

# MRI Systems III: Gradients

M219 - Principles and Applications of MRI

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1/29/2024

# Course Overview

- 2024 course schedule
  - [https://mrrl.ucla.edu/pages/m219\\_2024](https://mrrl.ucla.edu/pages/m219_2024)
- Assignments
  - Homework #1 is due today
  - Homework #2 is out
- TA office hours, Weds 4-6pm
- Office hours, Fridays 10-12pm

# Bloch Equations - Lab Frame

$$\frac{d\vec{M}}{dt} = \underbrace{\vec{M} \times \gamma\vec{B}}_{\text{Precession}} - \underbrace{\frac{M_x\hat{i} + M_y\hat{j}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\hat{k}}{T_1}}_{\text{Longitudinal Relaxation}}$$

- Precession
  - Magnitude of M unchanged
  - Phase (rotation) of M changes due to B
- Relaxation
  - $T_1$  changes are slow O(100ms)
  - $T_2$  changes are fast O(10ms)
  - Magnitude of M can be ZERO

# Free Precession in the Rotating Frame

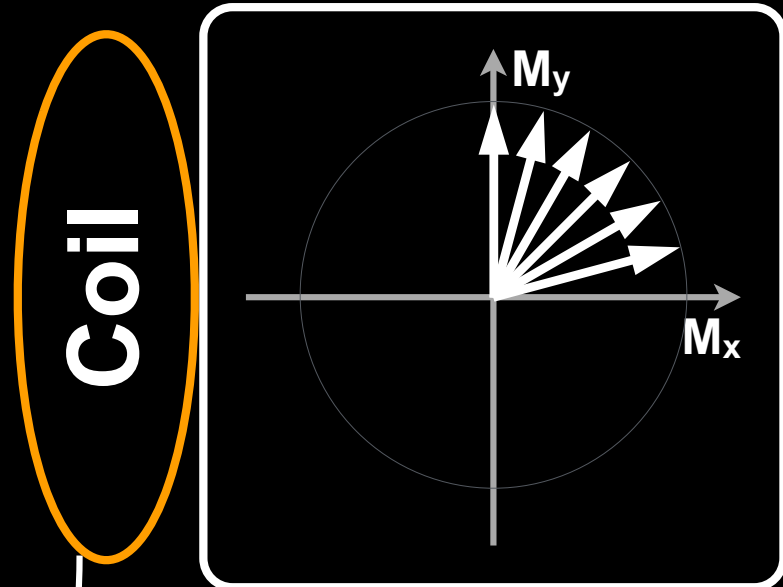
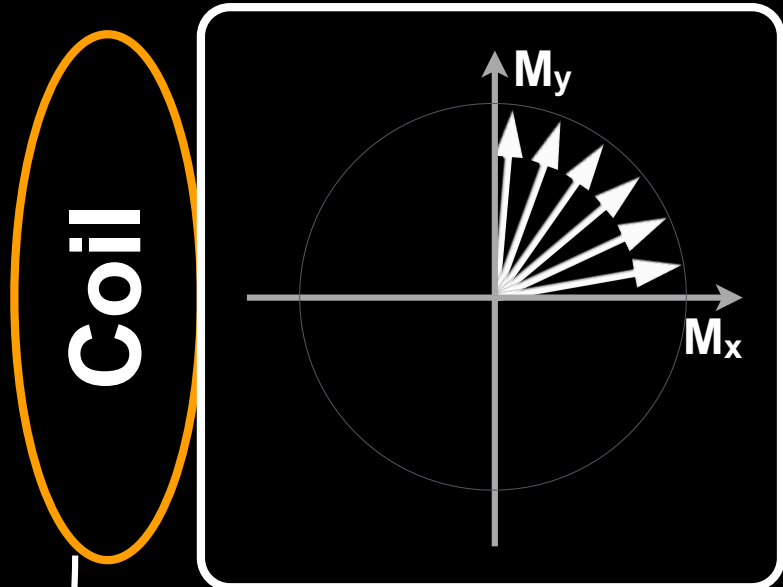
$$\frac{\partial \vec{M}_{rot}}{\partial t} = - \underbrace{\frac{M_{x'} \vec{i}' + M_{y'} \vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0) \vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

- **No precession**
- **T<sub>1</sub> and T<sub>2</sub> Relaxation**
- **Drop the diffusion term**
- **System of first order, linear, separable ODEs!**

# $T_2$ versus $T_2^*$

$T_2$  Decay

$T_2^*$  Decay



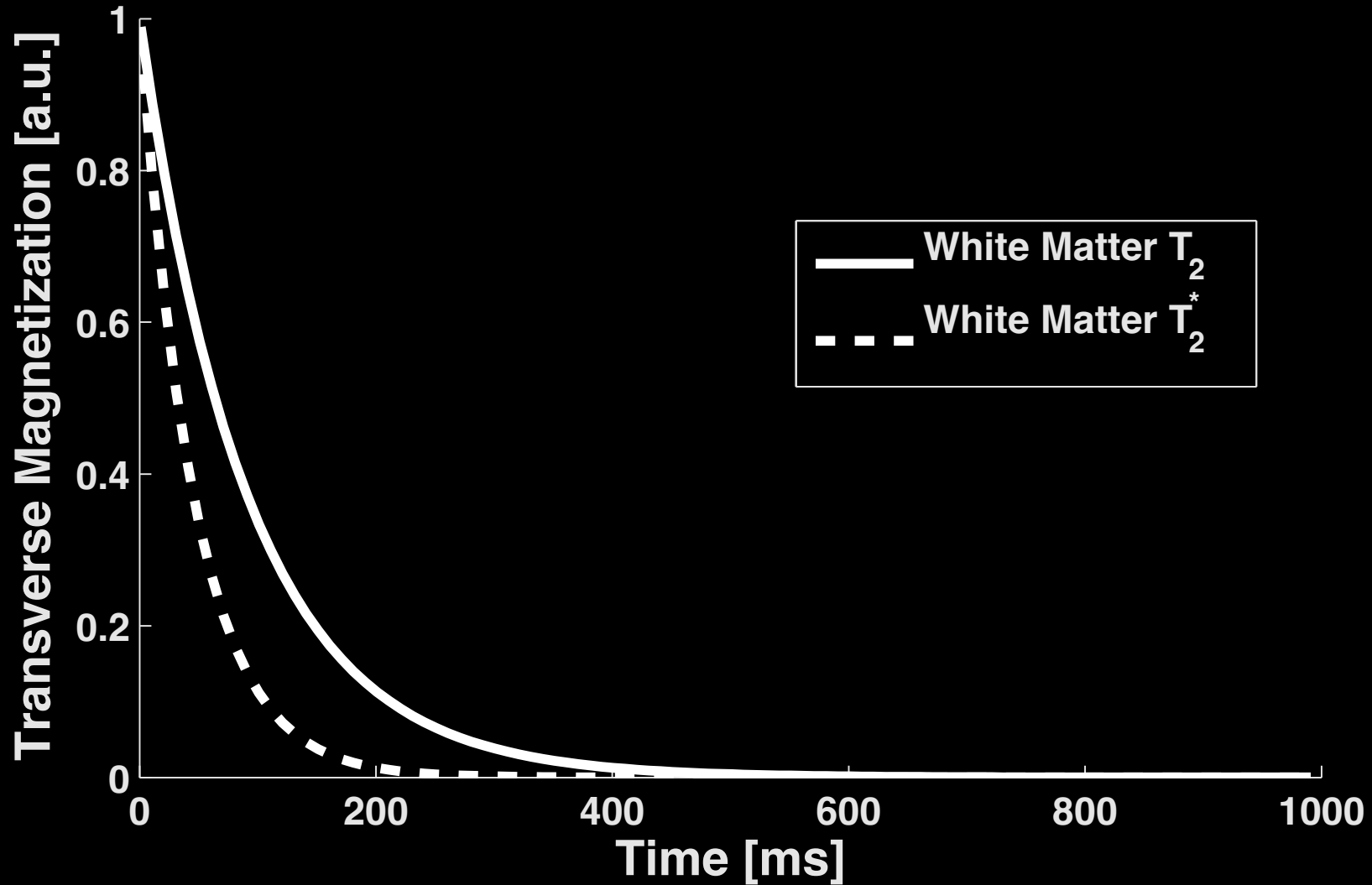
Signal loss from spin-spin interaction.

Signal loss from spin-spin interaction and off-resonance dephasing and  $T_2^*$ .



$T_2^*$  is signal loss from spin dephasing and  $T_2$

$T_2^* < T_2$  (always!)



# Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'} \vec{i}' + M_{y'} \vec{j}'}{T_2} - \frac{(M_{z'} - M_0) \vec{k}'}{T_1}$$

$$\vec{B}_{eff} = B_1^e(t) \hat{i}'$$

- **B1 induced nutation**
- **T<sub>1</sub> and T<sub>2</sub> Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, coupled PDEs!**
- **When does this equation apply?**

# Types of RF Pulses

- Excitation Pulses
- Inversion Pulses
- Refocusing Pulses
- Saturation Pulses
- Spectrally Selective Pulses
- Spectral-spatial Pulses
- Adiabatic Pulses



# Gradient Fields & Spins

# Gradients

Gradients are a special kind of inhomogeneous field whose z-component varies linearly along a specific direction called the gradient direction.

$$(\vec{G} \cdot \vec{r}) \hat{k} = (G_x \cdot x + G_y \cdot y + G_z \cdot z) \hat{k}$$

$$\vec{B}(\vec{r}, t) = (B_0 + \vec{G}(t) \cdot \vec{r}) \hat{k}$$

$$B_G(\vec{r}, t)$$

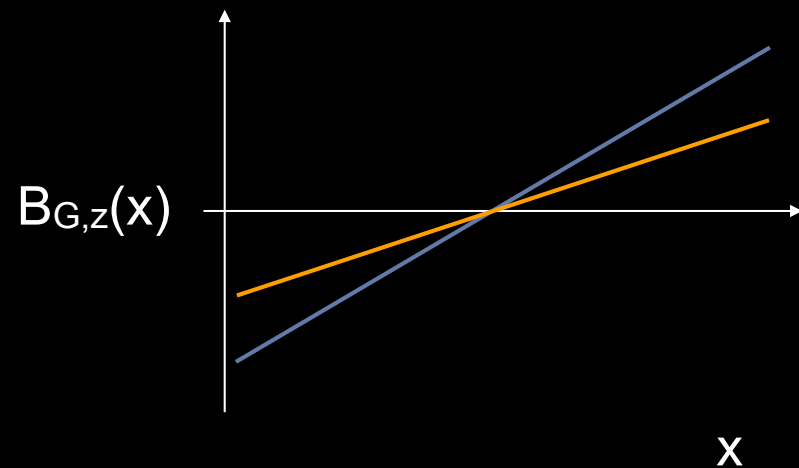
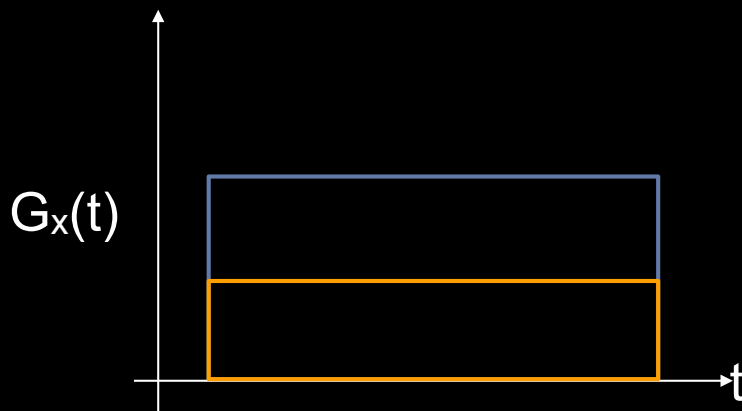
- Each gradient coil can be activated independently and simultaneously

# Gradient Induced B-Fields

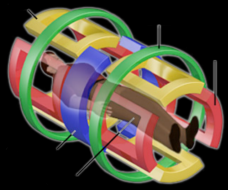
$$B_G(x) = G_x x \quad \text{x-gradient}$$

$$B_G(y) = G_y y \quad \text{y-gradient}$$

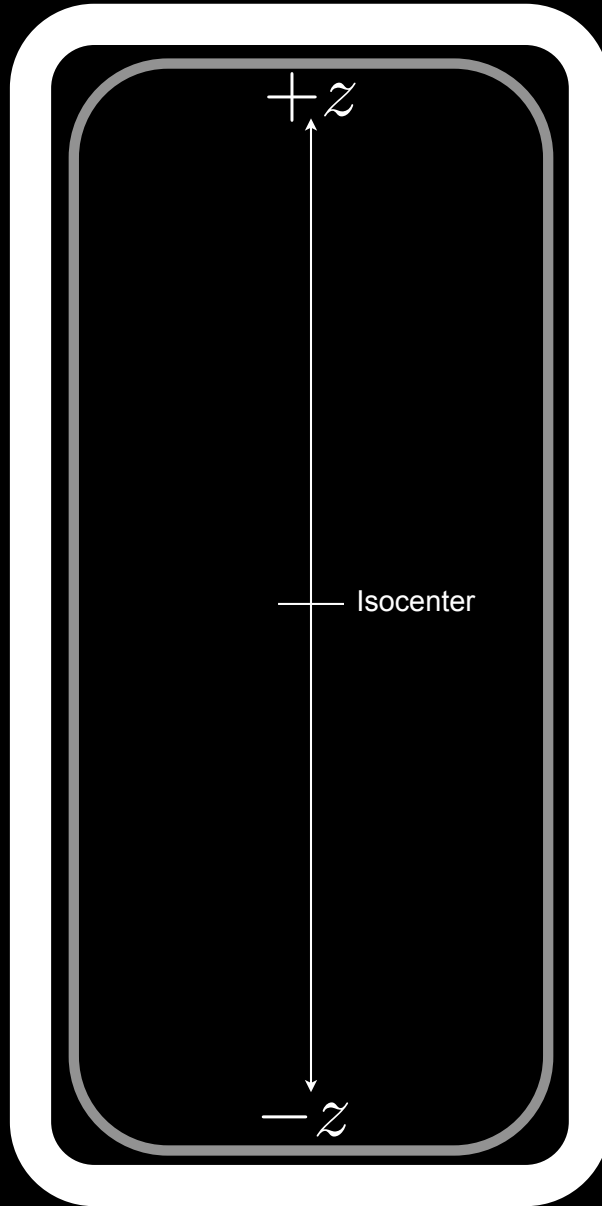
$$B_G(z) = G_z z \quad \text{z-gradient}$$



The magnetic field at a position depends on the magnitude of the applied gradient.



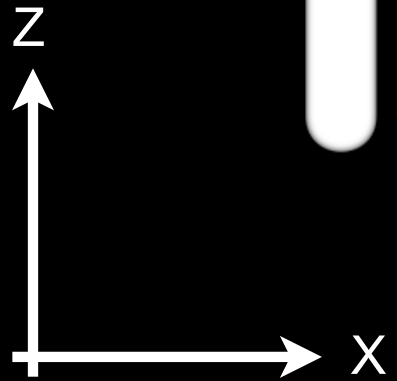
# No Gradients Turned On



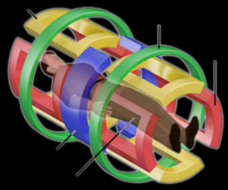
$B_0$

$B_0$

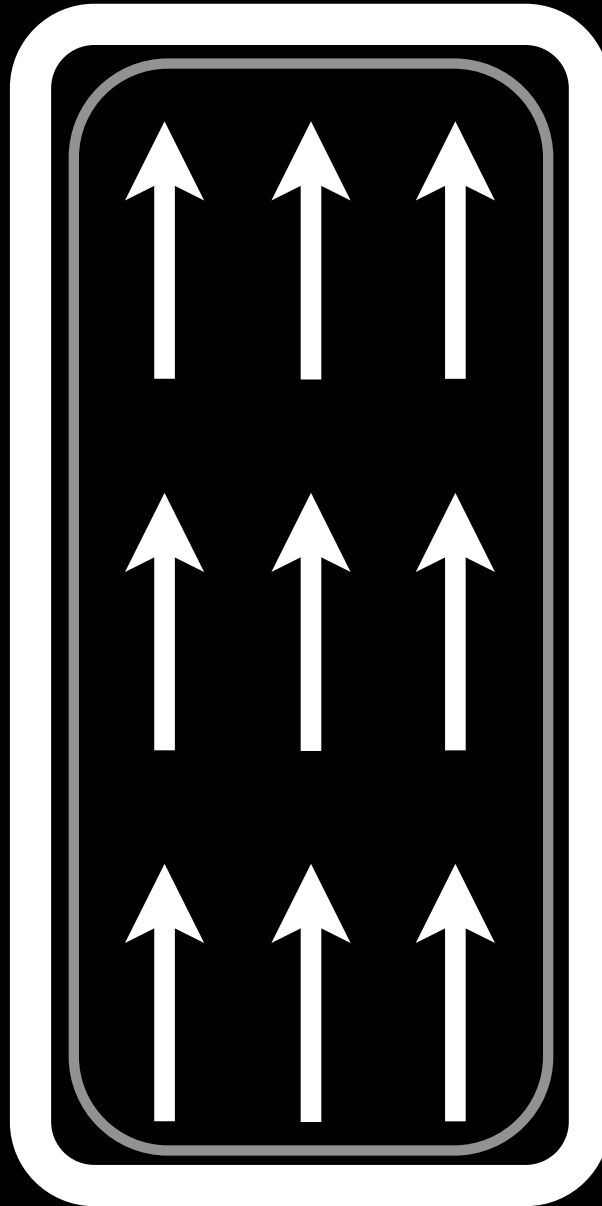
$B_0$



$$\omega = \gamma B_0$$



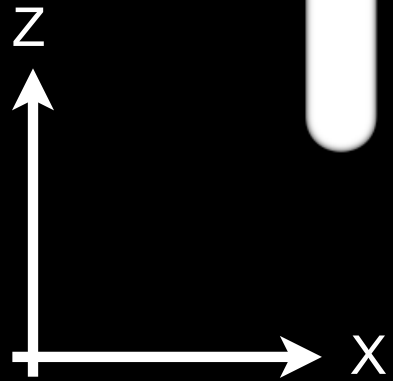
# No Gradients Turned On



$B_0$

$B_0$

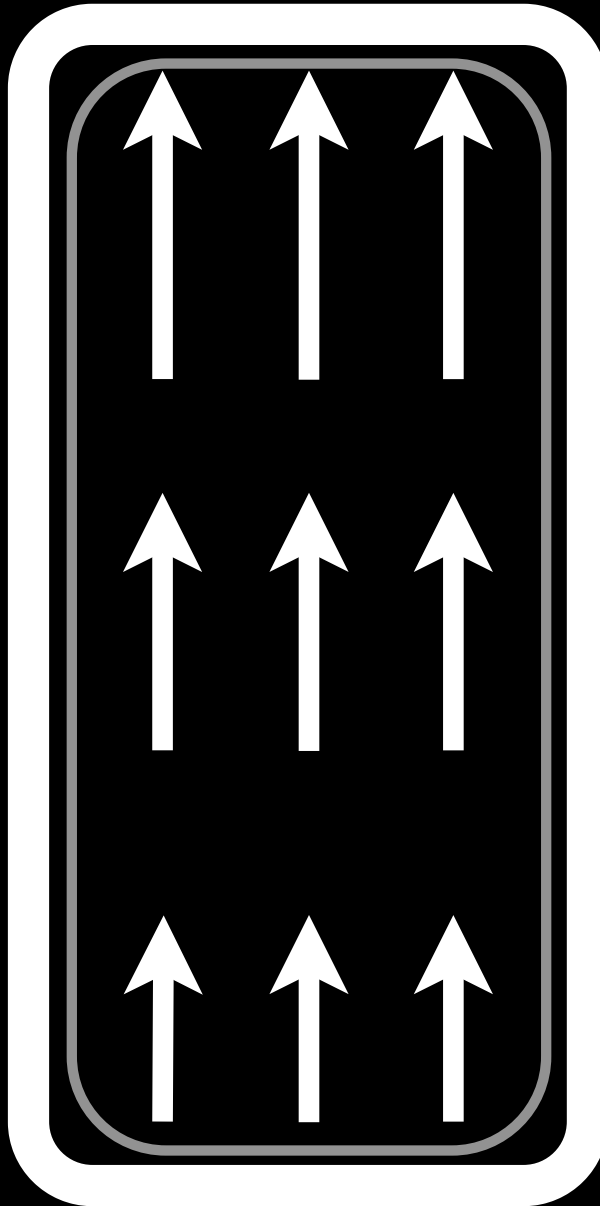
$B_0$



$$\omega = \gamma B_0$$

*Length of arrow indicates strength of local field.*

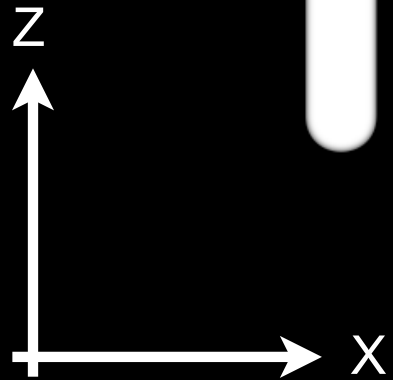
# Z-Gradients



$$B_0 + \delta B_0$$

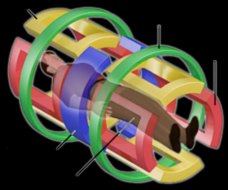
$$B_0$$

$$B_0 - \delta B_0$$

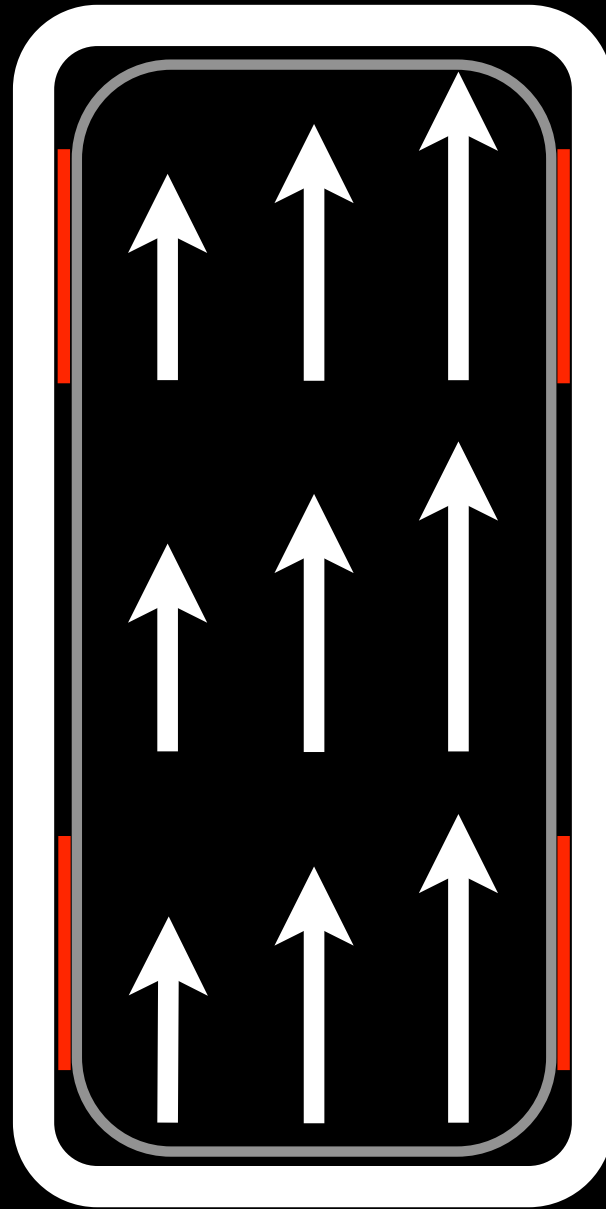


$$\omega(x, z) = \gamma(B_0 + G_z \cdot z)$$

What coordinate frame are we in?

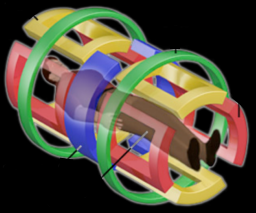


# X-Gradients

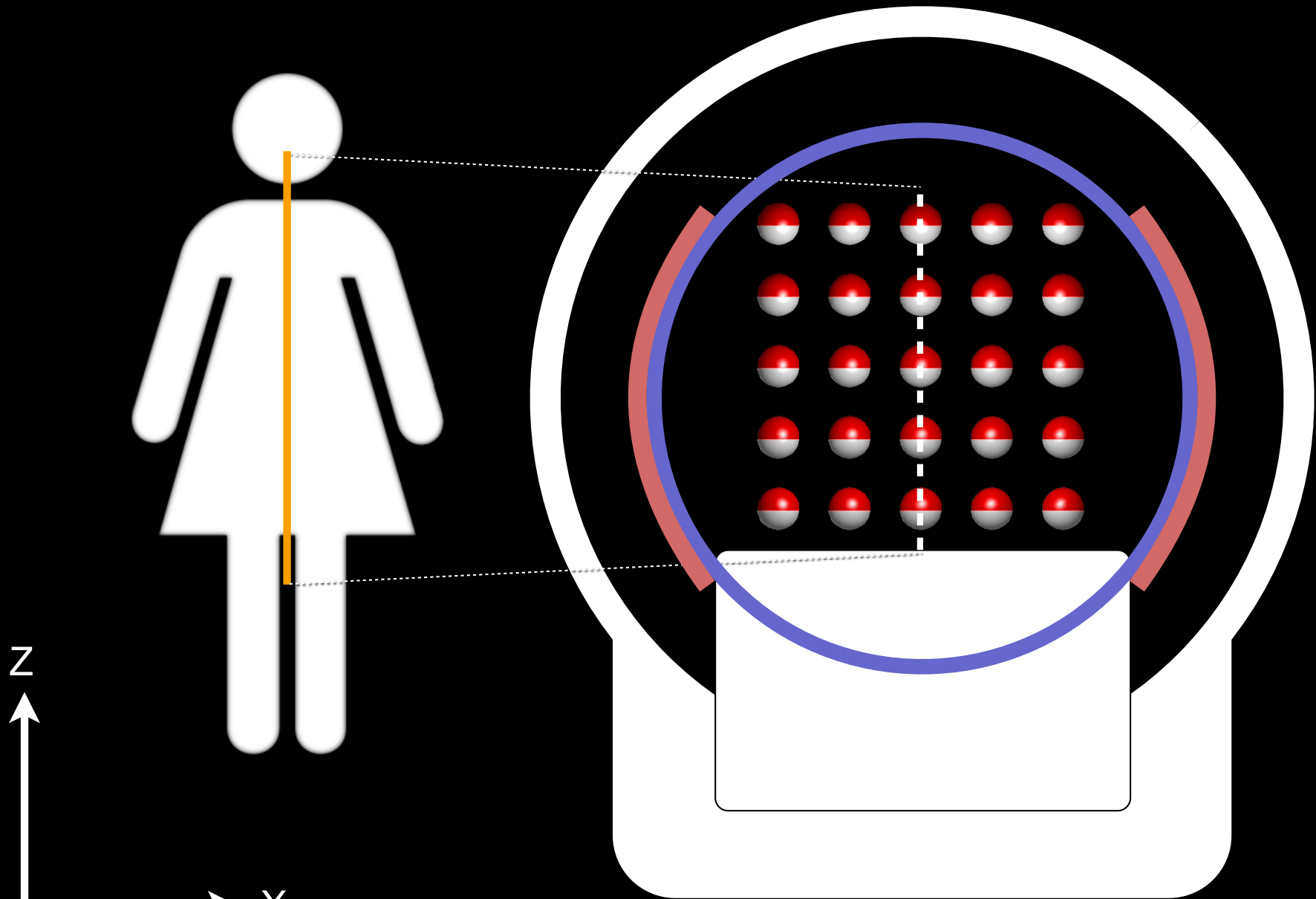


$x$   $\omega(x, z) = \gamma(B_0 + G_x \cdot x + G_z \cdot z)$

$B_0 - \delta B_0$     $B_0$     $B_0 + \delta B_0$

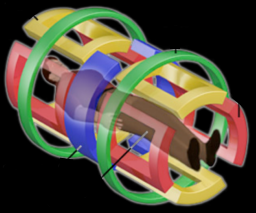


# Spins and X-Gradients

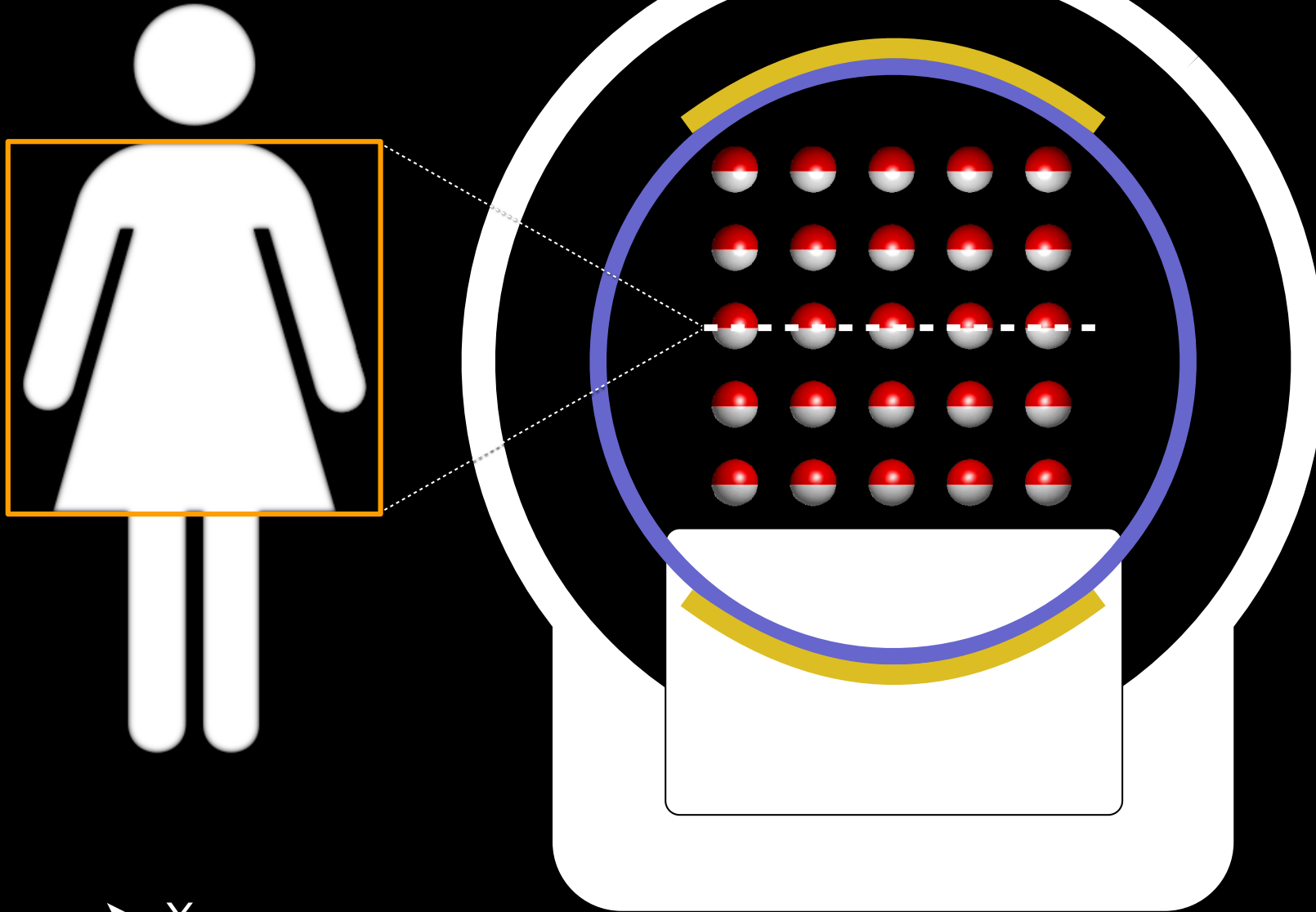


Gradients give rise to isochromats (planes of common frequency).

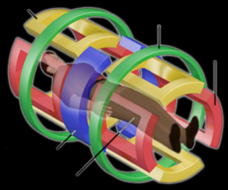




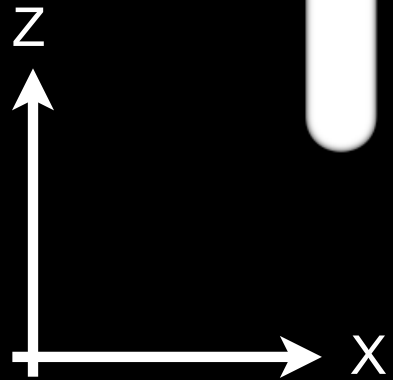
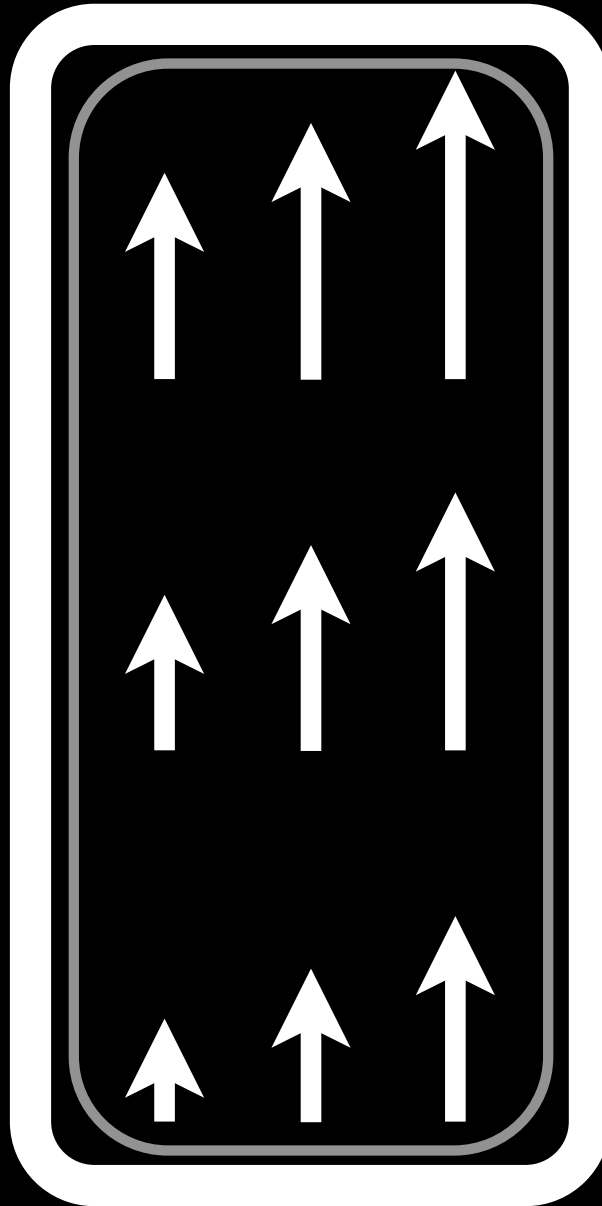
# Spins and Y-Gradients

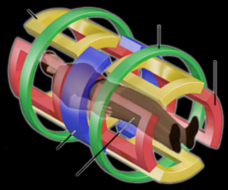


Gradients add/subtract to  $B_0$  along a specific direction.



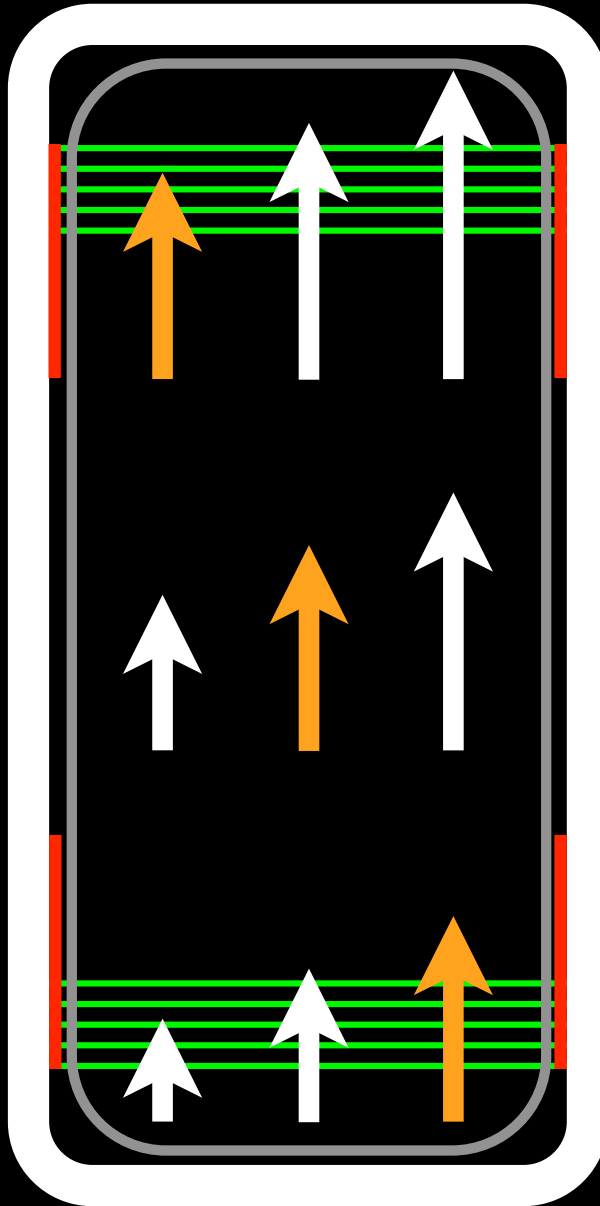
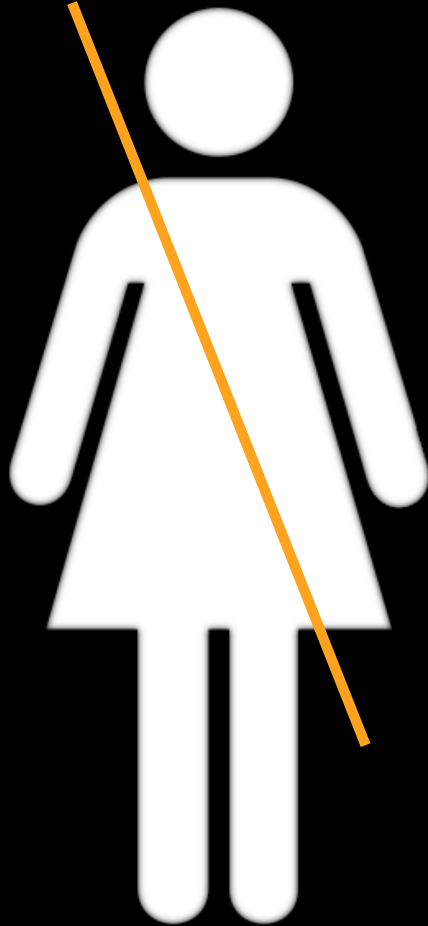
# How do we do this?



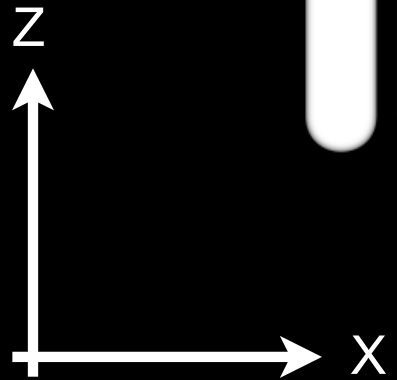


# X+Z-Gradients

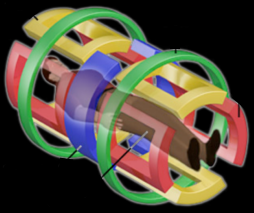
Possible Slice



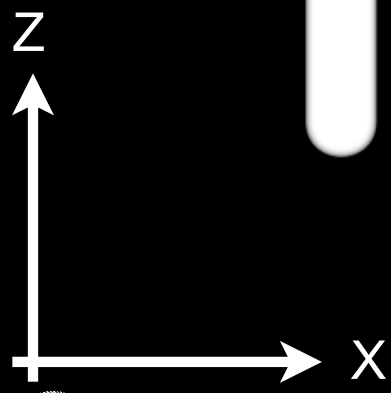
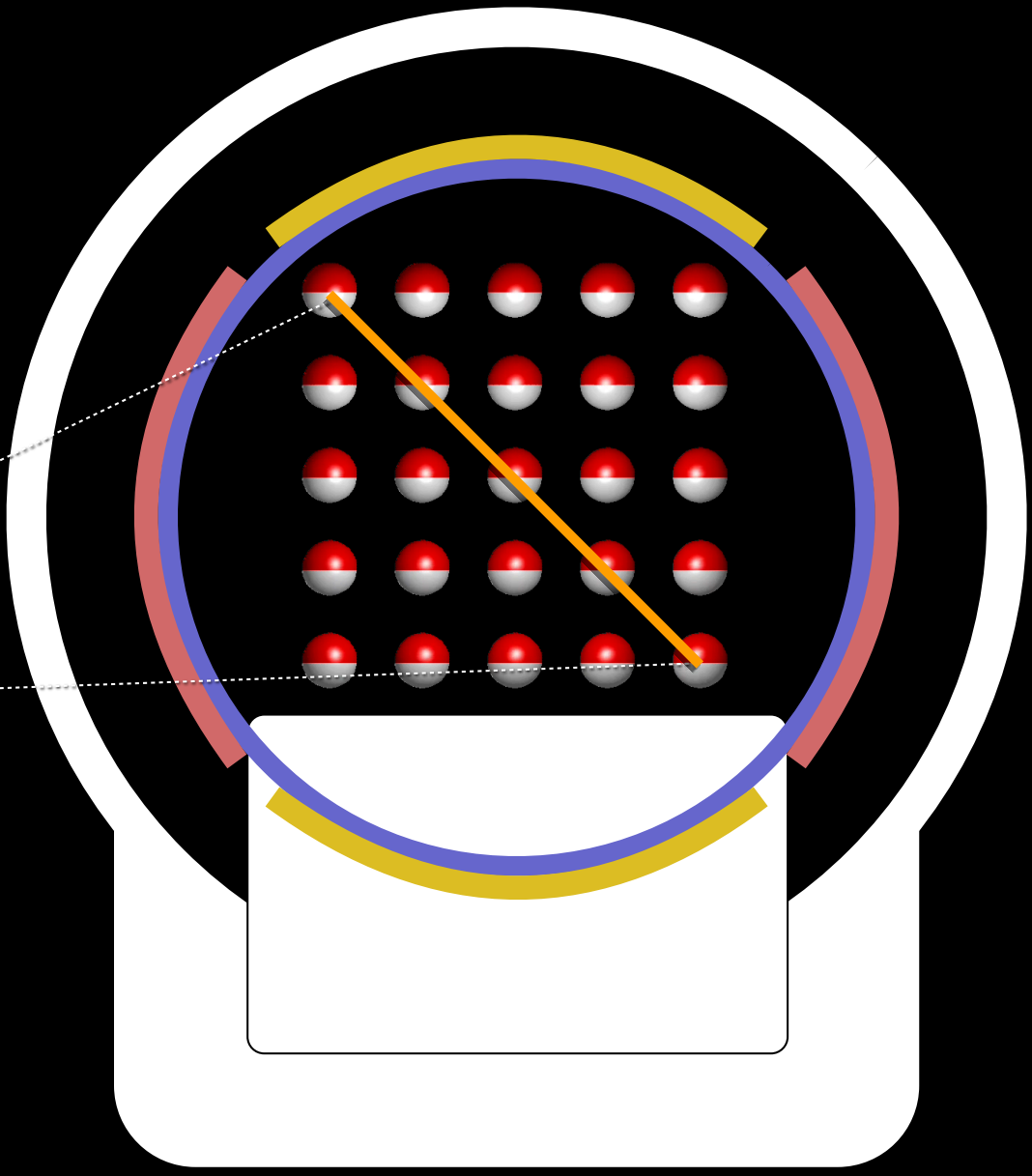
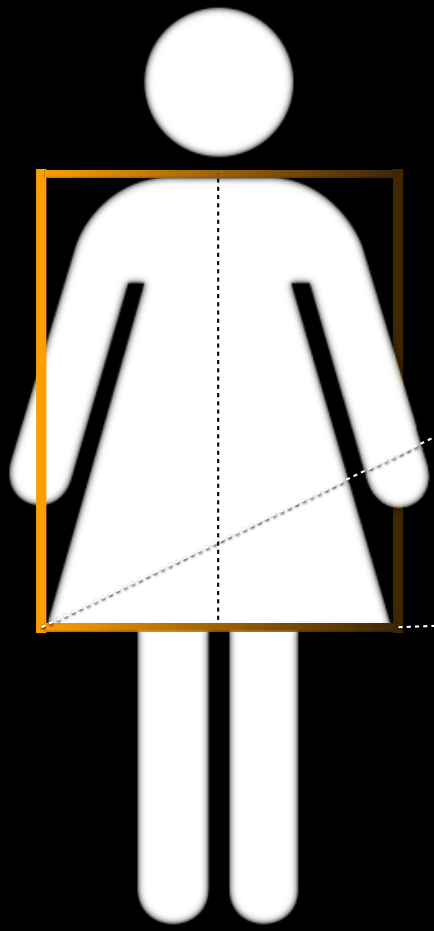
Spin Isochromat



$$\omega(x, z) = \gamma(B_0 + G_x \cdot x + G_z \cdot z)$$



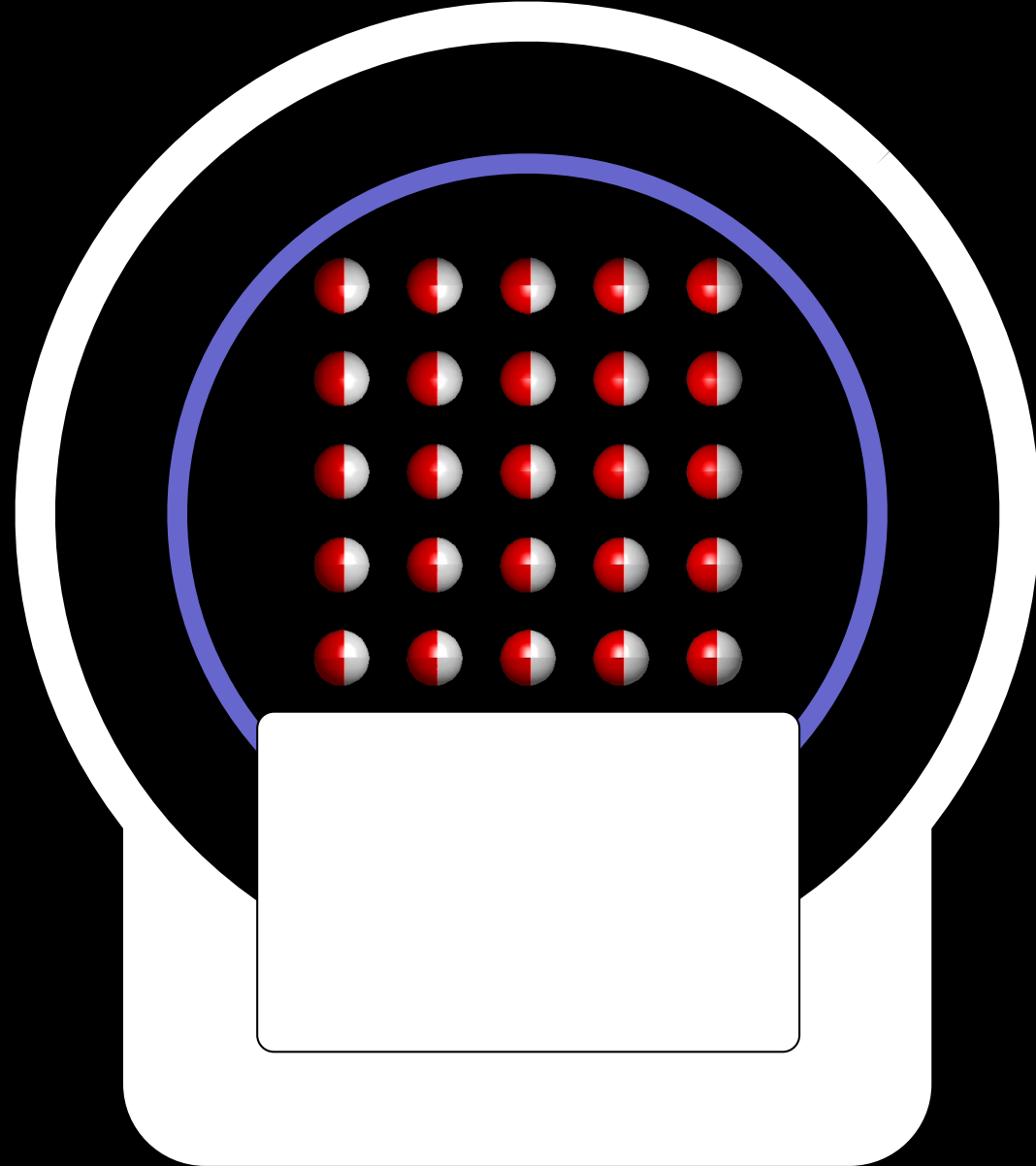
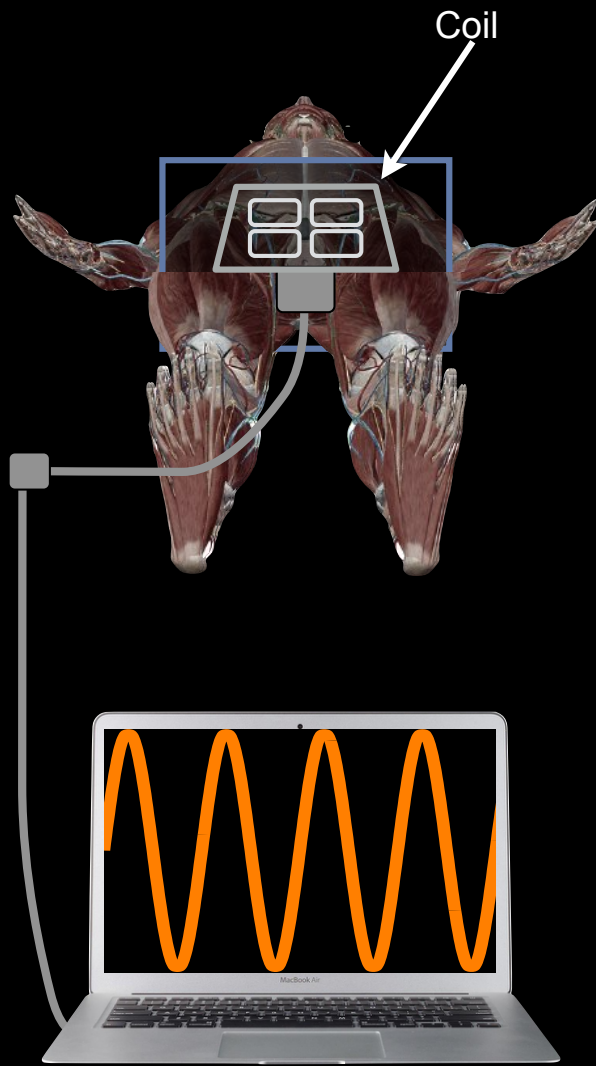
# Spins and X- & Y-Gradients



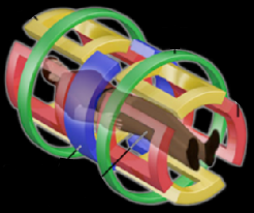
Simultaneous gradients create an arbitrary isochromat plane.

How do we measure  $M_{xy}$ ?

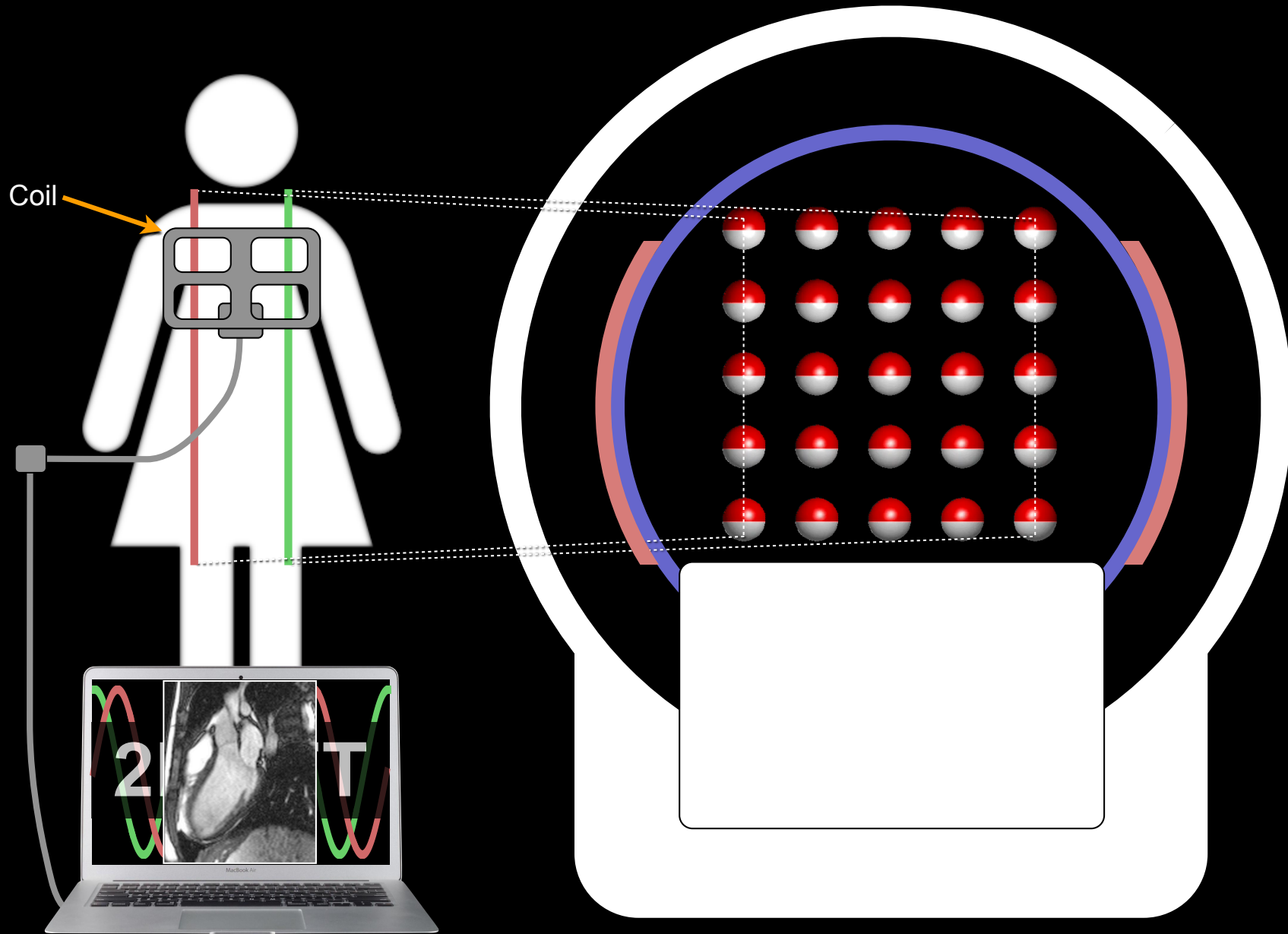
# Faraday's Law of Induction



Precessing spins *induce* a current in a nearby coil.



# Faraday's Law of Induction



The trick is to encode spatial information and image contrast in the echo.

# Basic Detection Principles

## Magnetic Flux Through The Coil – *Reciprocity*

$$\Phi(t) = \int_{object} \vec{B}_r(\vec{r}) \cdot \vec{M}(\vec{r}, t) d\vec{r}$$

↑  
Magnetic  
Flux

↑  
Coil  
Sensitivity

↑  
Bulk  
Magnetization

Eqn. 5.38

**What happens if the coil has poor sensitivity?**

**What happens if the coil's sensitivity is perpendicular to the bulk magnetization? How would that happen?**



# Basic Detection Principles

We get here

$$S(t) = \int_{\text{object}} M_{xy}(\mathbf{r}, 0) e^{-i\gamma\Delta B(\mathbf{r})t} d\mathbf{r}$$

From Here

$$V(t) = -\frac{\partial\Phi(t)}{\partial t} = -\frac{\partial}{\partial t} \int_{\text{object}} \vec{B}(\vec{r}) \cdot \vec{M}(\vec{r}, t) d\vec{r}$$

with 25 pages of Math!

# Basic Detection Principles

$$S(t) = \int_{\text{object}} M_{xy}(\mathbf{r}, 0) e^{-i\gamma\Delta B(\mathbf{r})t} d\mathbf{r}$$

## Observations

**Detected signal is the vector sum of all transverse magnetizations in the “rotating frame” within the imaging volume.**

**The Larmor frequency precession (Lab frame rotation) is necessary for detection, although only the baseband signal matters for imaging**

To the Board

# Gradient Hardware

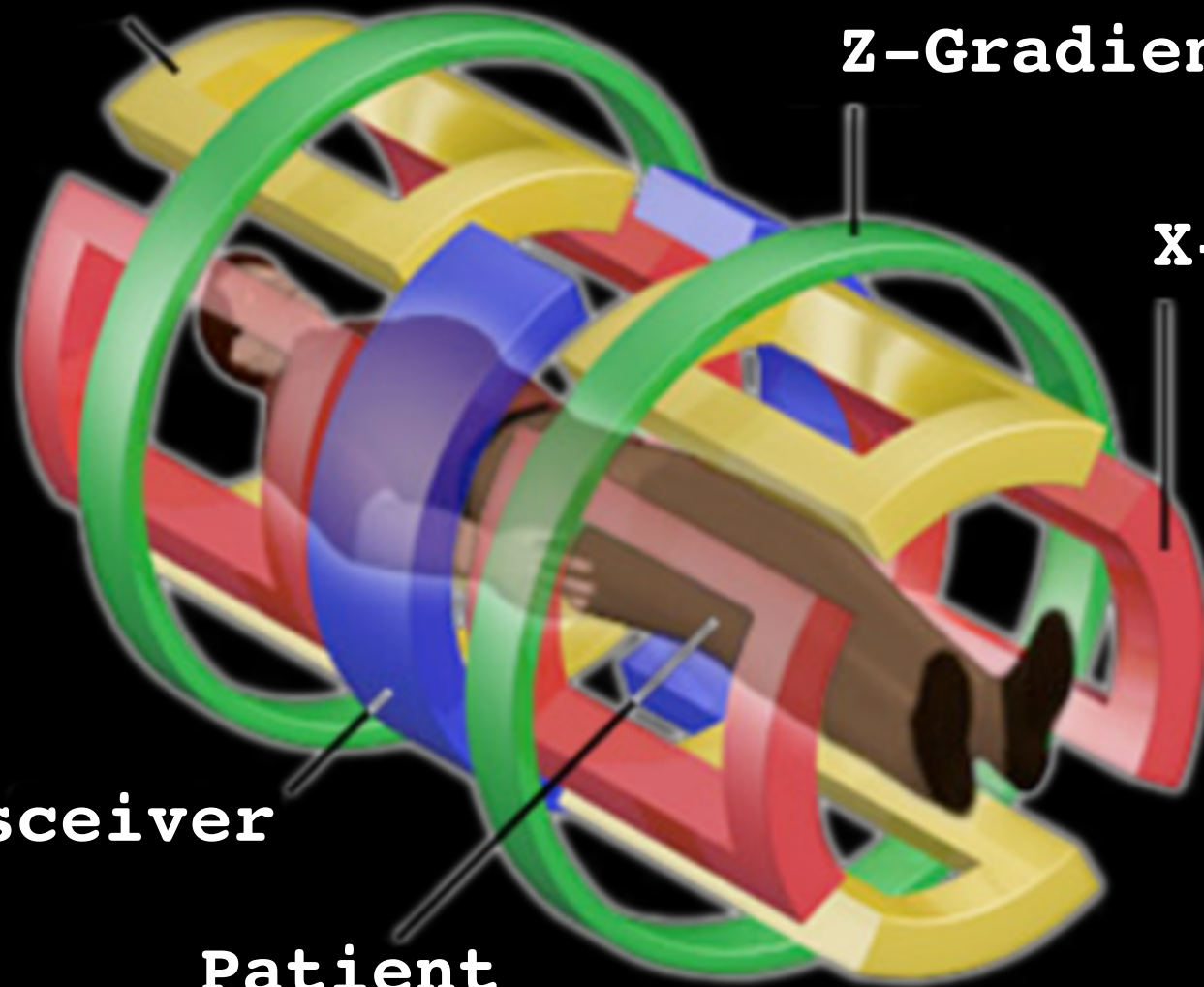
**Y-Gradient**

**Z-Gradient**

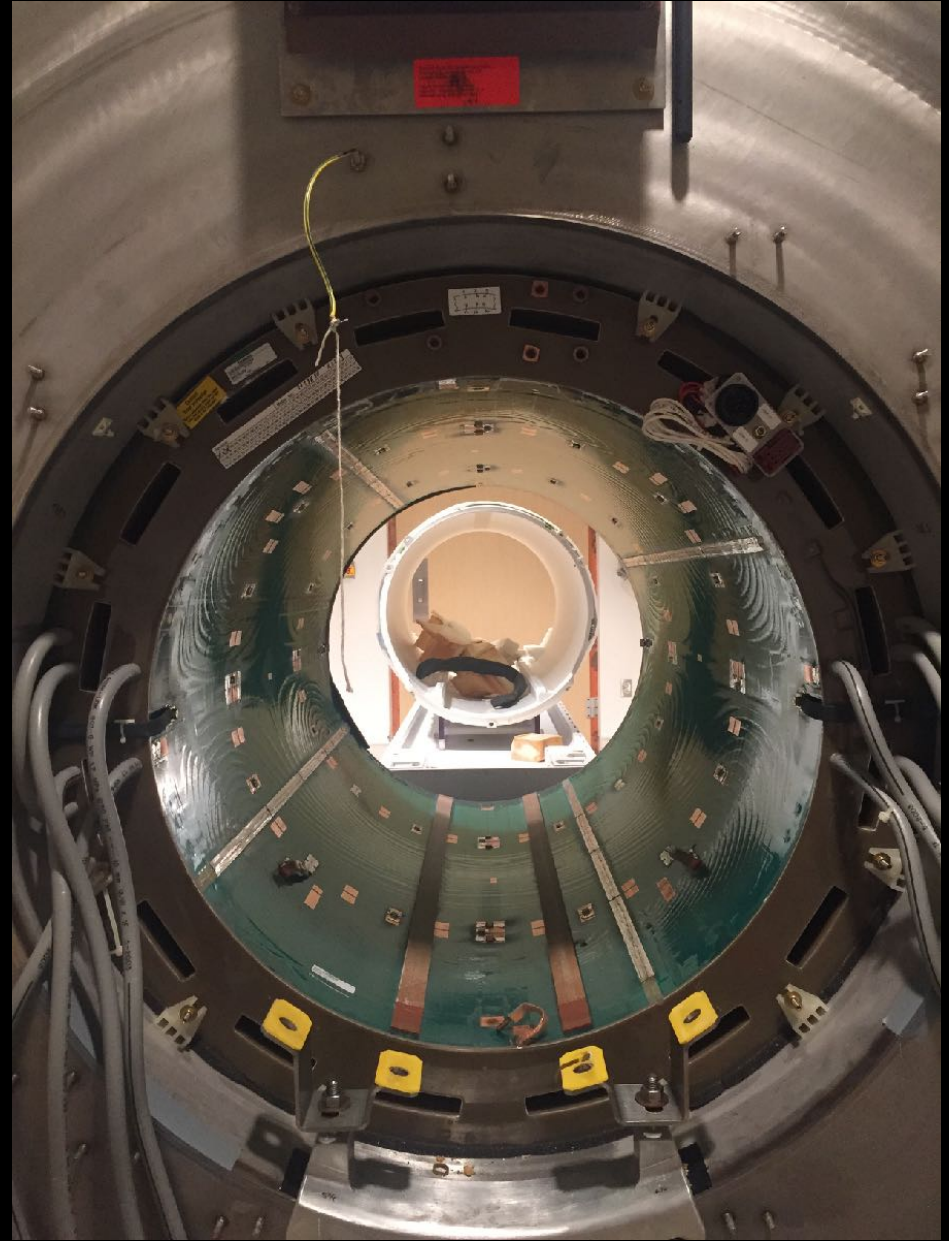
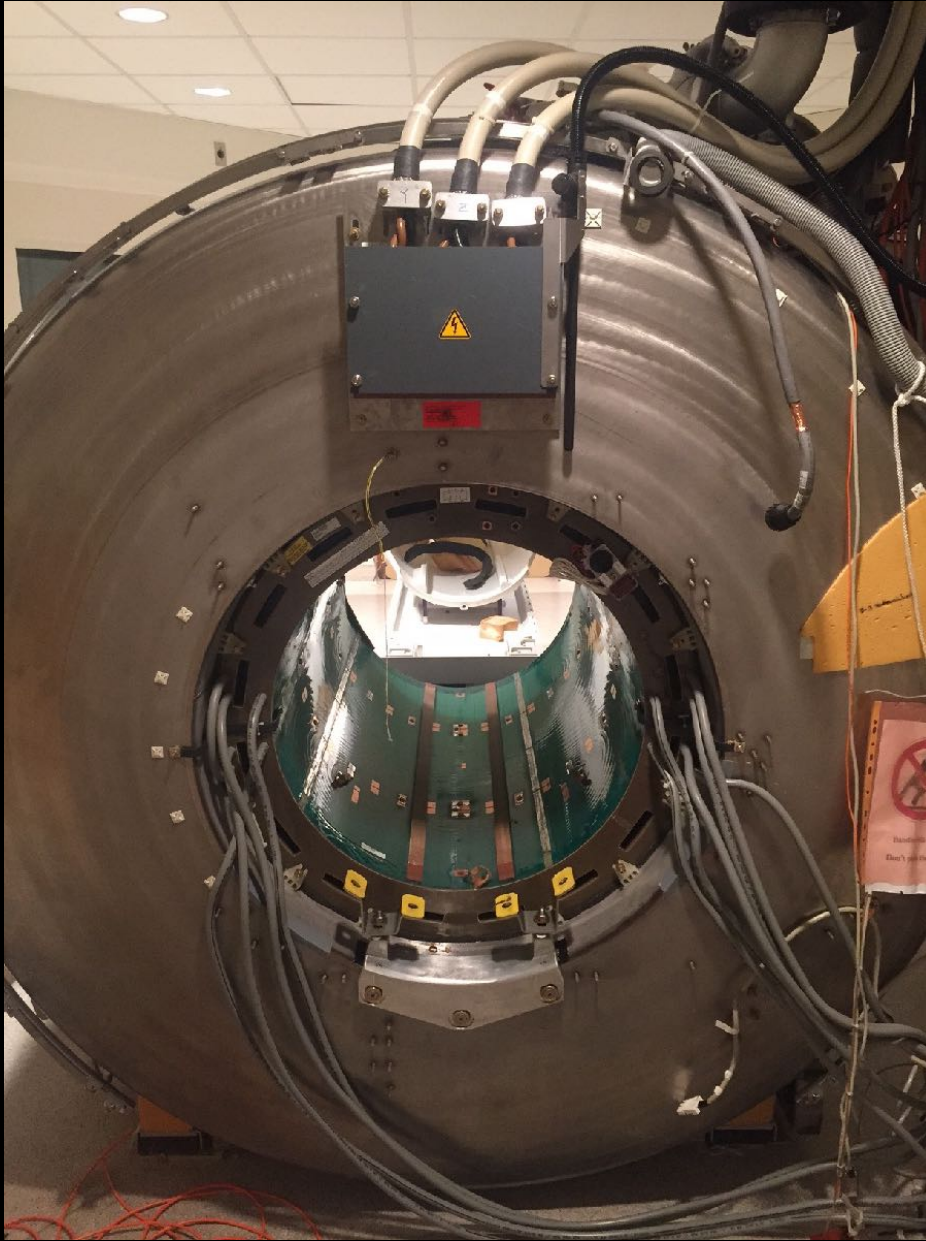
**X-Gradient**

**Transceiver**

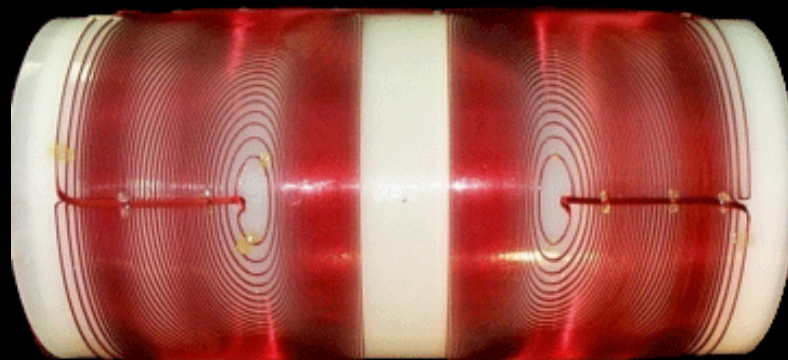
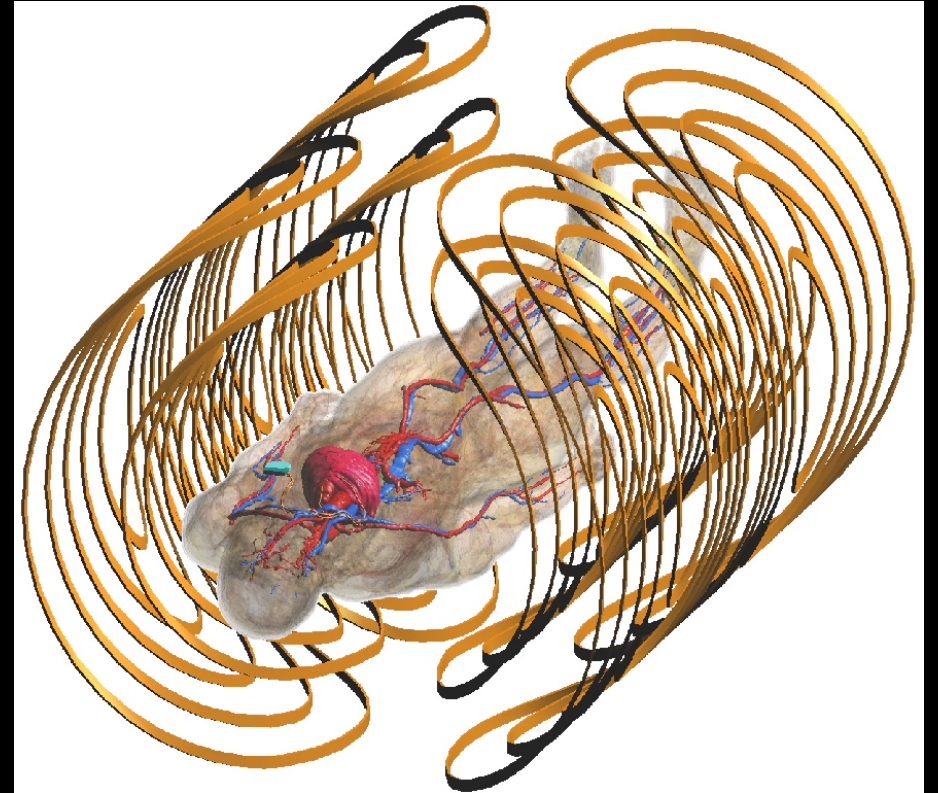
**Patient**



# Gradient Hardware

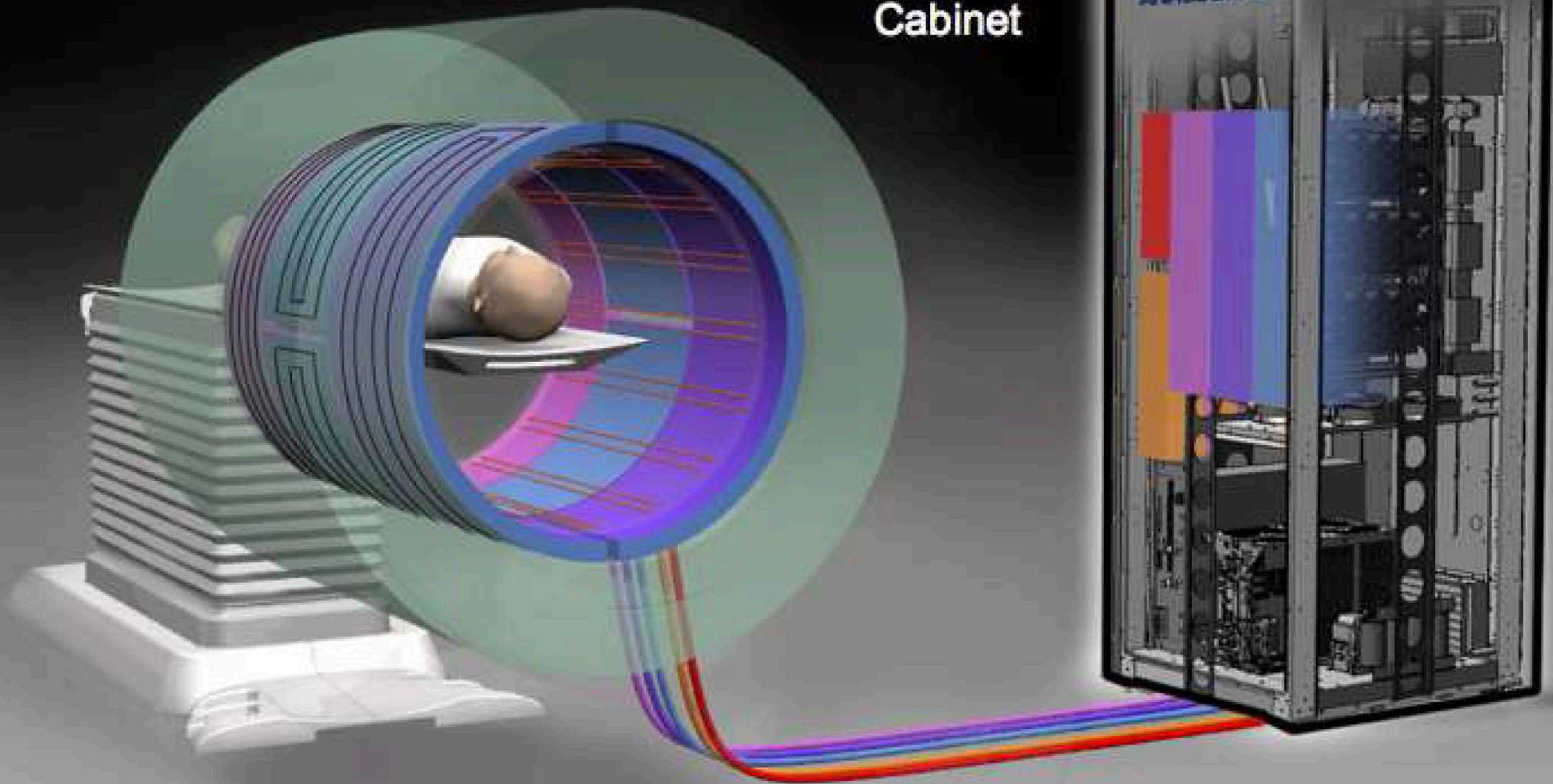


# Gradient Hardware



# Gradient Hardware

Integrated  
MR Power  
Cabinet



# Gradients

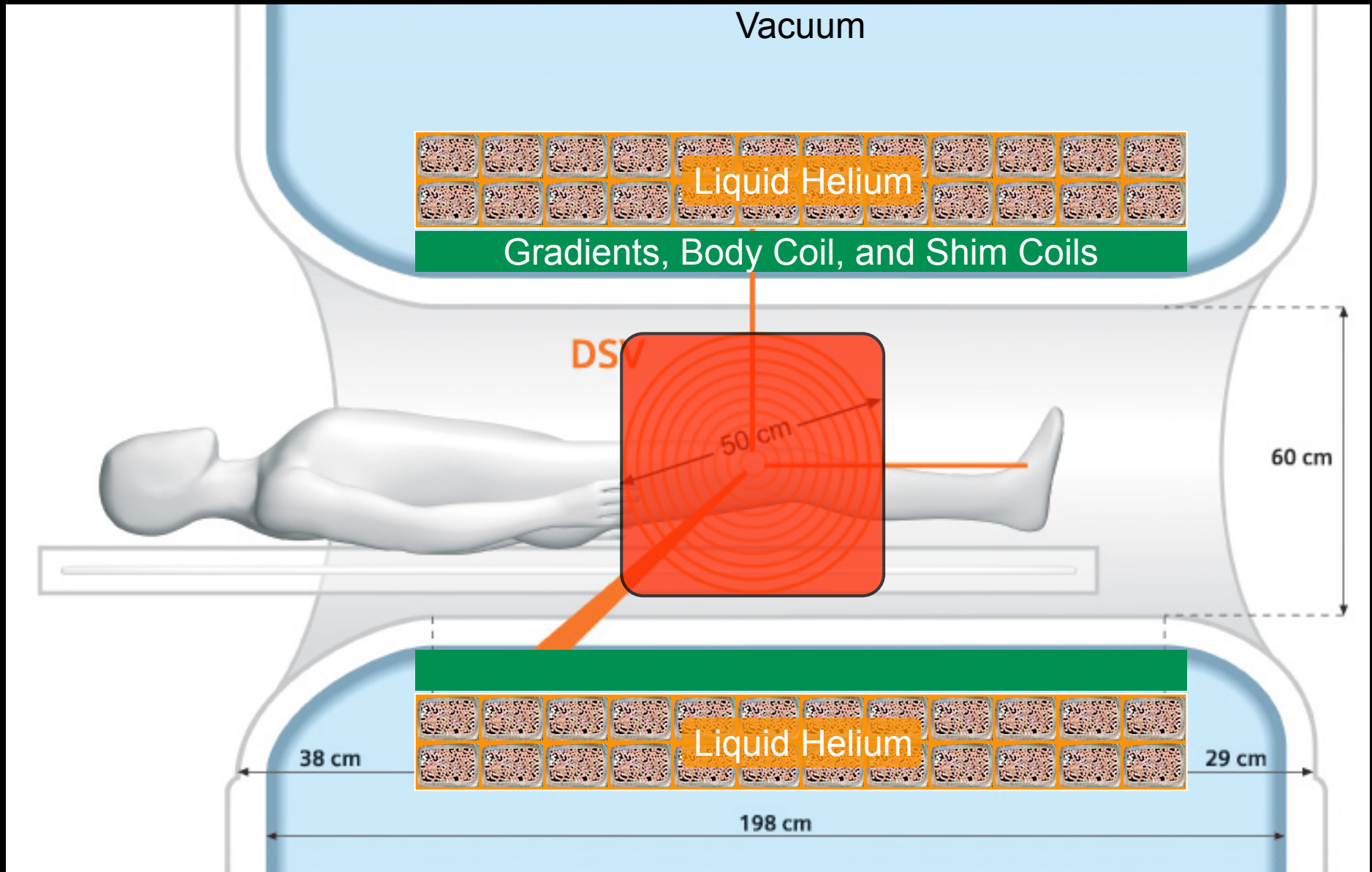
- Primary function
  - Encode spatial information
    - Slice selection
    - Phase encoding
    - Frequency encoding
- Secondary functions
  - Sensitize/de-sensitize images to motion
  - Minimize artifacts (crushers & spoilers)
  - Magnetization **re**-phasing in slice selection
  - Magnetization **de**-phasing during readout



# Gradients

- Gradients are a:
  - Small
    - $<5\text{G/cm}$  ( $<0.0075\text{T}$  @ edge of 30cm FOV)
  - Spatially varying
    - Linear gradients
    - Adds to  $B_0$  only in Z-direction
  - Time varying
    - Slewrate Max.  $\sim 150\text{-}200\text{mT/m/ms}$
  - Magnetic field
    - Adds/Subtracts to the  $B_0$  field
  - Parallel to  $B_0$
- Gradients are NOT:
  - Fields perpendicular to  $B_0$

# Gradients



Gradients are “linear” over ~40-50cm on each axis.

# B-Field Assumptions in MRI

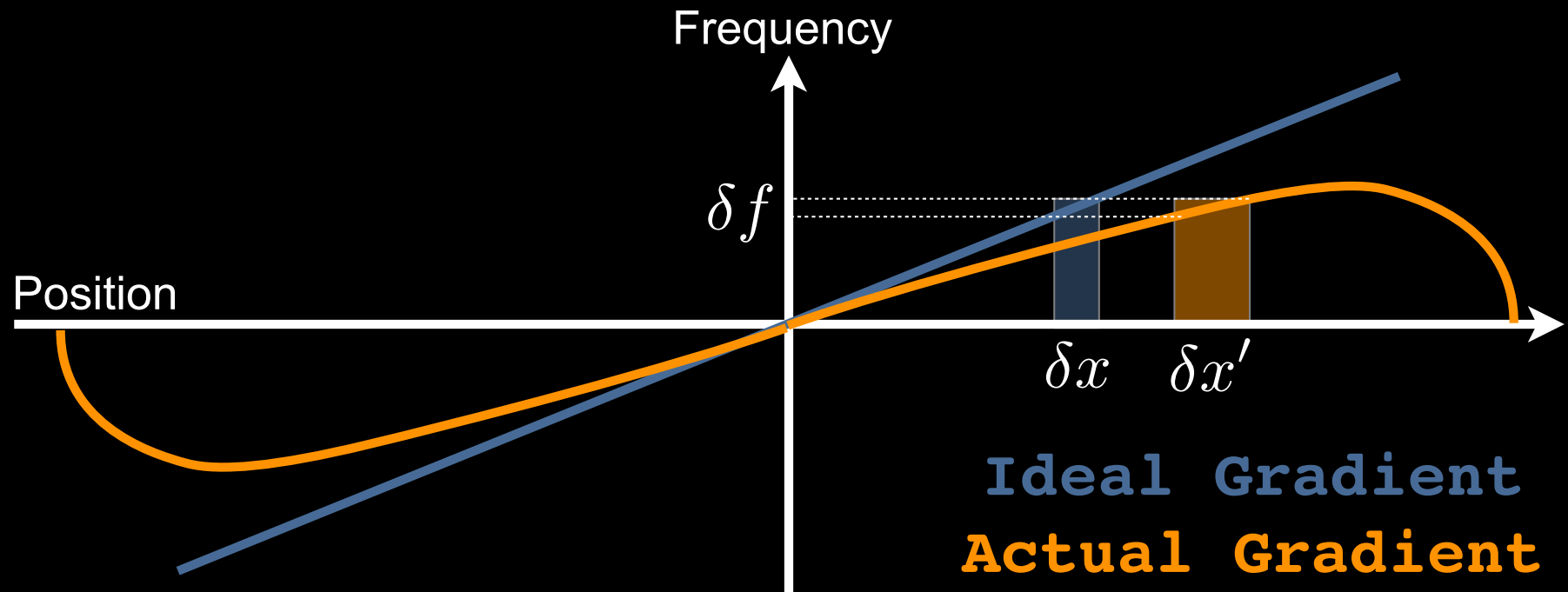
- **B<sub>0</sub>-field is:**
  - Perfectly uniform over space.
    - “B<sub>0</sub> homogeneity”
  - Perfectly stable with time.
- **B<sub>1</sub>-field is:**
  - Perfectly uniform over space.
    - “B<sub>1</sub> homogeneity”
  - Temporally modulated exactly as specified.
- **Gradient Fields are:**
  - Perfectly linear over space.
    - “Gradient linearity”
  - Temporally modulated exactly as specified

# Imperfections of Gradient Fields

- Gradient coils aren't perfect
  - Non-linearity
  - Eddy Currents
  - Maxwell terms (Concomitant fields)
  - But they are small
    - Much smaller than  $B_0$
    - We will ignore them...but they exist...

# Gradient Non-linearity

# Gradient Non-linearity

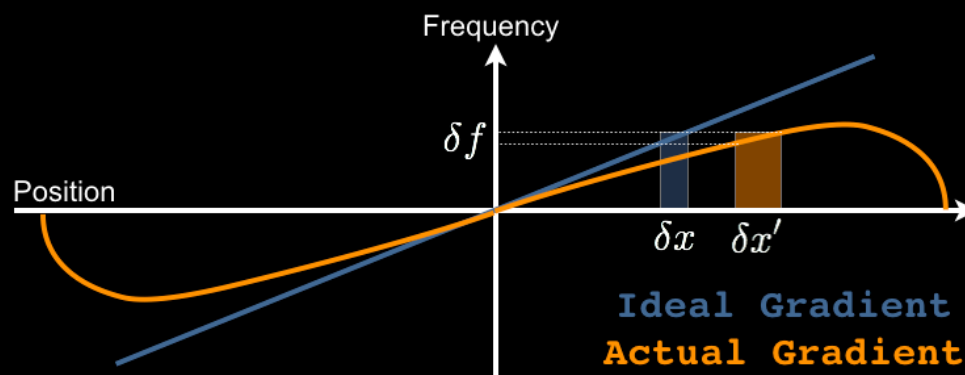


Ideally spatial position is linearly related to frequency.

# Gradient Non-linearity

- **Basic assumption in MRI is that the z-component of the B-field created by the gradient coils varies linearly with x, y, or z over the FOV.**
- **Higher gradient amplitudes and slewrates can be achieved by compromising on spatial linearity.**
- **Gradient non-linearity causes geometric and intensity distortions.**

# Gradient Non-linearity





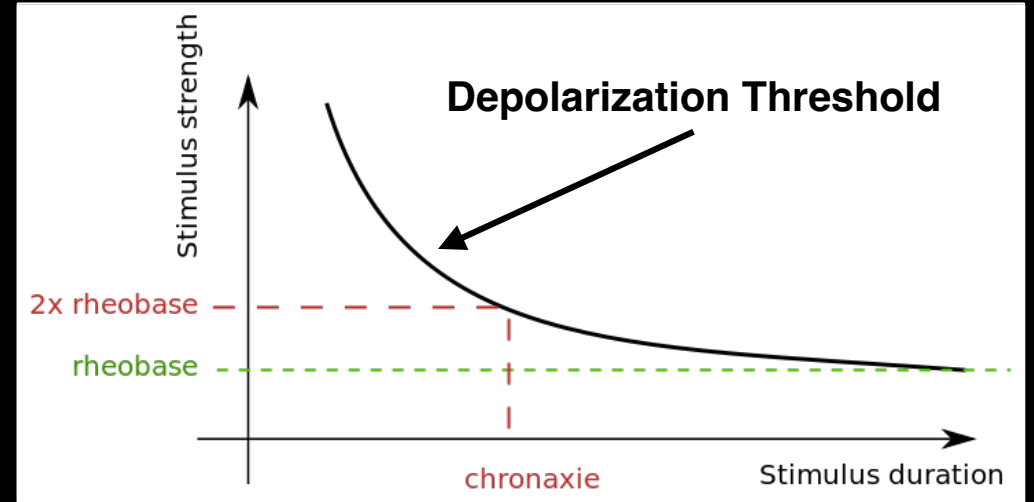
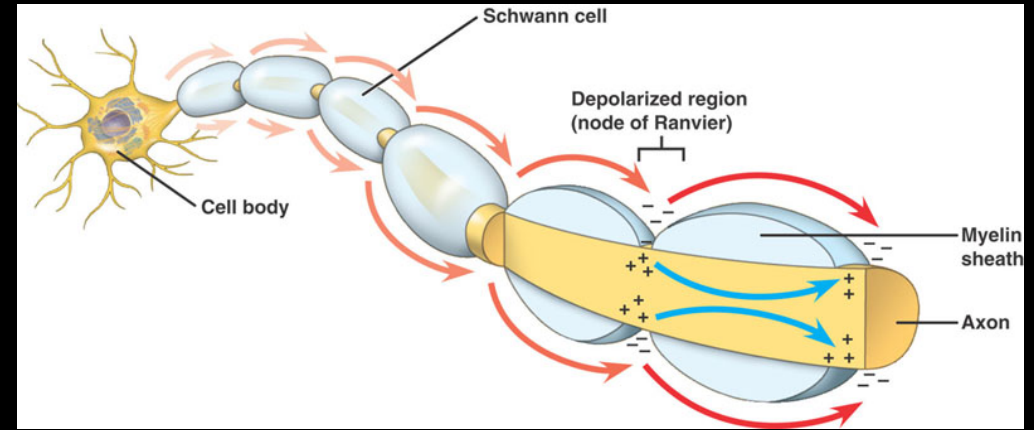
# Solution

- **Improve hardware and linearity!**
- **Pay attention to FOV!**
- **Image warping parameters that are system specific and applied to all images.**
  - **Works well qualitatively.**
  - **Can be problematic quantitatively.**

# Gradient Safety

# Gradient Safety

- Noise
- Peripheral nerve stimulation (PNS)



Solution: De-rate gradient slew rates, but this increases scan time.



Solution: Ear plugs



Head phones

**Time-varying gradients induce mechanical vibrations and PNS.**

# MRI Gradient Noise



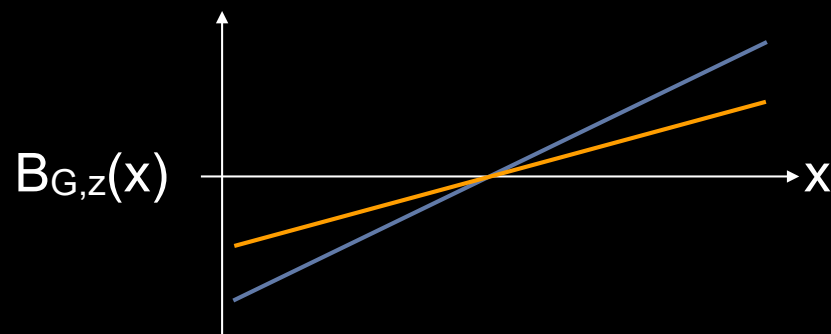
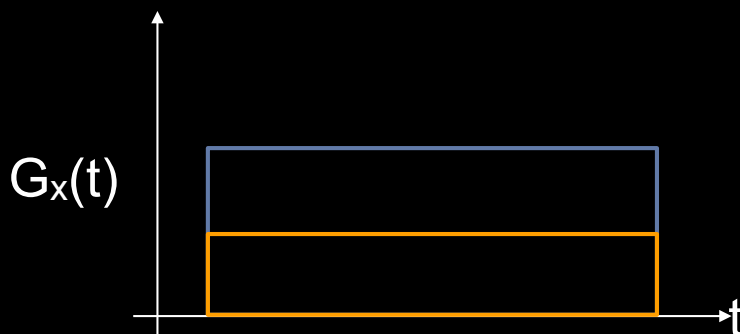
# Gradient Noise

- Jet take-off @ 25m ~150 dB (eardrum rupture)
- Car horn @ 1m ~110 dB (borderline painful)
- Live rock band ~100 dB
- **MRI gradients full load** ≤99 dB
- Garbage disposal ~80 dB
- **MRI gradients basic load** ≤75 dB
- Radio or TV Audio ~70dB



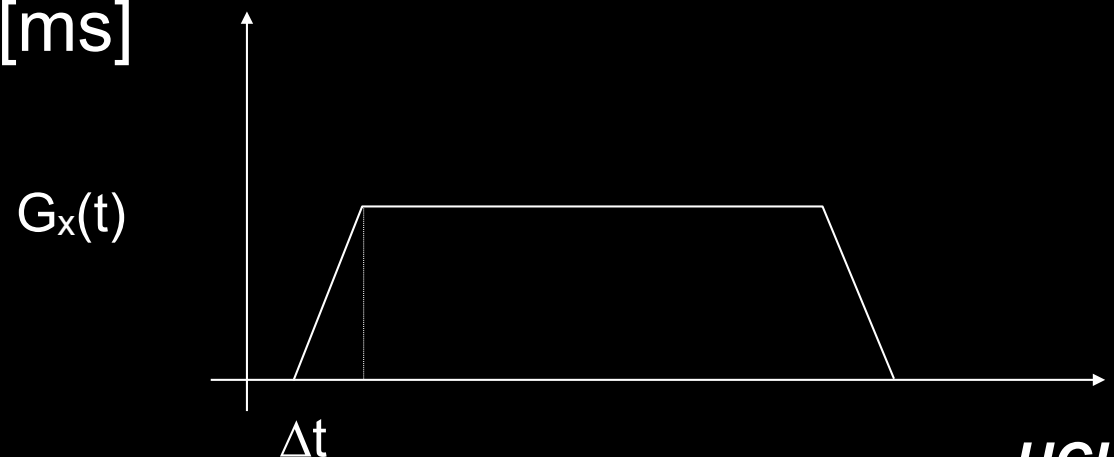
# Gradient Safety – $G_{\text{Max}}$

- $G_{\text{max}}$  limitations:
  - Concern: None known.
    - $B_0$  is already pretty big.
  - Conventional Gradients
    - $G_{\text{Max}} = 4$  to  $5\text{G/cm}$  ( $=50\text{mT/m}$ )
  - Cutting Edge Gradients
    - $G_{\text{Max}} = 8\text{G/cm}$  ( $=80\text{mT/m}$ )
  - Connectome Gradients
    - $G_{\text{Max}} = 30\text{G/cm}$  ( $=300\text{mT/m}$ )
  - Consider the  $\Delta B$  contributed by a gradient...



# Gradient Slewrate

- **Gradient slew rate**
  - T/m/s (or G/cm/s)
  - $dG/dt$  – Rate of change of gradient amplitude
- **Slew rate limited by dB/dt:**
  - Concern: Peripheral Nerve Stimulation
  - Regulated by FDA
  - Normal Mode:  $dB/dt = 16 \text{ T/s} \cdot (1 + 0.36/\beta)$
  - First Level Mode:  $dB/dt = 20 \text{ T/s} \cdot (1 + 0.36/\beta)$
  - $\beta = \text{stimulus duration [ms]}$



# Questions?

- Related reading materials
  - Nishimura - Chap 5

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<http://mrri.ucla.edu/sunglab>