

(1)

* Faraday's Law of Induction
electromotive force (E)

$$\mathcal{E} = - \frac{d\Phi}{dt}$$

$$d\mathcal{E} = - \frac{dM(\vec{r}, t)}{dt} \cdot dV \quad (\text{see Eq. 5.38})$$

$$\int_V d\mathcal{E} dV = S_r(t) = -k \int_V \frac{d}{dt} M(\vec{r}, t) dV$$

$$= -k \int_V M(\vec{r}, 0) \left[-\hat{n}(\omega_0 + \gamma \vec{G}(t) \cdot \vec{r}) \right]$$

$$e^{-i\omega_0 t} e^{-i\gamma \int_0^t \vec{G}(\tau) \cdot \vec{r} d\tau}$$

(ignore T_2 decay, $\frac{d}{dt} e^{at} = a e^{at}$)

in general, $\omega_0 \gg \gamma \vec{G} \cdot \vec{r}$

$$S_r(t) = k \hat{n} \omega_0 \int_V M(\vec{r}, 0) e^{-i\omega_0 t} e^{-i\gamma \int_0^t \vec{G}(\tau) \cdot \vec{r} d\tau}$$

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$$S_r(t) = \bar{n} \omega_0 K \int_V M(\vec{r}, t) e^{-i\omega_0 t} e^{-i\delta \int_0^t \vec{G}(\vec{r}) \cdot \vec{r} dz} dV$$

$$\equiv \bar{n} \omega_0 K \iint_{xy} M_{xy}(\vec{r}) e^{-i\omega_0 t} e^{-i\delta \int_0^t \vec{G}(\vec{r}) \cdot \vec{r} dz} dxdy$$

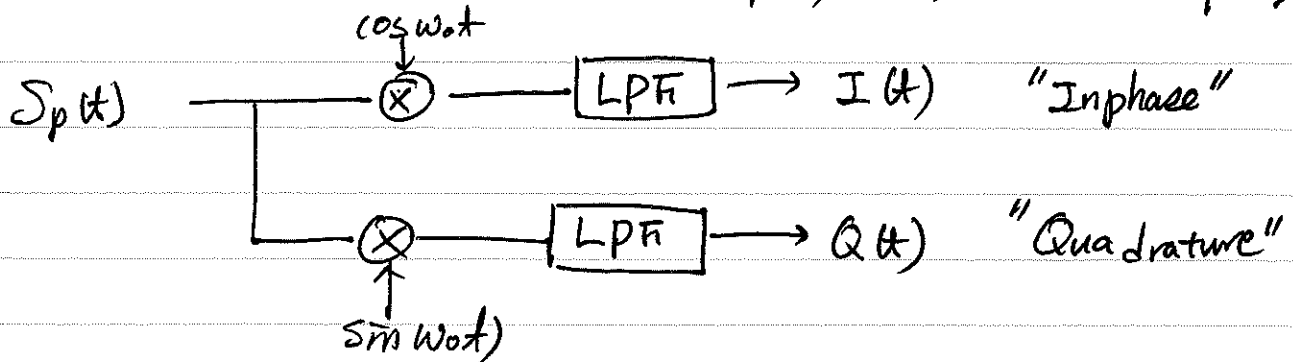
$$S_r(t) = S(t) e^{-i\omega_0 t} = \rho(t) e^{-i[\omega_0 t + \phi(t)]}$$

$$S(t) = S_r(t) e^{i\omega_0 t} = \rho(t) e^{-i\phi(t)}$$

* Single receive coil; sensitive to the rate of change of M only along one axis

$$S_p(t) = \text{Re}\{S_r(t)\} = \rho(t) \cos(\omega_0 t + \phi(t))$$

$$= \rho(t) \cos(\omega_0 t) \cos\phi(t) - \rho(t) \sin(\omega_0 t) \sin\phi(t)$$



$$I(t) = \left(\rho(t) \cos(\omega_0 t + \phi(t)) \cdot \cos \omega_0 t \right) * \text{LPF}$$

$$\implies \rho(t) \cos \phi(t)$$

$$Q(t) = -\rho(t) \sin \phi(t)$$

$$S(t) = I(t) + i Q(t) = \rho(t) e^{-i\phi(t)}$$

(2)

3 simplifications

1) 2D imaging

$$\text{def. } m(x, y) = \int_{z - \frac{\Delta z}{2}}^{z + \frac{\Delta z}{2}} m(x, y, z) dz$$

2) ignore T_2 decay

3) demodulate by ω_0

$$\text{def. } S(t) = S_r(t) \cdot e^{i\omega_0 t}$$

"baseband"

Signal Equation

$$S(t) = \iint_{x, y} \underbrace{m(x, y)}_{\text{desme}} e^{-i\gamma \int_0^t G(\tau) \cdot r d\tau} dx dy$$

$$= \iint_{x, y} m(x, y) e^{-i\gamma \left[\left(\int_0^t G_x(\tau) d\tau \right) \cdot x + \left(\int_0^t G_y(\tau) d\tau \right) \cdot y \right]} dx dy$$

$$= \iint_{x, y} m(x, y) e^{-i\gamma \pi \left[\underbrace{\left(\frac{\gamma}{\pi} \int_0^t G_x(\tau) d\tau \right)}_{\cong K_x(t)} \cdot x + \underbrace{\left(\frac{\gamma}{\pi} \int_0^t G_y(\tau) d\tau \right)}_{\cong K_y(t)} \cdot y \right]} dx dy$$

$$S(t) = \iint_{x, y} m(x, y) e^{-i\gamma \pi (K_x(t) \cdot x + K_y(t) \cdot y)} dx dy$$

2D FT of $m(x, y)$

$$= M(K_x(t), K_y(t))$$

3

- $s(t)$ equals values of M along trajectory in FT space "K space"
- $G_x G_y$ control path in k-space
- to image $m(x, y)$ acquire set samples $\{s(t)\}$ to cover k-space "sufficiently"