Bloch Equations and Relaxation II

M219 - Principles and Applications of MRI

Kyung Sung, Ph.D.

1/24/2024

Course Overview

- Course website
  - https://mrrl.ucla.edu/pages/m219
- 2024 course schedule
  - https://mrrl.ucla.edu/pages/m219_2024
- TA - Ran Yan, RanYan@mednet.ucla.edu
  - Wed 4-6pm (Ueberroth, 1417)
- Assignments
  - Homework #1 is due on 1/29
- Office hours, Fridays 10-12pm
  - In-person (Ueberroth, 1417B) or Zoom

Relationship Between Lab and Rotating Frames

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{matrix} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{matrix}$$

$$\overrightarrow{M}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{M}_{rot}(t)$$

Bulk magnetization components in the rotating frame.

$$\overrightarrow{B}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{B}_{rot}(t)$$

Applied B-field components in the rotating frame.

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \Rightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Nishimura - Chap 6 (Appendix I)

Bloch Equation (Rotating Frame)

$$\overrightarrow{B}(t) = B_0\hat{k} + B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$$\overrightarrow{B}_{lab}(t) = \begin{pmatrix} B_1^e(t)\cos(\omega_{RF}t + \theta) \\ -B_1^e(t)\sin(\omega_{RF}t + \theta) \\ B_0 \end{pmatrix} \qquad \overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t)\cos\theta \\ -B_1^e(t)\sin\theta \\ B_0 \end{pmatrix}$$

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

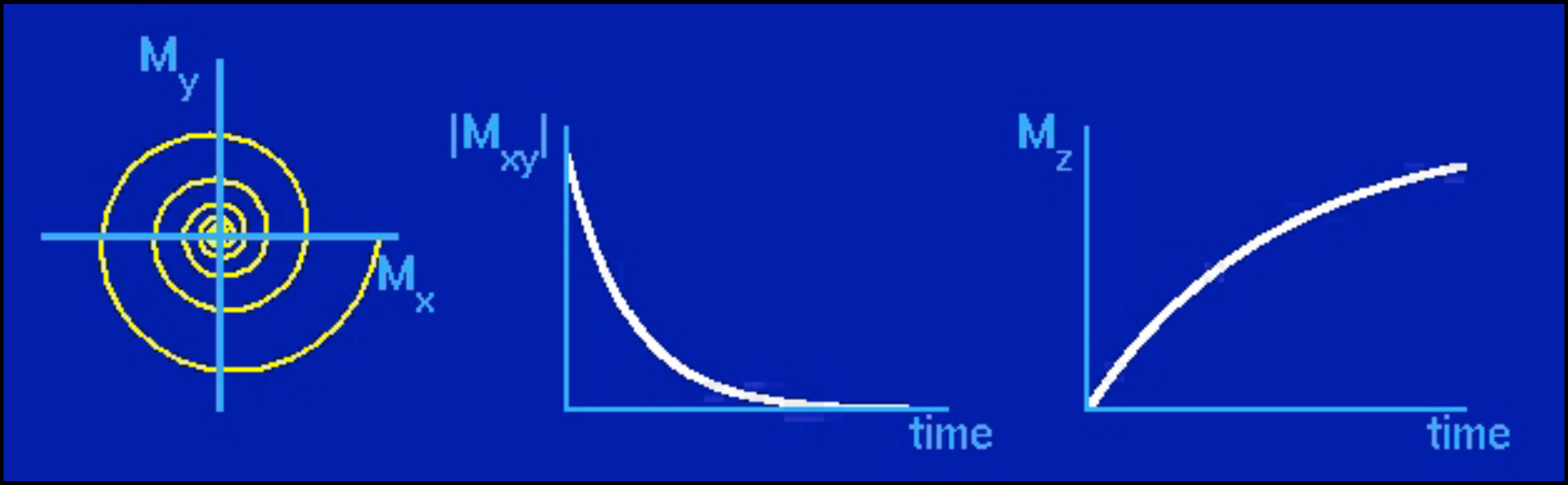
Assume no RF phase (θ = 0)

$$\overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

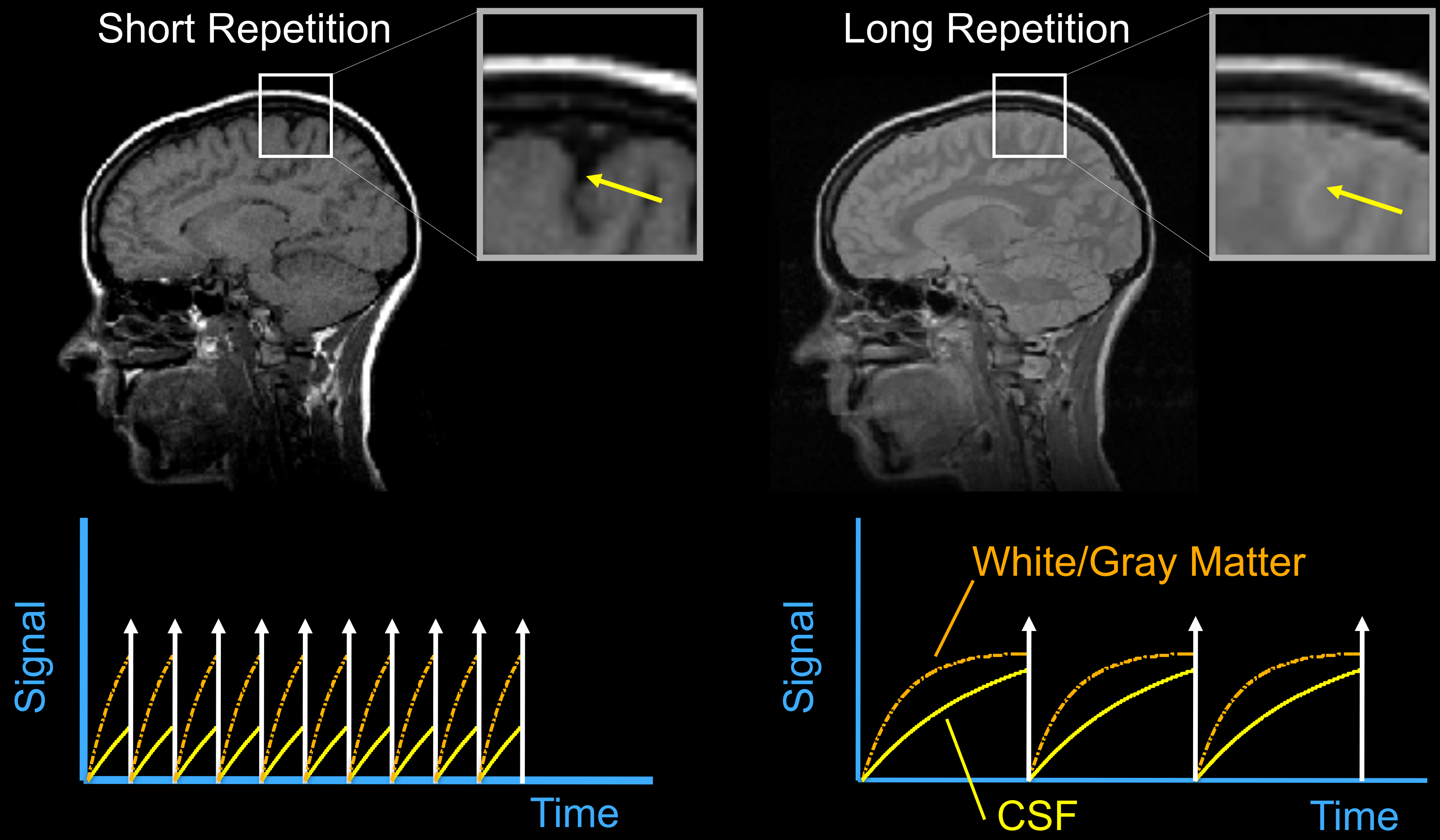
$$\overrightarrow{B}_{eff}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} \end{pmatrix}$$

Relaxation

- Magnetization returns exponentially to equilibrium:
  - Longitudinal recovery time constant is T1
  - Transverse decay time constant is T2
- Relaxation and precession are independent

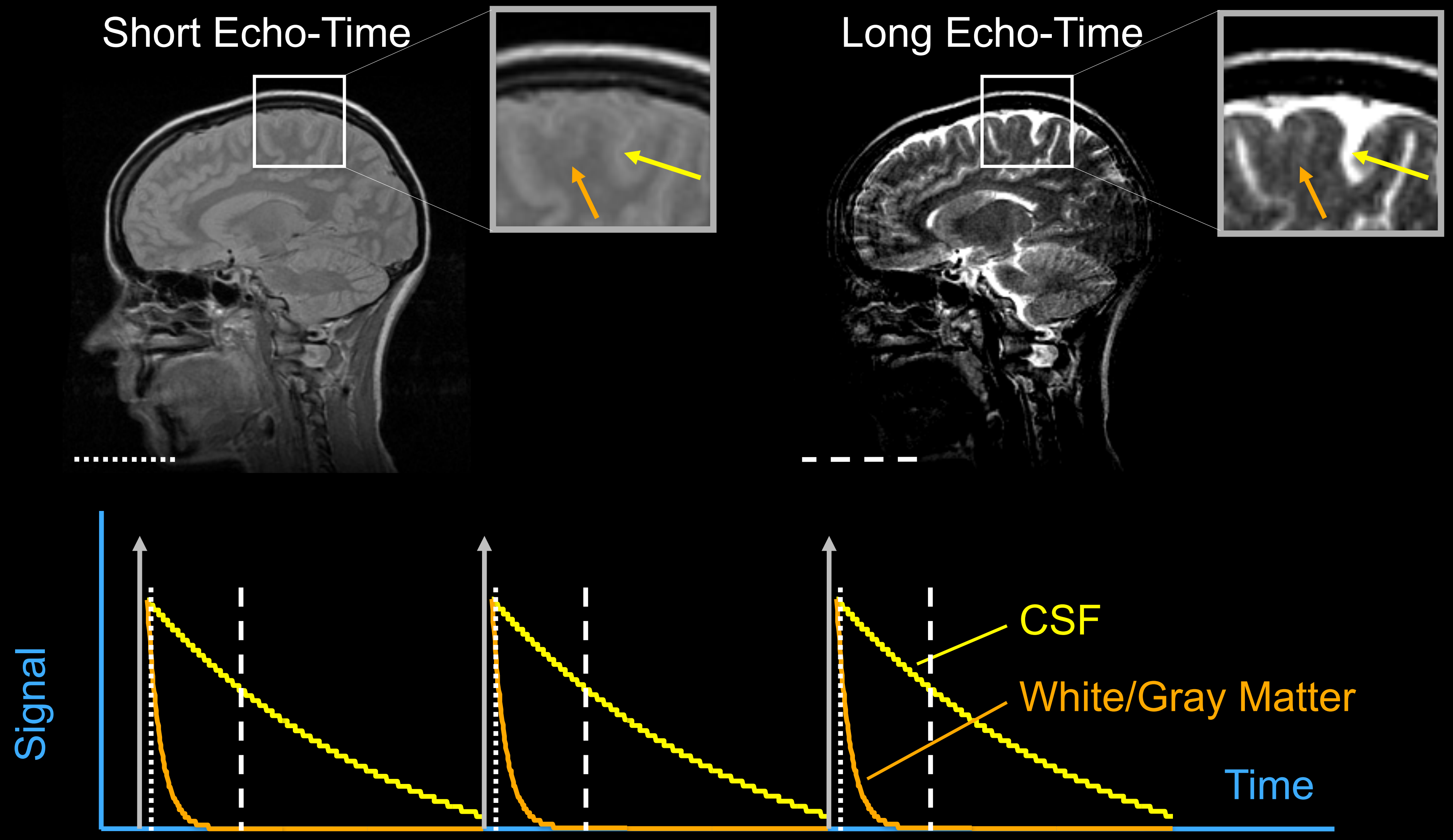


T_1 Contrast



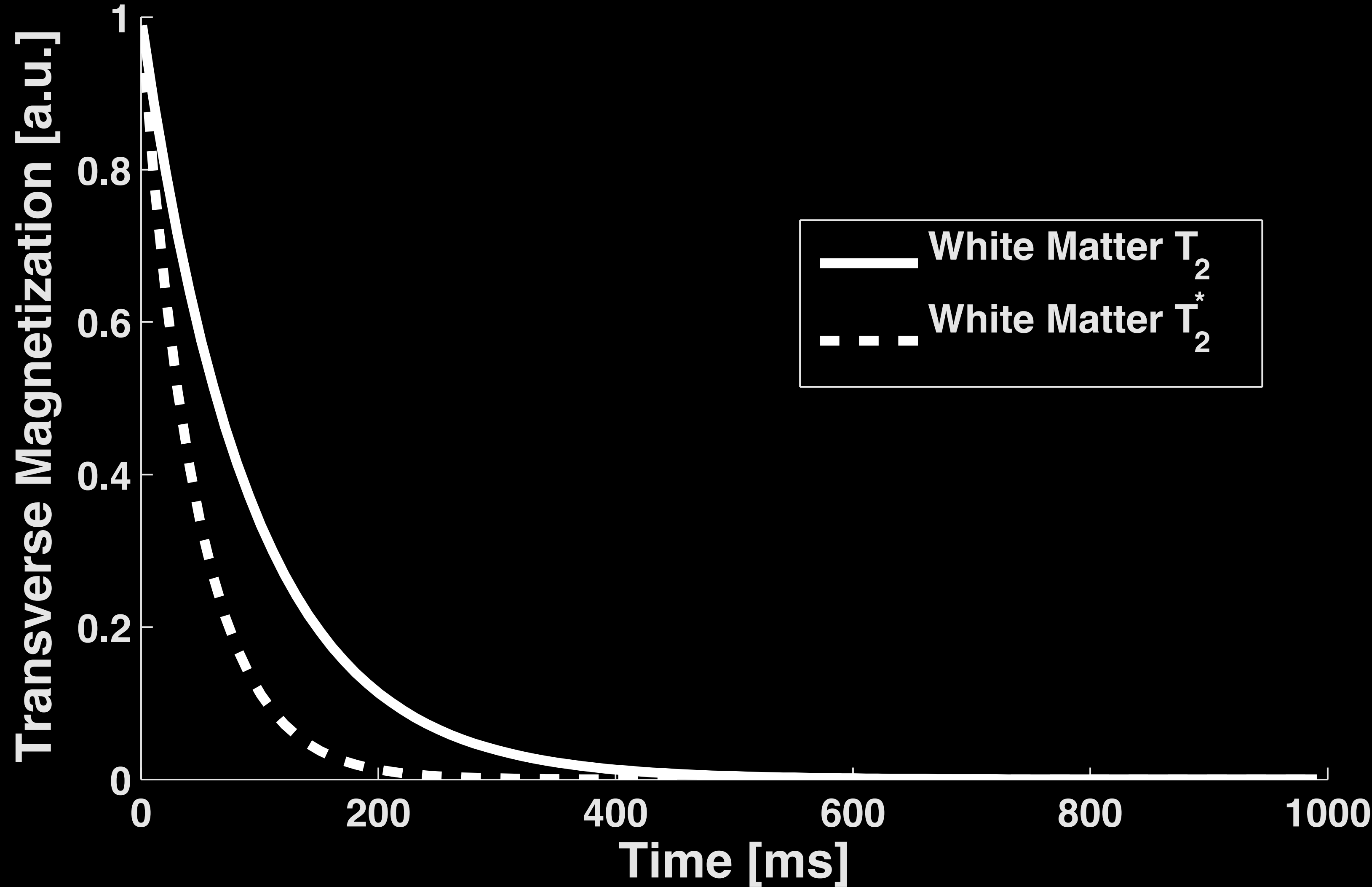
Short Repetition
Long Repetition
Signal
Time
Signal
White/Gray Matter
CSF
Time

T2 Contrast



Short Echo-Time
Long Echo-Time
CSF
White/Gray Matter
Signal
Time

T2*<T2 (always!)



Transverse Magnetization [a.u.]
1
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
Time [ms]
White Matter T_2
White Matter T_2^*

Bloch Equations with Relaxation

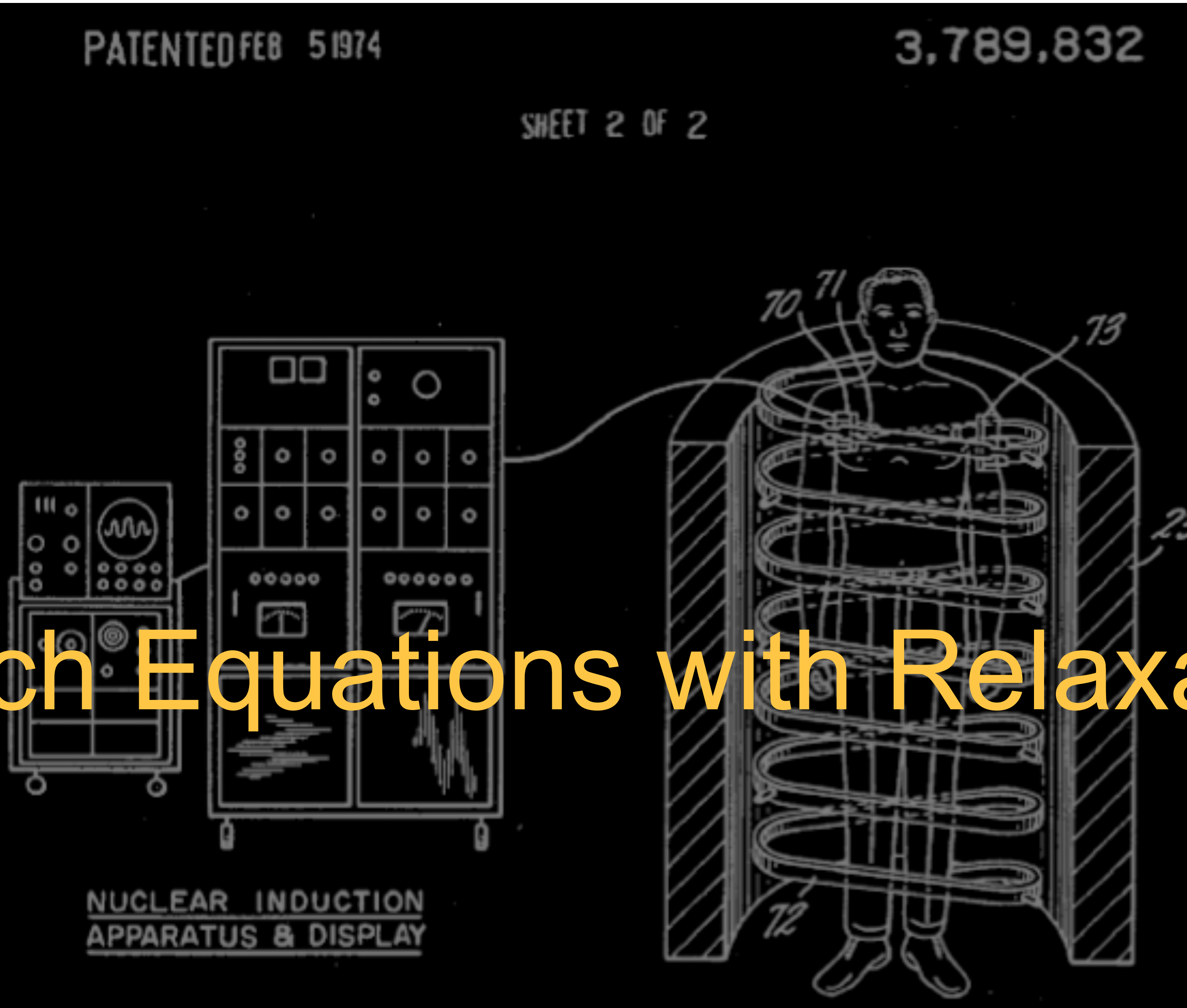


FIG. 2

David Geffen
School of Medicine

UCLA
Radiology

Bloch Equations with Relaxation

$$\frac{d\vec{\mathrm{M}}}{dt} = \vec{\mathrm{M}} \times \gamma \vec{\mathrm{B}} - \frac{M_x \hat{\mathrm{i}} + M_y \hat{\mathrm{j}}}{T_2} - \frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}$$

- **Differential Equation**
  - Ordinary, Coupled, Non-linear
- **No analytic solution, in general.**
  - Analytic solutions for simple cases.
  - Numerical solutions for all cases.
- **Phenomenological**
  - Exponential behavior is an approximation.

Bloch Equations - Lab Frame

$$\frac{d\vec{\mathrm{M}}}{dt} = \underbrace{\vec{\mathrm{M}} \times \gamma \vec{\mathrm{B}}}_{\text{Precession}} - \underbrace{\frac{M_x \hat{\mathrm{i}} + M_y \hat{\mathrm{j}}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}}_{\text{Longitudinal Relaxation}}$$

- Precession
  - Magnitude of M unchanged
  - Phase (rotation) of M changes due to B
- Relaxation
  - T_1 changes are slow O(100ms)
  - T_2 changes are fast O(10ms)
  - Magnitude of M can be ZERO

Bloch Equations – Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \underbrace{\gamma \vec{M}_{rot} \times \vec{B}_{eff}}_{\text{"Precession"}} - \underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

- $\vec{B}_{eff}$: Effective B-field that M experiences in the rotating frame
- $\frac{\vec{\omega}}{\gamma}$: Fictitious field created by the rotating frame that demodulates the apparent effect of B_0
- $\vec{B}_{rot}$: The applied B_0 and B_1 field in the rotating frame

Free Precession in the Rotating Frame with Relaxation

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{\mathrm{B}}_{rot} = \mathrm{B}_0 \hat{k}$$

$$\vec{B}_{eff} = \vec{0}$$

$$\frac{\partial \vec{M}_{rot}}{\partial t} = - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

- **No precession**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, separable ODEs!**

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

Transverse Relaxation

Longitudinal Relaxation

Solution:

$$M_{z'}(t) = M_z^0 e^{-t/T_1} + M_0\left(1 - e^{-t/T_1}\right)$$

$$M_{x'y'}(t) = M_{x'y'}(0_+)e^{-t/T_2}$$

The precessional term drops out in the rotating frame.

Forced Precession
in the Rotating Frame
with Relaxation

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{B}_{rot} = B_0 \hat{k} + B_1^e(t)\hat{i}'$$

$$\vec{B}_{eff} = B_1^e(t)\hat{i}'$$

The precessional term ***does not*** drop out in the rotating frame.

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} = B_1^e(t)\hat{\vec{i}}'$$

- **B1 induced nutation**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, coupled PDEs!**
- **When does this equation apply?**

The precessional term ***does not*** drop out in the rotating frame.

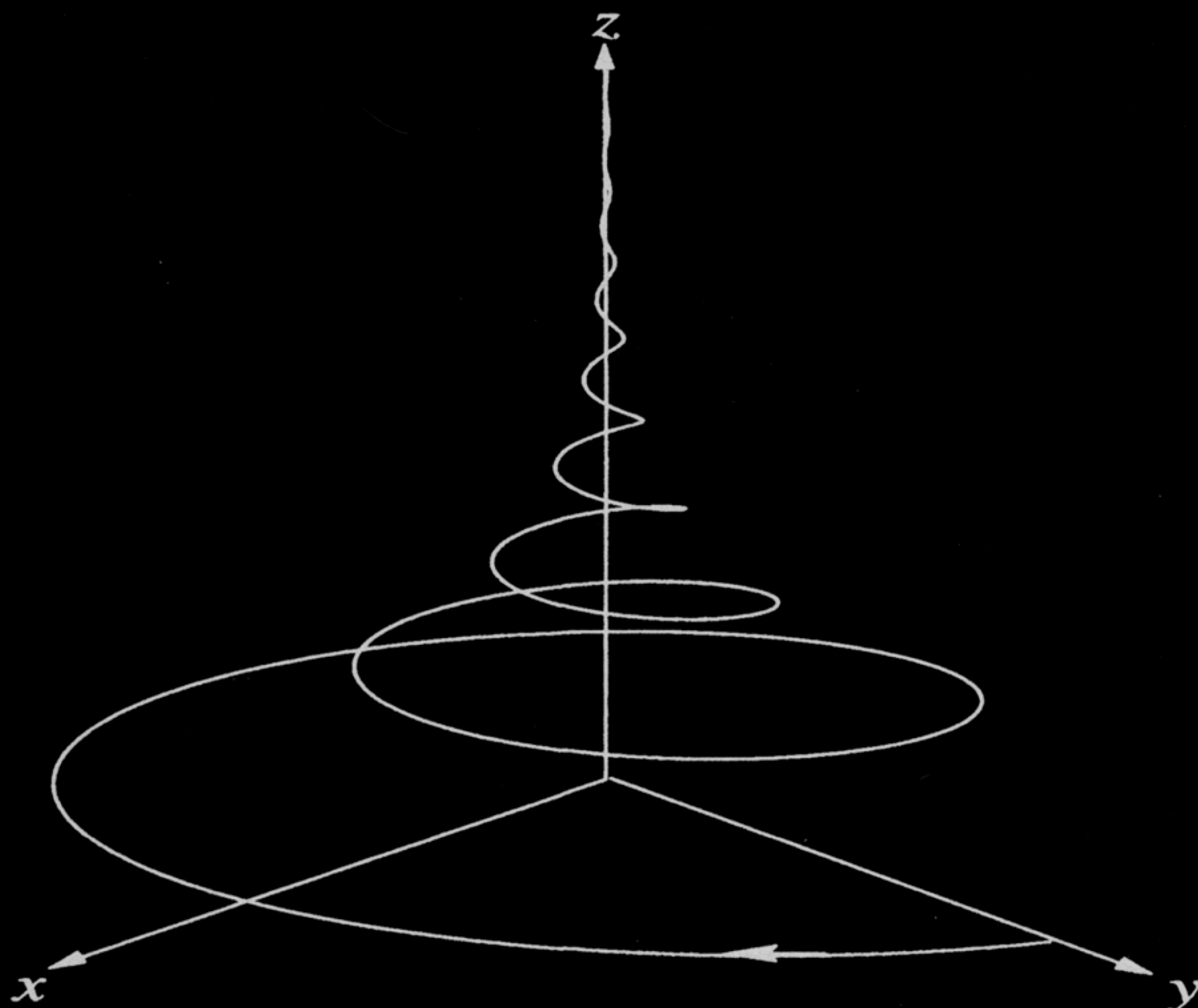
Forced Precession in the Rotating Frame with Relaxation

- RF pulses are short
  - 100μs to 5ms
- Relaxation time constants are long
  - T_1 O(100s) ms
  - T_2 O(10s) ms
- Complicated Coupling
- Best suited for simulation

Free? Forced? Relaxation?

- We've considered all combinations of:
  - Free and forced precession
  - With and without relaxation
  - Laboratory and rotating frames
- Which one's concern M219 the most?
  - Free precession in the rotating frame with relaxation
  - Forced precession in the rotating frame without relaxation.
- We can, in fact, simulate all of them…

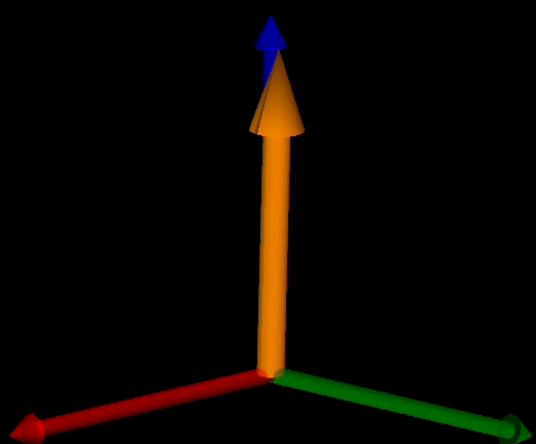
Spin Gymnastics - Lab Frame



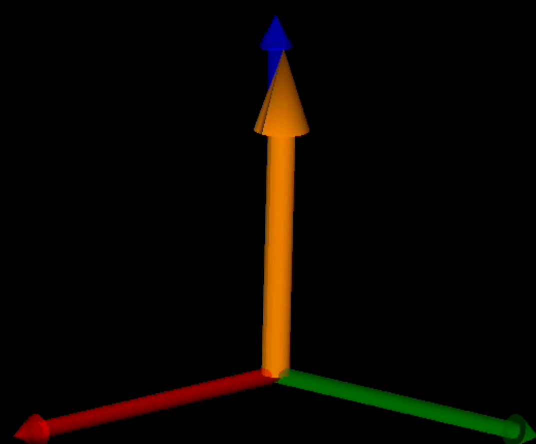
Spin Gymnastics - Rotating Frame

$$M_Z(t) = M_Z^0 e^{-\frac{t}{T_1}} + M_0\left(1 - e^{-\frac{t}{T_1}}\right)$$

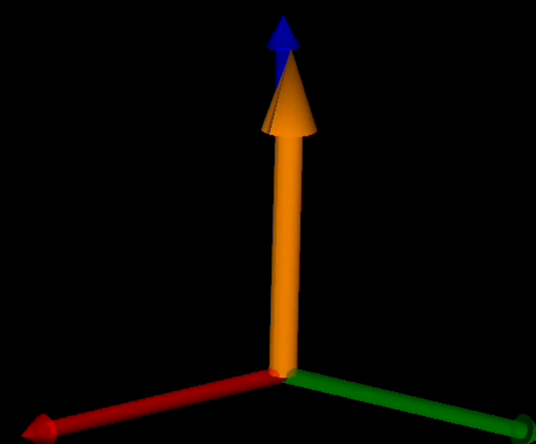
$$M_{xy}(t) = M_{xy}^0 e^{-t/T_2}$$



90° RF



135° RF



180° RF

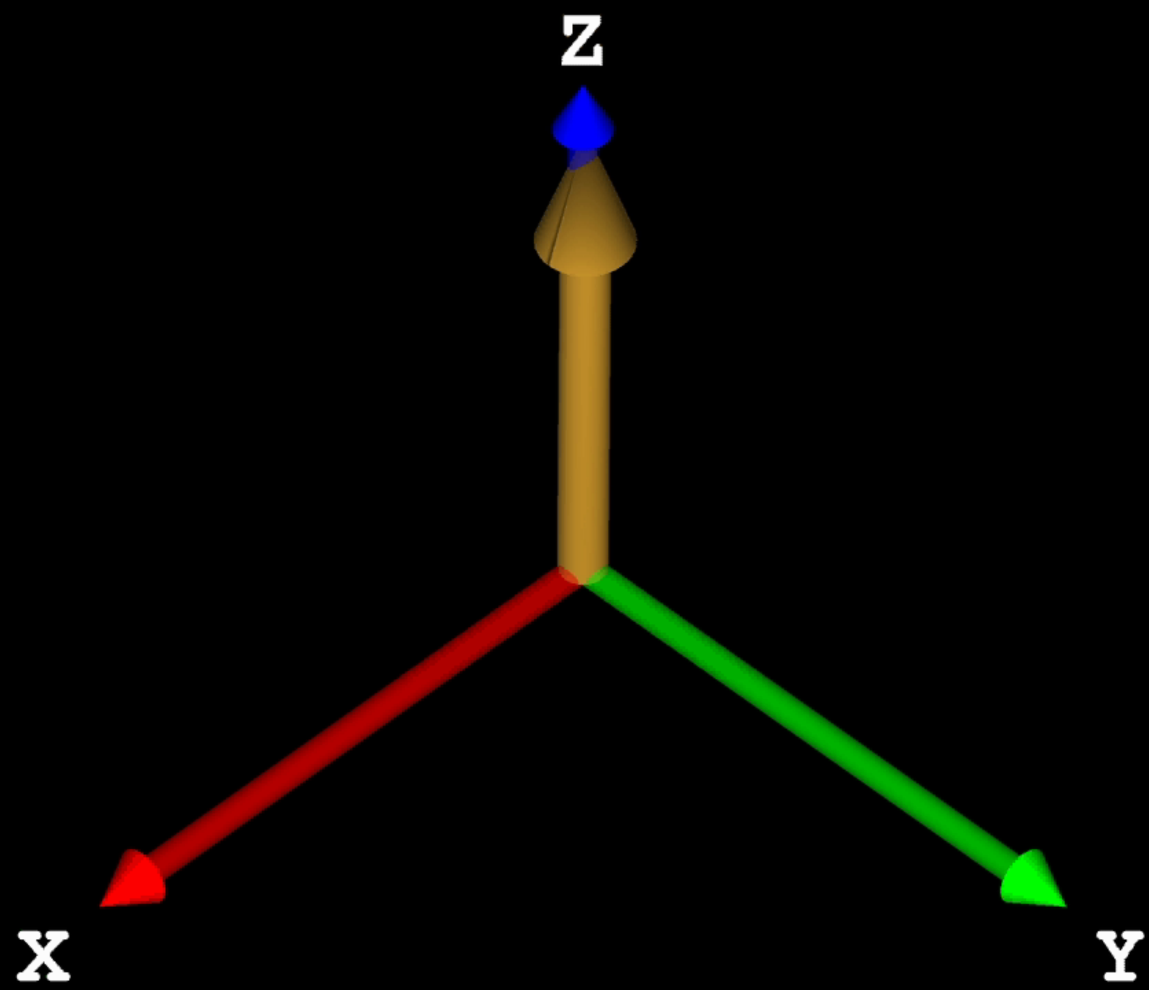
Types of RF Pulses

- Excitation Pulses
- Inversion Pulses
- Refocusing Pulses
- Saturation Pulses
- Spectrally Selective Pulses
- Spectral-spatial Pulses
- Adiabatic Pulses

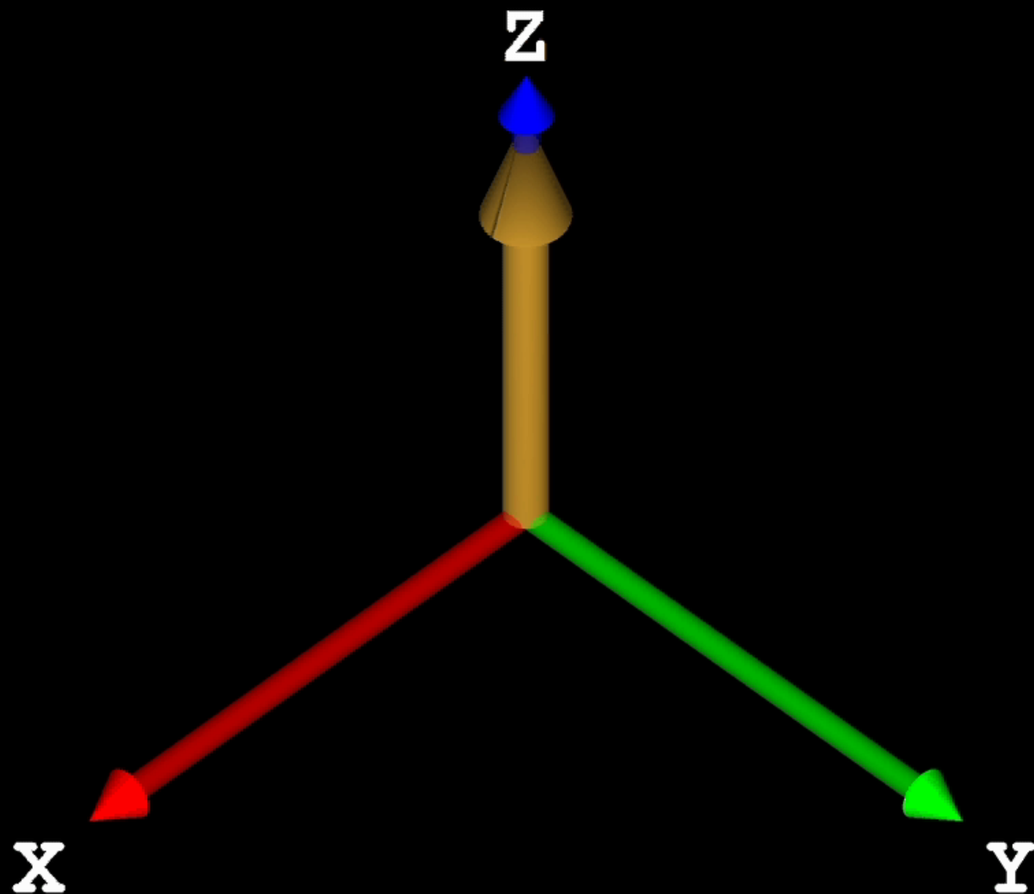
Excitation Pulses

- Tip M_z into the transverse plane
- Typically 200μs to 5ms
- Non-uniform across slice thickness
  - Imperfect slice profile
- Non-uniform within slice
  - Termed B_1 inhomogeneity
  - Non-uniform signal intensity across FOV

90° Excitation Pulse



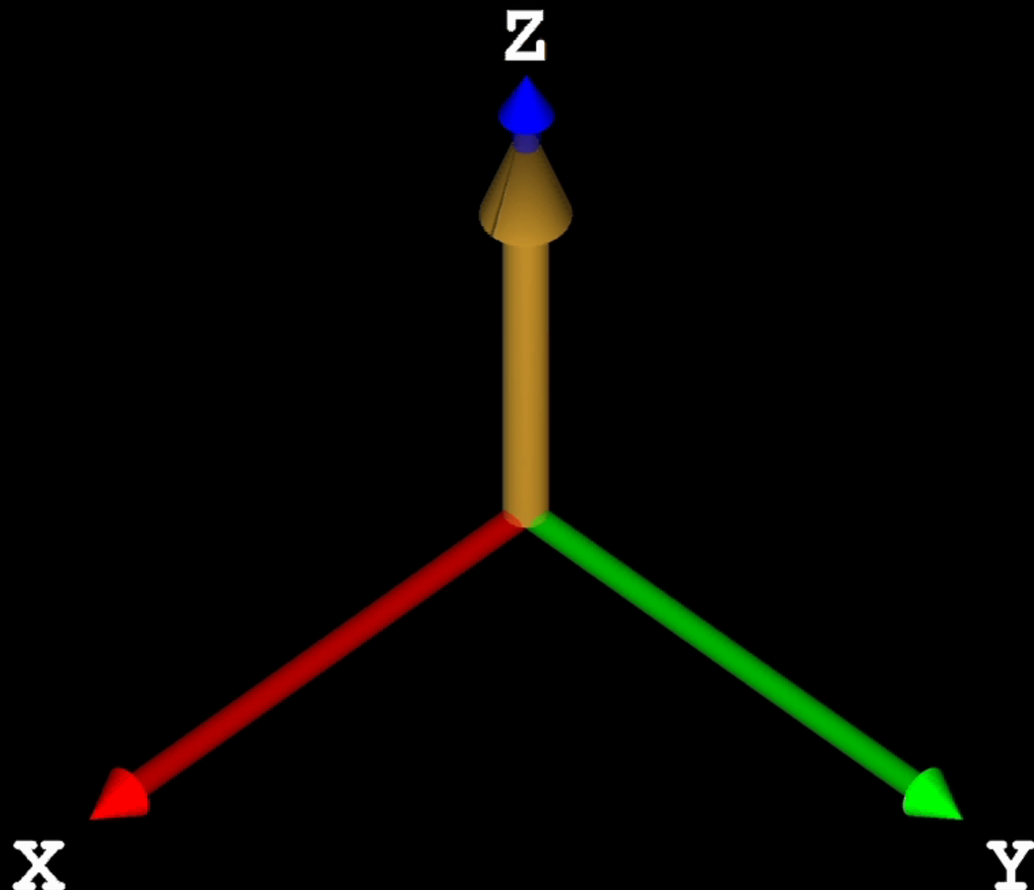
Small Flip Angle Excitation



Inversion Pulses

- Typically, 180° RF Pulse
  - non-180° that still results in $-M_Z$
- Invert M_Z to $-M_Z$
  - Ideally produces no M_{XY}
- Hard Pulse
  - Constant RF amplitude
  - Typically non-selective
- Soft (Amplitude Modulated) Pulse
  - Frequency selective
  - Spatially Selective

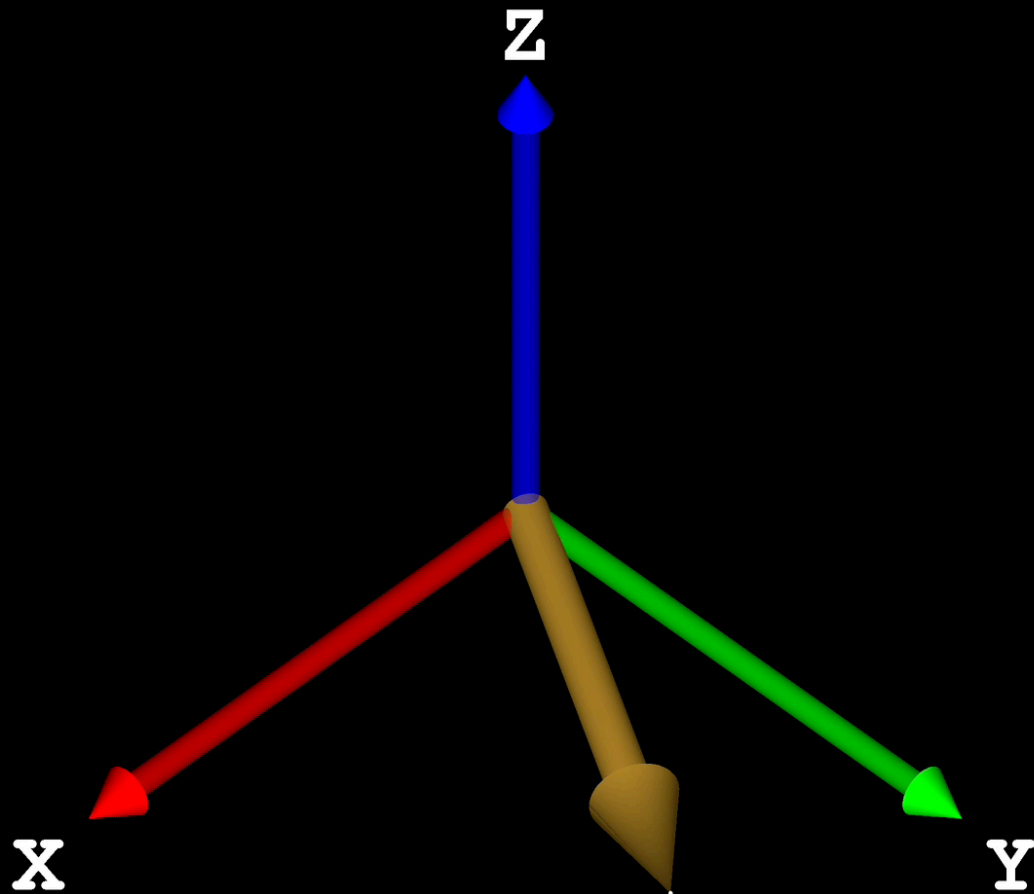
Inversion Pulses



Refocusing Pulses

- Typically, 180° RF Pulse
  - Provides optimally refocused M_{XY}
  - Largest spin echo signal
- non-180°
  - Partial refocusing
  - Lower SAR
  - Multiple non-180° produce stimulated echoes
- Refocus spin dephasing due to
  - imaging gradients
  - local magnetic field inhomogeneity
  - magnetic susceptibility variation
  - chemical shift

Refocusing Pulses



Frequency Selectivity of RF Pulses

Matlab Demo

Questions?

- Related reading materials
  - Nishimura - Chap 4 and 5
  - Nishimura - Chap 6 (Appendix I)

Kyung Sung, Ph.D.
KSung@mednet.ucla.edu
http://mrrl.ucla.edu/sunglab