





Lecture #5 Learning Objectives

- Explain what the most important equations of motion are for describing spin systems for MRI.
- Understand the assumptions about the B₀, B₁, and Gradient fields.
- Be able to define the magnetic field at any point in space when a gradient is applied.
- Appreciate the gradients change the phase or frequency of the bulk magnetization as a function of spatial position.
- Describe the safety concerns for gradients.

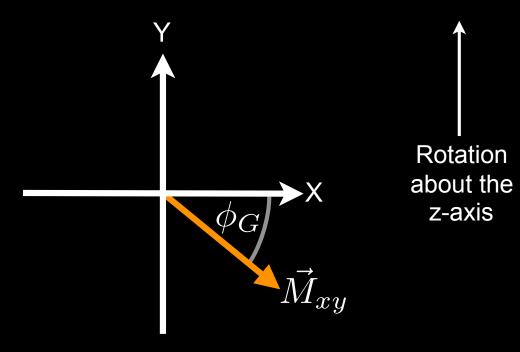




Phase from a Gradient

$$\phi_G(\vec{r},t) = -\gamma \int_0^t \vec{G}(\tau) \cdot \vec{r}(\tau) d\tau$$

$$R_G = \begin{bmatrix} \cos(\phi_G) & -\sin(\phi_G) & 0\\ \sin(\phi_G) & \cos(\phi_G) & 0\\ 0 & 1 \end{bmatrix}$$





Homogeneous Coordinate Gradient Operator

$$\mathbf{G}_{\vec{G}}^{\vec{r}} = \begin{bmatrix} \cos\left(\gamma\int_0^{\tau}\vec{G}(t)\cdot\vec{r}(t)dt\right) & -\sin\left(\gamma\int_0^{\tau}\vec{G}(t)\cdot\vec{r}(t)dt\right) & 0 & 0\\ \sin\left(\gamma\int_0^{\tau}\vec{G}(t)\cdot\vec{r}(t)dt\right) & \cos\left(\gamma\int_0^{\tau}\vec{G}(t)\cdot\vec{r}(t)dt\right) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{M}_H(0_+) = G_{\vec{G}}^{\vec{r}} \vec{M}_H(0_-)$$





Gradient Operator

```
% This function returns the 4x4 homogenous coordinate expression for a
% gradient (G) acting on a spin at position (p) for a duration (dt) for a
% particular gyromagnetic ratio (gamma).
% SYNTAX: dG=PAM Grad op(gamma,G,dt,p)
% INPUTS: gamma - gyromagnetic ratio [Hz/T]

    Gradient vector

                                    [G/cm] (3x1)
              time step
                                     [s]
               - spin position
                                    [cm] (3x1)
% OUTPUT: RF - RF pulse operator [4x4]
% DBE@UCLA 2015.01.21
function dG=PAM Grad op(gamma,G,dt,p)
% Angle of rotation imparted on the spin by the gradient
phi=-2*pi*gamma*(dot(G,p)/10000)*dt % Convert gauss to Tesla
% Homogeneous expression for the GRADIENT MATRIX
dG=[cos(phi) -sin(phi) 0 0;
    sin(phi) cos(phi) 0 0;
                      10;
                      0 1];
return
```



Lecture #6 Learning Objectives

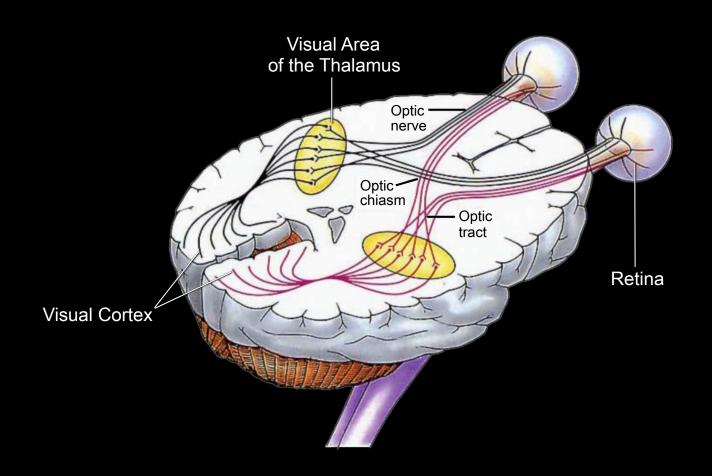
- Appreciate the definition of image contrast.
- Explain what a T1 or T2-weighted image is.
- Describe what a pulse sequence is.
- Understand the saturation recovery pulse sequence and the saturation condition.
- Describe the inversion recover sequence.
- Distinguish between STIR and FLAIR.





Image Contrast

Why Image Contrast?

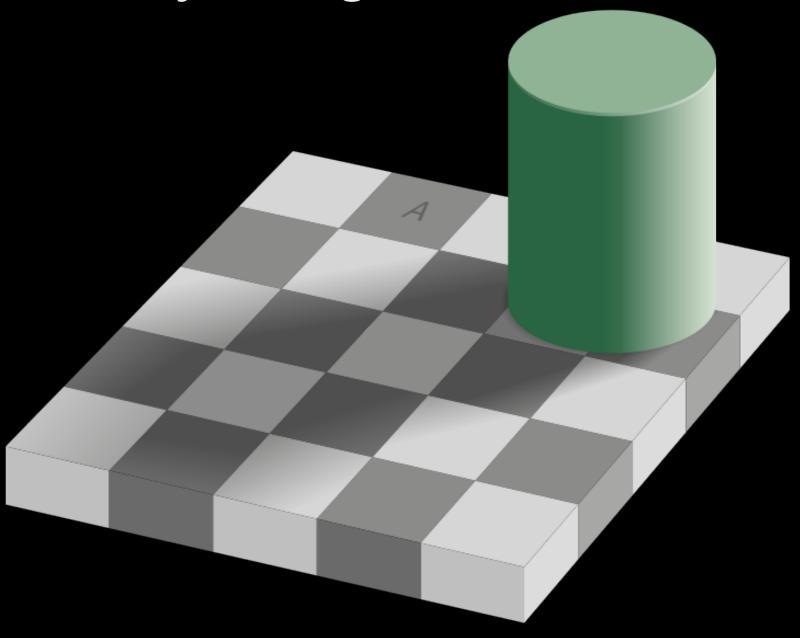


The human visual system is more sensitive to contrast than absolute luminance.





Why Image Contrast?



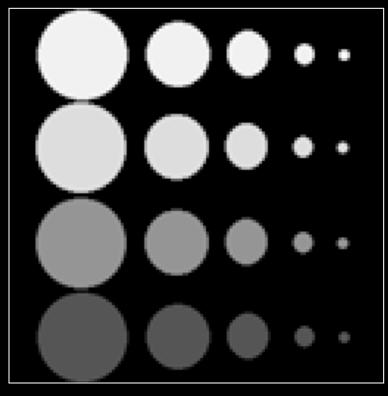
Which is brighter A or B?



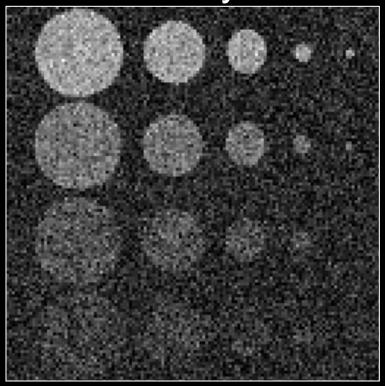


CNR, Object Size, and Noise

Noise Free



Noisy



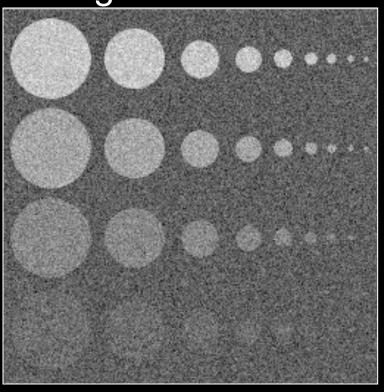
Large high-contrast objects are easier to see in the presence of noise.



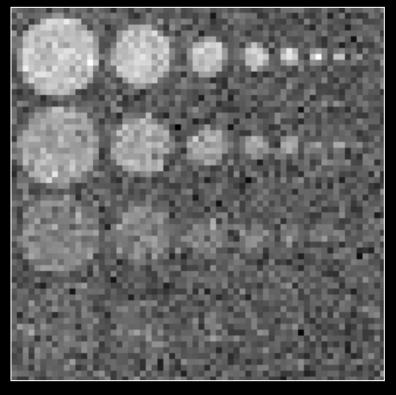


CNR, Resolution, and Noise

High Resolution



Low Resolution



Small low-contrast objects are easier to see with higher resolution.





Image Contrast

$$\mathcal{C}_{AB} = \frac{|I_A - I_B|}{I_{ref}}$$

$$C_{AB} = f(\rho, T_1, T_2, T_2^*, D, ...)$$

$$C_{AB} \approx f(T_1)$$
 $C_{AB} \approx f(T_2)$

$$C_{AB} \approx f(T_2)$$

A central goal in MRI is to limit image contrast to a single mechanism.



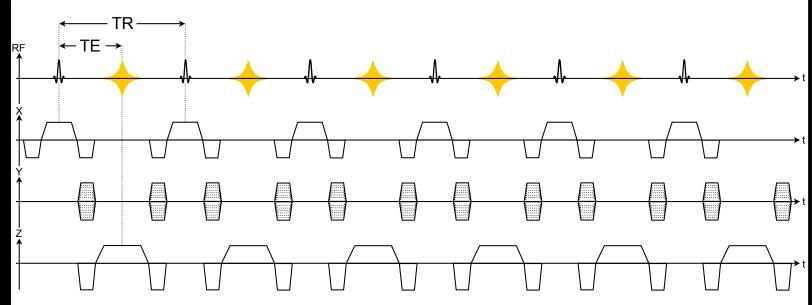


Pulse Sequences

What is a pulse sequence?



Sheet music is a *timing diagram* for playing the piano.





Pulse Sequences

Gradients & RF Pulses

Contrast Module

Imaging Module

> time



Inversion Recovery
T2-preparation
Fat Saturation
Tagging



Spoiled Gradient Echo SPGR, FLASH, T1 FFE

bSSFP

FIESTA, Balanced FFE, True FISP

Spin Echo

Fast or Turbo Spin Echo





How do we keep track of the magnetization's history?

Pulse Sequence Definitions

$$\mathcal{M}_{z}^{(n)}\left(0_{-}\right)$$

Longitudinal magnetization before the nth event.

$$\mathcal{M}_{z}^{(n)}\left(0_{+}\right)$$

Longitudinal magnetization *after* the *n*th event.

$$\mathcal{M}_{xy}^{(n)}\left(0_{-}\right)$$

Transverse magnetization **before** the *n*th event.

$$\mathcal{M}_{xy}^{(n)}\left(0_{+}\right)$$

Transverse magnetization after the nth event.





Free? Forced? Relaxation?

- We've considered all combinations of:
 - Free or forced precession
 - With or without relaxation
 - Laboratory or rotating frames
- Which one's concern M219 the most?
 - Rotating frame
 - Free precession with relaxation

$$M_z(t) = M_z^0 e^{-\frac{t}{T_1}} + M_0 \left(1 - e^{-\frac{t}{T_1}}\right)$$
 $M_{xy}(t) = M_{xy}^0 e^{-t/T_2}$

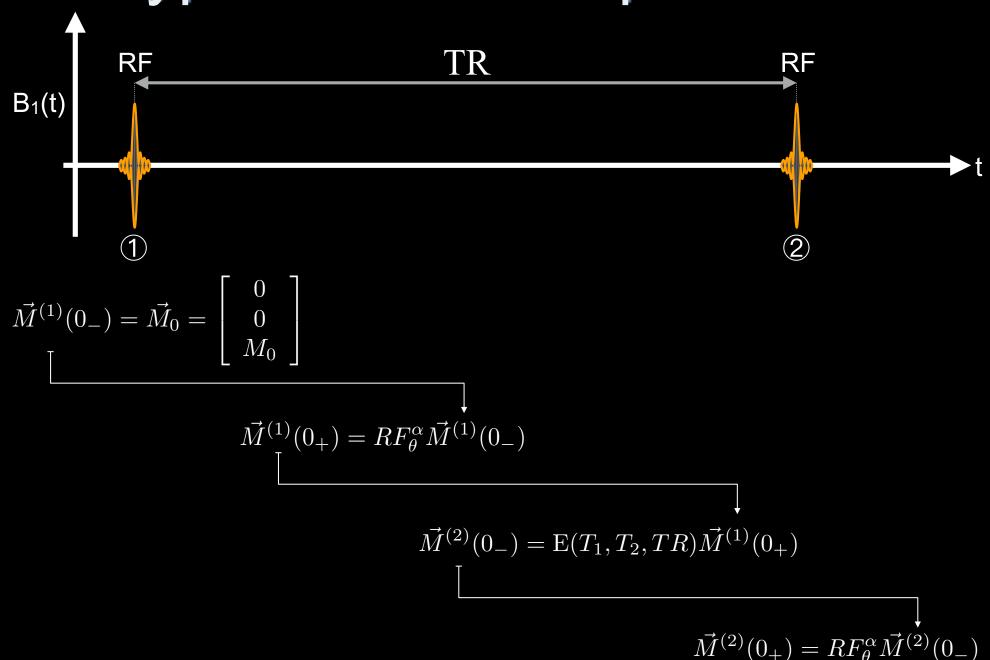
Forced precession without relaxation

$$\vec{M}^{(n)}(0_{+}) = \begin{bmatrix} c^{2}\theta + s^{2}\theta c\alpha & c\theta s\theta - c\theta s\theta c\alpha & -s\theta s\alpha \\ c\theta s\theta - c\theta s\theta c\alpha & s^{2}\theta + c^{2}\theta c\alpha & c\theta s\alpha \\ s\theta s\alpha & -c\theta s\alpha & c\alpha \end{bmatrix} \vec{M}^{(n)}(0_{-})$$





Typical Pulse Sequence...





UCLA Radiology

Saturation Recovery

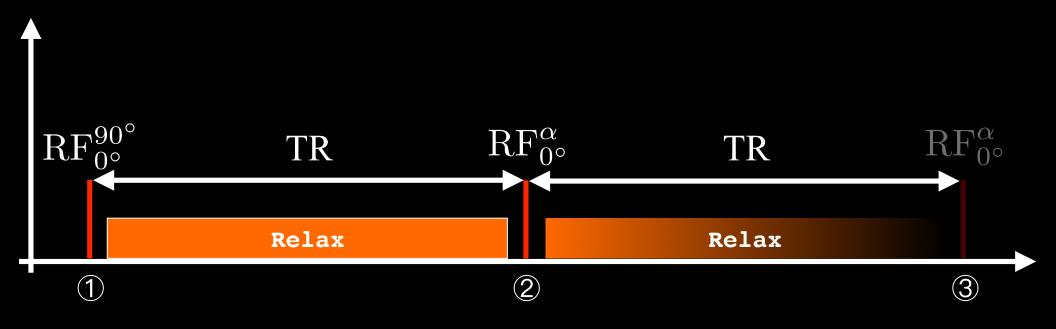
Pulse Sequence Definitions

- TR Repetition Time
 - Duration of basic pulse sequence repeating block
 - At least one echo acquired per TR
- TE Echo Time
 - Time from excitation to the maximum of the echo
 - Data is recorded at time TE to form an image





Saturation Recovery



$$(90^{\circ} - \text{TR})_{N}$$

To The Board...





The saturation condition states:

$$M_z^{(n)}(0_+) = 0, n \ge 1$$

Mz is ZERO after the event (RF pulse).



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- This is true if the M_{xy} is "gone" before the next 90° RF-pulse is applied:
 - No M_{xy} to convert to M_z
 - How? TR>>T2



• The *saturation condition* states:

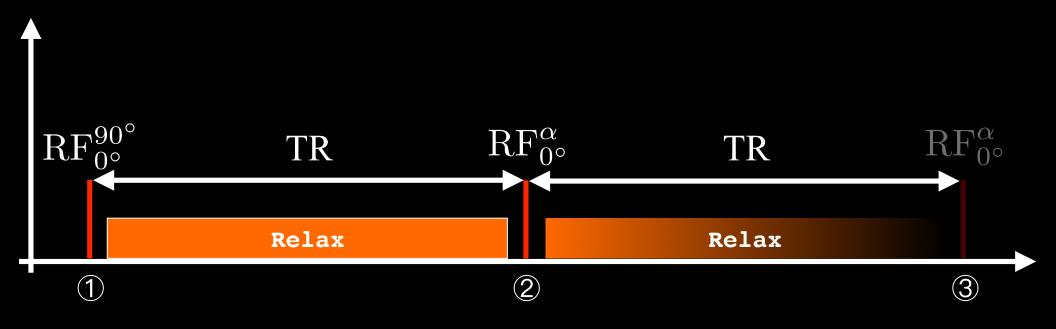
$$M_z^{(n)}(0_+) = 0, n \ge 1$$

M_z is ZERO after the event (RF pulse).

- This is true if the M_{xy} is "gone" before the next 90° RF-pulse is applied:
 - No M_{xy} to convert to M_z
 - How? TR>>T2
- What if $TR < ~3T_2$?
 - M_{xy} can be converted back to M_z
 - Corrupts/complicates image contrast
 - Solution? Spoiler gradients to disperse M_{xy}
- Steady-state solution arises if the saturation conditions are met/enforced



Saturation Recovery



$$(90^{\circ} - \text{TR})_{N}$$

To The Board...





SR Contrast

$$A_{fid} \propto {
m M}_z^0 \left(1-e^{-TR/T_1}
ight) \propto
ho \left(1-e^{-TR/T_1}
ight)$$
 Eqn. 7.13

- A_{fid} Signal amplitude immediately after the 90°.
- ρ proton density.
- If the process of imaging doesn't perturb the magnetization:

$$I\left(ec{r}
ight) \propto
ho\left(ec{r}
ight) \left(1-e^{-TR/T_{1}\left(ec{r}
ight) }
ight)$$
 Eqn. 7.14





SR Contrast

$$I(\vec{r}) \propto \rho(\vec{r}) \left(1 - e^{-TR/T_1(\vec{r})}\right)$$

The final image is the product of $\rho(r)$ and $f(T_1(r))$.

$$I\left(\vec{r}\right)_{TR\to\infty}\propto\rho\left(\vec{r}\right)$$

The image pure pure $\rho(r)$ contrast under this limit.

- Note only one parameter adjusts contrast
 - Longer T₁s appear darker with short TRs
- Long T1 will be dark.
- Short T1 will be bright.





SR Contrast

$$I(\vec{r})_{TR \to TR_{opt}} \propto \text{Maximum } T_1 \text{ contrast}$$

$$TR_{opt} = rac{\ln\left(rac{T_{1,A}}{T_{1,B}}\right)}{rac{1}{T_{1,B}} - rac{1}{T_{1,A}}}$$

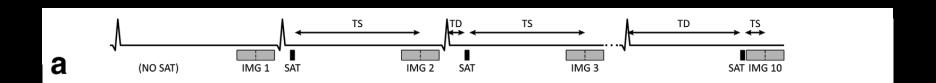
Eqn. 7.19





Saturation Recovery - Applications

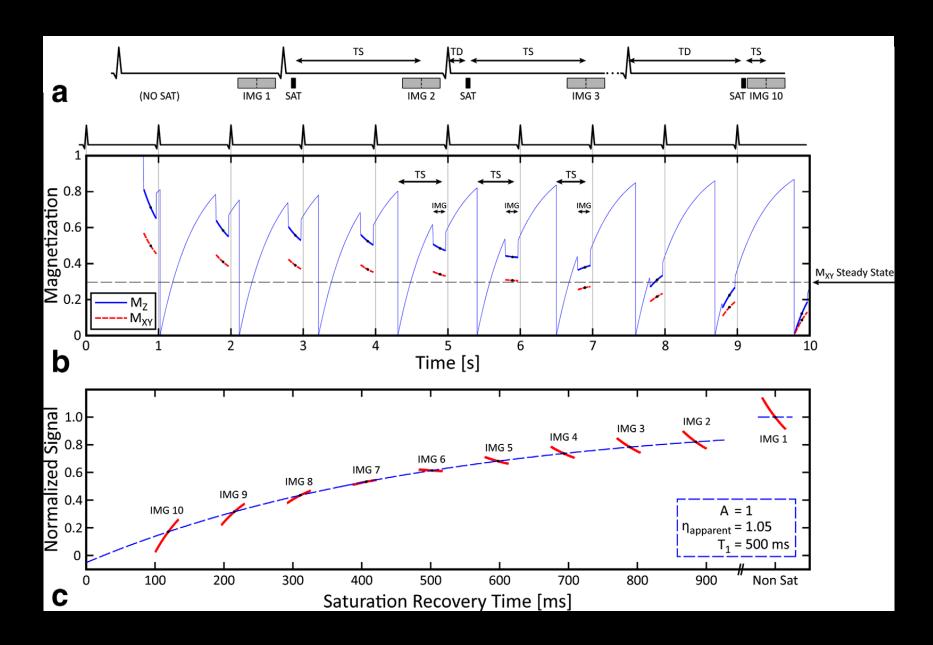
SAturation recovery single-SHot Acquisition (SASHA)







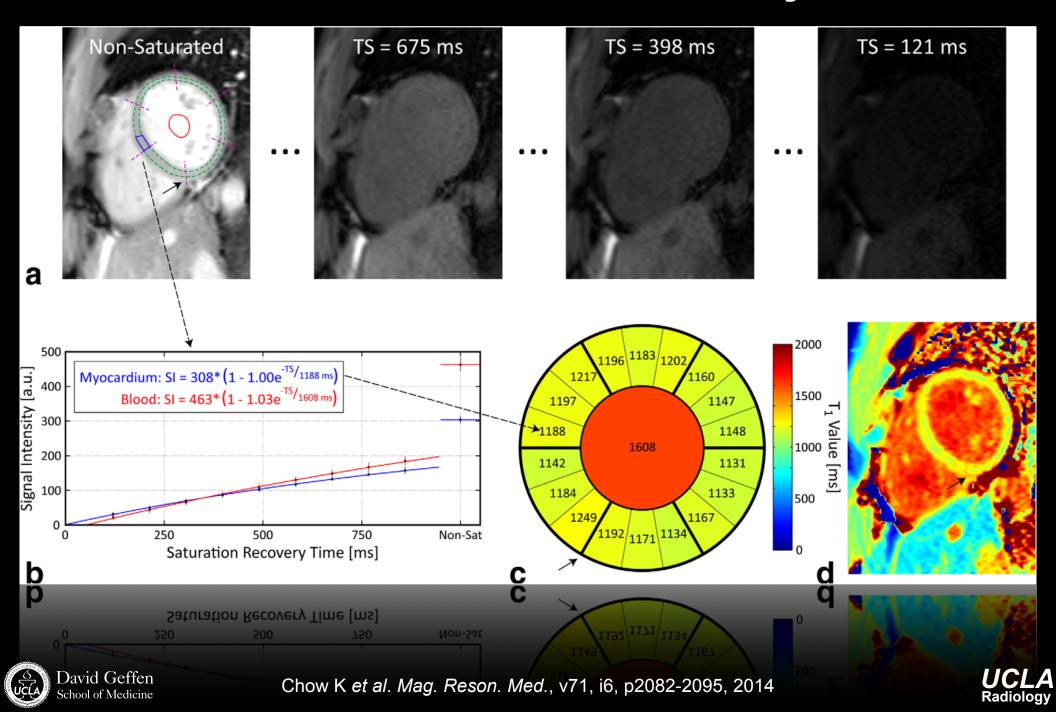
SAturation recovery single-SHot Acquisition (SASHA)



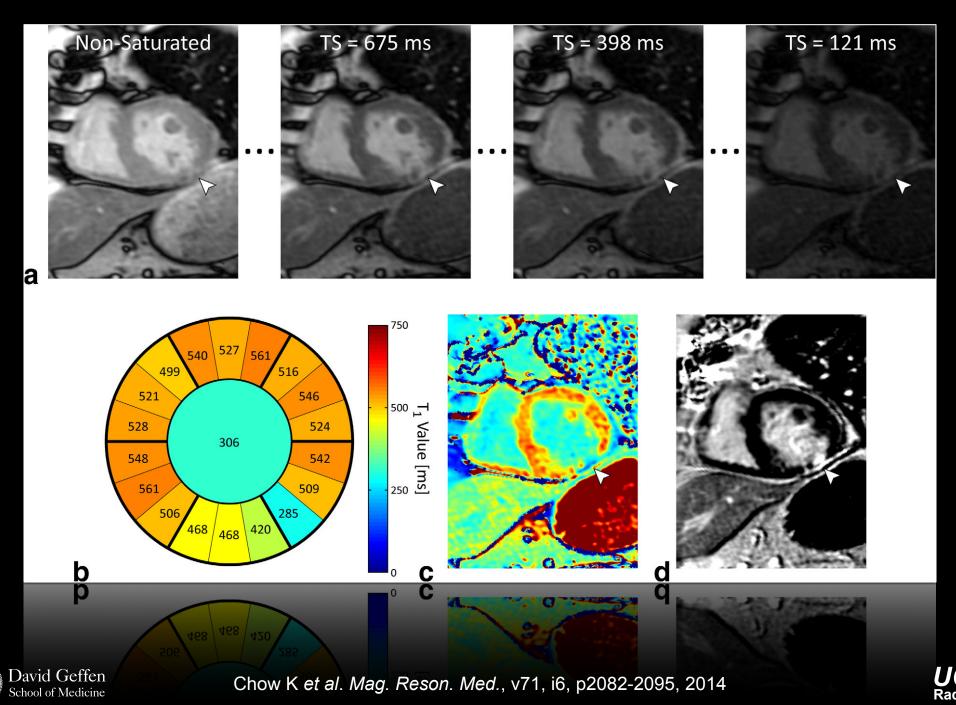




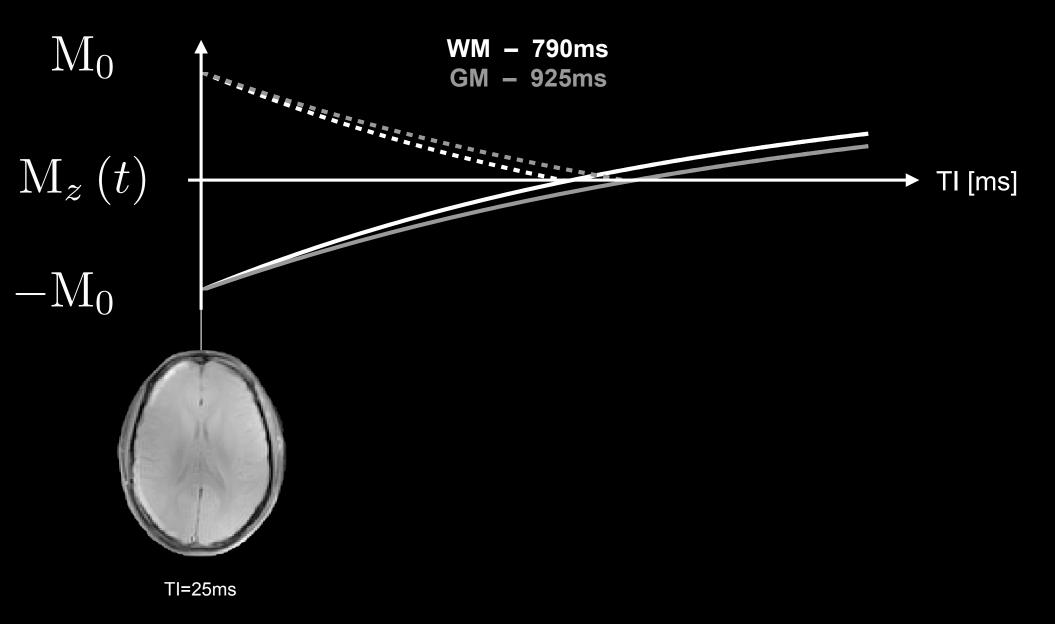
SASHA - Normal Subject



SASHA - Myocardial Infarct

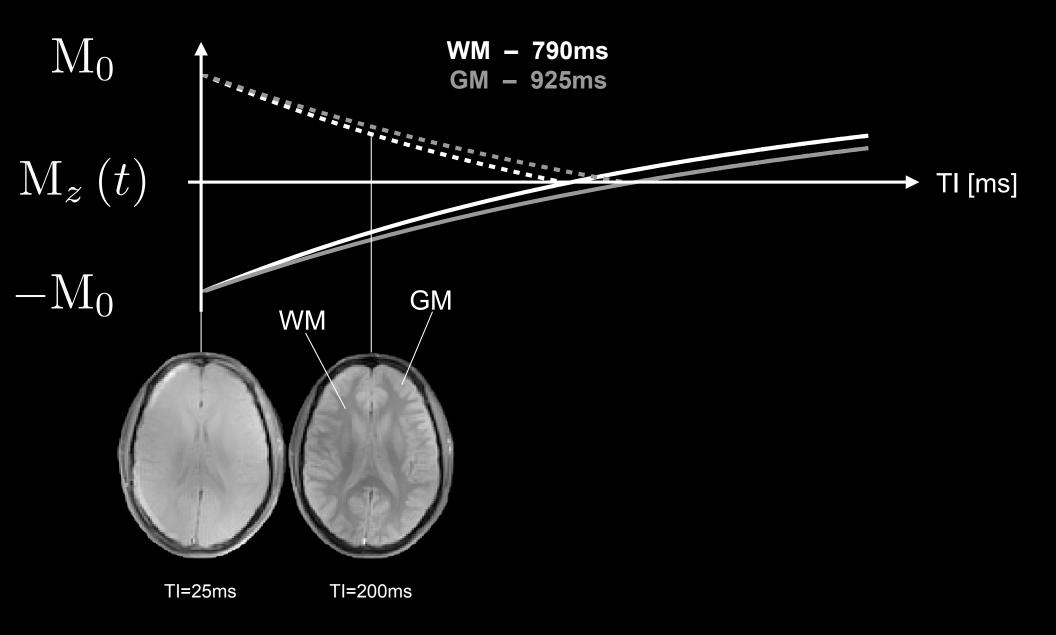


Inversion Recovery



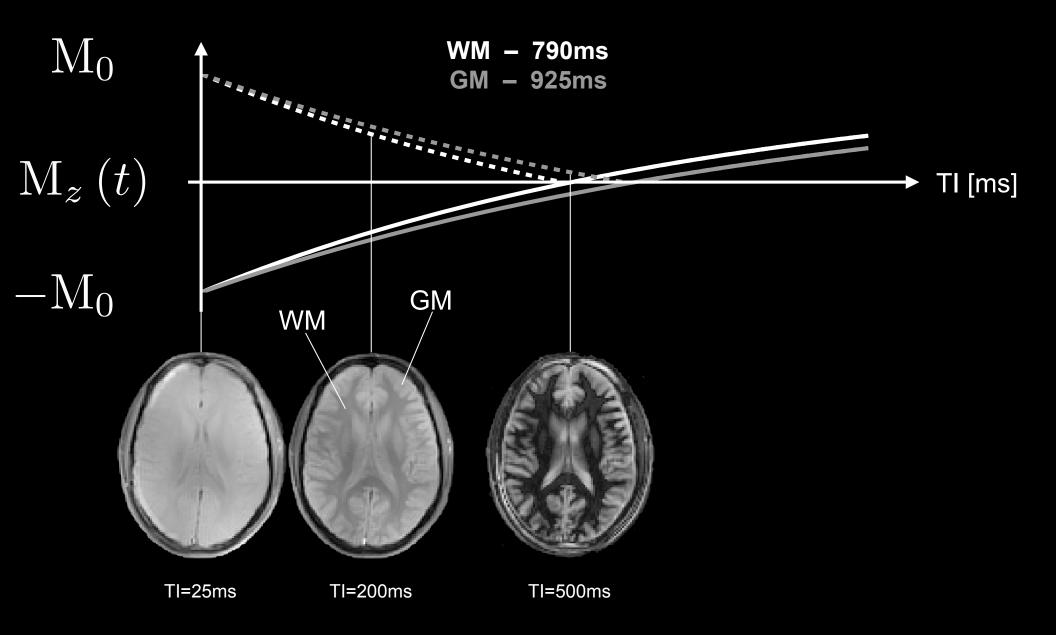






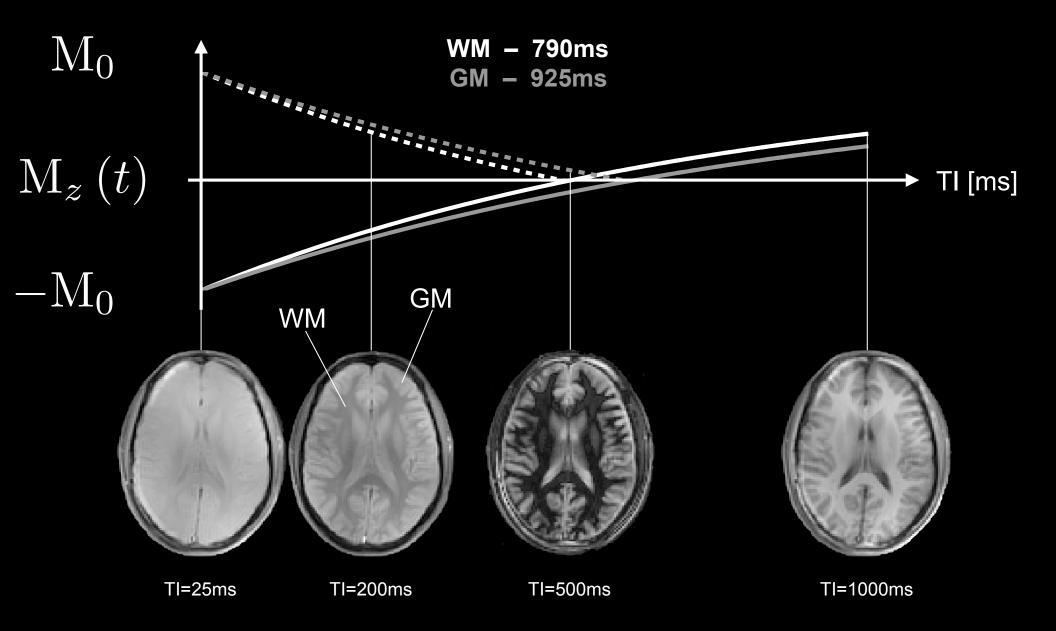
















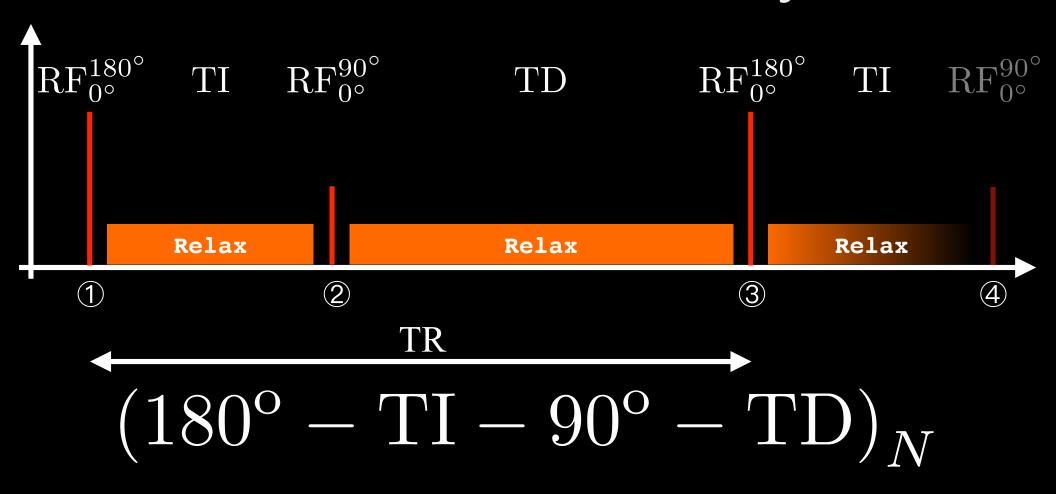




TI=200ms TI=500ms TE=12ms, TR=2000ms

TI=1000ms

Inversion Recovery



To The Board...





IR Contrast

$$A_{fid} \propto \rho \left(1-2e^{-TI/T_1}+e^{-TR/T_1}\right)$$

$$I\left(\vec{r}\right) \propto \rho \left(\vec{r}\right) \left(1-2e^{-TI/T_1(\vec{r})}+e^{-TR/T_1(\vec{r})}\right)$$
 Eqn. 7.21

The final image is the product of $\rho(r)$ and $f(T_1(r))$.

The final image contrast is controlled by TI and TR.





IR Signal Nulling Effect

$$TI_{null} = \left[\ln 2 - \ln \left(1 + \exp^{-TR/T_1^0}\right)\right] T_1^0$$

$$TI_{null} = [\ln 2] T_1^0$$
, if TR $\longrightarrow \infty$

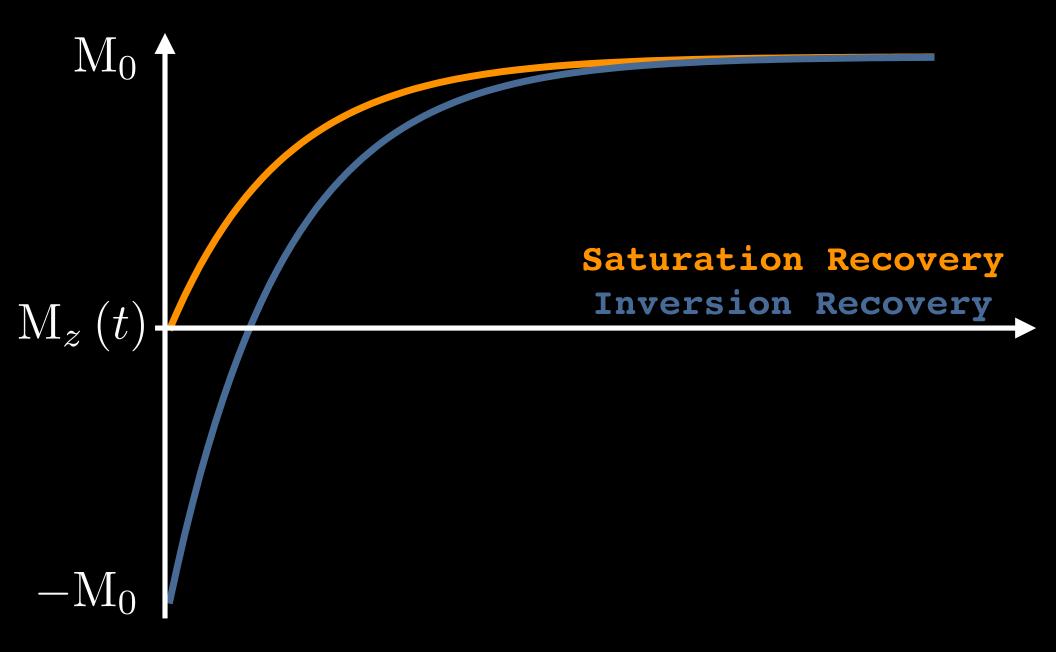
$$I(\vec{r}) = 0$$
, if $T_1(\vec{r}) = T_1^0(\vec{r})$





Target T₁

SR vs. IR







Inversion Pulse - Applications

- Greater T₁ contrast than SR
- T₁ species nulling/attenuation
 - FLAIR (Fluid Attenuated Inversion Recovery)
 - STIR (Short Tau Inversion Recovery)
- IR is better than SR for generating contrast when:
 - $\rho(A) = \rho(B)$ and $T_2(A) = T_2(B)$
 - AND
 - $T_1(A)$ and $T_1(B)$ are slightly different
- Quantitative T₁ mapping

$$I\left(\overrightarrow{r}
ight) \propto
ho\left(\overrightarrow{r}
ight) \left(1-2e^{-TI/T_{1}\left(\overrightarrow{r}
ight) }+e^{-TR/T_{1}\left(\overrightarrow{r}
ight) }
ight)$$
 Eqn. 7.21

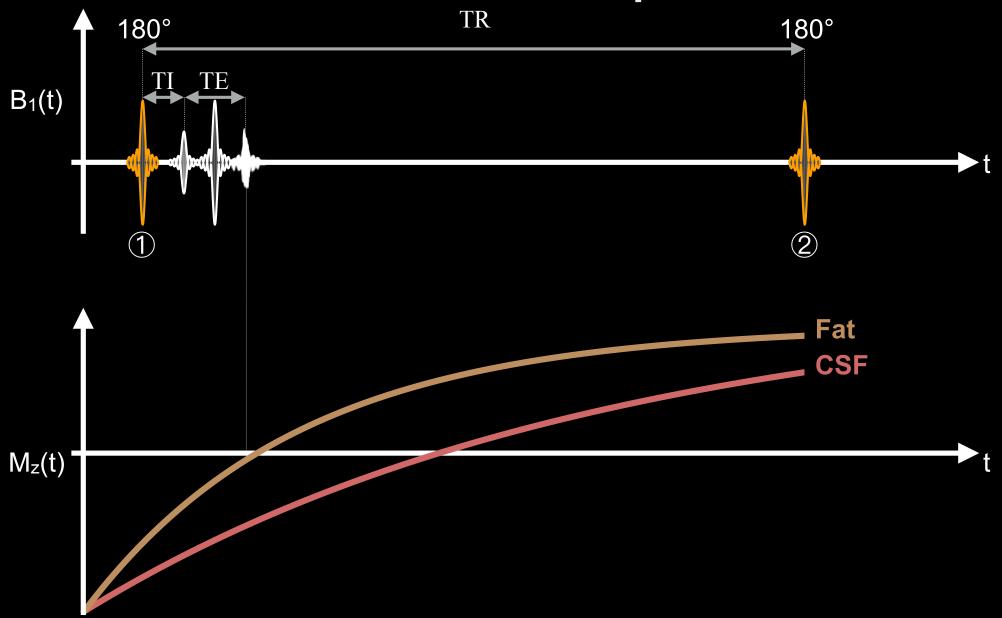
The final image is the product of $\rho(r)$ and $f(T_1(r))$.

The final image contrast is controlled by TI and TR.





STIR Pulse Sequence

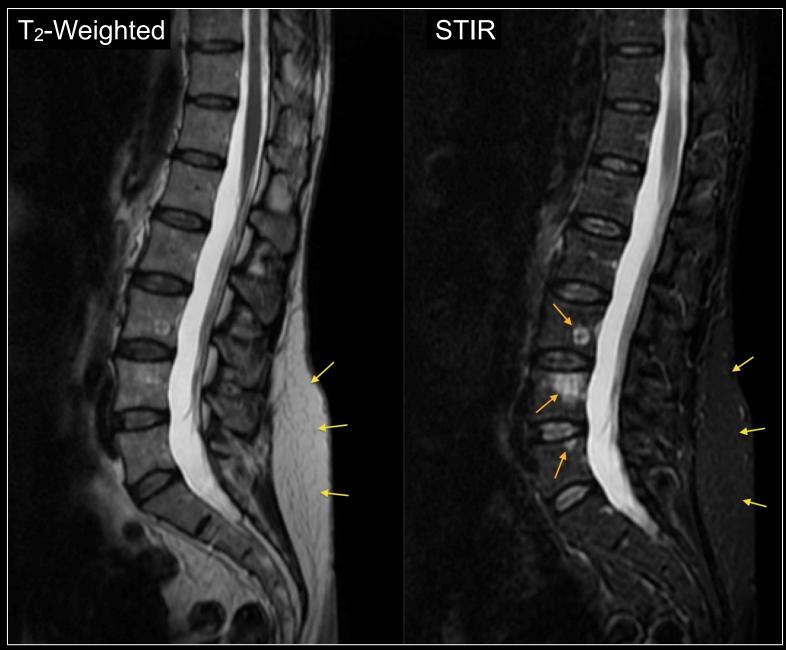


Short Tau Inversion Recovery (STIR) is used to null fat.





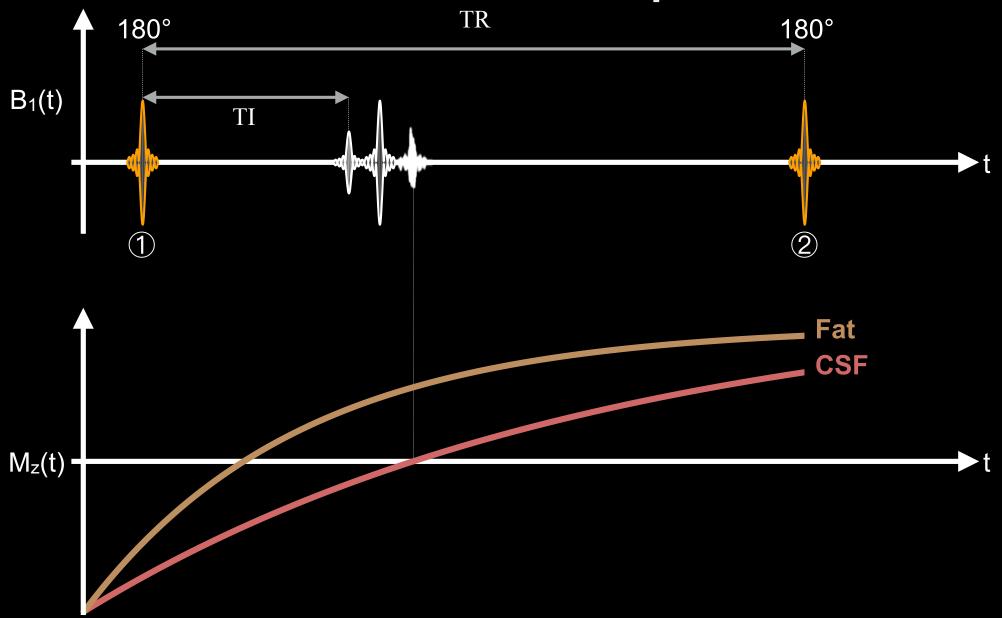
STIR Images







FLAIR Pulse Sequence



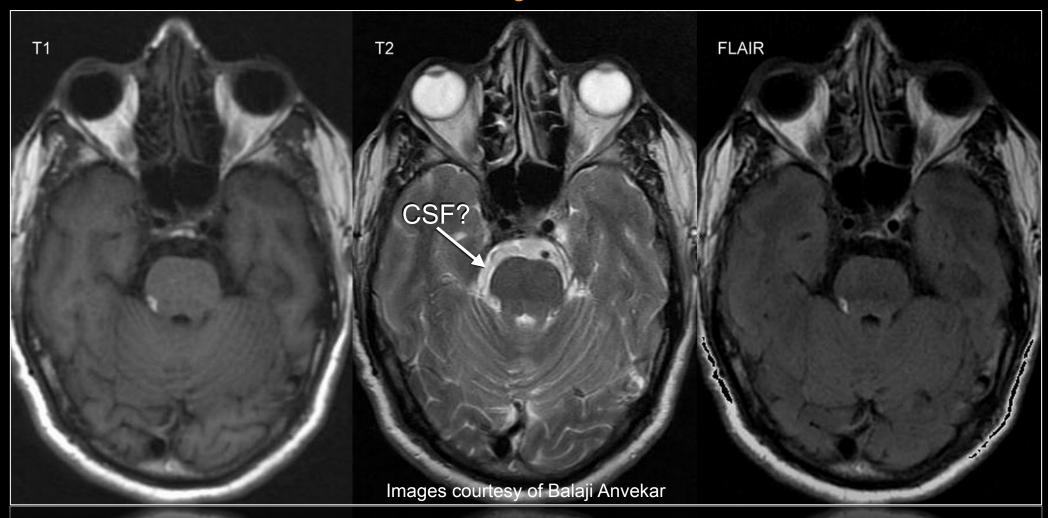
FLuid Attenuated Inversion Recovery (STIR) is used to CSF.





FLAIR Images

FLAIR can distinguish fat from CSF.







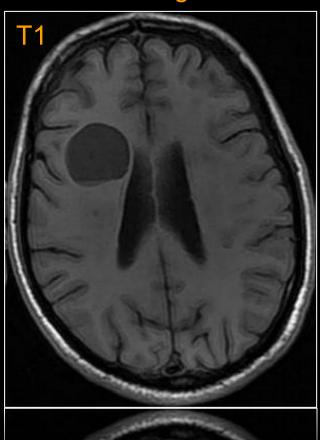
FLAIR Images

Long T2 is bright on T2w.

Short T1 is bright on T1w.

Long T1 is dark on FLAIR.







Lesion has long T2 and intermediate T1. Not fat. Not CSF. Cerebral hydatid.





Thanks



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