Spatial Localization - II

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Course Business

- **HW #1 14.6±3**
- **HW #2 12.7±1.8**
- **HW #3 Due Thursday (2/22) @ 10pm**
- **Lab #1 <u>Wednesday</u> (2/21) 6-9pm**
- **Lab #2 <u>Wednesday</u> (3/7) 6-9pm**
- **6:00-7:30pm Lab Groups**
  - Avanto - Michael Lauria, Chang Gao, and Brad Stiehl
  - Prisma - Tyler Cork, Yeun Kim, and James Zhang
  - Skyra - Caffi Meyer, Caroline Colbert, and Yeun Kim
- **7:30-9:00pm Lab Groups**
  - Avanto - Ruiming Cao, Pei Han, and Zhehao Hu
  - Prisma - Yubin Cai, David Lee, Joseph Park, Xinheng Zhang
  - Skyra - Aidan Pearigan, Peter Pellionisz, Yudi Sang

Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.

Slice Selective Excitation

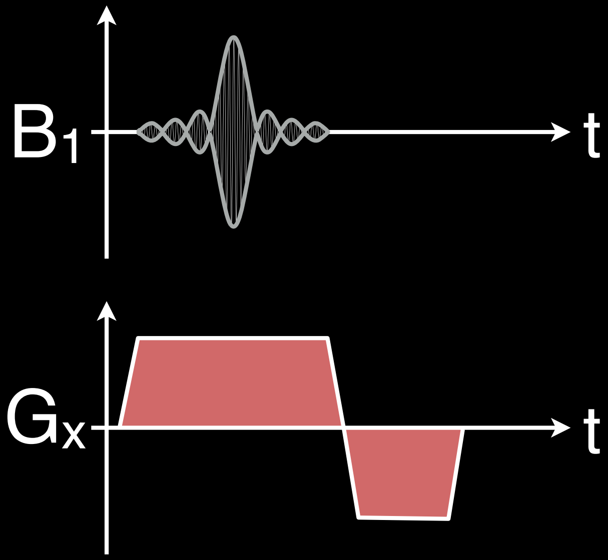
- **What factors control slice selection?**

$B_1^e(t)$ **Pulse envelope function**

– $B_{1,\max}$, $\Delta\omega$ (bandwidth), flip angle

ω_{RF} **Excitation carrier frequency**

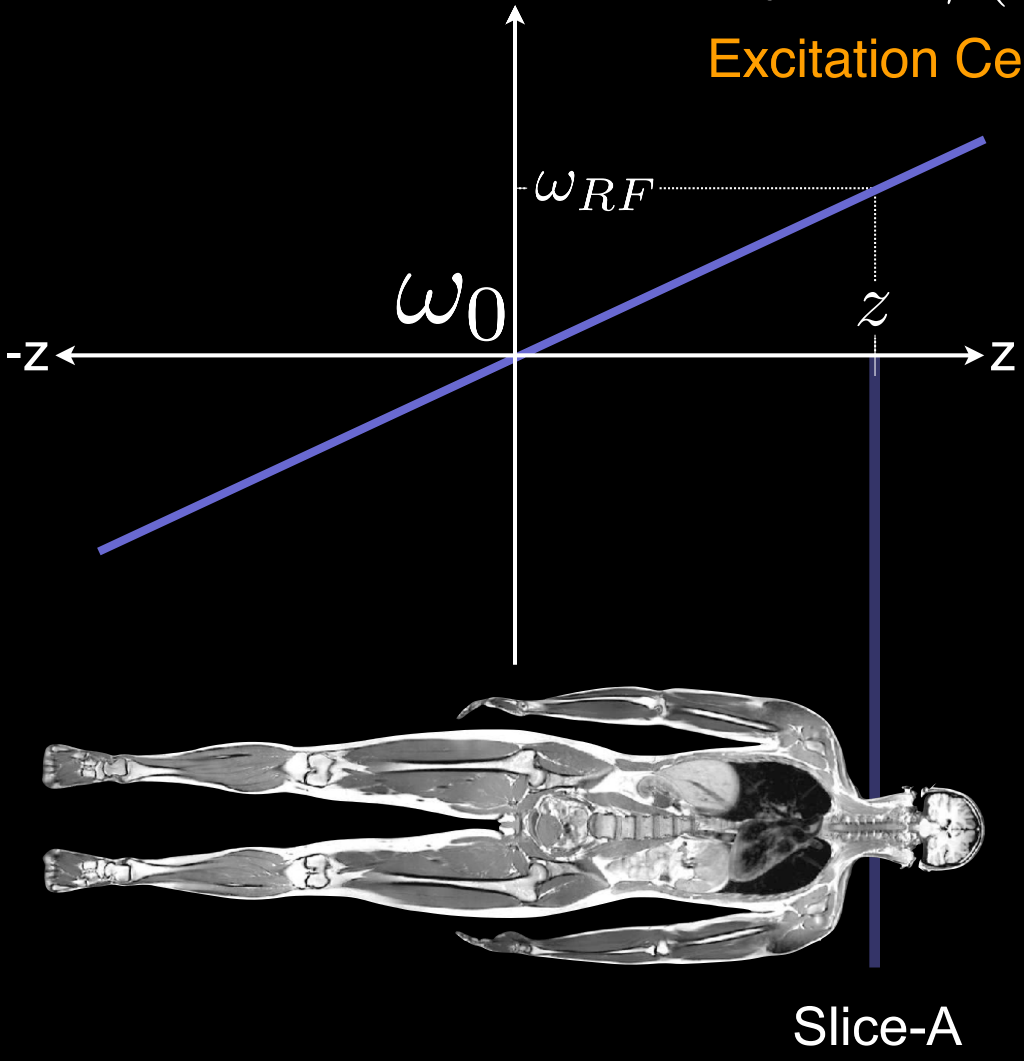
$\vec{G}$ **Gradient amplitude**



How to pick ω_{RF}?

$$\omega_{RF} = \gamma\left(B_0 + G_z \cdot z\right)$$

Excitation Center Frequency

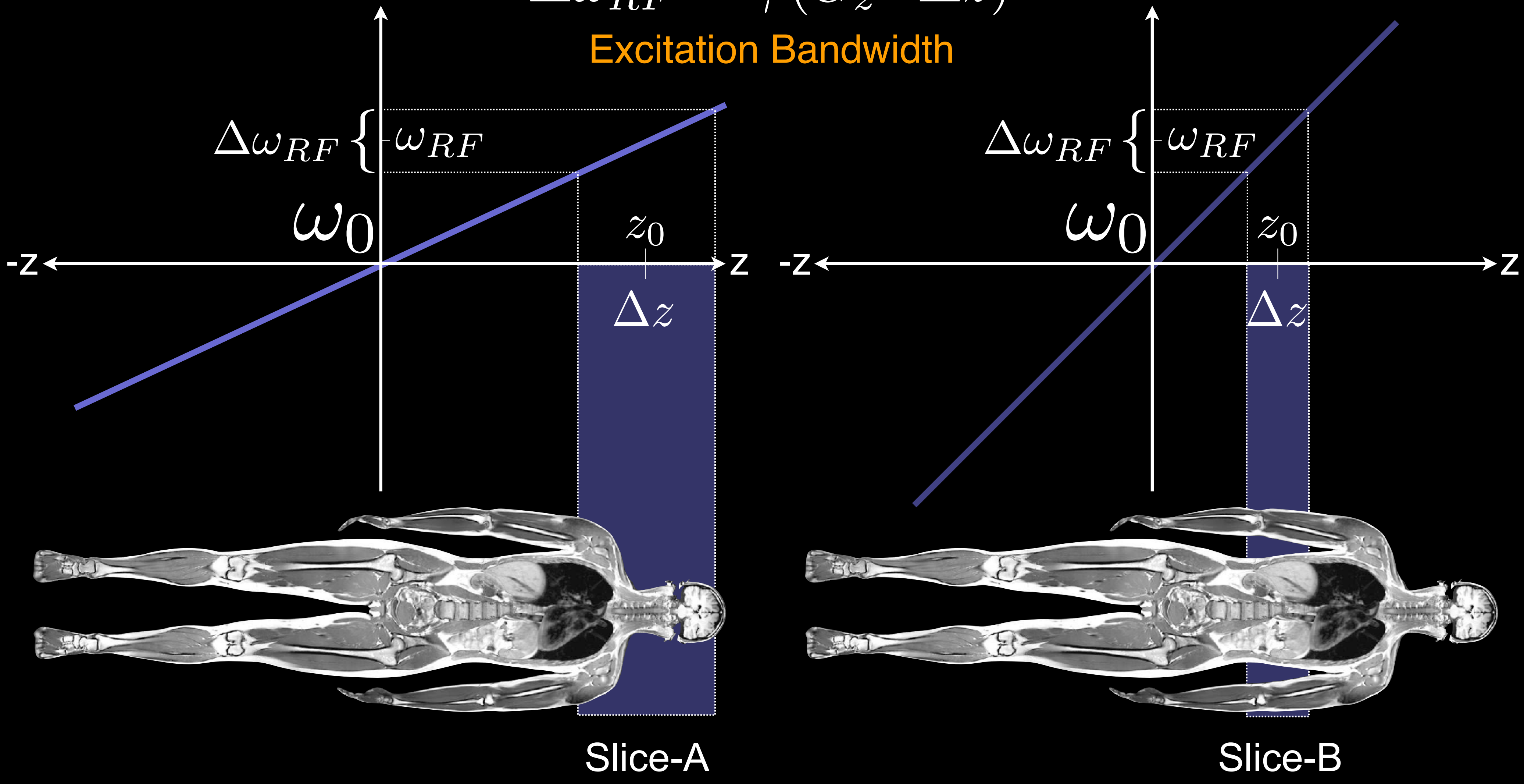


This frequency excites a slice at position *z* when G_z is turned on.

How to pick $\Delta\omega_{RF}$?

$$\Delta\omega_{RF} = \gamma\left(G_z \cdot \Delta z\right)$$

Excitation Bandwidth



How do you move the slice along ±z?

Compare $\Delta\omega$ and ω_{RF} for Slice-A and Slice-B.

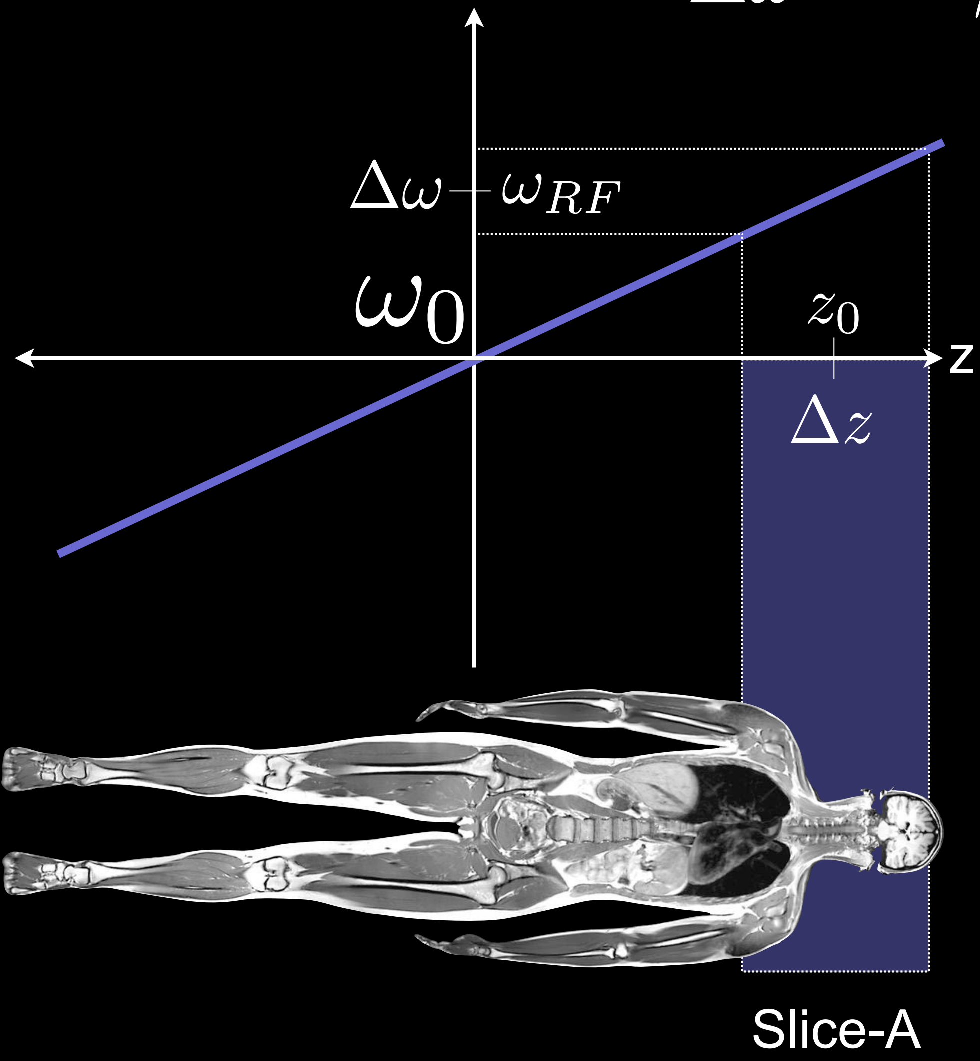
Do we usually acquire $\omega_{RF} > \omega_0$?

Time Bandwidth Product (TBW)

- **Time bandwidth (TBW) product:**
  - **Pulse Duration [s] x Pulse Bandwidth [Hz]**
  - **Unitless**
  - **# of zero crossings**
  - **High TBW**
    - Large # of zero crossings ∴ fewer truncation artifacts
    - Longer duration pulse
- **Examples:**
  - **TBW = 4, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?
  - **TBW = 16, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?

Slice Selective Excitation - Example

$$\Delta\omega = -\gamma\left(G_z \cdot \Delta z\right)$$ Excitation (BW_{RF}) Bandwidth



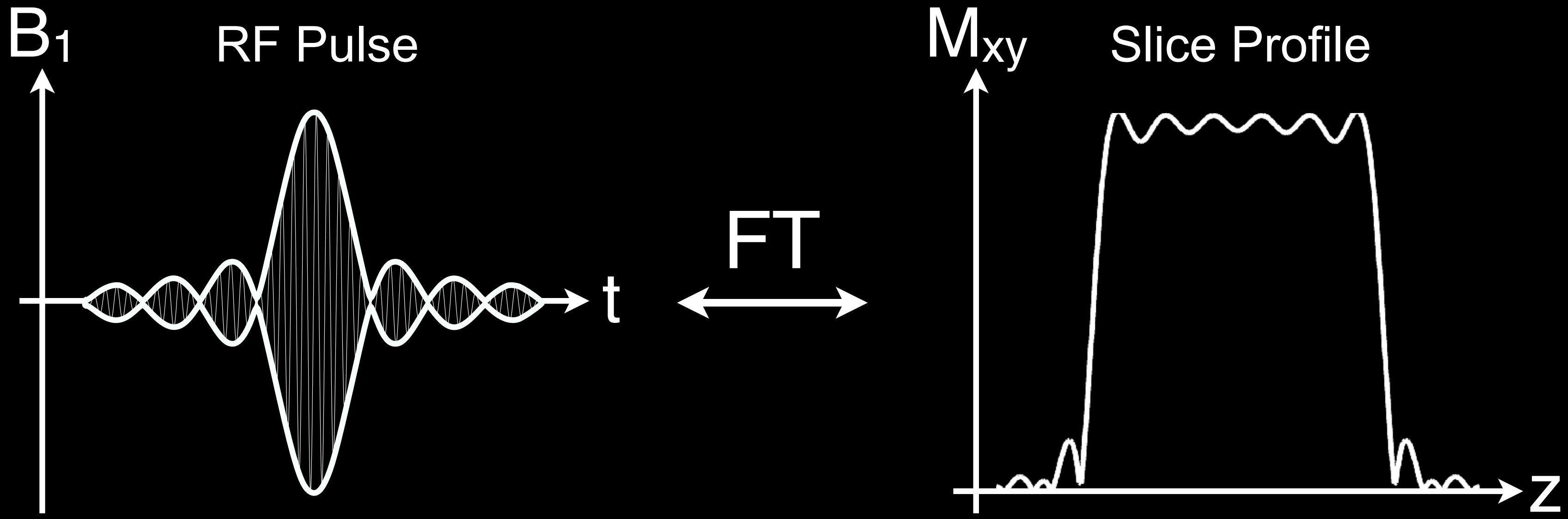
$$\mathrm{TBW} = \tau_{RF} \cdot \mathrm{BW}_{RF}$$

$$\mathrm{BW}_{RF} = \frac{\mathrm{TBW}}{\tau_{RF}}$$
$$= \frac{4}{1\mathrm{ms}}$$
$$= 4\mathrm{kHz}$$

$$G_z = \frac{\Delta f}{\gamma \Delta z}$$
$$= \frac{4000\mathrm{Hz}}{42.57e6\,\frac{Hz}{T}\,\frac{1\mathrm{T}}{10000\mathrm{G}} \cdot 10\mathrm{mm}}$$
$$= 0.94\,\frac{\mathrm{G}}{\mathrm{cm}}$$

How do we pick the envelop function?

- $B_1^e(t)$ **determines the "slice profile".**
- **What is the ideal slice profile?**
- **Changing the shape (envelope function) of the pulse affects the *excitation bandwidth*.**
- **How do we know which shape to use?**
  - **Small Tip Angle Approximation**
    - ➡ **Slice profile depends on the FT of the shape.**



Post-Excitation Refocusing

Remember

$$M_{xy}(\tau_p, \omega) = iM_0 \underbrace{\exp\left[-\frac{i\omega\tau_p}{2}\right]} \int_{-\infty}^{\infty} B_x(t) e^{i\omega t} \mathrm{d}t$$

Frequency dependent phase-shift from the gradients

$$M_{xy}\left(\tau_p, \gamma \mathbf{G}(z - z_0)\right) \propto i\gamma M_0 \exp\left[-i\gamma \mathbf{G}(z - z_0) \cdot \tau_p/2\right]$$

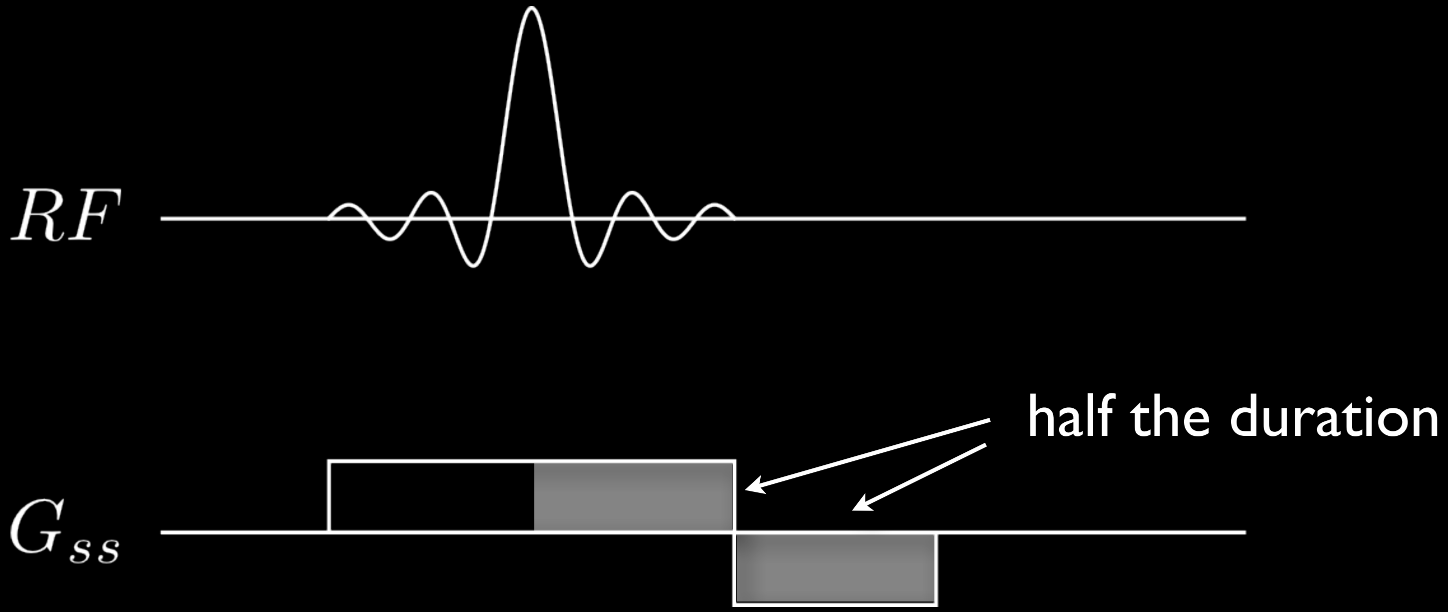
This can be fixed by multiplying the magnetization by $\exp\left[+i\gamma \mathbf{G}(z - z_0) \cdot \tau_p/2\right]$

Post-Excitation Refocusing

Multiply by $\exp\left[+i\gamma\mathbf{G}(z - z_0) \cdot \tau_p/2\right]$

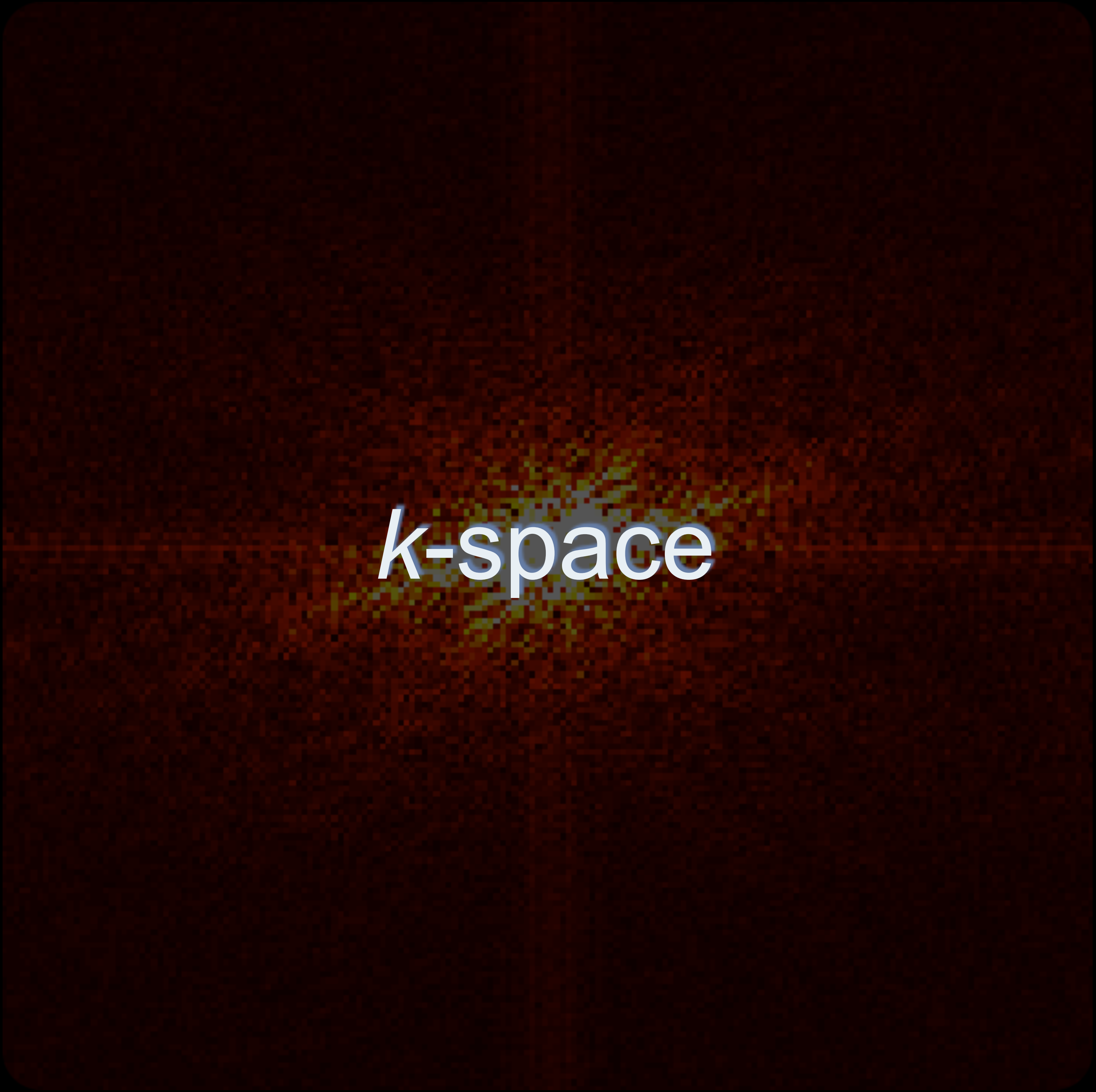
How is this achieved?

Apply a gradient with opposite polarity for $\tau_p/2$



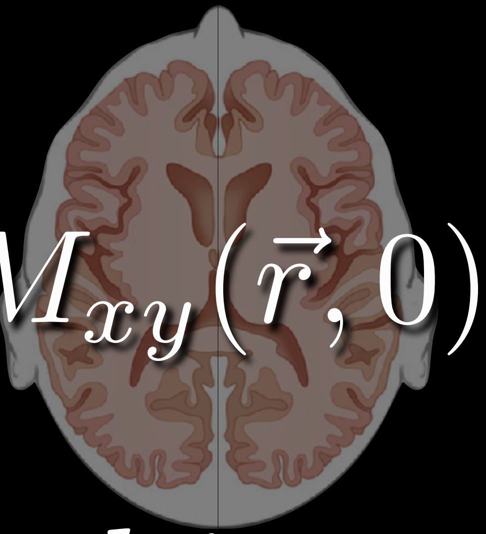
Lecture #11 - Learning Objectives

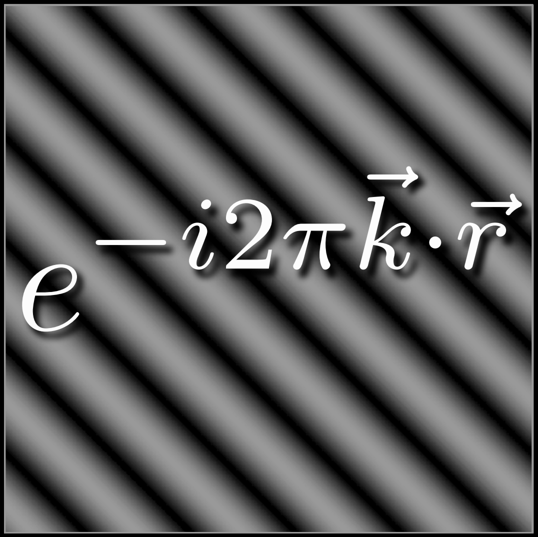
- **Understand the MRI signal equation.**
- **Appreciate what different points in *k*-space represent.**
- **Understand the connection between Fourier encoding and image acquisition.**
- **Be able to describe qualitatively and quantitatively phase and frequency encoding.**



k-space

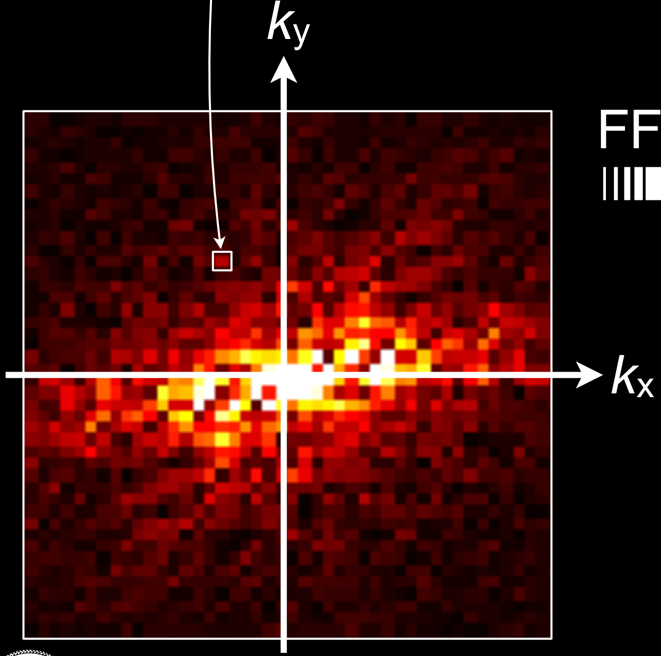
MRI Signal Equation



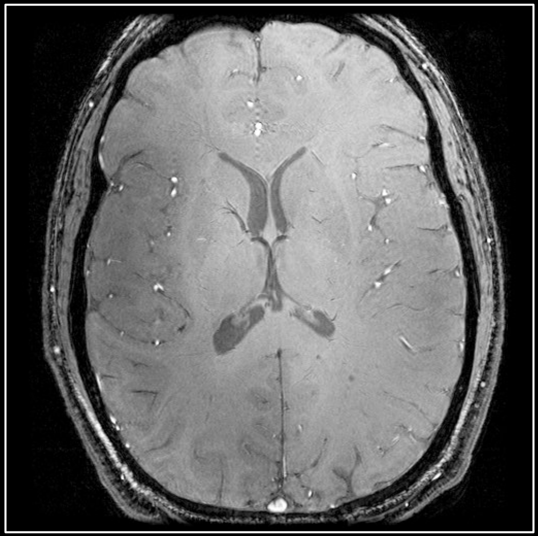


$$S(\vec{k}) = \int M_{xy}(\vec{r},0)\; e^{-i2\pi\vec{k}\cdot\vec{r}}\; d\vec{r}$$

$object$

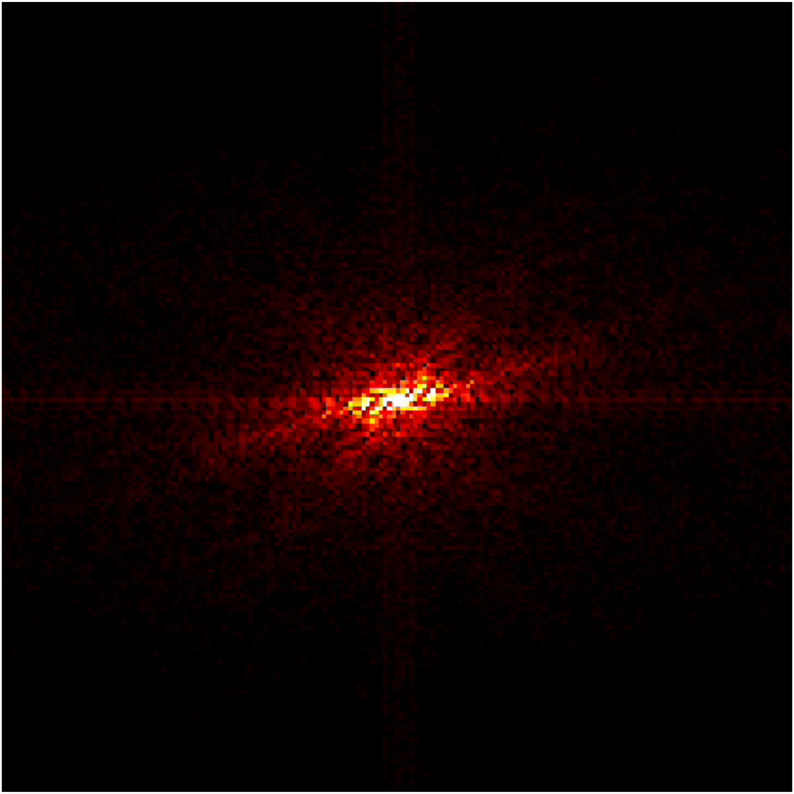


FFT



k-space spikes

k-space

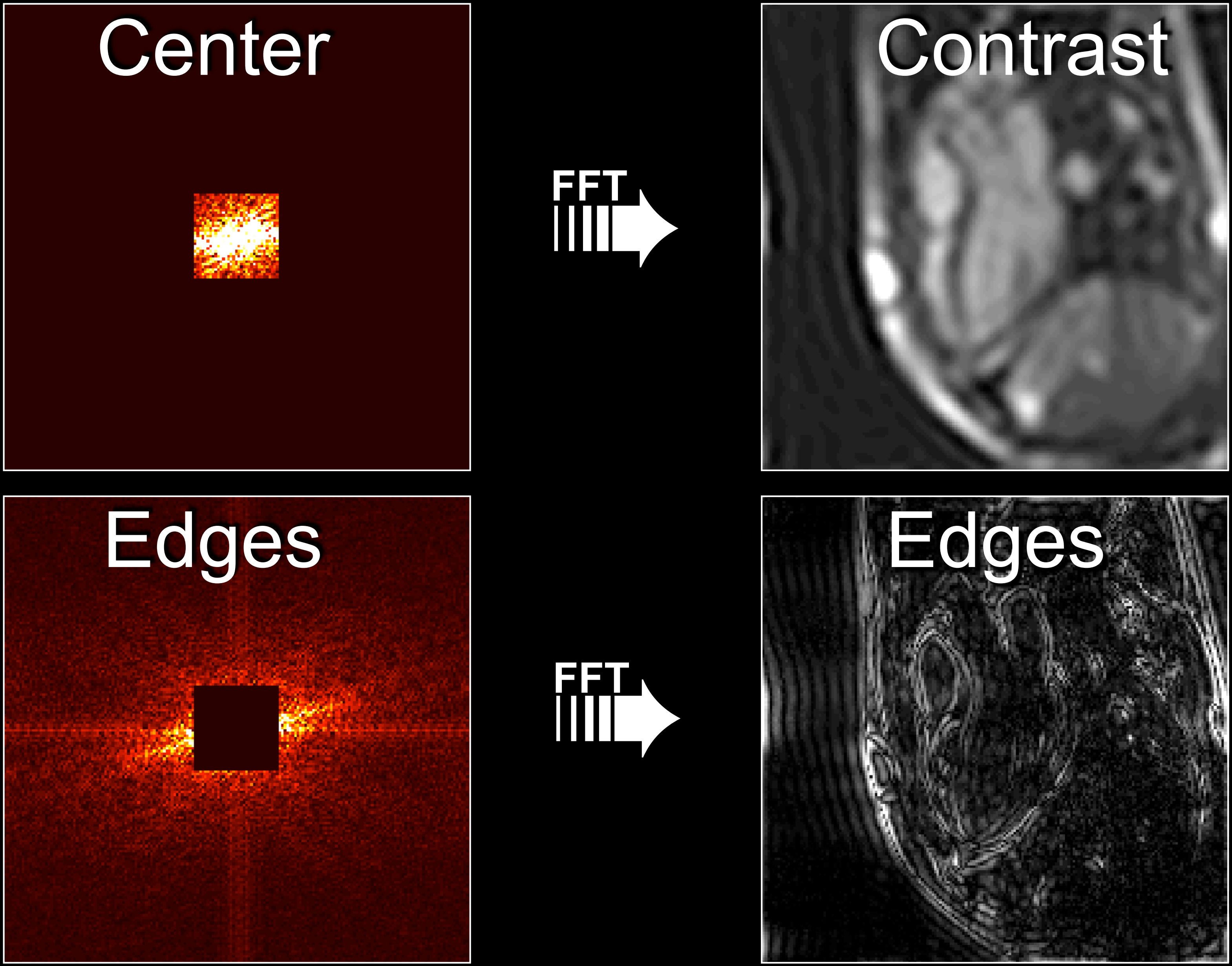


FFT

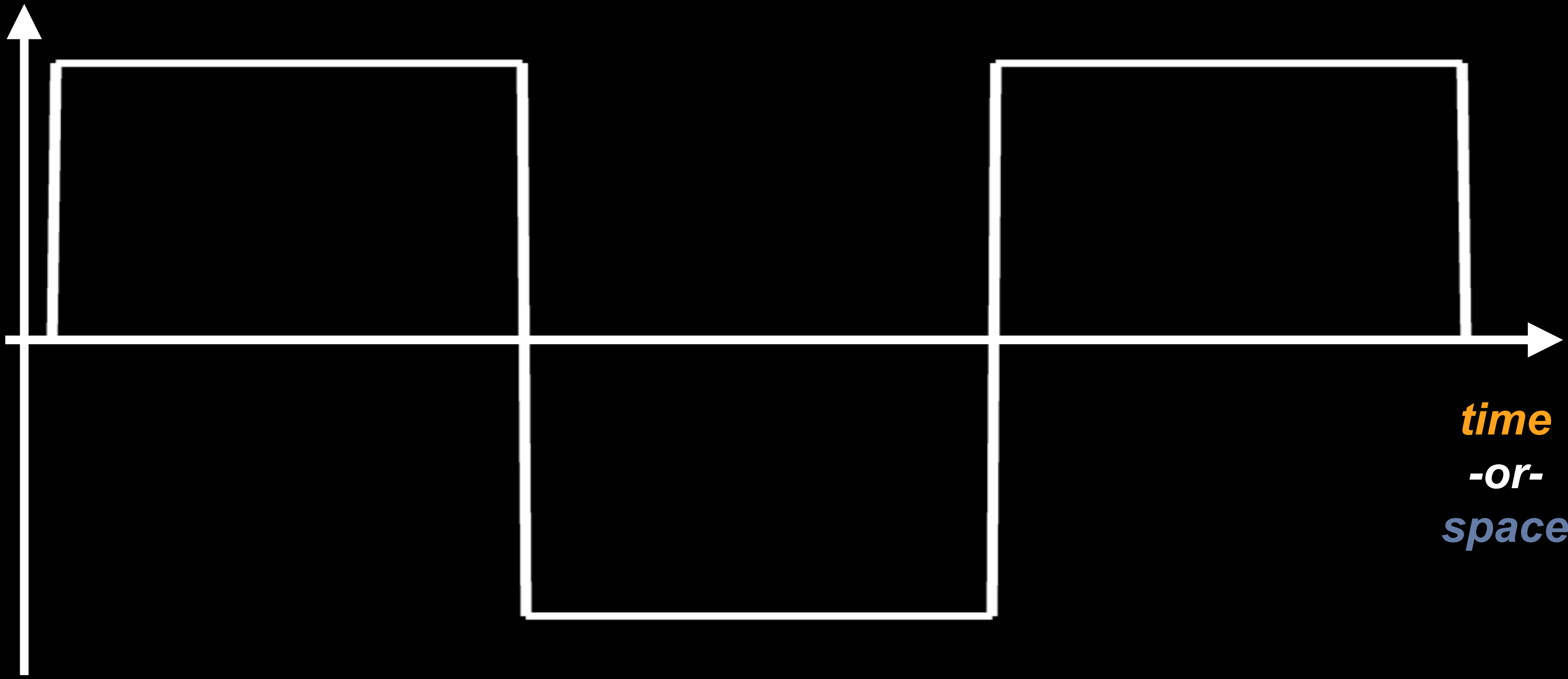
image space

A *k*-space spike creates a banding artifact.

What is *k*-space?

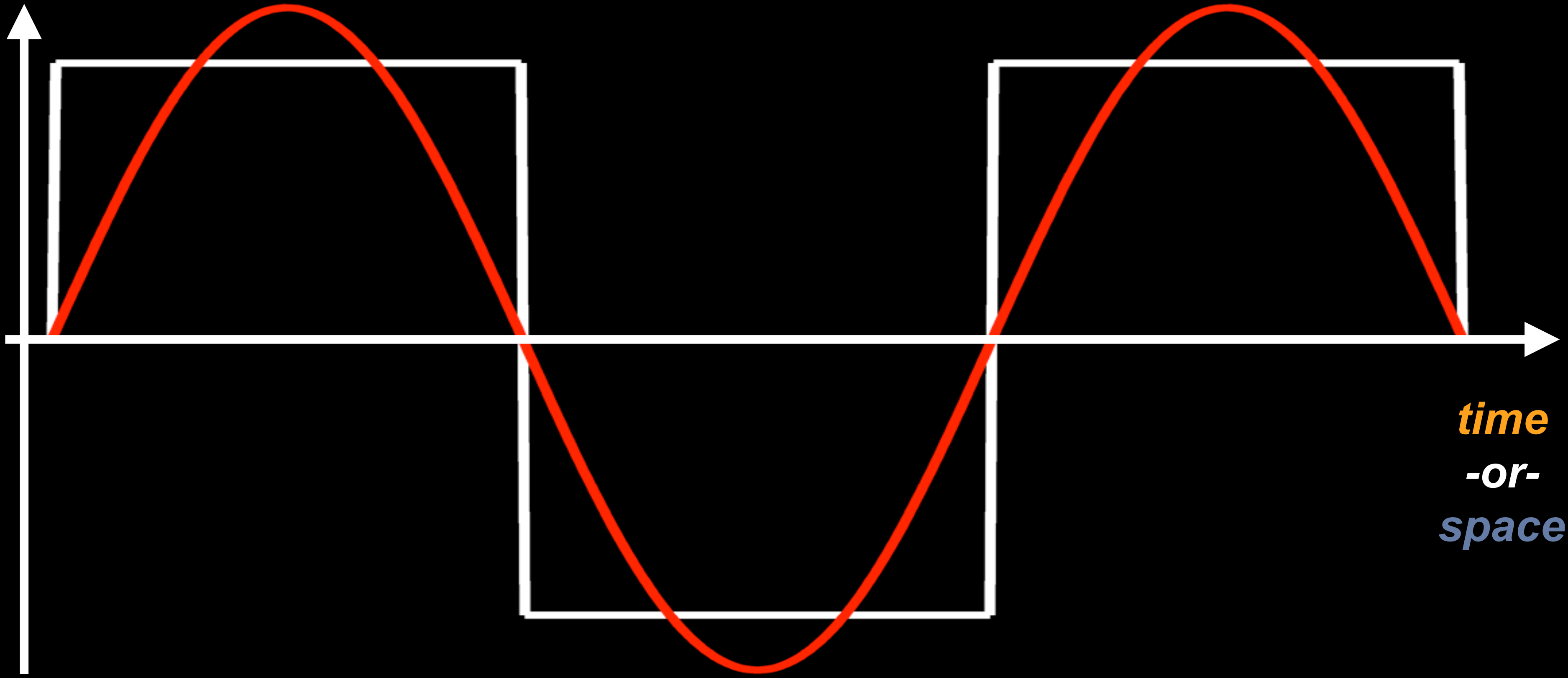


1D *k*-space



Any signal/image can be decomposed into a summation of sine waves of appropriate amplitude.

1D *k*-space



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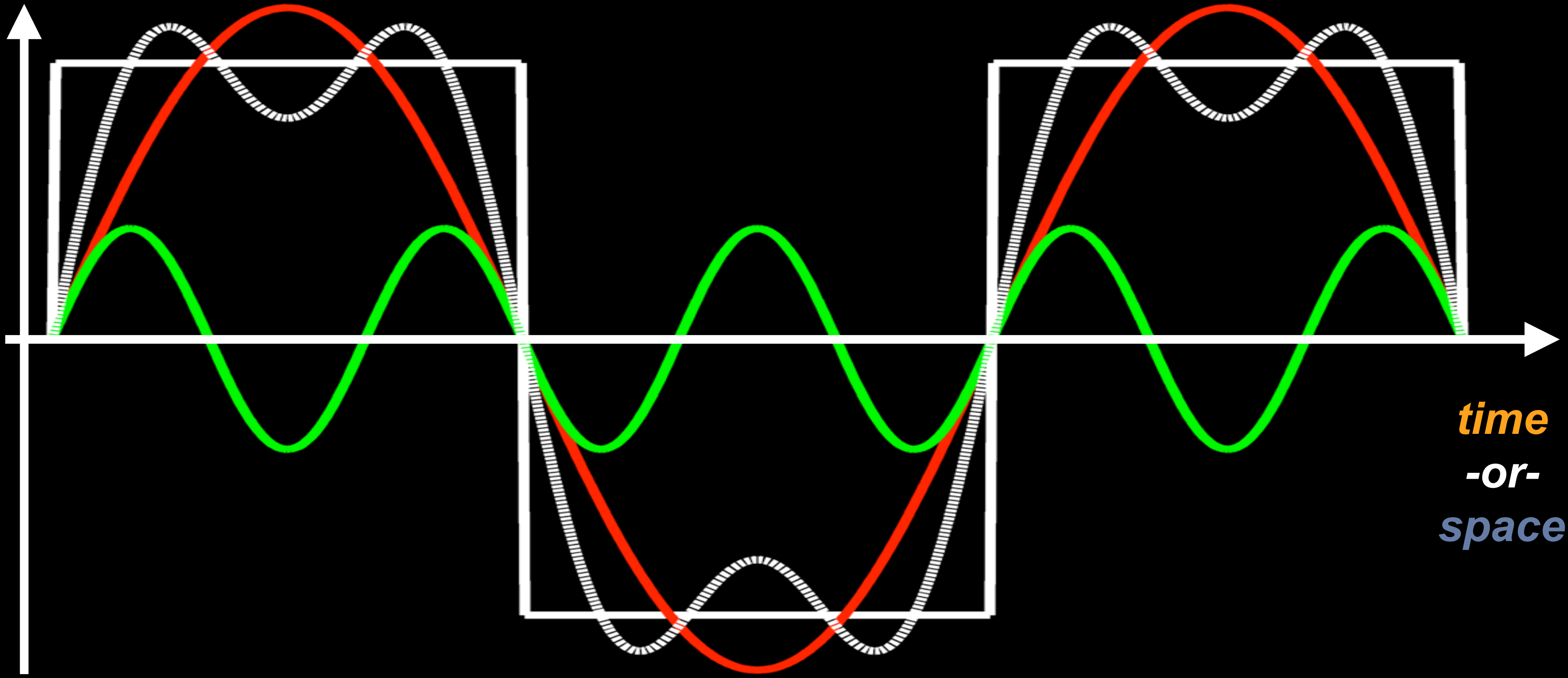
1D *k*-space



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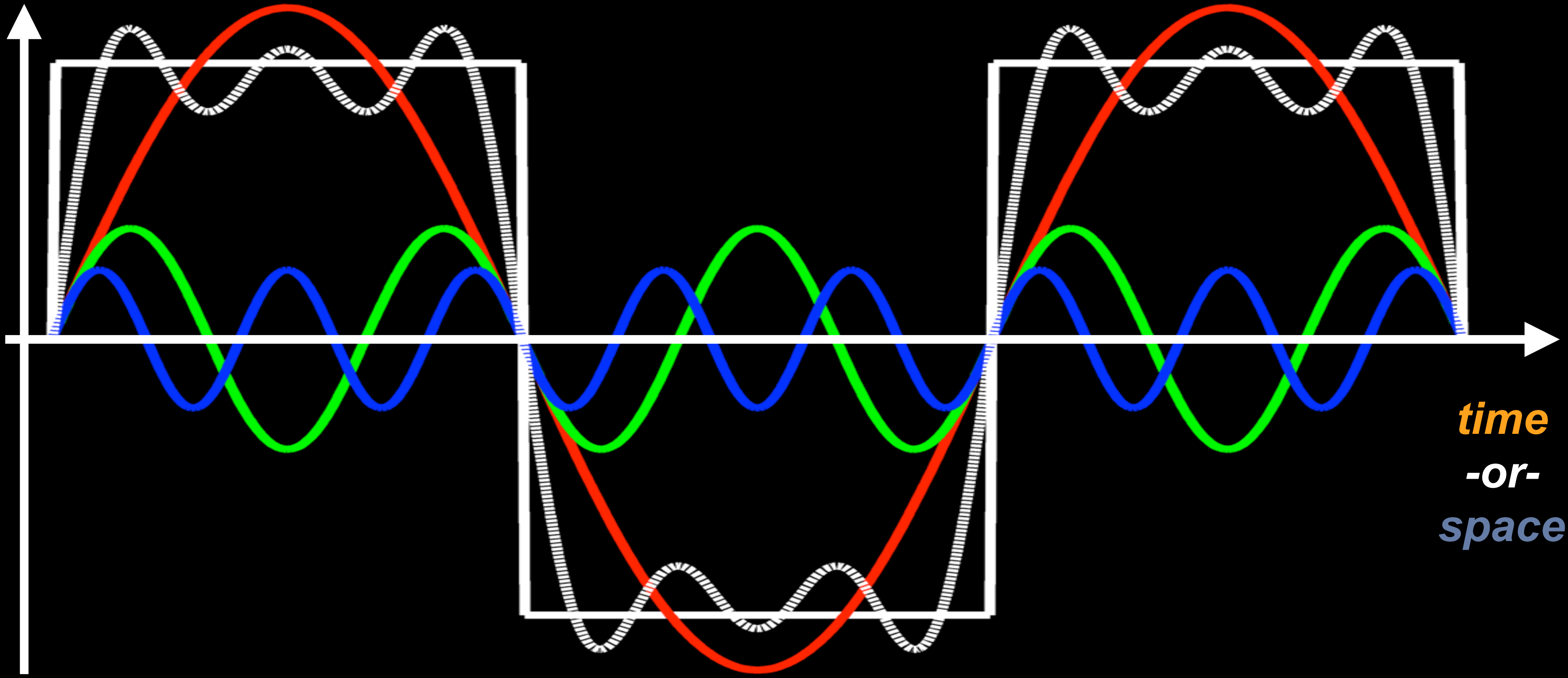
1D *k*-space



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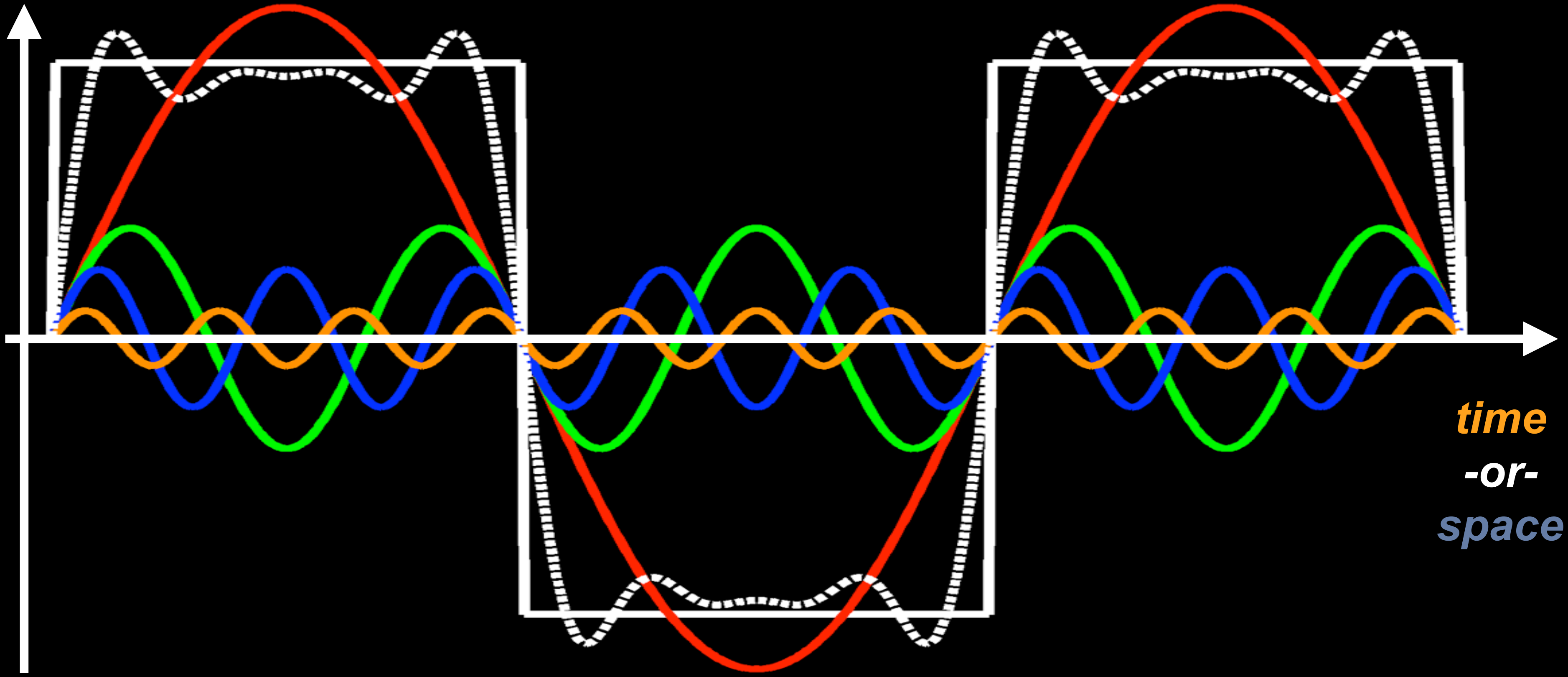
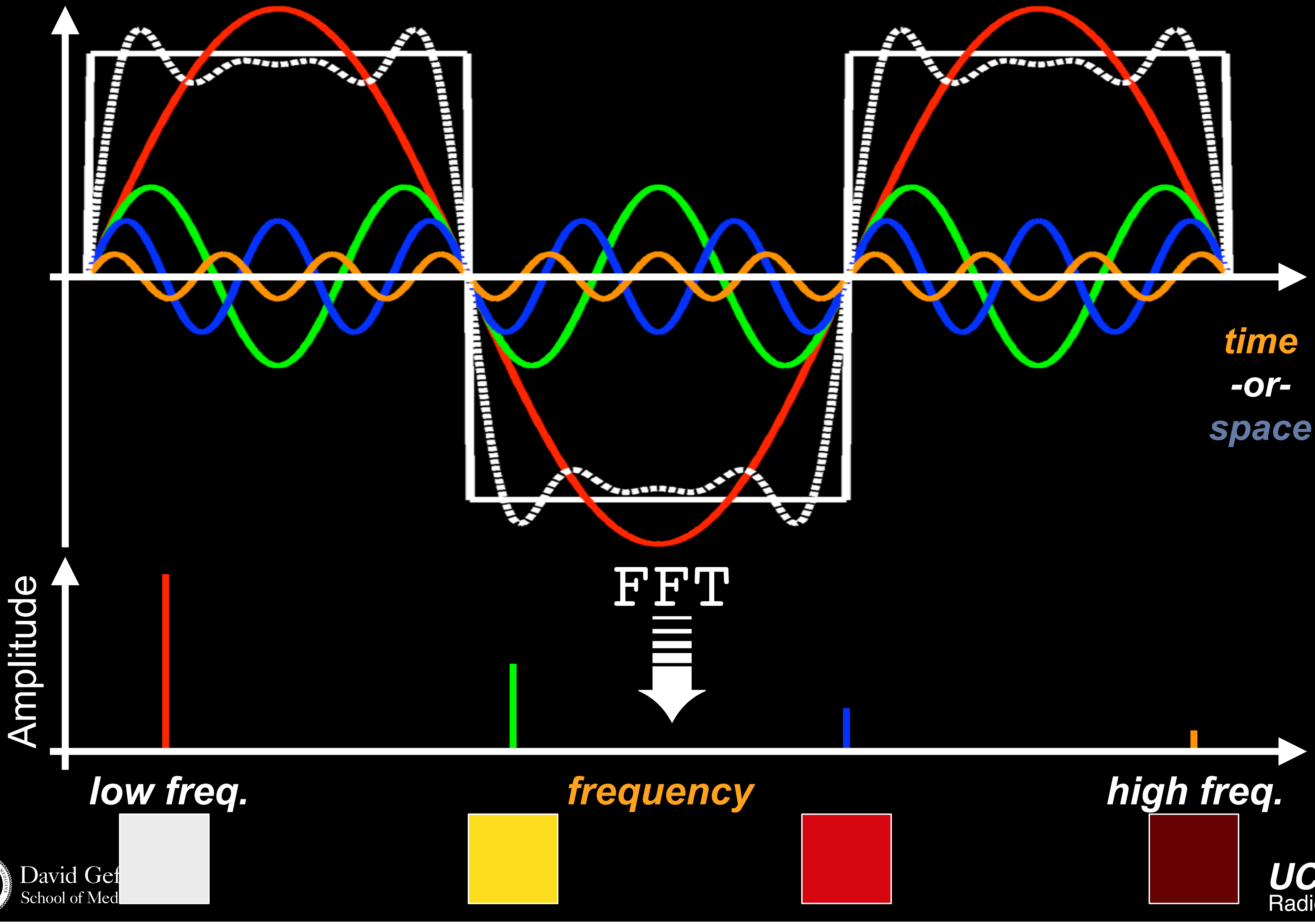
1D *k*-space

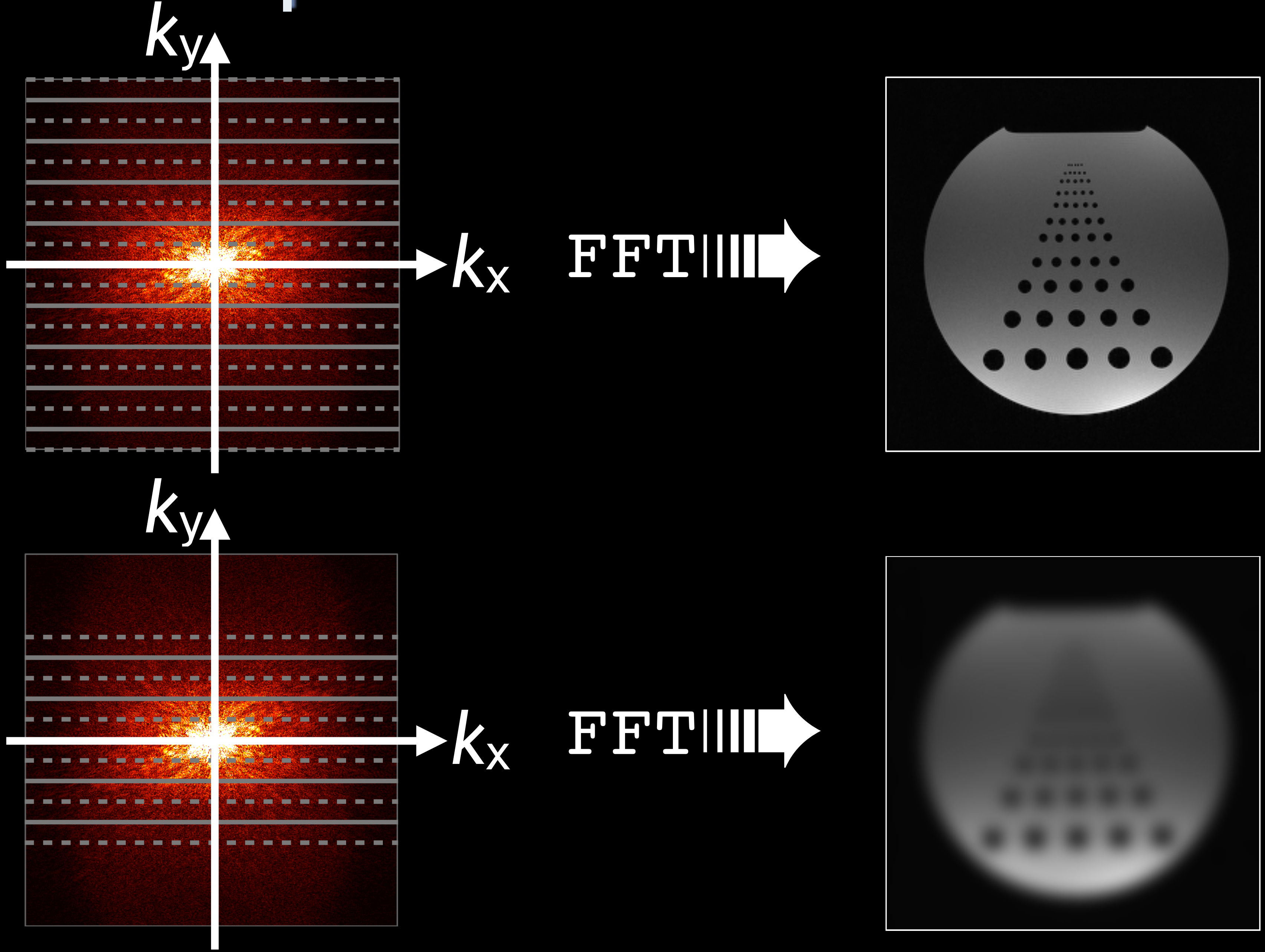


Any signal/image can be decomposed into a summation of sine waves of appropriate amplitude.

Fourier Representation

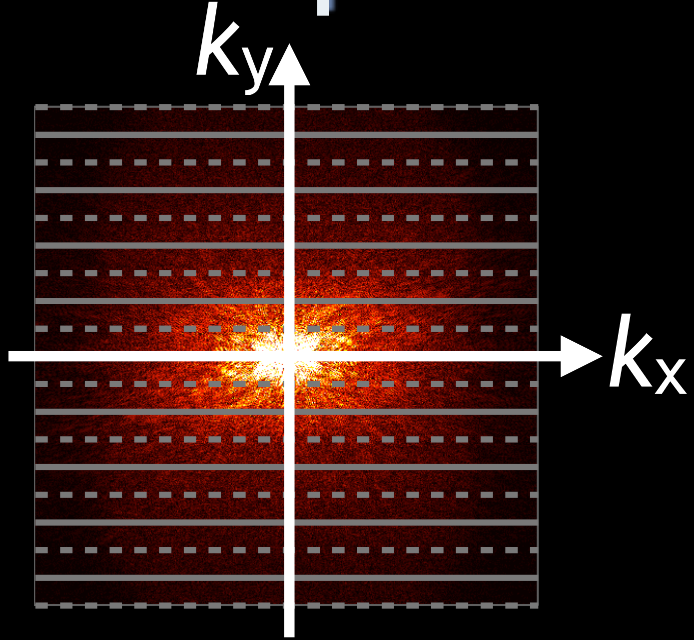


k-space and Resolution

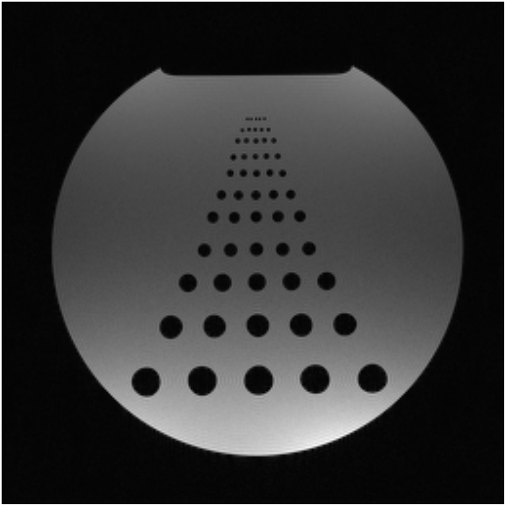


Acquiring fewer high phase encodes decreases resolution.

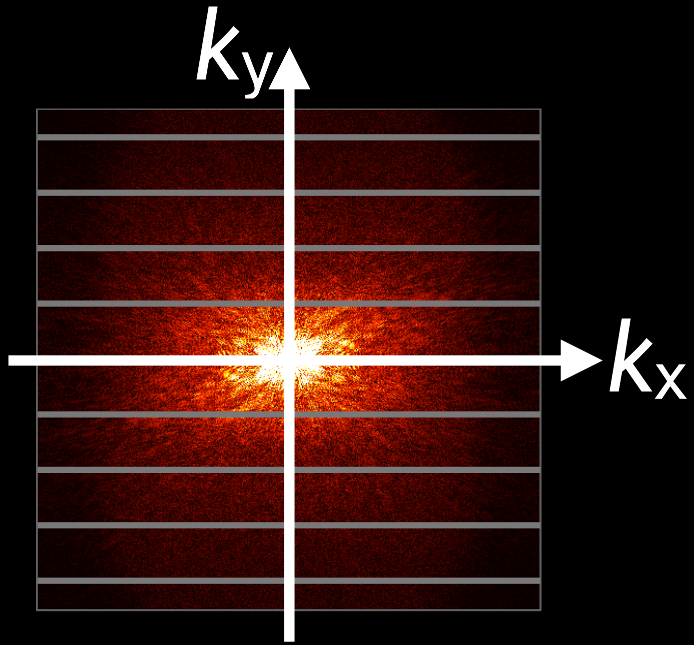
k-space and Field of View



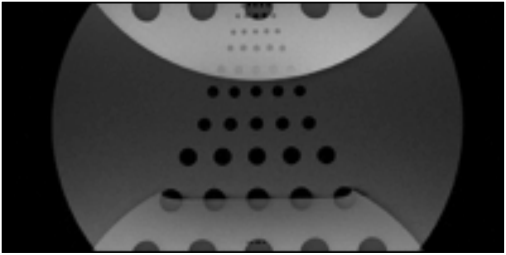
FFT



$$FOV = \frac{1}{\Delta k}$$



FFT

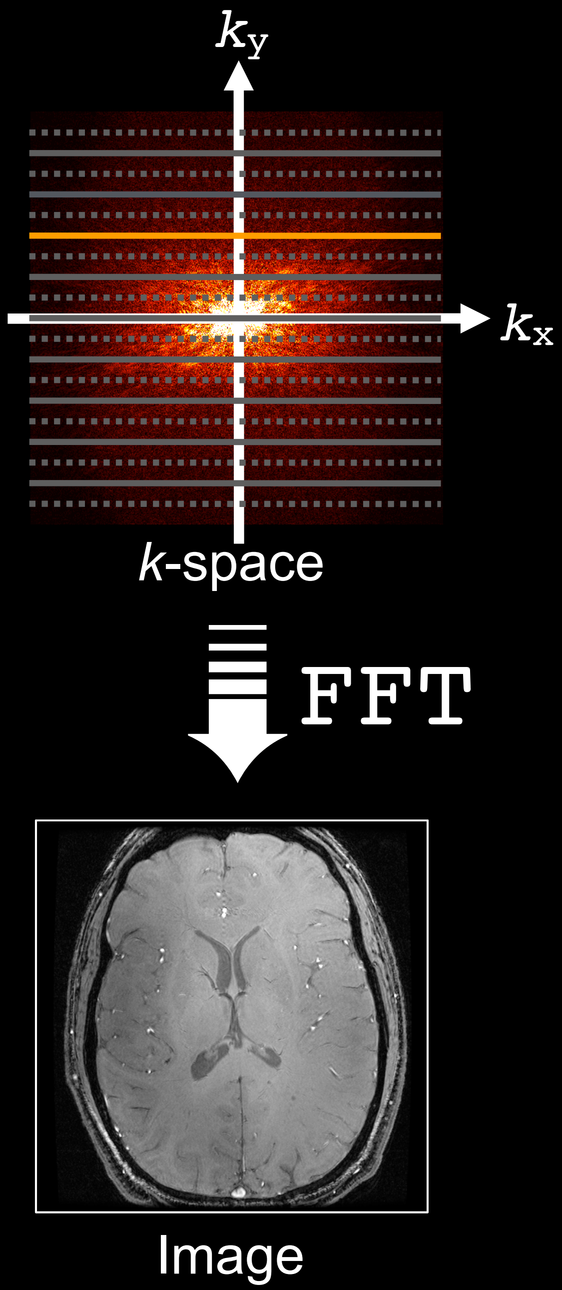


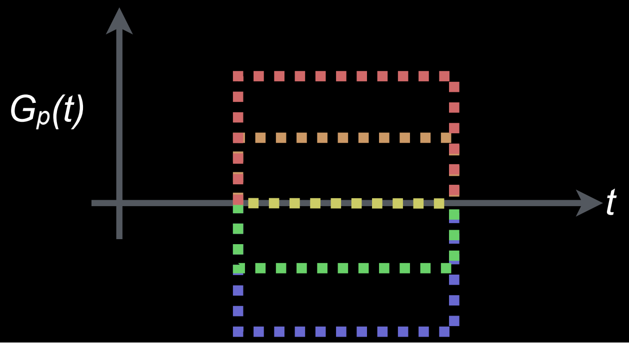
Uniformly skipping lines in *k*-space causes ***aliasing***.

Phase Encoding

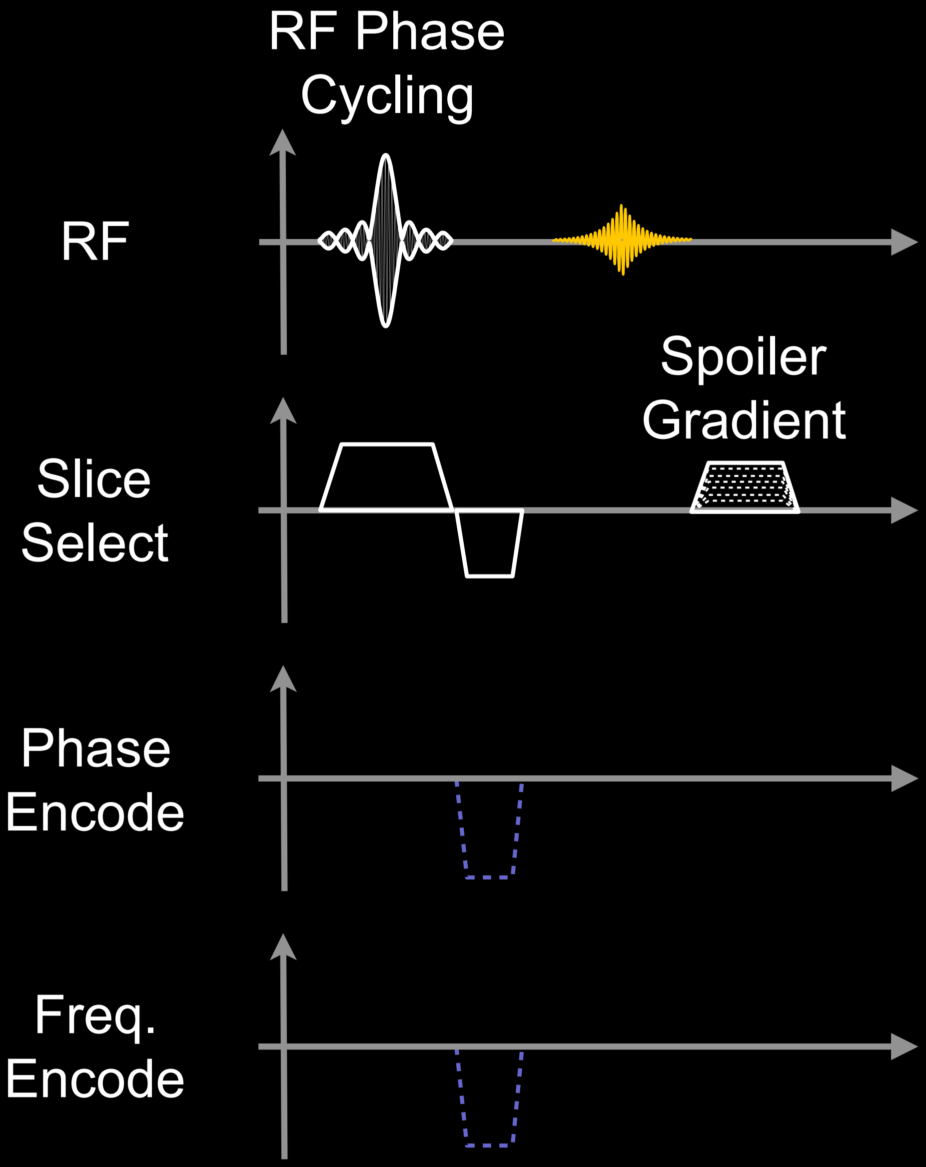
Phase Encoding

- **Consists of:**
  - **Phase encoding gradient**
    - **Magnitude changes with each TR**
    - **Can be played with other gradients**
      - **Crushers, Slice-selection rephaser, readout dephasing**
- **Used with Cartesian imaging**
- **After excitation, before readout**
- **Adds linear spatial variation of phase**
- **Phase encode in**
  - **one direction for 2D imaging**
  - **two directions for 3D imaging**
- **Only one PE step per echo**





Where am I in *k*-space?



RF Phase Cycling

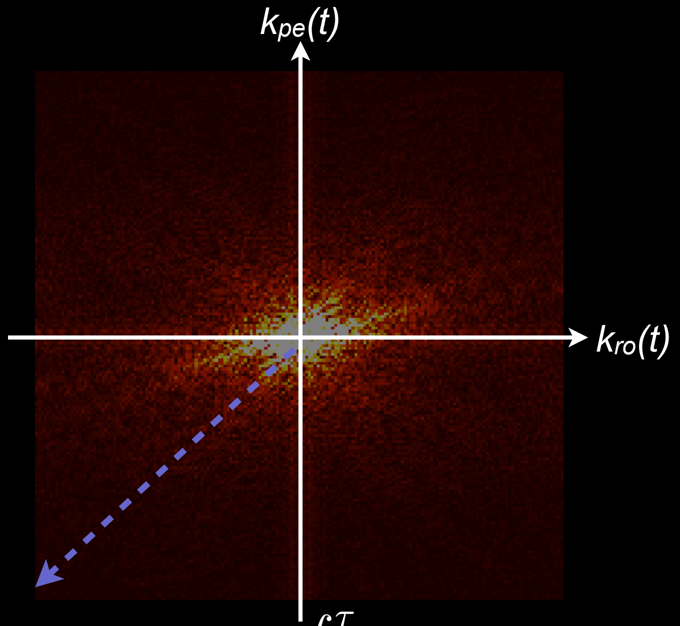
RF

Spoiler Gradient

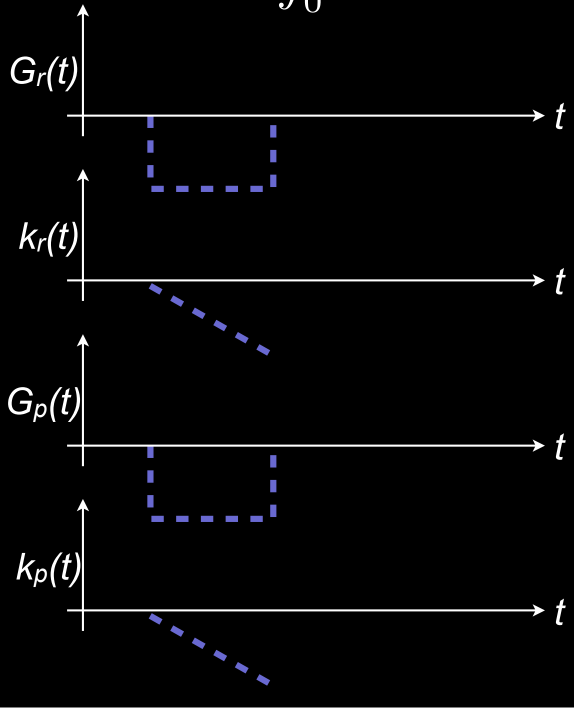
Slice Select

Phase Encode

Freq. Encode

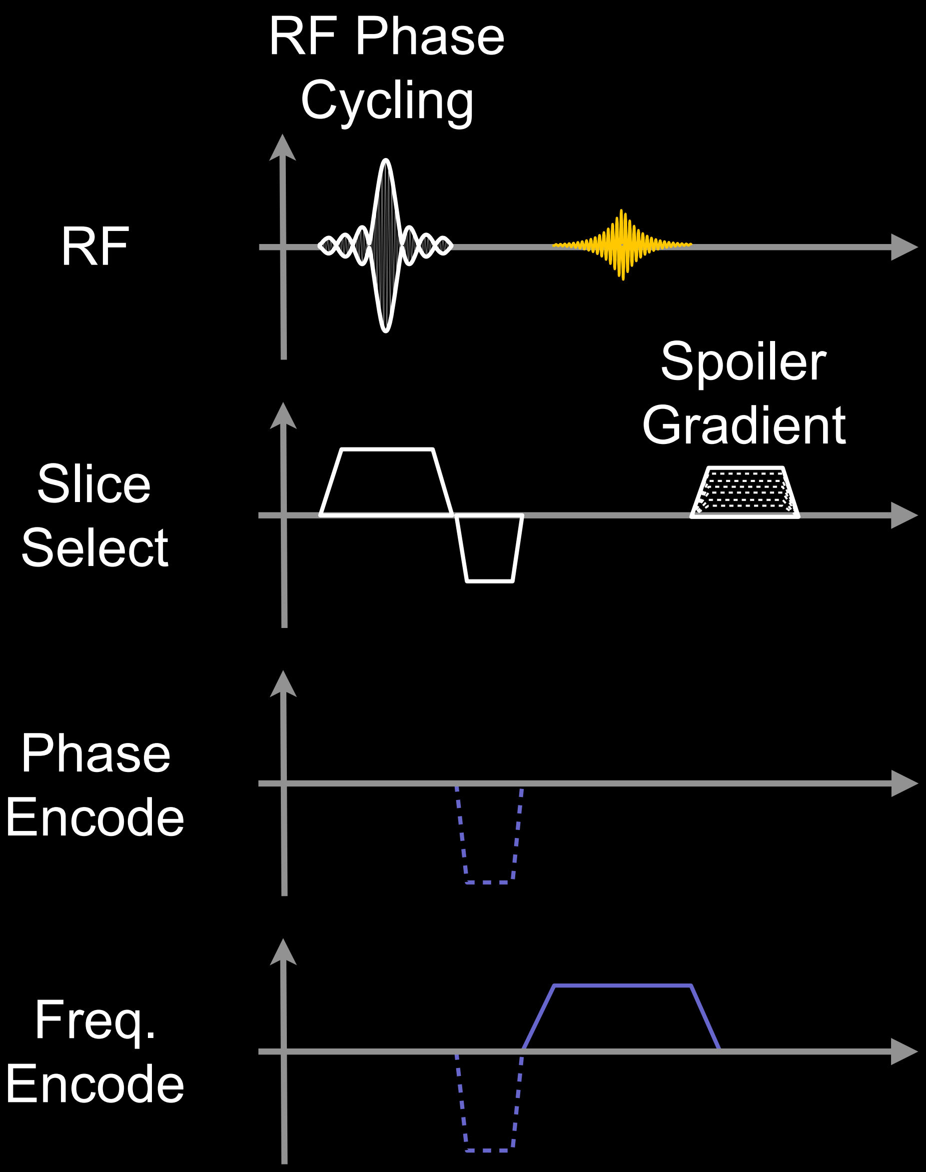


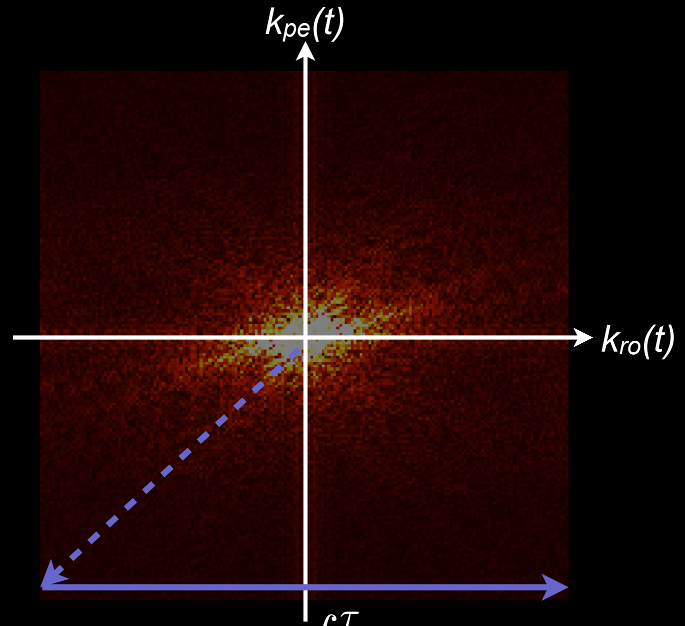
$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\,d\tau$$



One phase encoded echo is acquired per TR.

Where am I in *k*-space?





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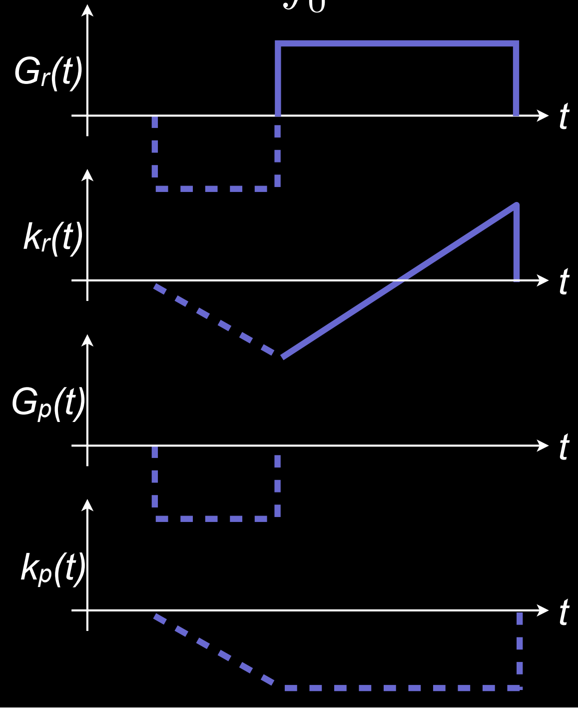
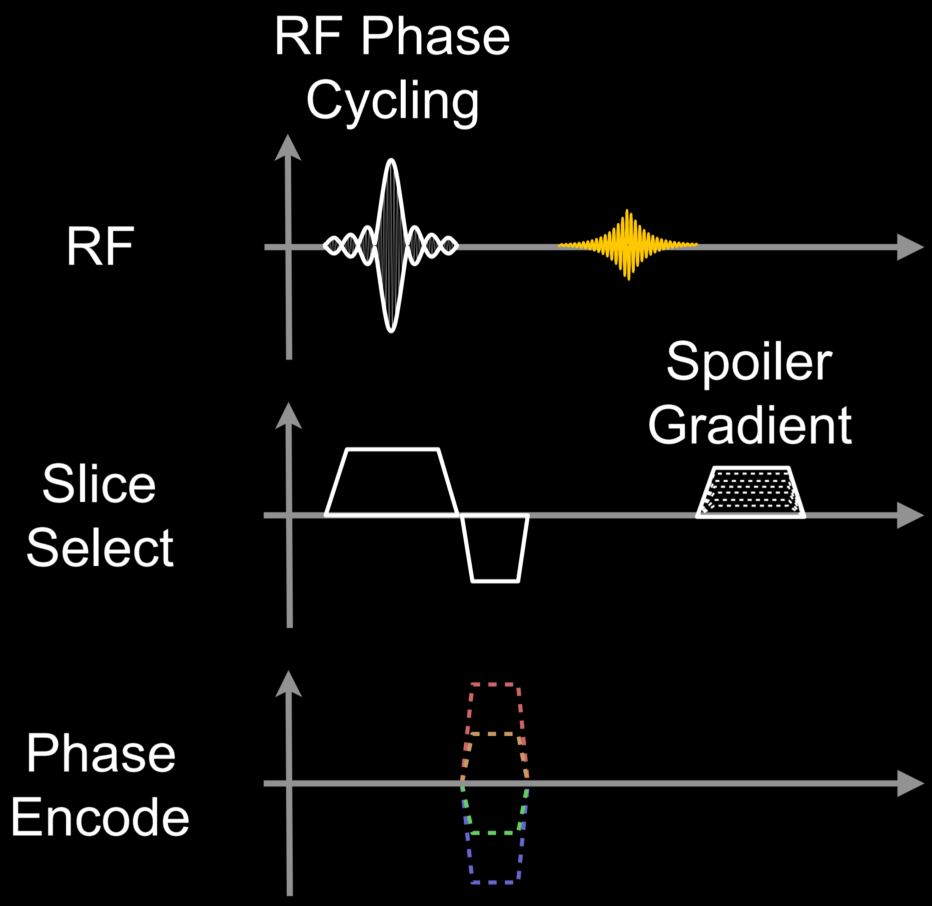
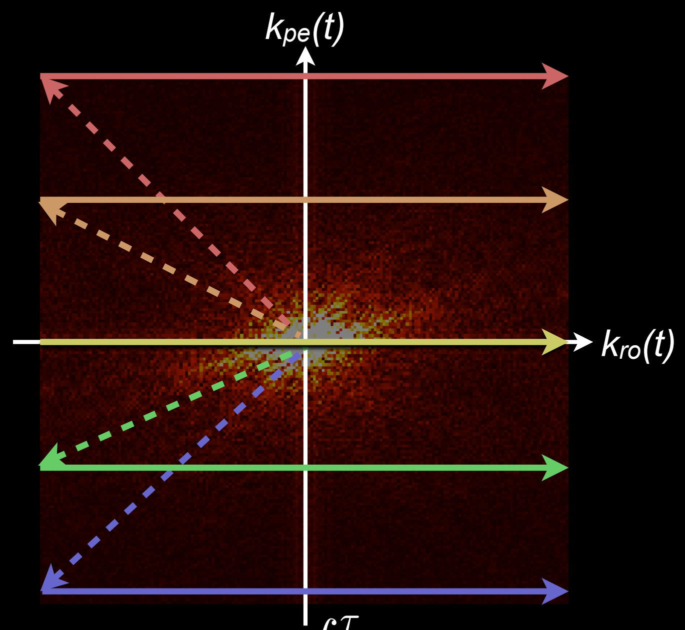


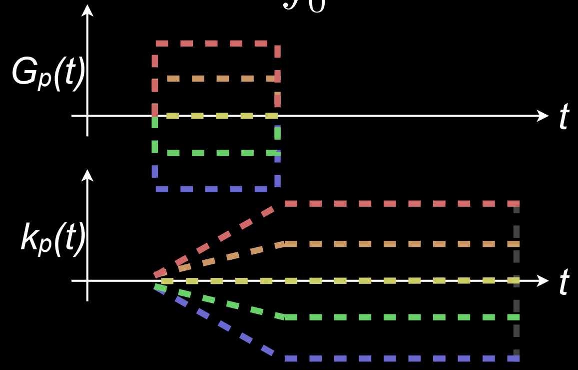
One phase encoded echo is acquired per TR.

Phase Encode Gradients



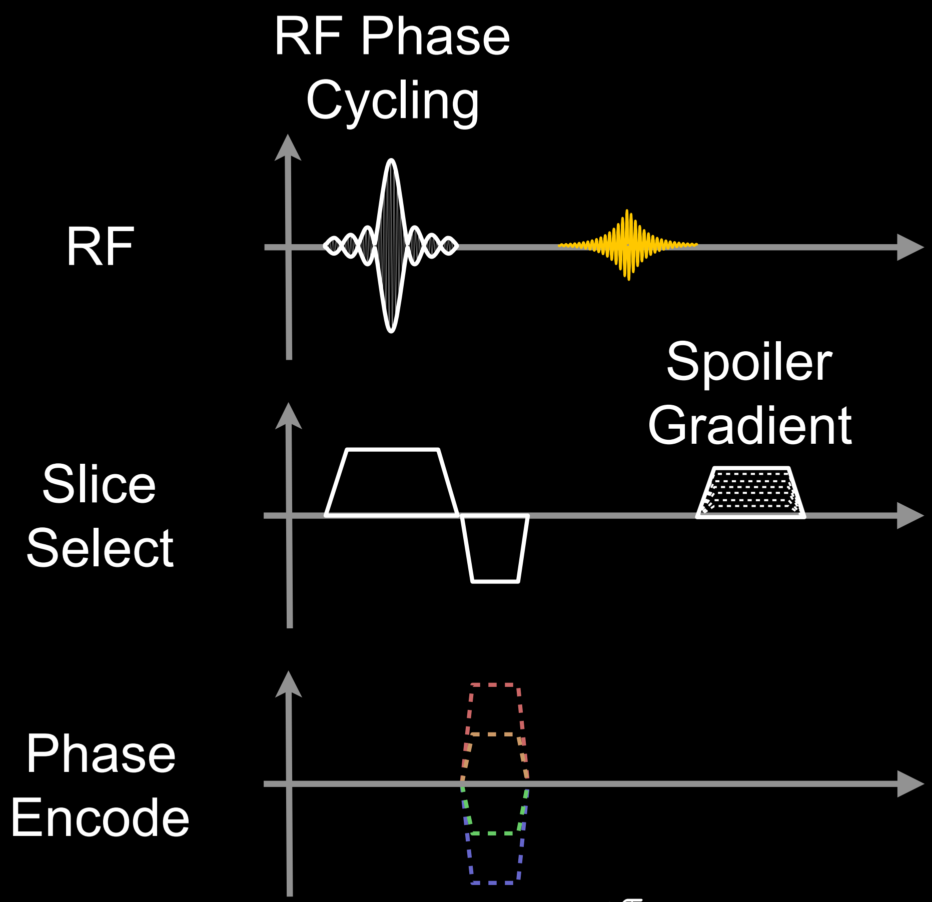


$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^\tau \vec{G}(t)\,d\tau$$

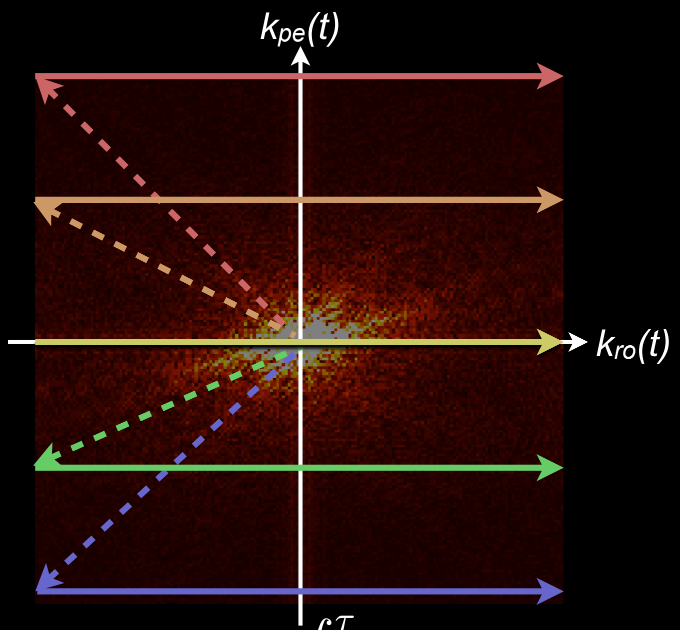


For sequence efficiency the slice-select rephasing gradient and the phase encode gradient can overlap.

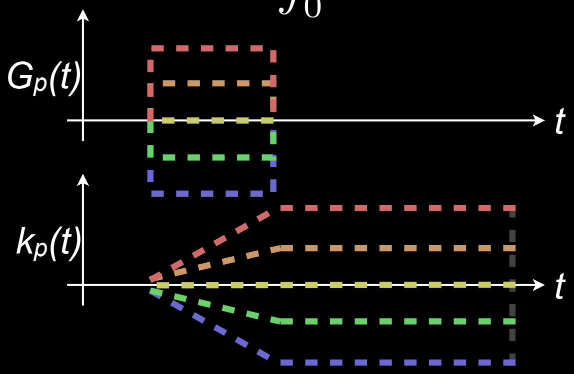
Phase Encode Gradients



$$
\begin{aligned}
\phi_{y,pe}(y) &= \int_0^{\tau_{PE}} \omega(y,t)dt \\
&= \int_0^{\tau_{PE}} \gamma G_{y,pe}(t) \cdot y dt \\
&= \gamma G_{y,pe} \cdot \tau_{PE} \cdot y \\
&= 2\pi k_y \cdot y
\end{aligned}
$$



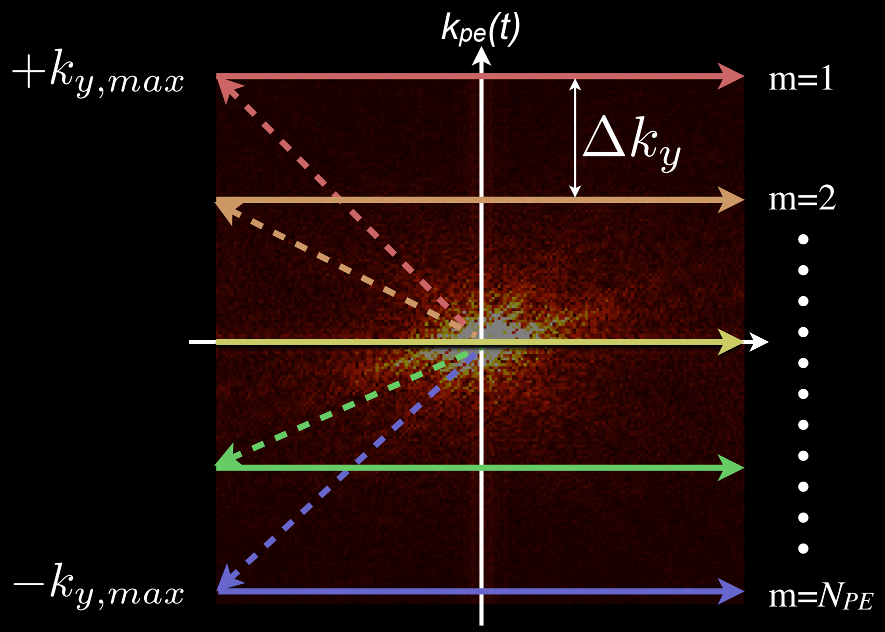
$$\vec{k}\,(t) = \frac{\gamma}{2\pi} \int_0^{\tau} \vec{G}\,(t)\, d\tau$$



Phase Encode Gradients

$FOV = \frac{1}{\Delta k_y}$, encoded with N_{PE} steps.

$$\Delta k_y = \frac{1}{N_{PE} \cdot \Delta y} = \frac{1}{128 \cdot 0.1\text{cm}} = 0.078\text{cm}^{-1}$$



$$k_{y,max} = \tfrac{1}{2}(N_{PE} - 1)\Delta k_y = \tfrac{1}{2}(128 - 1) \cdot 0.078\text{cm}^{-1} = 4.95\text{cm}^{-1}$$

↑ 2x Nyquist

In general, $k_y(m) = \left(\frac{N_{PE}-1}{2} - m\right) \Delta k_y$

Phase Encode Gradients

- **How do we design the steps?**
  - **Calculate $k_{y,max}$ from defined N_{PE} and FOV**
    - Defines largest PE step (e.g. largest gradient)
  - **Design shortest gradient for $k_{y,max}$**
  - **Linear scaling of gradient area for all other steps**
    - Keeps sequence timing constant TR to TR

$$\Delta k_y = \frac{1}{\mathrm{FOV}_y} = \gamma \Delta \mathbf{G}_y T_{pe}$$ Eqn. 5.123

$$k_{y,max} = \left(\frac{N_{PE}-1}{2}\right)\Delta k_y$$
$$= \left(\frac{N_{PE}-1}{2}\right)\frac{\gamma}{2\pi}\Delta G_{PE}\tau_{PE}$$

Let, $G_{PE,max} = \left(\frac{N_{PE}-1}{2}\right)\Delta G_{PE}$

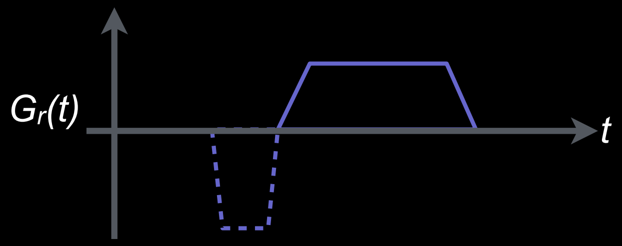
- Use the maximum available gradient strength.
- Calculate the duration, τ_{PE}.

$$\tau_{PE} = \frac{2\pi k_{y,max}}{\gamma G_{max}}$$
$$= \frac{4.95\mathrm{cm}^{-1}}{4248\frac{\mathrm{Hz}}{\mathrm{G}}\cdot 4\frac{\mathrm{G}}{\mathrm{cm}}}$$
$$= 0.290\mathrm{ms}$$

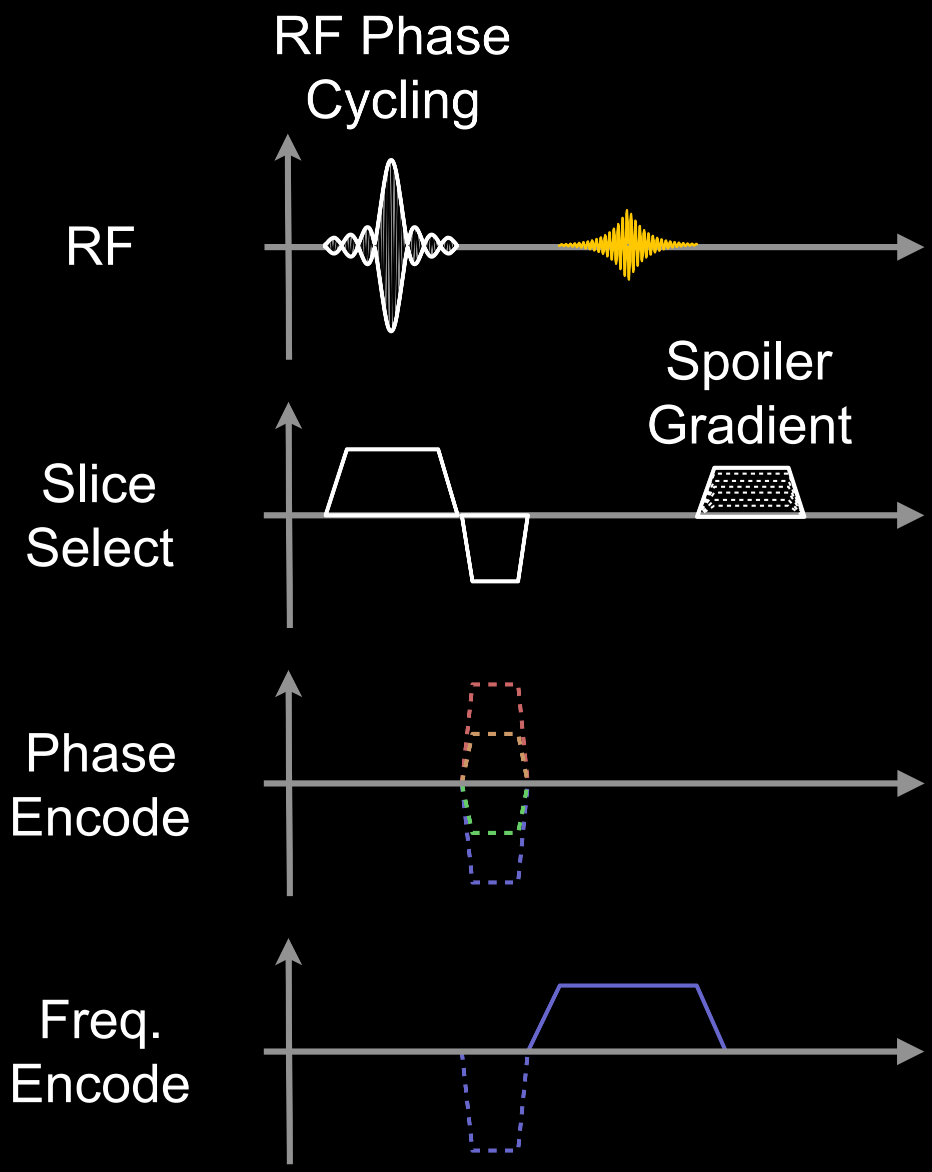
Frequency Encoding

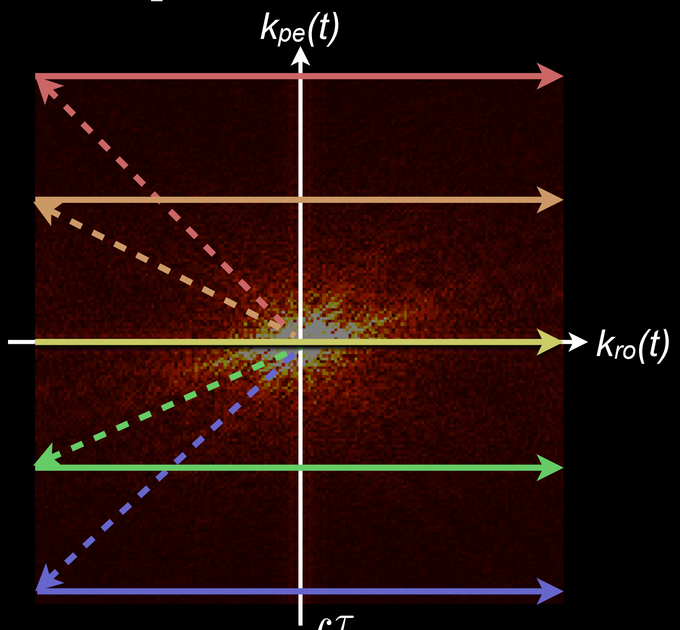
Frequency Encoding

- Consists of:
  - Frequency encoding gradient
    - Constant magnitude for Cartesian imaging
  - No simultaneous
    - RF (B_1)
    - Other gradients
      - phase encoding, slice encoding, crushers
  - Readout pre-phasing gradient
    - Prepares spin phase so peak echo amplitude occurs at middle of readout (TE)
    - AKA “readout de-phasing gradient”
- Adds linear spatial variation of frequency
- Helps form an *echo*

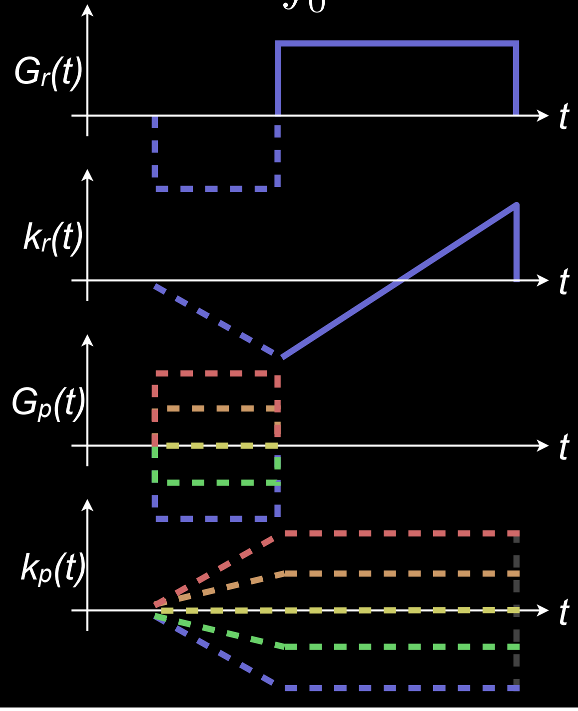


Gradient Echo Sequence



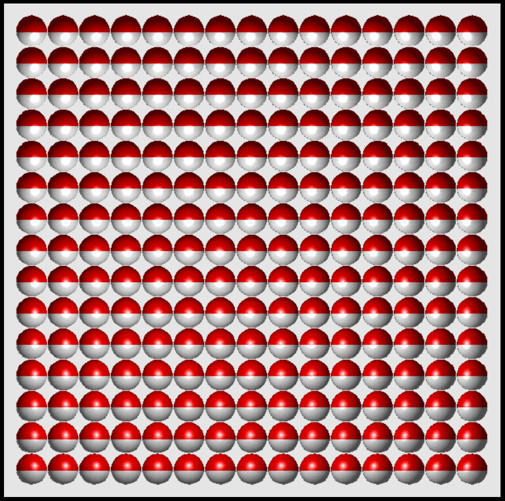


$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$

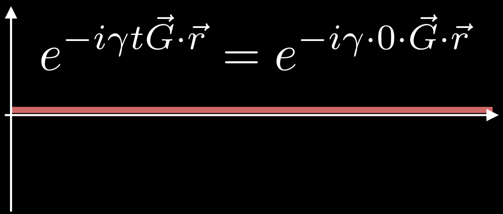


One phase encoded echo is acquired per TR.

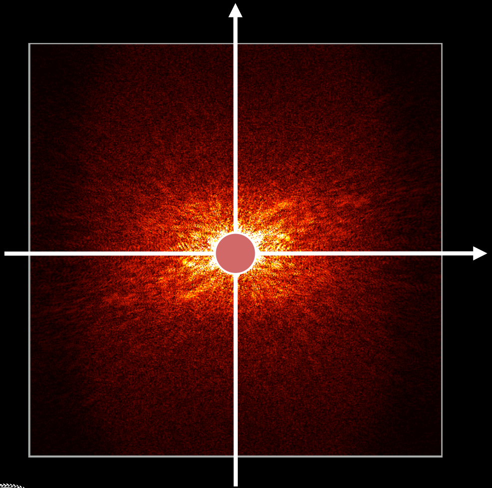
Frequency Encoding



G_{Freq}=0



$$e^{-i\gamma t\vec{G}\cdot\vec{r}} = e^{-i\gamma\cdot 0\cdot\vec{G}\cdot\vec{r}}$$



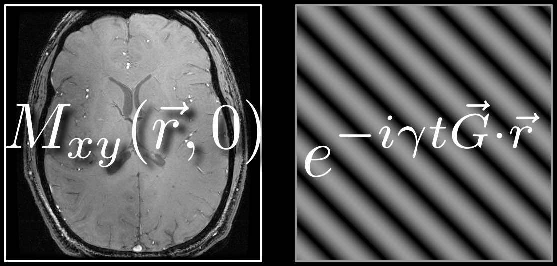
$$\vec{k}\left(t\right) = \frac{\gamma}{2\pi}\int_0^{\tau}\vec{G}\left(t\right)d\tau$$

In general…

$$2\pi\vec{k}\left(t\right) = \gamma\vec{G}t$$

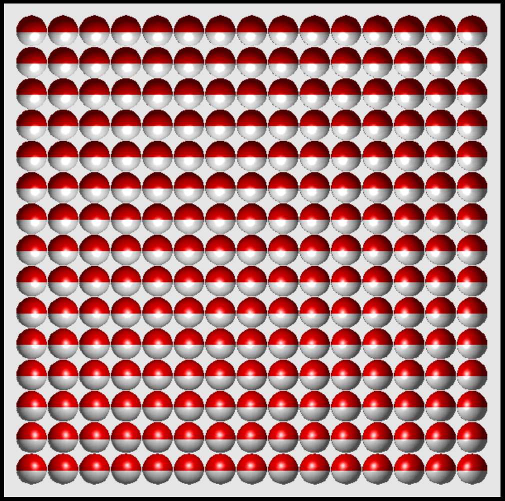
For a constant amplitude gradient…

$$S(\vec{k}) = \int \underbrace{M_{xy}(\vec{r},0)}_{object}\, e^{-i2\pi\vec{k}\cdot\vec{r}}\, d\vec{r}$$

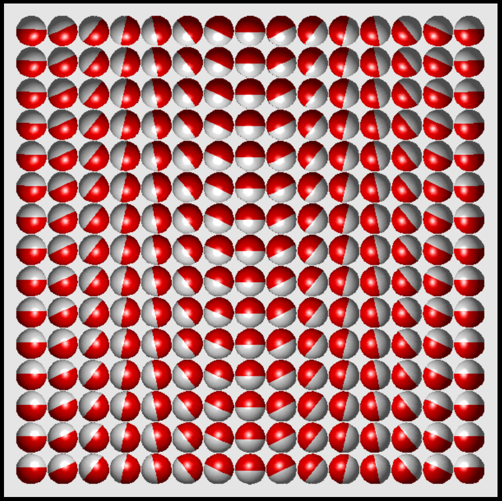


$$\int \underbrace{M_{xy}(\vec{r},0)}_{object}\, e^{-i\gamma t\vec{G}\cdot\vec{r}}\, d\vec{r}$$

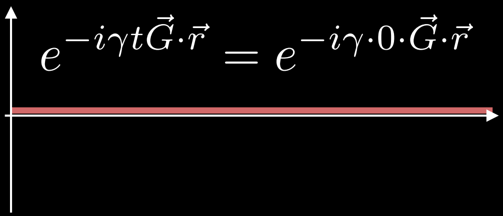
Frequency Encoding



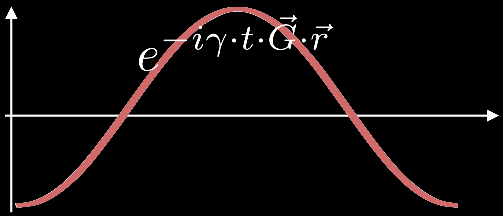
$G_{Freq}=0$



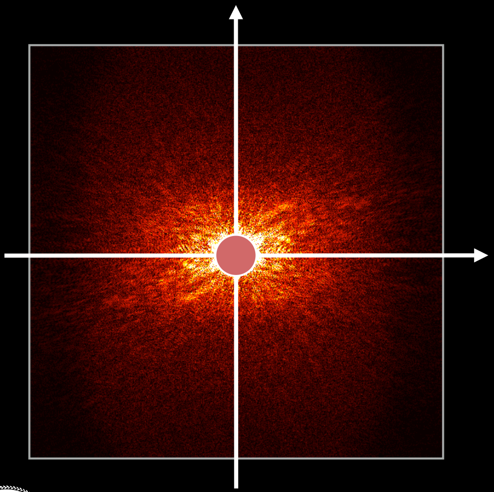
$G_{Freq}=G\cdot t$

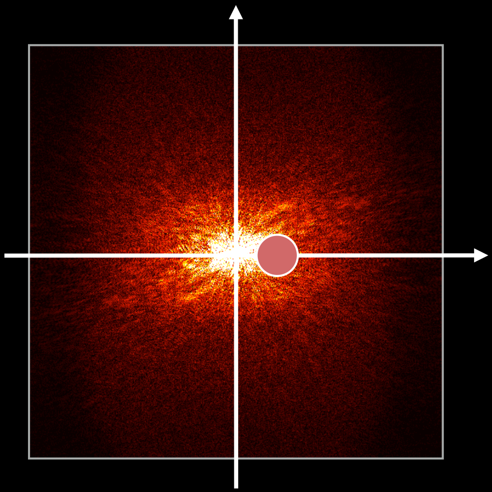


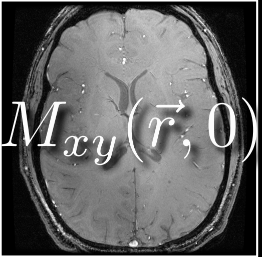
$$e^{-i\gamma t\vec{G}\cdot\vec{r}}=e^{-i\gamma\cdot 0\cdot\vec{G}\cdot\vec{r}}$$

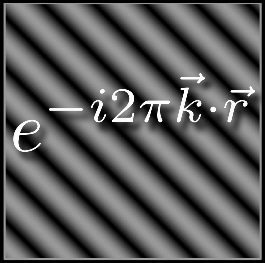


$$e^{-i\gamma\cdot t\cdot\vec{G}\cdot\vec{r}}$$



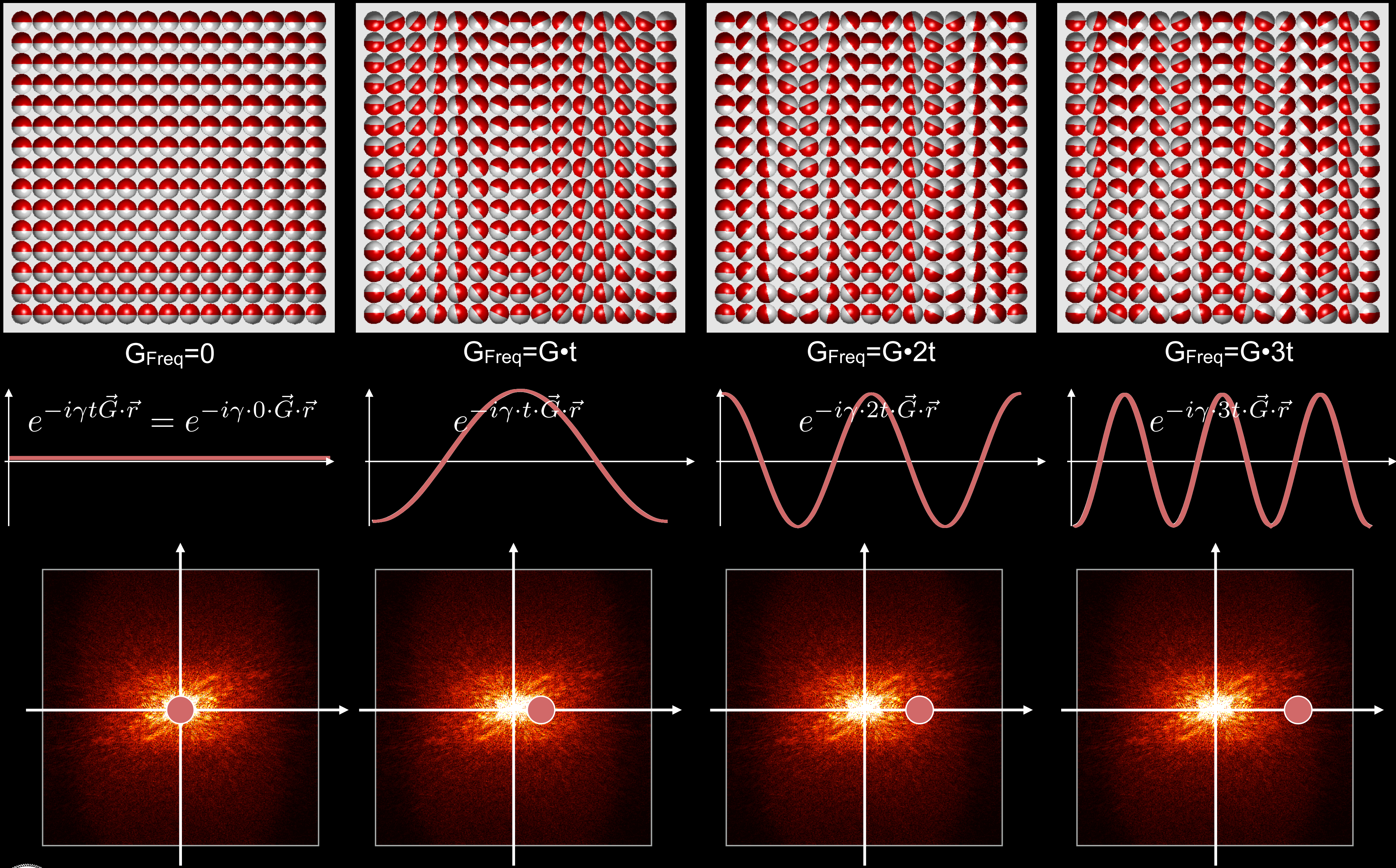






$$S(\vec{k})=\int \underbrace{M_{xy}(\vec{r},0)}_{object}\, e^{-i2\pi\vec{k}\cdot\vec{r}}\, d\vec{r}$$

Frequency Encoding



$G_{Freq}=0$

$G_{Freq}=G\cdot t$

$G_{Freq}=G\cdot 2t$

$G_{Freq}=G\cdot 3t$

$$e^{-i\gamma t\vec{G}\cdot\vec{r}} = e^{-i\gamma\cdot 0\cdot\vec{G}\cdot\vec{r}}$$

$$e^{-i\gamma\cdot t\cdot\vec{G}\cdot\vec{r}}$$

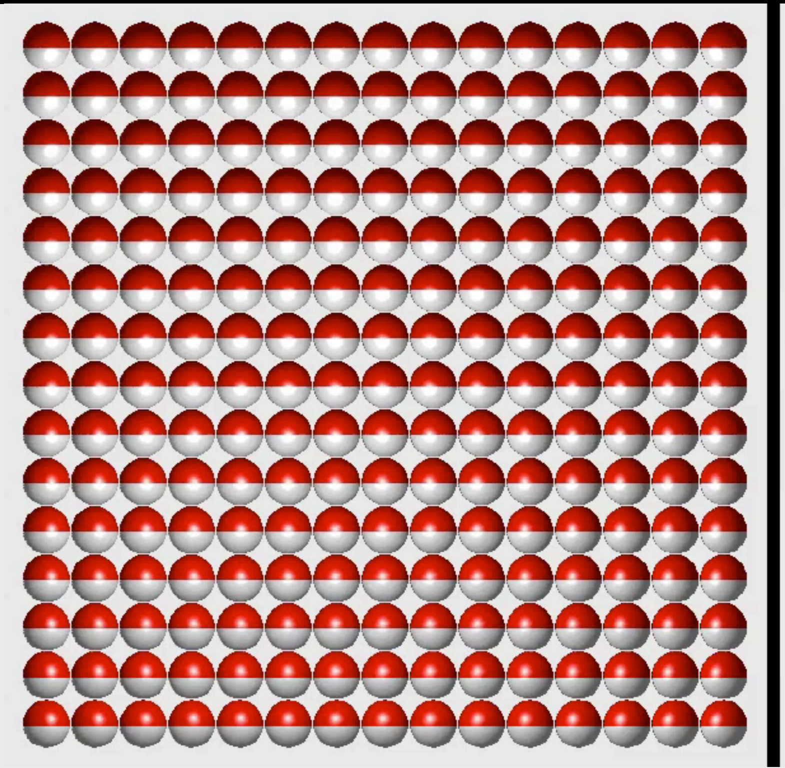
$$e^{-i\gamma\cdot 2t\cdot\vec{G}\cdot\vec{r}}$$

$$e^{-i\gamma\cdot 3t\cdot\vec{G}\cdot\vec{r}}$$

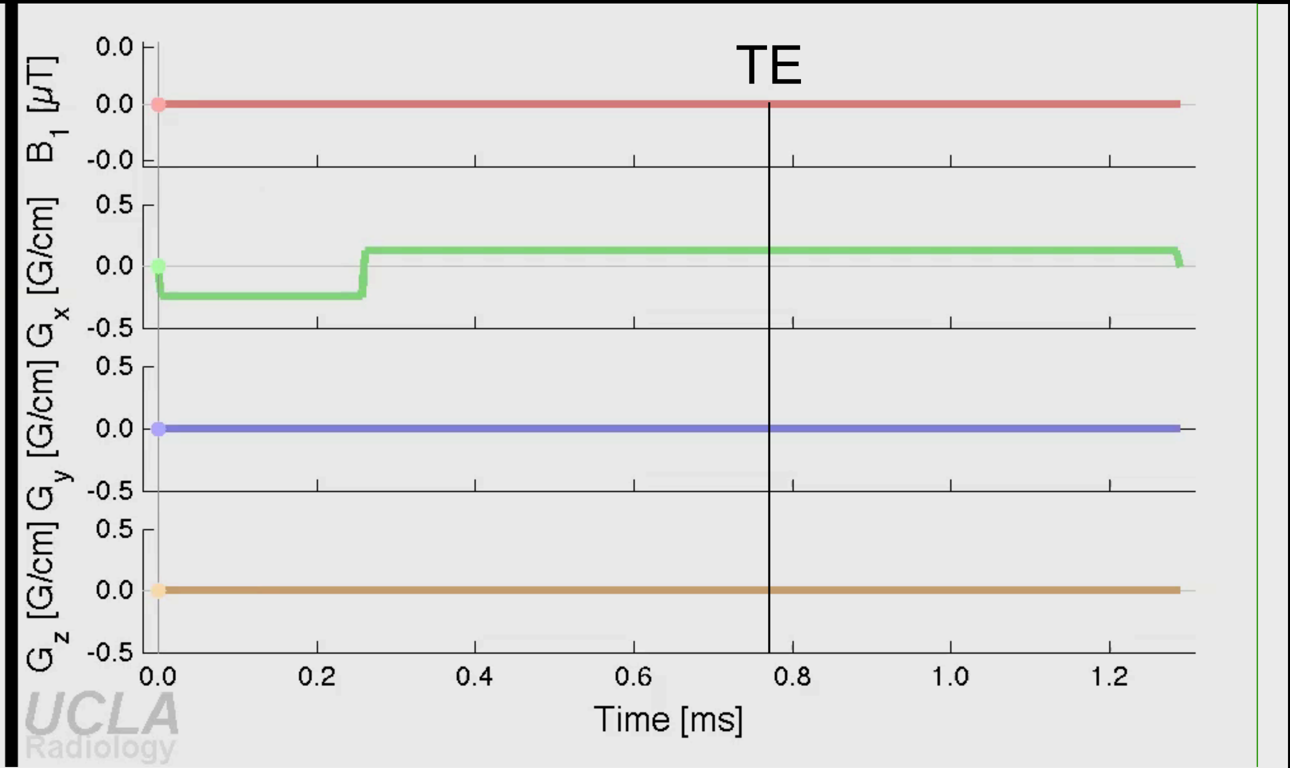
At each time in this process the signal can be measured.

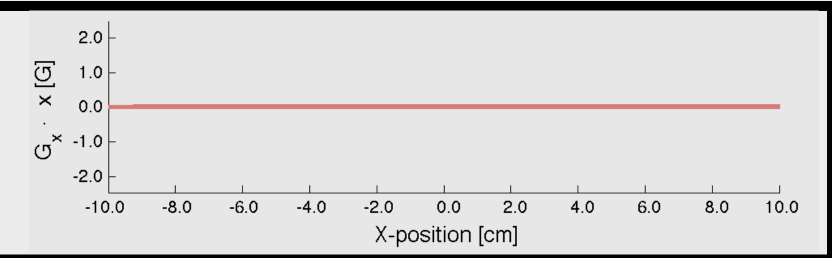
Frequency Encoding

$B_0 - G_x \cdot x$ B_0 $B_0 + G_x \cdot x$



Frequency Encoding





Applied Magnetic Field

Receiver Bandwidth

- **High Receiver Bandwidth (RBW, Δf)**
  - Stronger gradients
  - Larger range of frequencies across the FOV (or pixel)
  - Less chemical shift (larger freq. difference per pixel)
  - Lower SNR (shorter acquisition time)
  - Shorter TE (move across *k*-space faster)

$$\Delta f = \frac{1}{2}\frac{\gamma}{2\pi} G_x \cdot FOV_x$$

User can pick 2 of 3 (Δf, G_x, FOV_x)

Temporal Nyquist Sampling Requires: $\Delta t = \frac{1}{2\Delta f}$

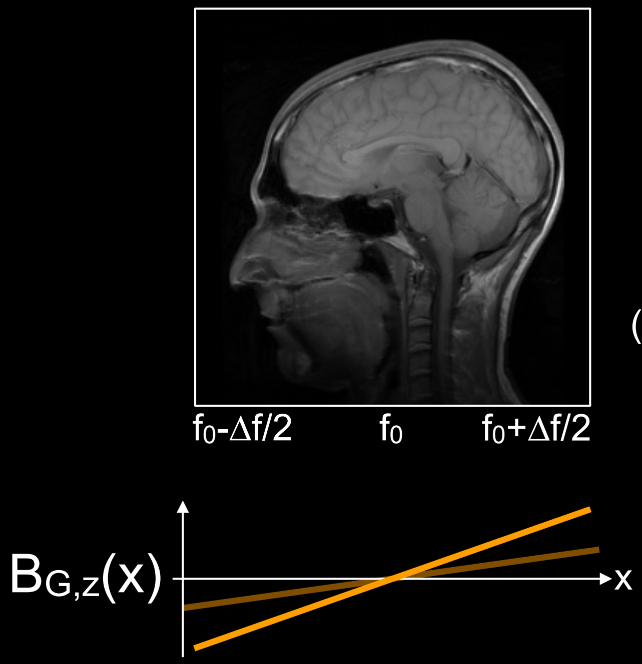
k-space Nyquist Sampling Requires: $\Delta k_x = \frac{\gamma}{2\pi} G_x \Delta t$

$$\Delta k_x = \frac{1}{FOV_x}$$

$$N_x \cdot \Delta k_x = \frac{N_x}{FOV_x} = \frac{1}{\Delta x}$$

Readout Gradient Amplitude

- **High Receiver Bandwidth (RBW, Δf)**
  - Stronger gradients
  - Larger range of frequencies across the FOV (or pixel)
  - Less chemical shift (larger freq. difference per pixel)
  - Lower SNR (shorter acquisition time)
  - Shorter TE (move across *k*-space faster)



$$\Delta f = \frac{1}{2}\frac{\gamma}{2\pi}G_x \cdot FOV_x$$

User can pick 2 of 3 (Δf, G_x, FOV_x)

Receiver Bandwidth (e.g. 32kHz)

Field of View (e.g. 30cm)

$$\begin{aligned} G_x &= \frac{2 \cdot \Delta f}{\gamma FOV_x} \\ &= \frac{2 \cdot 32000\mathrm{Hz}}{4258\,\frac{\mathrm{Hz}}{\mathrm{G}} \cdot 30\mathrm{cm}} \\ &= 0.501\,\frac{\mathrm{G}}{\mathrm{cm}} \end{aligned}$$

Readout Gradient Duration

- **High Receiver Bandwidth (RBW, Δf)**
  - Stronger gradients
  - Larger range of frequencies across the FOV (or pixel)
  - Less chemical shift (larger freq. difference per pixel)
  - Lower SNR (shorter acquisition time)
  - Shorter TE (move across *k*-space faster)

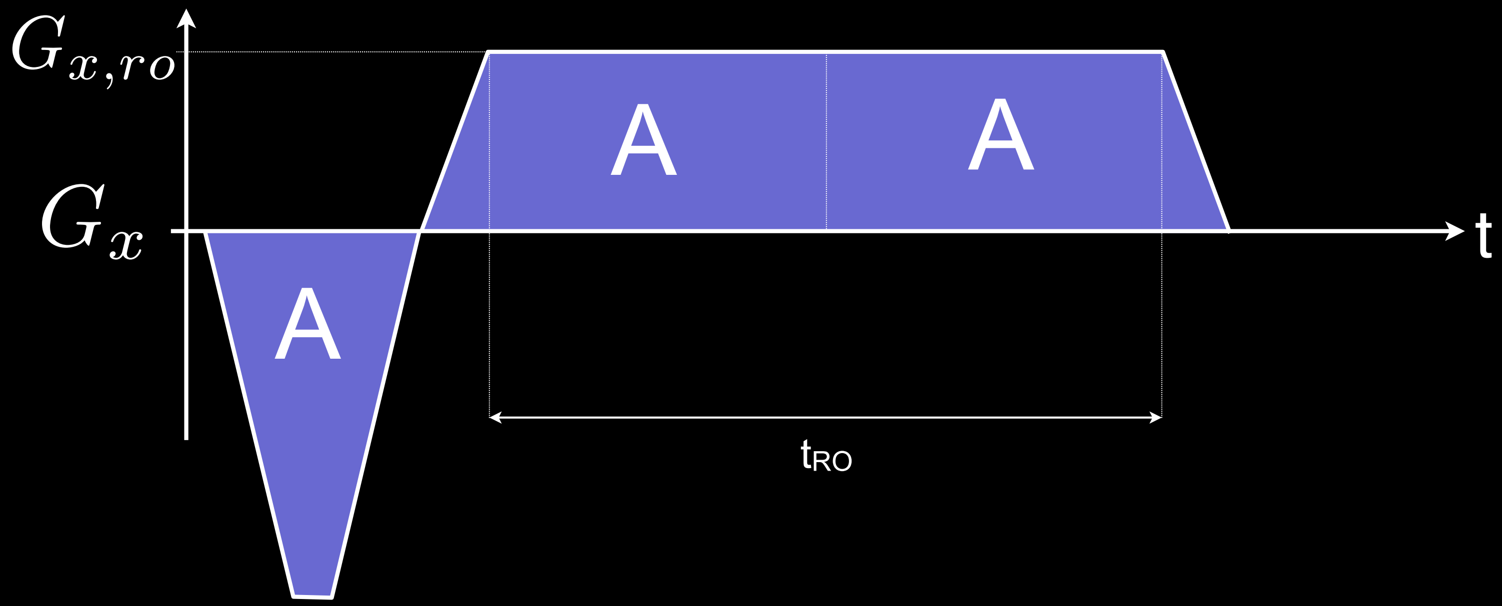
Temporal Nyquist Sampling Requires: $\Delta t = \frac{1}{2\Delta f}$

$$\begin{aligned}\Delta t &= \frac{1}{2\Delta f}\\ &= \frac{1}{2 \cdot 32000\mathrm{Hz}}\\ &= 15.625\mu\mathrm{S}\end{aligned}$$

$$\begin{aligned}\tau_{RO} &= N_{read} \cdot \Delta t\\ &= 128 \cdot 15.625\mu\mathrm{S}\\ &= 2000\mu\mathrm{S}\end{aligned}$$

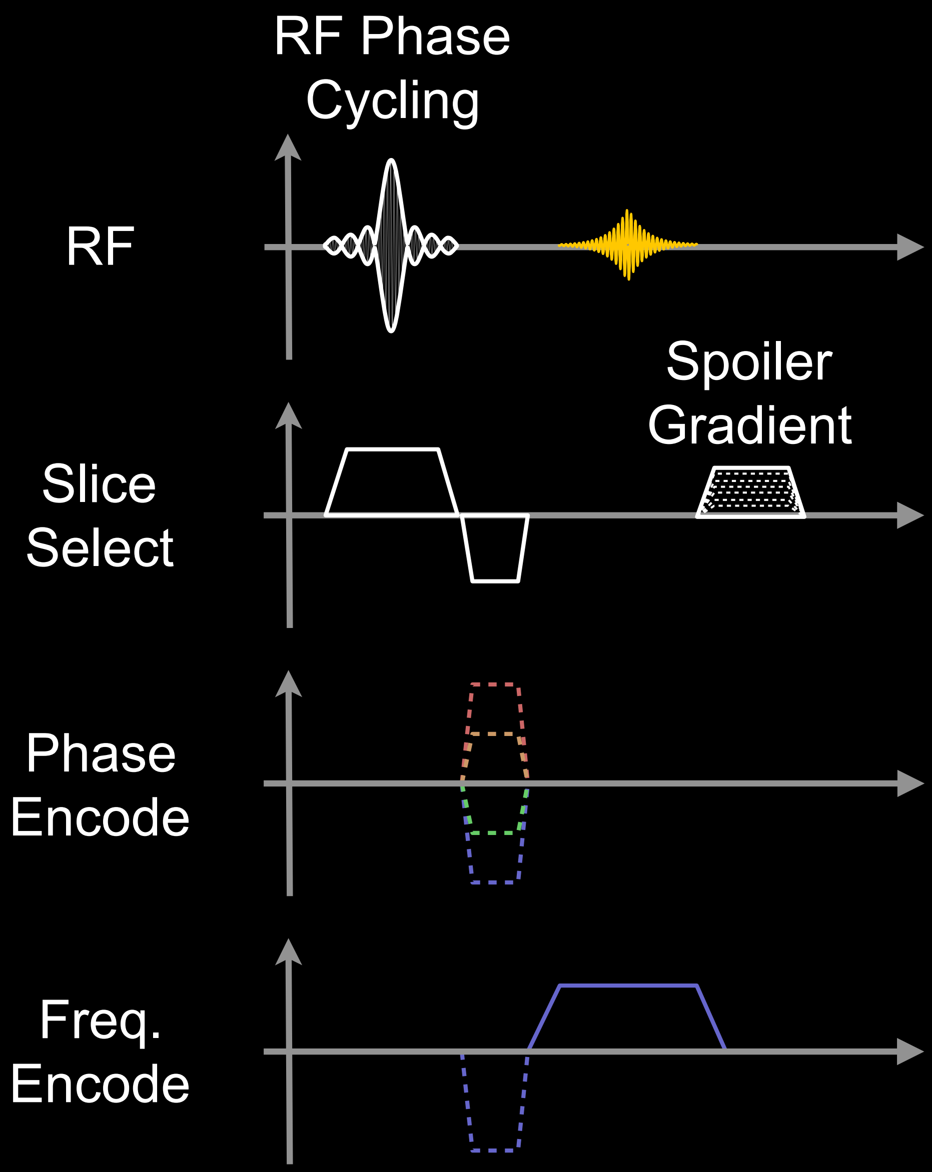
Readout Gradient Pre-Phaser

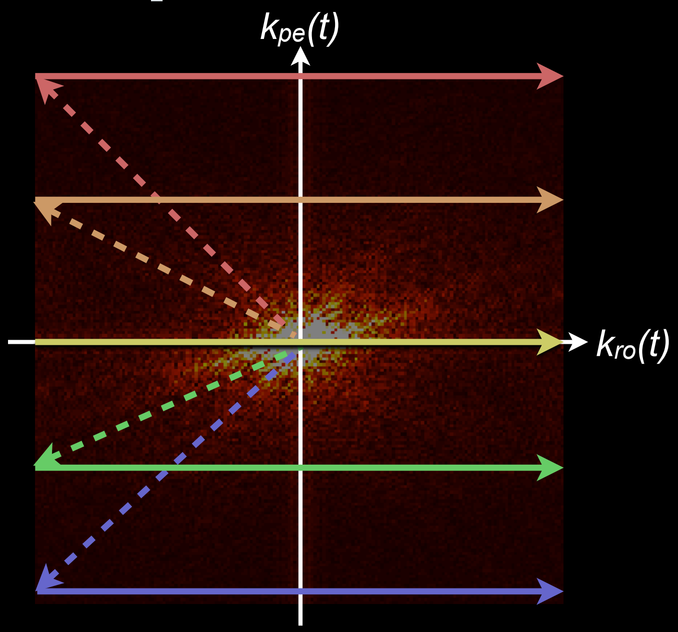
A’=A for symmetric k-space coverage.



The readout pre-phasing gradient area is half the readout gradient area.

Gradient Echo Sequence

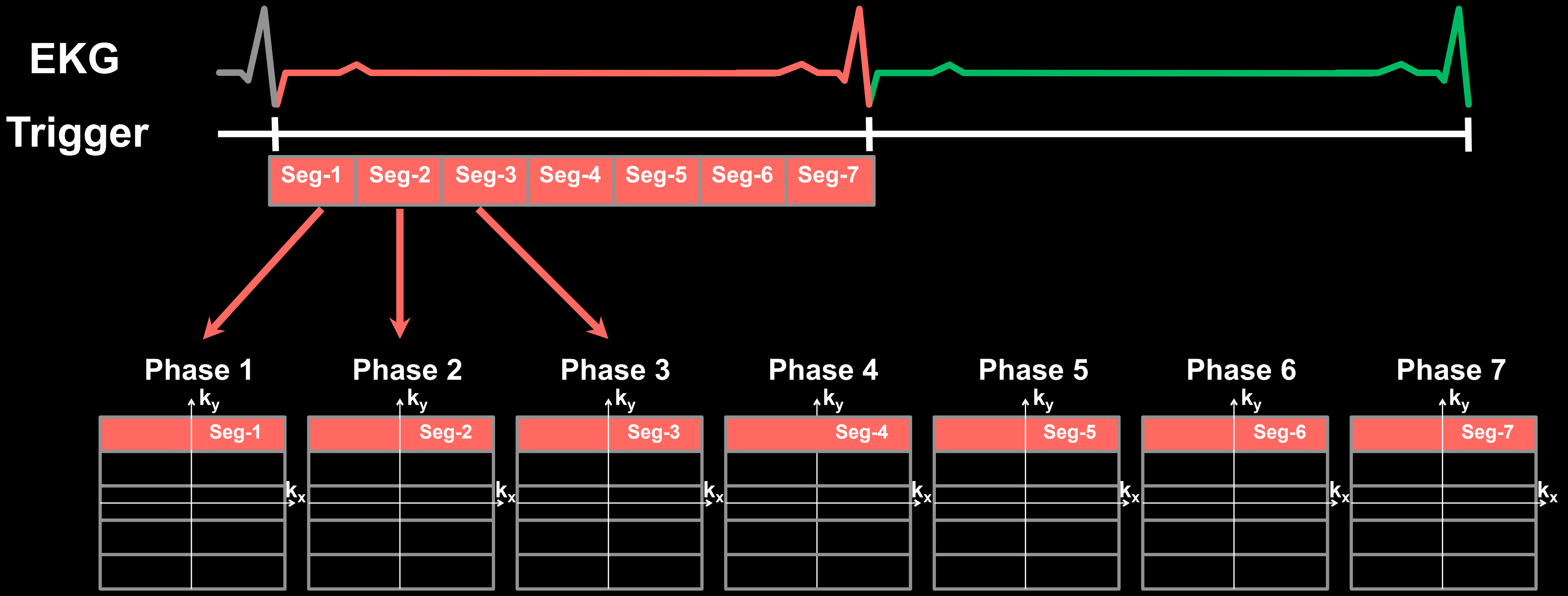


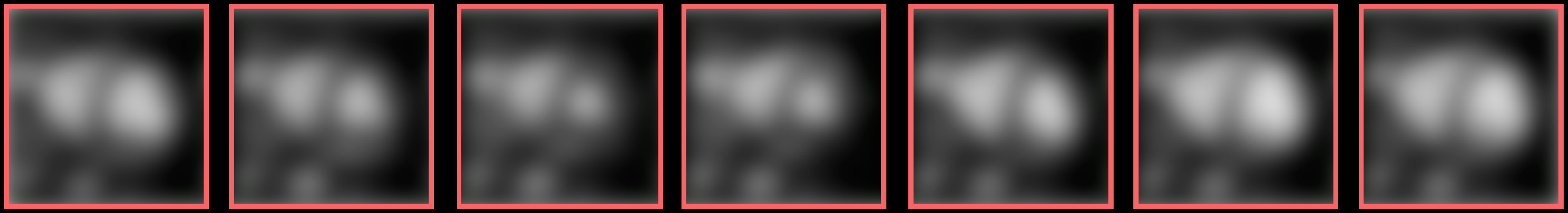


For sequence efficiency several gradients can overlap.

MRI is slow.
How do we make movies?

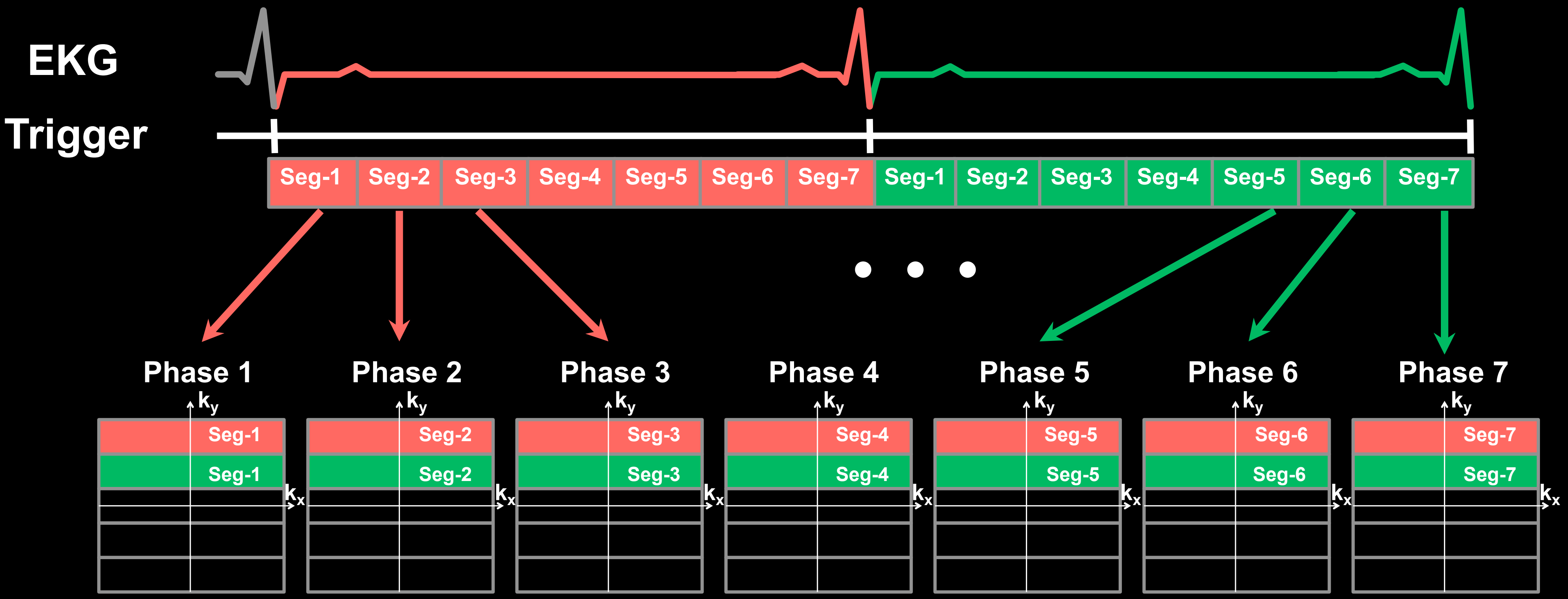
Segmented Cardiac Imaging





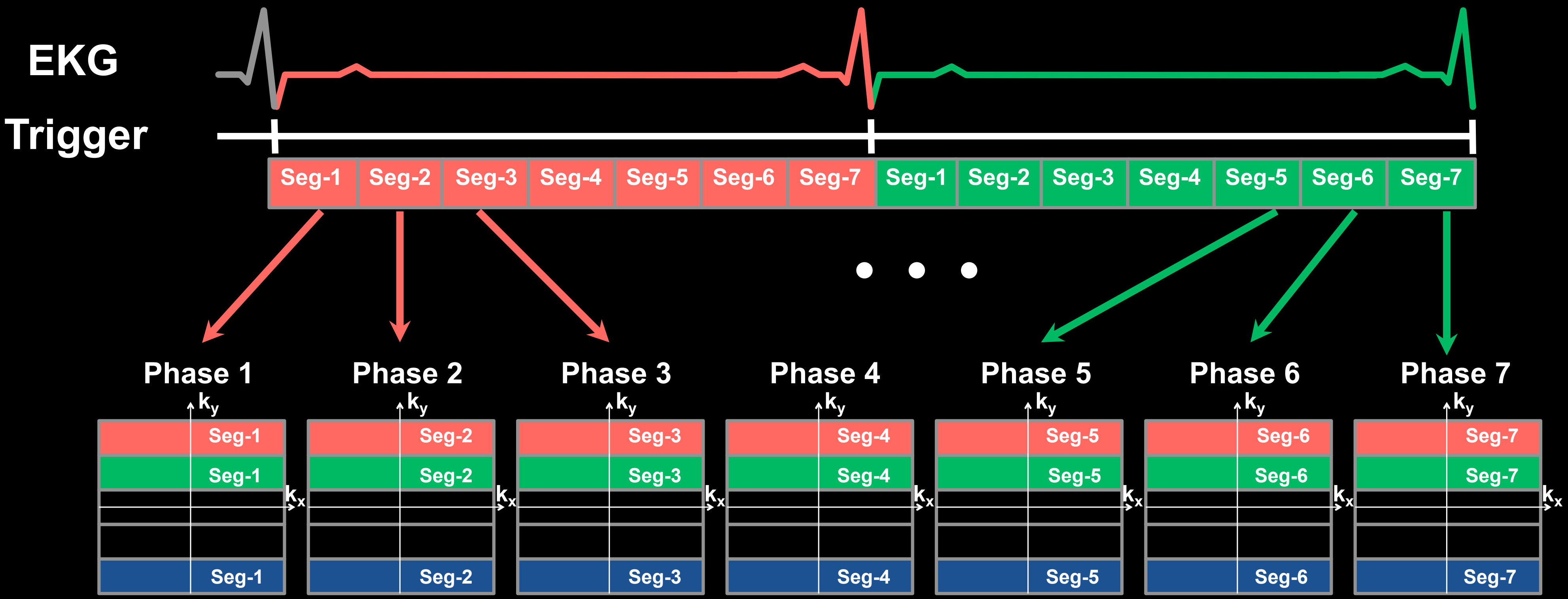
A *k*-space ***segment*** is a few lines of k-space.

Segmented Cardiac Imaging



Each heartbeat acquires a unique *k*-space segment.

Segmented Cardiac Imaging



EKG

Trigger

Seg-1 Seg-2 Seg-3 Seg-4 Seg-5 Seg-6 Seg-7 Seg-1 Seg-2 Seg-3 Seg-4 Seg-5 Seg-6 Seg-7

• • •

Phase 1
k_y
Seg-1
Seg-1
k_x
Seg-1

Phase 2
k_y
Seg-2
Seg-2
k_x
Seg-2

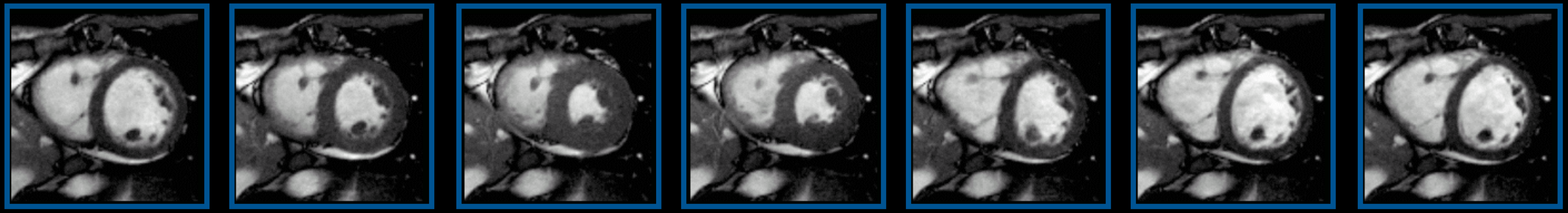
Phase 3
k_y
Seg-3
Seg-3
k_x
Seg-3

Phase 4
k_y
Seg-4
Seg-4
k_x
Seg-4

Phase 5
k_y
Seg-5
Seg-5
k_x
Seg-5

Phase 6
k_y
Seg-6
Seg-6
k_x
Seg-6

Phase 7
k_y
Seg-7
Seg-7
k_x
Seg-7



Once all heartbeats are acquired a movie can be played.

Thanks

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