M219 Mid-Quarter Review

Lectures 7 to 10

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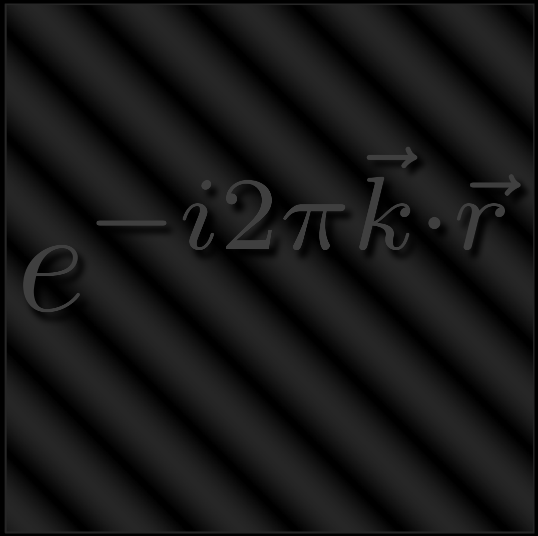
$$S(\vec{k}) = \int \underbrace{M_{xy}(\vec{r}, 0)}_{object} \; e^{-i2\pi\vec{k}\cdot\vec{r}} \; d\vec{r}$$



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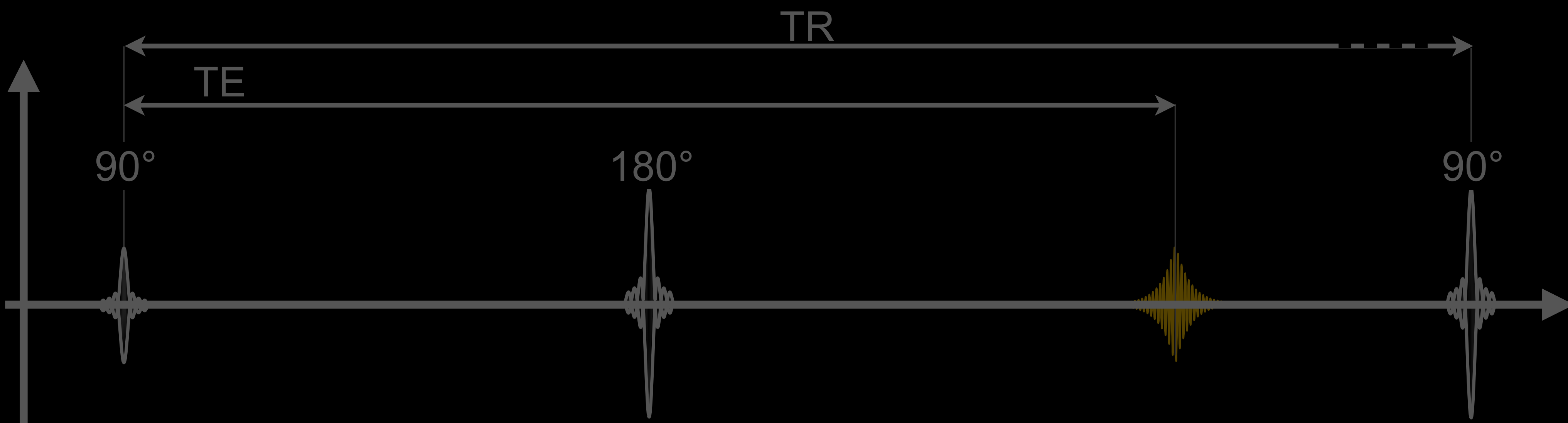
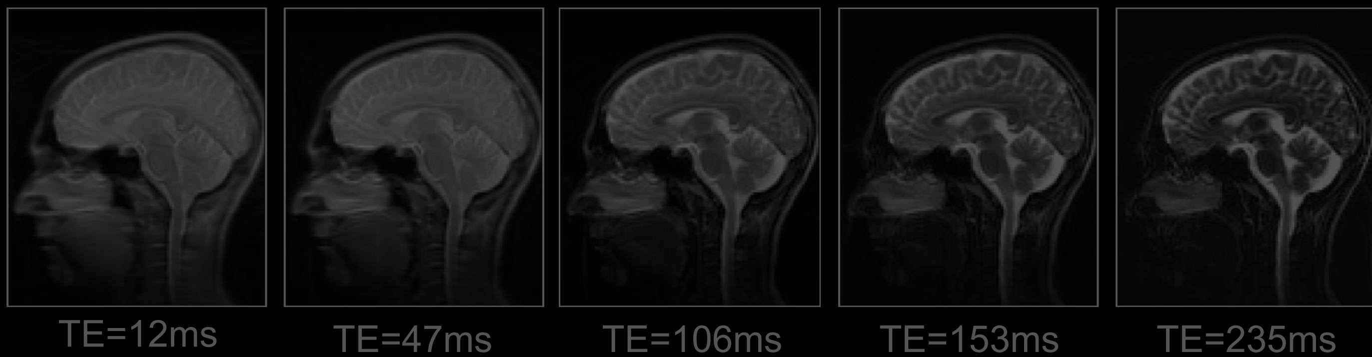


Basic Pulse Sequences II - Spin Echoes



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Lecture #7 Learning Objectives

- Describe four sources of off-resonance.
- Explain why off-resonance gives rise to apparent signal decay and why it is reversible.
- Be able to explain the difference between T2 and T2*.
- Understand the free induction decay signal and possible applications.
- Be able to define an spin echo and the utility of a refocusing pulse.
- Describe how to obtain proton-density, T1-weighted, and T2-weighted images with spin echoes.

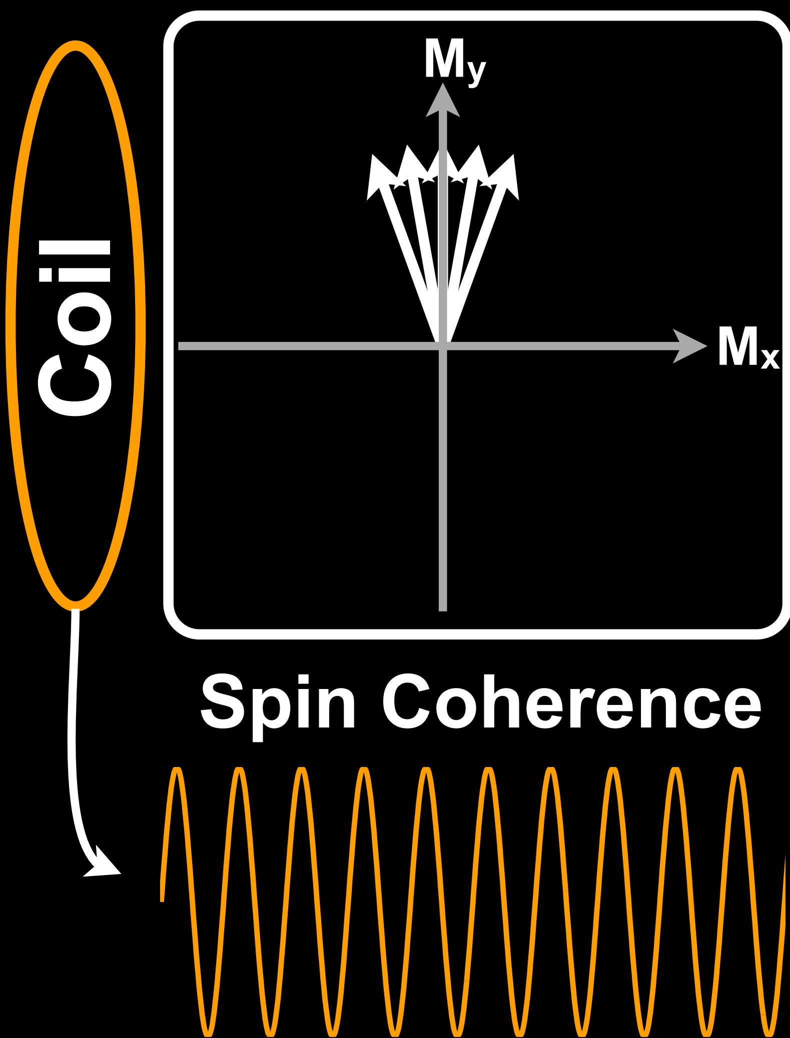
Off Resonance

Spin Dephasing

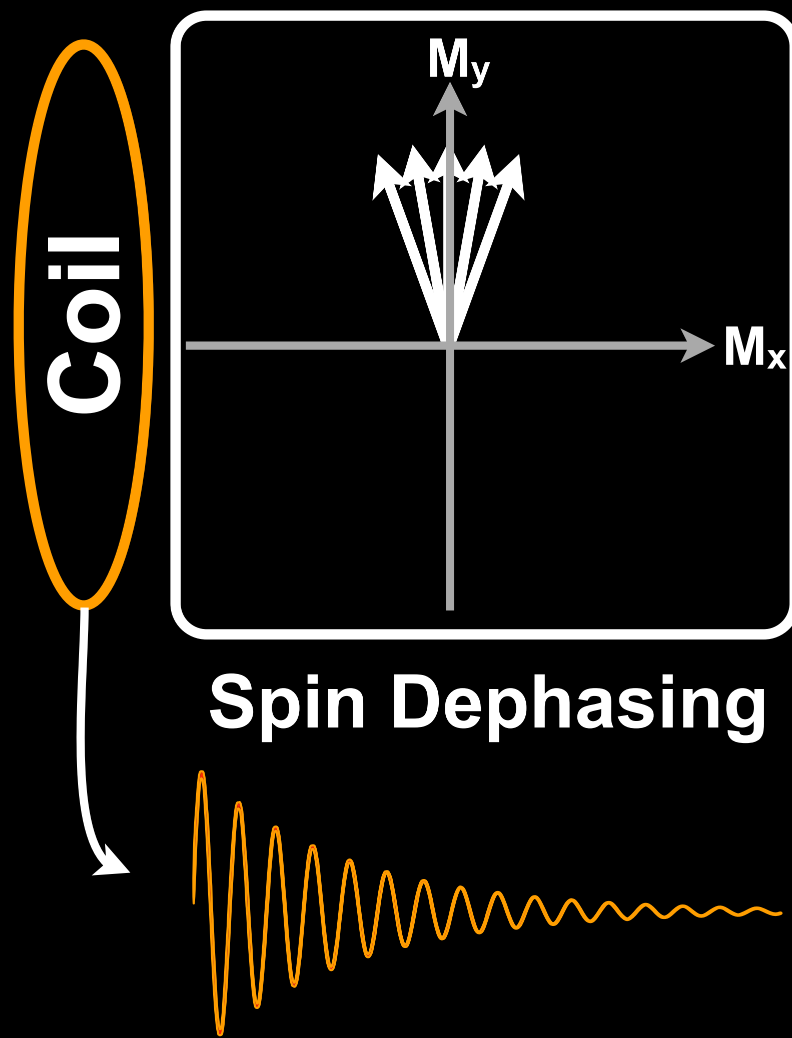
- Intravoxel spin dephasing from:
  - **Off-resonance**
    - B_0 inhomogeneity
    - Chemical shift effects
    - Susceptibility differences (macro and micro)
      - Blood products (***iron***)
      - Blood oxygenation levels
  - Applied gradients
    - Strong gradients produce more spin dephasing
- ... leads to:
  - Loss of spin phase coherence
  - Usually within a voxel
  - Leads to a decreased echo amplitude.
- Minimized by:
  - Field shimming
  - Susceptibility manipulation
  - Refocusing pulses

Intravoxel Spin Dephasing

Homogenous Intravoxel Field



Inhomogenous Intravoxel Field



Signal loss from spin dephasing and T_2*.

Echoes

Why echoes?

- Free Induction Decay
  - Signal decays rapidly
    - T_2
      - Spin-spin interaction
    - Spectral (frequency) distribution
      - Micro-scale B-field heterogeneity (T_2*)
      - Macro-scale B-field heterogeneity (T_2**)
  - Imaging requires certain “delays”
    - Slice-selective rephasing
    - Phase encoding
    - Read-out pre-phasing
  - Echoes let us buy some time

What are echoes?

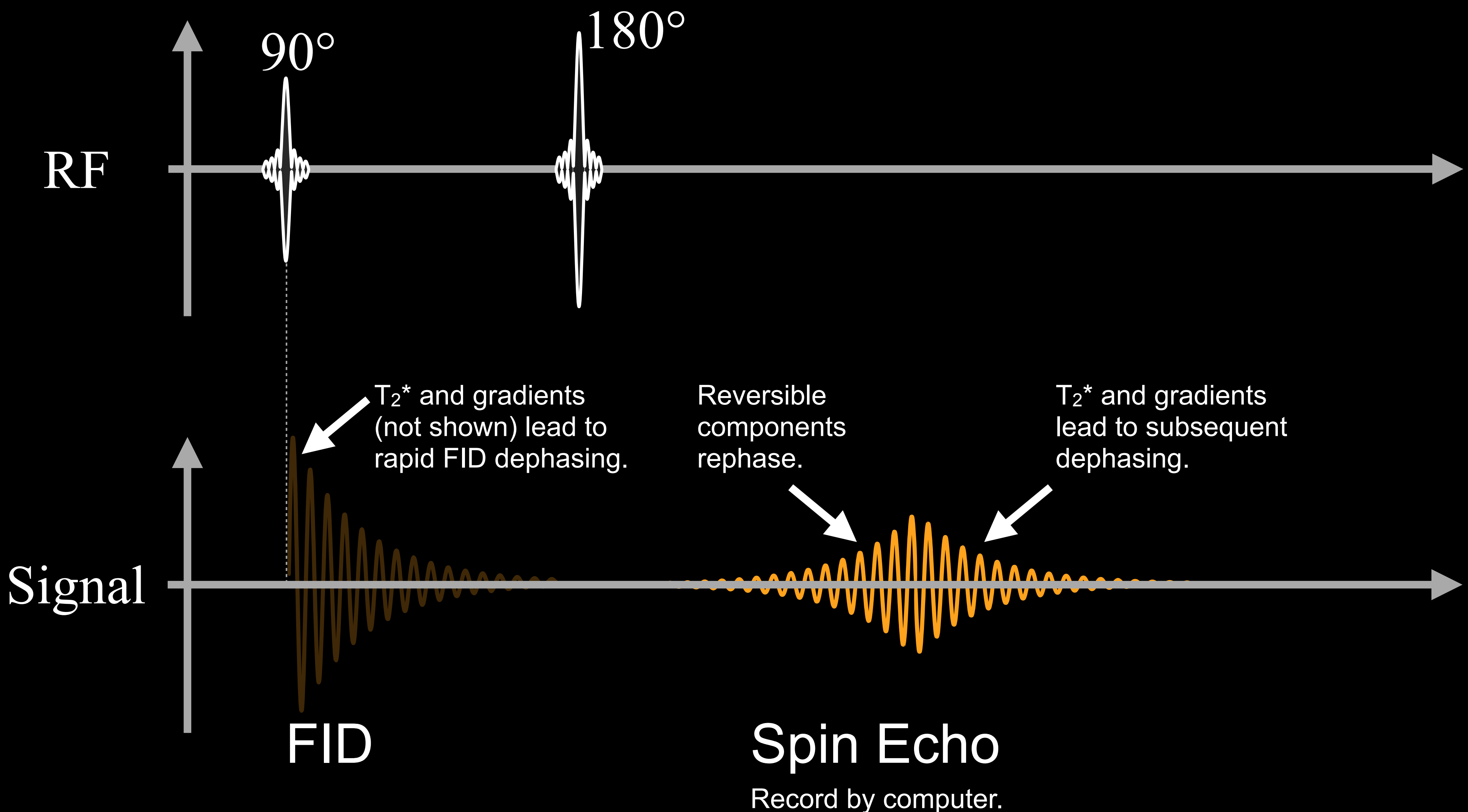
- **Two-sided NMR signals**
  - First half from re-focusing
  - Second half from de-phasing
- **Radiofrequency Echoes**
  - Arise from multiple RF-pulses
- **Gradient Echoes**
  - Arise from magnetic field gradient reversal

"it is easier to generate an echo than to ignore it in multiple-pulse MR experiments"

--Liang & Lauterbur, Page 114

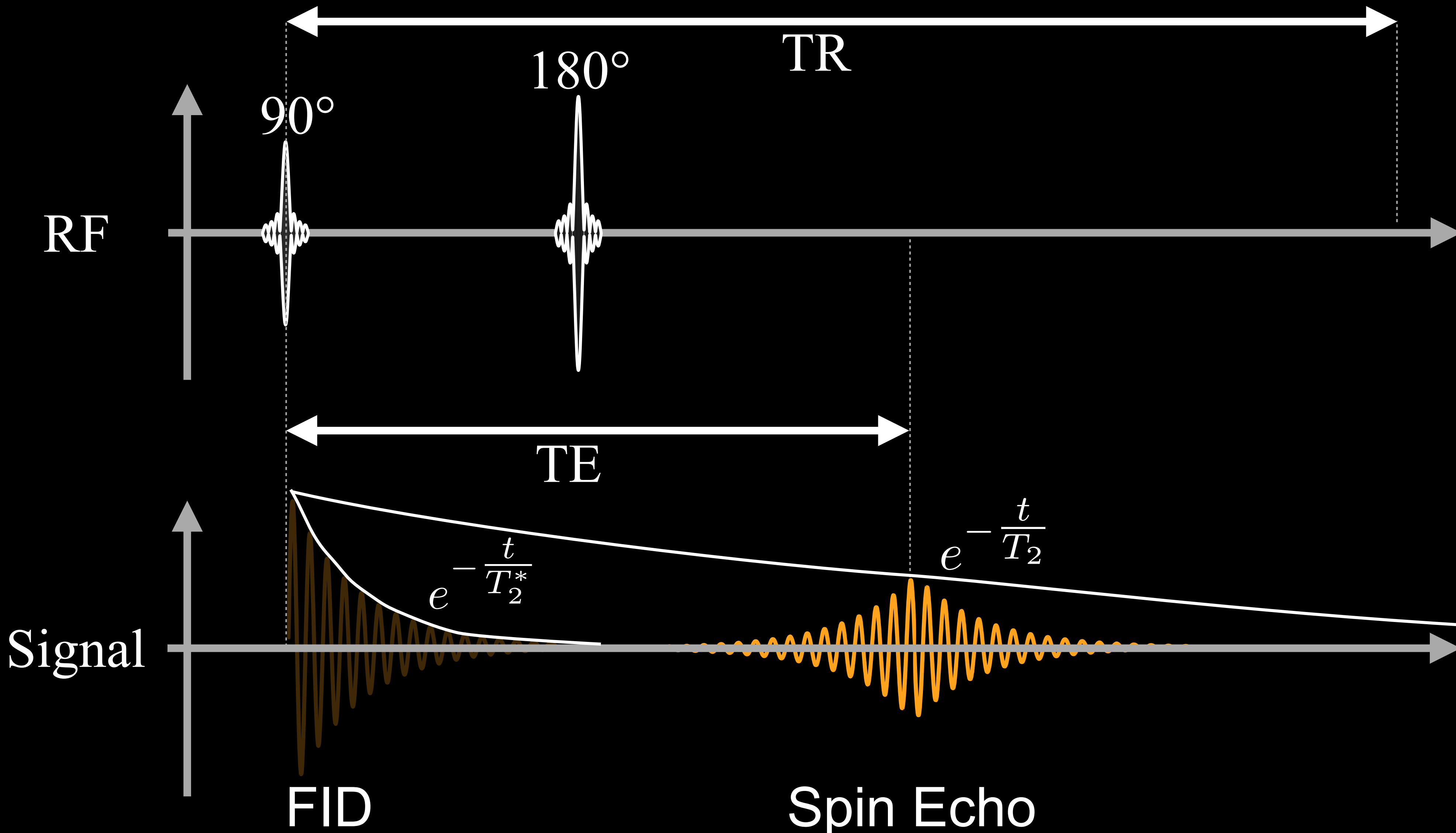
Spin Echo Imaging

Spin Echo



90°
180°
RF
Signal
T_2* and gradients (not shown) lead to rapid FID dephasing.
Reversible components rephase.
T_2* and gradients lead to subsequent dephasing.
FID
Spin Echo
Record by computer.

Spin Echo



How do you adjust the TR?

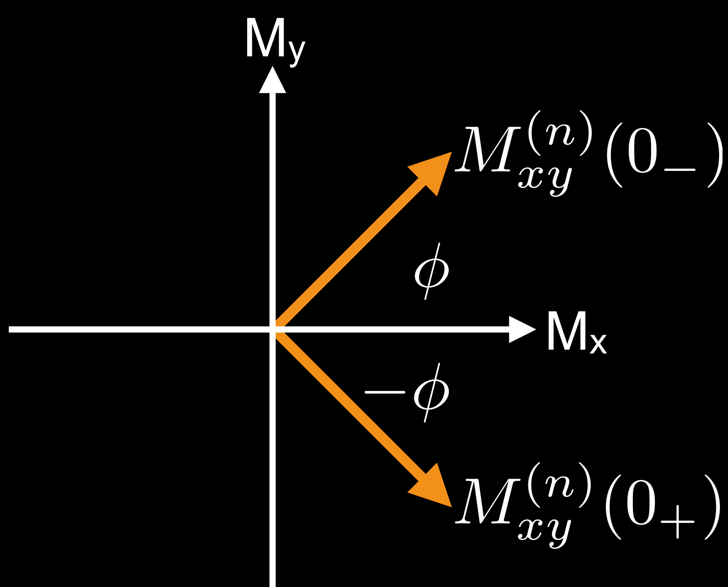
How do you adjust the TE?

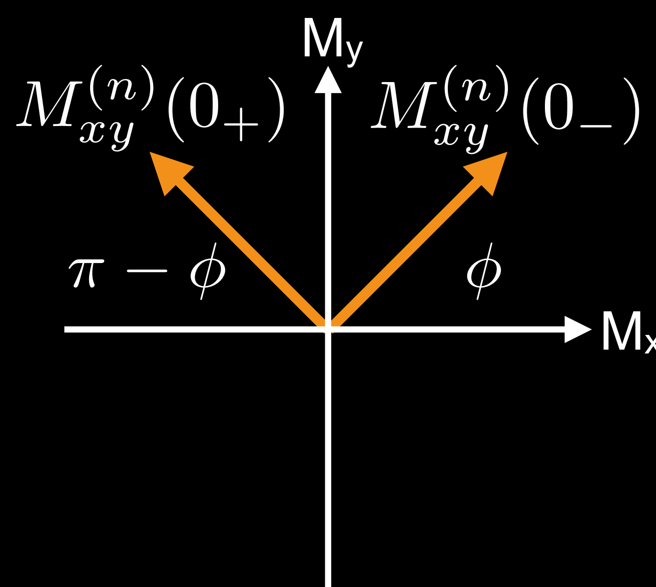
Hard Refocusing Pulses

$$\mathrm{RF}_\theta^\alpha = \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha \end{bmatrix}$$

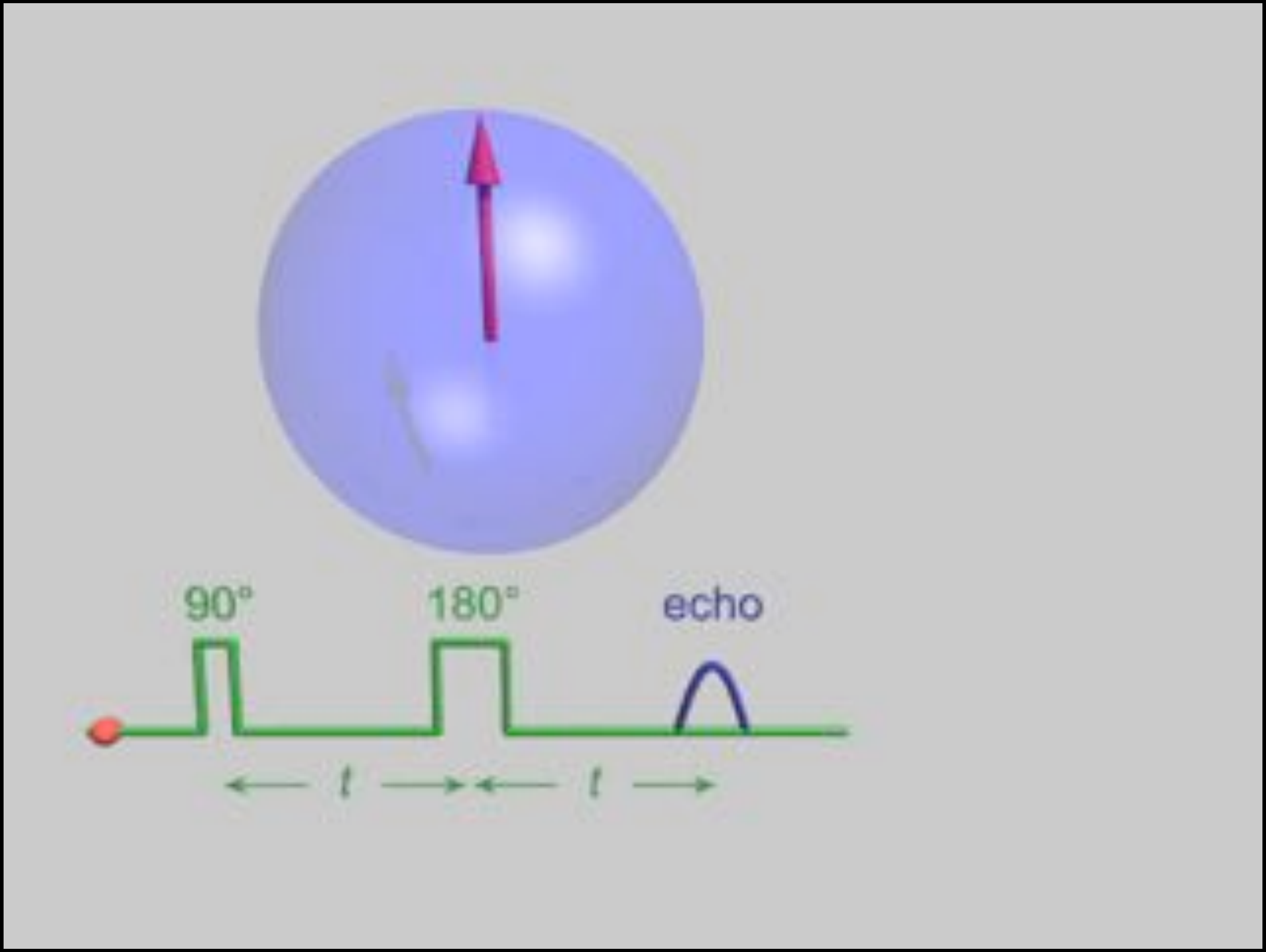
$$\mathrm{RF}_\theta^\alpha = \mathrm{RF}_0^\pi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$\mathrm{RF}_\theta^\alpha = \mathrm{RF}_{\pi/2}^\pi = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$



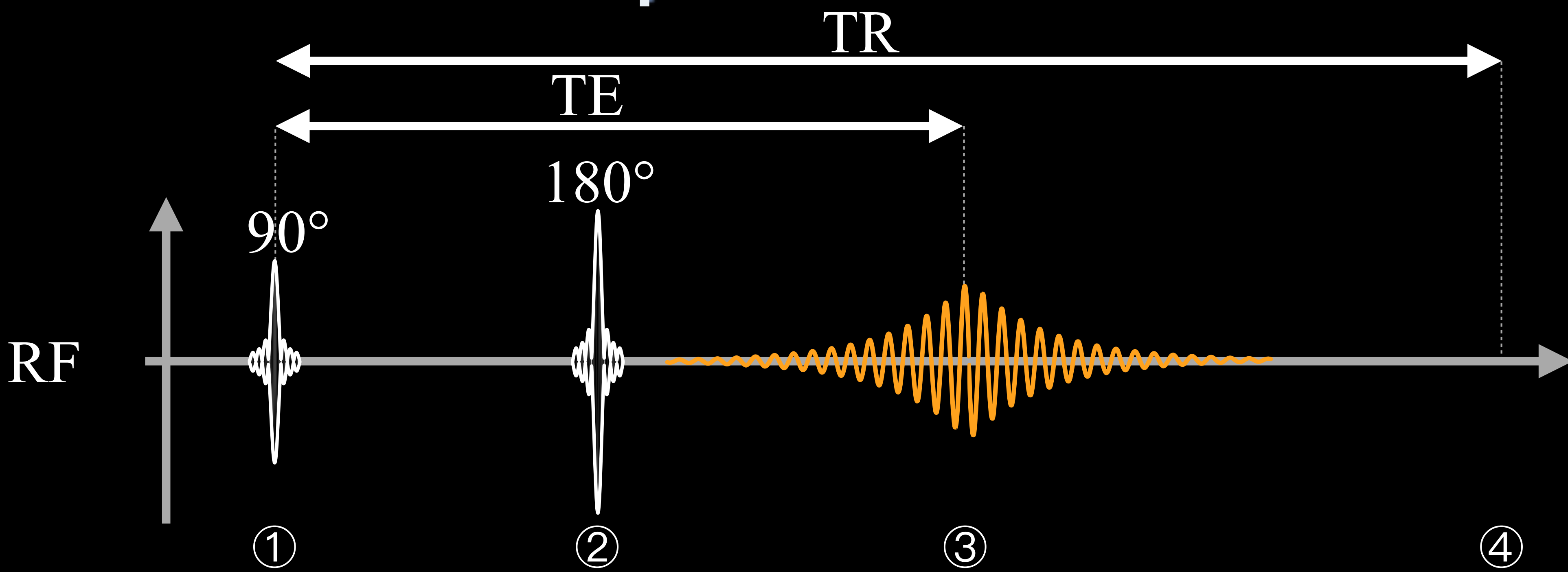


Spin Echo - Contrast



http://en.wikipedia.org/wiki/File:HahnEcho_GWM.gif

Spin Echo



$$M_{z'}^{(4)}\left(0_-\right) = M_z^0\left(1 - 2e^{-(TR-TE/2)/T_1} + e^{-TR/T_1}\right)$$

This becomes the initial condition for the subsequent TR. Eqn. 7.24

$$A_{Echo} \propto \rho\left(1 - 2e^{-(TR-TE/2)/T_1} + e^{-TR/T_1}\right)e^{-TE/T_2}$$

This the signal at time-point #3 for the second TR. Eqn. 7.25

Spin Echo

$$A_{Echo} \propto \rho \left(1 - e^{-TR/T_1}\right) e^{-TE/T_2}$$

This the signal at time-point #3 for the second TR when TE<<TR.

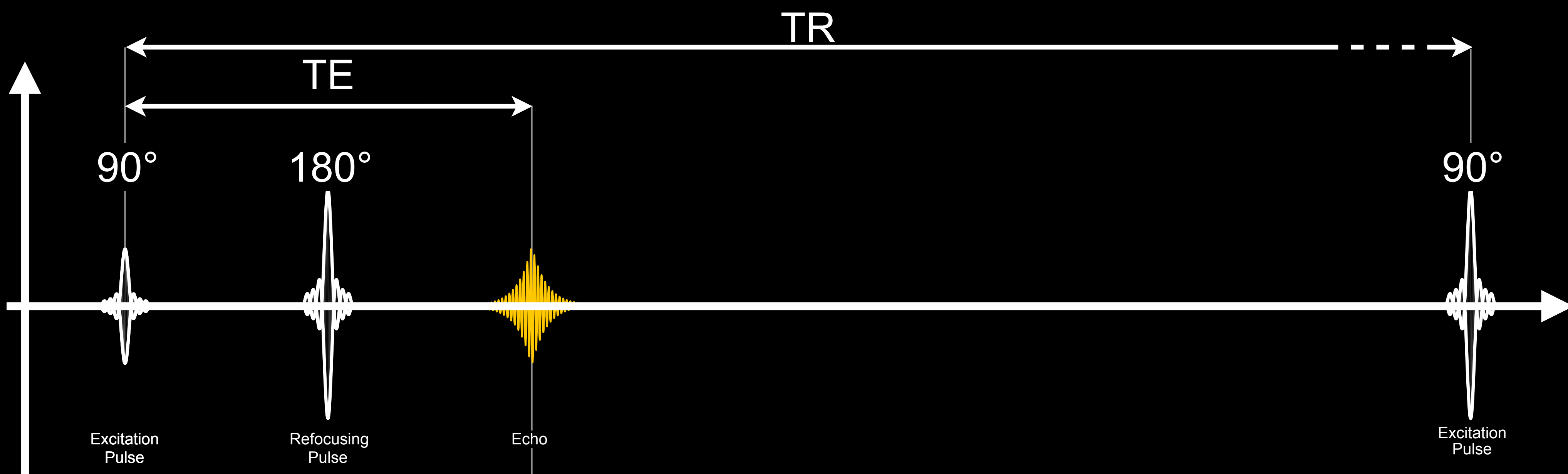
TE=12ms

Short TE and Long TR is proton density weighted.

Spin Echo: TR=6500ms (ETL=12)

Spin Echo

$$A_{Echo} \propto \rho \left(1 - e^{-TR/T_1}\right) e^{-TE/T_2}$$



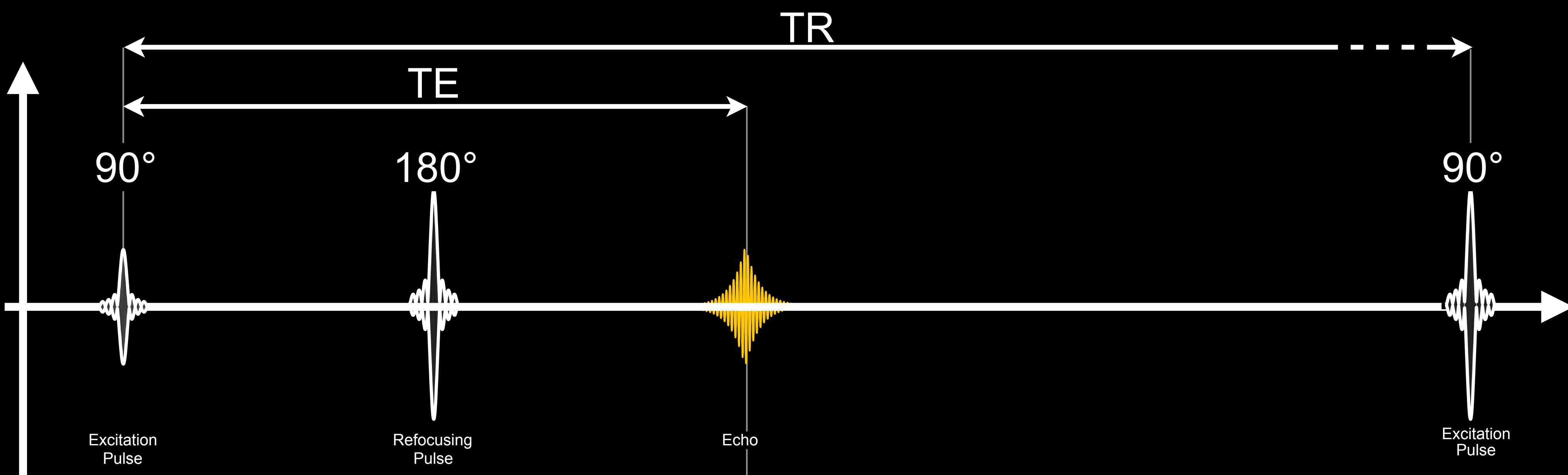

TE=12ms


TE=47ms

Delaying the 180° refocusing pulse delays the TE.

Spin Echo

$$A_{Echo} \propto \rho \left(1 - e^{-TR/T_1}\right) e^{-TE/T_2}$$



TE=12ms

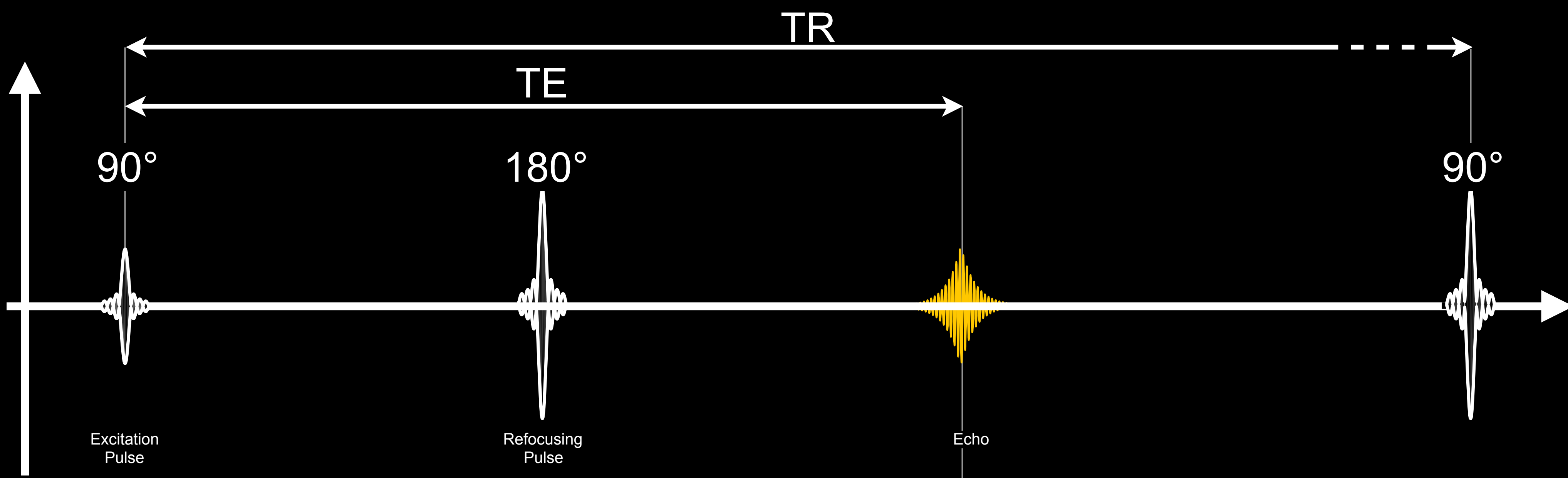
TE=47ms

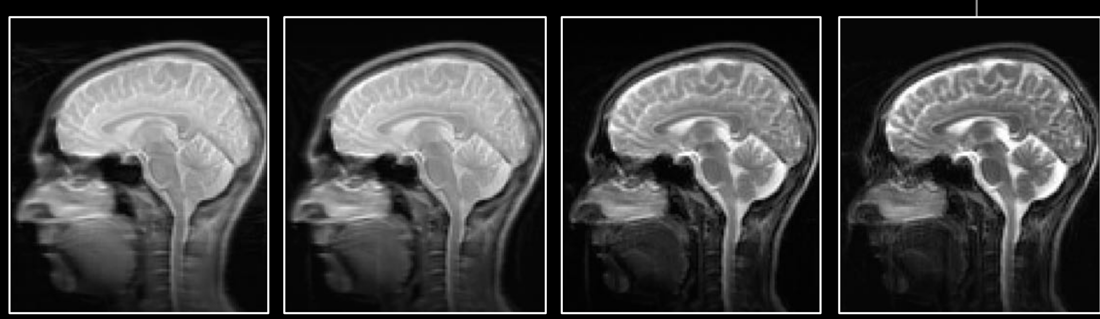
TE=106ms

Longer TEs produce more T_2-weighting.

Spin Echo

$$A_{Echo} \propto \rho \left(1 - e^{-TR/T_1}\right) e^{-TE/T_2}$$





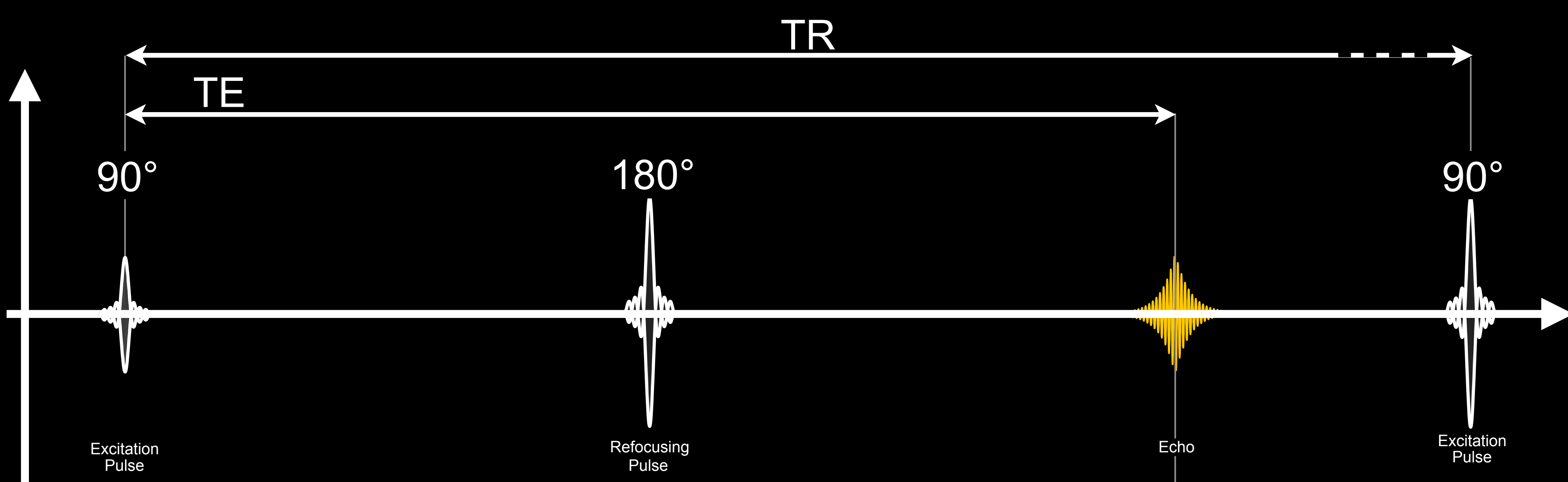
TE=12ms TE=47ms TE=106ms TE=153ms

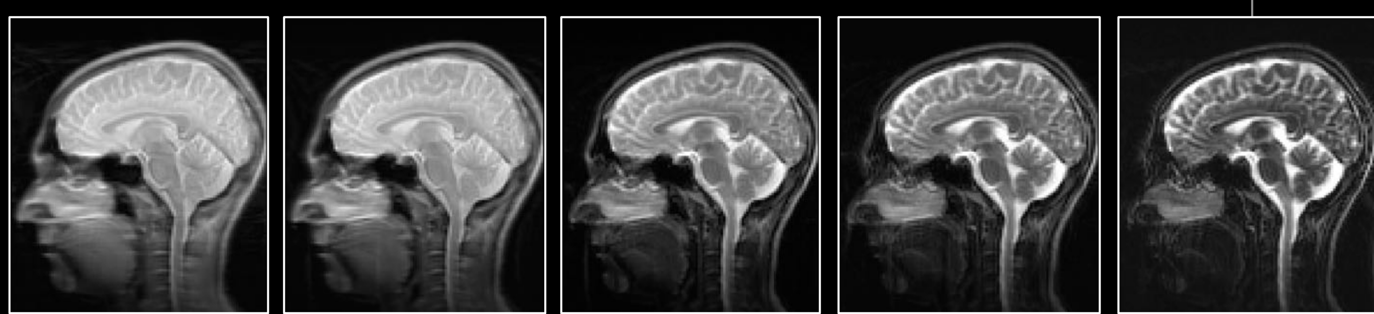
Longer TEs produce more T_2-weighting.

Spin Echo: TR=6500ms (ETL=12)

Spin Echo

$$A_{Echo} \propto \rho \left(1 - e^{-TR/T_1}\right) e^{-TE/T_2}$$





TE=12ms TE=47ms TE=106ms TE=153ms TE=235ms

Long T_2 is bright on T_2-weighted (long TE) images.

Basic Pulse Sequences III
Gradient Echoes

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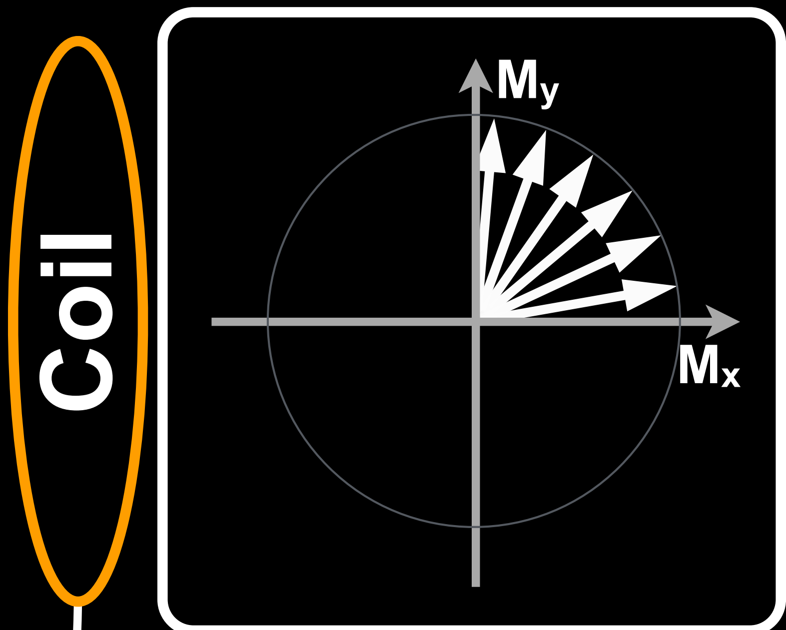
Lecture #8 Learning Objectives

- **Describe the pros and cons of a GRE acquisition, especially in comparison to a spin-echo sequence.**
- **Understand why GRE can't acquire true T2 contrast.**
- **Explain why spoilers are typically used with GRE.**
- **Understand how to calculate scan time.**
- **Be able to derive the optimal flip angle for a GRE sequence, and understand why we might not use that value (contrast).**
- **Describe methods of fat-water separation and their utility.**

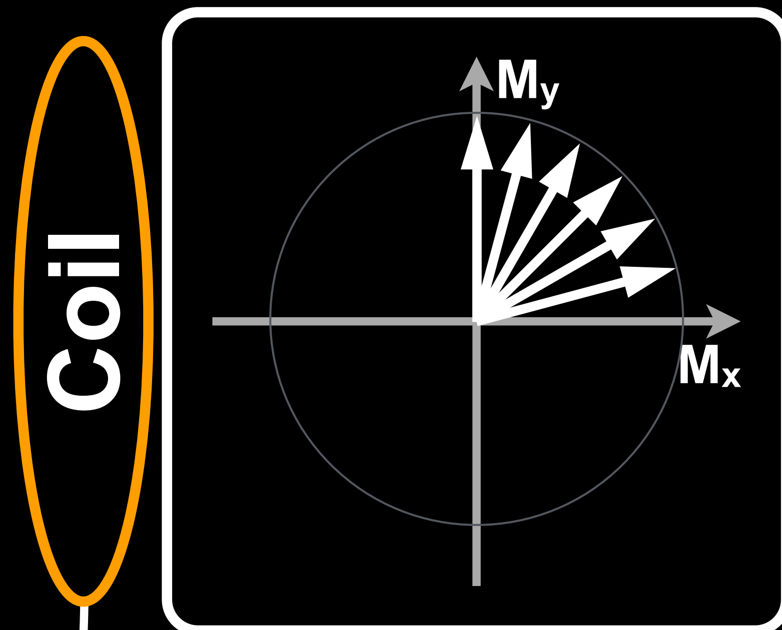
T_2 versus T_2^*

T_2 Decay



Signal loss from spin-spin interaction.

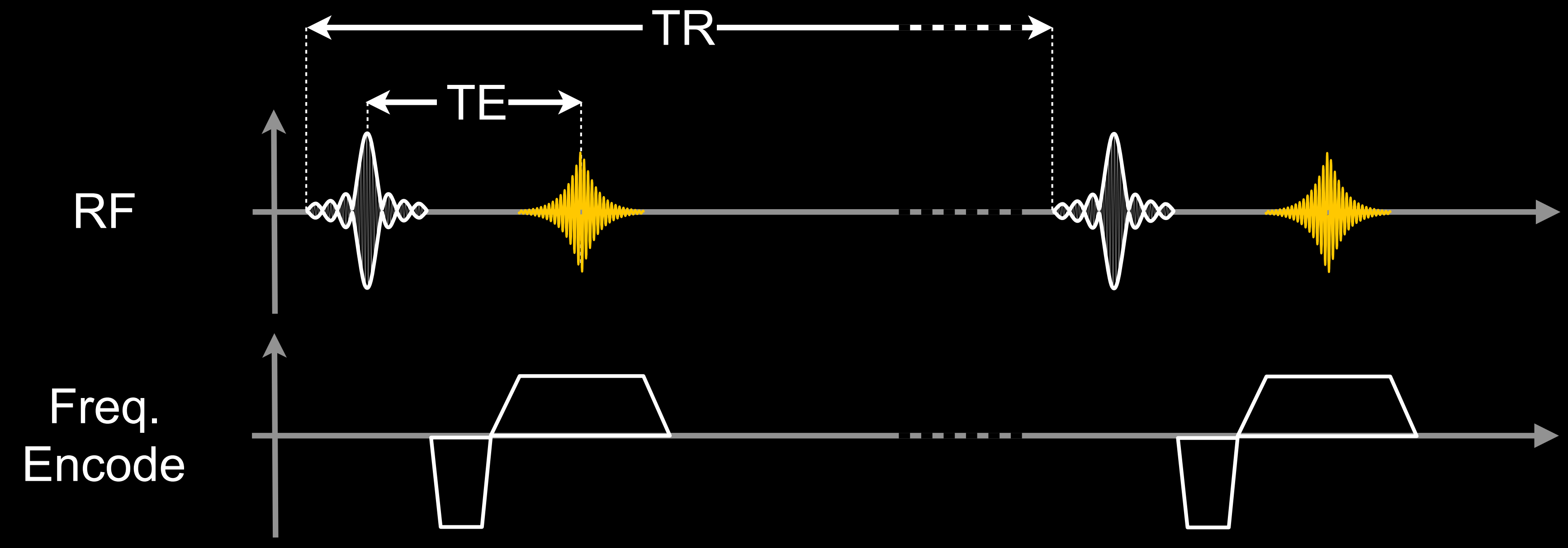
T_2^* Decay



Signal loss from spin-spin interaction and off-resonance dephasing and T_2^*.

Signal loss from spin dephasing and T_2^*.

Gradient Echo



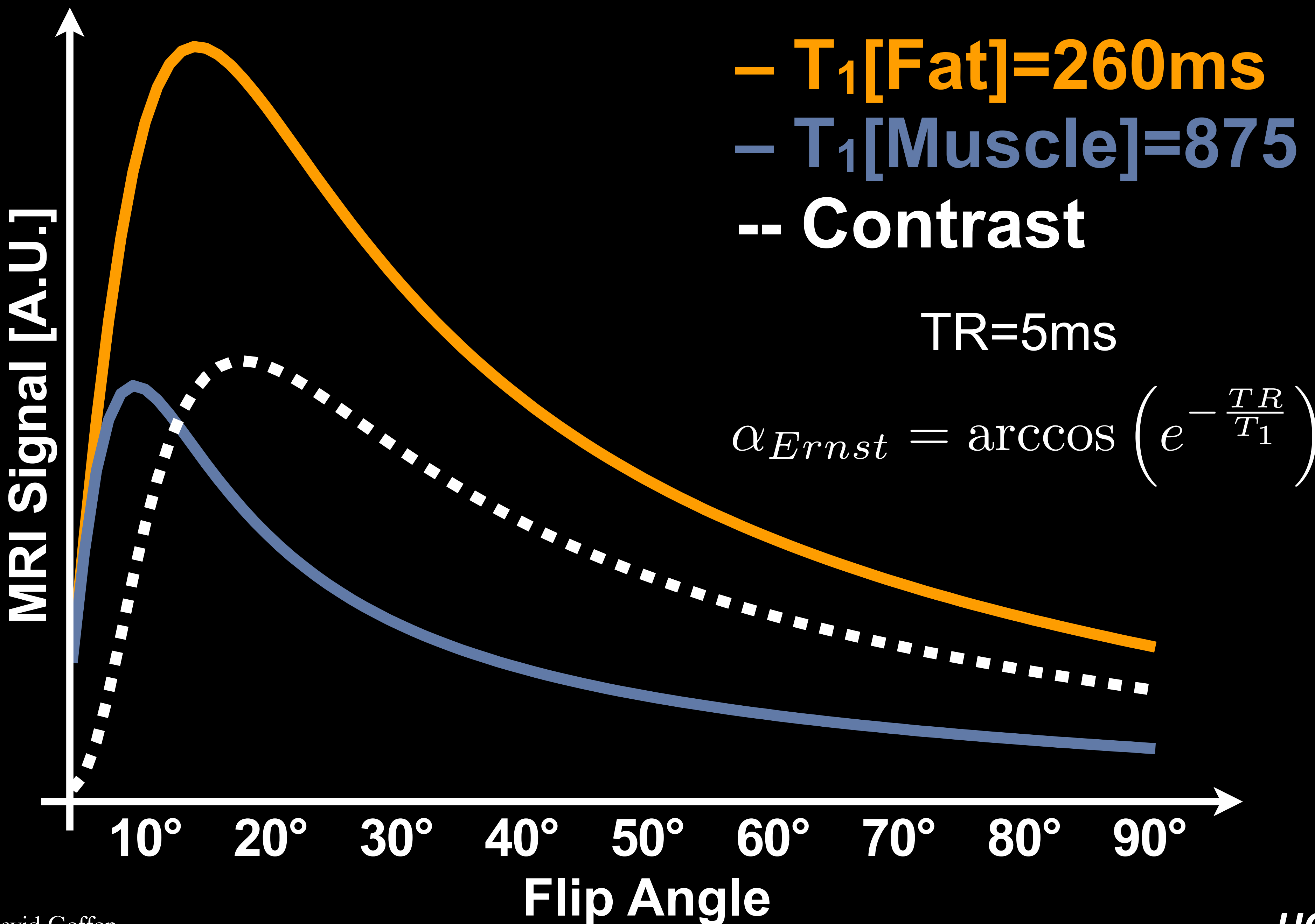
Spoiled Gradient Echo Contrast

$$\mathrm{M}_z^{ss} = \frac{\mathrm{M}_0\left(1 - e^{-TR/T_1}\right)}{1 - \cos\alpha e^{-TR/T_1}}$$

$$A_{echo} \propto \frac{\rho\left(1 - e^{-TR/T_1}\right)}{1 - \cos\alpha e^{-TR/T_1}} \sin\alpha e^{-TE/T_2^*}$$

Contrast adjusted by changing flip angle, TE and TR.

Spoiled GRE & Ernst Angle



Spin Echo Contrast

$$A_{Echo} \propto \rho \left(1 - 2e^{-(TR-TE/2)/T_1} + e^{-TR/T_1}\right) e^{-TE/T_2}$$

$$\text{If } TE \ll TR, \text{ then}$$

$$A_{Echo} \propto \rho \left(1 - e^{-TR/T_1}\right) e^{-TE/T_2}$$

Gradient Echo Contrast

$$A_{echo} \propto \frac{\rho\left(1 - e^{-TR/T_1}\right)}{1 - \cos\alpha e^{-TR/T_1}} \sin\alpha e^{-TE/T_2^*}$$

RF pulse and gradient timing encode image contrast in the echo (M_{xy}).

A major challenge in MRI is encoding spatial information in the echo.

Spin vs. Gradient Echo Contrast

Gradient Echo Parameters

Type of Contrast	TE	TR	Flip Angle
Spin Density	Short	Long	Small
T_1-Weighted	Short	Intermediate	Large
T_2^*-Weighted	Intermediate	Long	Small

Spin Echo Parameters

Spin Density	Short	Long
T_1-Weighted	Short	Intermediate
T_2-Weighted	Intermediate	Long

Spin vs. Gradient Echo Contrast

Gradient Echo Parameters

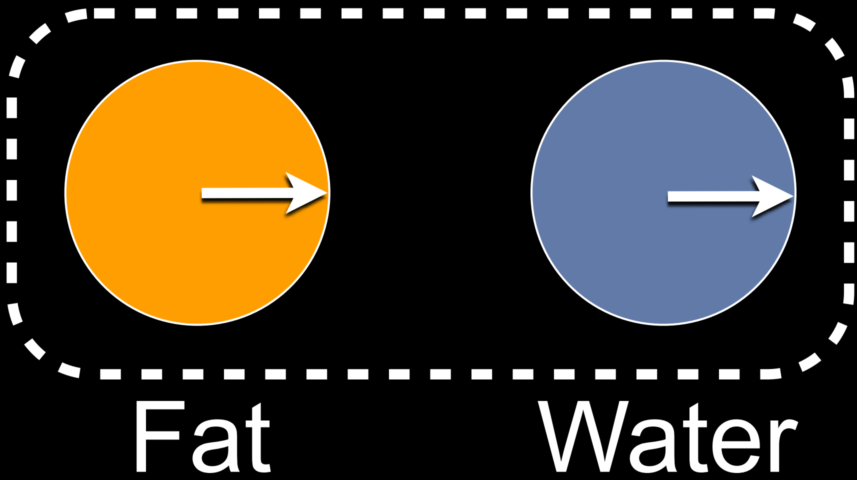
Type of Contrast	TE	TR	Flip Angle
Spin Density	<5ms	>100ms	<10°
T_1-Weighted	<5ms	<50ms	>30°
T_2^*-Weighted	>20ms	>100ms	<10°

Spin Echo Parameters

Spin Density	10-30ms	>2000ms
T_1-Weighted	10-30ms	450-850ms
T_2-Weighted	>60ms	>2000ms

GRE and Fat/Water Phase

- Pixels are frequently a mixture of fat and water
- Pixel intensity is the vector sum of fat and water



In-Phase

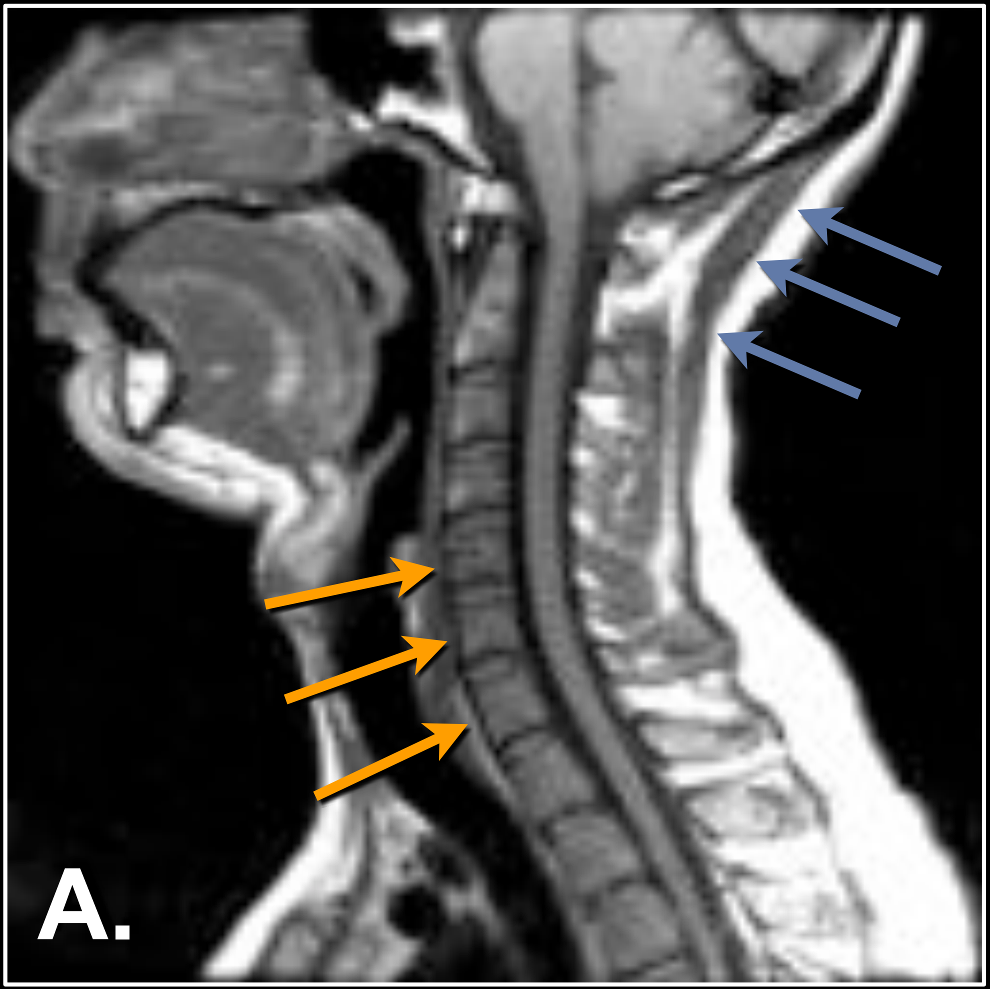
$\longrightarrow + \longrightarrow\ >0$

Opposed-Phase

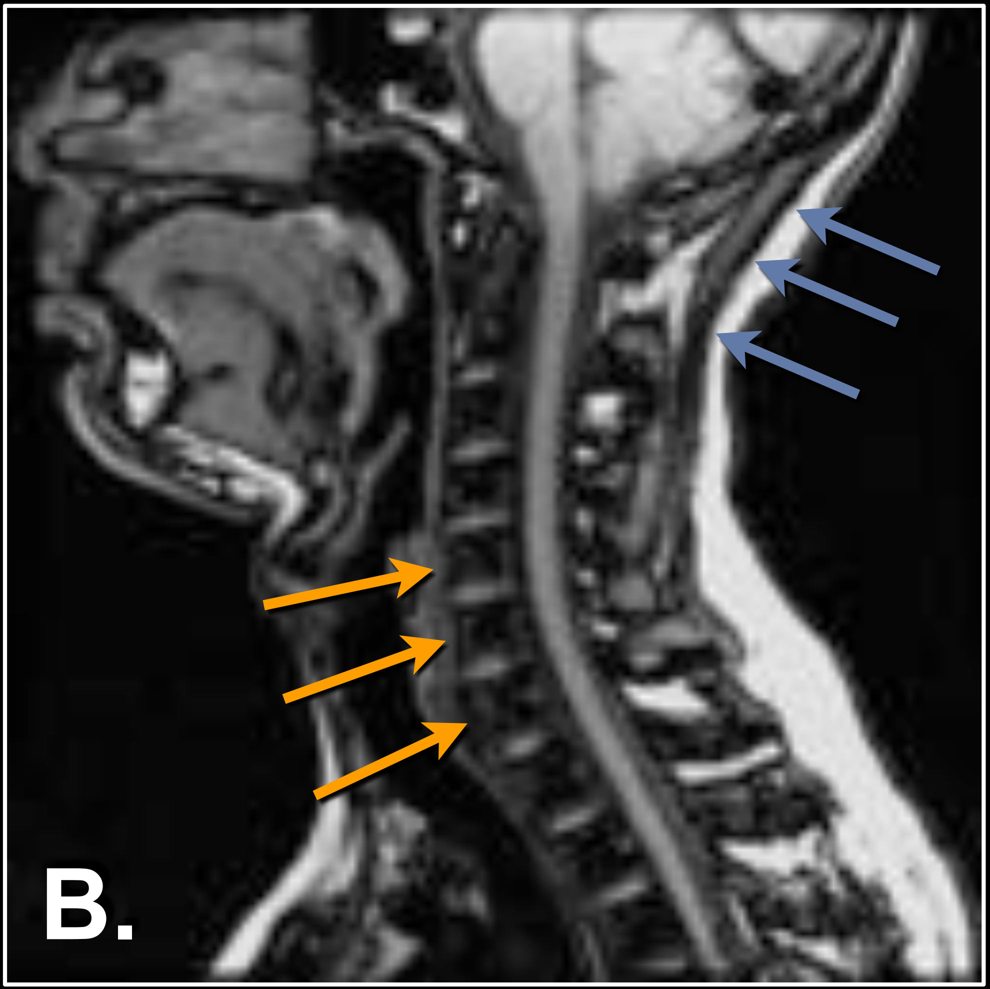
$\longleftarrow + \longrightarrow\ =0$

The TE controls the phase between fat and water.

Which image is the in-phase image?

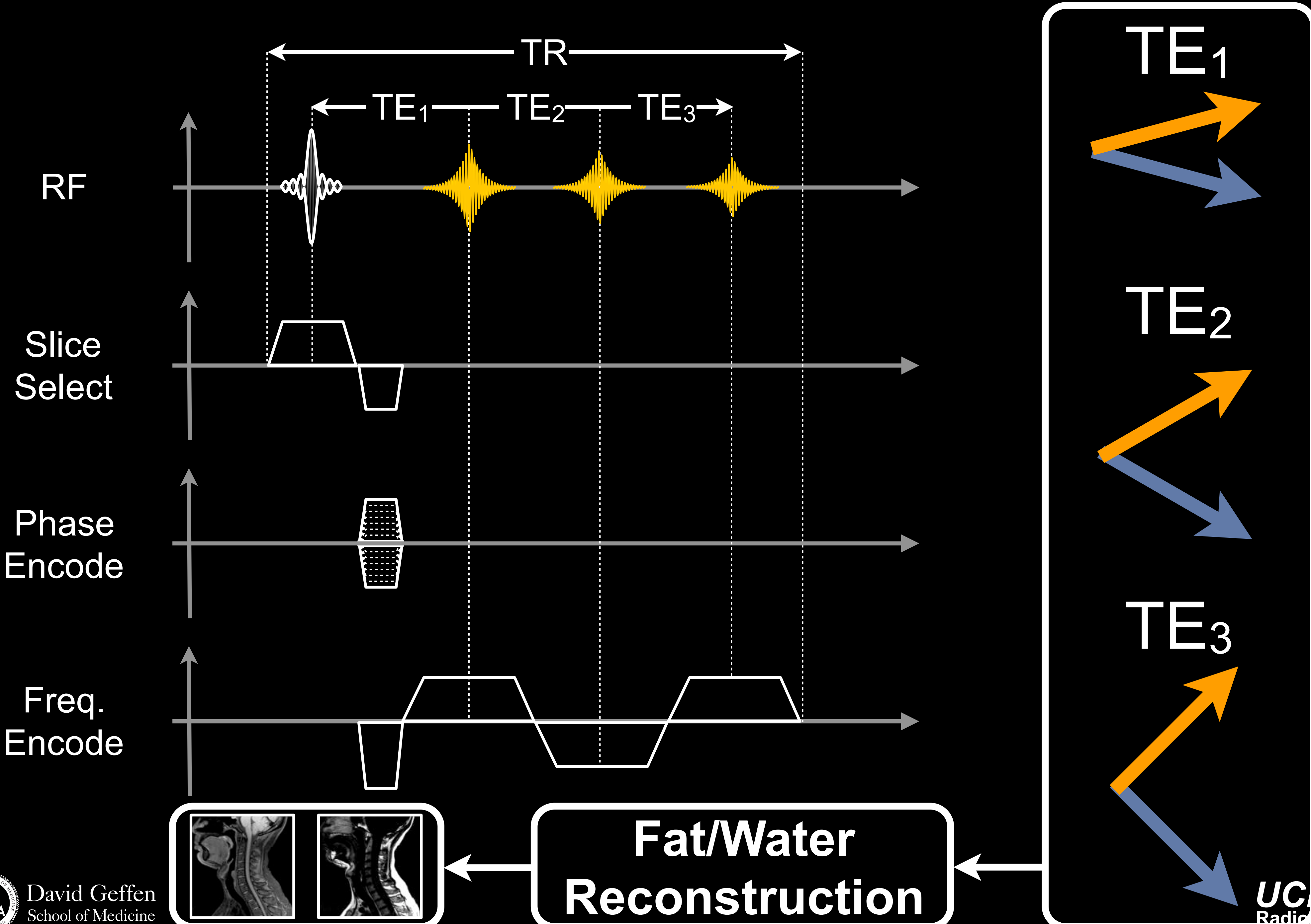


In-Phase



Opposed-Phase

GRE & Fat/Water Separation - How?



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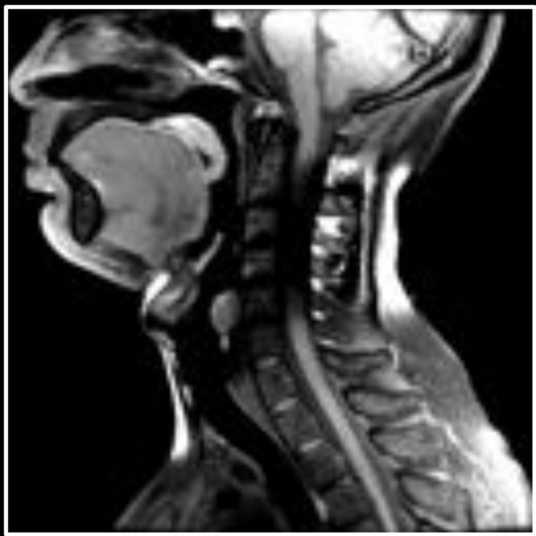
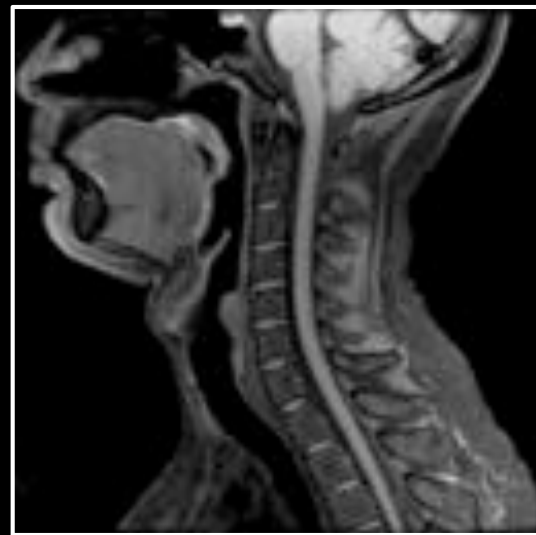
Gradient Echoes & Fat/Water Separation



Imperfect Fat Sat



IDEAL water image

IDEAL fat image

in-phase

opposed-phase

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Images Courtesy of Dr. Scott Reeder

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The MRI Signal Equation

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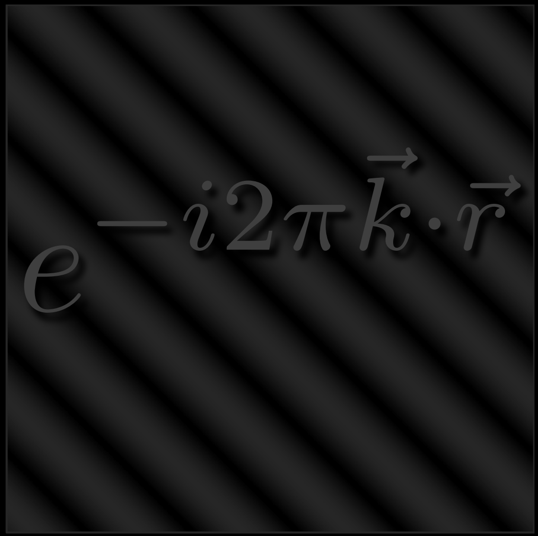
$$S(\vec{k}) = \int_{object} M_{xy}(\vec{r}, 0)\, e^{-i2\pi\vec{k}\cdot\vec{r}}\, d\vec{r}$$



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Lecture #9 Learning Objectives

- Understand that SE and GRE control image contrast at the echo time.
- Appreciate that gradients move us through k-space.
- Describe how to calculate scan time.
- Explain the concept of “coil sensitivity.”
- Explain why MRI is not directly sensitive to M_z.
- Understand the role of phase sensitive detection.
- Describe the importance of quadrature detection.
- Be able to define the MRI signal equation and each term.

Dipoles to Images

$\vec{\mu}$	Magnetic Moment	
$\downarrow$		B_0
$\vec{M}$	Bulk Magnetization	
$\downarrow$		B_1
$\vec{M}_{xy}(t)$	Transverse Magnetization	
$\downarrow$		$G_{Encoding}$
$\vec{M}_{xy}(\vec{r}, t)$	Spatially Encoded Magnetization	
$\downarrow$		Coil
$V(t)$	Received Voltage	
$\downarrow$		PSD
$S(t)$	Complex Signal	
$\downarrow$		$G_{Decoding}$
$S\left(\vec{k}\right)$	k-space signal	
$\downarrow$		Reconstruction
$I(\vec{r})$	Image	

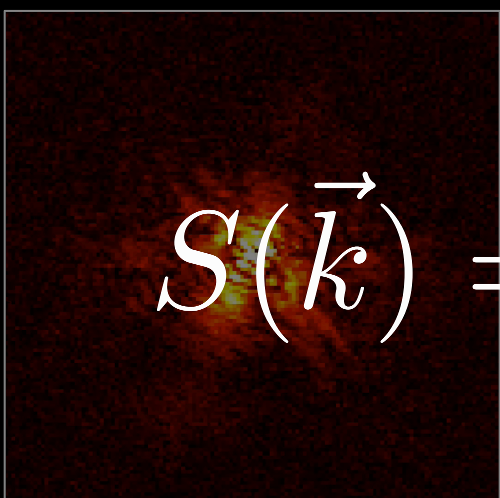
The MRI Signal Equation

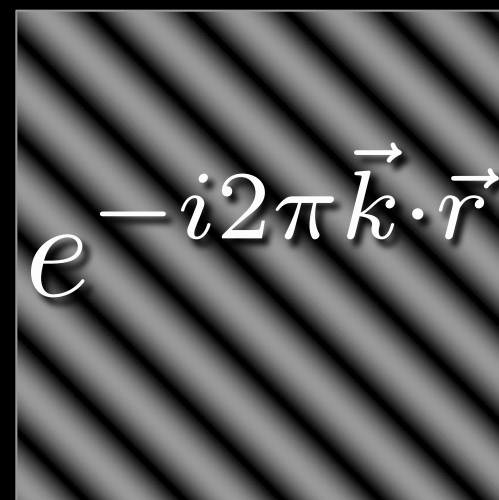
Signal at a point in *k*-space

Faraday's Law of Induction

State of M_{xy} in the *object*

k-space sampling function



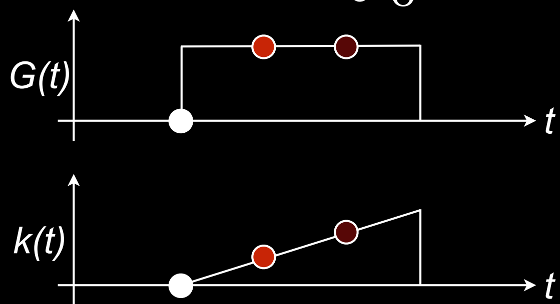


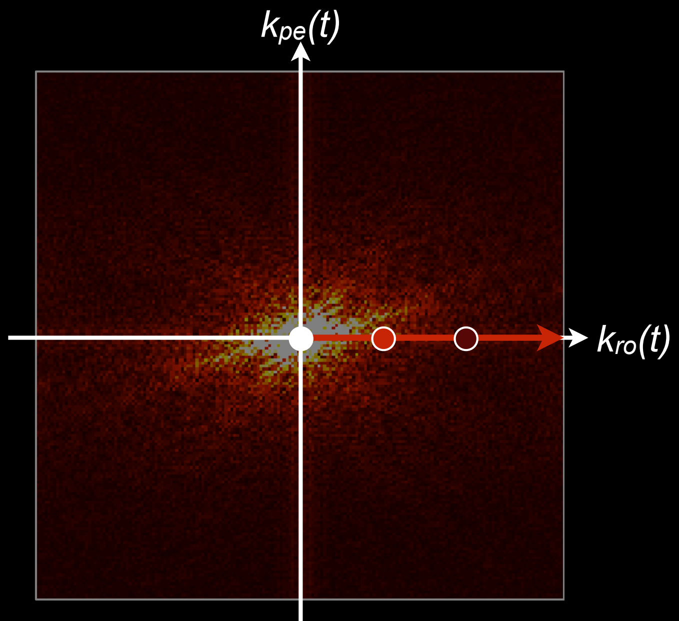
$$S(\vec{k}) = \int_{Object} M_{xy}(\vec{r}, 0)\, e^{-i2\pi\vec{k}\cdot\vec{r}}\, d\vec{r}$$

MRI *acquires* point-wise the Fourier Transform of the object.

What is *k*-space?

$$\vec{k}(\tau) = \frac{\gamma}{2\pi}\int_0^\tau \vec{G}(t)dt$$



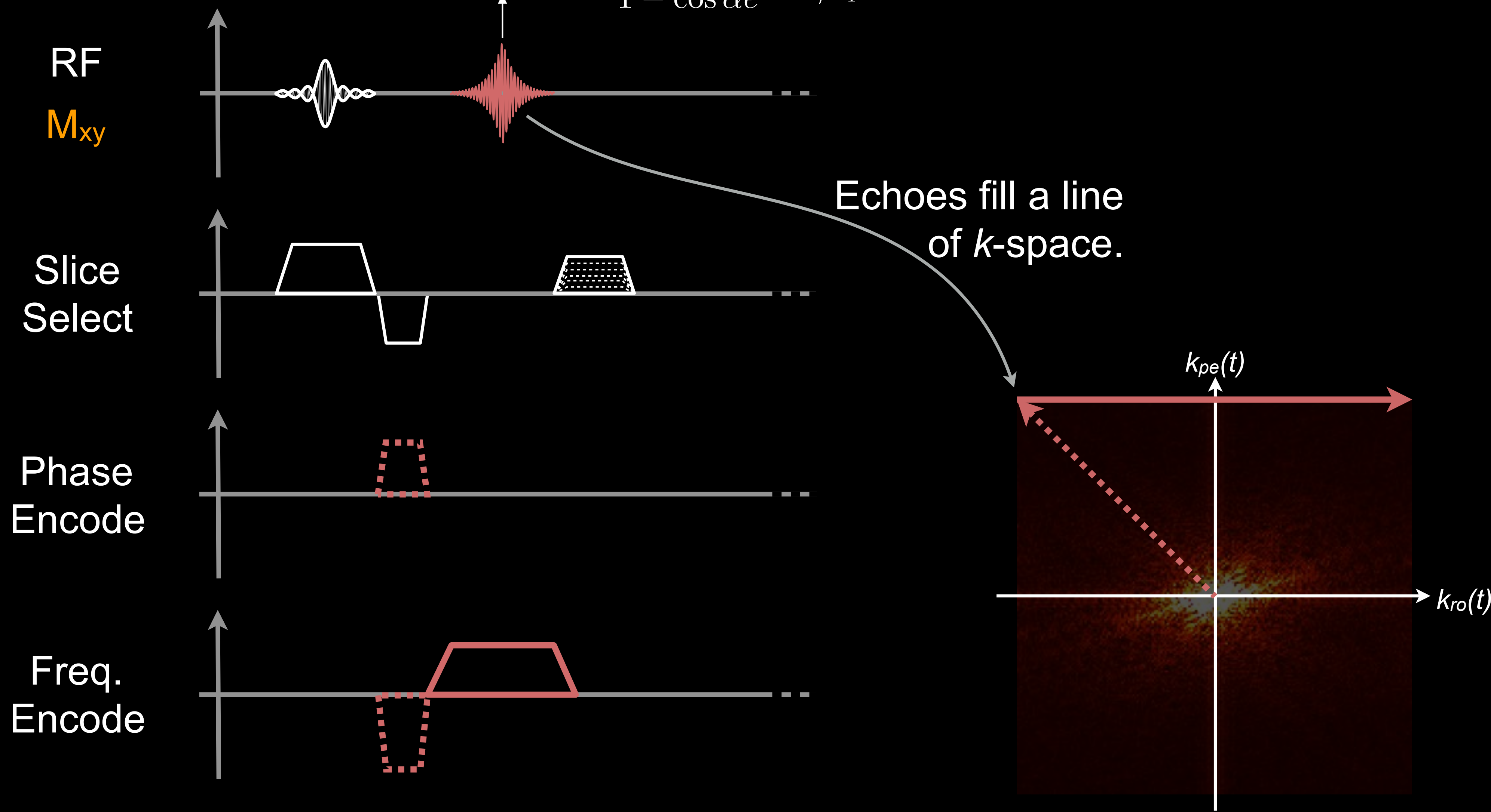


Gradients move us through *k*-space.

Spoiled Gradient Echo

$$A_{echo} \propto \frac{\rho\left(1 - e^{-TR/T_1}\right)}{1 - \cos\alpha e^{-TR/T_1}} \sin\alpha e^{-TE/T_2^*}$$

Magnitude of M_{xy}



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Gradients move us through *k*-space!

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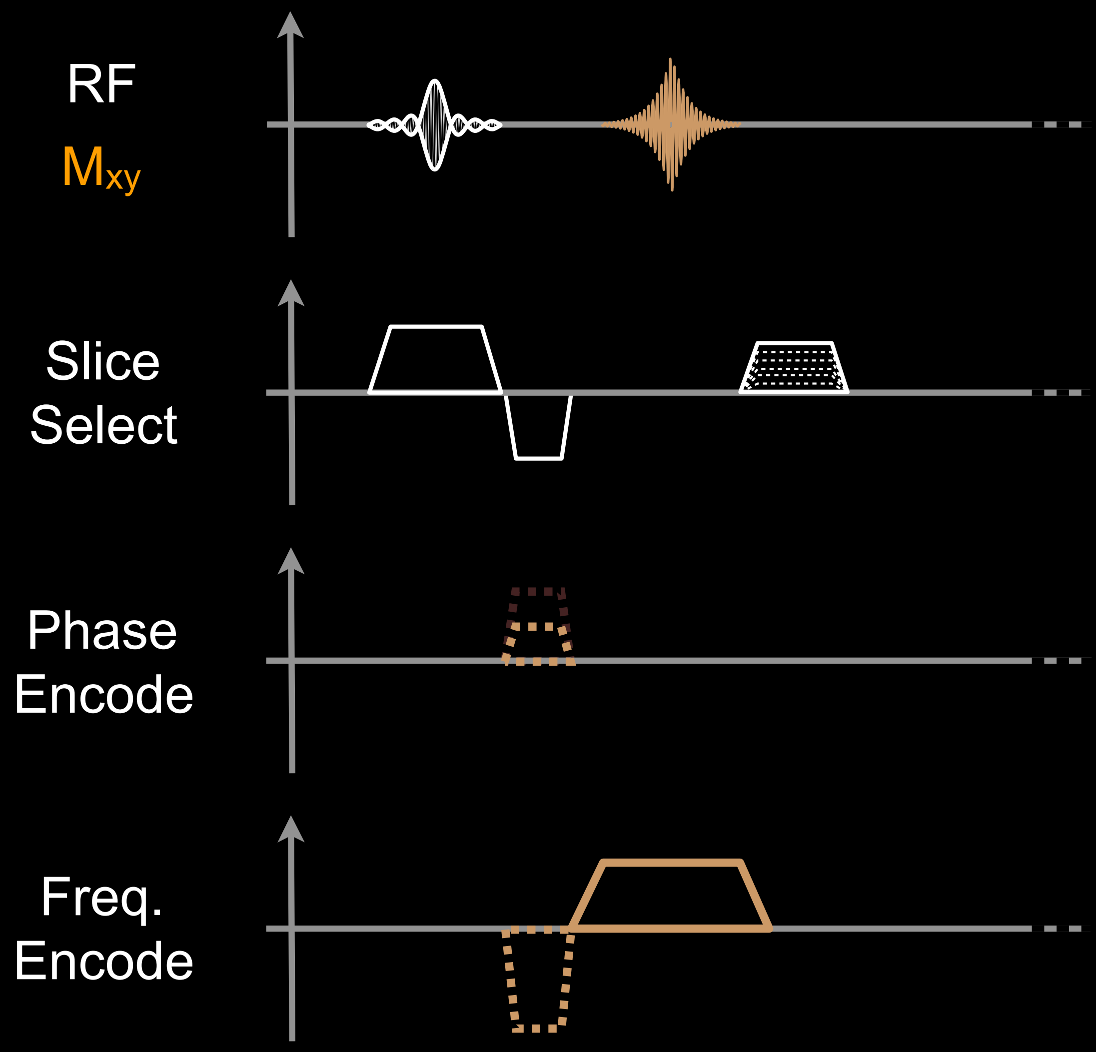
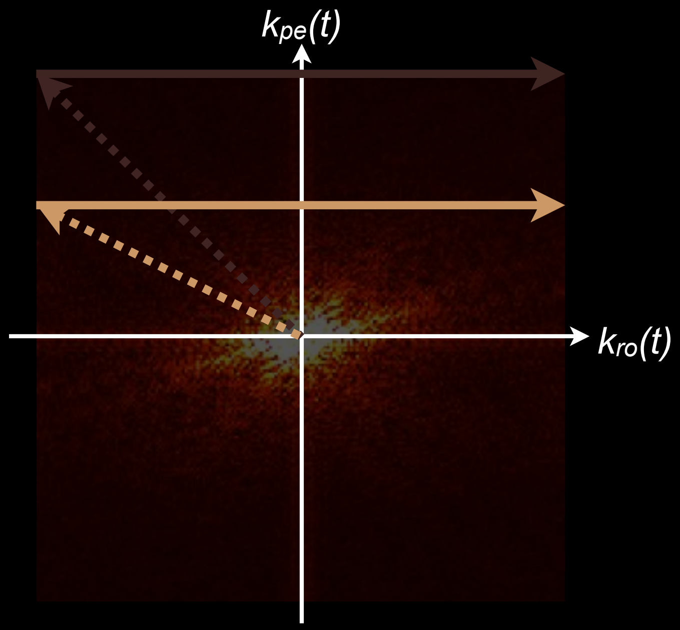
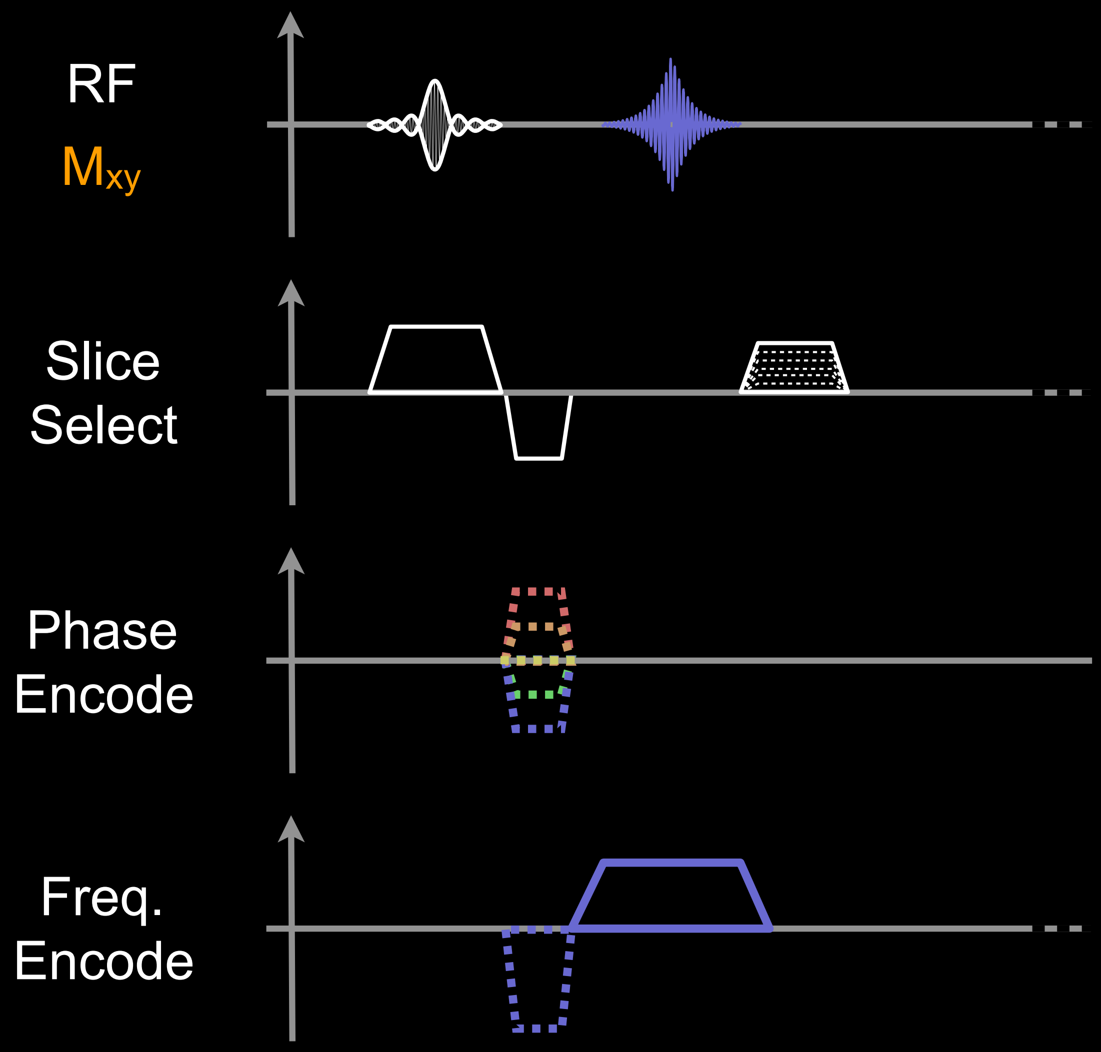
Spoiled Gradient Echo

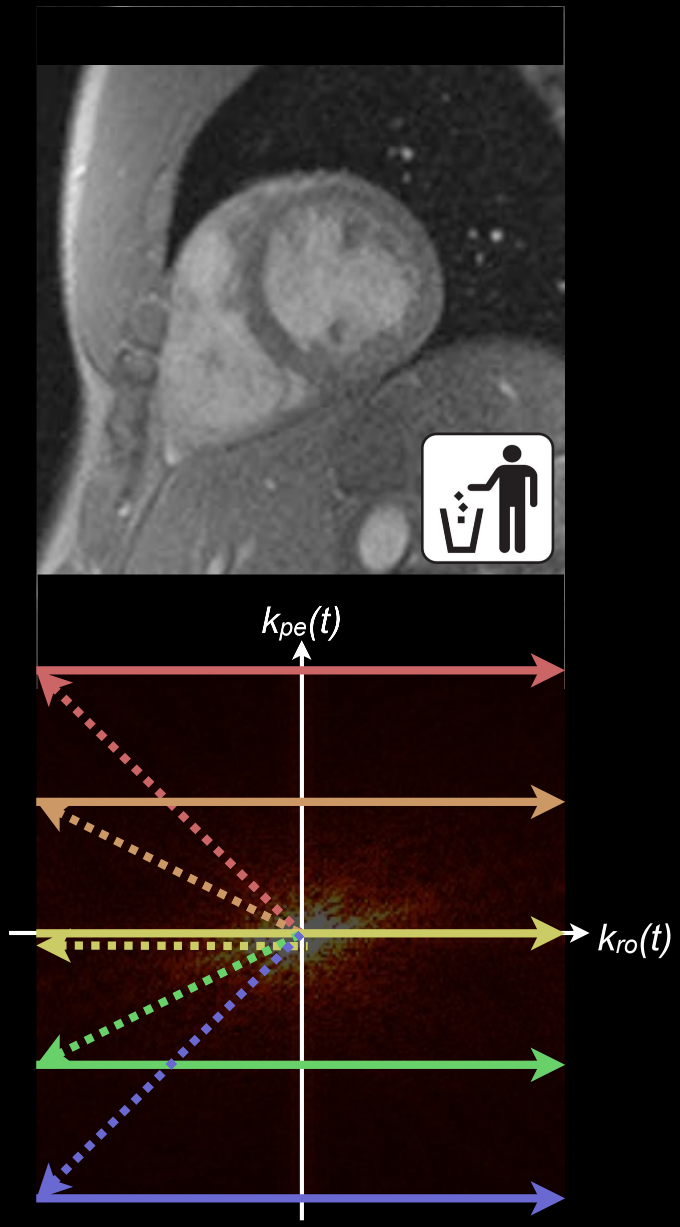




Gradients move us through *k*-space!

Spoiled Gradient Echo



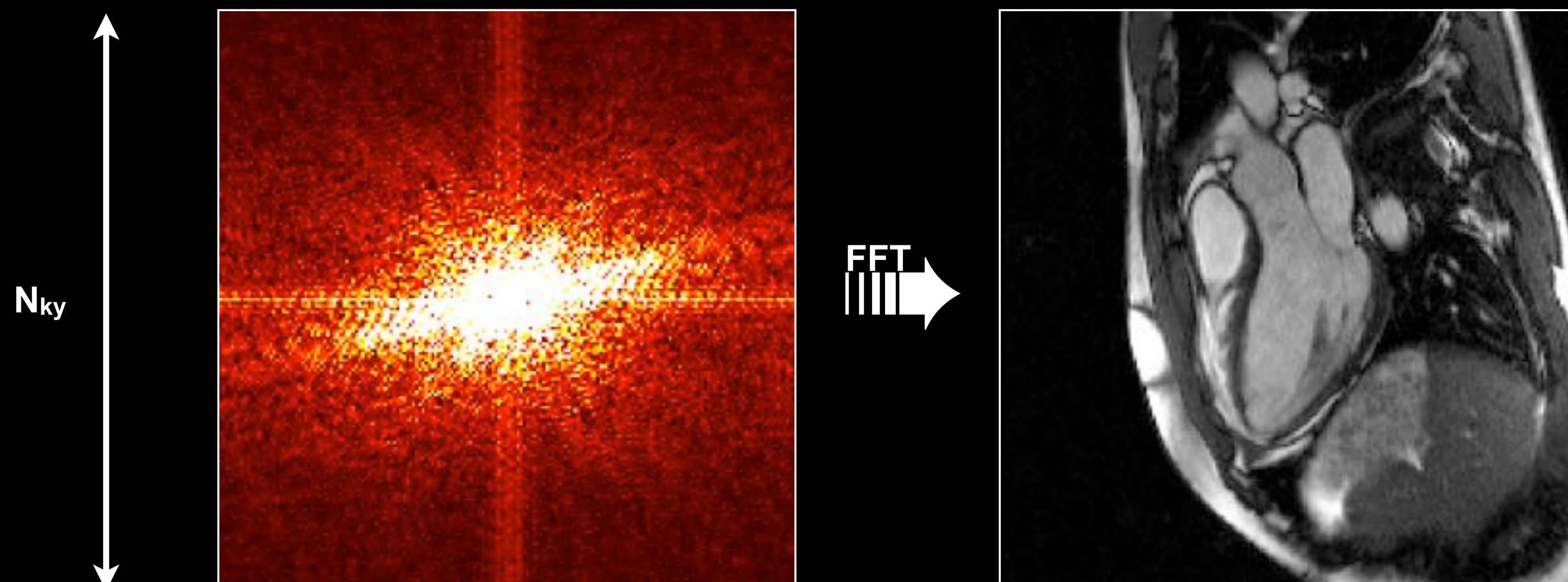


Gradients move us through *k*-space!

MRI is slow...

$$T_{Scan} = TR \cdot N_{PE} \cdot N_{Avg}$$

- **One phase encode step per TR.**
  - **Each phase encode step acquires one echo.**
- **~128 echoes (N_{ky}=# phase encodes) per image.**
- **T_{Scan}=TR•N_{ky}**
  - **T_{Scan}=2500ms•128=5:20 (MM:SS)**



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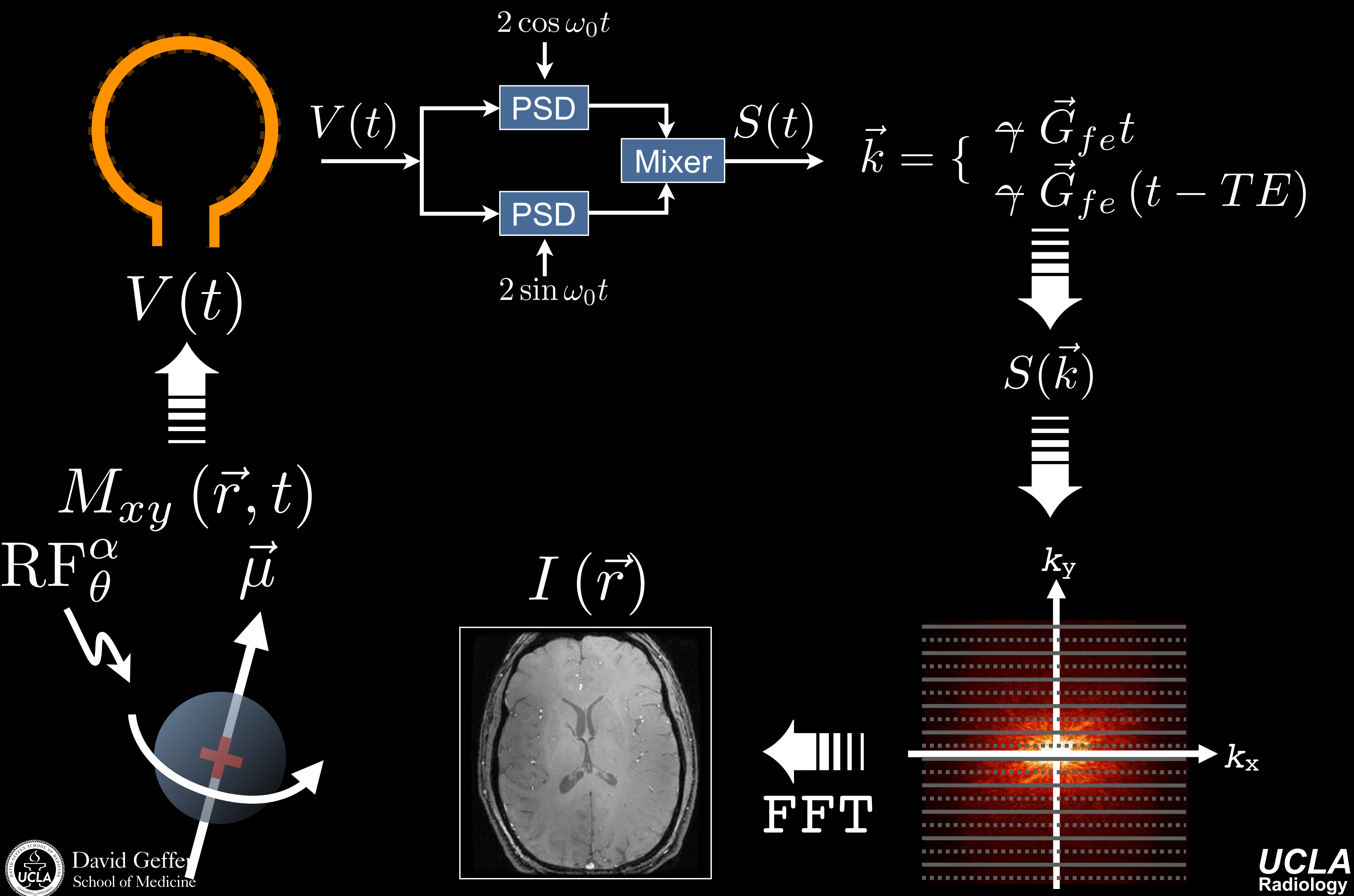
Where does the MRI signal equation come from?

Signals in MRI



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Voltage Equation

$$V\left(t\right)=-\frac{\partial\Phi\left(t\right)}{\partial t}=-\frac{\partial}{\partial t}\int_{object}\vec{B}\left(\vec{r}\right)\cdot\vec{M}\left(\vec{r},t\right)d\vec{r}$$

Coil Sensitivity

Bulk Magnetization

$\frac{dM_z}{dt}\approx 0$ $$B_{r,xy}(\vec{r})=|B_{r,xy}(\vec{r})|e^{-i\phi_r(\vec{r})}$$

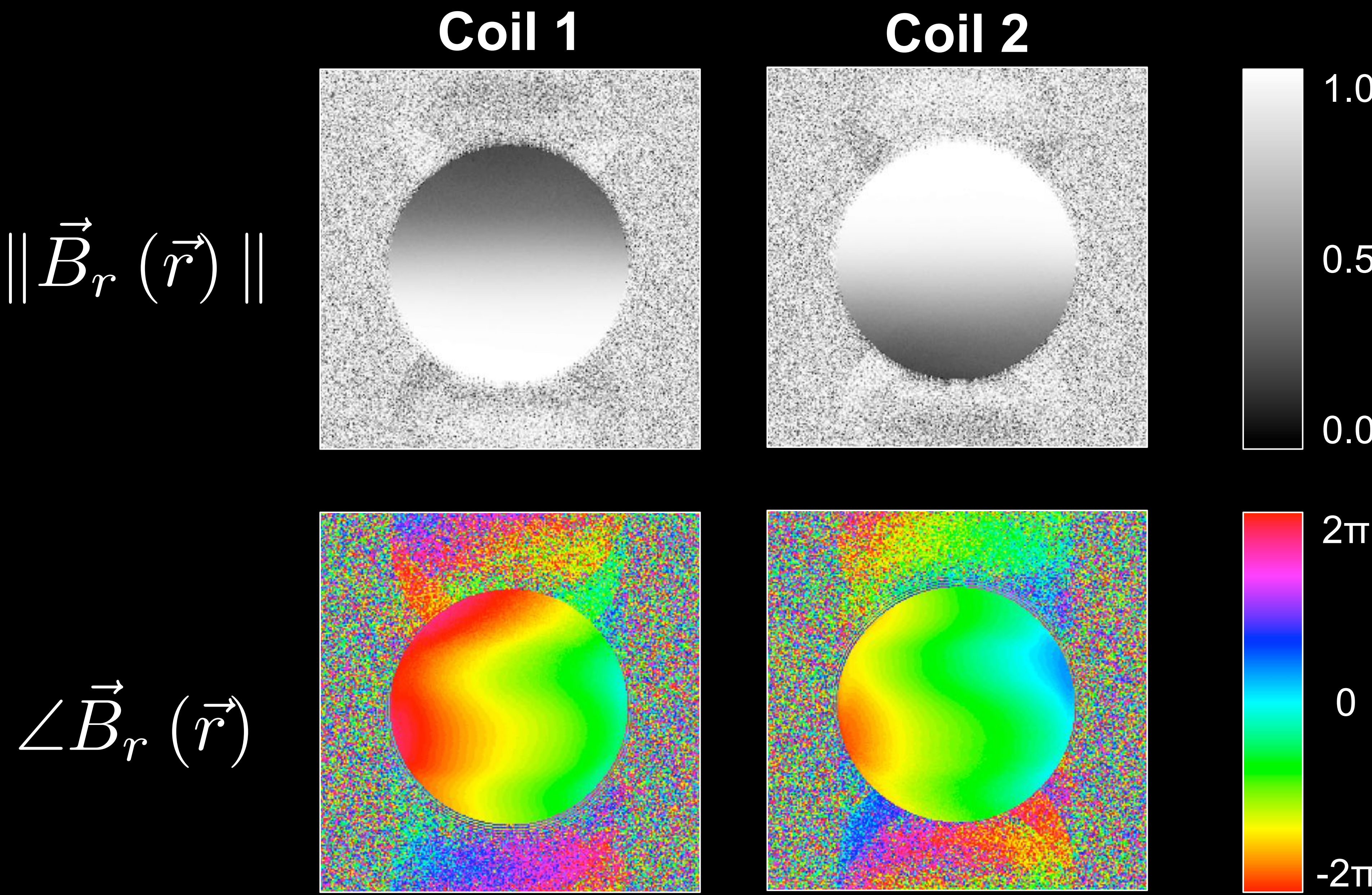
Free Precession $$M_{xy}(\vec{r},t)=|M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2}}e^{-i\omega_0 t}e^{-i\phi_{RF}}$$

Lots of trigonometry and algebra…

$$V(t)=\int_{object}\omega(\vec{r})|B_{r,xy}(\vec{r})||M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2(\vec{r})}}\cos\left(-\omega(\vec{r})t+\phi_e(\vec{r})-\phi_r(\vec{r})+\frac{\pi}{2}\right)d\vec{r}$$

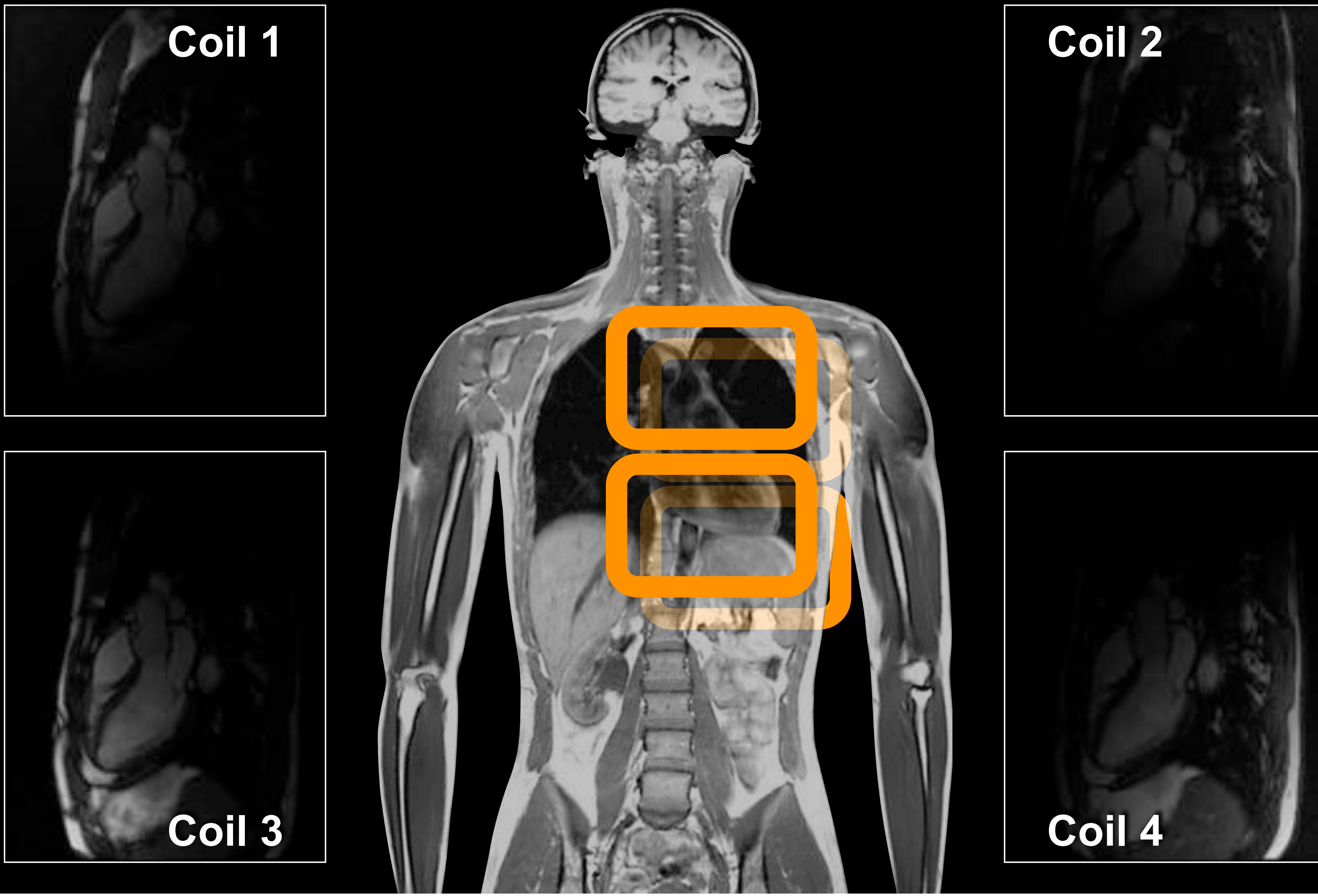
High frequency voltage signal.

Multi-coil Magnitude & Phase



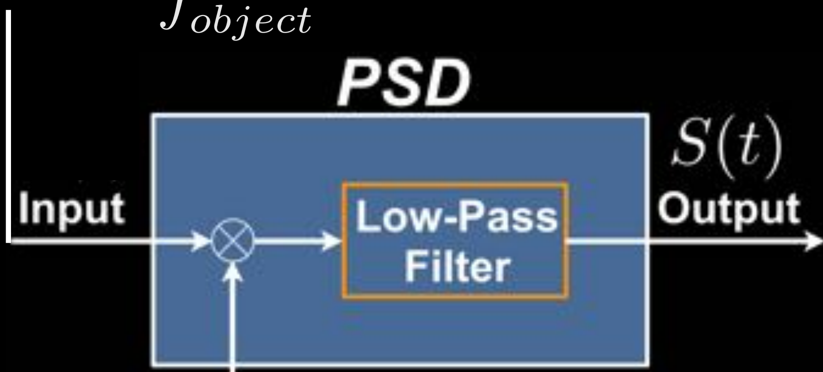
4-Channel Cardiac Coil

Each coil element has a unique sensitivity profile.



Voltage to *k*-space

$$V(t) = \int_{object} \omega(\vec{r})|B_{r,xy}(\vec{r})||M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2(\vec{r})}} \cos\left(-\omega(\vec{r})t + \phi_e(\vec{r}) - \phi_r(\vec{r}) + \frac{\pi}{2}\right) d\vec{r}$$



$$V_{PSD}^{\cos}(t) = \omega_0 \int_{object} |B_{r,xy}(\vec{r})||M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2(\vec{r})}} \cos\left(-\Delta\omega(\vec{r})t + \phi_e(\vec{r}) - \phi_r(\vec{r}) + \frac{\pi}{2}\right) d\vec{r}$$

-or-

$$V_{PSD}^{\sin}(t) = \omega_0 \int_{object} |B_{r,xy}(\vec{r})||M_{xy}(\vec{r},0)|e^{-\frac{t}{T_2(\vec{r})}} \sin\left(-\Delta\omega(\vec{r})t + \phi_e(\vec{r}) - \phi_r(\vec{r}) + \frac{\pi}{2}\right) d\vec{r}$$

Spatial Frequency Encoding

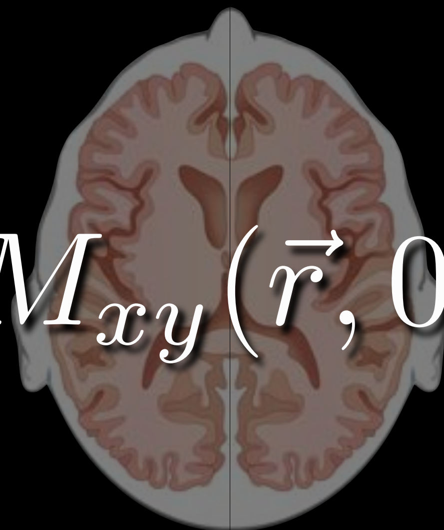
$$\begin{aligned} S(t) &= V_{PSD}^{\cos} + iV_{PSD}^{\sin} \\ &= \omega_0 e^{i\pi/2} \int_{Object} B_{r,xy}^*(\vec{r}) M_{xy}(\vec{r},0) e^{-i\Delta\omega(\vec{r})t} d\vec{r} \end{aligned}$$

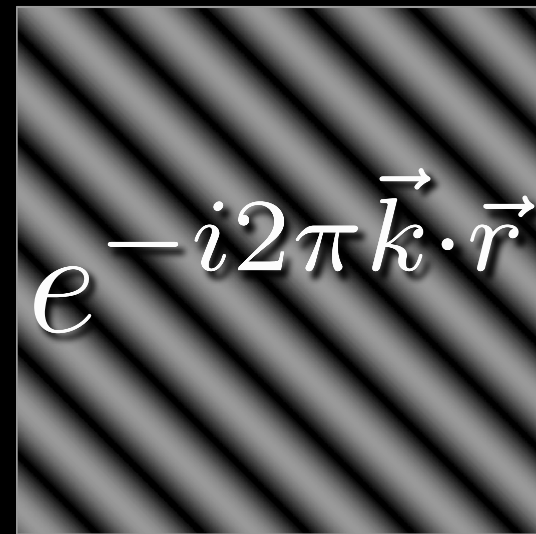
$$\Delta\omega(\vec{r})t = \gamma\vec{G}\cdot\vec{r} = 2\pi\vec{k}\cdot\vec{r}$$ Definition of k-space

$$S(\vec{k}) = \int_{Object} M_{xy}(\vec{r},0) e^{-i2\pi\vec{k}\cdot\vec{r}} d\vec{r}$$ MRI Signal Equation

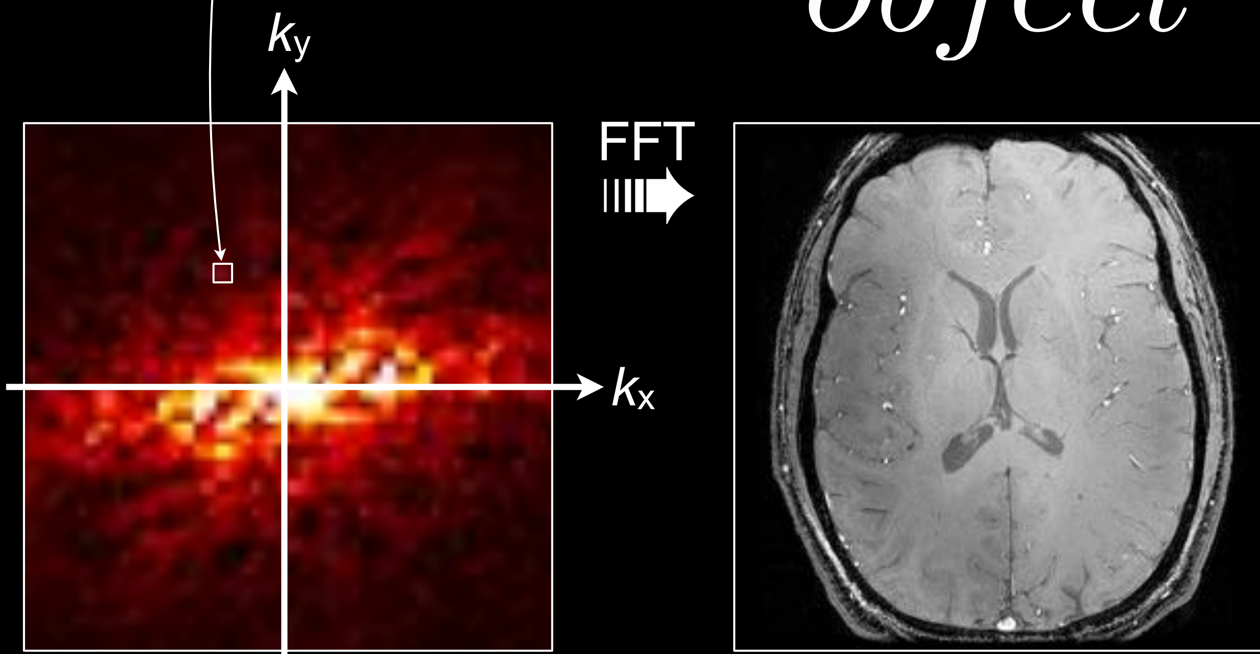
MRI Signal Equation

$$S(\vec{k}) = \int M_{xy}(\vec{r},0)\; e^{-i2\pi\vec{k}\cdot\vec{r}}\; d\vec{r}$$





object



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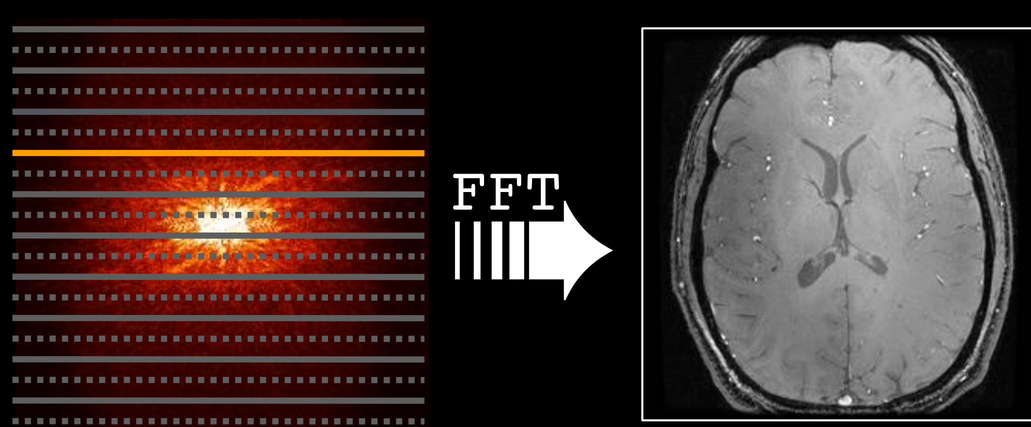
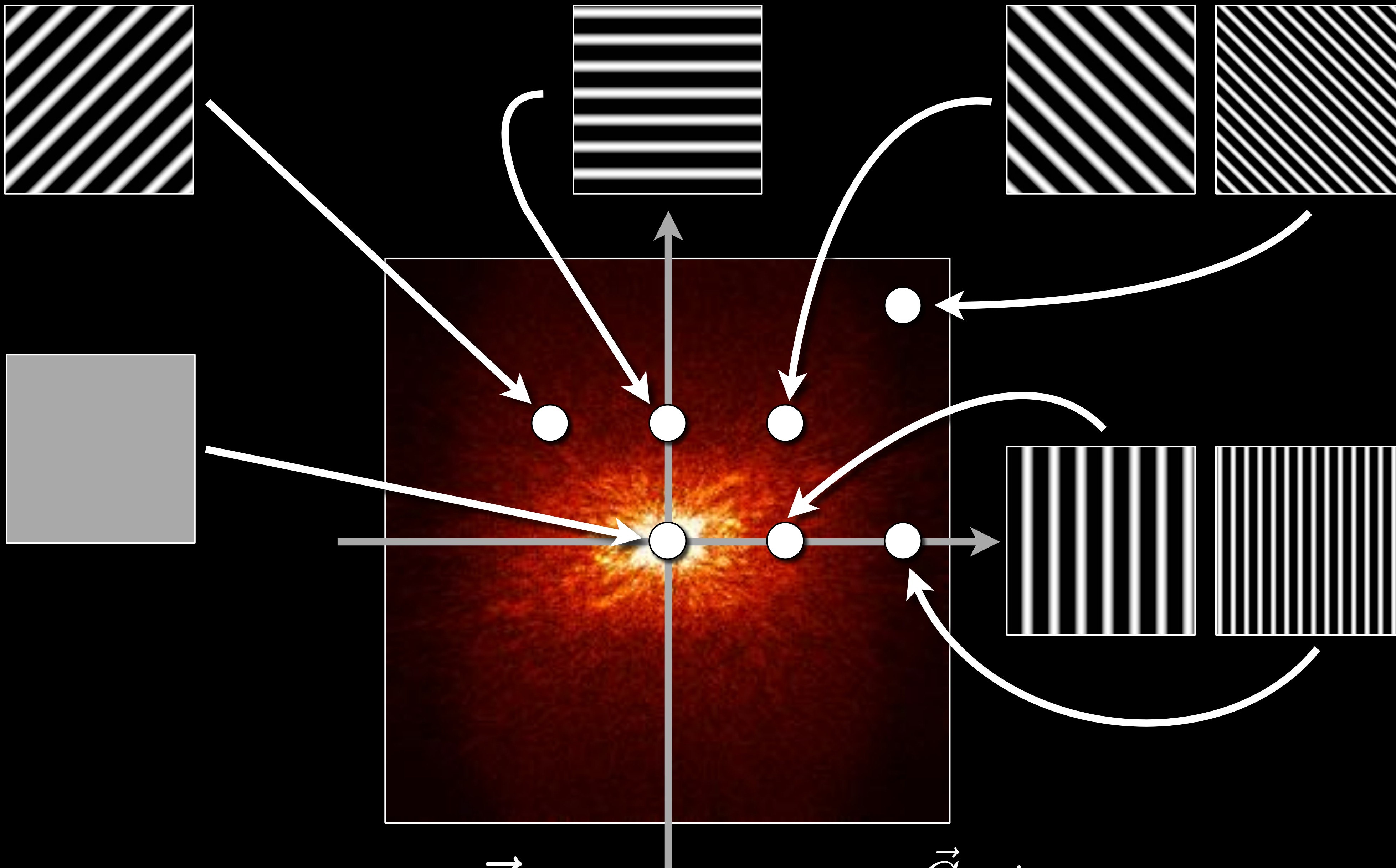
What is *k*-space?

- ***k*-space is the raw data collected by the scanner.**
  - A point in k-space tells us about the presence/absence of a spatial frequency (pattern) in the acquired image.
  - Each echo measures *many* of the spatial frequencies that comprise the object.
  - *k*-space has units of cm^{-1} or mm^{-1}
    - Audio signals have units of Hertz (s^{-1})
- **Gradients**
  - Help extract spatial frequency information
  - Move us around in k-space
- **A line of *k*-space is filled by an echo**



k-space

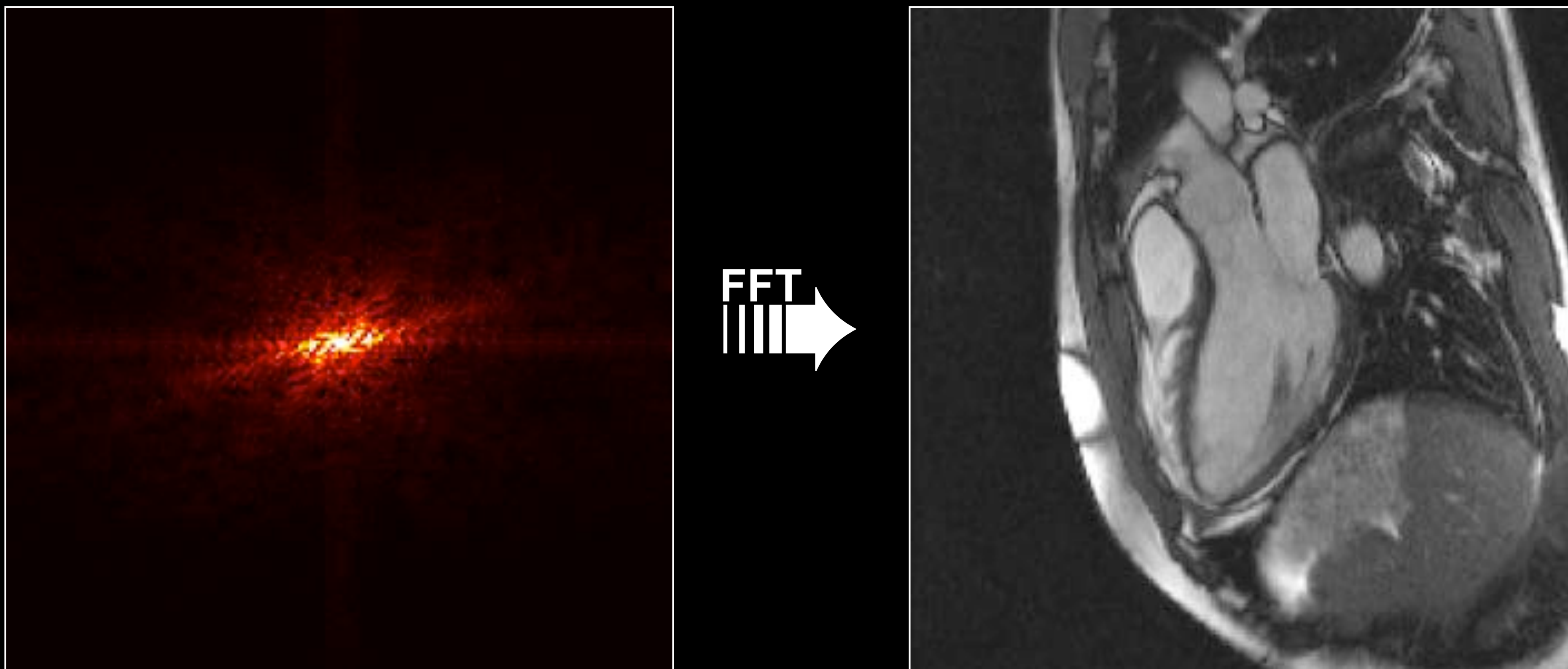


$$e^{-i2\pi\vec{k}\cdot\vec{r}}$$

$$\vec{k} = \begin{cases} \gamma \vec{G}_{fe} t \\ \gamma \vec{G}_{fe}(t - TE) \end{cases}$$

k-space spikes

k-space image space

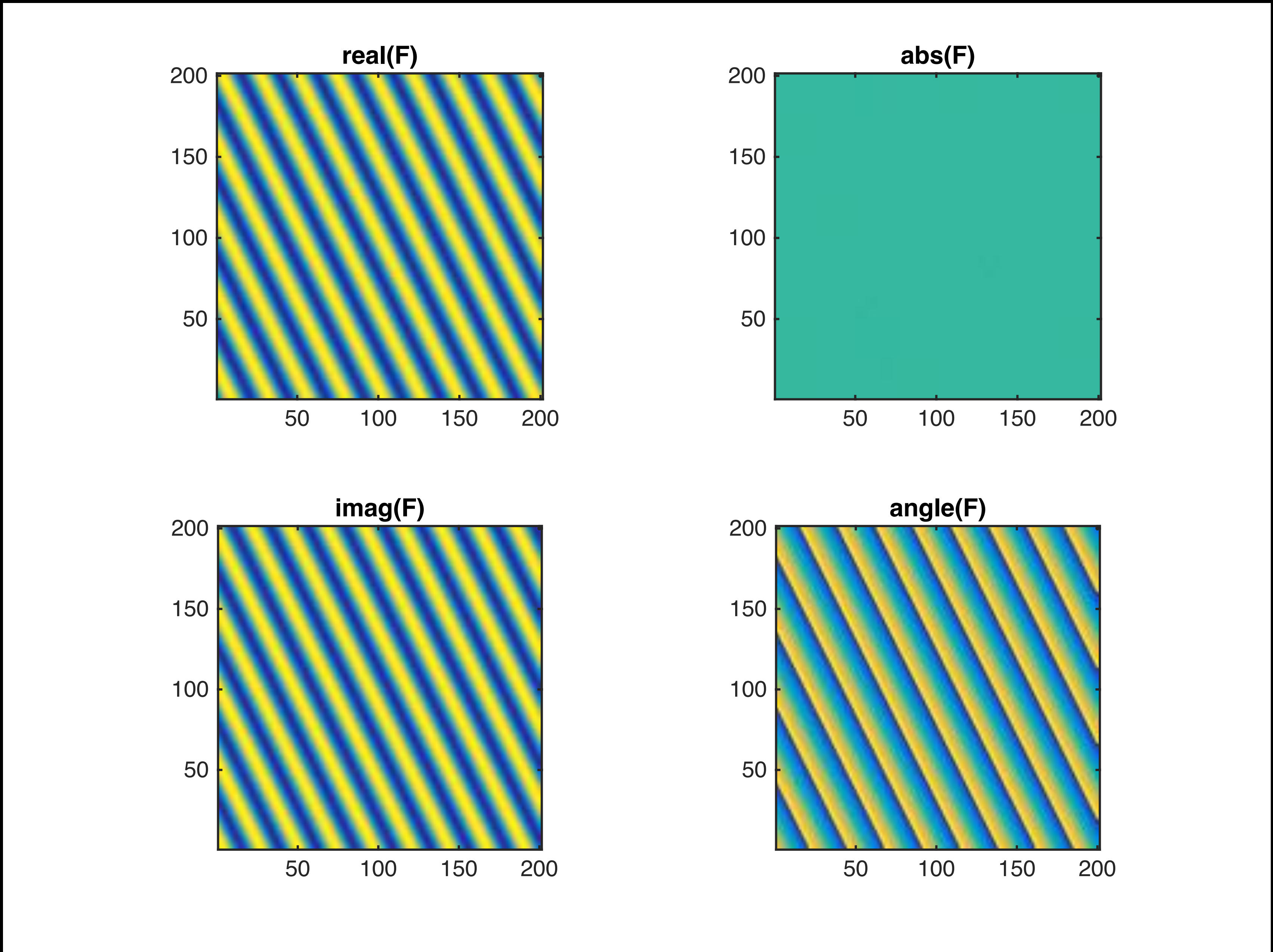


A *k*-space spike creates a banding artifact.

k-space

```
%% Define and display some Fourier sampling functions...
gamma_bar=4257.7480;       % Gyromagnetic ratio, [Hz/G]
Gx=1;                      % [Gauss/cm]
Gy=1;                      % [Gauss/cm]
dt=1.0e-3;                 % [s]

kx=gamma_bar*Gx*dt;        % Kx-space component
ky=gamma_bar*Gy*dt;        % Ky-space component

[X,Y]=ndgrid(-1:0.01:1,-1:0.01:1); % Define some positions in space [cm]

F=exp(-1i*2*pi*(kx*X+ky*Y)); % Fourier sampling functions

%% Display the sampling function
figure; hold on;
subplot(2,2,1);
  imagesc(real(F));
  title('real(F)'); axis image xy;
subplot(2,2,3);
  imagesc(imag(F));
  title('imag(F)'); axis image xy;
subplot(2,2,2);
  imagesc(abs(F));
  title('abs(F)'); axis image xy;
subplot(2,2,4);
  imagesc(angle(F));
  title('angle(F)'); axis image xy;
```

k-space



Spatial Localization - I

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Magnetic Resonance Research Labs

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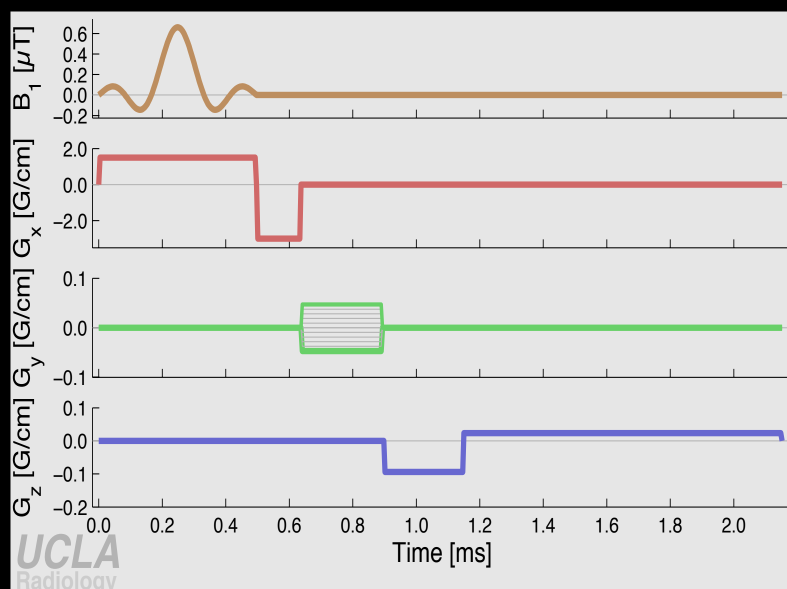
Lecture #10 - Learning Objectives

- Describe the three steps required for spatial localization.
- Be able to explain the role of RF and gradients during slice selection.
- Learn to define B_{eff} for various combinations of B-fields (B_0, gradients, and RF).
- Identify the complexity of the Bloch equations for forced precession in the presence of a gradient field.
- Understand the small tip angle approximation.
- Appreciate that the small tip angle approximation works for intermediate flip angles!
- Understand what truncation artifacts are and one way to reduce them.

Spatial Localization

Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.



Slice Selection

Phase Encoding

Frequency Encoding

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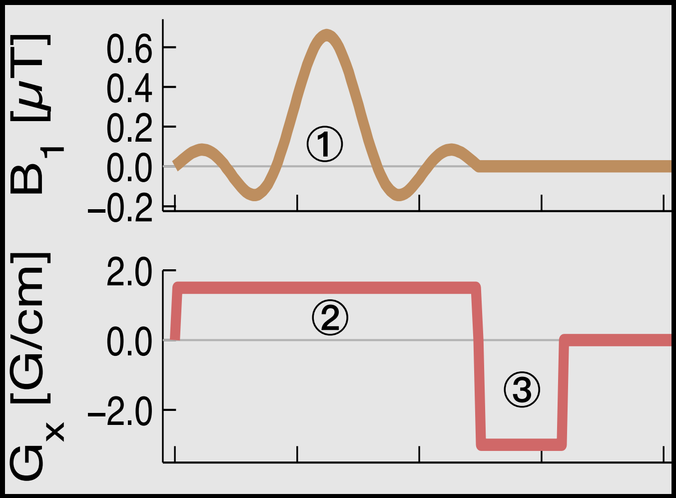
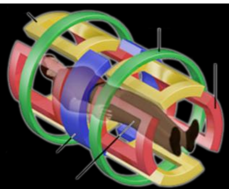
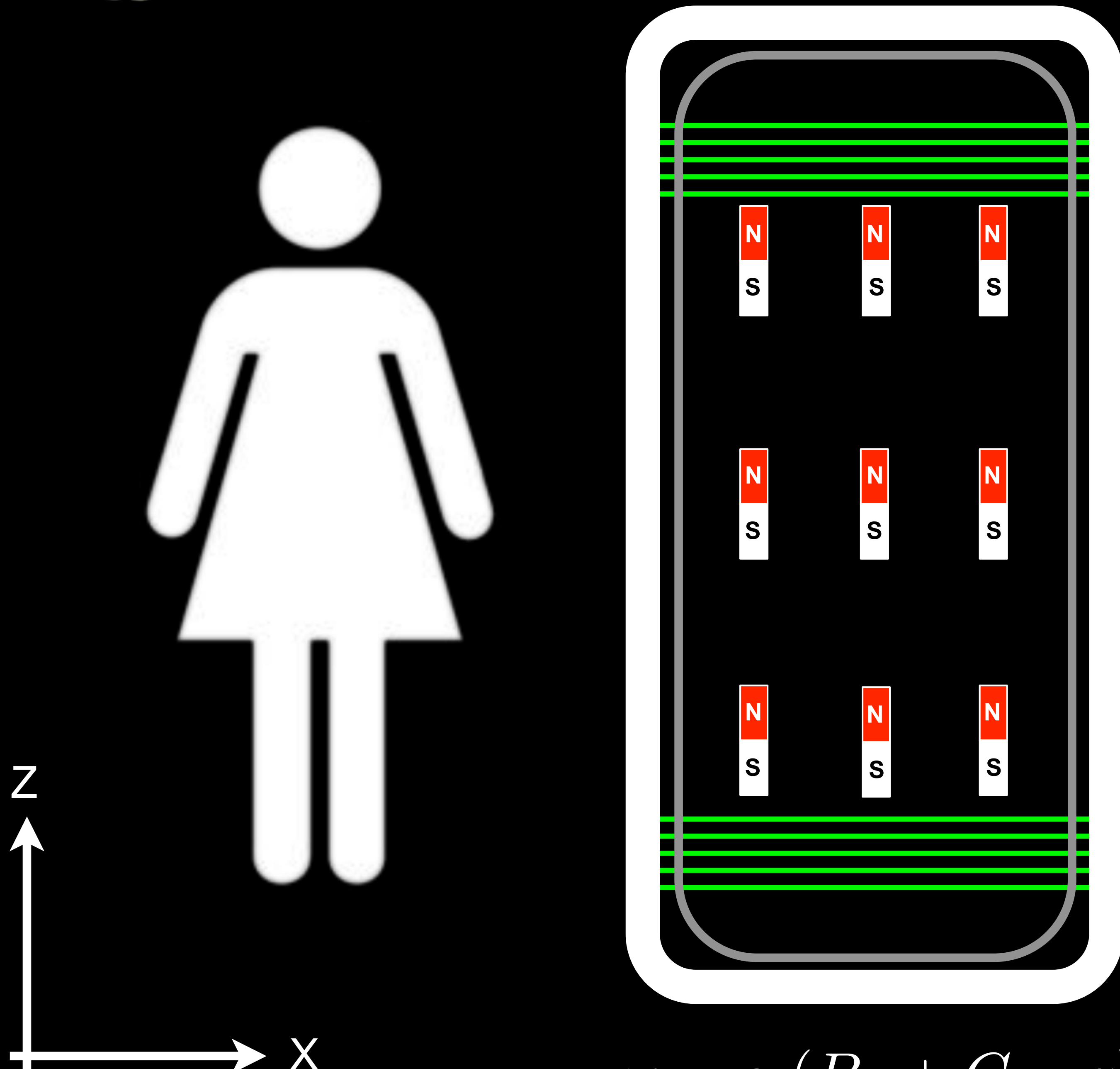
Slice Selection

- **Consists of:**
  1. **RF (B_1) Pulse**
     - **Contains frequencies matched to slice of interest**
  2. **Slice selection gradient**
     - **Constant magnitude**
  3. **Slice-select re-phasing gradient**
     - **Increases SNR**
     - **Re-phases spins within slice**
     - **AKA "slice refocusing gradient"**



Z-Gradients is ON





$$B_0 + \delta B_0$$

$$B_0$$

$$B_0 - \delta B_0$$

$$\omega = \gamma \left(B_0 + G_z \cdot z \right)$$

This frequency excites a slice at position z when G_z is turned on.

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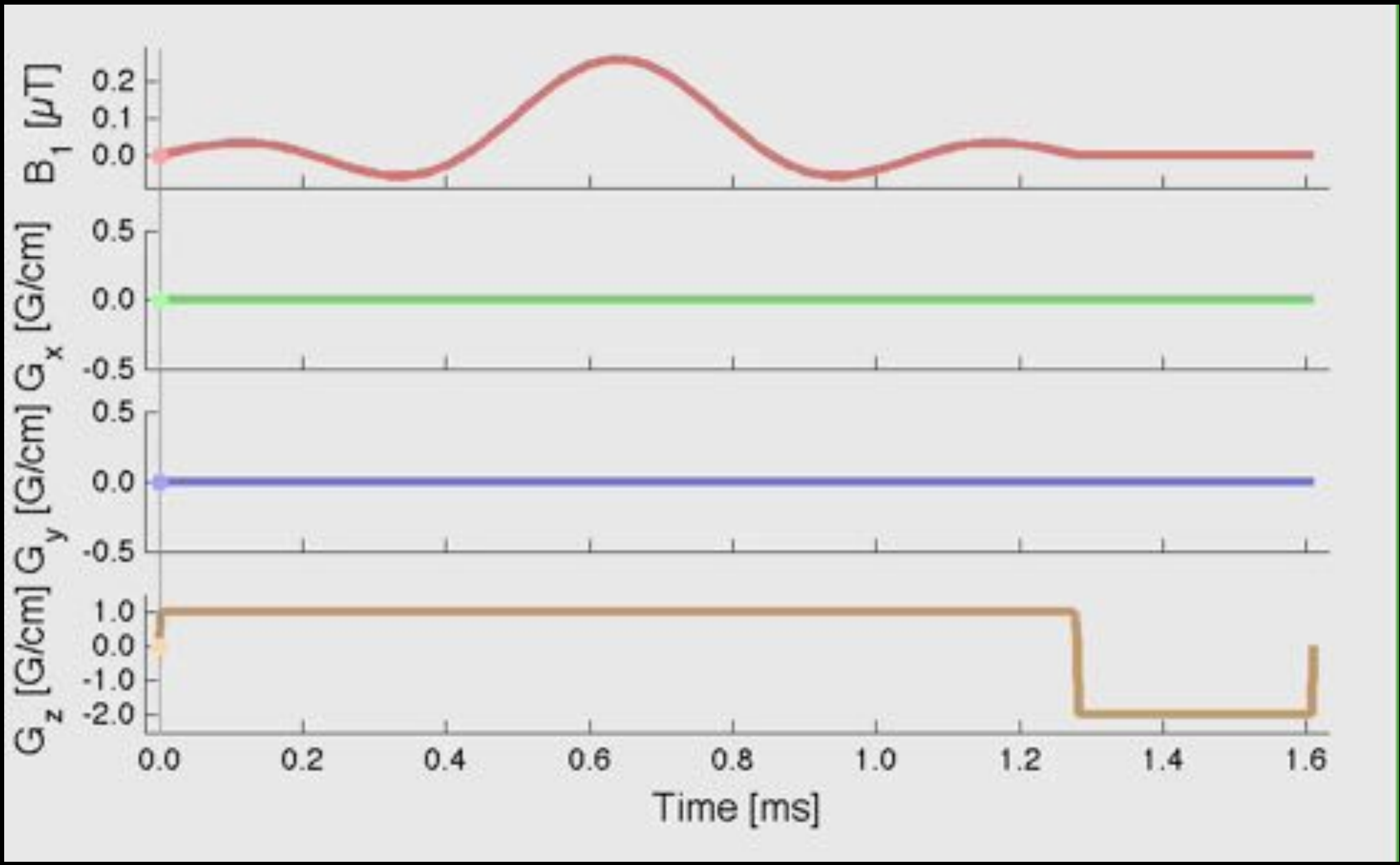
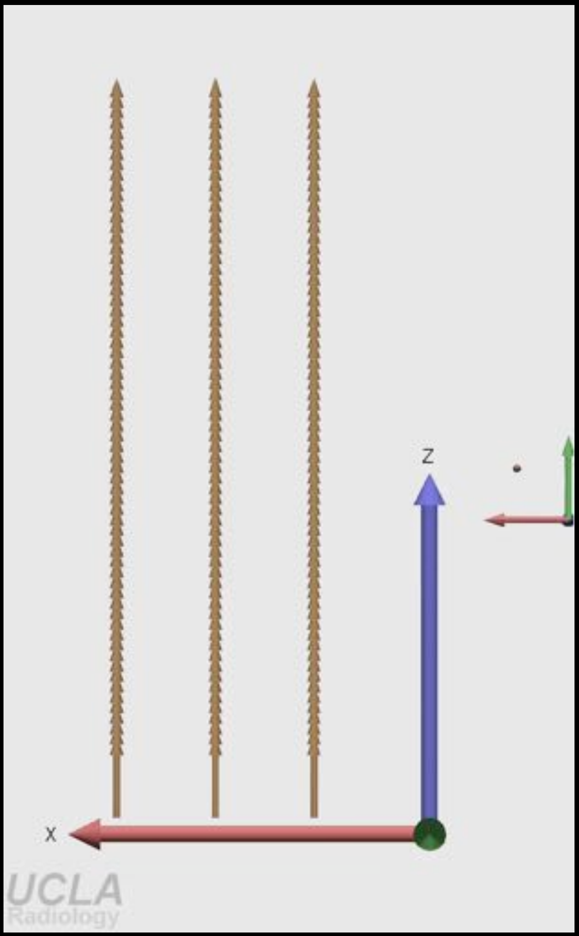
Slice Selection & Rephasing



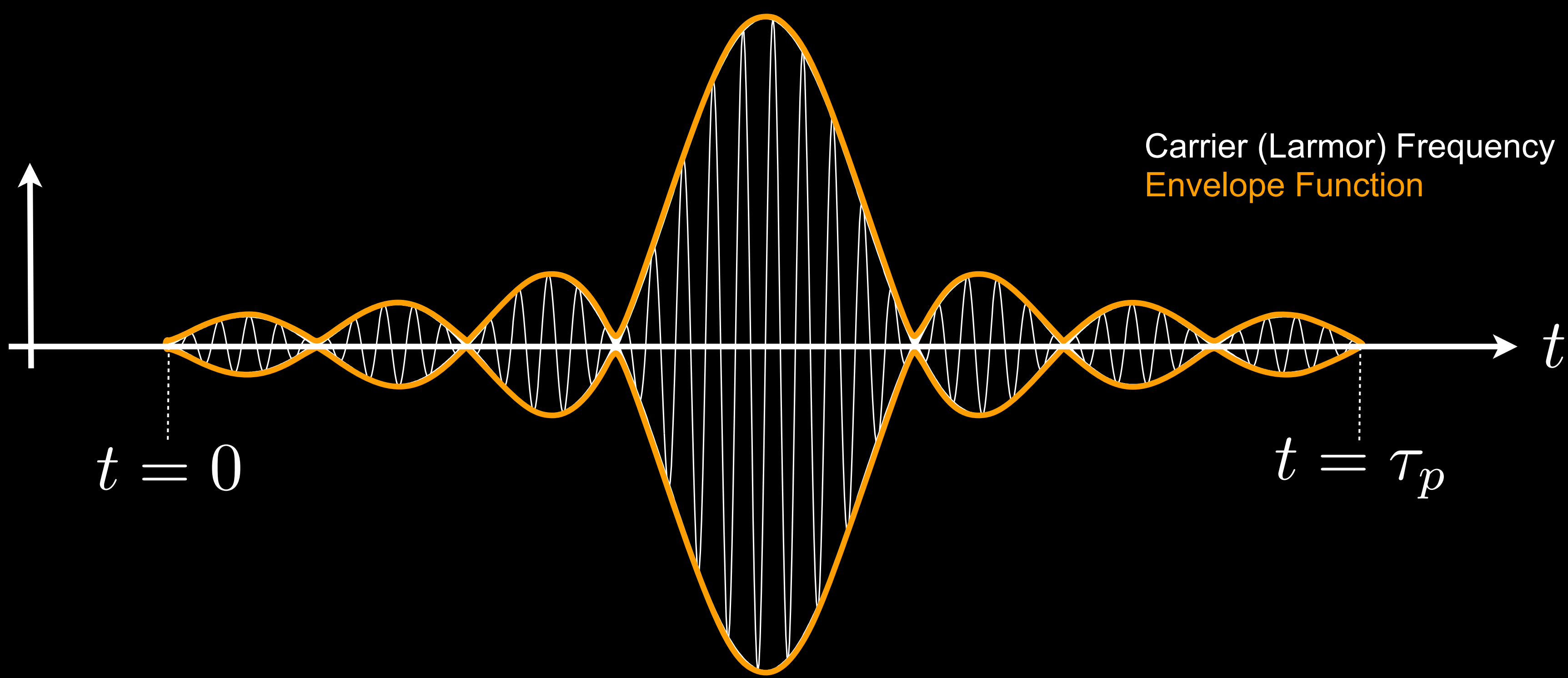


Excitation Pulses

Sinc Envelope Function

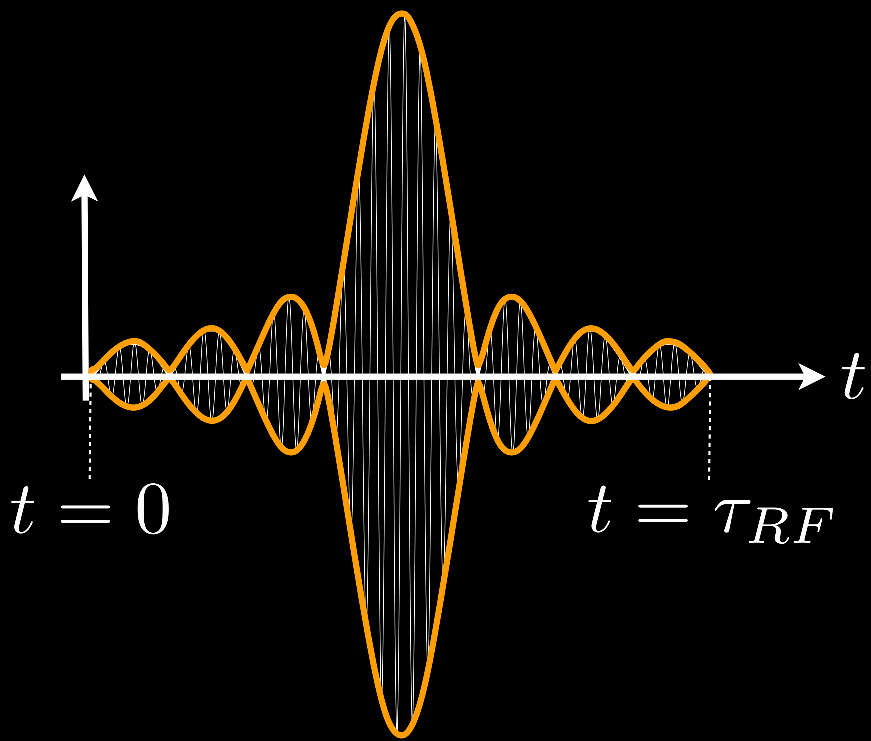
$$B_1(t) = B_1^e(t)\left[\cos\left(\omega_{RF} t\right)\hat{i} - \sin\left(\omega_{RF} t\right)\hat{j}\right]$$ Laboratory Frame

$$B_1^e(t) = \begin{cases} B_1 \text{sinc}\left[\pi f_\omega \left(t - \tau_p/2\right)\right], & 0 \le t \le \tau_p \\ 0, & otherwise \end{cases}$$

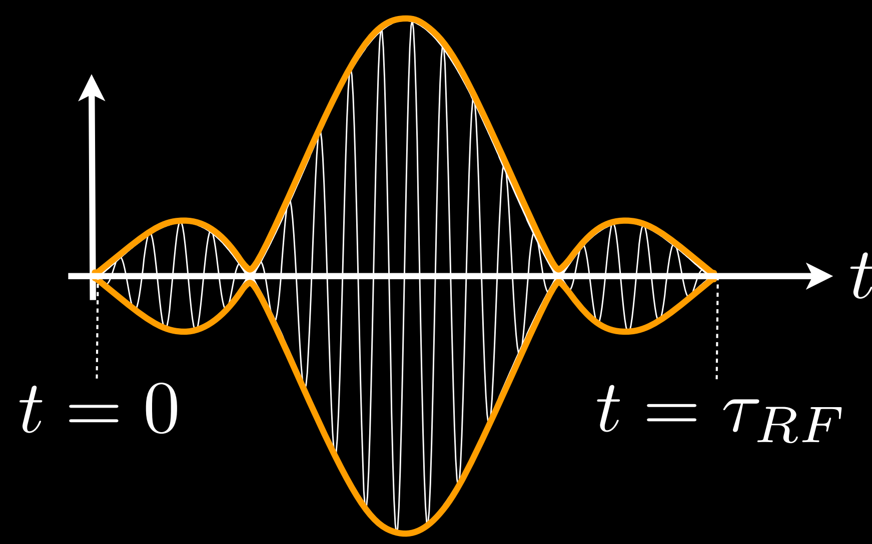


SINC functions are used to excite a narrow band of frequencies.

Sinc Envelope Function



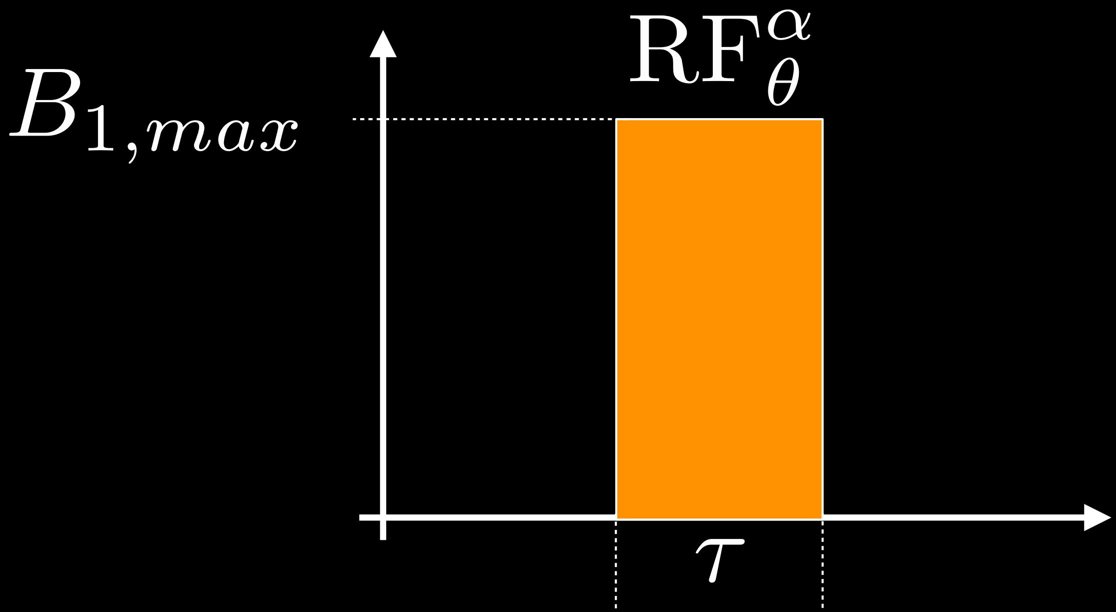
TBW=8, τ_{RF}=1ms
BW=8kHz



TBW=4, τ_{RF}=1ms
BW=4kHz

SINC functions are used to excite a narrow band of frequencies.

How to determine α?

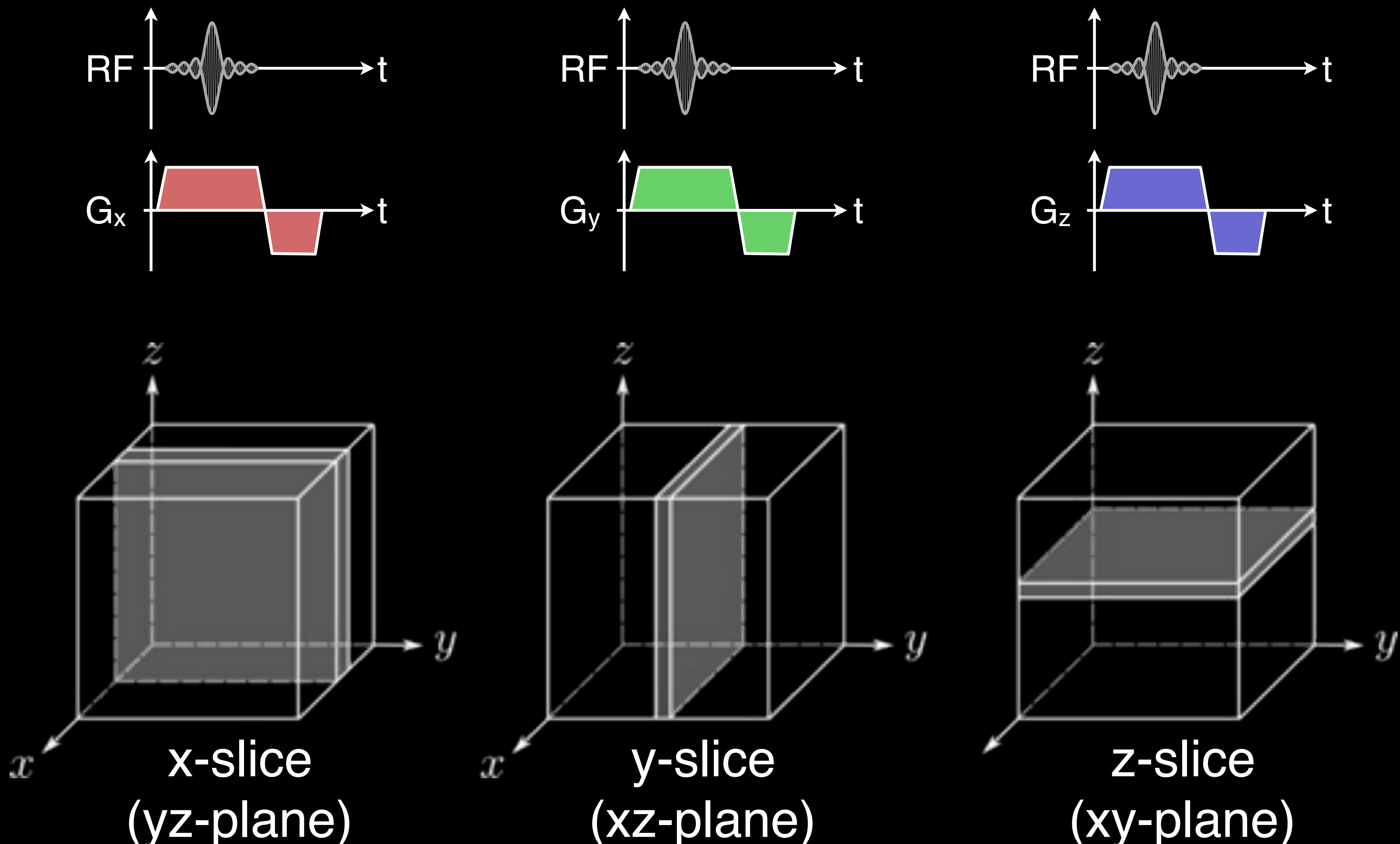


$$\alpha = \gamma \int_0^{\tau_p} B_1^e\left(t\right) dt$$

Rules: 1) Specify α [radians]
2) Use $B_{1,max}$ if we can
3) Shortest duration pulse

Slice Selective Excitation

Slice Selective Excitation



Slice selection requires a simultaneous RF pulse and gradient.

B_0 and Gradients

$$
\begin{aligned}
B_{G,z}\vec{k} &= (G_x x + G_y y + G_z z)\vec{k} \\
&= (\vec{G} \cdot \vec{r})\vec{k}
\end{aligned}
$$

Total applied gradient field.

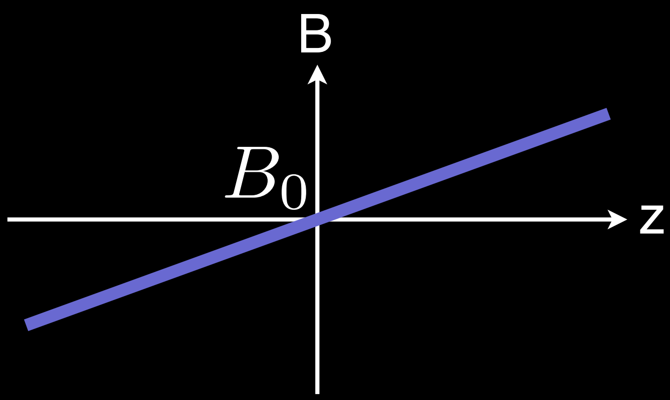
$$
\begin{aligned}
\vec{B}\left(\vec{r}, t\right) &= \left(B_0 + B_{G,z}\right)\vec{k} \\
&= \left(B_0 + \vec{G}\left(t\right) \cdot \vec{r}\right)\vec{k}
\end{aligned}
$$

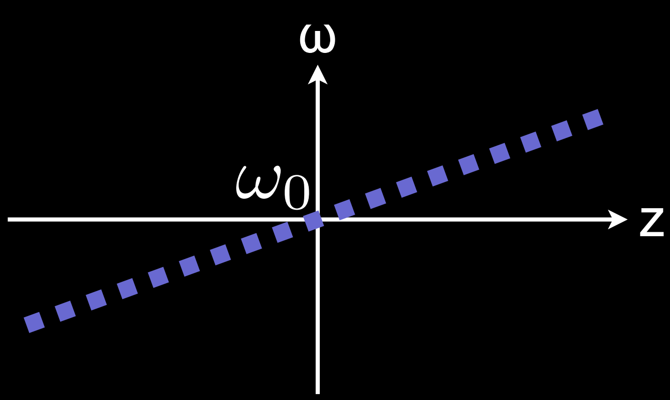
Total applied magnetic field.

Gradients

- **Gradients produce a spatial distribution of frequencies**

$$\vec{B}(z) = (B_0 + G_z \cdot z)\,\hat{k} \qquad \vec{\omega}(z) = -\gamma \vec{B}(z) = -\gamma (B_0 + G_z \cdot z)\,\hat{k}$$

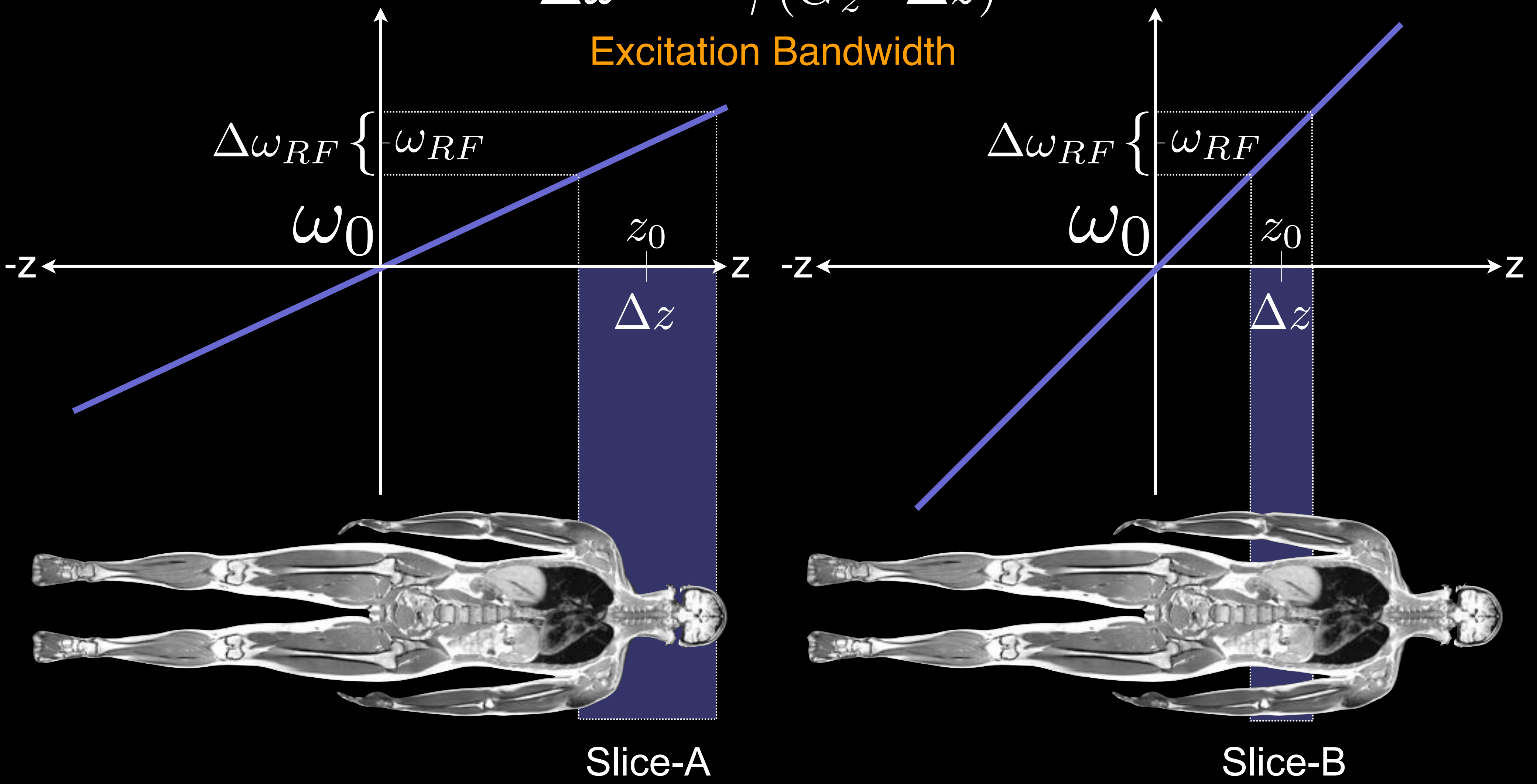




Gradients create a direct correspondence between frequency and spatial position.

Slice Selective Excitation

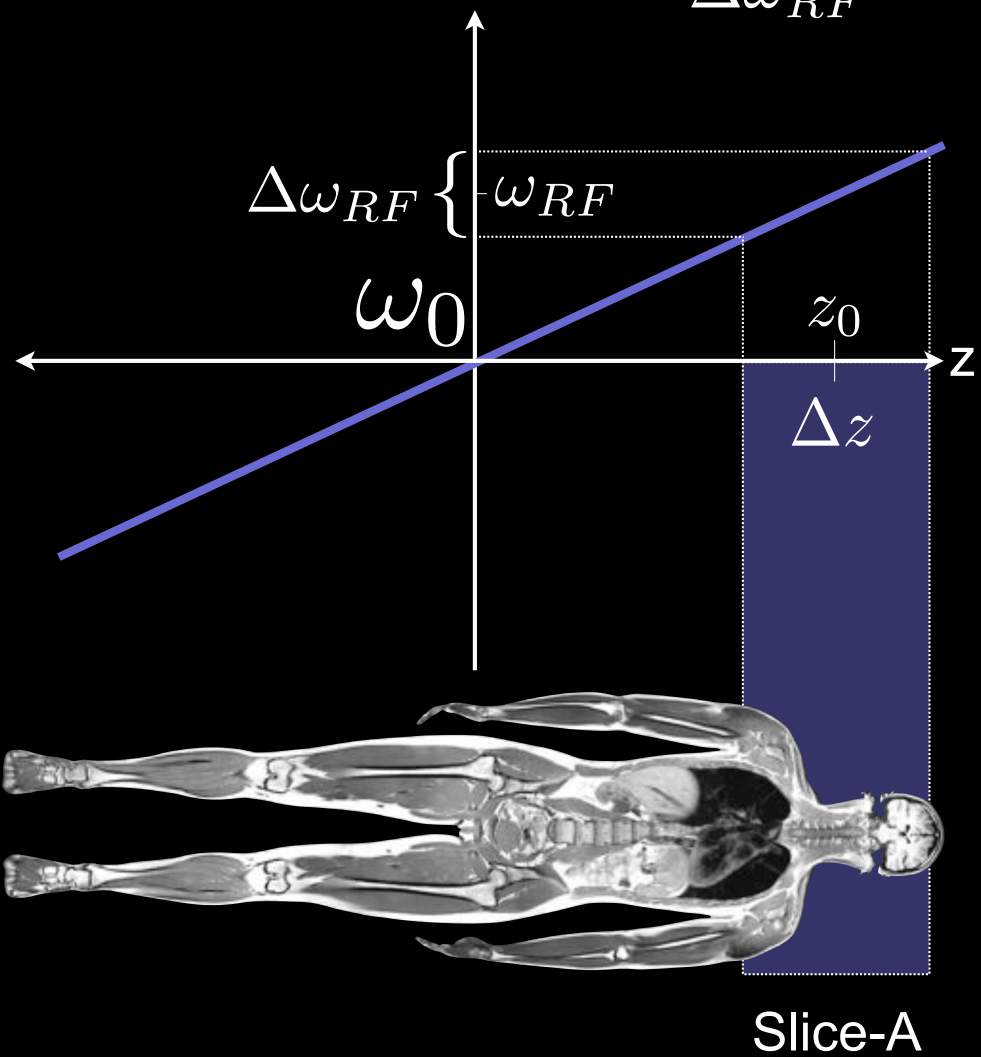
$$\Delta\omega = -\gamma\left(G_z \cdot \Delta z\right)$$



How do you move the slice along ±z?
Compare $\Delta\omega$ and ω_{RF} for Slice-A and Slice-B.
Do we usually acquire $\omega_{RF} > \omega_0$?

Slice Selective Excitation - Example

$$\Delta\omega_{RF} = -\gamma\left(G_z \cdot \Delta z\right)$$ Excitation Bandwidth



$$\mathrm{TBW} = \tau_{RF} \cdot \Delta\omega_{RF}$$

$$\Delta\omega_{RF} = \frac{\mathrm{TBW}}{\tau_{RF}} = \frac{4}{1\mathrm{ms}} = 4\mathrm{kHz}$$

$$G_z = \frac{\Delta\omega_{RF}}{\gamma \Delta z} = \frac{4000\mathrm{Hz}}{42.57\mathrm{e}6\,\frac{\mathrm{Hz}}{\mathrm{T}}\,\frac{1\mathrm{T}}{10000\mathrm{G}} \cdot 10\mathrm{mm}} = 0.94\,\frac{\mathrm{G}}{\mathrm{cm}}$$

Selective Excitation

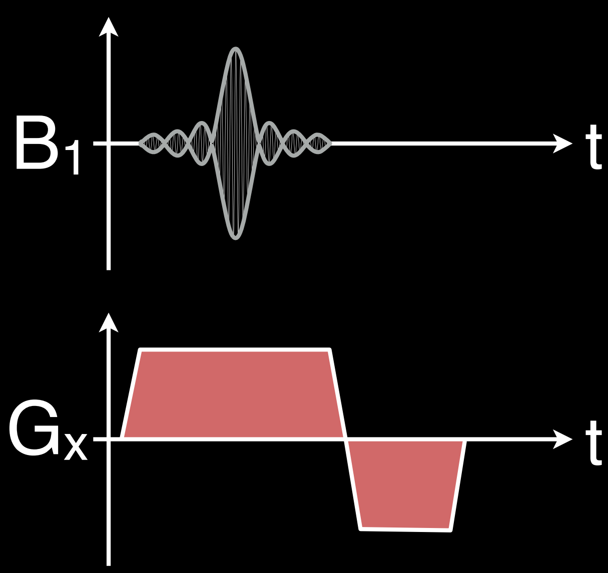
- **What factors control slice selection?**

$$B_1^e(t)$$ **Pulse envelope function**

(e.g. $B_{1,max}$ and $\Delta\omega$)

$$\omega_{RF}$$ **Excitation carrier frequency**

$$\vec{G}$$ **Gradient amplitude**



Forced Precession with a Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff}(z,t) = \begin{bmatrix} B_1(t) \\ 0 \\ \cancel{B_0 +} G_z \cdot z \cancel{- \frac{\omega_{RF}}{\gamma}} \end{bmatrix}$$

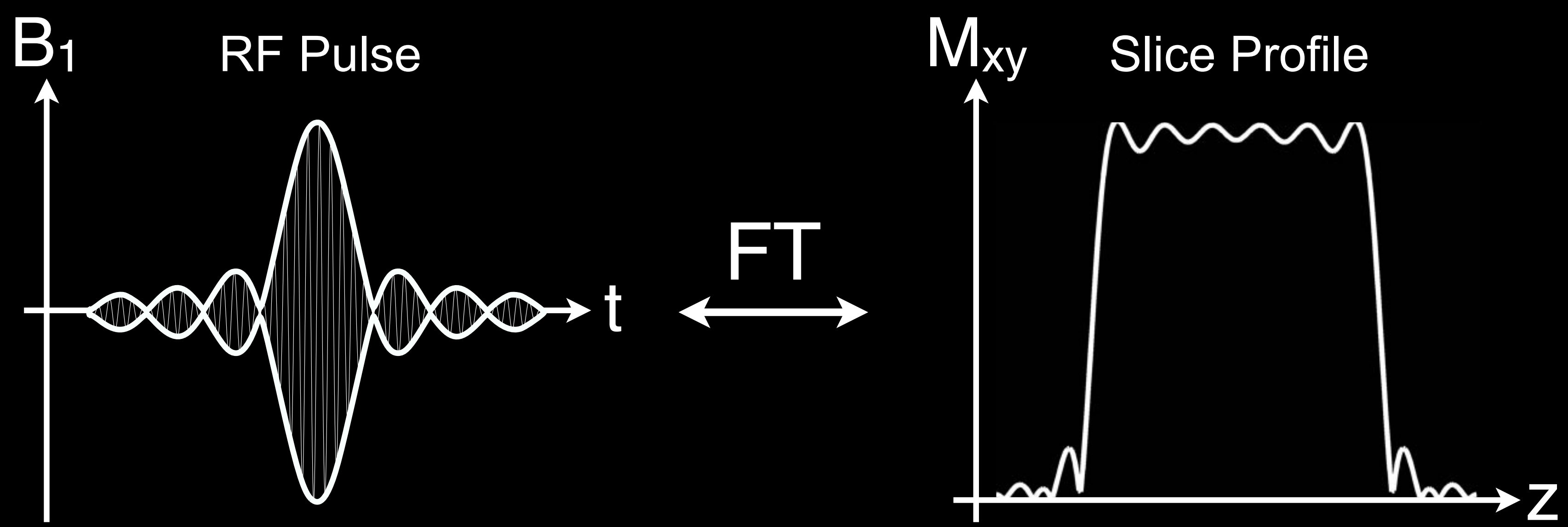
Effective B-Field in the Rotating Frame

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \gamma B_1(t) & 0 & \gamma G_z \cdot z \end{vmatrix} \Rightarrow \begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \omega_1(t) & 0 & \omega(z) \end{vmatrix}$$

Coupled system of differential equations!

Slice Selective Excitation

- What is the ideal slice profile?
- Changing the shape (envelope function) of the pulse affects the ***excitation bandwidth***.
- How do we know which shape to use?
  - Small Tip Angle Approximation
    - ➡ Slice profile depends on the FT of the shape.



Small Tip Angle Approximation

Small Tip Approximation

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \omega_1(t) & 0 & \omega(z) \end{vmatrix}$$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

Solving a first order linear differential equation:

$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\omega(z)(t-s)} ds$$

To the board ...

Summary for Small Tip

Assuming carrier frequency = resonance frequency

$$\omega_{RF} = \omega_0$$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix}$$

$M_z \approx M_0$ small tip-angle approximation

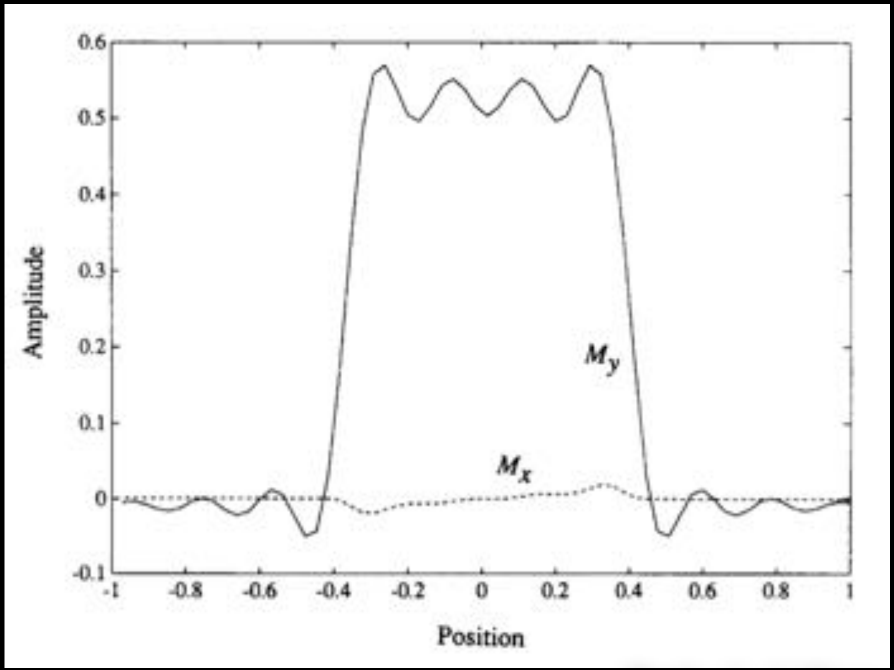
$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \mid_{f=-(\gamma/2\pi)G_z z}$$

Small Tip Approximation

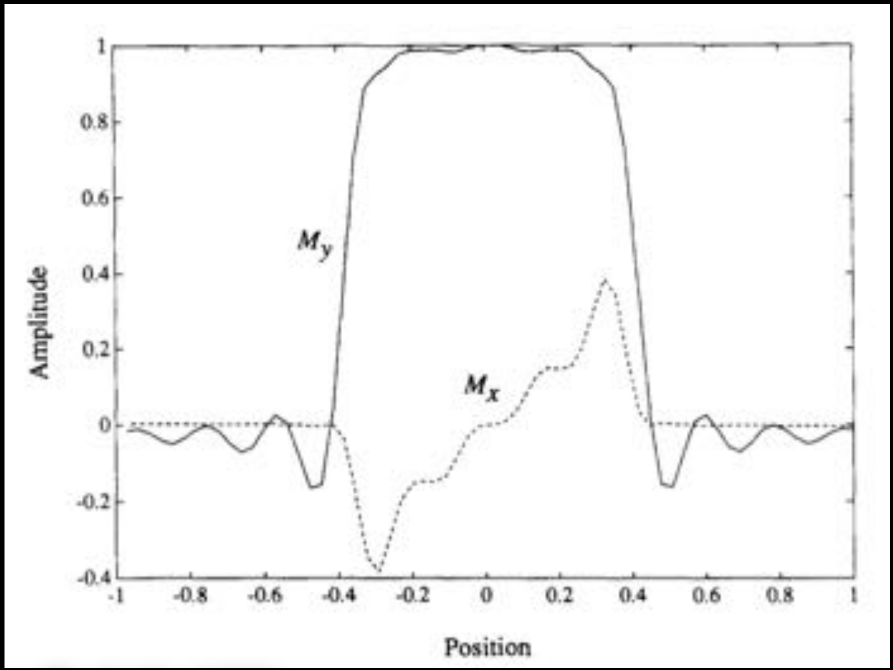
1. The excitation profile, within the small angle approximation, is just the Fourier transform of the pulse.
2. Remember that the Bloch equations are non-linear and thus cannot be expected to behave linearly.
3. The approximation works surprisingly well even for flip angles up to 90°!

Shaped Pulses

30°



90°



Pauly, J. *J. Magn. Reson.* **81** 43-56 (1989)

The small flip angle approximation still works reasonably well for flip angles that aren't necessarily "small".

Truncation Artifacts

In MRI we want pulses to be as short as possible:

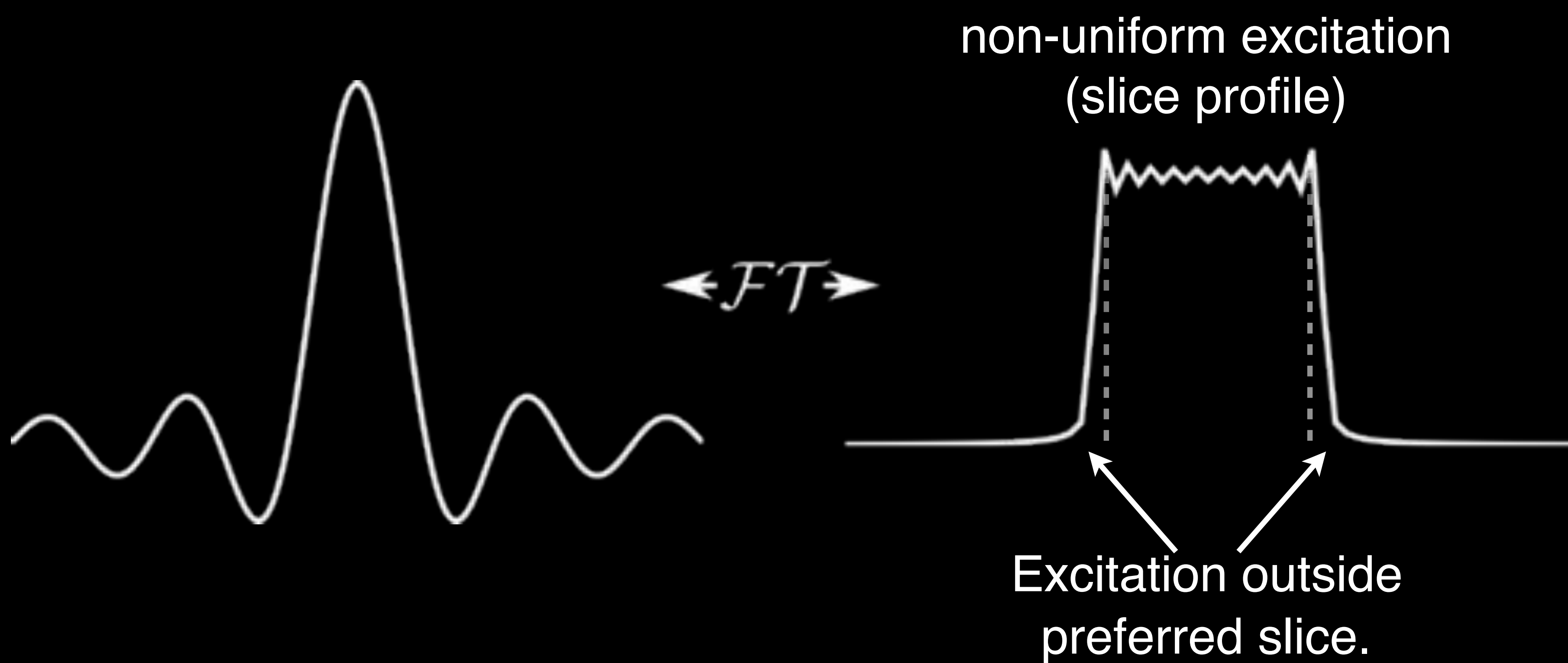
1) To avoid relaxation effects.
2) To improve scan efficiency.

The ***sinc*** function is defined over all time, which is impractical in any experiment.

The ***sinc*** pulse needs to be truncated to be appropriate for clinical scans.

Truncation Artifacts

What happens when we truncate our pulses?



Deviations from the ideal slice profile are known as truncation artifacts.

Reducing Truncation Artifacts

Alternative Pulse Shapes

$$B_x(t) = A\exp\left[-a(t-\tau/2)^2\right] \quad \text{Gaussian}$$

Reduced side-lobes, but not as flat of a slice profile.

Thanks

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