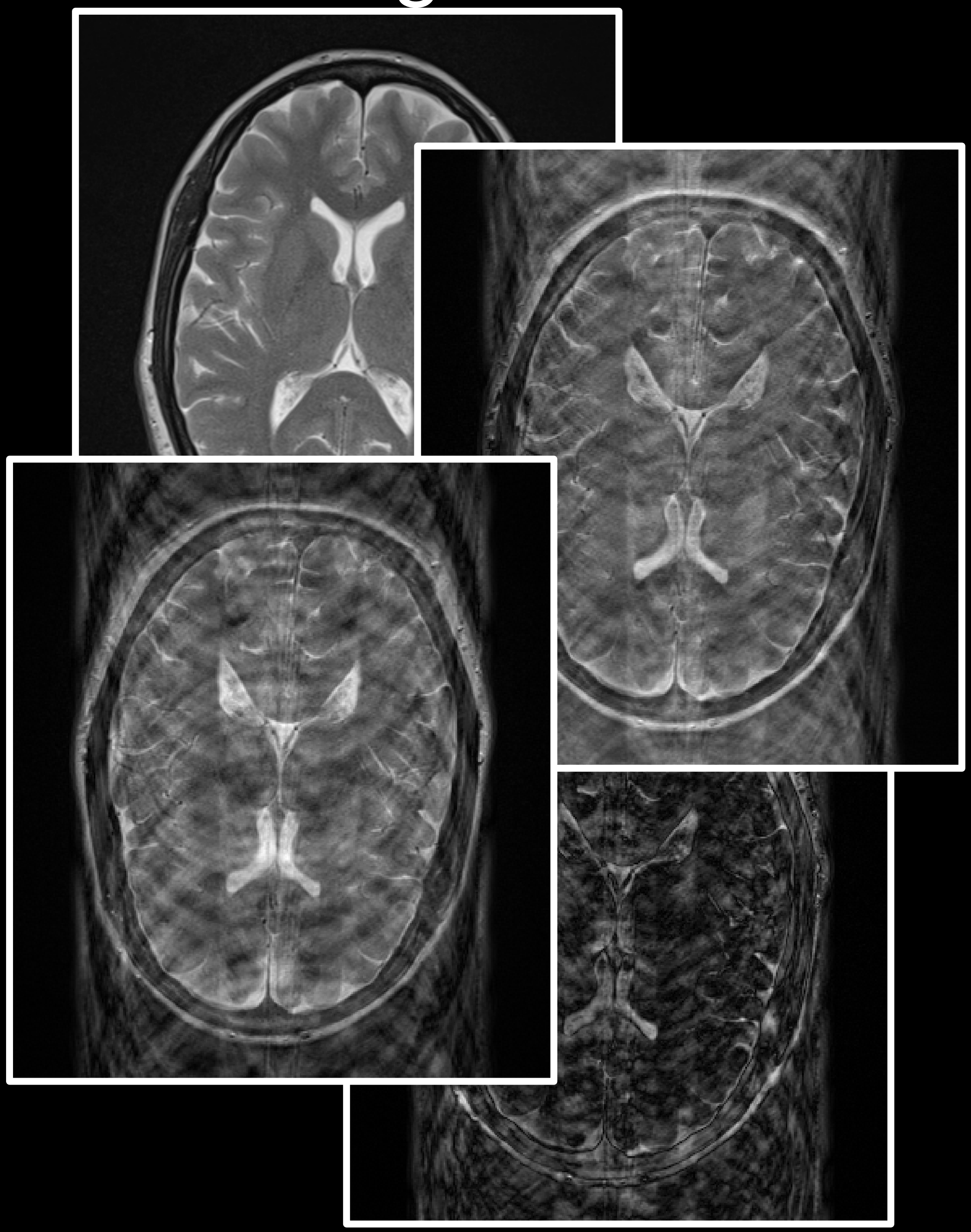
k-space



~~Inverse Fourier Transform Φ^{-1}~~

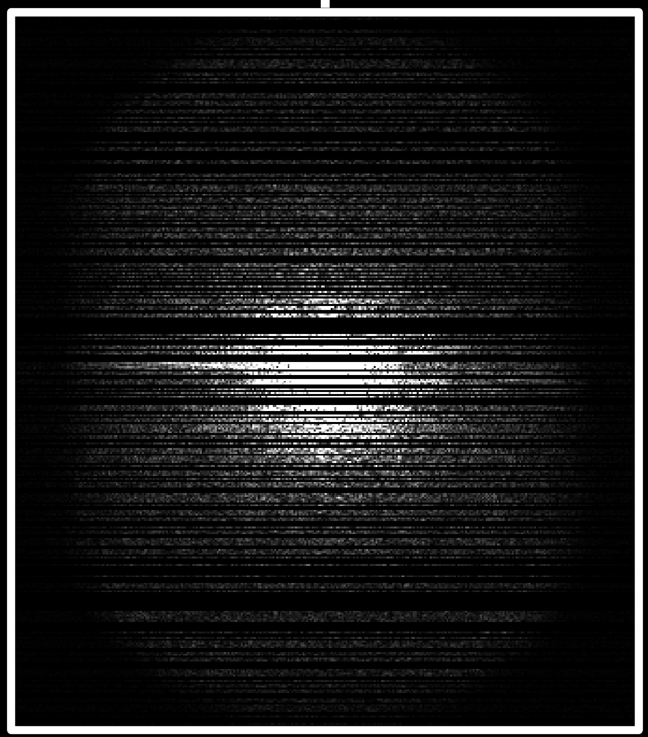
$$\cancel{x = \Phi^{-1} y}$$

Image



Compressed Sensing MRI

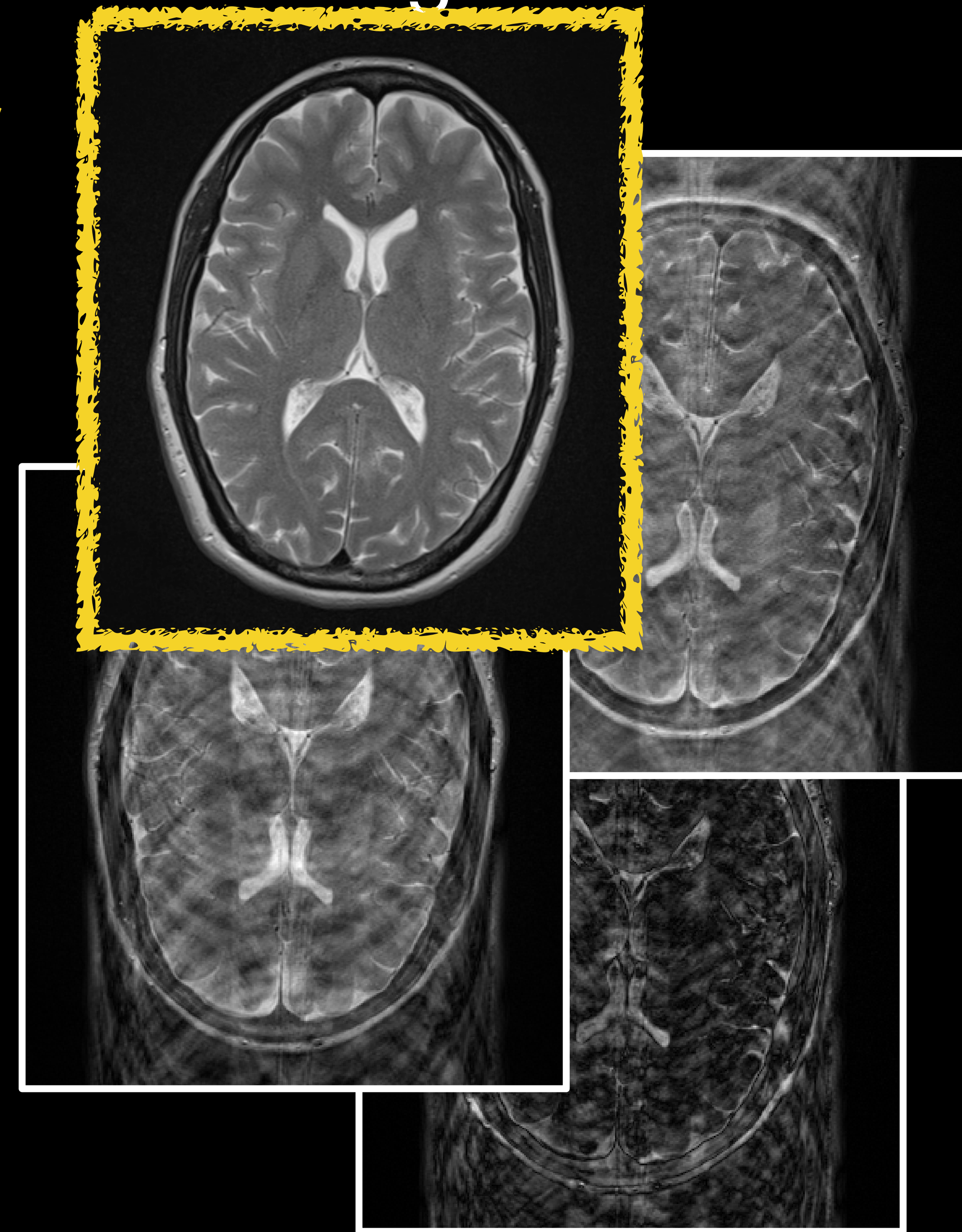
k-space



~~Inverse Fourier Transform~~ $\cancel{\Phi^{-1}}$

$$\cancel{x = \Phi^{-1} y}$$

Image

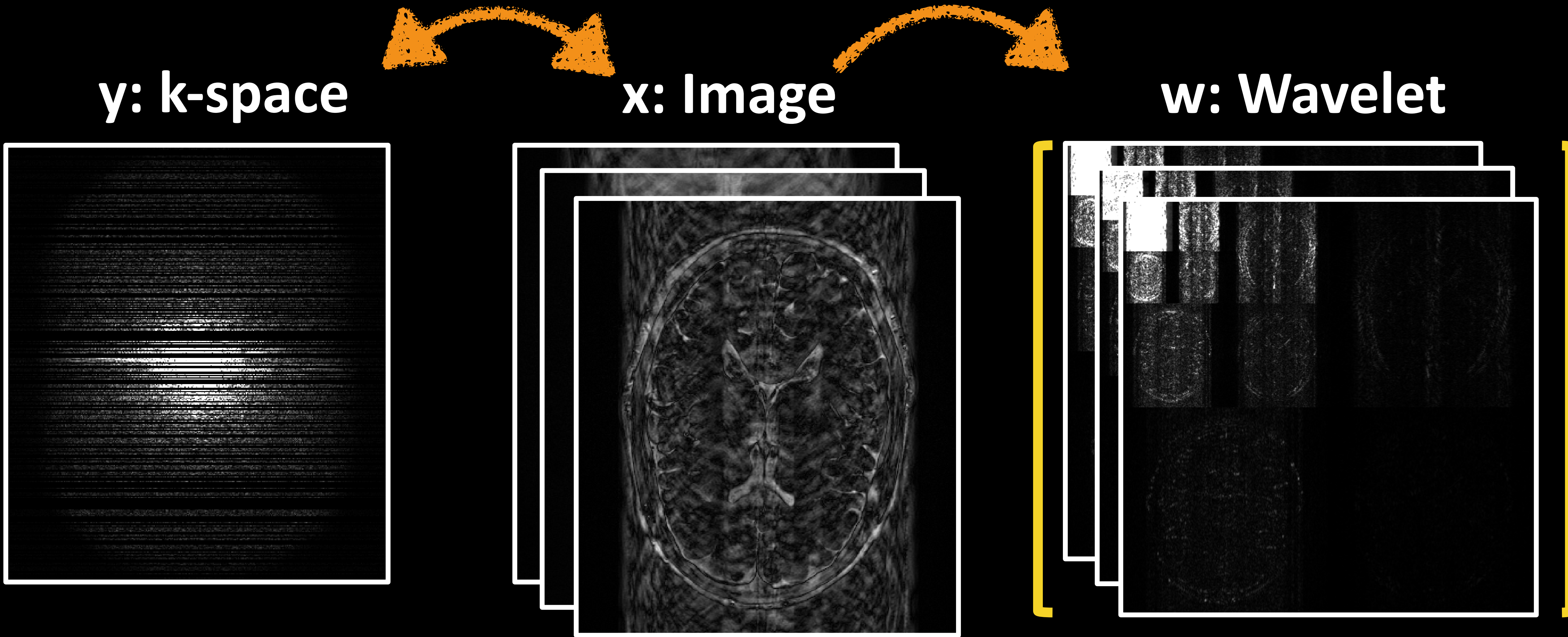


Choose the most compressible image matching the acquired data (*systematic optimization*)

CS-MRI Reconstruction

$|y - \Phi x|^2 < \mathcal{E}$

$w = \Psi x$

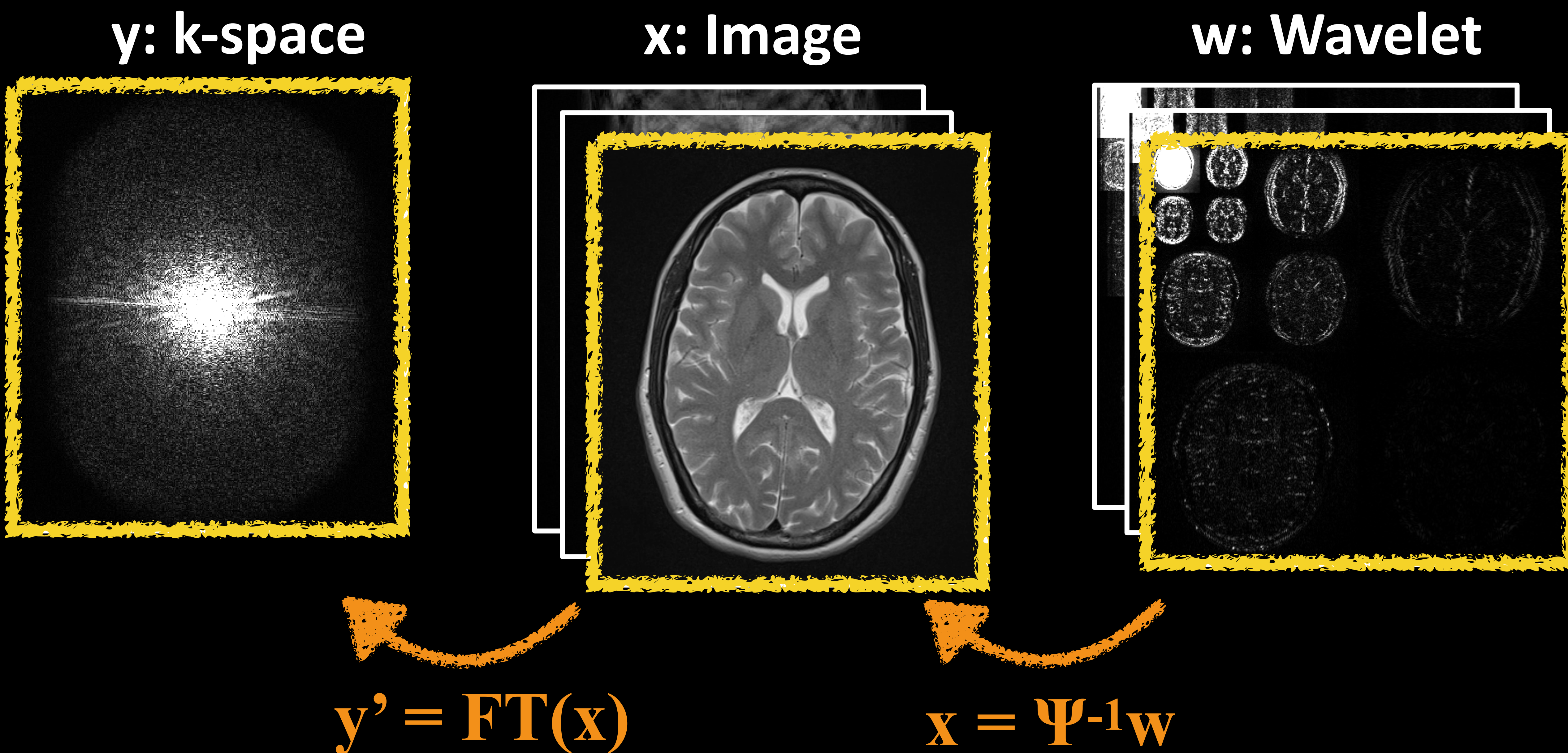


L1-norm

minimize $|\Psi x|_1$

CS-MRI Reconstruction

minimize $F(x)$: $|y - \Phi x|^2 + R(x)$



Three Tenets of CS

minimize $F(x): \underbrace{|y - \Phi x|_2^2}_{\text{Data Consistency}} + \underbrace{R(x)}_{\text{Compressibility Constraint}}$

- Three key elements of Compressed Sensing:

Compressibility

Incoherence

Nonlinear Reconstruction

CS-MRI Reconstruction

minimize $F(x): |y - \Phi x|_2^2 + R(x)$

- Minimizing F(x) is non-trivial since R(x) is not <u>differentiable</u>
  - Linear programming is challenging due to high computational complexity
- Simple gradient-based algorithms have been developed:
  - Re-weighted L1 / FOCUSS
  - IST / IHT / AMP / FISTA
  - Split Bregman / ADMM

I.F. Gorodnitsky, et al., J. Electroencephalog. Clinical Neurophysiol. 1995 Daubechies I, et al. Commun. Pure Appl. Math. 2004
Elad M, et al. in Proc. SPIE 2007
T. Goldstein, S. Osher, SIAM J. Imaging Sci. 2009

State-of-the-Art CS-MRI

- Reducing possible reconstruction failure
  - Improve sparse transformations
  - Develop k-space undersampling schemes
- Integrating CS with DL/parallel imaging
  - Develop compatible undersampling patterns
  - Develop reconstruction methods

State-of-the-Art CS-MRI

- Methods to evaluate CS reconstructed images
  - RMSE / SSIM / Mutual Information
- Reducing reconstruction time
  - Reduce computational complexity
  - Parallelize reconstruction problems
- Developing stable reconstruction algorithms
  - Minimize / avoid the number of regularization parameters

Thanks!

- Interested in more? M229 in Spring
  - Fast imaging sequences
  - Fast sampling trajectories
  - Parallel imaging
  - Constrained reconstruction
  - Deep learning-based methods

Thanks!

- Acknowledgments
  - Dr. Daniel Ennis
  - Dr. Peng Hu
  - Dr. Kyung Sung

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