HW3 Solutions Winter 2018 - Problem 1

```
% set a range of values to simulate
TR = 10:10:10000;
TE = 5:0.1:100;

alpha = 30*pi/180; % fip angle, deg

% Tissue A
T1a  = 2000;      % T1, ms
T2a  = 40;        % T2, ms
T2as = 25;        % T2*, ms
% Tissue B
T1b  = 500;       % T1, ms
T2b  = 40;        % T2, ms
T2bs = 25;        % T2*, ms

A_GRE = zeros(length(TR),length(TE),2);
A_SE  = zeros(length(TR),length(TE),2);

% cycle through each TE and TR
for j = 1:length(TR)
    for k = 1:length(TE)
        TRtmp = TR(j);
        TEtmp = TE(k);

        % GRE contrast for Tissue A
        A_GRE(j,k,1) = ( 1-exp(-TRtmp/T1a) )*( sin(alpha)*exp(-TEtmp/
T2as) )/( 1-cos(alpha)*exp(-TRtmp/T1a) );
        % GRE contrast for Tissue B
        A_GRE(j,k,2) = ( 1-exp(-TRtmp/T1b) )*( sin(alpha)*exp(-TEtmp/
T2bs) )/( 1-cos(alpha)*exp(-TRtmp/T1b) );

        % SE contrast for Tissue A
        A_SE(j,k,1)  = ( 1-exp(-TRtmp/T1a) )*exp(-TEtmp/T2a);
        % SE contrast for Tissue B
        A_SE(j,k,2)  = ( 1-exp(-TRtmp/T1b) )*exp(-TEtmp/T2b);

    end
end

% find the point of maximum contrast for both SE and GRE

TRsearch = repmat(TR',[1,length(TE)]);
TEsearch = repmat(TE,[length(TR),1]);

C_GRE = abs(A_GRE(:,:,1)-A_GRE(:,:,2));
C_SE  = abs(A_SE(:,:,1)-A_SE(:,:,2));

Cmax_GRE = max(C_GRE(:));
C_GRE(C_GRE<Cmax_GRE) = 0;
```

```
TR_opt_GRE = TRsearch(C_GRE~=0);
TE_opt_GRE = TEsearch(C_GRE~=0);

Cmax_SE = max(C_SE(:));
C_SE(C_SE<Cmax_SE) = 0;

TR_opt_SE = TRsearch(C_SE~=0);
TE_opt_SE = TEsearch(C_SE~=0);

fprintf('Gradient Echo : Optimal TE = %2.1fms\n',TE_opt_GRE);
fprintf('                Optimal TR = %2.1fms\n\n',TR_opt_GRE);
fprintf('Spin Echo     : Optimal TE = %2.1fms\n',TE_opt_SE);
fprintf('                Optimal TR = %2.1fms\n\n',TR_opt_SE);

fprintf('For optimal T1 contrast, the spin echo sequence is %2.2fx
 longer! \n',TR_opt_SE/TR_opt_GRE);
```

```
Gradient Echo : Optimal TE = 5.0ms
                Optimal TR = 140.0ms

Spin Echo     : Optimal TE = 5.0ms
                Optimal TR = 920.0ms

For optimal T1 contrast, the spin echo sequence is 6.57x longer!
```

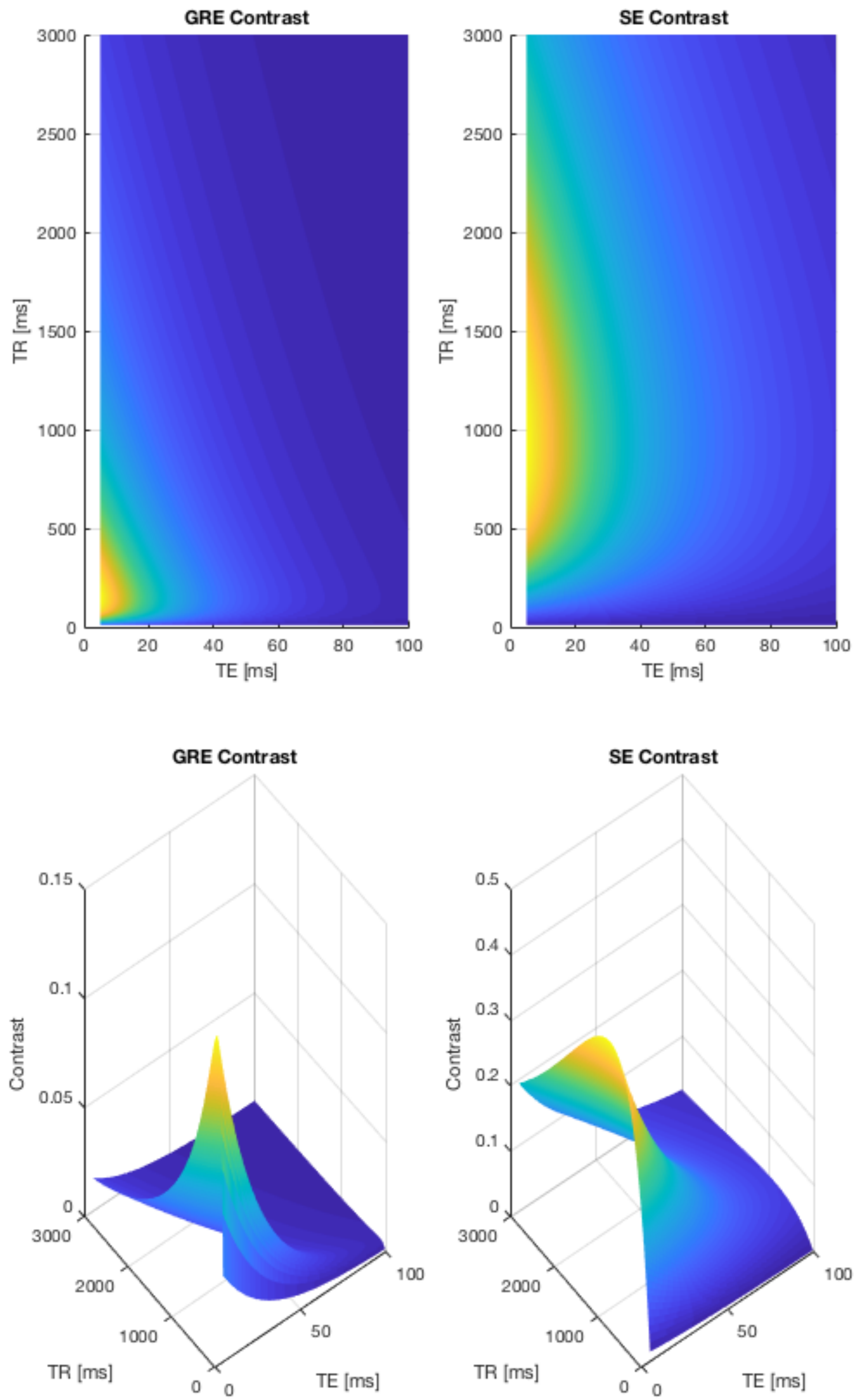
view plots of contrast for each parameter

```
figure(1);
subplot(1,2,1);
surf(TE,TR,abs(A_GRE(:,:,1)-A_GRE(:,:,2)),'LineStyle','none');
 view(0,90);  title('GRE Contrast');
xlabel('TE [ms]'); ylabel('TR [ms]'); zlabel('Contrast'); ylim([0
 3000]);

subplot(1,2,2);
surf(TE,TR,abs(A_SE(:,:,1)-A_SE(:,:,2)),'LineStyle','none');
 view(0,90);  title('SE Contrast');
xlabel('TE [ms]'); ylabel('TR [ms]'); zlabel('Contrast'); ylim([0
 3000]);

figure(2);
[xx yy] = meshgrid(TE,TR);
subplot(1,2,1); surf(xx,yy,abs(A_GRE(:,:,1)-
A_GRE(:,:,2)),'edgecolor', 'none');  title('GRE Contrast');
xlabel('TE [ms]'); ylabel('TR [ms]'); zlabel('Contrast'); ylim([0
 3000]);

subplot(1,2,2); surf(xx,yy,abs(A_SE(:,:,1)-
A_SE(:,:,2)),'edgecolor', 'none');   title('SE Contrast');
xlabel('TE [ms]'); ylabel('TR [ms]'); zlabel('Contrast'); ylim([0
 3000]);
```


GRE Contrast
SE Contrast
TR [ms]
TE [ms]
3000
2500
2000
1500
1000
500
0
0
20
40
60
80
100
GRE Contrast
SE Contrast
Contrast
0.15
0.1
0.05
0.5
0.4
0.3
0.2
0.1
3000
2000
1000
0
50
100
TR [ms]
TE [ms]

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Problem 2

A.

$f_0 = \gamma B_0$ = (42.576 MHz/T)*(3.0T) = **127.728 MHz**

BW = $\Delta f_0 = \gamma G_z \Delta z$ = (42.576 MHz/T)*(8 G/cm * 1T/10,000G)*(0.3cm) = **10.2 kHz**

B.

$f_0 = \gamma(B_0 + G_Z{*}z)$ = (42.576 MHz/T)*(3.0T + (8G/cm)*(1T/10,000G)*(1cm)) = **127.762 MHz**

BW = $\Delta f_0 = \gamma G_z \Delta z$ = (42.576 MHz/T)*(8 G/cm * 1T/10,000G)*(3mm) = **10.2 kHz**

C.

γ_{31P} = 17.235 MHz/T

$f_0 = \gamma B_0$ = (17.235 MHz/T)*(3.0T) = **51.705 MHz**

BW = $\Delta f_0 = \gamma G_z \Delta z$ = (17.235 MHz/T)*(8 G/cm*1T/10,000G)*Δz = 10.2 **kHz**

Δz = (0.0102 MHz) / (17.235 MHz/T)*(8 G/cm*1T/10,000G) = **0.74 cm The slice gets *thicker***

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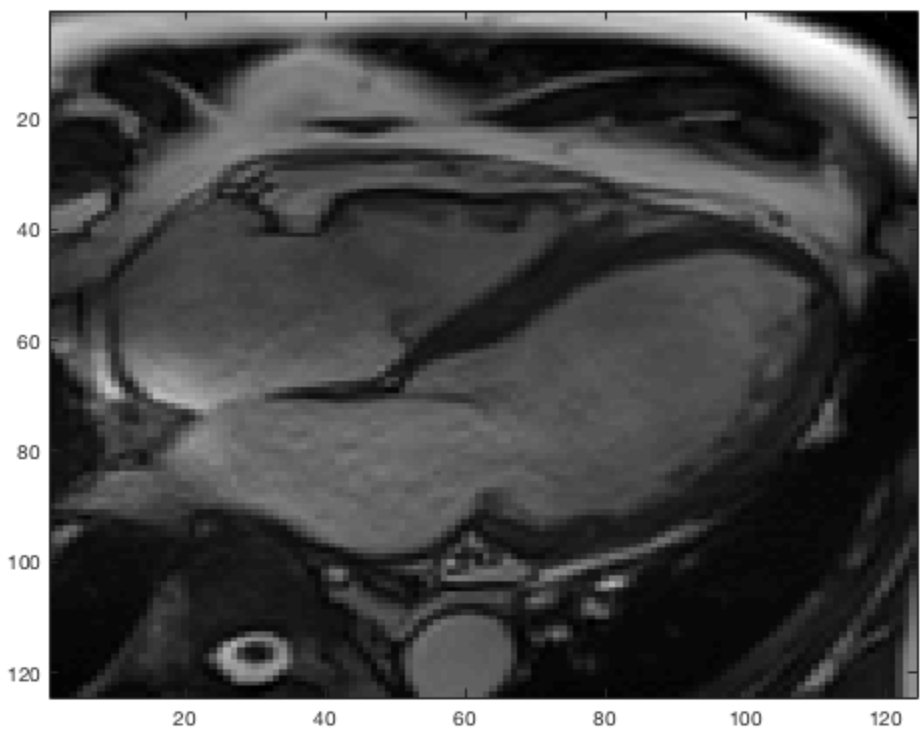
HW3 Solutions Winter 2018 - Problem 3

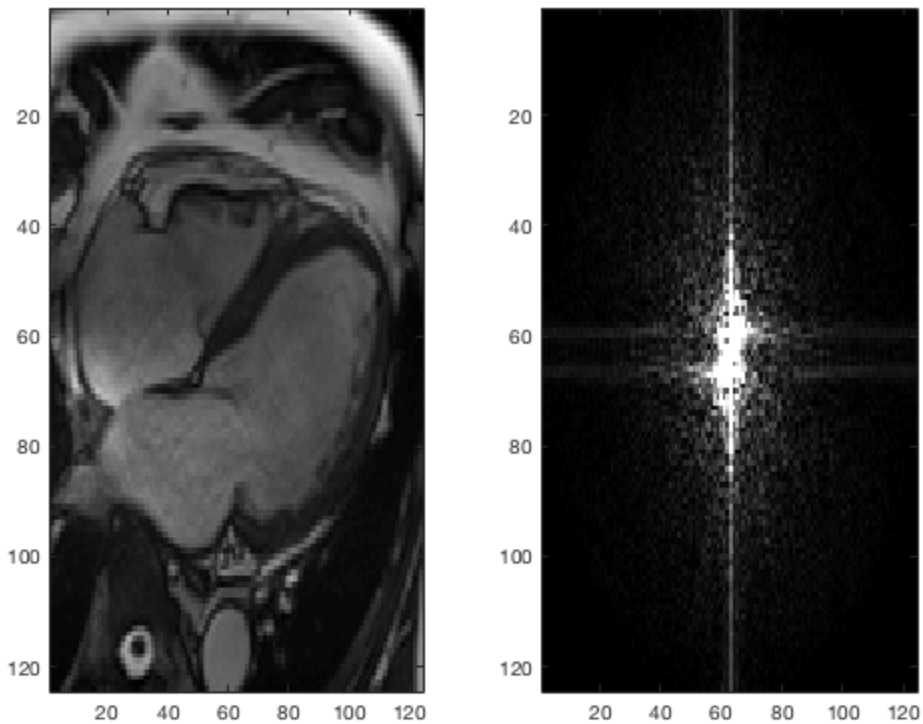
```
load '/Users/nyashagracious/Box Sync/M219_TA_18/
M219_2017_HW03_Solutions/heart.mat'

imagesc(IM); colormap gray;
```



A. view fourier transform

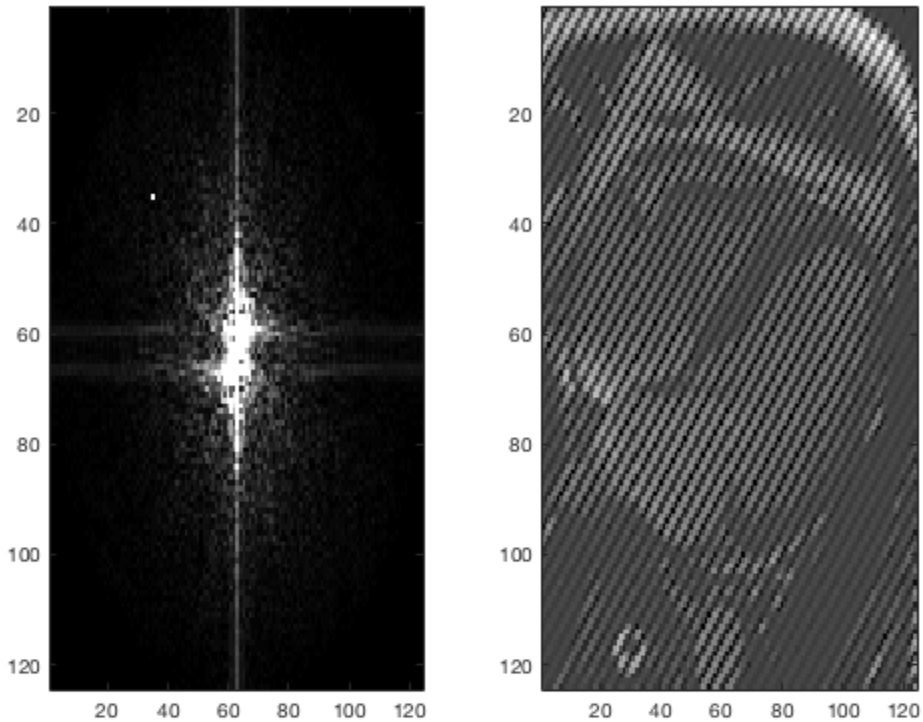
```
FourierIM = fftshift(fft2(IM));

figure;
```

```
subplot(1,2,1);
imagesc(IM); colormap gray;
subplot(1,2,2);
imagesc(abs(FourierIM),[0 1e5]); colormap gray;
```



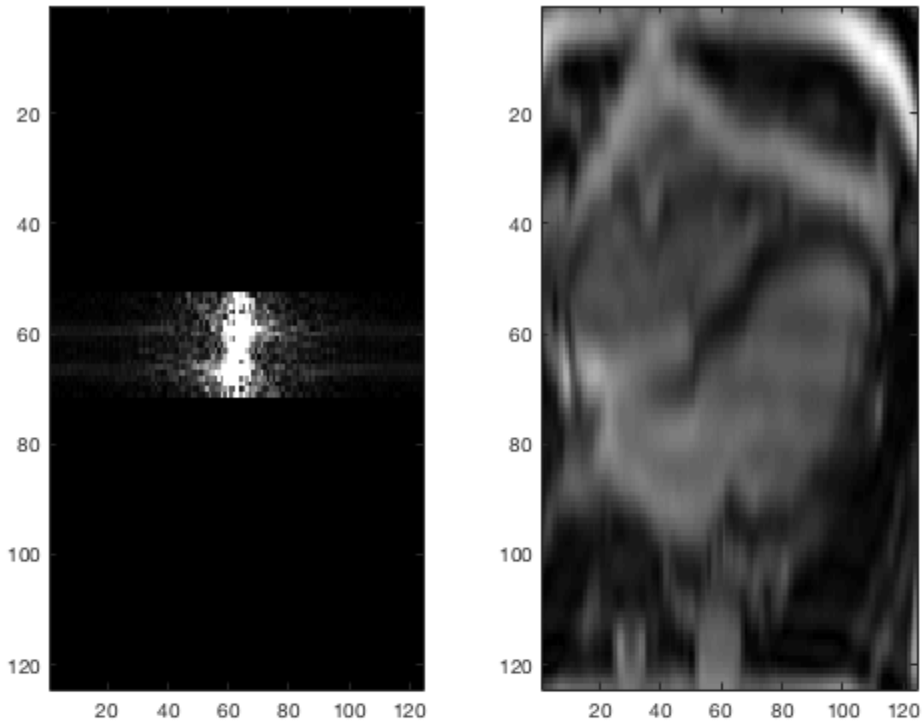
B. add a noisy spike

```
NoisySpike = FourierIM;
NoisySpike(35,35) = NoisySpike(35,35)*1000;

NoisySpikeIM = ifft2(NoisySpike);

figure;
subplot(1,2,1);
imagesc(abs(NoisySpike),[0 1e5]); colormap gray;
subplot(1,2,2);
imagesc(abs(NoisySpikeIM)); colormap gray;
```



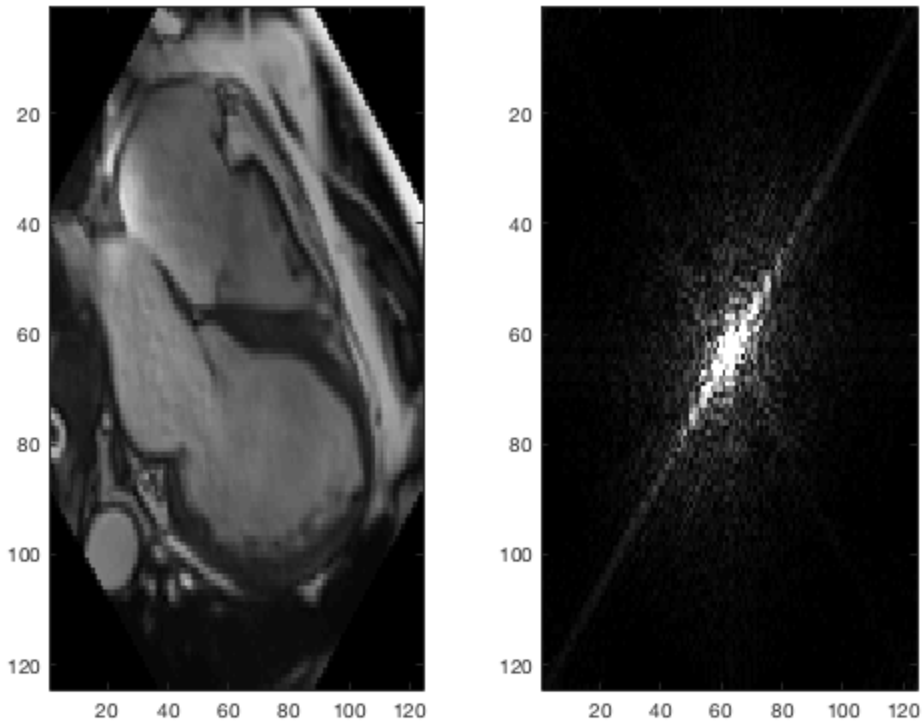
C. remove all but middle of k-space

```
PartialFFT = FourierIM;
PartialFFT(1:52,:) = 0;
PartialFFT(72:end,:) = 0;


PartialIM = ifft2(PartialFFT);

figure;
subplot(1,2,1);
imagesc(abs(PartialFFT),[0 1e5]); colormap gray;
subplot(1,2,2);
imagesc(abs(PartialIM)); colormap gray;
```



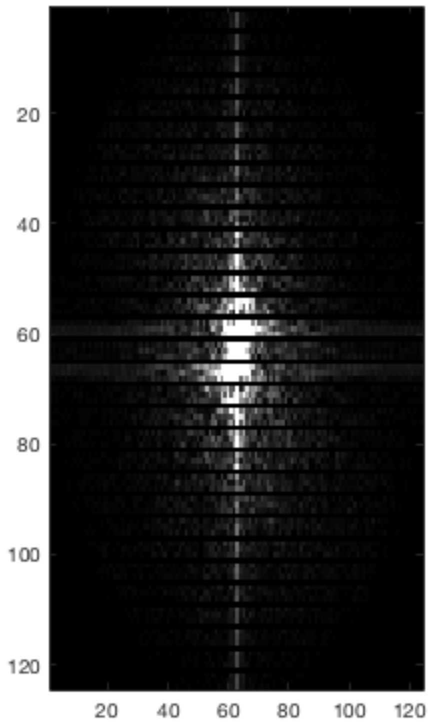
D. rotate image by -45deg

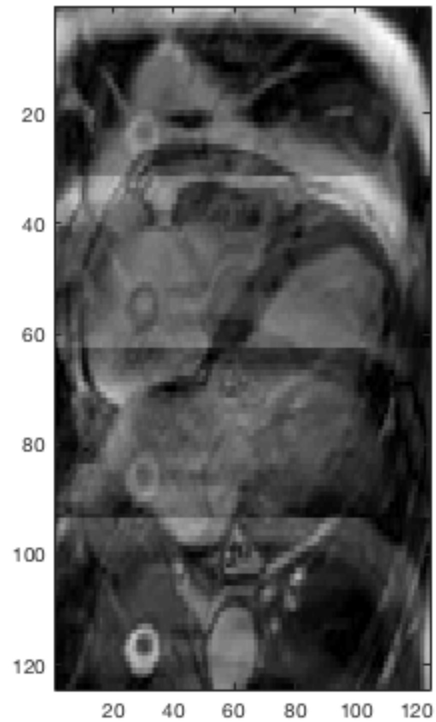
```
RotateIM = imrotate(IM,-45,'bilinear','crop');
FourierRotate = fftshift(fft2(RotateIM));

figure;
subplot(1,2,1);
imagesc(RotateIM); colormap gray;
subplot(1,2,2);
imagesc(abs(FourierRotate),[0 1e5]); colormap gray;
```



E. remove every fourth line

```
PartialFFT = FourierIM;

PartialFFT(1:4:end,:) = 0;

PartialIM = ifft2(PartialFFT);

figure;
subplot(1,2,1);
imagesc(abs(PartialFFT),[0 1e5]); colormap gray;
subplot(1,2,2);
imagesc(abs(PartialIM)); colormap gray;
```





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Problem 4.

A. Given $\Delta x \Delta k = 1/N$:

$\Delta x = 1/(N*\Delta k)$

since $\Delta k = \gamma*G*\Delta t$ …

$\mathbf{\Delta x = 1/(N*\gamma*G*\Delta t)}$

B. Since $FOV = N*\Delta x = N/(N*\gamma*G*\Delta t)$

and $\Delta f = G*FOV*\gamma$ …
$\Delta f = \gamma*G*(N/(N*\gamma*G*\Delta t))$

$\mathbf{\Delta f = 1/\Delta t}$

C. γ = 42.576 MHz/T, N=128, Δx=2mm

G = 20mT/m:
$\Delta f = G*N*\Delta x*\gamma$ = (20 mT/m * 1T/1000mT * 1m/100cm)*(128)*(0.2 cm)*(42.576 MHz/T)
= 217.99 kHz
$\Delta t = 1/\Delta f$ = 4.59 μs

G = 40mT/m:
$\Delta f = G*N*\Delta x*\gamma$ = (40 mT/m * 1T/1000mT * 1m/100cm)*(128)*(0.2 cm)*(42.576 MHz/T)
= 435.98 kHz
$\Delta t = 1/\Delta f$ = 2.29 μs

D. At 3.0T, $f_0 = \gamma*B_0$ = (42.576 MHz/T)(3.0T) = 127.728 MHz

So, 127.728 x10^6 cycles of precession are completed per second.

In dwell time, Δt, the number of rotations, N_{rot} is given by:

N_{rot} = (127.728 x10^6 cycles/s)*(Δt)

for Δt = 4.59 μs:
= (127.728 cycles/μs) * (4.59 μs)
N_{rot} = 586 cycles

for Δt = 2.29 μs:
= (127.728 cycles/μs) * (2.29 μs)
N_{rot} = 292 cycles