MRI Systems III: Gradients

M219 - Principles and Applications of MRI

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1/27/2025

Course Overview

- 2025 course schedule
  - https://mrrl.ucla.edu/pages/m219_2025
- Assignments
  - Homework #1 due on 1/29

- TA office hours, Mon 4-6pm
- Office hours, Fri 10-11am

Bloch Equations - Lab Frame

$$\frac{d\vec{\mathrm{M}}}{dt} = \underbrace{\vec{\mathrm{M}} \times \gamma \vec{\mathrm{B}}}_{\text{Precession}} - \underbrace{\frac{M_x \hat{\mathrm{i}} + M_y \hat{\mathrm{j}}}{T_2}}_{\substack{\text{Transverse} \\ \text{Relaxation}}} - \underbrace{\frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}}_{\substack{\text{Longitudinal} \\ \text{Relaxation}}}$$

- Precession
  - Magnitude of M unchanged
  - Phase (rotation) of M changes due to B
- Relaxation
  - T_1 changes are slow O(100ms)
  - T_2 changes are fast O(10ms)
  - Magnitude of M can be ZERO

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

- **No precession**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, separable ODEs!**

The precessional term drops out in the rotating frame.

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} = B_1^e(t)\hat{i}'$$

- B1 induced nutation
- T_1 and T_2 Relaxation
- Drop the diffusion term
- System or first order, linear, coupled PDEs!
- When does this equation apply?

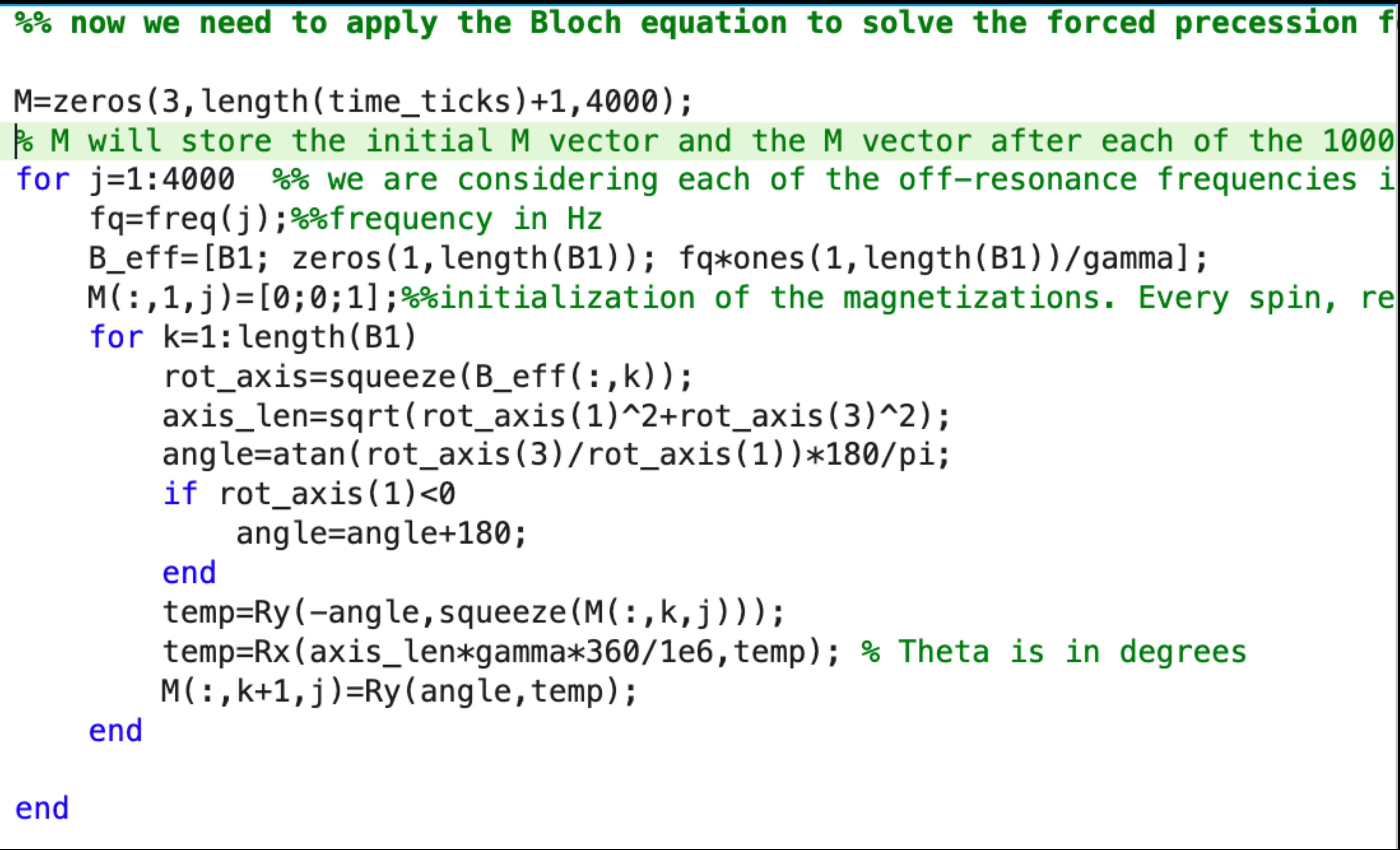
David Geffen
School of Medicine

The precessional term ***does not*** drop out in the rotating frame.

UCLA
Radiology

Free? Forced? Relaxation?

- **We've considered all combinations of:**
  - **Free and forced precession**
  - **With and without relaxation**
  - **Laboratory and rotating frames**
- **Which one's concern M219 the most?**
  - **Free precession in the rotating frame with relaxation**
  - **Forced precession in the rotating frame without relaxation.**
- **We can, in fact, simulate all of them…**



```
%% now we need to apply the Bloch equation to solve the forced precession f

M=zeros(3,length(time_ticks)+1,4000);
% M will store the initial M vector and the M vector after each of the 1000
for j=1:4000  %% we are considering each of the off-resonance frequencies i
    fq=freq(j);%%frequency in Hz
    B_eff=[B1; zeros(1,length(B1)); fq*ones(1,length(B1))/gamma];
    M(:,1,j)=[0;0;1];%%initialization of the magnetizations. Every spin, re
    for k=1:length(B1)
        rot_axis=squeeze(B_eff(:,k));
        axis_len=sqrt(rot_axis(1)^2+rot_axis(3)^2);
        angle=atan(rot_axis(3)/rot_axis(1))*180/pi;
        if rot_axis(1)<0
            angle=angle+180;
        end
        temp=Ry(-angle,squeeze(M(:,k,j)));
        temp=Rx(axis_len*gamma*360/1e6,temp); % Theta is in degrees
        M(:,k+1,j)=Ry(angle,temp);
    end

end
```

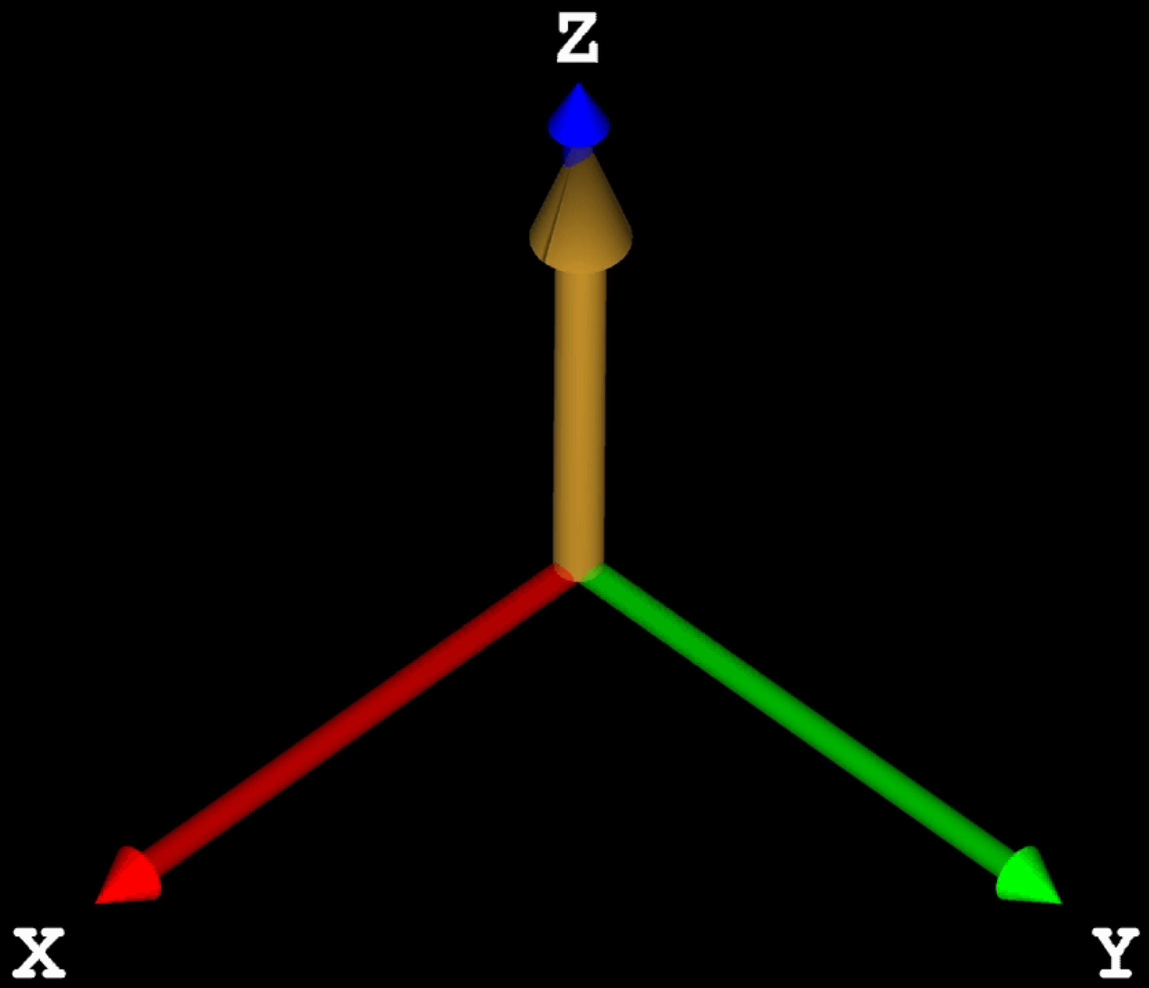
Types of RF Pulses

- Excitation Pulses
- Inversion Pulses
- Refocusing Pulses
- Saturation Pulses
- Spectrally Selective Pulses
- Spectral-spatial Pulses
- Adiabatic Pulses

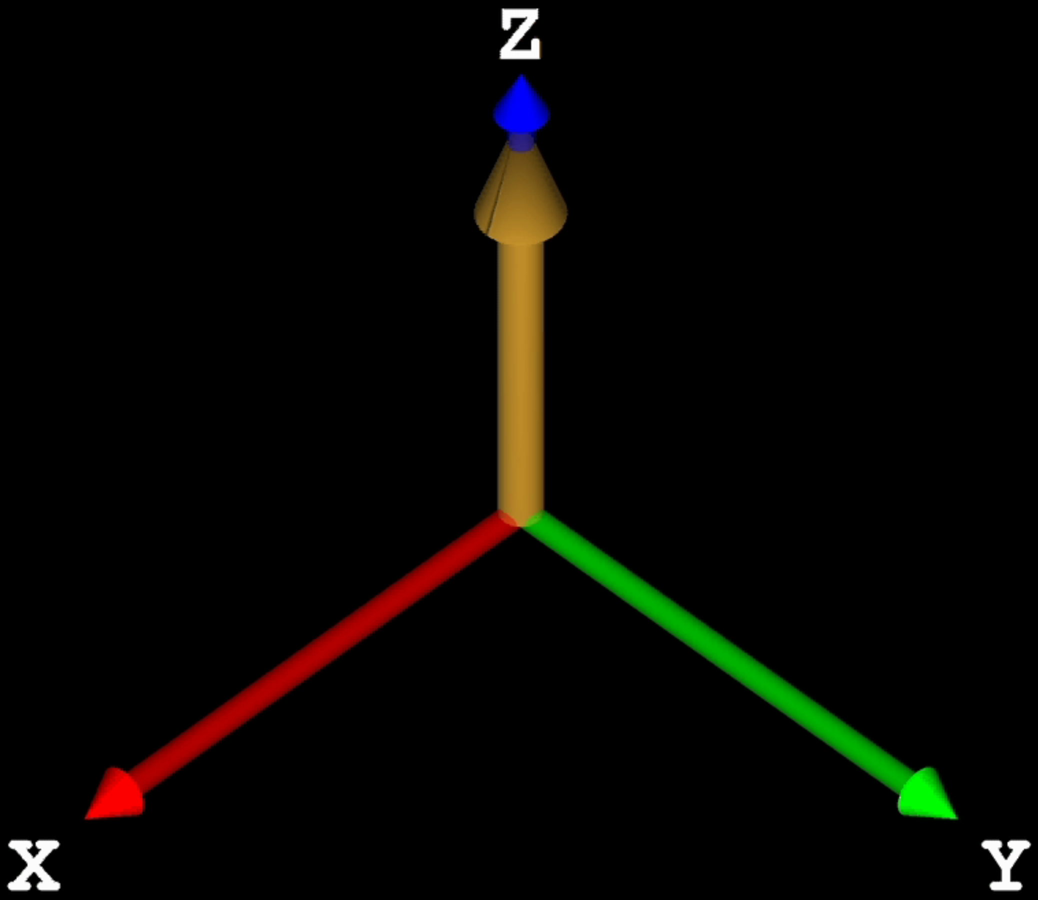
Excitation Pulses

- Tip M_z into the transverse plane
- Typically 200μs to 5ms
- Non-uniform across slice thickness
  - Imperfect slice profile
- Non-uniform within slice
  - Termed B_1 inhomogeneity
  - Non-uniform signal intensity across FOV

90° Excitation Pulse



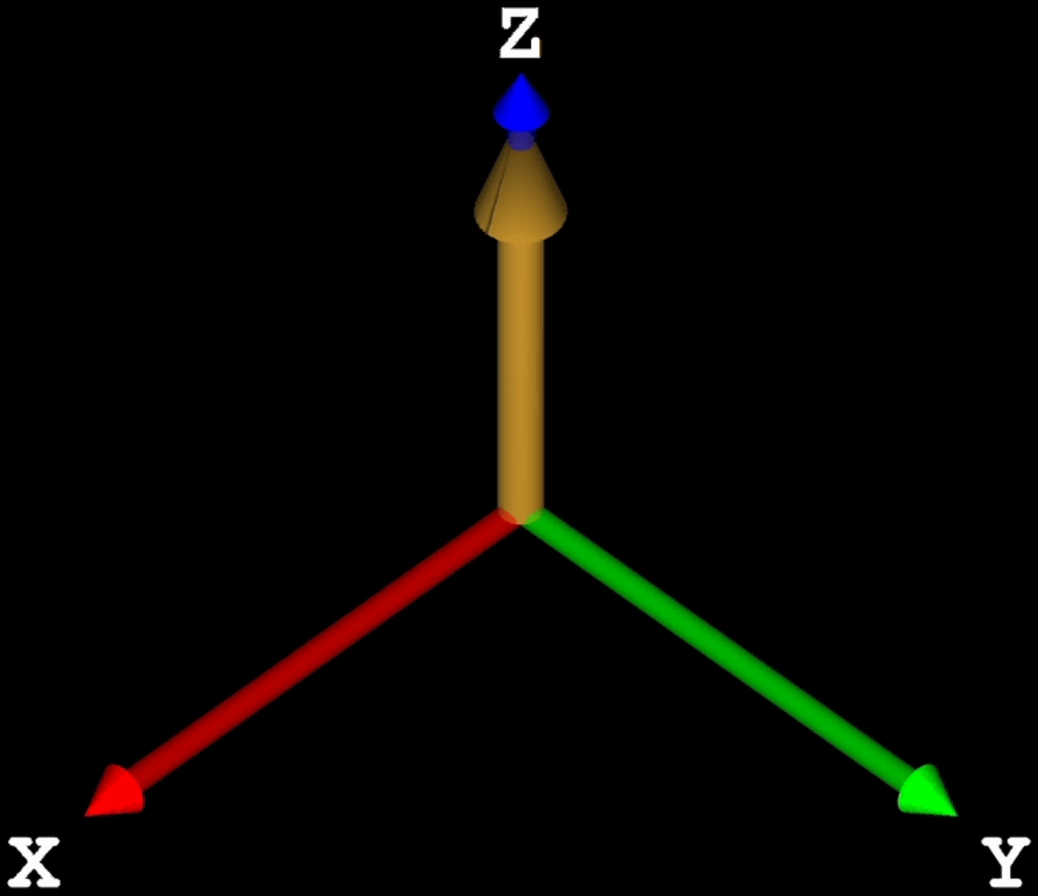
Small Flip Angle Excitation



Inversion Pulses

- Typically, 180° RF Pulse
  - non-180° that still results in $-M_Z$
- Invert M_Z to $-M_Z$
  - Ideally produces no M_{XY}
- Hard Pulse
  - Constant RF amplitude
  - Typically non-selective
- Soft (Amplitude Modulated) Pulse
  - Frequency selective
  - Spatially Selective

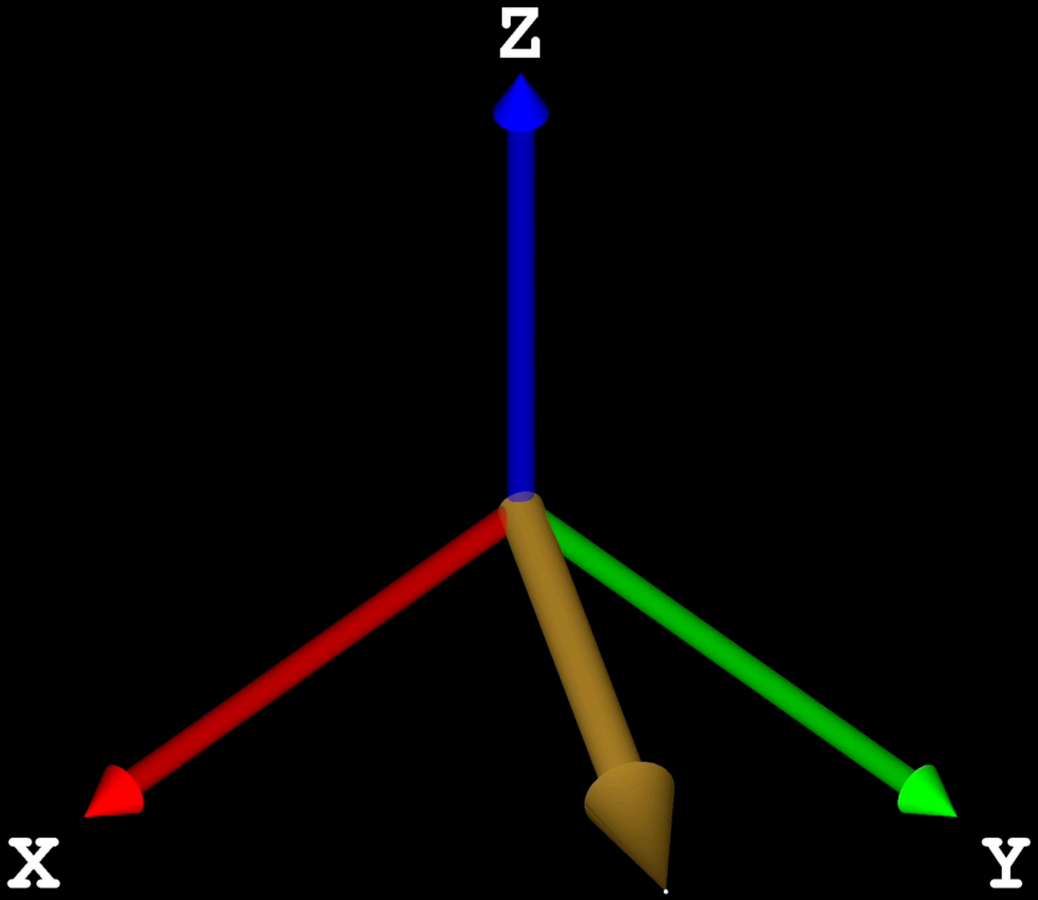
Inversion Pulses



Refocusing Pulses

- Typically, 180° RF Pulse
  - Provides optimally refocused M_{XY}
  - Largest spin echo signal
- non-180°
  - Partial refocusing
  - Lower SAR
  - Multiple non-180° produce stimulated echoes
- Refocus spin dephasing due to
  - imaging gradients
  - local magnetic field inhomogeneity
  - magnetic susceptibility variation
  - chemical shift

Refocusing Pulses



Gradient Fields & Spins

Gradients

Gradients are a special kind of inhomogeneous field whose z-component varies linearly along a specific direction called the gradient direction.

$$(\vec{G}\cdot\vec{r})\ \hat{k} = (G_x\cdot x + G_y\cdot y + G_z\cdot z)\ \hat{k}$$

$$\vec{B}(\vec{r},t) = (B_0 + \underbrace{\vec{G}(t)\cdot\vec{r}}_{B_G(\vec{r},t)})\ \hat{k}$$

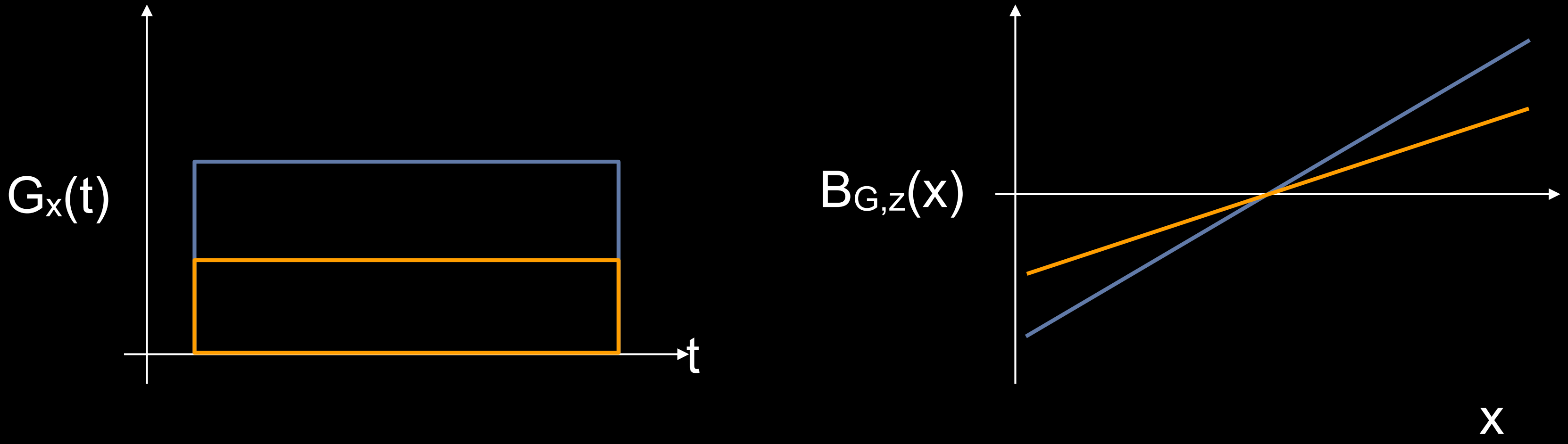
- Each gradient coil can be activated independently and simultaneously

Gradient Induced B-Fields

$$B_G(x) = G_x x$$ x-gradient

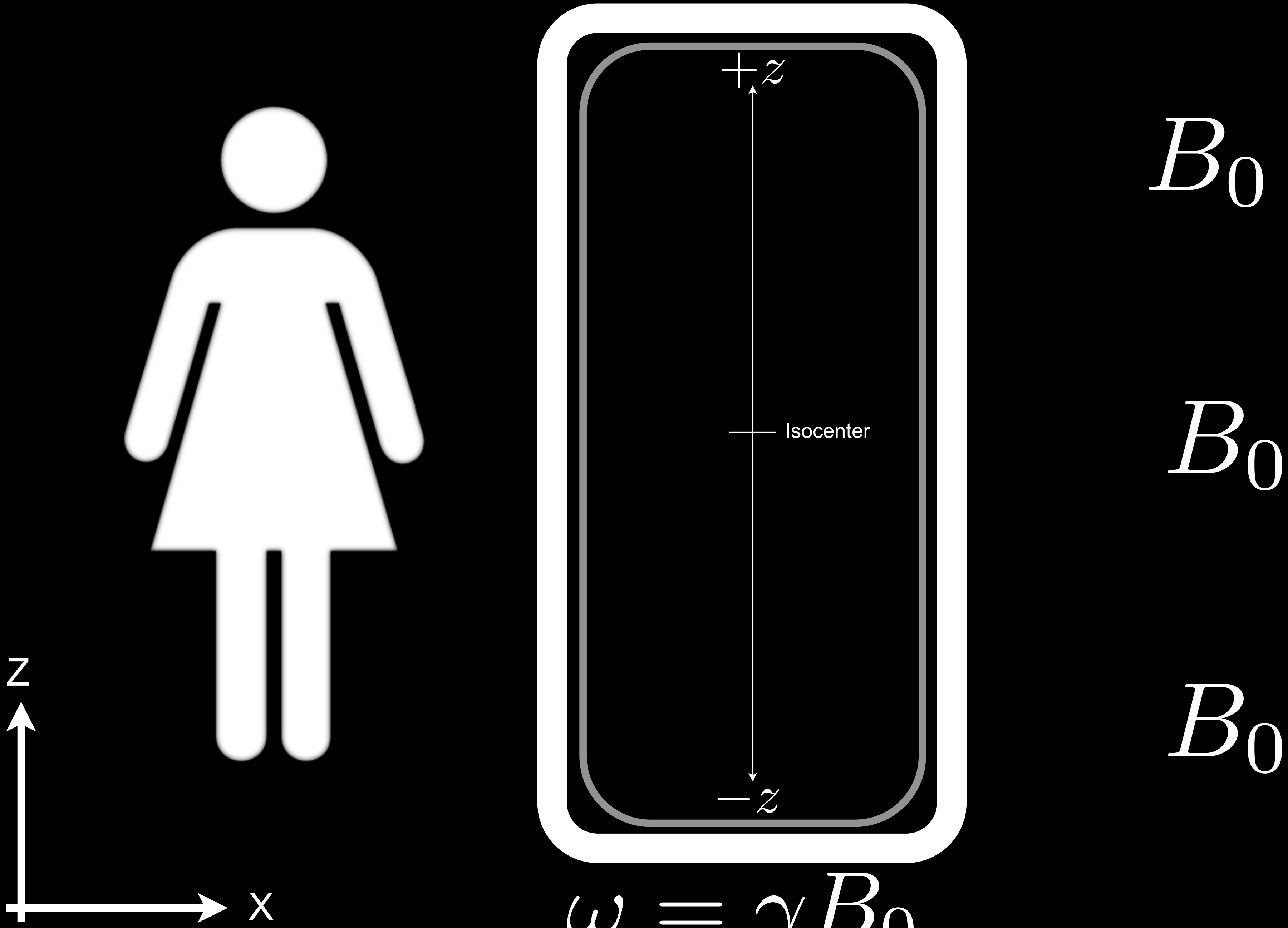
$$B_G(y) = G_y y$$ y-gradient

$$B_G(z) = G_z z$$ z-gradient



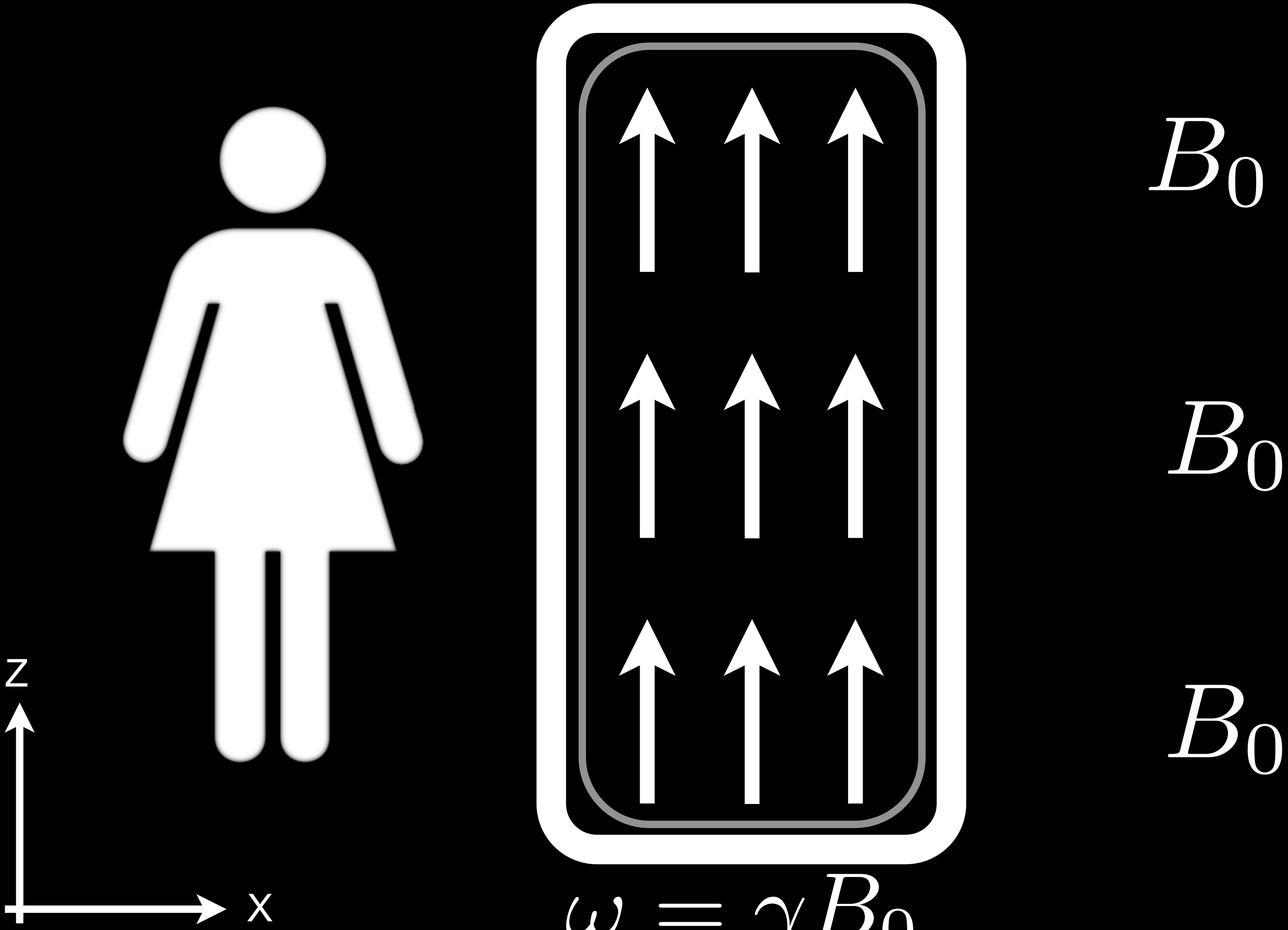
The magnetic field at a position depends on the magnitude of the applied gradient.

No Gradients Turned On



$$\omega = \gamma B_0$$

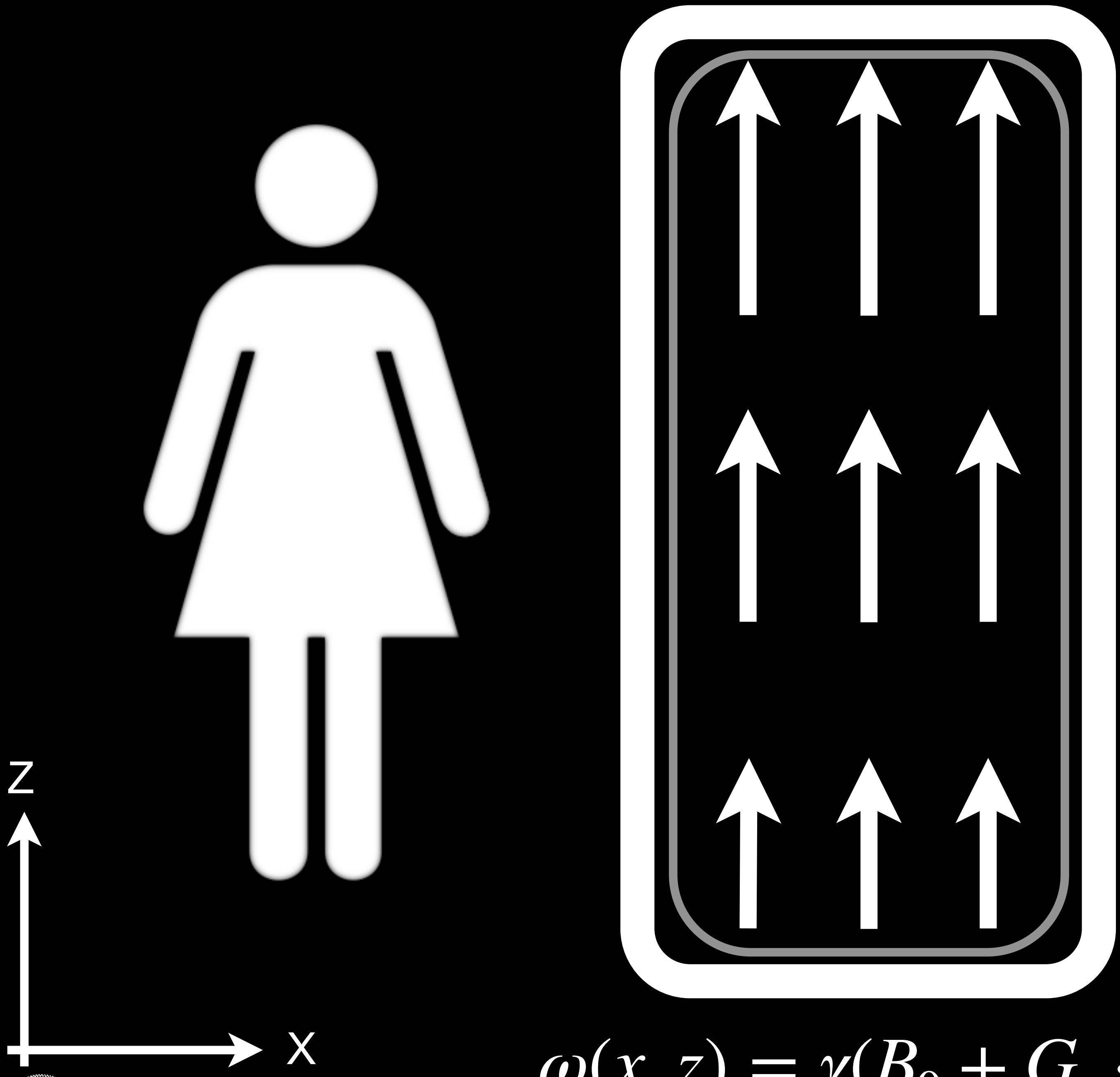
No Gradients Turned On



$$\omega = \gamma B_0$$

Length of arrow indicates strength of local field.

Z-Gradients



$$B_0 + \delta B_0$$

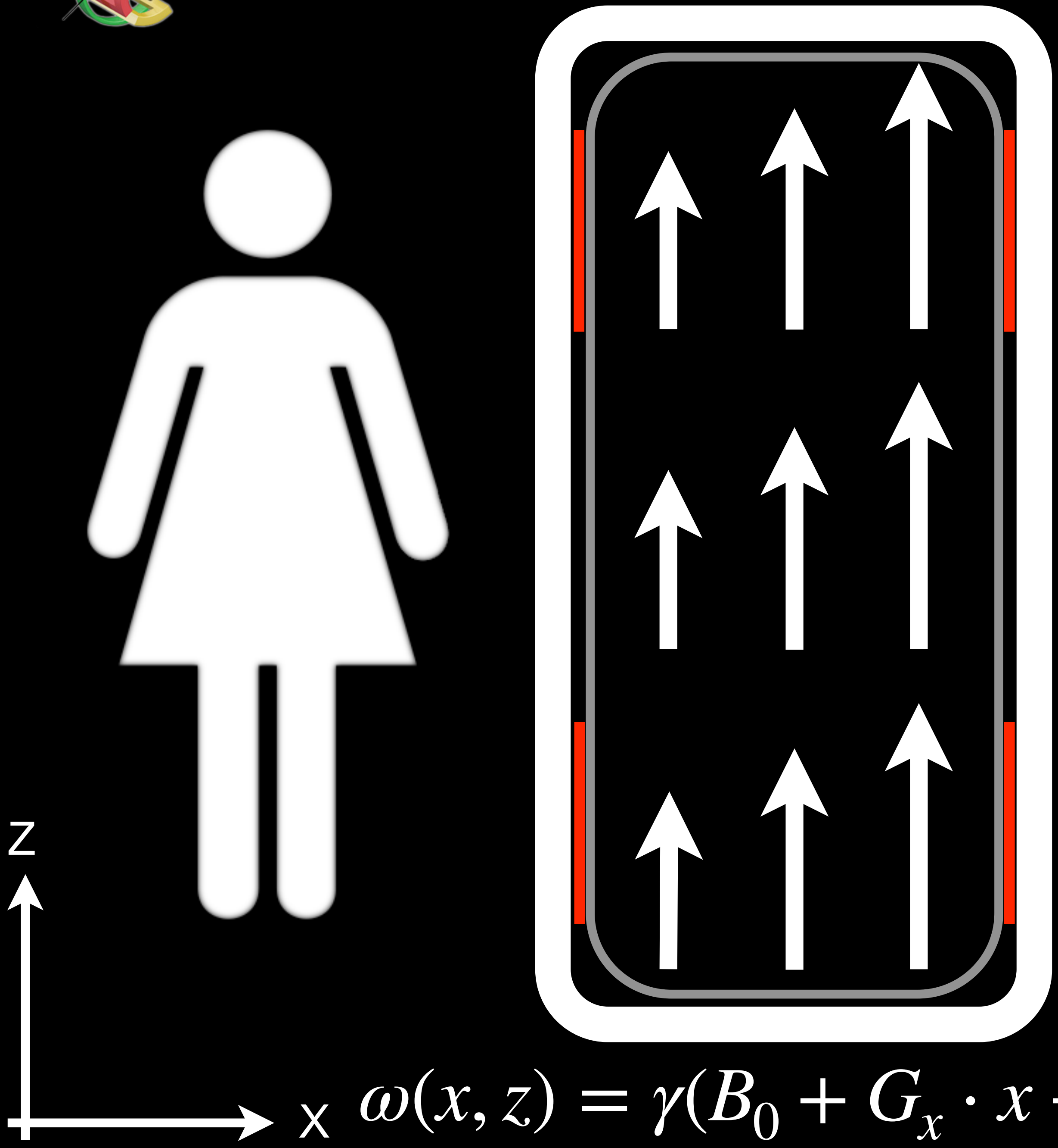
$$B_0$$

$$B_0 - \delta B_0$$

$$\omega(x, z) = \gamma(B_0 + G_z \cdot z)$$

What coordinate frame are we in?

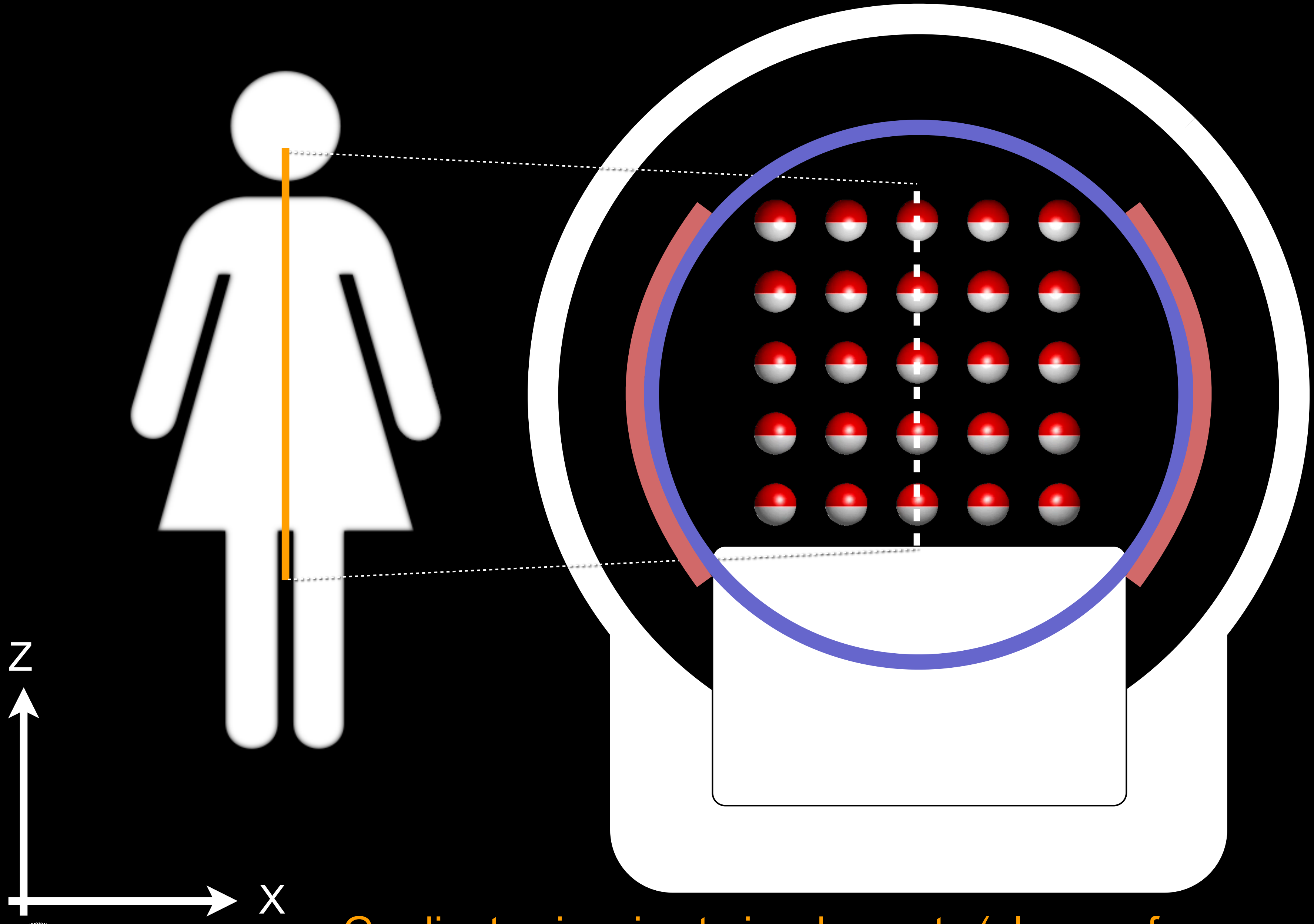
X-Gradients



$$\omega(x, z) = \gamma(B_0 + G_x \cdot x + G_z \cdot z)$$

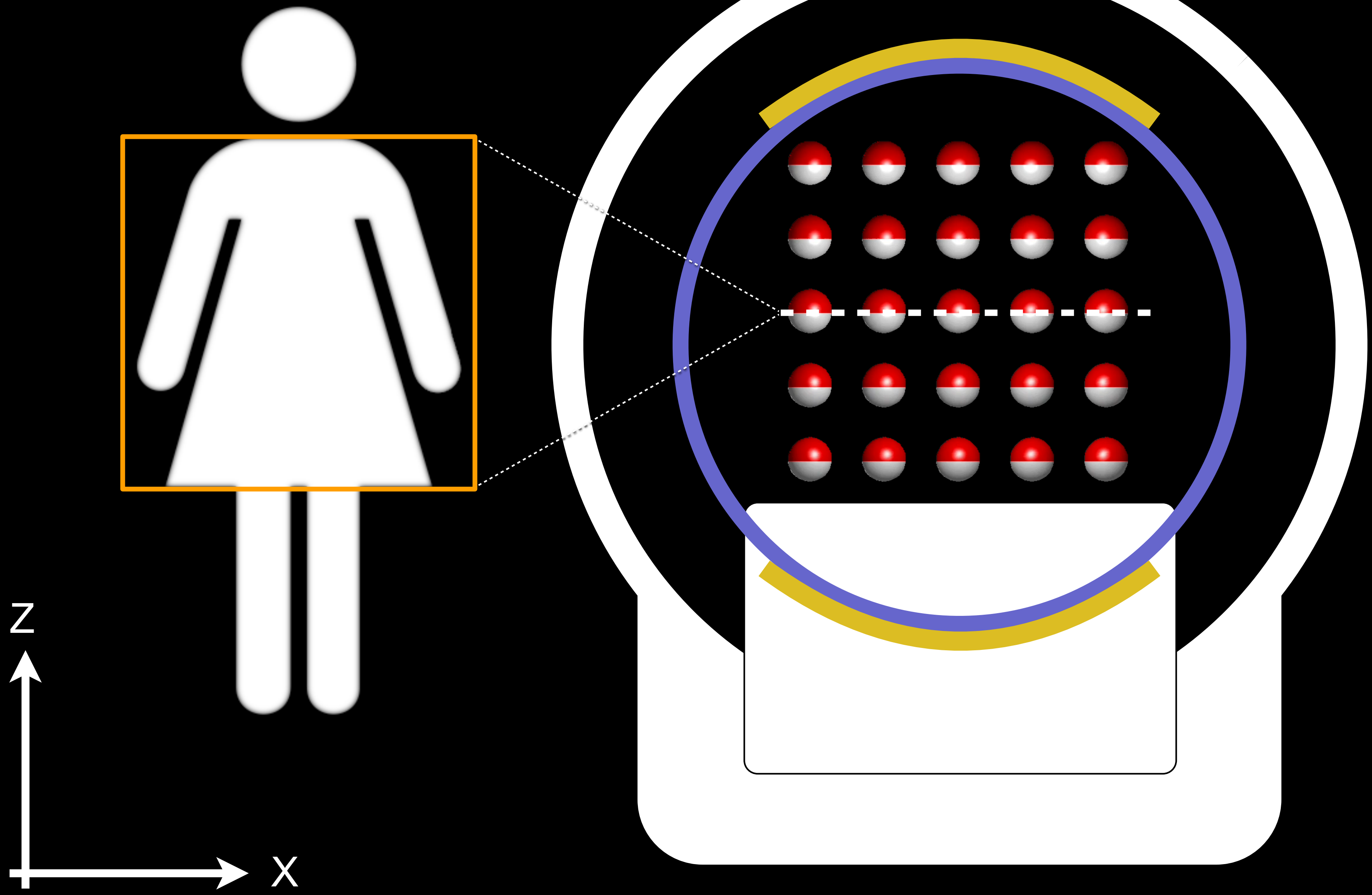
$B_0 - \delta B_0$ B_0 $B_0 + \delta B_0$

Spins and X-Gradients



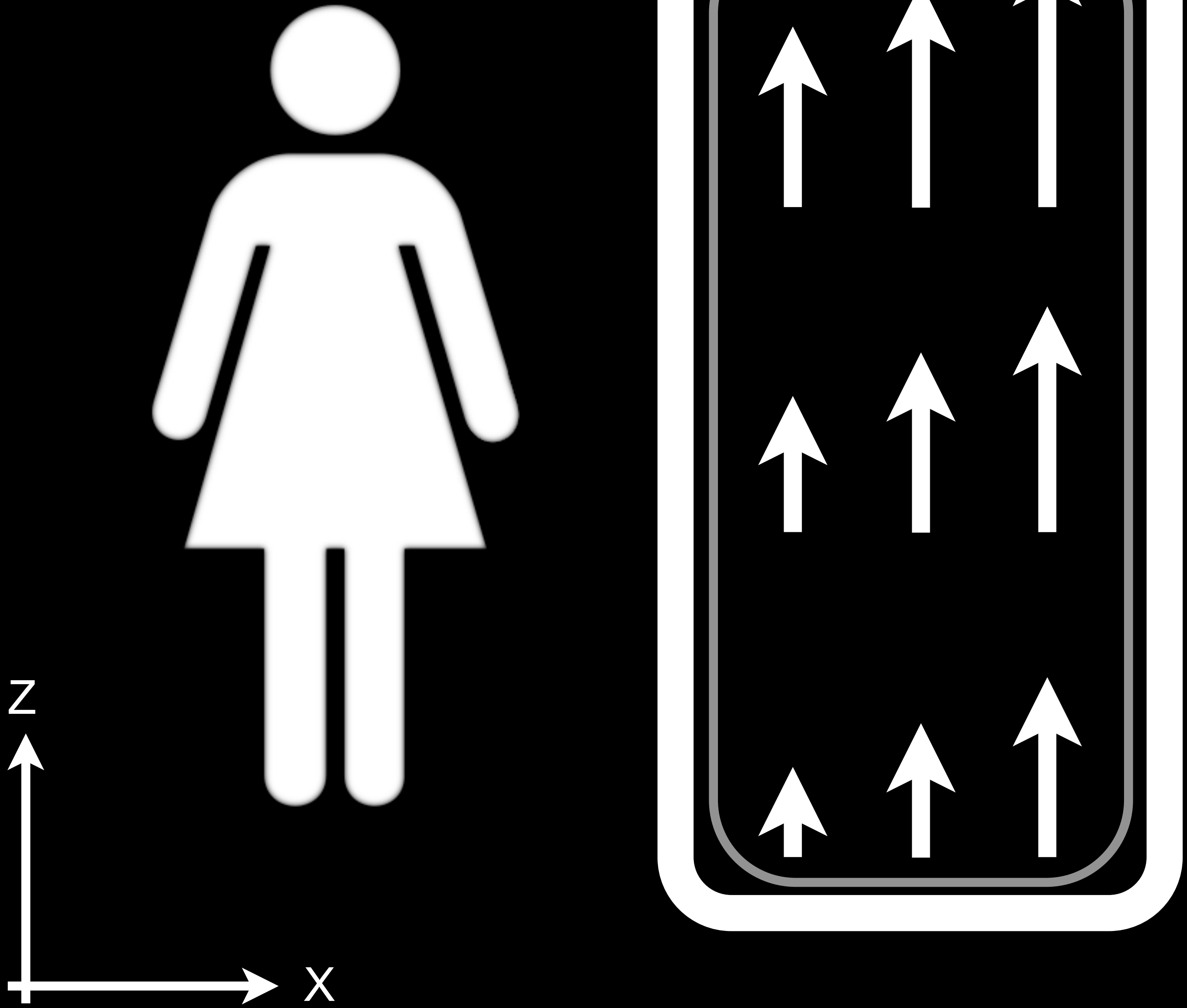
Gradients give rise to isochromats (planes of common frequency).

Spins and Y-Gradients

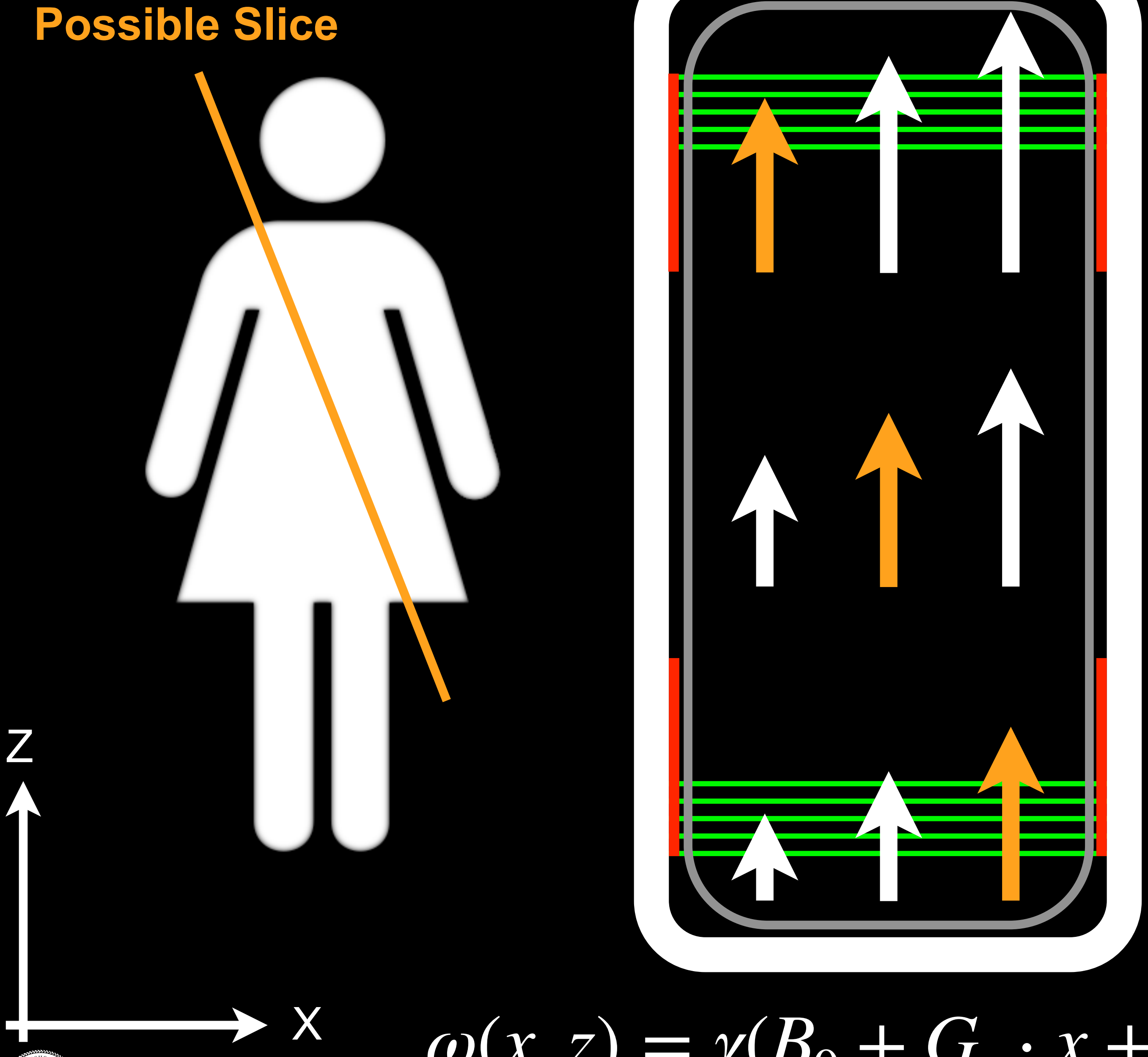


Gradients add/subtract to B_0 along a specific direction.

How do we do this?



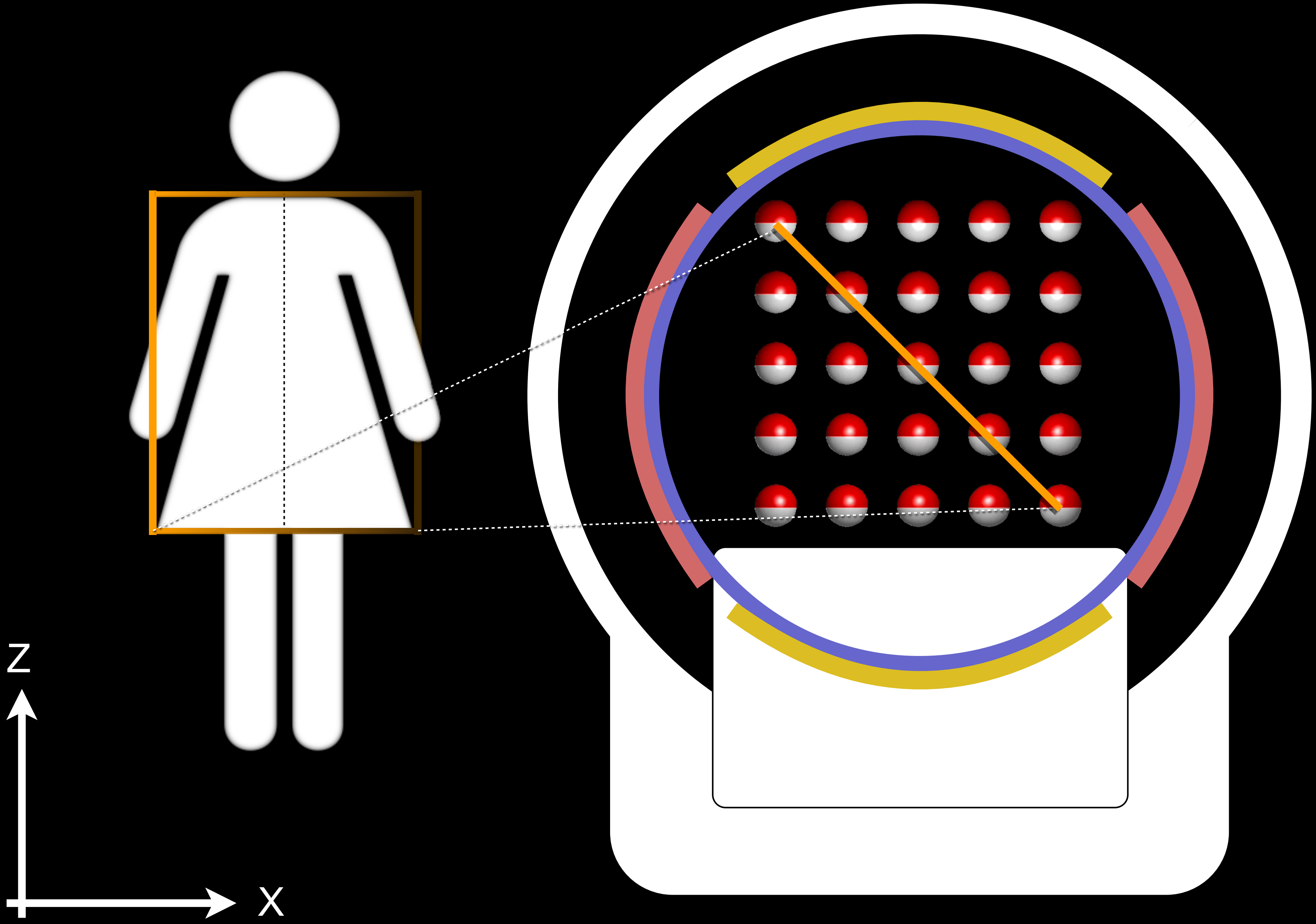
X+Z-Gradients



Spin Isochromat

$$\omega(x,z) = \gamma(B_0 + G_x \cdot x + G_z \cdot z)$$

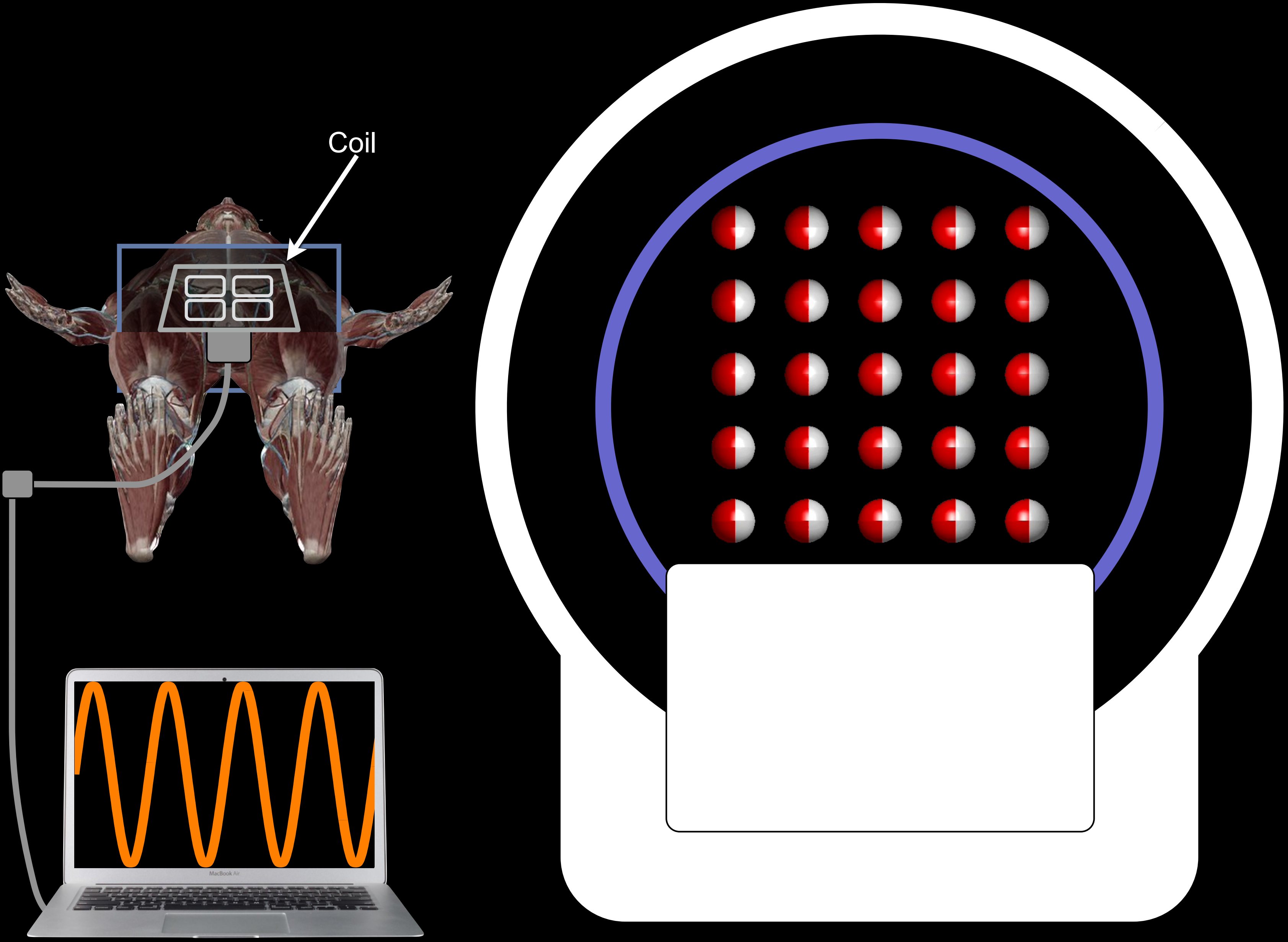
Spins and X- & Y-Gradients



Simultaneous gradients create an arbitrary isochromat plane.

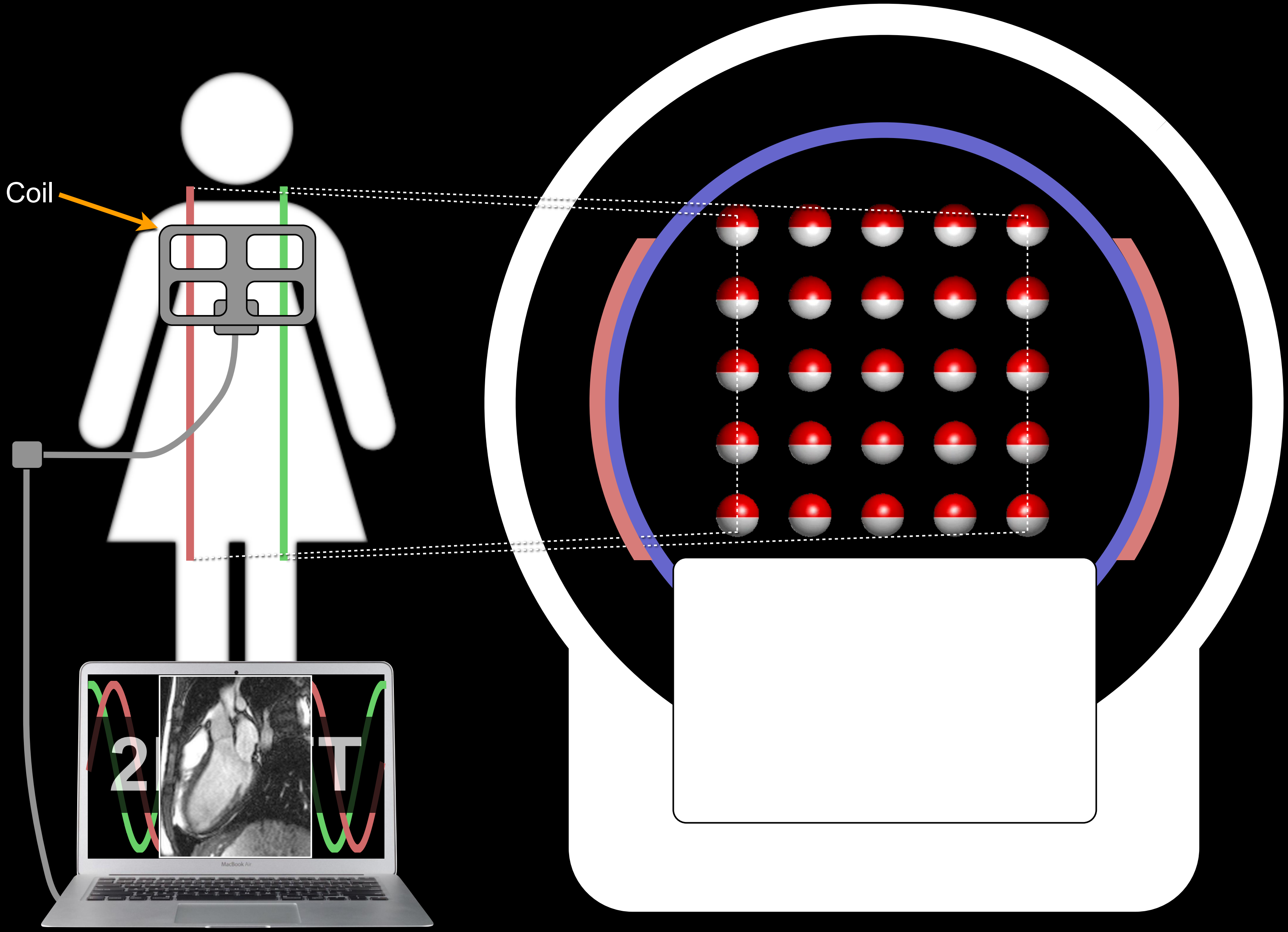
How do we measure M_{xy}?

Faraday’s Law of Induction



Precessing spins *induce* a current in a nearby coil.

Faraday's Law of Induction



The trick is to encode spatial information and image contrast in the echo.

Basic Detection Principles

Magnetic Flux Through The Coil – *Reciprocity*

$$\Phi\left(t\right) = \int_{object} \vec{B}_r\left(\vec{r}\right) \cdot \vec{M}\left(\vec{r}, t\right) d\vec{r}$$

Eqn. 5.38

Magnetic
Flux

Coil
Sensitivity

Bulk
Magnetization

What happens if the coil has poor sensitivity?

What happens if the coil's sensitivity is perpendicular to the bulk magnetization? How would that happen?

Basic Detection Principles

We get here

$$S(t) = \int_{\text{object}} M_{xy}(\mathbf{r}, 0) e^{-i\gamma \Delta B(\mathbf{r}) t} d\mathbf{r}$$

From Here

$$V(t) = -\frac{\partial \Phi(t)}{\partial t} = -\frac{\partial}{\partial t} \int_{object} \vec{B}(\vec{r}) \cdot \vec{M}(\vec{r}, t) \, d\vec{r}$$

with 25 pages of Math!

Basic Detection Principles

$$S(t) = \int_{\text{object}} M_{xy}(\boldsymbol{r},0)e^{-i\gamma\Delta B(\boldsymbol{r})t}d\boldsymbol{r}$$

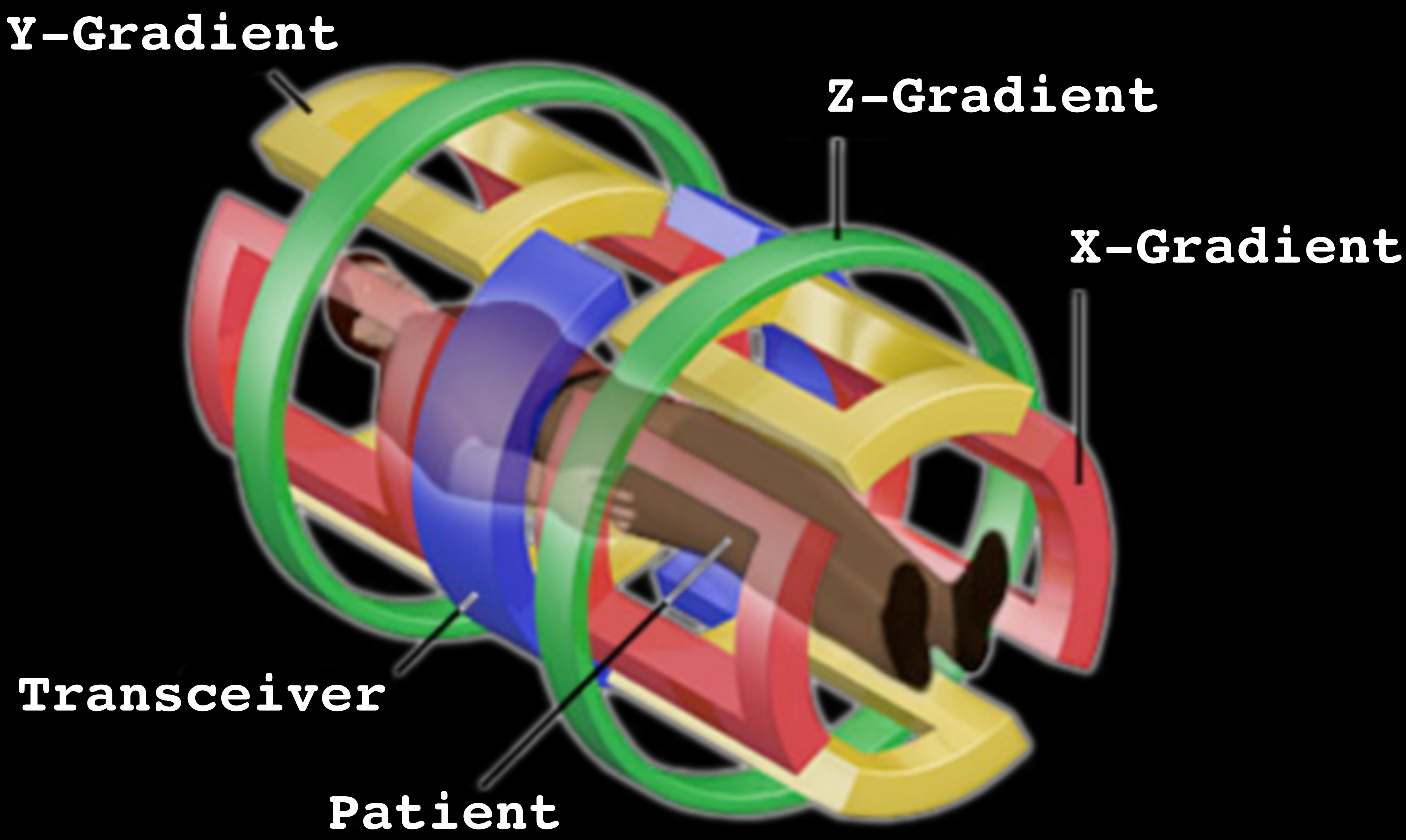
Observations

Detected signal is the vector sum of all transverse magnetizations in the “rotating frame” within the imaging volume.

The Larmor frequency precession (Lab frame rotation) is necessary for detection, although only the baseband signal matters for imaging

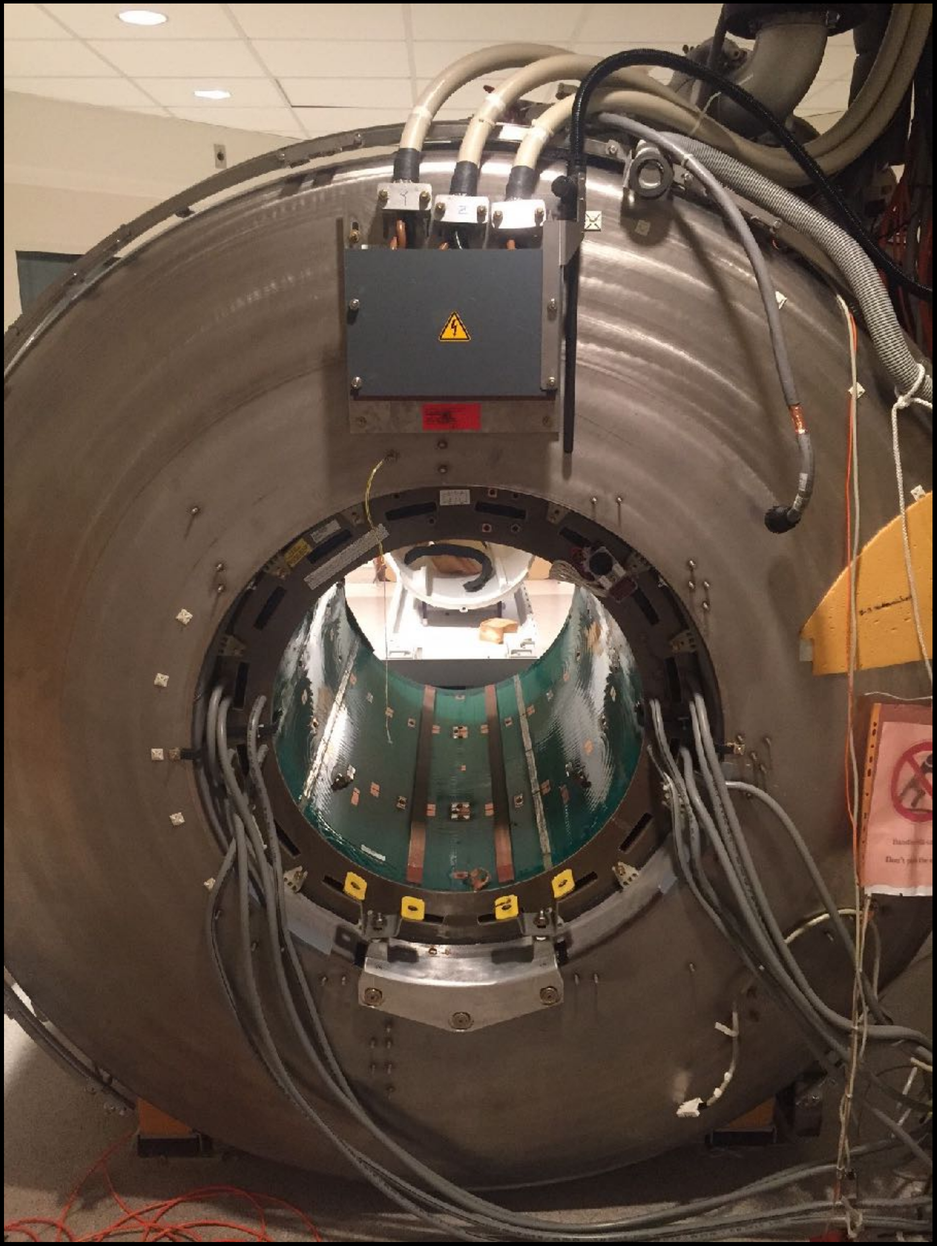
To the Board

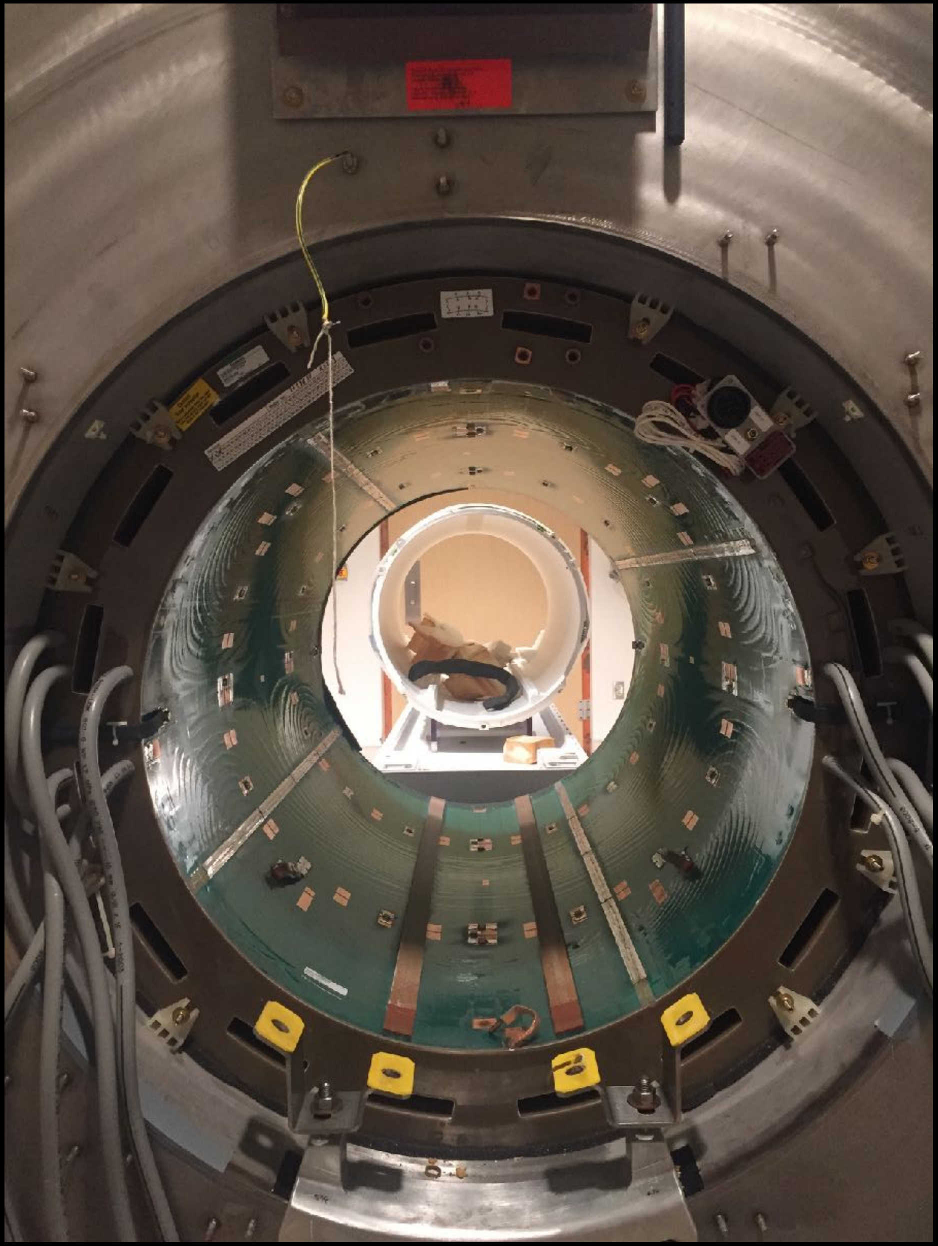
Gradient Hardware



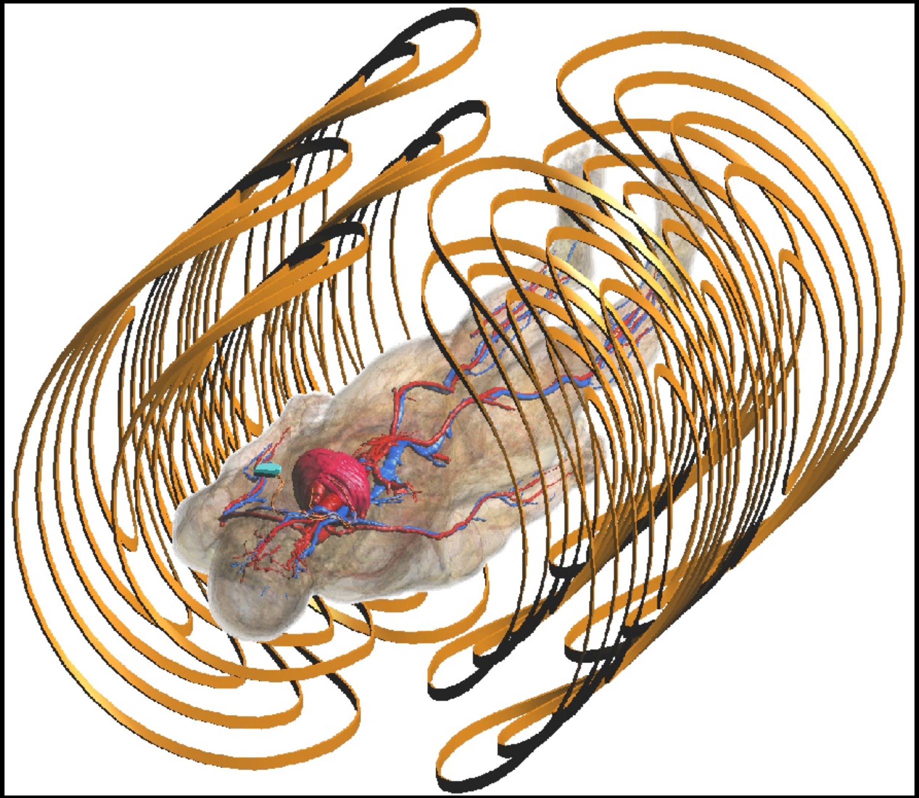
http://www.magnet.fsu.edu

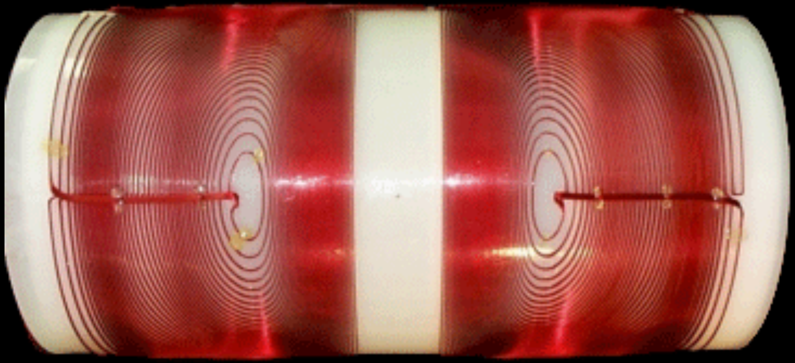
Gradient Hardware



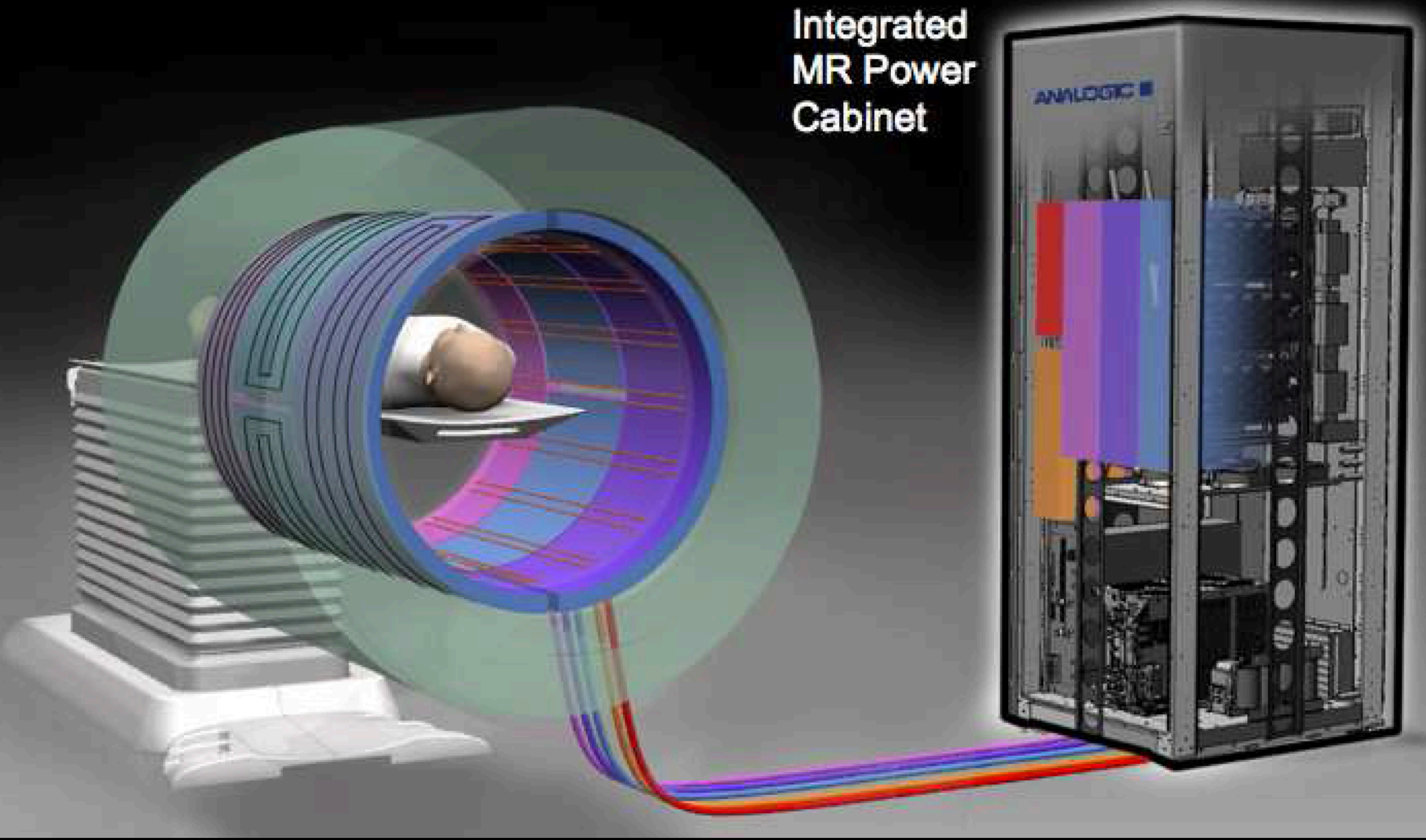


Gradient Hardware





Gradient Hardware



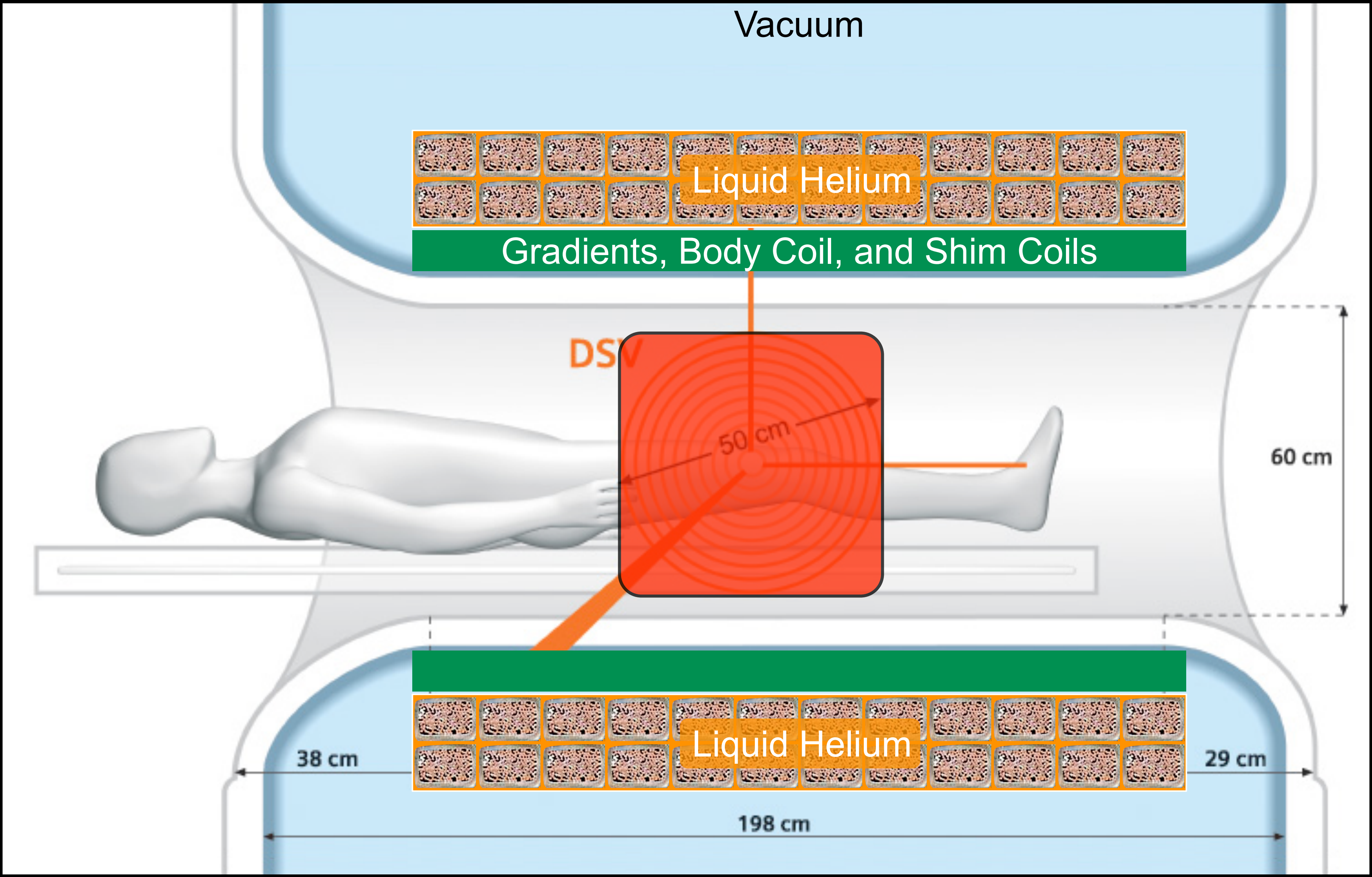
Gradients

- Primary function
  - Encode spatial information
    - Slice selection
    - Phase encoding
    - Frequency encoding
- Secondary functions
  - Sensitize/de-sensitize images to motion
  - Minimize artifacts (crushers & spoilers)
  - Magnetization re-phasing in slice selection
  - Magnetization de-phasing during readout

Gradients

- Gradients are a:
  - Small
    - <5G/cm (<0.0075T @ edge of 30cm FOV)
  - Spatially varying
    - Linear gradients
    - Adds to B_0 only in Z-direction
  - Time varying
    - Slewrate Max. ~150-200mT/m/ms
  - Magnetic field
    - Adds/Subtracts to the B_0 field
  - Parallel to B_0
- Gradients are NOT:
  - Fields perpendicular to B_0

Gradients



Gradients are “linear” over ~40-50cm on each axis.

B-Field Assumptions in MRI

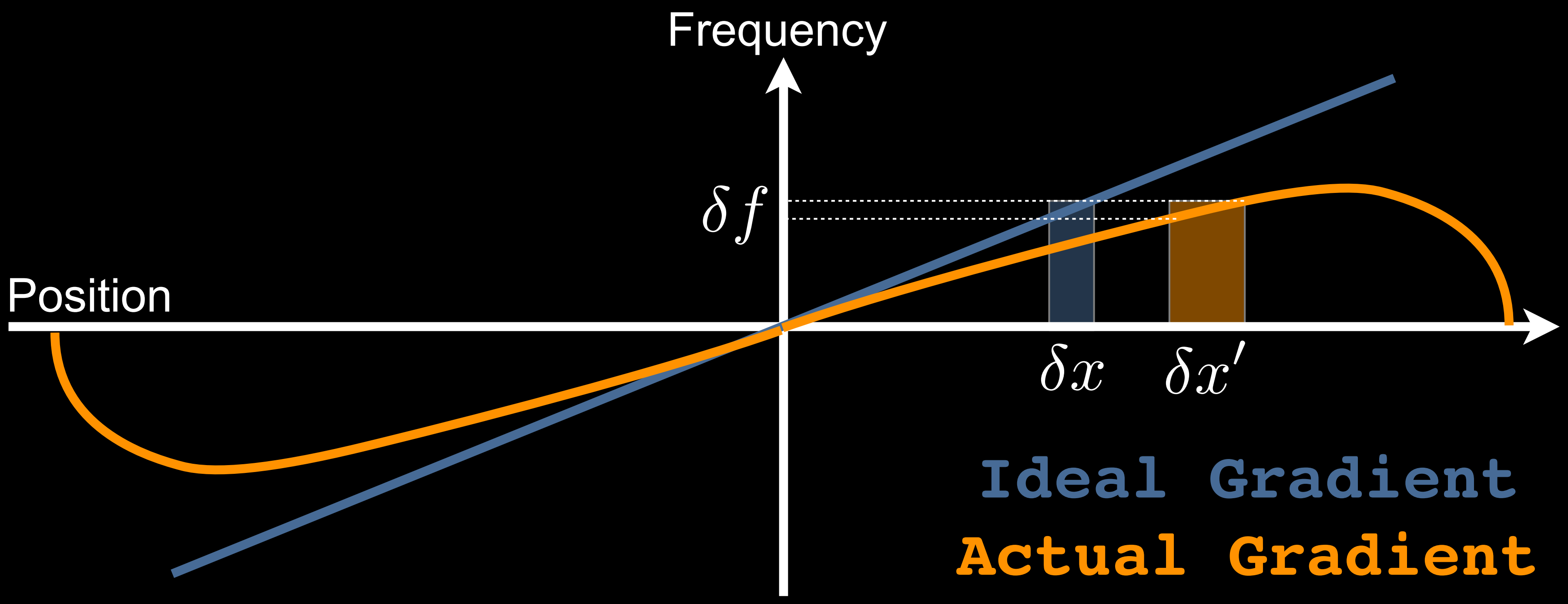
- **B_0-field is:**
  - Perfectly uniform over space.
    - “B_0 homogeneity”
  - Perfectly stable with time.
- **B_1-field is:**
  - Perfectly uniform over space.
    - “B_1 homogeneity”
  - Temporally modulated exactly as specified.
- **Gradient Fields are:**
  - Perfectly linear over space.
    - “Gradient linearity”
  - Temporally modulated exactly as specified

Imperfections of Gradient Fields

- Gradient coils aren't perfect
  - Non-linearity
  - Eddy Currents
  - Maxwell terms (Concomitant fields)
  - But they are small
    - *Much* smaller than B_0
    - We will ignore them...but they exist...

Gradient Non-linearity

Gradient Non-linearity

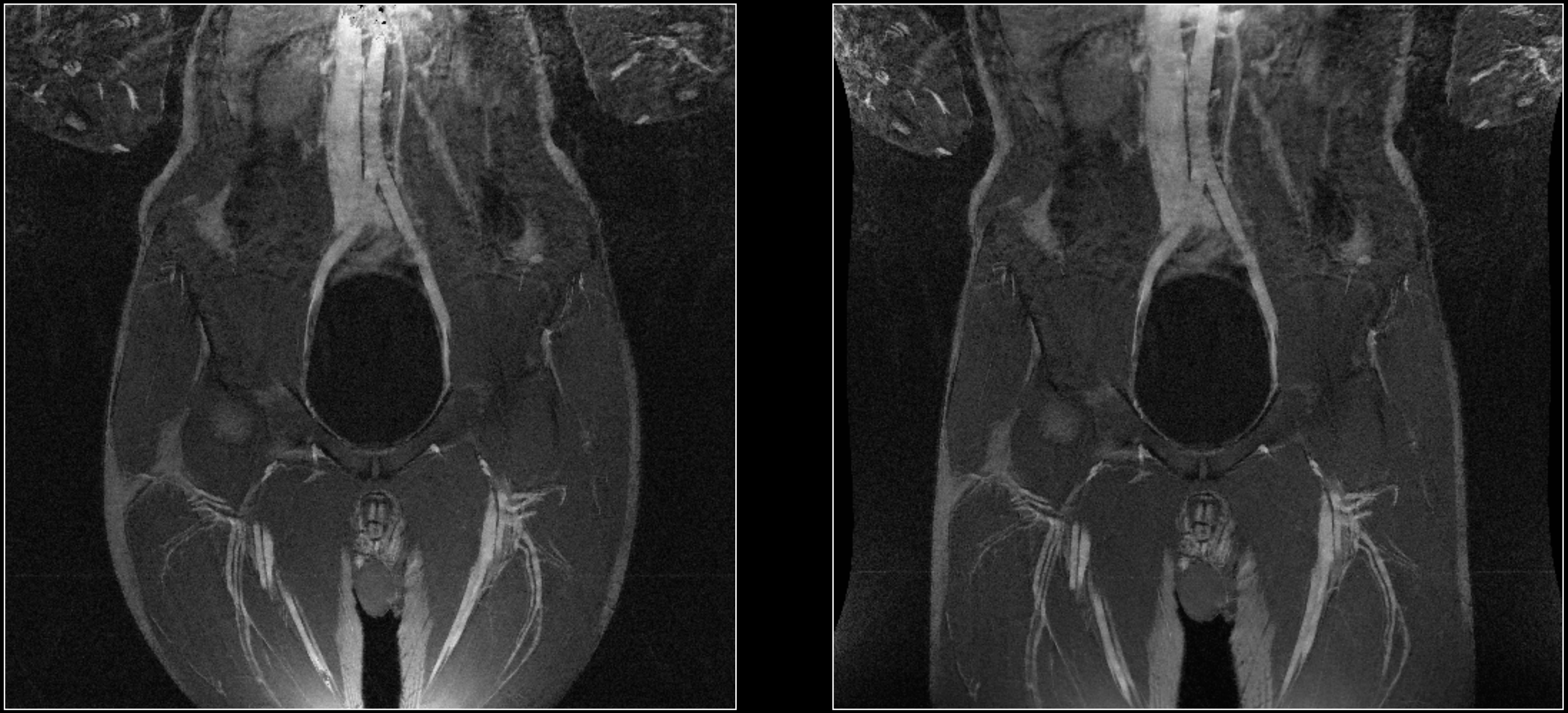


Ideally spatial position is linearly related to frequency.

Gradient Non-linearity

- **Basic *assumption* in MRI is that the z-component of the B-field created by the gradient coils varies linearly with x, y, or z over the FOV.**
- **Higher gradient amplitudes and slewrates can be achieved by compromising on spatial linearity.**
- **Gradient non-linearity causes geometric and intensity distortions.**

Gradient Non-linearity



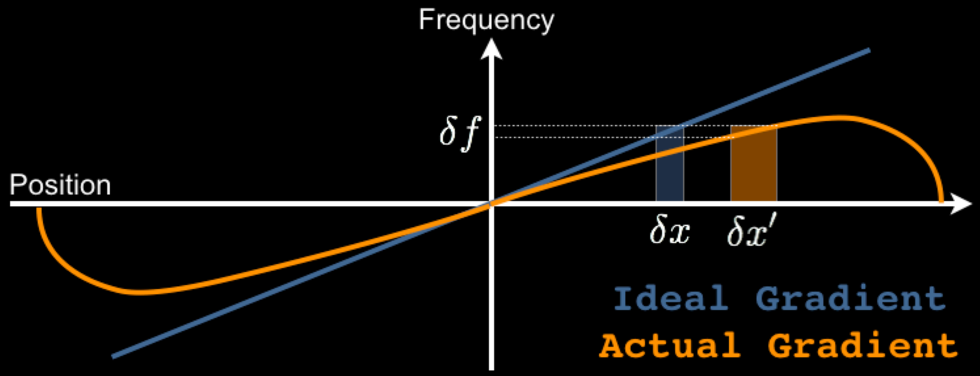


Image Courtesy of M.T. Alley & B.A. Hargeaves

Solution

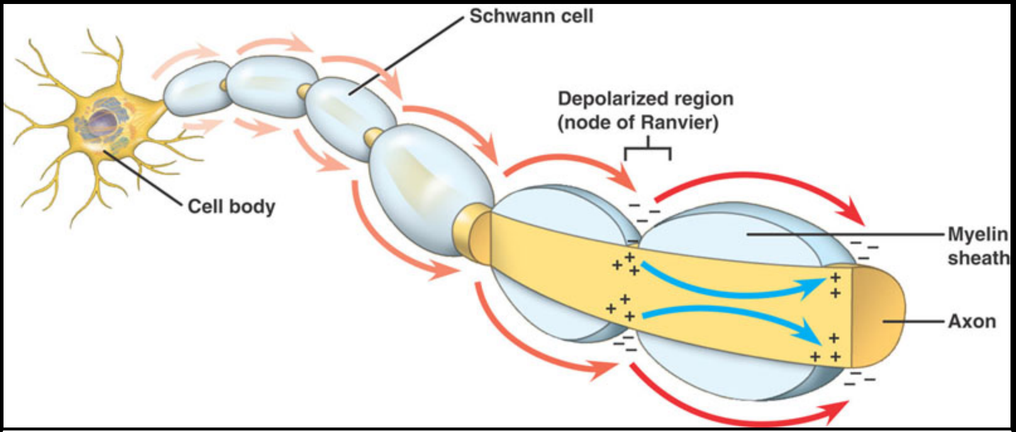
- Improve hardware and linearity!
- Pay attention to FOV!
- Image warping parameters that are system specific and applied to all images.
  - Works well qualitatively.
  - Can be problematic quantitatively.

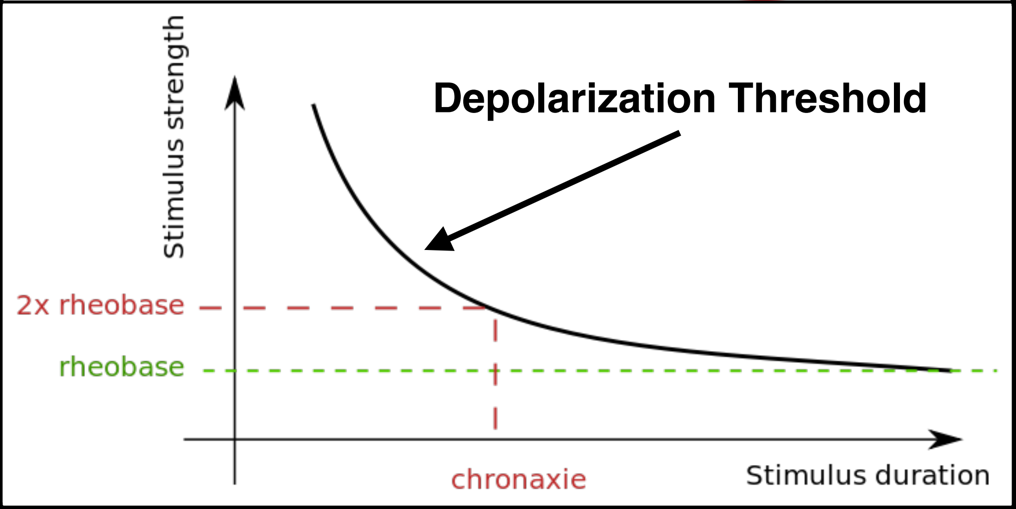
Gradient Safety

Gradient Safety

- **Noise**
- **Peripheral nerve stimulation (PNS)**

Solution: Ear plugs Head phones





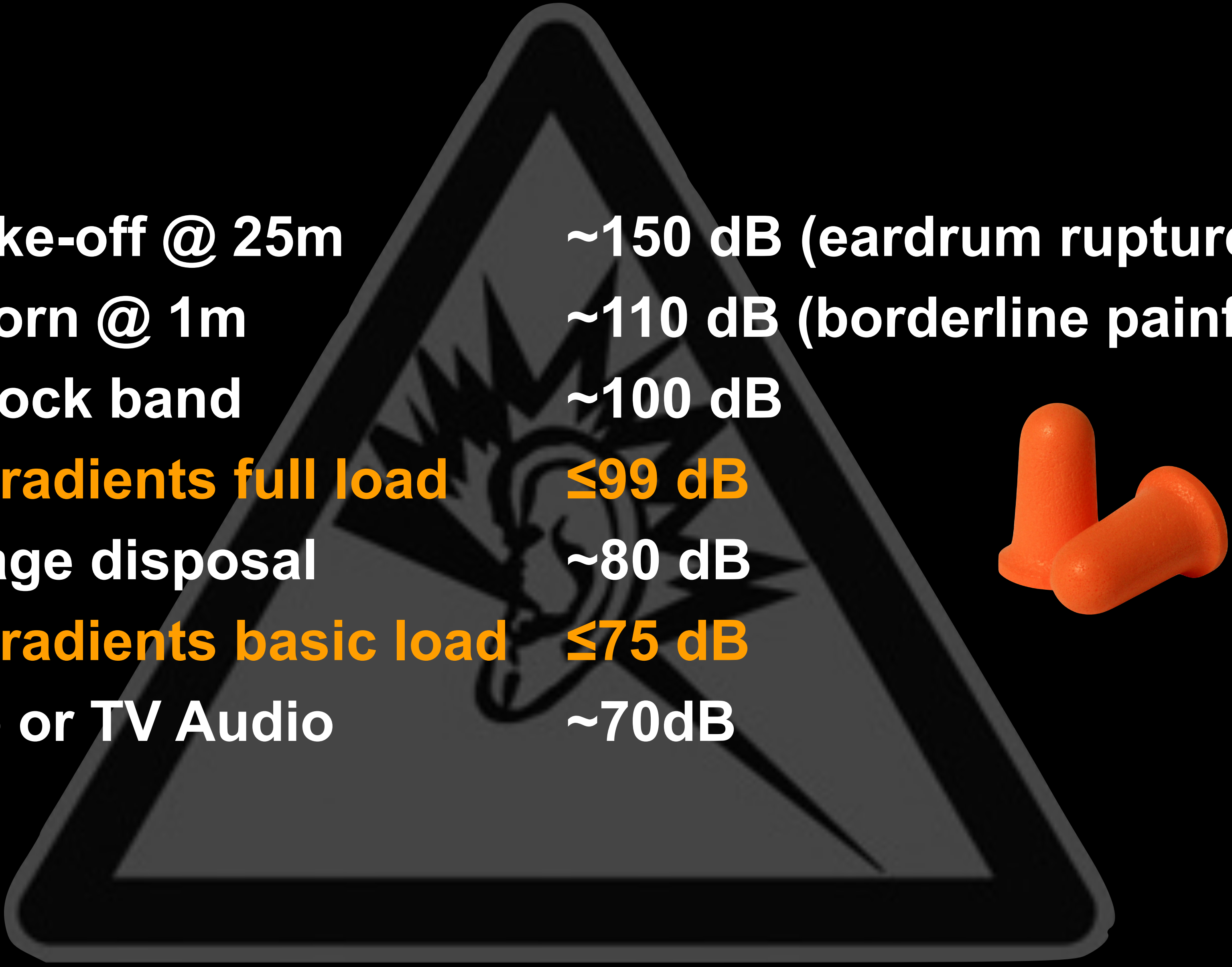
Solution: De-rate gradient slew rates, but this increases scan time.

Time-varying gradients induce mechanical vibrations and PNS.

MRI Gradient Noise

Switching the gradients on ms time scales (kHz) generates acoustic noise.

Gradient Noise

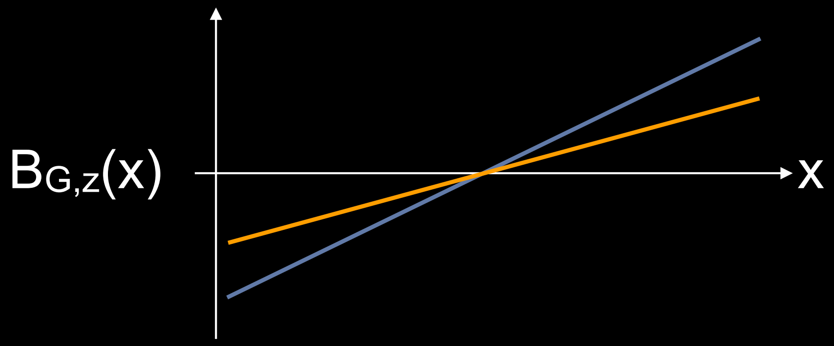


Jet take-off @ 25m	~150 dB (eardrum rupture)
Car horn @ 1m	~110 dB (borderline painful)
Live rock band	~100 dB
MRI gradients full load	**≤99 dB**
Garbage disposal	~80 dB
MRI gradients basic load	**≤75 dB**
Radio or TV Audio	~70dB

Siemens: mri-magnetom-essenza-environmental_product_declaration-00079271.pdf

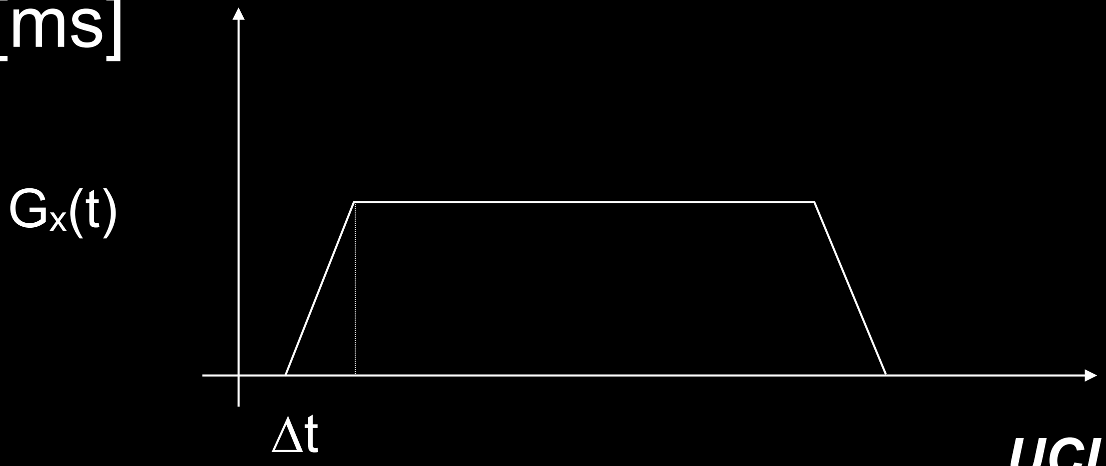
Gradient Safety – G_{Max}

- **G_{max} limitations:**
  - Concern: None known.
    - B_0 is already pretty big.
  - Conventional Gradients
    - G_{Max} = 4 to 5G/cm (=50mT/m)
  - Cutting Edge Gradients
    - G_{Max} = 8G/cm (=80mT/m)
  - Connectome Gradients
    - G_{Max} = 30G/cm (=300mT/m)
  - Consider the ΔB contributed by a gradient...



Gradient Slewrate

- **Gradient slew rate**
  - T/m/s (or G/cm/s)
  - dG/dt – Rate of change of gradient amplitude
- **Slew rate limited by dB/dt:**
  - Concern: Peripheral Nerve Stimulation
  - Regulated by FDA
  - Normal Mode: dB/dt=16 T/s•(1+0.36/ß)
  - First Level Mode: dB/dt=20 T/s•(1+0.36/ß)
  - ß=stimulus duration [ms]



Questions?

- Related reading materials
  - Nishimura - Chap 5

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