Gradient Waveform Design

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Radiology

Class Business

- **Tuesday (3/7) from 6-9pm**
  - **6:00-7:30pm Groups**
    - **Avanto**
      - Sara Said, Yara Azar, April Pan
    - **Skyra**
      - Timothy Marcum, Diana Lopez, Zhaohuan Zhang
    - **Prisma**
      - Daisong Zhang, Jingwen Yao, Fang-Chu Lin, Andy Vuong
  - **7:30-9:00pm Groups**
    - **Avanto**
      - Binru Chen, Junjie Chen, Yuhua Chen
    - **Skyra**
      - Jie Fu, Qihui Lyu, Cass Wong
    - **Prisma**
      - Nyasha Maforo, Fadil Ali, Vahid Ghodrati

Lecture #15 - Learning Objectives

- Distinguish Type-1 and Type-2 chemical shift artifacts, their origin, and mitigation.
- Describe advantages and disadvantages of two partial fourier acquisition methods.
- Explain the advantages and disadvantages of multi-slice imaging.
- Explain the advantages and disadvantages of multi-echo imaging.
- Identify ways to improve imaging protocols.

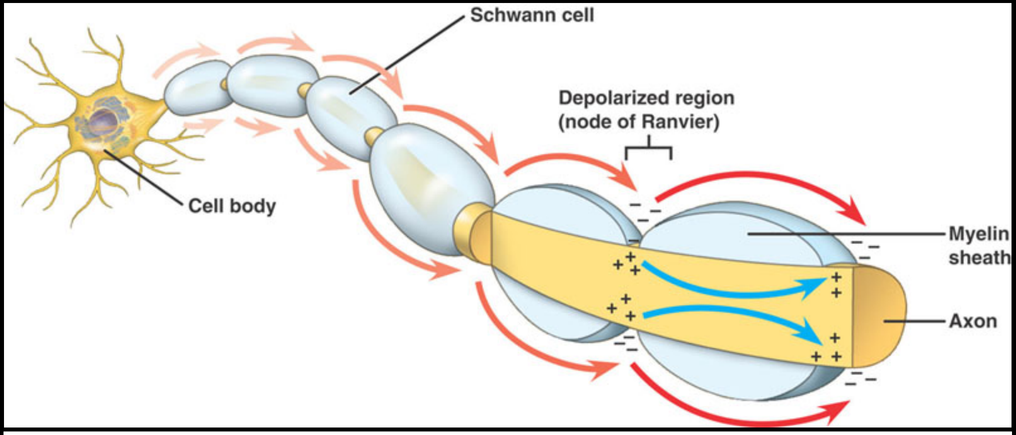
Lecture #16 - Learning Objectives

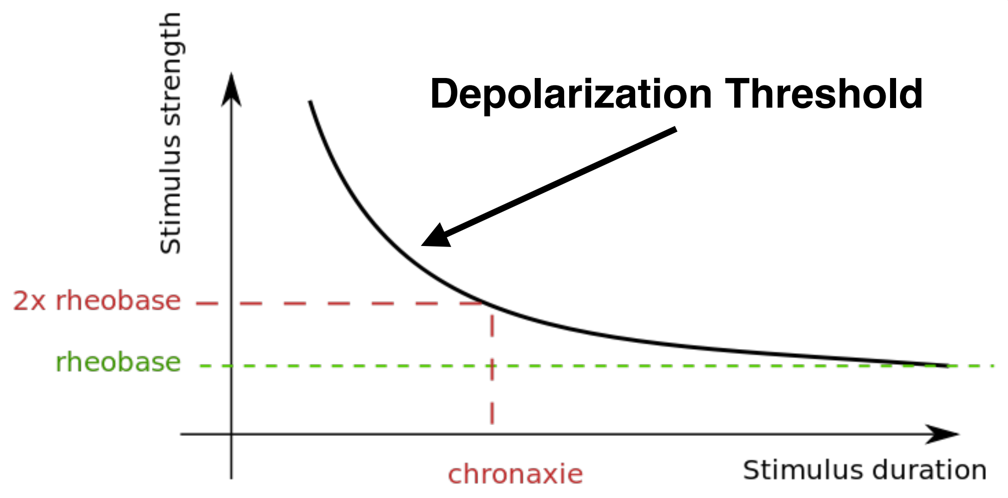
- Describe the safety concerns that relate to the use of magnetic field gradients.
- Know how to calculate the parameters for the slice selection gradient.
- Appreciate the importance of the slice-select re-phasing gradient.
- Learn how to design the phase encode gradient waveforms.
- Understand which gradient waveforms can overlap and which can not.
- Learn how to design the frequency encode gradient waveforms.
- Understand the origin and impact of eddy currents.

Gradient Safety

Gradient Safety

- **Noise**
- **Peripheral nerve stimulation (PNS)**





Solution: De-rate gradient slew rates, but this increases scan time.

Solution: Ear plugs Head phones

Time-varying gradients induce mechanical vibrations and PNS.

MRI Gradient Noise

Switching the gradients on ms time scales (kHz) generates acoustic noise.

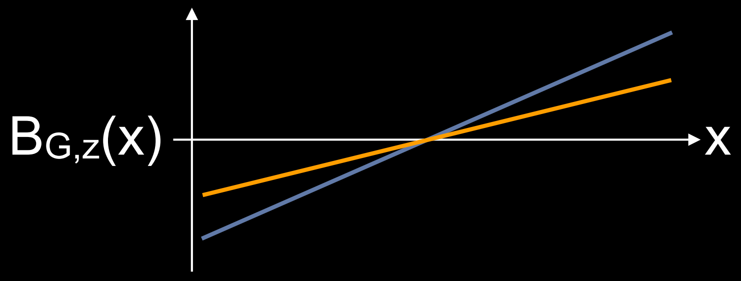
Gradient Noise

Jet take-off @ 25m	~150 dB (eardrum rupture)
Car horn @ 1m	~110 dB (borderline painful)
Live rock band	~100 dB
MRI gradients full load	≤99 dB
Garbage disposal	~80 dB
MRI gradients basic load	≤75 dB
Radio or TV Audio	~70dB

Siemens: mri-magnetom-essenza-environmental_product_declaration-00079271.pdf

Gradient Safety – G_{Max}

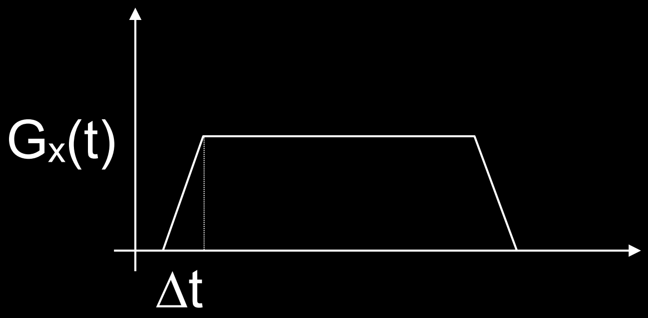
- **G_{max} limitations:**
  - Concern: None known.
    - B_0 is already pretty big.
  - Conventional Gradients
    - G_{Max} = 4 to 5G/cm (=50mT/m)
  - Cutting Edge Gradients
    - G_{Max} = 8G/cm (=80mT/m)
  - Connectome Gradients
    - G_{Max} = 30G/cm (=300mT/m)
  - Consider the ΔB contributed by a gradient...



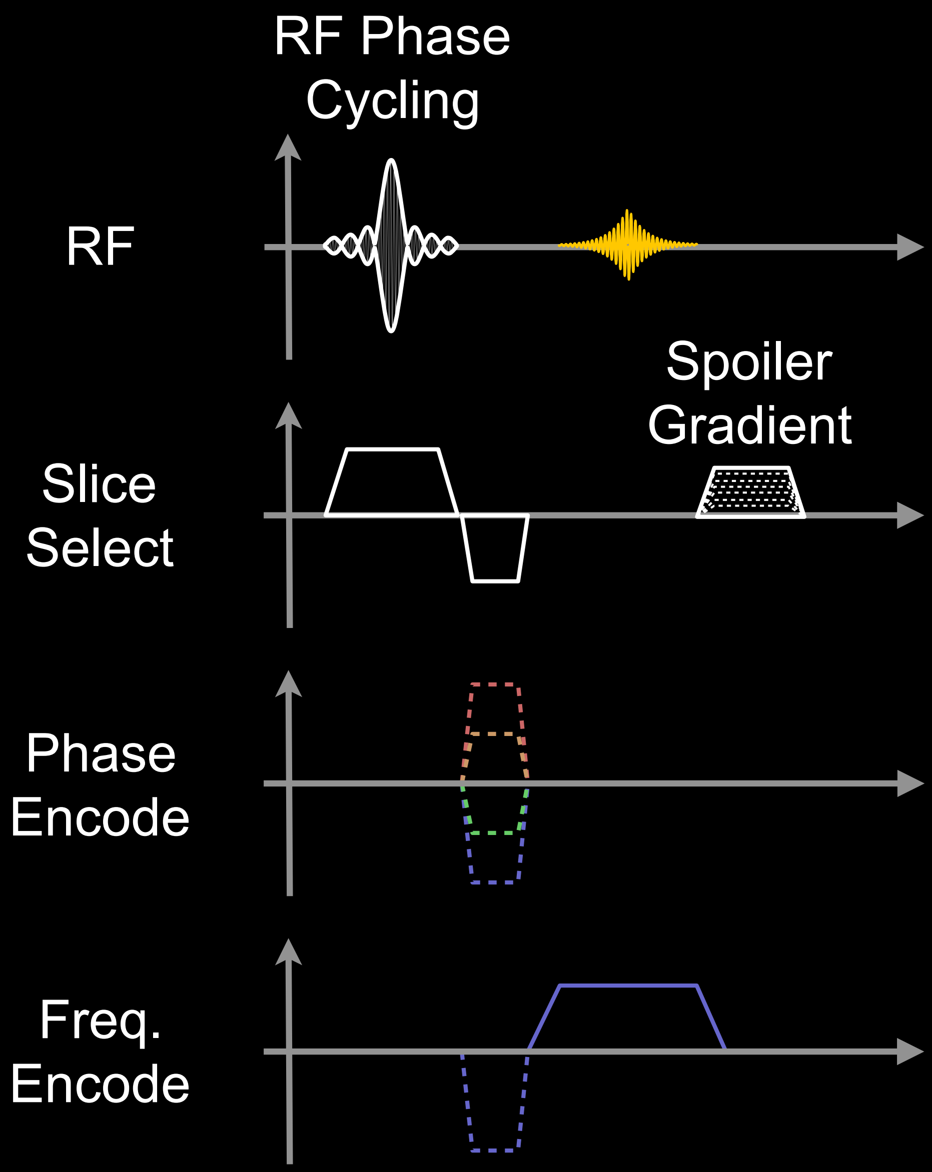
http://www.aapm.org/meetings/02AM/pdf/8356-48054.pdf

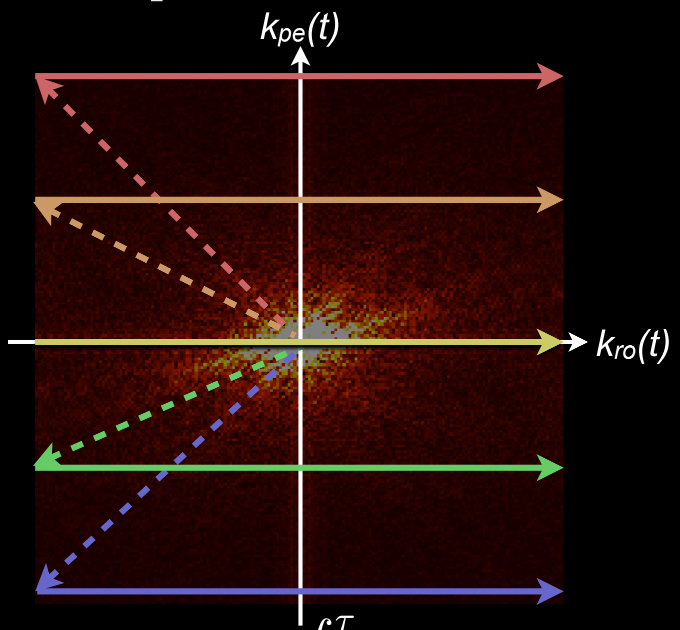
Gradient Slewrate

- **Gradient slew rate**
  - T/m/s (or G/cm/s)
  - dG/dt – Rate of change of gradient amplitude
- **Slew rate limited by dB/dt:**
  - Concern: Peripheral Nerve Stimulation
  - Regulated by FDA
  - Normal Mode: dB/dt=16 T/s•(1+0.36/ß)
  - First Level Mode: dB/dt=20 T/s•(1+0.36/ß)
  - ß=stimulus duration [ms]

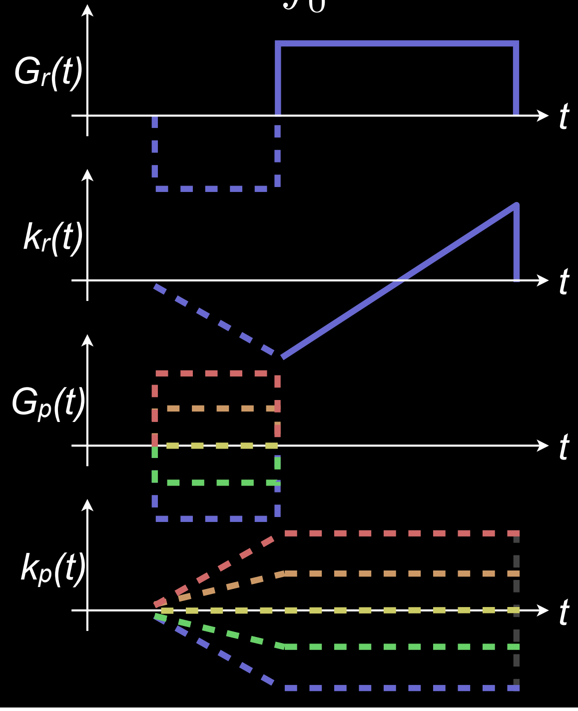


Gradient Echo Sequence





$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\, d\tau$$



One phase encoded echo is acquired per TR.

Slice Select Gradients

Selective Excitation

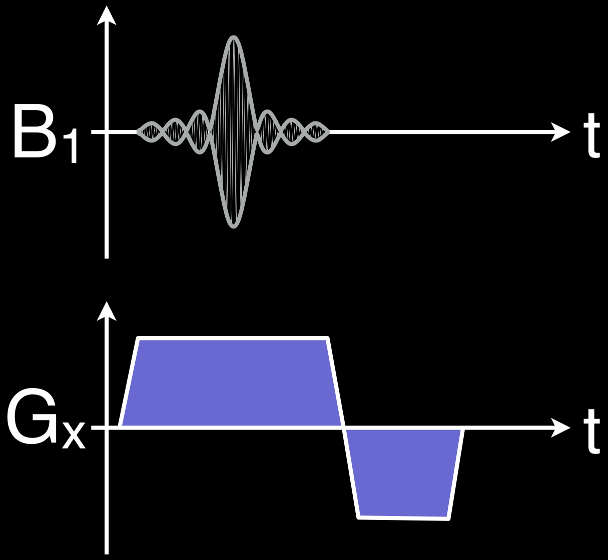
- **What factors control slice selection?**

$B_1^e(t)$ **Pulse envelope function**

(e.g. $B_{1,max}$ and $\Delta\omega$)

ω_{RF} **Excitation carrier frequency**

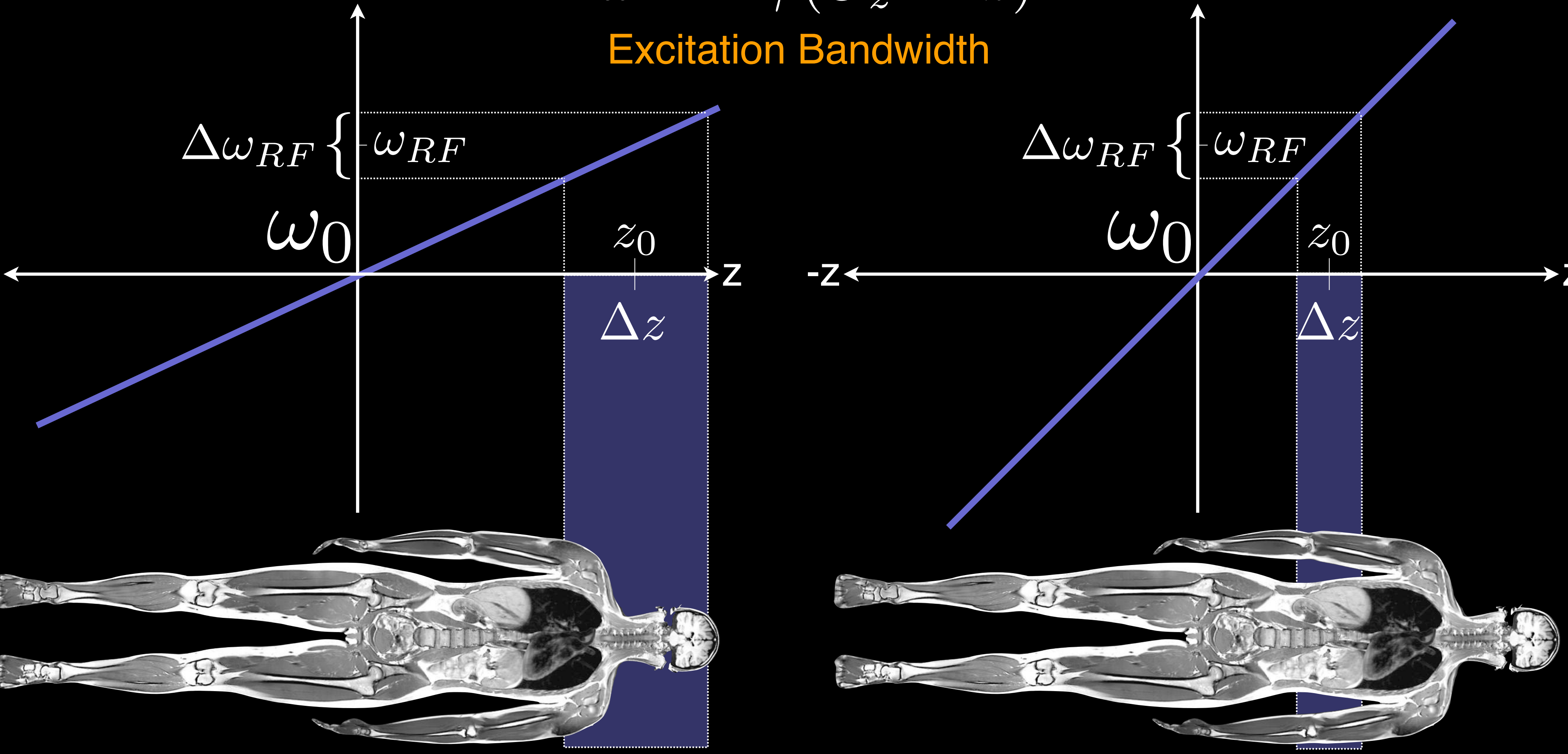
$\vec{G}$ **Gradient amplitude**



Slice Selective Excitation

$$\Delta\omega = -\gamma\left(G_z \cdot \Delta z\right)$$

Excitation Bandwidth



Slice-A

Slice-B

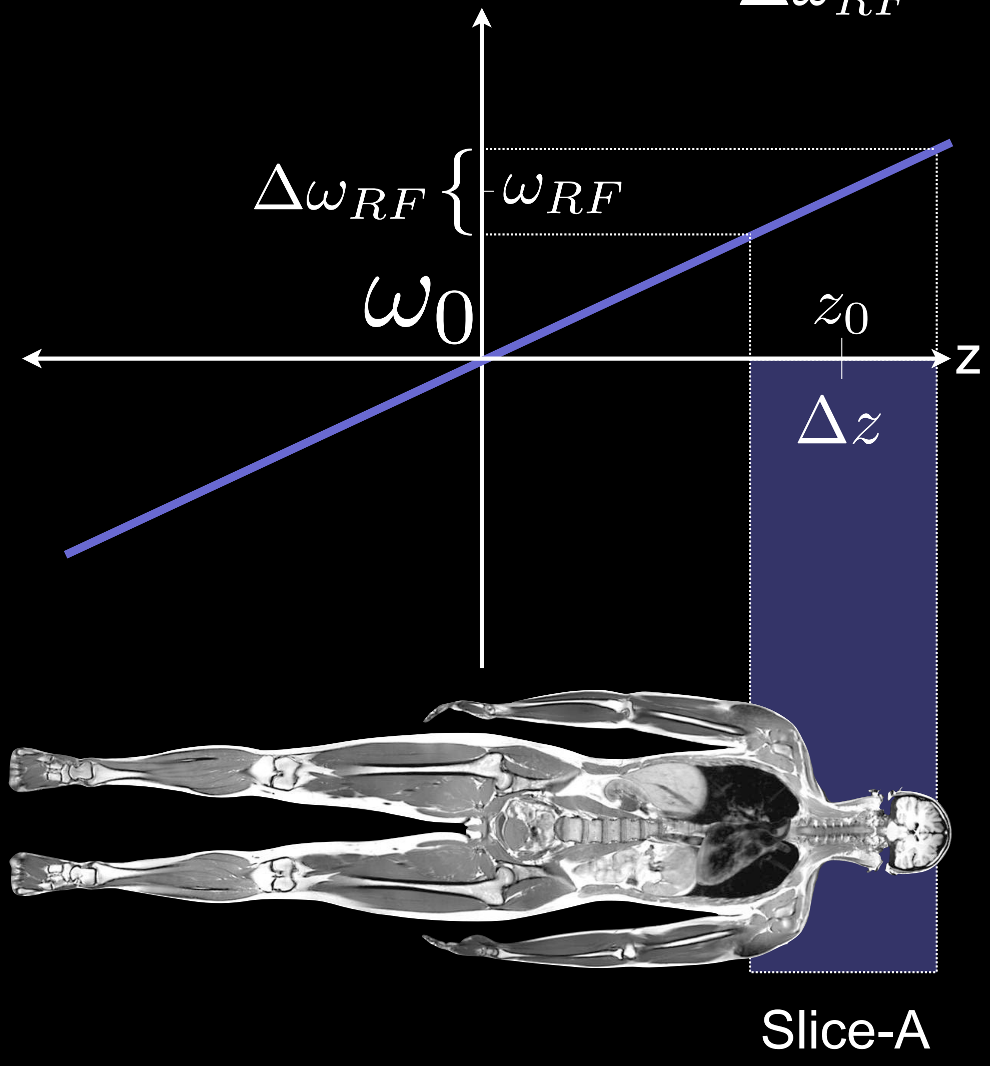
How do you move the slice along ±z?

Compare $\Delta\omega$ and ω_{RF} for Slice-A and Slice-B.

Do we usually acquire $\omega_{RF}>\omega_0$?

Slice Selective Excitation - Example

$$\Delta\omega_{RF} = -\gamma\left(G_z \cdot \Delta z\right)$$ Excitation Bandwidth

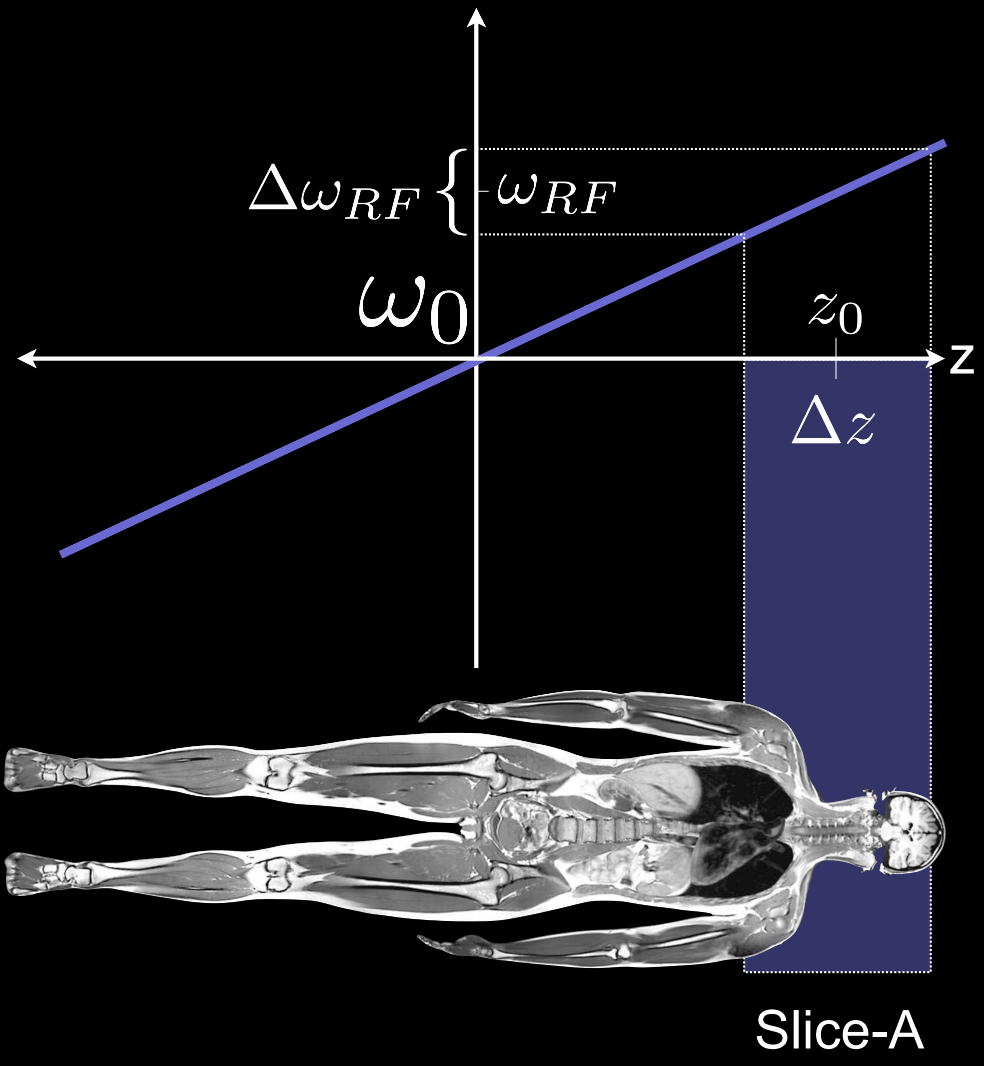


$$\mathrm{TBW} = \tau_{RF} \cdot \Delta\omega_{RF}$$

$$\Delta\omega_{RF} = \frac{\mathrm{TBW}}{\tau_{RF}} = \frac{4}{1\mathrm{ms}} = 4\mathrm{kHz}$$

$$G_{z,ss} = \frac{\Delta\omega_{RF}}{\gamma\Delta z} = \frac{4000\mathrm{Hz}}{42.57e6\,\frac{\mathrm{Hz}}{\mathrm{T}}\,\frac{1\mathrm{T}}{10000\mathrm{G}} \cdot 10\mathrm{mm}} = 0.94\,\frac{\mathrm{G}}{\mathrm{cm}}$$

Slice Selective Excitation - Example



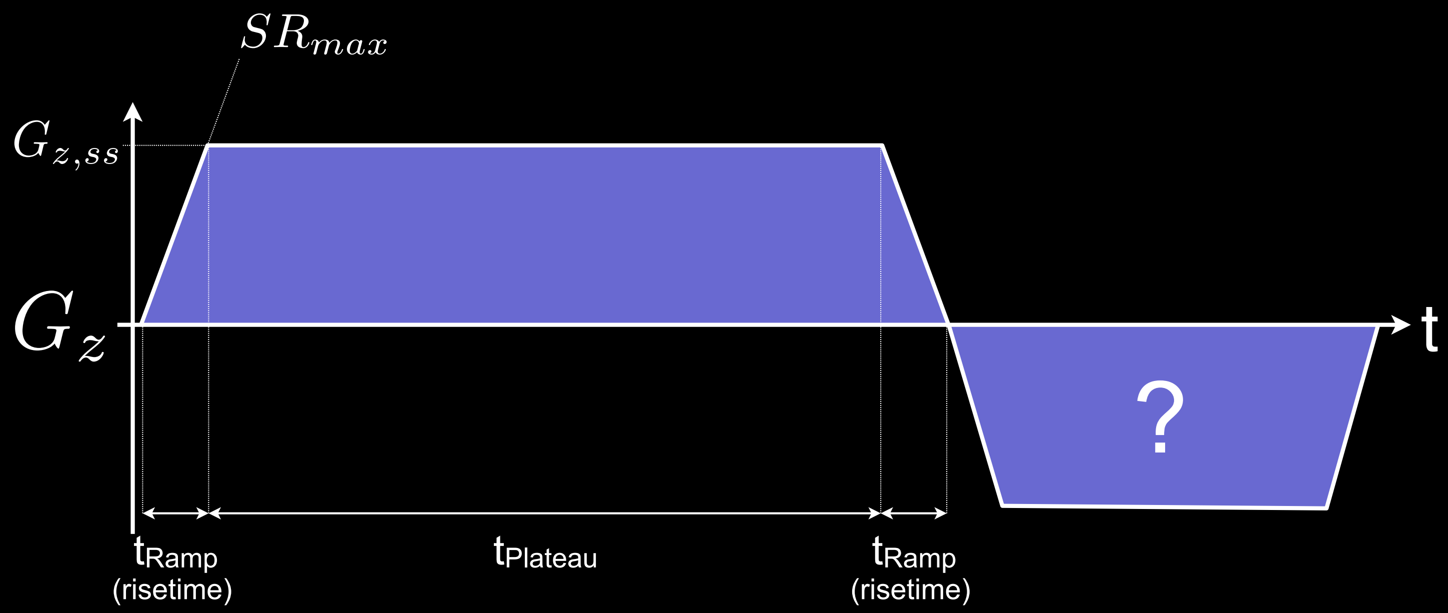
- **What is the thinnest slice possible?**

$$\Delta z = \frac{\Delta\omega_{RF}}{\gamma G_z}$$
$$= \frac{TBW}{\gamma G_z \cdot \tau_{RF}}$$

- Smallest TBW
  - Slice profile limited
- Maximum G_z
  - Hardware limited
- Longest τ_{RF}
  - Acceptable duration

Slice Selective Gradient Design

- **Ramp as fast as possible until $G_{z,ss}$**
- **Plateau for duration ($t_{plateau}$)of RF pulse**
- **Ramp down as fast as possible**



$$t_{ramp} = \frac{G_{z,ss}}{SR_{max}}$$

$$= \frac{0.95\frac{\mathrm{G}}{\mathrm{cm}}}{20\mathrm{G/cm/ms}}$$

$$= 0.0475\mathrm{ms}$$

$$t_{plateau} = \tau_{RF}$$

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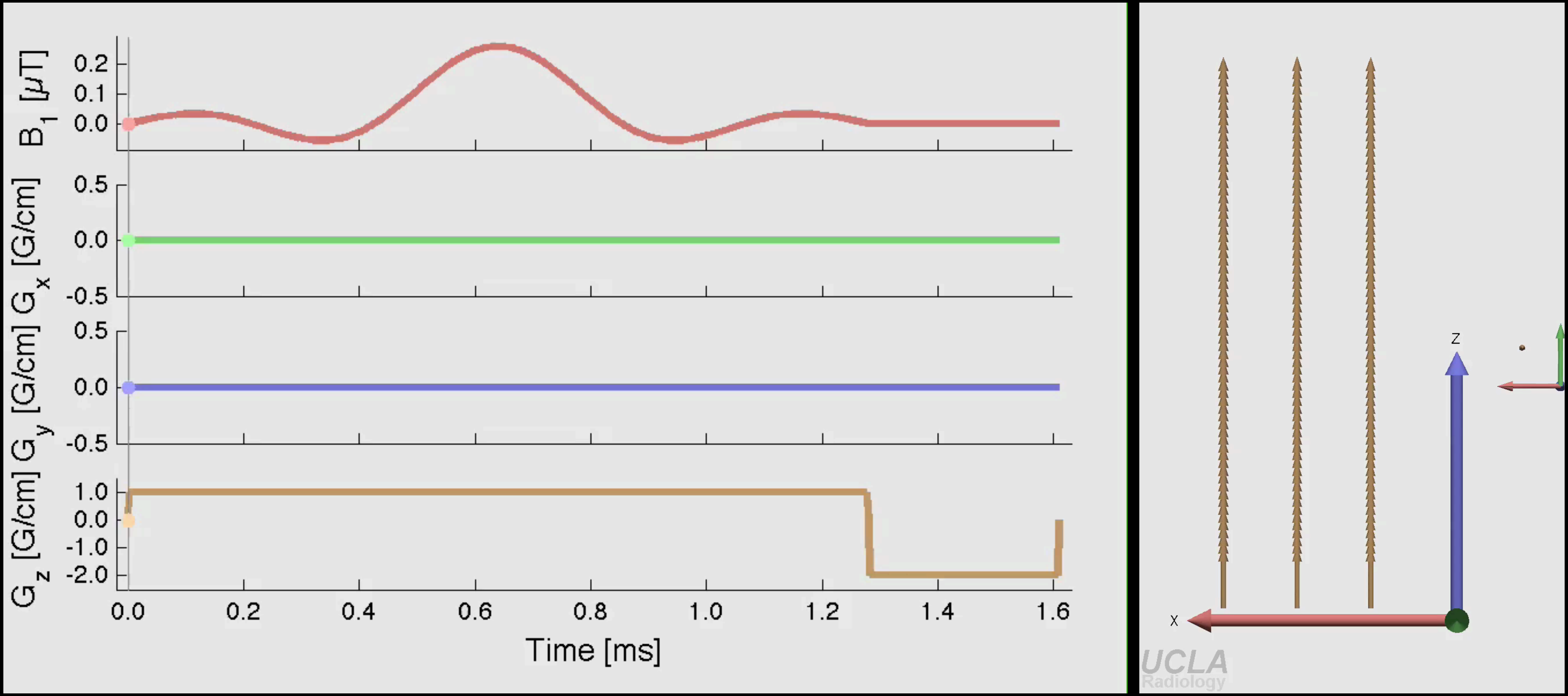
Slice Selection & Rephasing

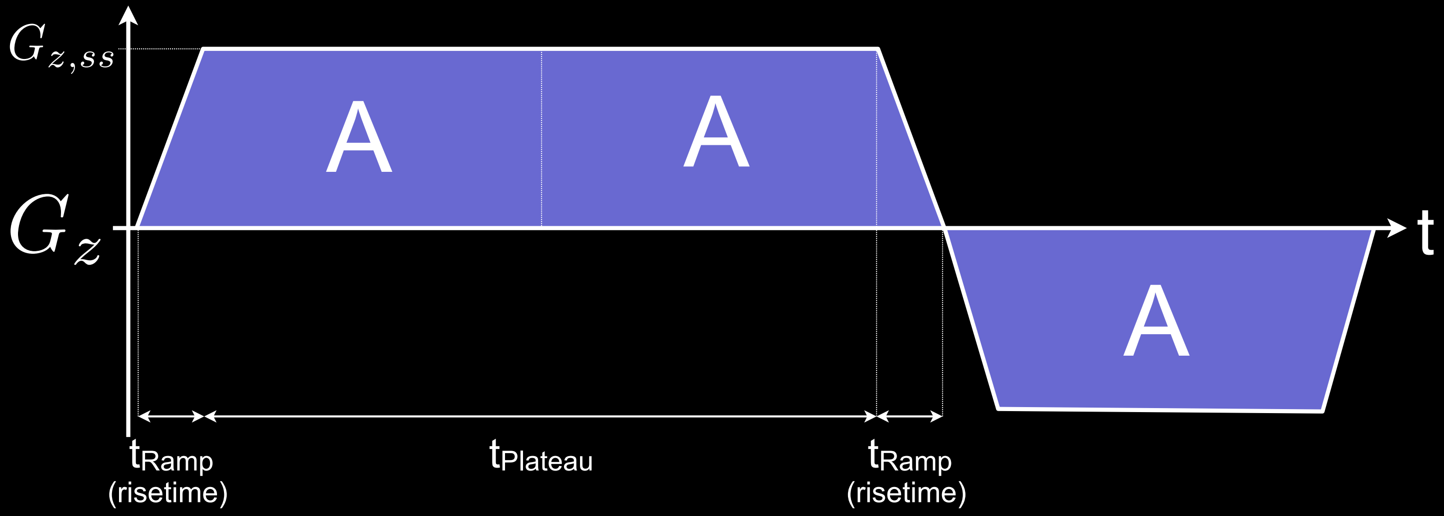


Slice Selective Gradient Design

- **How do you design the slice selective re-phasing gradient?**

$$M_{xy} = iM_0 \underbrace{e^{-i\omega(z)\frac{\tau_{RF}}{2}}}_{\substack{\text{Through-plane}\\ \text{de-phasing at the}\\ \text{end of slice selection.}}} \mathcal{F}_{1D}\{\omega_1(t+\tfrac{\tau_{RF}}{2})\}$$

Small Tip Angle Approximation



$$\omega(z,t) = \gamma G_{z,ss}(t) \cdot z$$

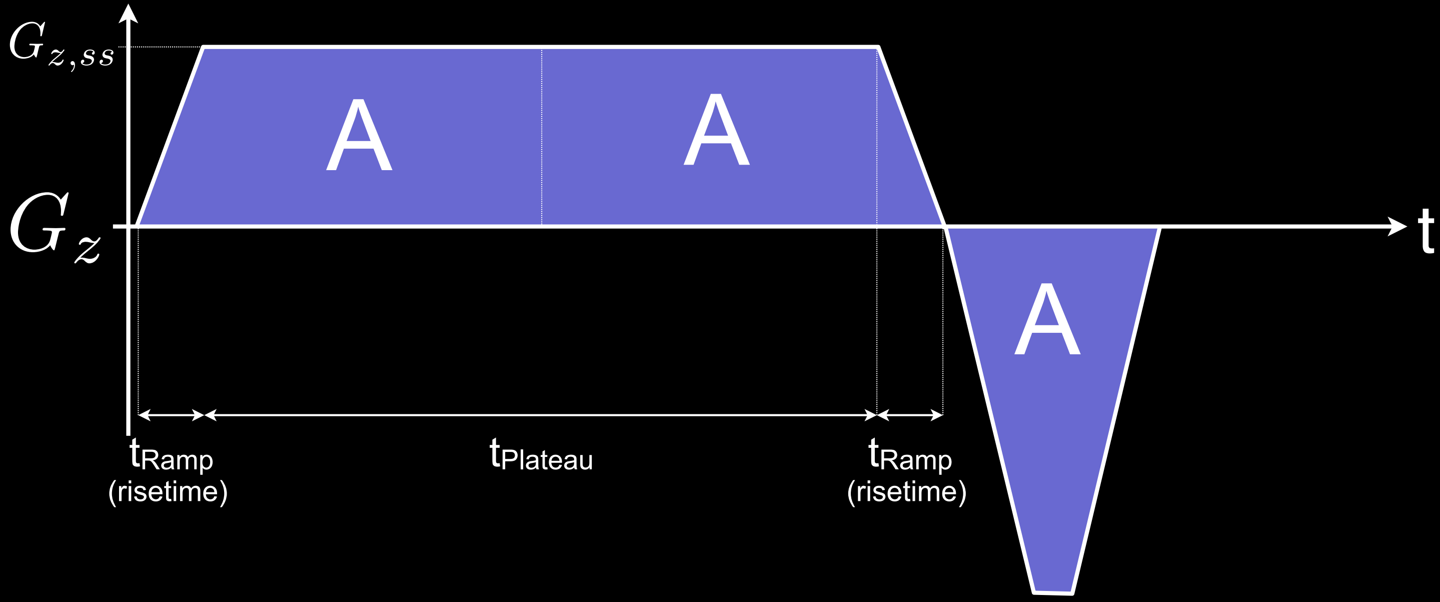
$$\begin{aligned}\phi_{z,ss} &= \int_0^{\frac{\tau_{RF}}{2}} \omega(z,t)dt \\ &= \int_0^{\frac{\tau_{RF}}{2}} \gamma G_{z,ss}(t) \cdot z dt \\ &= \gamma G_{z,ss} \cdot \frac{\tau_{RF}}{2} \cdot z\end{aligned}$$

Slice Selective Gradient Design

- **How do you design the slice selective re-phasing gradient?**

$$M_{xy} = iM_0 \underbrace{e^{-i\omega(z)\frac{\tau_{RF}}{2}}}_{\text{Through-plane de-phasing at the end of slice selection.}} \mathcal{F}_{1D}\{\omega_1(t+\frac{\tau_{RF}}{2})\}$$

Small Tip Angle
Approximation



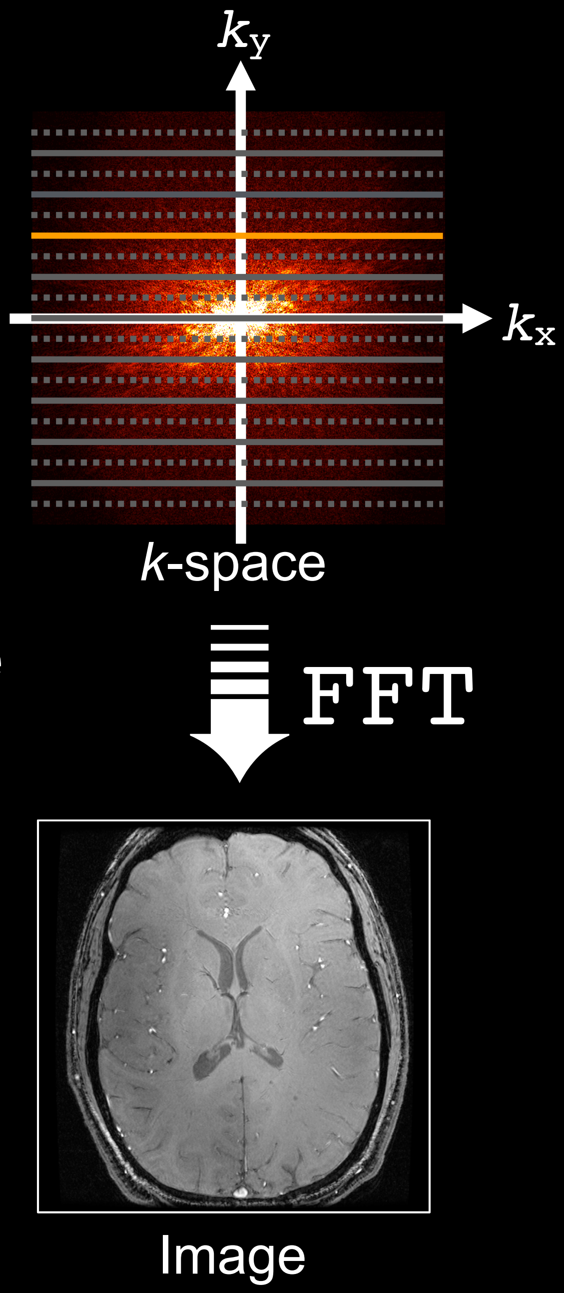
Slice select refocusing gradient should be as short as possible.

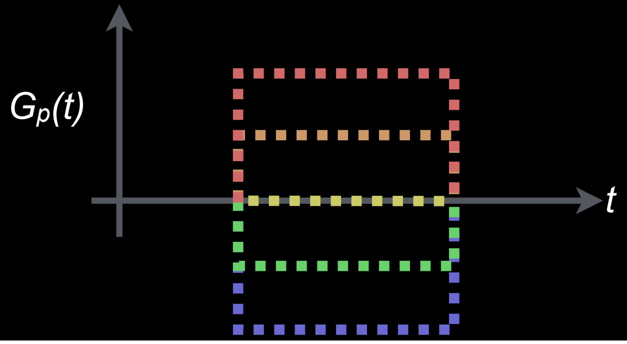
G_{Max} and SR_{Max} limited…

Phase Encode Gradients

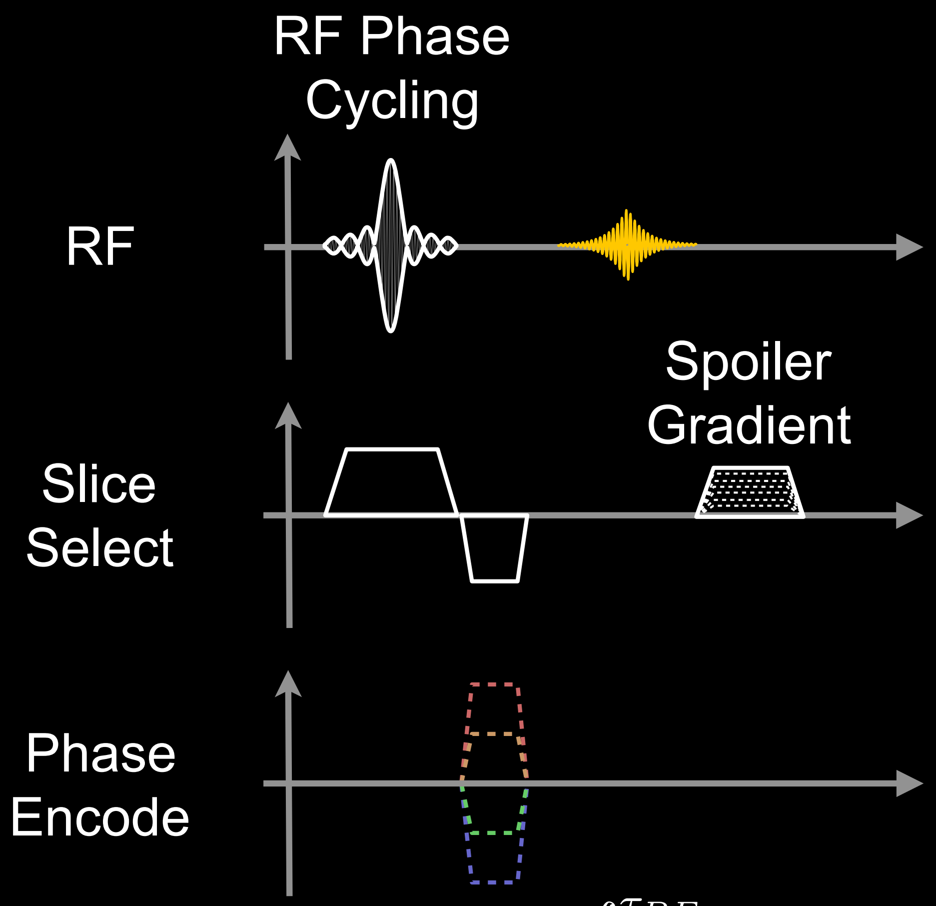
Phase Encoding

- **Consists of:**
  - **Phase encoding gradient**
    - **Magnitude changes with each TR**
    - **Can be played with other gradients**
      - **Crushers, Slice-selection rephaser, readout dephasing**
- **Used with Cartesian imaging**
- **After excitation, before readout**
- **Adds linear spatial variation of phase**
- **Phase encode in**
  - **one direction for 2D imaging**
  - **two directions for 3D imaging**
- **Only one PE step per echo**





Phase Encode Gradients

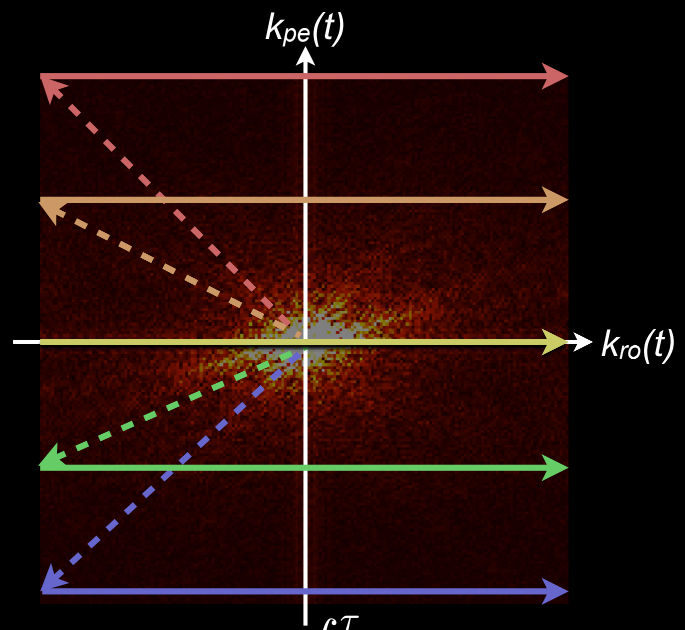


$$\phi_{y,pe}(y) = \int_0^{\tau_{PE}} \omega(y,t)dt$$

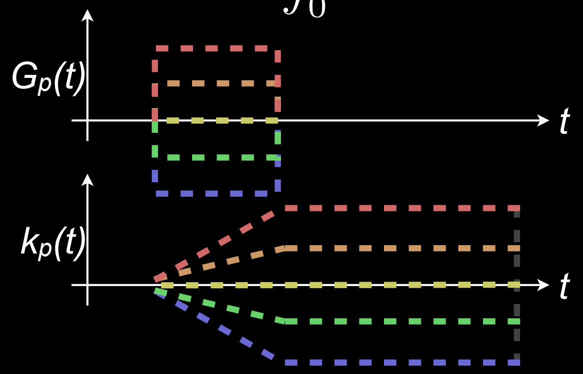
$$= \int_0^{\tau_{PE}} \gamma G_{y,pe}(t) \cdot y dt$$

$$= \gamma G_{y,pe} \cdot \tau_{PE} \cdot y$$

$$= 2\pi k_y \cdot y$$

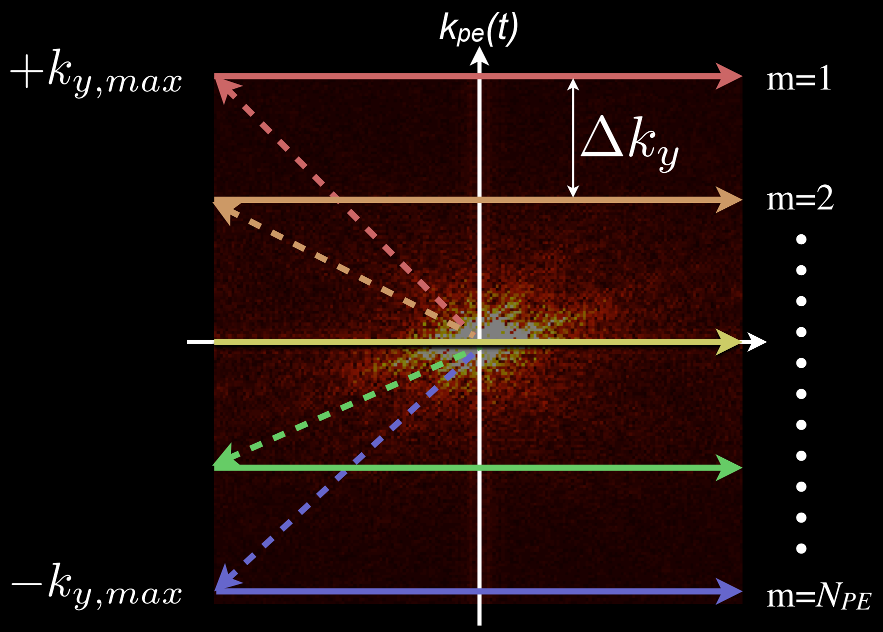


$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$



Phase Encode Gradients

$FOV = \frac{1}{\Delta k_y}$, encoded with N_{PE} steps.



$$\begin{aligned}\Delta k_y &= \frac{1}{N_{PE}\cdot\Delta y}\\ &= \frac{1}{128\cdot 0.1\text{cm}}\\ &= 0.078\text{cm}^{-1}\end{aligned}$$

$$\begin{aligned}k_{y,max} &= \tfrac{1}{2}(N_{PE}-1)\Delta k_y\\ &= \tfrac{1}{2}(128-1)\cdot 0.078\text{cm}^{-1}\\ &= 4.95\text{cm}^{-1}\end{aligned}$$

$\uparrow$

2x Nyquist

In general, $k_y(m) = \left(\frac{N_{PE}-1}{2} - m\right)\Delta k_y$

Phase Encode Gradients

- **How do we design the steps?**
  - **Calculate $k_{y,max}$ from defined N_{PE} and FOV**
    - Defines largest PE step (e.g. largest gradient)
  - **Design shortest gradient for $k_{y,max}$**
  - **Linear scaling of gradient area for all other steps**
    - Keeps sequence timing constant TR to TR

$$\Delta k_y = \frac{1}{\mathrm{FOV}_y} = \gamma \Delta \mathbf{G}_y T_{pe}$$ Eqn. 5.123

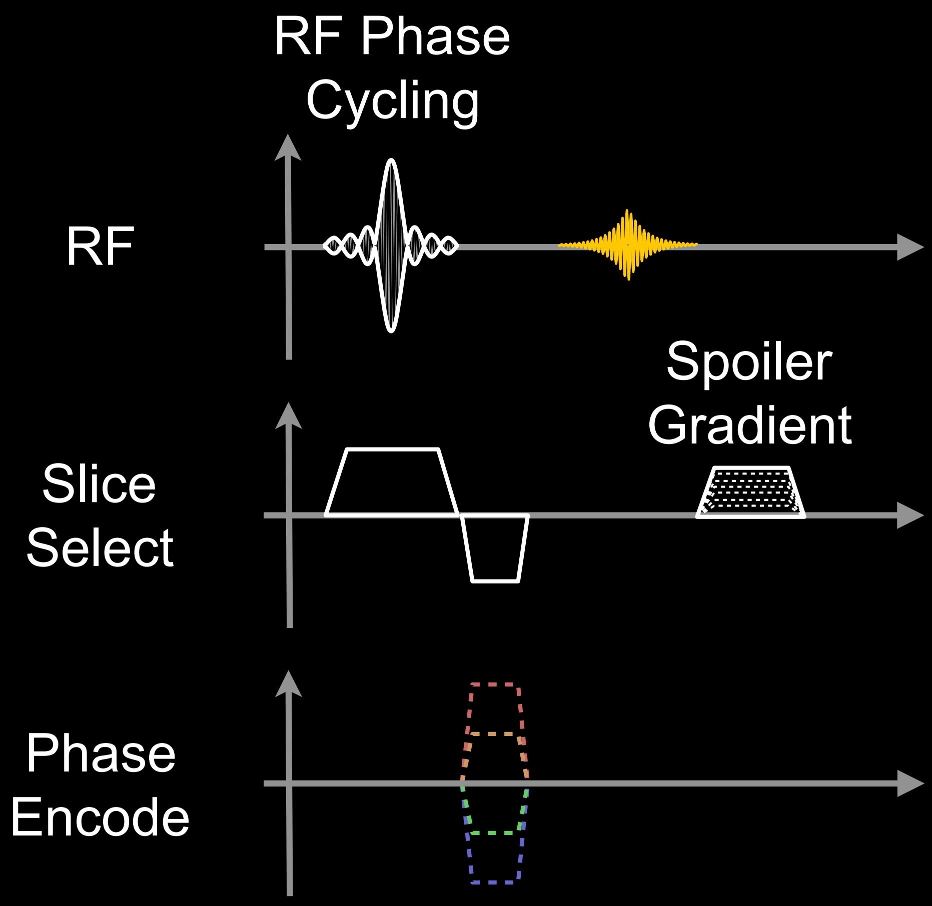
$$k_{y,max} = \left(\frac{N_{PE}-1}{2}\right)\Delta k_y$$
$$= \left(\frac{N_{PE}-1}{2}\right)\frac{\gamma}{2\pi}\Delta G_{PE}\tau_{PE}$$

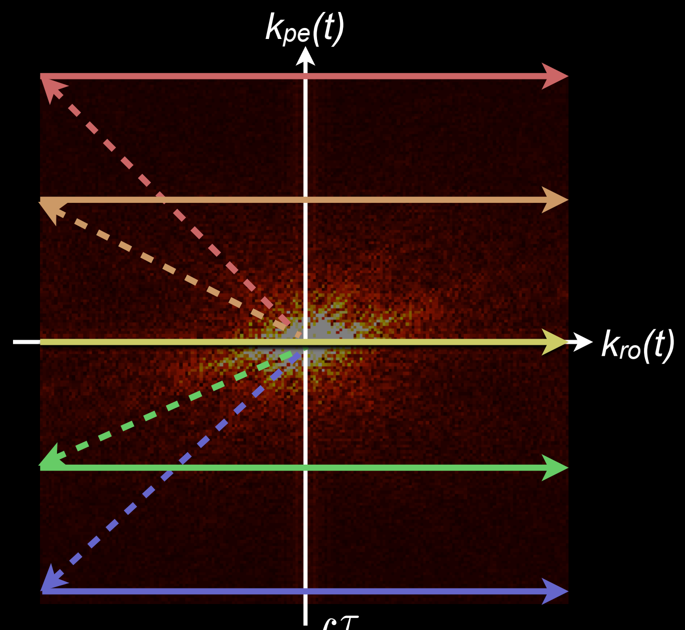
Let, $G_{PE,max} = \left(\frac{N_{PE}-1}{2}\right)\Delta G_{PE}$

- Use the maximum available gradient strength.
- Calculate the duration, τ_{PE}.

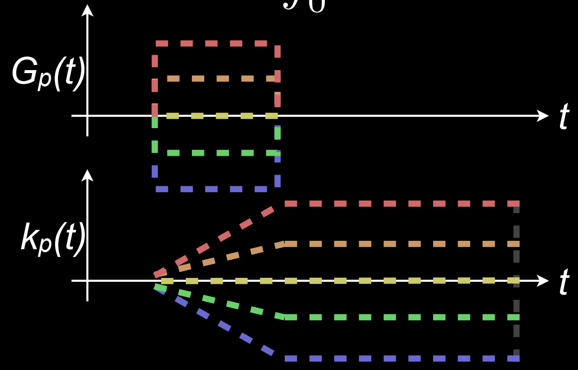
$$\tau_{PE} = \frac{2\pi k_{y,max}}{\gamma G_{max}}$$
$$= \frac{4.95\mathrm{cm}^{-1}}{4248\frac{\mathrm{Hz}}{\mathrm{G}}\cdot 4\frac{\mathrm{G}}{\mathrm{cm}}}$$
$$= 0.290\mathrm{ms}$$

Phase Encode Gradients





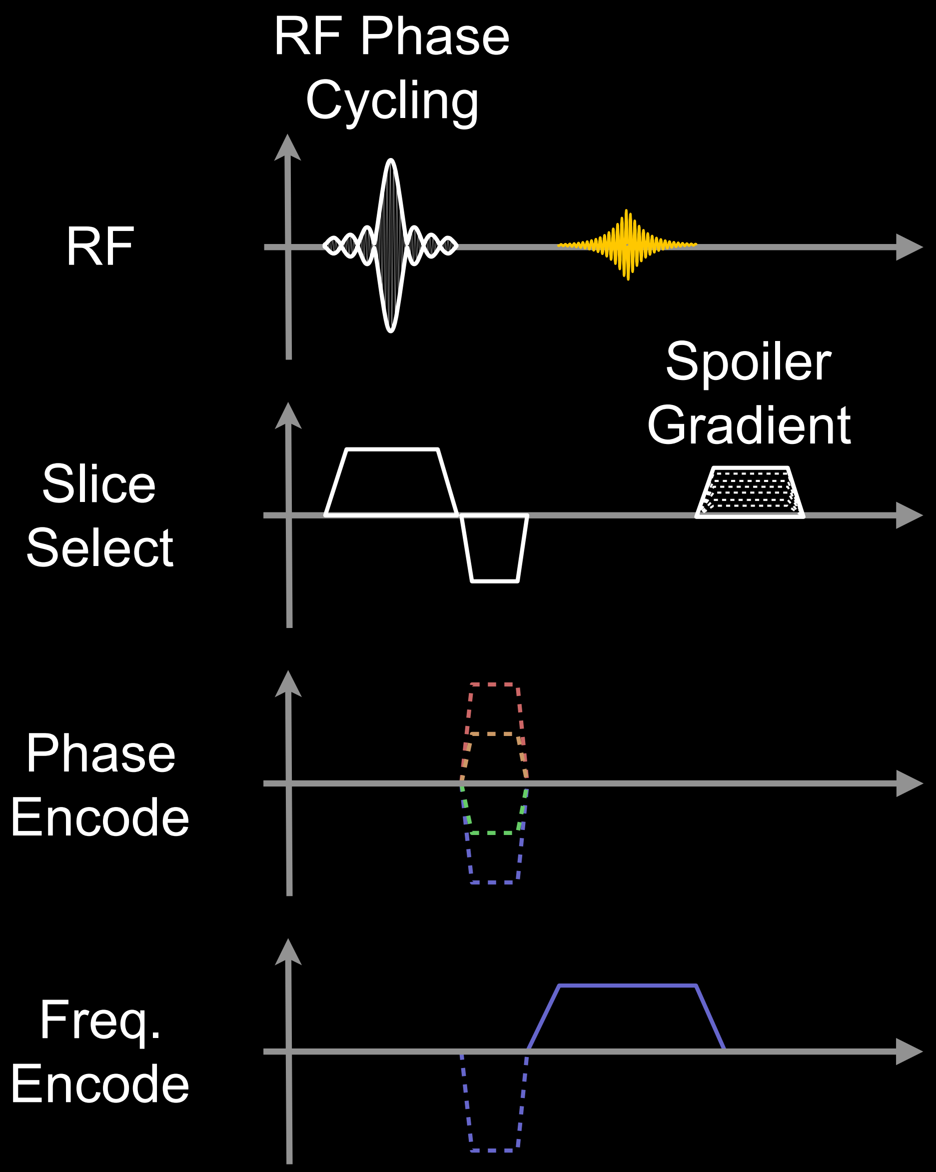
$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\, d\tau$$



For sequence efficiency the slice-select rephasing gradient and the phase encode gradient can overlap.

Readout Gradients

Gradient Echo Sequence



RF Phase Cycling

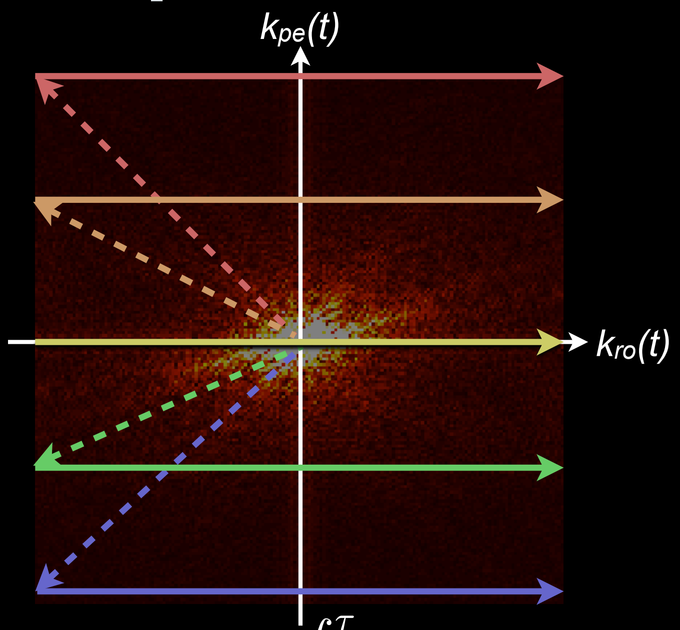
RF

Spoiler Gradient

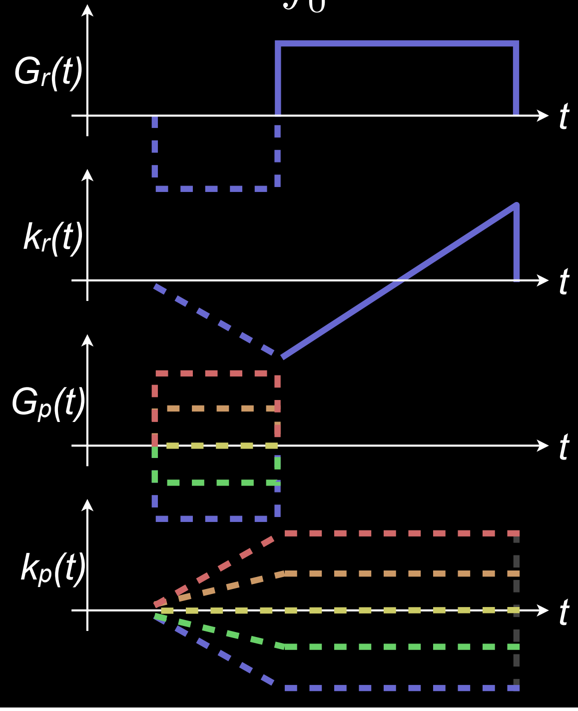
Slice Select

Phase Encode

Freq. Encode



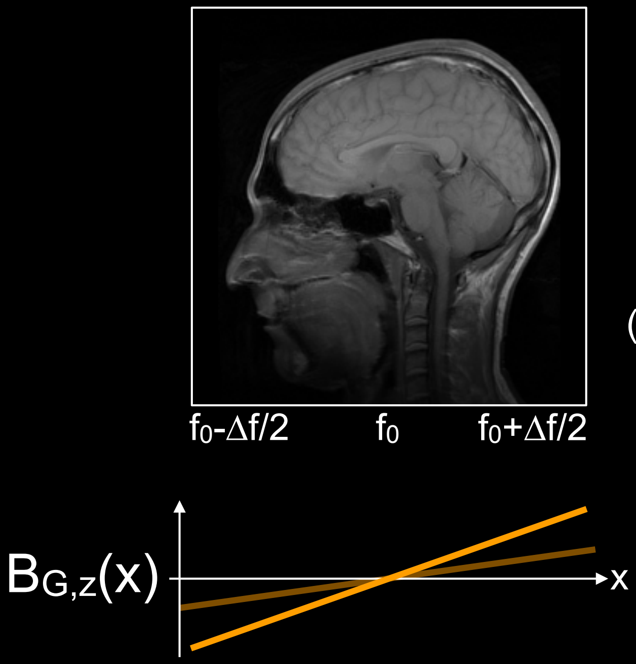
$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$



One phase encoded echo is acquired per TR.

Readout Gradient Amplitude

- **High Receiver Bandwidth (RBW, Δf)**
  - Stronger gradients
  - Larger range of frequencies across the FOV (or pixel)
  - Less chemical shift (smaller freq. difference per pixel)
  - Lower SNR (shorter acquisition time)
  - Shorter TE (move across *k*-space faster)



$$\Delta f = \frac{1}{2}\frac{\gamma}{2\pi}G_x \cdot FOV_x$$

User can pick 2 of 3 (Δf, G_x, FOV_x)

Receiver Bandwidth (e.g. 32kHz)

Field of View (e.g. 30cm)

$$G_x = \frac{4\pi\Delta f}{\gamma FOV_x} = \frac{4\pi \cdot 32000\mathrm{Hz}}{4258\frac{\mathrm{Hz}}{\mathrm{G}} \cdot 30\mathrm{cm}} = 3.128\frac{\mathrm{G}}{\mathrm{cm}}$$

Readout Gradient Duration

- **High Receiver Bandwidth (RBW, Δf)**
  - Stronger gradients
  - Larger range of frequencies across the FOV (or pixel)
  - Less chemical shift (smaller freq. difference per pixel)
  - Lower SNR (shorter acquisition time)
  - Shorter TE (move across *k*-space faster)

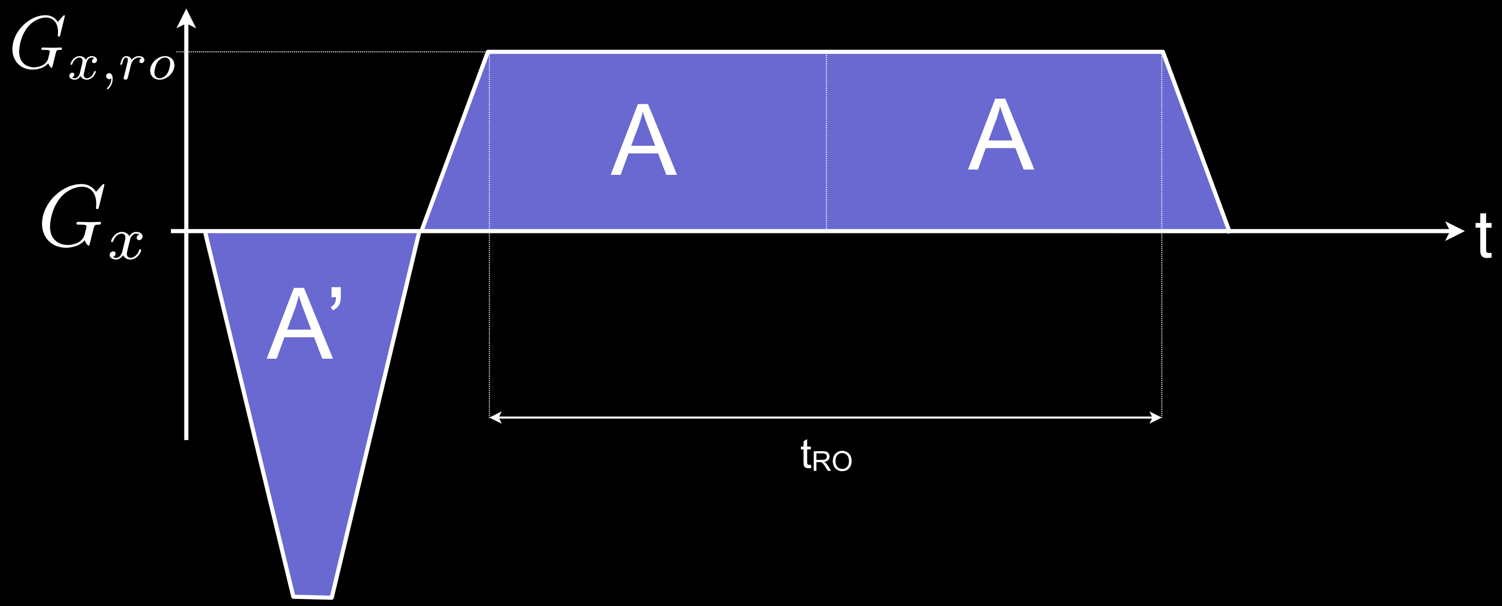
Temporal Nyquist Sampling Requires: $\Delta t = \dfrac{1}{2\Delta f}$

$$
\begin{aligned}
\Delta t &= \frac{1}{2\Delta f} \\
&= \frac{1}{2 \cdot 32000\text{Hz}} \\
&= 15.625\mu\text{S}
\end{aligned}
$$

$$
\begin{aligned}
\tau_{RO} &= N_{read} \cdot \Delta t \\
&= 128 \cdot 15.625\mu\text{S} \\
&= 2000\mu\text{S}
\end{aligned}
$$

Readout Gradient Pre-Phaser

A’=A for symmetric k-space coverage.



For sequence efficiency the readout pre-phasing gradient and the phase encode gradient can overlap.

Eddy Currents

Eddy Current Origins: Hardware

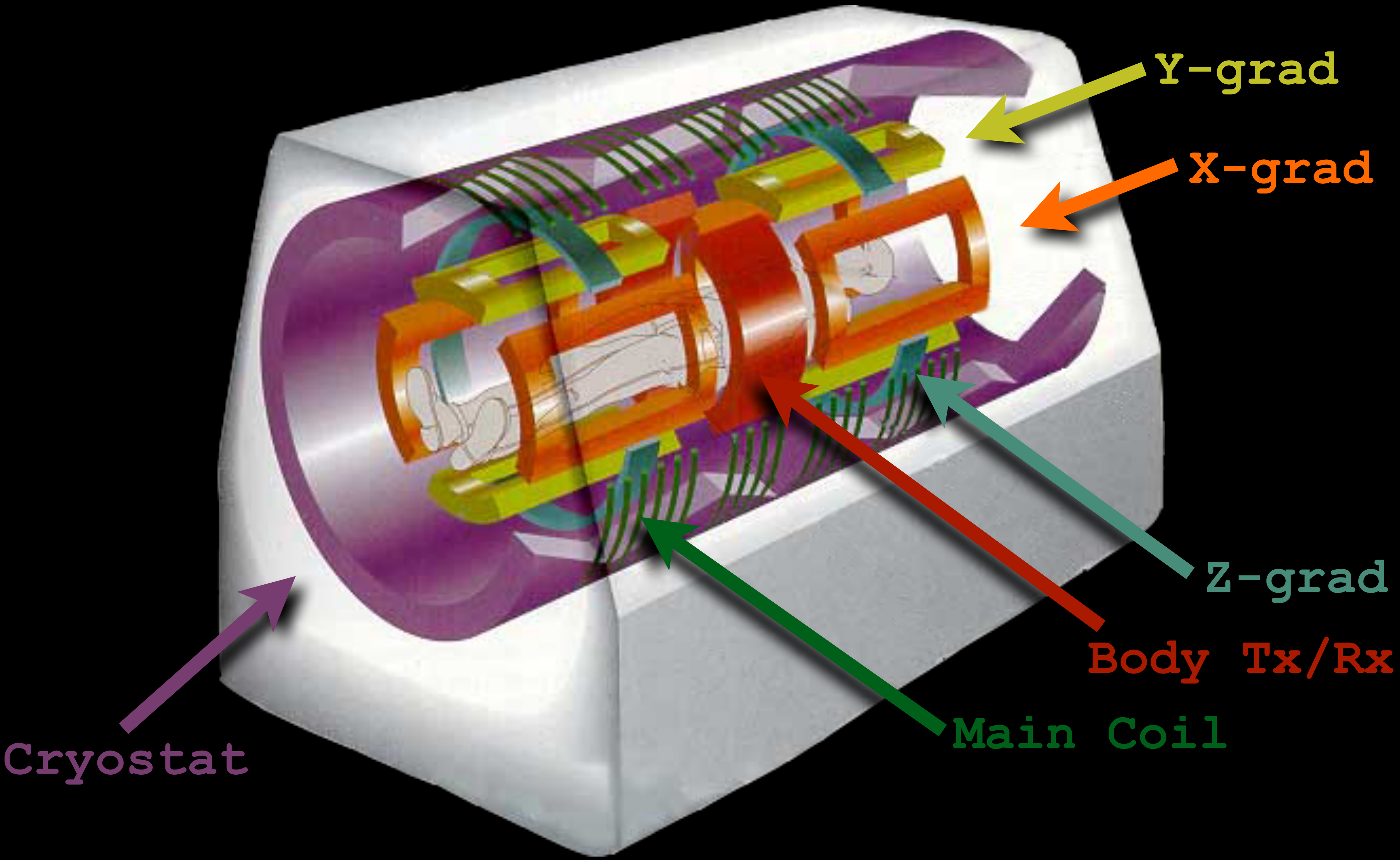
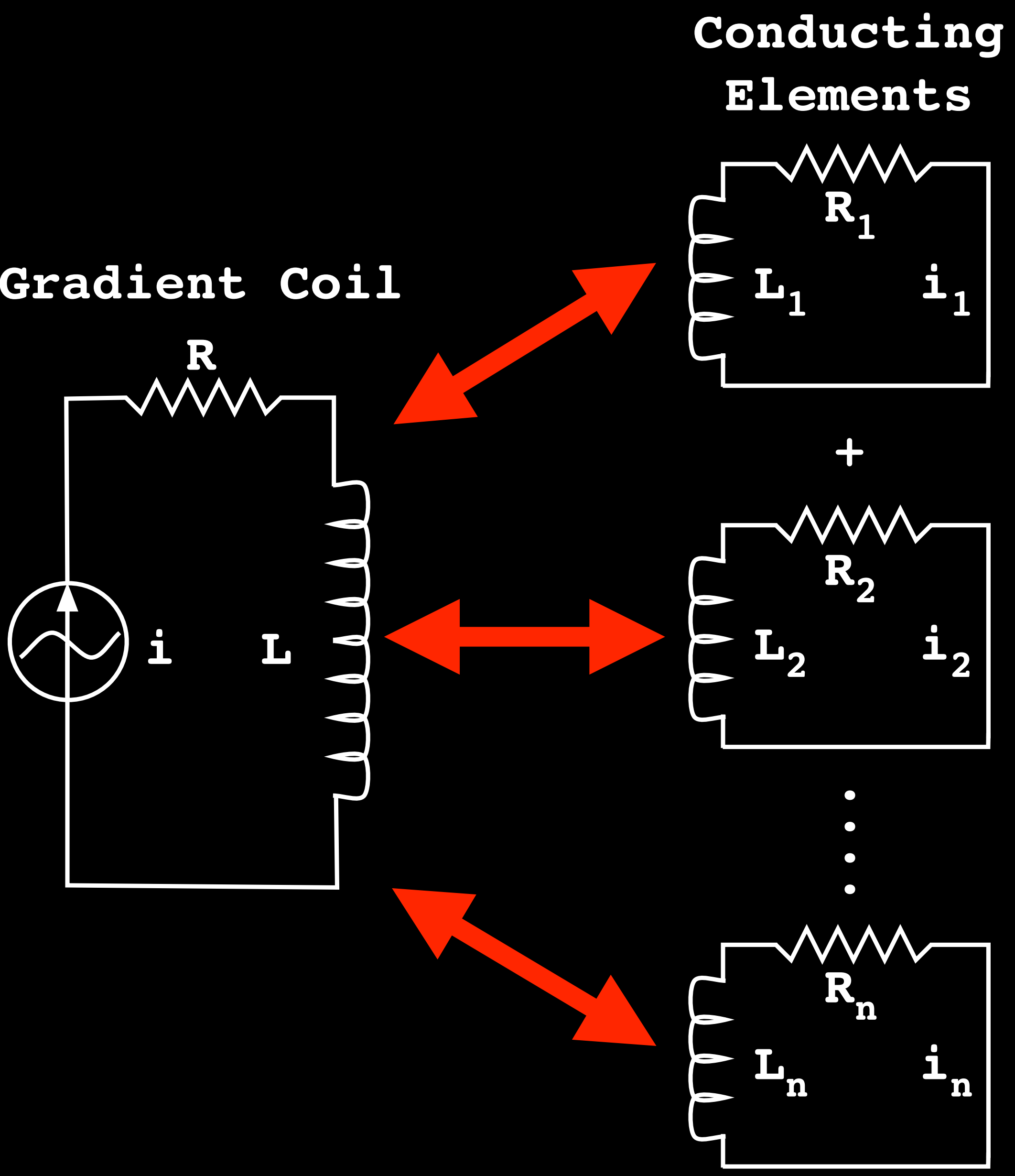


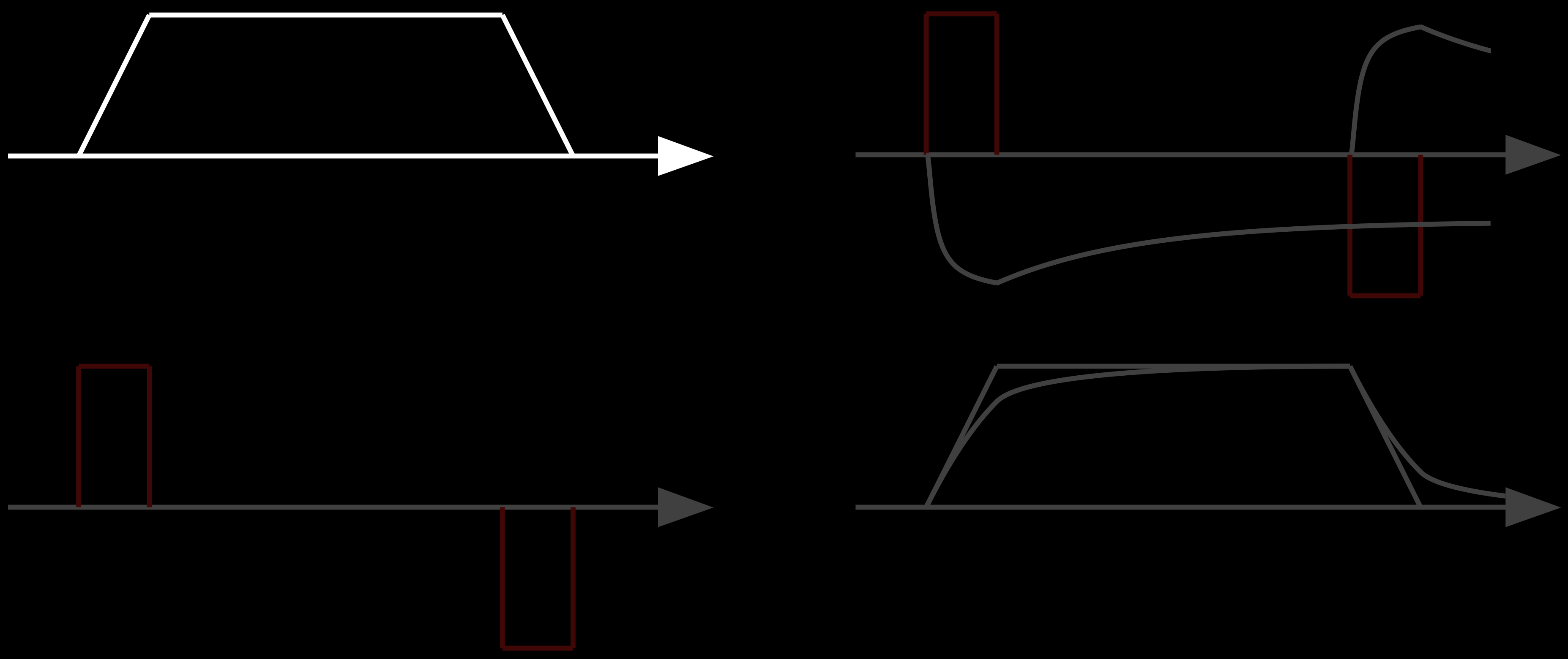
Image Adapted From: http://www.ee.duke.edu/~jshorey

Eddy Current Origins: Diagram



The gradient coil induces currents in nearby structures while *slewing*.

Eddy Current Gradient Distortion



Ideal Gradient Waveform

Eddy Current Gradients

Slewrate Waveform

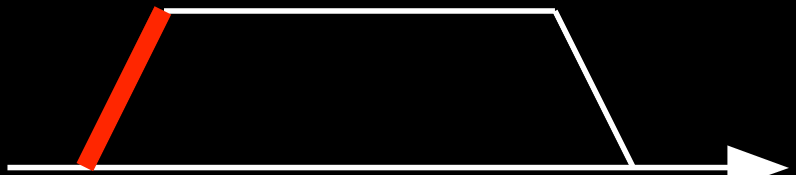
Actual Gradient Waveform

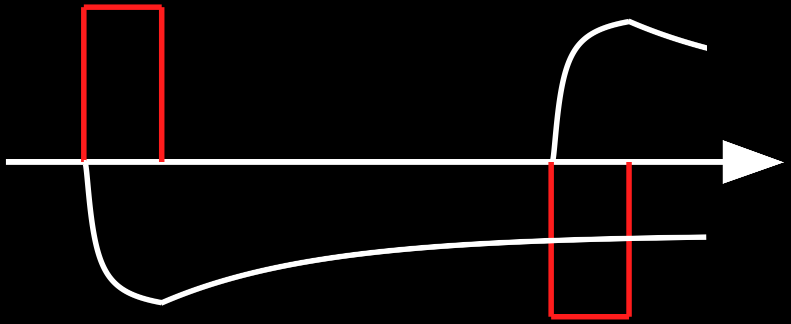
Eddy Current Origins: Mathematics

$$V_e = \text{\textcircled{$-$}} \oint_{\vec{A}} \frac{\partial G}{\partial t} \cdot d\vec{A}$$

Faraday's Law

Lenz's Law





$$I_0\left(t\right) = I_f\left(1 - e^{-\frac{Rt}{L}}\right)$$

Ohm's Law

$$I_e\left(t\right) = I_0\left(t_r\right) e^{-Rt/L}$$

Source-Free
***RL* Circuit**

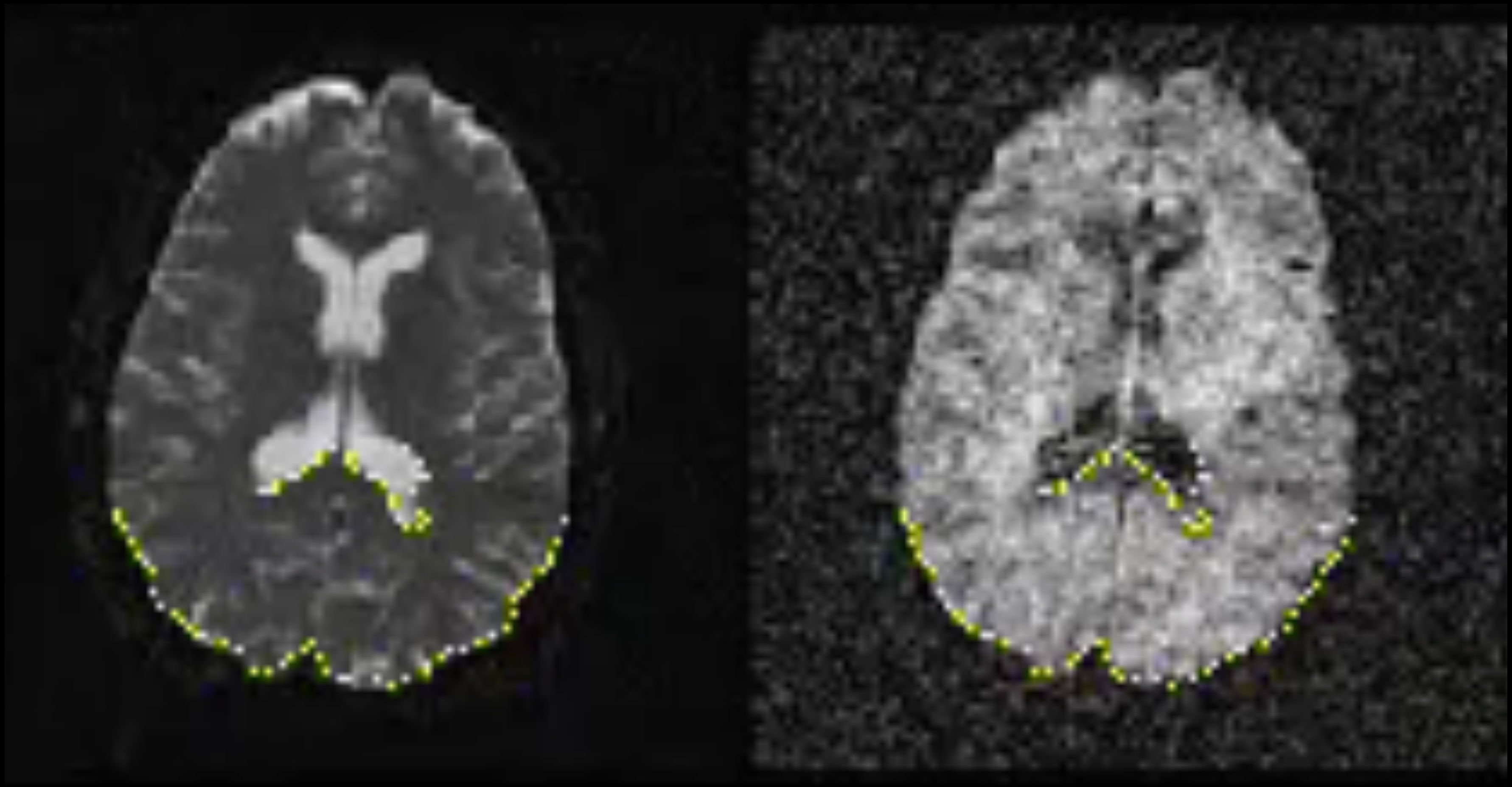
$$B_e\left(t\right) \propto I_e(t)$$

Eddy Current
Induced Field

Eddy Current Artifacts

- Ghosting in EPI
- Distortions in DTMRI
- Velocity Errors in PC

Eddy Current Artifacts: DWI



Movie Courtesy of Stefan Skare

Eddy Current Compensation

- **Hardware**
  - **Shielded Gradient Coils**
  - **Waveform Pre-emphasis**
- **Pulse Sequence**
  - **Slewrate de-rating**
  - **Twice Re-focused Spin Echo**
- **Reconstruction**
  - **Measure & Subtract (PC)**
  - **Predict & Subtract**

Thanks

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