Table of Contents


```
% Filename: M219_Lec01_Bz_Uniformity.m
%
% Demonstrate the axial uniformity of the B-field for a solenoid.
%
% DBE@UCLA 2014.12.12
```

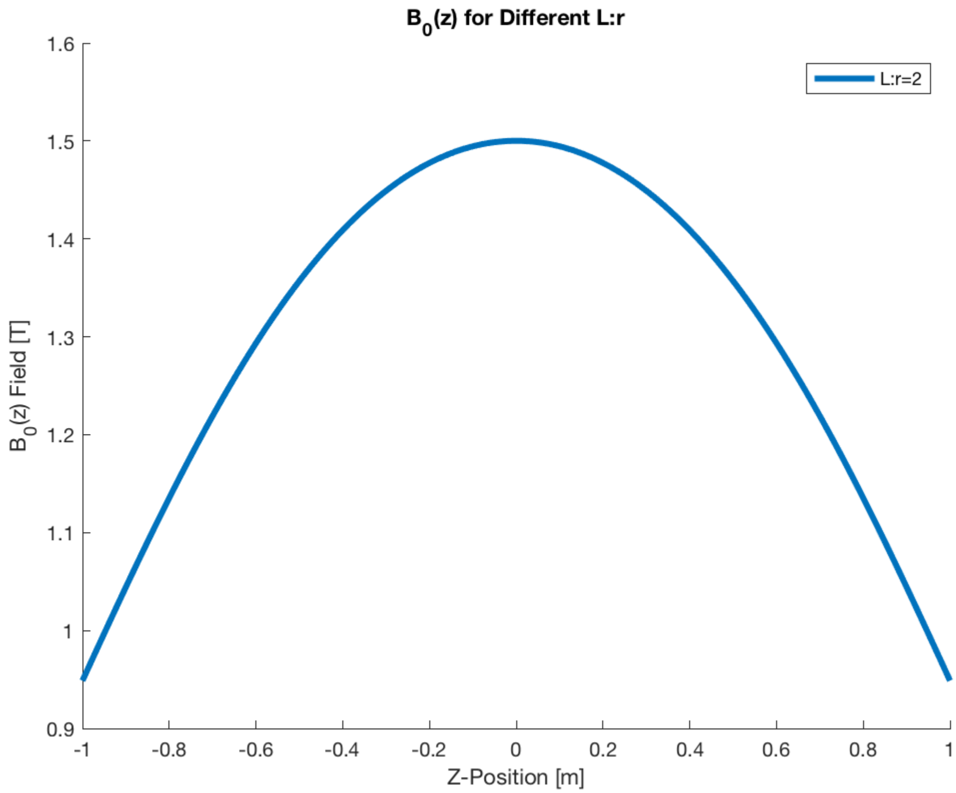
Define some constants

```
mu=4*pi*1e-7;    % Air [T.m.A-1]
I=675.3;        % Current [amps]
L=2;             % Length [meters]
N=2500;          % Number of windings [#]
r=1;             % Radius [m]
```

Problem 1A -- Calculate Bz(z) and plot the results

```
figure; hold on;
for ind=1:numel(L)                                       % Loop over
 designs
  z=linspace(0,L(ind),1000);                             % Z-distance to
 span
  alpha1=atan2(r,z);                                     % Calculate
 alpha1
  alpha2=pi-atan2(r,L(ind)-z);                           % Calculate
 alpha2
  Bz=(mu*I*N/2) * (cos(alpha1)-cos(alpha2));             % Calculate Bz
  p(ind)=plot(z-L(ind)/2,Bz);                            % Plot each Bz(z)
end

set(p,'LineWidth',3);                                    % Increase plot
 thickness
xlabel('Z-Position [m]');                                % Add the x-label
ylabel('B_0(z) Field [T]');                              % Add the y-label
title('B_0(z) for Different L:r');                       % Add the title
l=legend('L:r=2','L:r=4','L:r=6','L:r=8','L:r=10');      % Add the legend
```

Warning: Ignoring extra legend entries.



Problem 1B

```
B050=find(abs((z-0.5))==min(abs(z-0.5)));              % Find the index
 at 0.5m
B150=find(abs((z-1.5))==min(abs(z-1.5)));              % Find the index
 at 1.5m
B0_max=max(Bz(B050:B150))                              % B0 Max over
 +/-50cm
B0_min=min(Bz(B050:B150))                              % B0 Min over
 +/-50cm
B0_mean=mean(Bz(B050:B150))                            % B0 Mean over
 +/-50cm
B0_PPM=1000000*(B0_max-B0_min)/B0_mean                 % B0 PPM over
 +/-50cm
Bz_vRMS=sqrt(mean((1.5-Bz(B050:B150)).^2))             % B0 vRMS in
 Tesla
```

```
B0_max =

       1.5001


B0_min =

       1.3573
```

```
B0_mean =

    1.4526


B0_PPM =

     98348


Bz_vRMS =

  0.063805
```

Published with MATLAB® R2017a

Table of Contents


```
alpha=pi/2;          % flip angle [radians]
theta=pi/4;          % RF phase [radians]
B1_max=20;            % [Gauss]
gamma_31P=1723;      % Gamma for 31P [Hz/Gauss]
B0=0.15;              % B0 [T]
```

Problem 2A

```
tau_RF=(alpha/(2*pi))/(gamma_31P*B1_max)    % [seconds]
```

```
tau_RF =

   7.2548e-06
```

Problem 2B

```
w_0=gamma_31P*10000*B0                      % [Hz]
```

```
w_0 =

     2584500
```

Problem 2C

```
w_1=gamma_31P*B1_max                        % [Hz]
```

```
w_1 =

       34460
```

Problem 2D

```
N_precession=w_0*tau_RF                     % [#]
```

```
N_nutation  =w_1*tau_RF                     % [#]

X=N_precession/N_nutation                   % Times faster...


N_precession =

        18.75


N_nutation =

         0.25


X =

    75
```

Problem 2E

```
N=1000;                                     % Number of simulation
 points
t=linspace(0,tau_RF,N);                     % Time vector [seconds]
dt=mean(diff(t));                           % Delta time [seconds]
dB1=PAM_B1_op(10000*gamma_31P,B1_max/10000,dt,theta);  % B1 rotation
 operator

% Initialize the magnetization in the laboratory frame (LF)
Mag_LF=zeros(4,N);
Mag_LF(3:4,1)=1;

% Rotate the (previous) magnetization by the B1 rotation operator
for n=2:N
  Mag_LF(:,n)=dB1*Mag_LF(:,n-1);
end

% Plot the results
figure; p=plot(t,Mag_LF(1:3,:)');
  set(p,'LineWidth',3);
  legend('M_x','M_y','M_z');
  xlabel('Time [s]');
  ylabel('Magnetization [A.U.]');
  title('Forced Precession in the Laboratory Frame');

Undefined function 'PAM_B1_op' for input arguments of type 'double'.

Error in PAM_HW01_Prob02_RF_Pulse_Solution (line 26)
dB1=PAM_B1_op(10000*gamma_31P,B1_max/10000,dt,theta);  % B1 rotation
 operator
```

Problem 2F

```
Mag_RF=zeros(4,N);
Mag_RF(3:4,1)=1;

dB0=PAM_B0_op(10000*gamma_31P,B0,dt);

% The RF phase has to be updated each step to "keep up" with B0
 precession.
% This is effectively "circular polarization."
for n=2:N
  dB1=PAM_B1_op(10000*gamma_31P,B1_max/10000,dt,theta-2*pi*w_0*dt*n)
  Mag_RF(:,n)=dB0*dB1*Mag_RF(:,n-1);
end

figure; p=plot(t,Mag_RF(1:3,:)');
  set(p,'LineWidth',3);
  legend('M_x','M_y','M_z');
  xlabel('Time [s]');
  ylabel('Magnetization [A.U.]');
  title('Forced Precession in the Rotating Frame');
```

Problem 2G

The torque exerted by B0 and the torque exerted by B1 are orthogonal to one another. Hence, they both torque the bulk magnetization, but about orthogonal axes. This is akin to, for example, a jet being propelled forward by a jet-engine, but also pushed aside by a crosswind.

Published with MATLAB® R2017a

Homework #1, Problem #3

(A) $M_z(t) = M_z^o e^{-t/T_1} + M_o(1 - e^{-t/T_1})$

After an inversion pulse $M_z^o = -M_o$ ∴

$M_z(t) = -M_o e^{-t/T_1} + M_o(1 - e^{-t/T_1})$

$= M_o(1 - 2e^{-t/T_1})$ + 1/4

Let tissue-A have T_1^A, T_2^A, and $M_z^A(t)$ (White Matter)

Let tissue-B have T_1^B, T_2^B, and $M_z^B(t)$ (Gray Matter)

Also, let $M_o^A = M_o^B = M_o$ (i.e. ρ-density is equivalent)

Contrast is (sometimes) defined as the signal difference: $C = M_z^A - M_z^B$

$$C = M_z^A(t) - M_z^B(t) = M_o(1 - 2e^{-t/T_1^A}) - M_o(1 - 2e^{-t/T_1^B})$$

$$= 2e^{-t/T_1^B} - 2e^{-t/T_1^A}$$ + 1/4

(B) To find the time (t) when the contrast is maximum take d/dt of both sides, set equal to zero, and solve for t.

+1/2 for right set-up

$$dC/dt = (-2/T_1^B)e^{-t/T_1^B} - (-2/T_1^A)e^{-t/T_1^A} = 0$$

$$e^{-t/T_1^B} = (-2/T_1^A)e^{-t/T_1^A} / (-2/T_1^B)$$

$$= (T_1^B/T_1^A)e^{-t/T_1^A}$$

$$\ln(e^{-t/T_1^B}) = \ln[(T_1^B/T_1^A)e^{-t/T_1^A}]$$

$$-t/T_1^B = \ln(T_1^B/T_1^A) - t/T_1^A$$

$$t/T_1^A - t/T_1^B = \ln(T_1^B/T_1^A)$$

$$t = \ln\left(T_1^B / T_1^A\right) / \left(1/T_1^A - 1/T_1^B\right)$$

$$t = T_1^A T_1^B \ln\left(T_1^B / T_1^A\right) / \left(T_1^B - T_1^A\right)$$ +1/2

$M_{xy}(t) = M_{xy}^0 e^{-t/T_2}$, After a saturation pulse $M_{xy}^0 = M_0$

$M_{xy}(t) = M_0 e^{-t/T_2}$

(D) $C = M_0 e^{-t/T_2^A} - M_0 e^{-t/T_2^B}$

$$= M_0 \left(e^{-t/T_2^A} - e^{-t/T_2^B}\right)$$ +1/2

(E) $dC/dt = \left(-M_0/T_2^A\right) e^{-t/T_2^A} - \left(-M_0/T_2^B\right) e^{-t/T_2^B} = 0$

$$e^{-t/T_2^A} = \left(T_2^A / T_2^B\right) e^{-t/T_2^B}$$ +1/2 for right set-up

$$\ln\left(e^{-t/T_2^A}\right) = \ln\left(T_2^A/T_2^B\right) + \ln\left(e^{-t/T_2^B}\right)$$

$$-t/T_2^A = \ln\left(T_2^A/T_2^B\right) - t/T_2^B$$

$$t/T_2^B - t/T_2^A = \ln\left(T_2^A/T_2^B\right)$$

$$t\left(1/T_2^B - 1/T_2^A\right) = \ln\left(T_2^A/T_2^B\right)$$

$$t = \ln\left(T_2^A/T_2^B\right) / \left(1/T_2^B - 1/T_2^A\right)$$

$$t = T_2^A T_2^B \ln\left(T_2^A/T_2^B\right) / \left(T_2^A - T_2^B\right)$$ +1/2

M219 2018 Homework Problem #3

Table of Contents

DBE@UCLA 01.13.2018

Define some constants

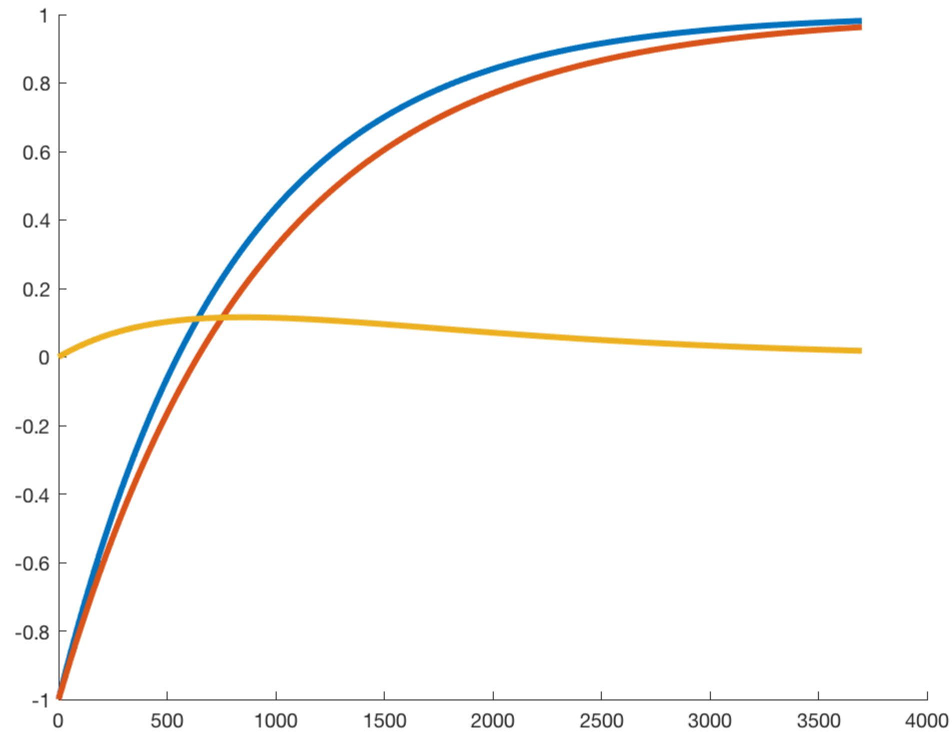
```
T1_WM=790;                      % T1 of white matter [ms]
T1_GM=925;                      % T1 of gray matter [ms]

T2_WM=92;                       % T2 of white matter [ms]
T2_GM=100;                      % T2 of gray matter [ms]
```

Calculate the logitudinal magnetization signal histories...

```
t=linspace(0,4*T1_GM,1000);     % Define a time vector [ms]
Mz_WM=1-2*exp(-t/T1_WM);
Mz_GM=1-2*exp(-t/T1_GM);
```

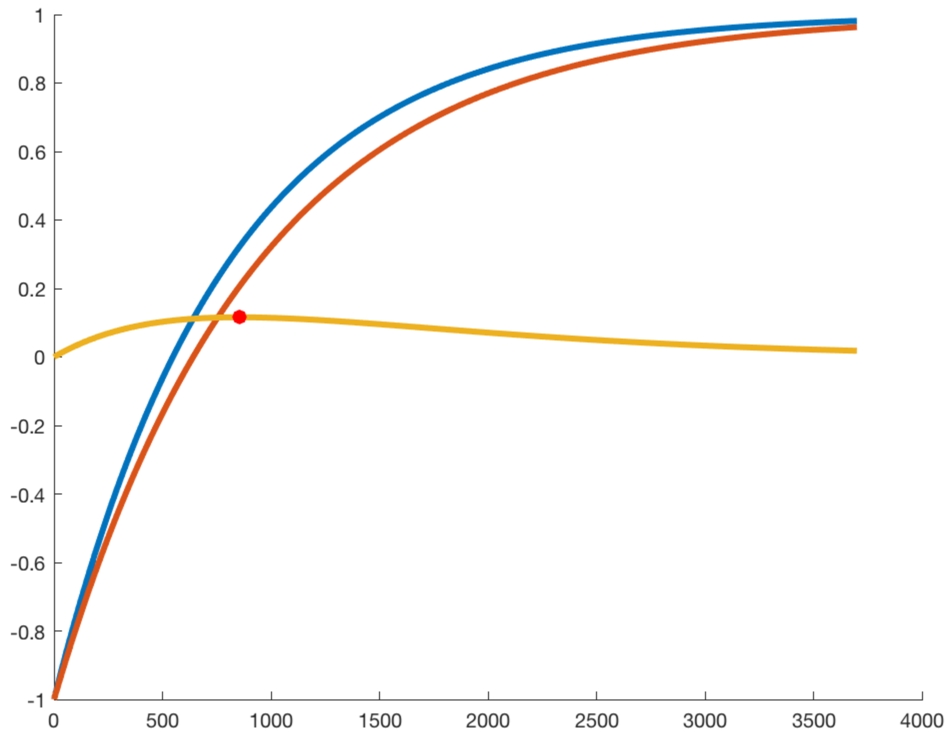
Generate the Mz figure

```
figure; hold on;
p(1)=plot(t,Mz_WM);             % Plot the Mz recovery of white matter
p(2)=plot(t,Mz_GM);             % Plot the Mz recovery of gray matter
p(3)=plot(t,Mz_WM-Mz_GM);       % Plot the signal difference
 (contrast)
  set(p(1:3),'LineWidth',3);
```



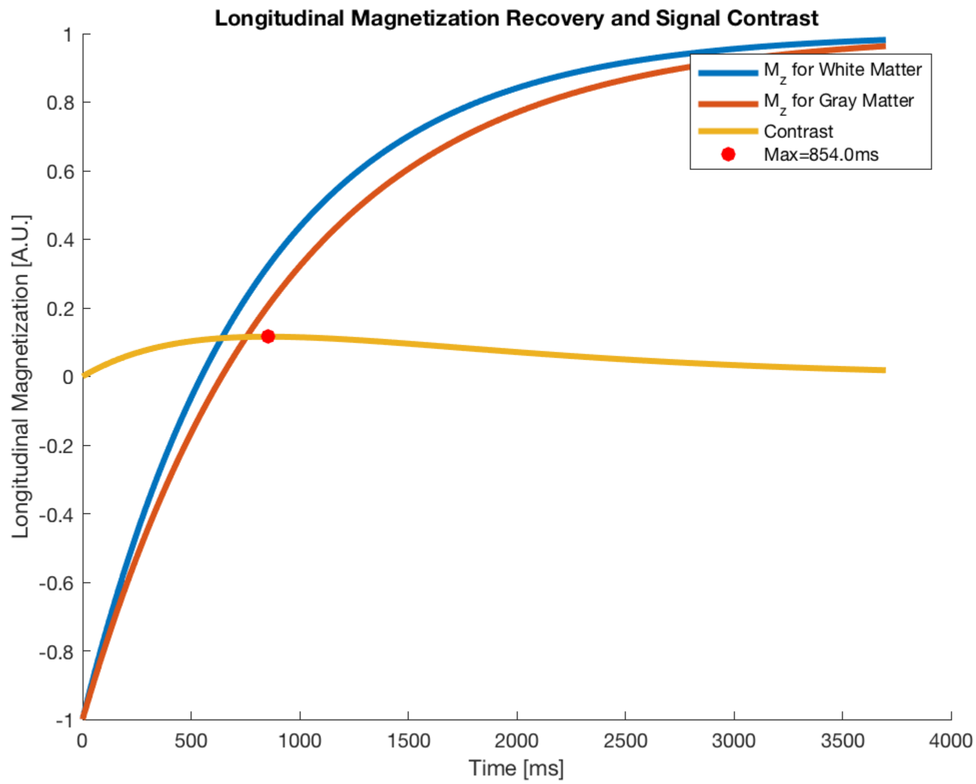
Calculate the time of MAXIMUM contrast

```
t_max=T1_WM*T1_GM*log(T1_GM/T1_WM)/(T1_GM-T1_WM); % This is the
 analytic solution
IND=find(abs(t-t_max)==min(abs(t-t_max)));        % Index of the max
 contrast

p(4)=plot(t(IND),Mz_WM(IND)-Mz_GM(IND),'r.');
  set(p(4),'Markersize',25);
```



Add labels

```
  xlabel('Time [ms]');
  ylabel('Longitudinal Magnetization [A.U.]');
  title('Longitudinal Magnetization Recovery and Signal Contrast');
  legend('M_z for White Matter','M_z for Gray Matter','Contrast',
['Max=',num2str(t_max,'%6.1f'),'ms']);
```

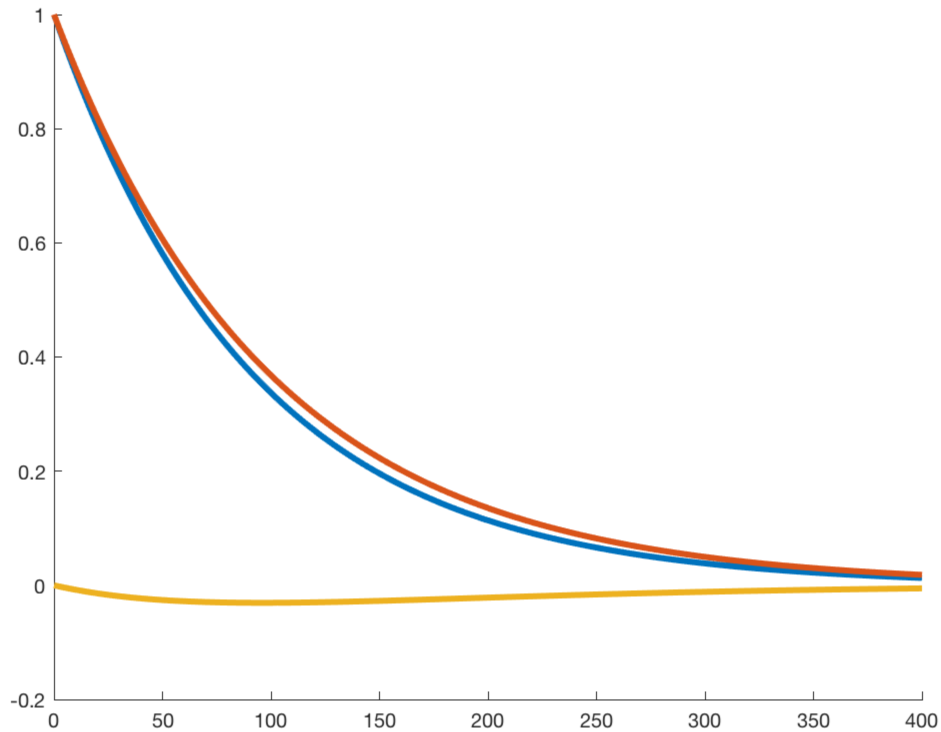


Calculate the transverse magnetization signal histories...

```
t=linspace(0,4*T2_GM,1000);      % Define a time vector [ms]
Mxy_WM=exp(-t/T2_WM);
Mxy_GM=exp(-t/T2_GM);
```

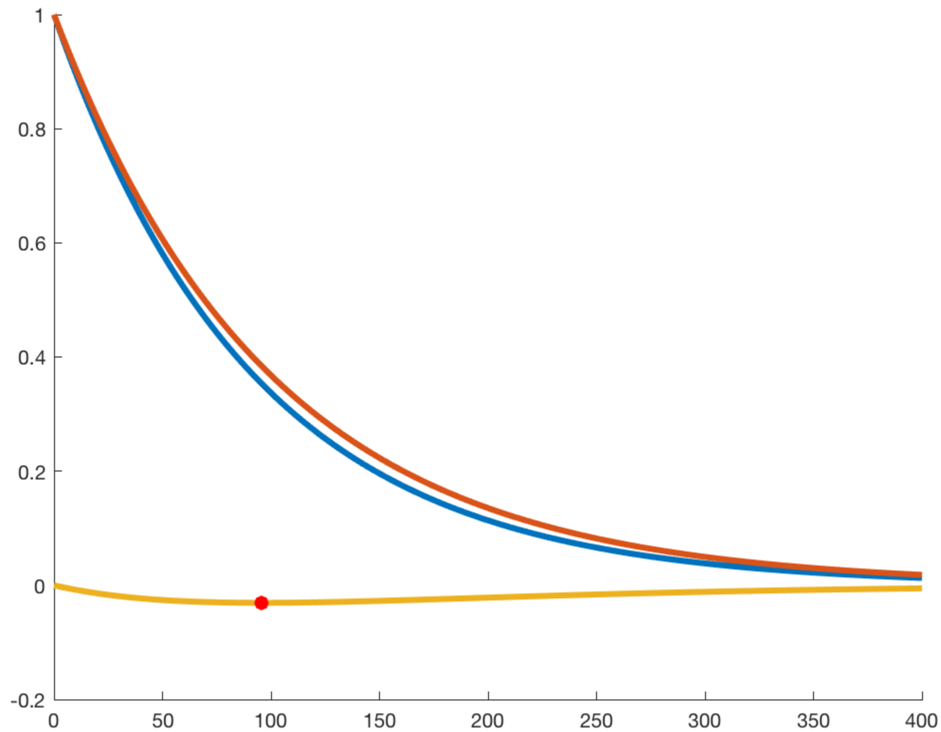
Generate the Mxy figure

```
figure; hold on;
q(1)=plot(t,Mxy_WM);              % Plot the Mxy recovery of white
 matter
q(2)=plot(t,Mxy_GM);              % Plot the Mxy recovery of gray
 matter
q(3)=plot(t,Mxy_WM-Mxy_GM);       % Plot the signal difference
 (contrast)
  set(q(1:3),'LineWidth',3);
```



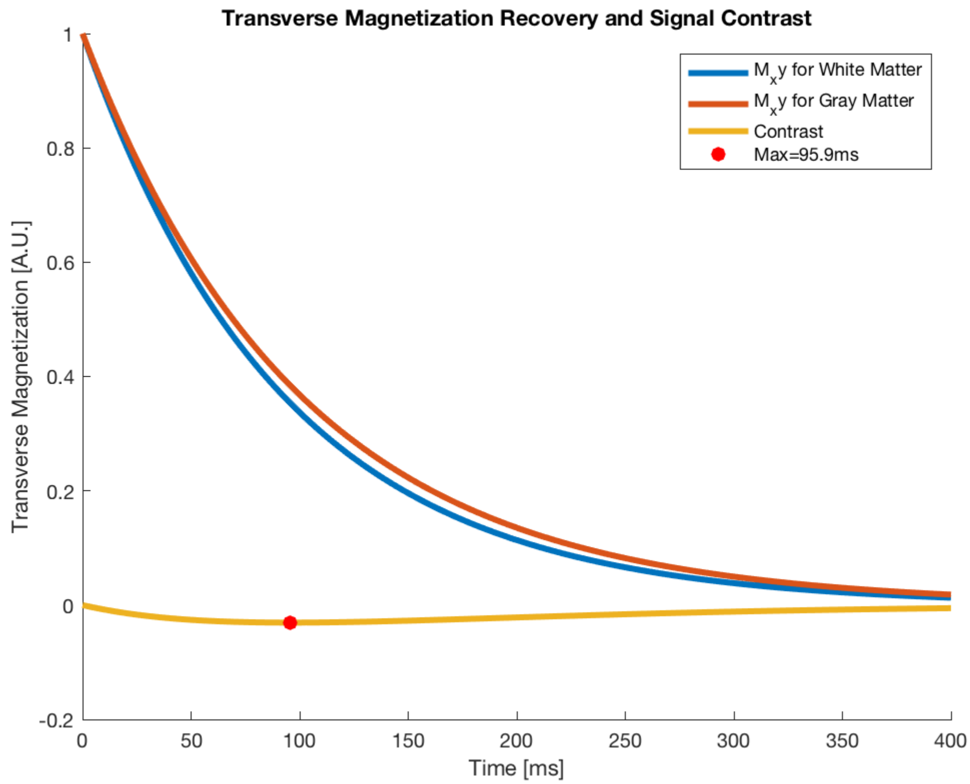
Calculate the time of MAXIMUM contrast

```
t_max=T2_WM*T2_GM*log(T2_WM/T2_GM)/(T2_WM-T2_GM); % This is the
 analytic solution
IND=find(abs(t-t_max)==min(abs(t-t_max)));       % Index of the max
 contrast

q(4)=plot(t(IND),Mxy_WM(IND)-Mxy_GM(IND),'r.');
  set(q(4),'Markersize',25);
```



Add labels

```
xlabel('Time [ms]');
ylabel('Transverse Magnetization [A.U.]');
title('Transverse Magnetization Recovery and Signal Contrast');
legend('M_xy for White Matter','M_xy for Gray Matter','Contrast',
['Max=',num2str(t_max,'%6.1f'),'ms']);
```



Published with MATLAB® R2017a